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Establishment and application of fractal capillary tube bundle model of porous media

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Abstract. In view of the problem of statistical regression constant in the model of capillary tube bundles in the porous media, a capillary bundle percolation model with fractal geometry was reconstructed. The function expressions of the fractal coefficient and Kozeny constant were deduced. The relationship between the macroscopic fractal properties of porous media and the fractal dimension and the micro pore parameters were obtained. Results show: Fractal coefficient is a function of fractal dimension, maximum pore radius and minimum pore radius; The macroscopic physical properties of porous media are a function of the fractal dimension and the radius of the capillary (the maximum capillary radius and the minimum capillary radius). The expression does not contain any empirical or experimental constants. In the fractal capillary percolation model, the relationship between the three kinds of surface volume, skeleton volume and pore volume are the same as the traditional equal diameter straight capillary bundle model. The Kozeny constant can be accurately described by the function expression of the z-h coefficient, which is used for correcting the difference between real and ideal porous media model.

1. Introduction

Since French mathematician B.B.Mandlbrot founded fractal geometry theory, scholars at home and abroad have confirmed that the microscopic pore structure of porous media (such as rock and metal materials) has obvious fractal geometry characteristics [1-9]. Based on the conventional capillary tube bundle model, the fractal capillary tube bundle model and capillary pressure model were established [10-13]. Although the fractal coefficient was given a physical meaning in the model (the physical meaning of the fractal coefficient is that it reflects the development degree of pores [10]), the expression of its definition is not given and actually was a constant obtained by parameter fitting.

This paper addresses the problems existing in the fractal capillary tube bundle model of porous media on the basis of fractal geometry theory. Fractal expressions of porosity, permeability and specific surface are established according to Poseuille's law and darcy's formula. And the expression of fractal coefficient definition is clarified, the fractal expression of permeability with respect to porosity and the fractal expression of permeability with respect to porosity and specific surface without any constant are derived.

2. Porous media fractal capillary bundle flow model

Boming Yu et al. [14] believes that islands and lakes of pores and surface of the surface as well as spots on the engineering surface of porous media all have fractal characteristics, and the measure and scale satisfy the fractal scaling law. The pore space of porous media is reduced to an unequal diameter parallel straight capillary bundle model with fractal geometry, and it is considered that the fractal scaling law is still satisfied between the number (measure) N and the radius (scale) r of the capillary on the cross section

$$N(r) = \left(\frac{r_{\max}}{r}\right)^D \tag{1}$$

D, fractal dimension ($1<D<2$ in two-dimensional space); $r_{\max}$, maximum capillary radius on cross section.

When the capillary radius is $r_{\min}$, from equation (1) the total number of capillaries on the cross section is

$$N_t = \left(\frac{r_{\max}}{r_{\min}}\right)^D \tag{2}$$

Due to the large number of pores in porous media, the size of capillary section can be considered continuous. So formula (1) can be approximately considered as continuous and differentiable. We take the derivative of formula (1) with respect to r, the number (measure) of capillaries with radius $r \sim r+$ dr on the cross section is obtained

$$-\mathrm{d}N = Dr_{\max}{}^{D} r^{-D-1}\mathrm{d}r \tag{3}$$

After divide equation (3) by equation (2), the probability density function of capillary distribution is obtained by simplification

$$f(r) = DR_{\max}^{D} r^{-D-1} \tag{4}$$

Equation (4) is compared with equation (2) in reference [10], the expression of fractal coefficient defined in reference [10] can be obtained

$$a = \frac{DR_{\max}^{D}}{r} \tag{5}$$

Suppose the cross section area of the capillary bundle model is A, the length is L, the pressure at the inlet end and the outlet end are p_1 and p_2 respectively, and the viscosity of the fluid is μ. The liquid flow rate in a single capillary tube and the pressure difference at both ends satisfy the Poseuille formula

$$q = \frac{\pi r^4 (p_1 - p_2)}{8\mu L} \tag{6}$$

Thus, the fluid flow through the entire section can be obtained

$$Q = \int \mathrm{d}\,q = \int_{r_{\min}}^{r_{\max}} \frac{\pi r^4 (p_1 - p_2)}{8\mu L} Dr_{\max}{}^{D} r^{-D-1}\mathrm{d}r = \frac{\pi Dr_{\max}{}^{D}}{8(4-D)} \frac{(p_1 - p_2)}{\mu L} (r_{\max}^{4-D} - r_{\min}^{4-D}) \tag{7}$$

According to darcy’s formula, the permeability of fractal porous media is obtained

$$k = \frac{\pi Dr_{\max}{}^{D}}{8A(4-D)} (r_{\max}^{4-D} - r_{\min}^{4-D}) = \frac{\pi Dr_{\max}{}^{4}}{8A(4-D)} \left[1 - \left(\frac{r_{\min}}{r_{\max}}\right)^{4-D}\right] \tag{8}$$

According to the definition of porosity, the porosity expression of the fractal porous media capillary model is obtained

$$\phi = \frac{V_{\mathrm{p}}}{V_b} = \frac{\int_{r_{\min}}^{r_{\max}} \pi r^2 L Dr_{\max}{}^{D} r^{-D-1}\mathrm{d}r}{AL} = \frac{\pi Dr_{\max}^2}{A(2-D)} \left[1 - \left(\frac{r_{\min}}{r_{\max}}\right)^{2-D}\right] \tag{9}$$

In fractal porous media, $r_{\min}/r_{\max}$ is generally far less than 0.01[15]. Meanwhile, in three-dimensional space, 2<D< 3, 1 < 4-d <2, -1< 2-d <0, therefore, $(r_{min}/r_{max})^{4-d}$ can be ignored, and $(r_{min}/r_{max})^{2-d}$ cannot be ignored. So the permeability expression of fractal porous media can be simplified

$$k=\frac{\pi D}{8A(4-D)}r_{max}{}^{4} \tag{10}$$

It can be seen from equation (5), equation (9) and equation (10) that the larger rmax (maximum capillary radius), the larger a (fractal coefficient) and the better the petro-physical property (*Φ*、*k*) under certain *r* and *D* conditions.

The fractal expressions of porous media specific surface are obtained according to the definition of surface area based on rock surface volume

$$S_b=\frac{A_0}{V_f}=\frac{\int_{r_{min}}^{r_{max}}2\pi rLDr_{max}{}^{D}r^{-D-1}\mathrm{d}r}{AL}=\frac{2\pi D}{A(1-D)}r_{max}\left[1-\left(\frac{r_{min}}{r_{max}}\right)^{1-D}\right] \tag{11}$$

Fractal expressions of porous media specific surface are obtained according to the definition of rock skeleton volume

$$S_s=\frac{A_0}{V_b-V_p}=\frac{\int_{r_{min}}^{r_{max}}2\pi rLDr_{max}{}^{D}r^{-D-1}\mathrm{d}r}{AL-\int_{r_{min}}^{r_{max}}\pi r^2LDr_{max}{}^{D}r^{-D-1}\mathrm{d}r}=\frac{2\pi D}{1-D}r_{max}\frac{1-(r_{min}/r_{max})^{1-D}}{A-\frac{\pi Dr_{max}^2}{2-D}[1-(r_{min}/r_{max})^{2-D}]} \tag{12}$$

Fractal expressions of porous media specific surface are obtained according to the definition of pore volume

$$S_p=\frac{A_0}{V_p}=\frac{\int_{r_{min}}^{r_{max}}2\pi rLDr_{max}{}^{D}r^{-D-1}\mathrm{d}r}{\int_{r_{min}}^{r_{max}}\pi r^2LDr_{max}{}^{D}r^{-D-1}\mathrm{d}r}=\frac{2(2-D)}{1-D}\frac{1}{r_{max}}\frac{1-(r_{min}/r_{max})^{1-D}}{1-(r_{min}/r_{max})^{2-D}} \tag{13}$$

3. Results and Discussion

3.1. The relationship between permeability and porosity

After equation (10) is divided by equation (9), the relationship between permeability and porosity of fractal porous media capillary bundle model is obtained through simplification

$$k=\frac{2-D}{8(4-D)}\frac{r_{max}^2}{1-(r_{min}/r_{max})^{2-D}}\phi \tag{14}$$

According to equation (14), Matlab software was used to draw a set of curves with r=0.01μm, R=1μm, porosities of 10%, 15%, 20%, 25% and 30%, and r=0.01μm, R=1μm, porosities of 10%, 15%, 20%, 25% and 30%, respectively, as shown in figure 1.

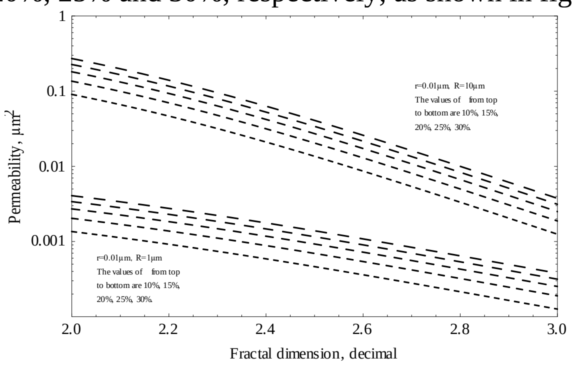

Figure 1. Porosity effect permeability

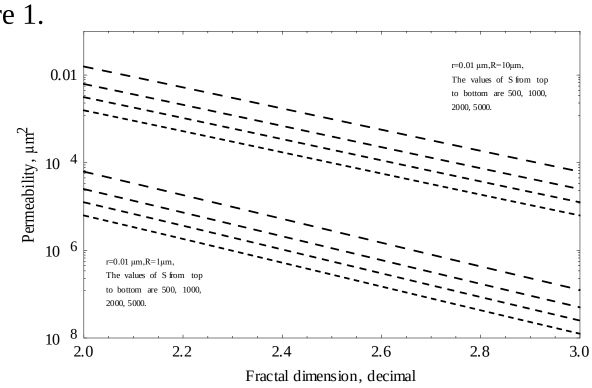

Figure 2. Specific surface effect permeability

From Figure 1. it appears that permeability decreases linearly with the increase of fractal dimension under certain porosity in semi-log coordinates, and the larger the porosity, the larger the permeability, and the maximum pore radius has the greatest influence on the permeability with a certain fractal dimension.

3.2. The relationship between permeability and specific surface

After equation (10) is divided by equation (11), the permeability relationship of the fractal porous media capillary bundle model with respect to the specific surface (based on the external volume) is obtained through simplification

$$k=\frac{r_{\max}^{3}(1-D)}{16(4-D)}\frac{1}{1-(r_{\min}/r_{\max})^{1-D}}s_{b} \tag{15-1}$$

Generally, the rock specific surface unit is cm^2/cm^3, and if it is converted to μm^2/μm^3, equation (14) can be deformed into

$$k=\frac{r_{\max}^{3}(1-D)}{16(4-D)}\frac{1}{1-(r_{\min}/r_{\max})^{1-D}}s_{b}\times10^{-4} \tag{15-2}$$

According to equation (15-2), Matlab software was used to draw a set of curves with r=0.01μm, R=1μm, S of 500, 1000, 2000, and 5000, and r=0.01μm, R=10μm, S of 500, 1000, 2000, and 5000, respectively, as shown in figure 2.

It can be seen from figure 2. that in semi-log coordinate system, permeability decreases linearly with the increase of fractal dimension when the specific surface is constant, and the larger the specific surface, the larger the permeability, and the maximum pore radius has the greatest influence on the permeability in the case of a certain fractal dimension.

3.3. Relationship between specific surface and porosity

After dividing equation (9) by equation (11), the relation between porosity and the specific surface based on external volume can be obtained through simplification

$$S_{b}=\frac{2(2-D)}{1-D}\frac{1}{r_{\max}}\frac{1-(r_{\min}/r_{\max})^{1-D}}{1-(r_{\min}/r_{\max})^{2-D}}\phi \tag{16}$$

When equation (8) is substituted into equation (12), the relation between the specific surface and porosity based on the skeleton volume is obtained through simplification

$$S_{\mathrm{s}}=\frac{2\pi D}{1-D}\frac{1}{1-\phi}\frac{r_{\max}}{A}[1-(r_{\min}/r_{\max})^{1-D}] \tag{17}$$

When equation (9) is substituted into equation (13), the relation between specific surface and porosity based on pore volume is obtained

$$S_{\mathrm{p}}=\frac{2\pi D}{1-D}\frac{1}{\phi}\frac{r_{\max}}{A}[1-(r_{\min}/r_{\max})^{1-D}] \tag{18}$$

According to equations (11), (17) and (18), the relationship between the specific surfaces of the surface volume, the skeleton volume, and the pore volume of fractal porous media can be obtained

$$S_{b}=(1-\phi)S_{\mathrm{s}}=\phi S_{\mathrm{p}} \tag{19}$$

It can be seen from equation (19) that the relation between the three specific surfaces obtained by the fractal capillary tube bundle model of porous media is consistent with that obtained by the traditional parallel straight capillary tube bundle model.

3.4. The relationship between permeability and porosity and specific surface

After the two ends of equation (16) are squared, the simplification can be obtained

$$r_{max}^{2}=\frac{4(2-D)^{2}}{(1-D)^{2}}\frac{[1-(r_{\min}/r_{\max})^{1-D}]^{2}}{[1-(r_{\min}/r_{\max})^{2-D}]^{2}}\frac{\phi^{2}}{S_{b}^{2}} \tag{20}$$

Equation (20) was substituted into equation (14), and the relation between the permeability of fractal porous media on porosity and specific surface (based on external volume) is

$$k=\frac{(2-D)^{3}}{2(4-D)(1-D)^{2}}\frac{[1-(r_{\min}/r_{\max})^{1-D}]^{2}}{[1-(r_{\min}/r_{\max})^{2-D}]^{3}}\frac{\phi^{3}}{S_{b}^{2}} \tag{21}$$

Equation (19) was substituted into equation (21), the relationship between porosity and specific surface (based on skeleton volume) of the fractal porous media capillary bundle model is

$$k = \frac{(2-D)^3}{2(4-D)(1-D)^2}\frac{[1-(r_{\min}/r_{\max})^{1-D}]^2}{[1-(r_{\min}/r_{\max})^{2-D}]^3}\frac{\phi^3}{(1-\phi)^2 S_s^2} \tag{22}$$

Equation (19) was substituted into equation (22), the relationship between permeability and porosity, specific surface (based on pore volume) in a fractal porous media capillary bundle model is

$$k = \frac{(2-D)^3}{2(4-D)(1-D)^2}\frac{[1-(r_{\min}/r_{\max})^{1-D}]^2}{[1-(r_{\min}/r_{\max})^{2-D}]^3}\frac{\phi}{S_p^2} \tag{23}$$

Make $\frac{(2-D)^3}{2(4-D)(1-D)^2}\frac{[1-(r_{\min}/r_{\max})^{1-D}]^2}{[1-(r_{\min}/r_{\max})^{2-D}]^3} = \zeta$, for the sake of ease of expression, ζ can be called the z-h coefficient. Then, according to equation (21), equation (22) and equation (23), it can be got that

$$k = \zeta\frac{\phi^3}{S_b^2} = \zeta\frac{\phi^3}{(1-\phi)^2 S_s^2} = \zeta\frac{\phi}{S_p^2} \tag{24}$$

The relationship between permeability and specific surface as represented by the general expression of Gauscny equation [16] is

$$K = \frac{\phi^3}{\xi S_b^2} = \frac{\phi^3}{\xi S_s^2(1-\phi)^2} = \frac{\phi}{\xi S_p^2} \tag{25}$$

By compared equation (24) and equation (25), the expression of Gauscny constant can be got

$$\xi = \frac{1}{\zeta} = \frac{2(4-D)(1-D)^2}{(2-D)^3}\frac{[1-(r_{\min}/r_{\max})^{2-D}]^3}{[1-(r_{\min}/r_{\max})^{1-D}]^2} \tag{26}$$

4. Conclusions and recommendations

(1) The function expression of fractal coefficient is derived, the physical meaning and influence factors of fractal coefficient are clarified, and the fractal coefficient is a function of fractal dimension, maximum pore radius and minimum pore radius.

(2) Macro-physical parameters of porous media are functions of fractal dimensions and capillary radius (maximum capillary radius, minimum capillary radius), and the expression does not contain any empirical or experimental constants.

(3) The relationship between three specific surfaces (based on external volume, skeleton volume, and pore volume) in the fractal capillary tube bundle model of porous media is consistent with that of the conventional linear capillary tube bundle model with equal diameter, all of them are relation of $S_b=(1-\Phi)\ S_s=\Phi S_p$.

(4) By compared the relation between permeability and specific surface, porosity in Gauscny equation and fractal capillary tube bundle model, it is found that the Gauscny constant which is used to correct the difference between real rock and ideal model can be accurately described by the function expression of z-h coefficient.

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References

[1] Katz A J, Thompson A H. (1985) Fractal sandstone pores : implications for conductivity and pore formation. Phys Rev Lett, 54: 1325-1328.
[2] Krohn C E, Thompson A H. (1986) Fractal sandstone pores: automated measurements using

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scanning-electron-microscope images. Phys Rev B, 33: 6366-6374.
[3] Young IM, Craw ford JW. (1991) The fractal structure of soil aggregations: its measurement and interpretation. J Soil Sc, 42:187-192.
[4] Jia F S, Shen P P, Li K W. (1995) Study on the Fractal Characteristies of Sand stone Pore Structure and Its APPlication. FAULT-BLOCK OIL & GAS FIELD, 2(1):1396-1408.
[5] Smidt J M, Monro D M. (1998) Fractal modeling applied to reservoir characterization and flow simulation. Fractals, 19:626-639.
[6] Yu B M, Li J H. (2001) Some fractal characters of porous media. Fractals, 18(9): 365-372.
[7] Ma X F, Zhang S C, Lang Z X. (2015) Fractal characterization of characteristic parameters of pore structure. PGRE, 12(6):34-36.
[8] Zhang He, Ren X J, Li J D, et al. (2011) The calculation method of the fractal dimension of waterflood front of micro-model. RESERVOIR EVALUATION AND DEVELOPMENT, 1(3):22-25.
[9] Gao Y，Mao L，Ma R. (2017) Pore structure characteristics and its fractal dimensions of transitional shales. Science Technology and Engineering, 17(33):77-85.
[10] LI L R, ZHAO Y Y, LI Z H, et al. (2004) Fractal characteristics of micropore structure of porous media and the meaning of fractal coefficient. Journal of the University of Petroleum, China (Edition of Natural Science), 28(3):105-107.
[11] LI L R, YUAN S Y, HU Y L. (2010) Simulation of the separation of liquid column in oil pipeline. Journal of Xi’an Shi you University (Natural Science Edition), 25(3):49-52.
[12] Zheng B, Li J H. (2015) A new fractal permeability model for porous media based on kozeny-carman equation. Natural Gas Geoscience, 26(1):193-198.
[13] Yuan Z，Liu P C. (2015) A New Capillary Pressure Model Using Fractal Geometry for Coal Porous Media. Science Technology and Engineering, 15(9):63-67.
[14] Yu B M, Ping C. (2002) A fractal permeability model for bi-dispersed porous media, Int. J. Heat and Mass Transfer, 45(14): 2983-2993.
[15] Yu B M, Li J H. (2008) Analysis of Flow in Fractal Porous Media. Applied Mechanics Reviews, 61(5):1239-1249.
[16] He G S, Tang H. (2011) Physics of Petroleum Reservoirs [M]. Petroleum Industry Press, Beijing.