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ORIGINAL ARTICLE

A New Ranking Function of Triangular Fuzzy Numbers for Solving Fuzzy Linear Programming Problems with Big -M Method

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Abstract: The objective of this paper is to introduce a new ranking function of Triangular Fuzzy Numbers for solving fuzzy linear programming problems in objective function and find the optimal solution of it by Big -M Method. A numerical example is given to illustrate the proposed method.

Keywords: Fuzzy number; Triangular fuzzy number; Big-M method; Ranking function

1. Introduction

Linear programming is an important technique of operational research and has many applications in optimization problem. Many researchers for treating problems in linear programming have proposed various methods. An application of a Simplex method proposed in[1] for the optimal solution while solving multi objective linear programming problem presented by[2] with the aid of interval arithmetic .Authors in[3] modified Lagrange method for special programming problems.

The FLP representation with the use of the product operator as well as the minimum operator[4-5]. In[6], a novel procedure for LP problems in an intuitionistic fuzzy environment was constructed. In[7], intuitionistic FLP problem was applied. Many other researchers suggested several techniques to solve fully FLP problems[8-10]. An efficient procedure for specific quadratic programming problems (QPP) presented in[11]. Fully fuzzy multi- objective LPP is solved by[12] while fuzzy multiobjective LPP is treated in[13] via fuzzy programming model. For other wok, one can see[14-23].

This work presents a novel ranking function of TFN for solving FLP in objective function and an optimal solution is obtained by Big -M method.

The section of the present paper is arranged as follows: some basic definitions are listed section 2 while section 3 reviews the algorithm of Big-M Method. The proposed ranking function of TFN is listed in section 5. The algorithm of ranking function for solving FLP problems with TFN is given in section 6. Section 7 contains numerical examples and the conclusion is listed in the last section.

2. Basic Definitions

Some important basics concepts concerning fuzzy number and fuzzy sets are presented through the following definitions

2.1 Fuzzy set[24]

Consider the nonempty set X , $\tilde{F}(x)$ is a fuzzy set and they are defined as $\tilde{F} = \{ x , \mu_{\tilde{F}}(x) / x \in X \}$ where $\mu_{\tilde{F}}(x)$ is named the membership function that maps each element of X to a value between 0 and 1.

doi: 10.18686/esta.v6i1.89

This weight is named the membership function. The convex normalized fuzzy set on the real line R is fuzzy number $\tilde{F}$ such that:

1. At least there exist one x ∈ R with $\mu_{\tilde{F}}(x) = 1$
2. $\mu_{\tilde{F}}(x)$ is piecewise continuous.

2.3 Triangular Fuzzy Numbers[25]

A fuzzy number $\tilde{a}$ is a TFN denoted by (a, b, c) where a, b and c are real numbers and its membership function $M_{\tilde{a}}(X)$ is given below:

$$M_{\tilde{a}}(X) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ 1 & x = b \\ \frac{c-x}{c-b} & b \le x \le c \end{cases}$$

3. The Big-M Method[21]

In order to solve a LPP involving artificial variables, a method developed by Charnes is used named the Big-M method. The steps lists of the procedure are listed below:

Step 1: The constraints are modified. So each constraint in the right-hand side is nonnegative (= or ≥ constraint).

Step 2: The inequality constraints are converted to standard form'

Step 3: Add an artificial variables for each ≥ or = constraint. Then add sign restriction $a_i \ge 0$.

Step 4: Add (for each artificial variable) a positive large number, maximum to minimum objective functions problem or minimum to maximum objective functions problem.

Step 5: Eliminate the artificial variables from row 0 and then begin with the simplex. Step 6: Solve the reduced problem by using simplex.

4. Proposed Ranking function of Triangular Fuzzy Numbers

A function $R: F(R) \to R$, which maps each fuzzy number into the real line, where natural order exists. Orders on $F(R)$ are defined by,

$\tilde{a} \ge \tilde{b}$ if and only if $F(\tilde{a}) \ge F(\tilde{b})$,

$\tilde{a} \le \tilde{b}$ if and only if $F(\tilde{a}) \le F(\tilde{b})$,

$\tilde{a} = \tilde{b}$ if and only if $F(\tilde{a}) = F(\tilde{b})$.

Now, a new ranking function is adopted with the following triangular membership:

$$M_{\tilde{A}}(X) = \begin{cases} \frac{w(x-a)}{b-a} & a \le x \le b \\ w & x = b \\ \frac{w(c-x)}{c-b} & b \le x \le c \end{cases}$$

Let $\alpha \in [0,1]$ then

$$\frac{w(x-a)}{b-a} = \alpha \to x = a + \frac{\alpha}{w}(b-a) = \inf \tilde{A}(\alpha).$$

$$\frac{w(c-x)}{c-b} = \alpha \to x = c - \frac{\alpha}{w}(c-b) = \sup \tilde{A}(\alpha).$$

By the ranking function $R(\tilde{A})$

$$R(\tilde{A}) = \frac{\frac{1}{2}\left[\int_0^w \alpha^3 \left[\inf \tilde{A}(\alpha) + \sup \tilde{A}(\alpha)\right] d\alpha\right]}{\alpha^3\, d\alpha}$$

$$R(\tilde{A}) = \frac{\frac{1}{2}\left[\int_0^w \alpha^3 \left[a + \frac{\alpha}{w}(b-a) + c - \frac{\alpha}{w}(c-b)\right] d\alpha\right]}{\alpha^3\, d\alpha}$$

$$R(\tilde{A}) = \frac{\frac{1}{2}\left[\int_0^w \left[\alpha^3 a + \frac{\alpha^4}{w}(b-a) + \alpha^3 c - \frac{\alpha^4}{w}(c-b)\, d\alpha\right]\right]}{\alpha^3\, d\alpha}$$

$$R(\tilde{A}) = \frac{\frac{1}{2}\left[\frac{\alpha^4}{4} a + \frac{\alpha^5}{5w}(b-a) + \frac{\alpha^4}{4} c - \frac{\alpha^5}{5w}(c-b)\Big|_0^w\right]}{\frac{\alpha^4}{4}\Big|_0^w}$$

$$R(\tilde{A}) = \frac{\frac{1}{2}\left[\frac{w^4}{4} a + \frac{w^5}{5w}(b-a) + \frac{w^4}{4} c - \frac{w^5}{5w}(c-b)\right]}{\frac{w^4}{4}}$$

$$R(\tilde{A}) = \frac{\frac{1}{2}\left[\frac{w^4}{4} a + \frac{w^4}{5}(b-a) + \frac{w^4}{4} c - \frac{w^4}{5}(c-b)\right]}{\frac{w^4}{4}}$$

$$R(\tilde{A}) = \frac{\frac{w^4}{2}\left[\frac{a}{4} + \frac{(b-a)}{5} + \frac{c}{4} - \frac{(c-b)}{5}\right]}{\frac{w^4}{4}}$$

$$R(\tilde{A}) = \frac{\frac{w^4}{2}\left[\frac{a+8*b+c}{20}\right]}{\frac{w^4}{4}}$$

$$R(\tilde{A}) = \frac{[a+8*b+c]}{10}$$

5. The Algorithm RF-FLP-TFN

The algorithm for treating FLP with TFN using RF can be summarized as follows:

Step1: The LP formulation

$Min\, z = cx$

$s.t\; Ax = b$

$x \ge 0$

where the parameters $c = (c_1\, c_2\, \ldots\, c_n)$ and $b = (b_1\, b_2 \ldots b_m)^T$, $m \le n$, and $A = [a_{ij}]_{m\times n}$.

Step2: The formulated LPP is converted into FLPP:

$Min\,\tilde{z} \approx \tilde{c}\,\tilde{x}$

$s.t.\ A\,\tilde{x} \geq b$

$\tilde{x} \geq 0$

where $A = (a_{ij})_{m\times n}$, $\tilde{c} = (c_j)_{1\times n}$, $b = (b_i)_{m\times 1}$, $\tilde{x} = (x_j)_{1\times n}$, $where\ a_{ij} \in R\ (i = 1,2,...,m,\ j = 1,2,...,n) and\ b_i,\ c_j,\ x_j \in F(R),\ b_i,\ \tilde{x}_j \geq 0$.

Step3: Using the proposed RF $R(\widetilde{A}) = \frac{[a+8*b+c]}{10}$, then the FLPP is reduced to a crisp linear programming problem.

Step4: An optimum solution is obtained by solving the crisp linear programming problem and win QSB program (Big – M method).

6. Test Examples

The following two examples are solved using the proposed algorithm RF-FLP-TFN.

Example 1

In this example the following LP problem is considered

$Min\,u = 2t_1 + t_2$

subject to

$t_1 + 3t_2 \geq 30$

$4t_1 + 2t_2 \geq 40$

$t_1, t_2 \geq 0$

We get the optimal solution by using win QSB program (Big – M method):

$t_1 = 6$, t_2=8, $Min\,u$ = 20.0000.

The formulated LPP is converted into FLPP:

Let Δ_1 = 0.5 and Δ_2 = 0.9 where (c_{ij} - Δ_1 , c_{ij} , c_{ij} + Δ_2) and $i = 1,2$, $j = 1,2$.

$Min\,u = (1.5,2,2.9)t_1 + (0.5,1,1.9)\,t_2$

subject to

$t_1 + 3t_2 \geq 30$

$4t_1 + 2t_2 \geq 40$

$t_1, t_2 \geq 0$

Using step (3) the proposed ranking function the FLPP is reduced to a crisp LPP.

$Min\,u = 2.04\,t_1 + 1.04\,t_2$

subject to

$t_1 + 3t_2 \geq 30$

$4t_1 + 2t_2 \geq 40$

$t_1, t_2 \geq 0$

We get the optimal solution by using win QSB program (Big – M method) of the above CLP problem is:

t_1 =6 , t_2=8, $Min\,u$ =20.5600.

Example 2

Consider the LP problem

$Min\,u = 3t_1 + 7t_2$

subject to

$t_1 + 6t_2 \geq 74$

$4t_1 + 2t_2 \geq 67$

$t_1, t_2 \geq 0$

We get the optimal solution by using win QSB program (Big – M method):

$t_1 = 11.5455$, t_2=10.4091, $Min\,u$ = 107.5000.

The formulated LPP is converted into FLPP:

Let Δ_1 = 0.7 and Δ_2 = 1.2 where (c_{ij} - Δ_1 , c_{ij} , c_{ij} + Δ_2) and $i = 1,2$, $j = 1,2$.

$Min\,u = (2.3,3,4.2)t_1 + (6.3,7,8.2)\,t_2$

subject to

$t_1 + 6t_2 \geq 74$

$4t_1 + 2t_2 \geq 67$

$t_1, t_2 \geq 0$

Using step (3) the proposed ranking function the FLPP is reduced to a crisp LPP.

$Min\,u = 3.05\,t_1 + 7.05\,t_2$

subject to

$t_1 + 6t_2 \geq 74$

$4t_1 + 2t_2 \geq 67$

$t_1, t_2 \geq 0$

We get the optimal solution by using win QSB program (Big – M method) of the above CLP problem is:

$t_1 = 11.5455$, t_2=10.4091, $Min\,u$ = 108.5977.

7. Conclusion

A new ranking function of triangular FN is constructed for solving FLP problems in objective function. The original problem is reduced to another problem named a crisp value problem by introducing a new ranking function. The best optimal solution is obtained by Big-M method. The numerical results show the effectiveness of the presented algorithm.

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