Layers of Logical Consequence:
Logical consequence as epistemically model-theoretic and metaphysically proof-theoretic

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Abstract

Model-theory and proof-theory are two long-standing alternative descriptions of logical consequence. Proof-theory characterises truth in terms of logical implication. Namely, according to proof-theory, the statement 'A implies B' is true iff there exists a proof from A to B. In contrast, model-theory characterises truth based on possible states of the world. By model-theory, 'A implies B' is true iff for any model m, if m satisfies A then m satisfies B. In this paper I argue that we can reconcile the views, by making an appropriate distinction between epistemic nature and metaphysical nature. Namely, I will argue that we can view logical consequence as epistemically model-theoretic and metaphysically proof-theoretic.

1 Introduction

When we say that something *logically implies* something else, we are appealing to the relationship of implication between a premise and a conclusion. This relationship is described by the theory of logical consequence. But how can we describe logical consequence itself? On what grounds can we say something logically implies something else? In other words, how should we think about the technical relationship between a premise and a conclusion? As of yet, there is not a universally accepted answer to this question, although the nature of logical consequence has been a subject of debate for decades.

Model theory and proof-theory have prevailed in parallel since the early days of the debate, as alternative descriptions of logical consequence. A model-theoretic view frames logical consequence in terms of truth preservation across cases, while a proof-theoretic view holds that consequence reduces to the existence of a formally valid argument between the premises and conclusion.

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It is difficult to refute either on purely technical grounds, and some authors have suggested adopting some combination of the two. It even seems reasonable to argue that we *should* be searching for a compromise: neither view seems entirely counter-intuitive, or technically flawed. In this paper I will identify the relevant compromise with a distinction between epistemic and metaphysical nature. My aim is to defend logical consequence as *epistemically* model-theoretic, and *metaphysically* proof-theoretic.

In §2, I will describe the technical frameworks of model-theory and proof-theory in more detail, describe existing compromise-style views, and emphasise that they tend to view proof-theory and model-theory as alternative but individually complete descriptions of logical consequence. In §3, I will present a view on which neither offers a complete description of logical consequence. Rather, I will show that we can take model-theory as a description of the epistemic side to logical consequence, and proof-theory as a description of its metaphysical side. These descriptions are individually incomplete—but when taken together, they offer a complete description of logical consequence. The aim of §3 is to show that it is possible to adopt such a view. The aim of §4 will be to show that it is plausible to adopt such a view, because of the link between epistemic access and the model-theoretic cases, and because we have reason to believe that these case-by-case relationships are instantiations of underlying deductive relationships.

2 Background

First of all, let me describe the existing frameworks of proof-theory and model-theory in more detail. Conceptually, we can think of model-theory as a case-by-case description: 'The *model-centred approach* to logical consequence takes the validity of an argument to be *absence of counterexample*' (Beall and Restall 2016). In technical terms, consequence is model-theoretic iff it is described by the following equivalence relation: B is a consequence of A iff for any model m, if m satisfies A then m satisfies B (Tarski 1983, 417).

Proof-theory instead relies on the existence of argument-based reasoning. Again appealing to Beall and Restall, 'On the *proof-centred approach* to logical consequence, the validity of an argument amounts to there being a *proof* of the conclusions from the premises' (2016). So a consequence is proof-theoretic on the condition that an argument from A to B is valid iff there is a proof of B from A.

As it is, the two theories tend to be seen as alternative rather than compatible views. This state of affairs is exhibited particularly clearly in Shapiro's article, 'Logical Consequence, Proof Theory and Model Theory', where he summarises the current state of the logical consequence debate. He writes, '[m]odel theory and proof theory each provide for a notion of logical consequence, but the notions employed by these two branches are quite different from each other, at least conceptually' (Shapiro 2007, 651). Later in the article he comments on the attitude philosophers commonly have towards these theories. He writes that there are, in general, two schools of thought. One takes model-theory to be the primary essence of logical consequence,

the other takes proof-theory to be the primary essence of logical consequence (2007, 668). Proof-theory and model-theory are taken to be 'different, but closely related, notions of logical consequence' (2007, 669).

Even within the diverse landscape of existing compromise-style theory, in which authors do not commit themselves to pure proof-theory or pure model-theory, the two views tend to be seen as individually complete descriptions of logical consequence. Logical pluralists hold that there are several correct notions of logical consequence; Beall and Restall, Cook and Shapiro, and Carnap have each defended a unique version of this view (Beall and Restall 2000; Cook 2002; Shapiro 2006, Carnap 1959). Relativists have suggested that the correct notion might instead be domain-relative—variable but determined by the field we are working in. Some versions of relativism even identify logical domain with culture, resulting in culture-dependent logic (Lokhorst 1998, Greiffenhagen and Sharrock 2006). In the pluralist version, proof-theory and model-theory are seen as two correct but individually complete descriptions. On the relativist view, each theory is a complete description of logical consequence, but only within a certain domain.

At first sight, it does seem natural to treat them this way, as alternative but complete descriptions—they propose two different ways to describe the same thing. However, in the following section I will argue that we are not forced to adopt this attitude. Instead, if we make an appropriate distinction between the metaphysical and epistemic nature of logical consequence, we can view proof-theory and model-theory as two separately incomplete components of a full description.

3 The technical details, and why they are possible

To address the epistemic and metaphysical nature of logical consequence, we must begin with a clear definition for each type of property. For my theory of logical consequence, a property is *epistemic* iff it concerns knowledge or justified belief. Metaphysics is slightly more abstract by nature, and this is sure to be reflected in any attempt at a definition. I will stick to the following admittedly elusive, but meaningful notion: a property is *metaphysical* iff it concerns the most abstract essence of a concept.

The general frameworks of model-theory and proof-theory, as described in the previous section, clearly establish a set of minimal conditions for my theory of logical consequence. At the very least, I must be able to identify the epistemic side of logical consequence with a set of possible models and a corresponding satisfaction relation. And I must be able to identify its metaphysical nature with existence, or absence, of proof. Here I merely aim to show that it is at least *possible* for logical consequence to fit the strict technical requirements of model-theory in its practical use, and fit the equivalent requirements of proof-theory in its metaphysical nature. In the next section I will argue that this is not only an option, but a *plausible* option.

So, how can model-theory fit with the epistemic side to logical consequence? What are the

models, and what is the satisfaction relation? As Varzi notes, logic aims to act as a universal requirement for theoretical reasoning. As a result, its models must be universally applicable in some sense; 'logic is a uniquely ambitious theory [...] It aims to be the theory included in every other theory [...] its models want to include the models of every other' (Varzi 2002, 199). I propose we can access this universality by appealing to Carnap's distinction between individual and general concepts. An individual concept is specific to a certain space-time coordinate, while the corresponding general concept has many possible space-time representations. Carnap's pet dog Luchs exists at a given location at a given instant. This individual concept is one among many possible representations of the corresponding concept *dog* (Carnap 1959, 247–48). The fact that Carnap's dog does not have some property X at that specific place and time is a representation of the general concept of X *not obtaining*.

We may adopt this distinction as a backbone for the epistemic nature of logical consequence, partly in virtue of its universality. I have claimed that epistemically, logical consequence can be described by satisfaction of models. I propose we should make the following identifications: *model* with the above-described notion of *individual concept*, satisfaction with representation, and subject of satisfaction with the above-described notion of *general concept*. Then my model-theoretic proposal for the epistemic side of logical consequence amounts to the claim that we observe individual concepts to come to *know and/or believe* consequence relationships between general concepts. Note the emphasis on observation, knowing, and believing here—these are all epistemically-charged terms, which is just what we are asking for since we are talking about the epistemically accessible side of logical consequence. In technical terms, this proposal means: a relationship of logical consequence from A to B is epistemically valid if and only if for all individual concepts m, if m is a representation of the general concept A, then m is a representation of the general concept B.

An immediate objection to this view might be that it fails to include a huge set of states of the world—namely, states of the world that do not explicitly depend on space or time, and therefore cannot be represented in the way I have described above. For such states, we must add to Carnap's conception. The individual concept in such cases cannot be the discrete state itself. Instead, we may identify the individual concept as our knowledge or experience of that discrete state. The fact that this involves connecting models with experience is not an issue, since we are only attempting to deal with the epistemic nature of logical consequence. Overall, the proposed individual-general concept distinction is universally applicable in an epistemic sense, if it includes this adjustment for discrete states without inherent space-time coordinates.

To get a further sense of how this works, consider $\neg(\neg X) \models X$, the statement that X is a logical consequence of $\neg(\neg X)$. Let's examine our epistemic access to this consequence relationship, on the view I have proposed above. The general concept on the left hand side of the implication is 'it fails to obtain that some property of the world, X, fails to obtain'. This general concept can be represented by individual concepts that fail to lack some property of the world, where individual concepts are specific in space and time, either themselves or through space and time-localised experience. An example of such an individual concept would be the space-time event of me looking out my window and failing to find a field of view in which there

is not a german shepherd on my front lawn. The right hand side of the implication $\neg(\neg X) \models X$ is the general concept 'some property of the world, X, obtains'. This might be represented by any number of individual concepts in which some property of the world obtains, for example by my observation that I hear a helicopters in the sky. My proposal for the epistemic nature of logical consequence claims that we can see our epistemic access to relationships like $\neg(\neg X) \models X$ by running through these individual concepts, and checking that every space-time specific and experientally accessible individual concept that represents 'it fails to obtain that some property of the world, X, fails to obtain' also represents 'X obtains of the world'. It should be noted here that we clearly cannot run through the infinite number of possible individual concepts available. This is an epistemic limitation. But we can satisfy ourselves with the idea that we gain knowledge of relationships like $\neg(\neg X) \models X$ by having experienced some large number of its individual concept instantiations. And the greater the number of instantiations we have experienced, the greater our confidence in the validity of $\neg(\neg X) \models X$. For example, when I look out of my window and fail to find a field of view in which there is not a German shepherd on my front lawn, at that same instant of time and space I observe that there is a German shepherd on my front lawn. This individual experiential and space-time specific concept can be seen as one of many such concepts that give me epistemic access to $\neg(\neg X) \models X$. When I listen to the sky and fail to avoid hearing the noise as helicopter engine noise, I am always simultaneously hearing helicopter engine noise. Having this experience, in addition to the previous experience, gives me even more confidence in the validity of $\neg(\neg X) \models X$.

So we have seen that it is possible to describe the epistemic nature of logical consequence model-theoretically, according to an experience-based description of models, and a representation-based description of the satisfaction relation. Next I need to show that it is possible to describe metaphysical nature of logical consequence proof-theoretically. This requirement is more straightforward to satisfy. Since a concept's metaphysical nature is its abstract essence, we merely need to show it is possible to think of logical consequence as being fundamentally represented by abstract proof from premises to a conclusion. But we can imagine the existence of an argument from a set of premises to the corresponding conclusion, in the same way we can imagine the existence of an abstract mathematical proof for $x + x = 2x$. The difficulty for the proof-theoretic side of my view is not whether we can *possibly imagine* a metaphysical relationship between proof and consequence. The difficulty will be to establish whether we can *plausibly assert* the existence of that relationship. That will be the aim of the next section.

A satisfactory proof must be both rigorous and formal. Beyond that basic requirement, I do not claim to subscribe to any particular type of proof when I suggest proof-theoretic metaphysical essence. Any discussion of that kind raises myriad problems of its own; and if I were to choose one type of proof, my entire proposal for logical consequence would become opaque for anyone unsympathetic to my choice. Instead I propose a view in which logical consequence is epistemically model-theoretic, in the sense of individual-general concept experience-based modelling outlined above, and metaphysically proof-theoretic, according to some rigorous and formal type of proof.

By specifying its epistemic and metaphysical nature, we must have provided a complete

description of logical consequence. If we have both a description for how we can come to know and believe about a concept, and a description for how that concept exists independently in the abstract world, what more could we ask for? Epistemic nature and metaphysical nature, taken together, exhaustively describe any concept. Therefore, the sub-descriptions I have offered come together naturally as individually incomplete descriptions that jointly offer a complete description of logical consequence. Of the two parts of logical consequence—its epistemic nature and its metaphysical nature—one can be described by model-theory and the other can be described by proof-theory. In this way, my proposal allows us to transition from a view of *alternative* theories to a view of *compatible* theories, each of which only describes one part of the full concept we are trying to characterise.

4 The technical details, and why they are plausible

Now that we have established that it is *possible* to provide a view on which logical consequence is epistemically model-theoretic and metaphysically proof-theoretic, we are in a position to establish whether this is a *plausible* view to adopt. I will argue that it is, for several reasons. First, I will examine our reasons for subscribing to the specific models and satisfaction relation I have proposed. Based on this technical framework, with the models and satisfaction relation as I have described, I will then explain why we should think logical consequence is epistemically model-theoretic in the way I have described. Finally, I will examine our reasons for believing that logical consequence is metaphysically proof-theoretic, according to some rigorous and formal kind of proof.

To begin with, why should we subscribe to the models and satisfaction relation I have described? Our epistemic access to logical consequence is defined by the limits on our actual inference process. And I claim that our actual inference process follows a representationalist model-theoretic program, where models represent possible states of reality (Sher 1996, 658–61). I make this claim in light of the intuitive dependency between experience and epistemic access to the world. Carnap notably supports such dependency, in his comprehensive work *The Logical Structure of the World*. In his words, 'I can make an "epistemic evaluation" of any experience I have had by stating to what extent this experience has added to my (theoretical) knowledge. This addition consists not only of the theoretical content of the experience itself, but also of whatever I can infer from this content with the aid of my earlier knowledge' (Carnap 2003, 309). As he notes, we come to know or believe through assessing the nature of the world based on real experience, or imaginary but believable extension of experience. And an experience is a snapshot of a possible state-of-the-world. Our models and satisfaction relation, if they are to describe our epistemic access to logical consequence, should respect this relationship between epistemic access and experience. And it does not take much to get from this requirement to the models and satisfaction relation I describe. I merely take a model to be a space and time-specific state of the world, accessible or hypothetically accessible via experience, and the satisfaction relation to be the relationship of representation that holds between these localised state of the world and unlocalised, general concepts. Thus, I argue that it is plausible

to accept the models and satisfaction relation I have proposed for the epistemic side of logical consequence.

Having granted these models and this satisfaction relation, why should we think logical consequence is epistemically model-theoretic in the first place? In other words, why should we think our knowledge and beliefs about logical consequence involve case-by-case analysis of representational relationships between different individual concepts? A point raised by Shapiro is particularly relevant here. He notes that, even if we are working within a proof-theoretic system, the only way we can intuitively check the correctness of that system is by checking that its rules of inference 'do not lead from truth to falsehood' (Shapiro 2007, 667). This checking is an epistemic endeavour—we are asking, how can we *know, and test the world* to conclude that our proof-theoretic system of logical reasoning is correct? There is only one clear way to do this test in an epistemically accessible way—namely, by running through possible hypothetical states of the world. This amounts to leaning on the model-theoretic version of logical consequence I have described. Thus, it *is* plausible to believe that epistemically, we are limited to treating logical consequence model-theoretically.

Logical consequence might still be *metaphysically* proof-theoretic. But why should we see it this way? To fully argue for the plausibility of proof-theoretic metaphysical nature for logical consequence, we need to firmly establish whether it even needs any kind of metaphysical description at all. This brings us to a challenge tied up with the ancient debate on realism versus antirealism towards the existence of abstract universals. Namely, if we come to know and believe using a certain notion of a concept, why should we think there exists anything more abstract or fundamental to that concept's full nature? A sceptic might claim: logical consequence does not have any metaphysical content beyond its epistemic nature, which we have already described as model-theoretic. I will argue that such scepticism is unwarranted—that we *do* have reason to believe in a non-trivial metaphysical essence for logical consequence. Then I will explain why we have reason to believe that this metaphysical essence is proof-theoretic.

We will be in a good position to identify gaps in the sceptic's reasoning if we first examine the source of their inspiration. Hume's treatise on causation and induction forcefully encourages doubt about the existence of abstract metaphysical essence for causal law. In *An Enquiry concerning Human Understanding*, he argues that we are fundamentally limited in our ability to identify causal relationships *a priori*, since all knowledge stems from case-by-case examination of experience (Hume 2008, 18–23). This position has since been widely acknowledged by scientists and philosophers alike (Carnap 2003, 265). Hume then settles on a description of cause-and-effect as mere cosmic regularity, questioning the notion of cause-and-effect as manifestation of abstract causal law (Armstrong 1993, 438; 1983, 4). Causation is not equivalent to logical consequence; but I believe we can construct an analogy here. Epistemic reliance on case-by-case method in science has led to widespread scepticism about the existence of any abstract scientific law beyond functional dependency. It seems that similar doubt could apply to existence of metaphysical essence for logical consequence - assuming its epistemic nature is model-theoretic, and therefore reliant on case-by-case relationships. Definitely, this model-theoretic epistemic nature restricts our epistemic access to any proof-based abstract nature that

might exist.

However, we cannot directly conclude from this that an abstract proof-based nature does not exist. I will argue that it likely *does* exist, beginning with an appeal to work in logic-as-modelling. I have emphasised that, if epistemic and metaphysical nature do both carry weight, they are not equivalent properties. One useful way to characterise this difference allows us to draw from Shapiro's work on logic-as-modelling. Namely, we can easily view the epistemic nature of logical consequence as a model. Here, I mean model in the colloquial sense of approximate representation—I am not referring to the individual-general concept introduced earlier for my treatment of model-theory. In general, epistemic nature is defined by the boundaries of knowledge; therefore, it is in some sense a knowledge-based approximation of a concept that *might* have deeper metaphysical qualities. Key here is the approximate in approximate representation. A model is not necessarily a complete representation of the concept it stands for; 'there is almost always a gap between a model and what it is a model of' (Shapiro 2006, 50). This point helps Shapiro to develop his logic-as-modelling view, but I will use the same point to claim that there is likely an underlying metaphysical nature to logical consequence, above and beyond its model-theoretic epistemic nature.

The above reasoning only works if the epistemic nature of logical consequence is in fact a knowledge-based approximation. So how do we know it is an approximation of this kind? Here I can use the sceptic's point for my own benefit. As the sceptic notes, if the epistemic nature of logical consequence is model-theoretic, it is limited in its access to *a priori* reasoning. Therefore it is a limited description. And a limited description is an approximate description. It is an approximate model, in the same way that 'a collection of point masses is a model of a system of physical objects, and the Bohr construction is a model of an atom' (Shapiro 2006, 49). However, every model models something—every approximate representation must represent some non-trivial exact concept. I suggest that this exact concept is precisely the metaphysical nature of logical consequence. Epistemic nature covers everything we are able to know about logical consequence relationships. And everything we are able to know creates a model of the non-trivial abstract metaphysical reality, which is outside the realm of our direct knowledge. Therefore, the sceptic's emphasis on our epistemic constraints offers indirect support for my defence of non-trivial metaphysical nature.

Furthermore, and crucially, there is no fundamental inconsistency in a view that includes non-trivial essence from both sides, epistemic and metaphysical. Here we are encouraged by various influential philosophers, who subscribe to Hume's work on scientific method yet maintain realism on the existence of universal law. Again, we work with an analogy, comparing the Humean notion of scientific method with epistemic model-theoretic essence, and the existence of fundamental causal law with the existence of non-trivial metaphysical essence.

From the early 20th century, Carnap and Russell are among many explicit advocates for Hume's view on the epistemic limits of *a priori* reasoning. Still they do not deny the existence of fundamental law, as metaphysical nature. See the following excerpt from Carnap on the metaphysical essence of causal correlation: '[h]ere we do not simply ask between what object

the relation obtains, but what it is between the correlated objects, by virtue of which they are connected' (Carnap 2003, 35). And, even more explicitly, 'the essence problems belong to *metaphysics*' (2003, 35). Still, he sees our epistemic experience of specific causal relationships as a manifestation of mere functional dependency (2003, 264). Russell, whose position on epistemic empiricism is steadfast, still does not rule out the existence of causal law as an abstract mathematical formulation (Russel 1913, 14). He writes, 'there is no *a priori* category of causality, but merely certain observed uniformities' (1913, 24). However, '[i]n all science we have to distinguish two sorts of laws: first, those that are empirically verifiable but probably only approximate; secondly, those that are not verifiable, but may be exact' (1913, 16). He clearly acknowledges the possibility of exact abstract law, in spite of its inherently unverifiable nature.

From the more modern tradition, Armstrong and Davidson hold a similar position. Both endorse Humean views on method but accept abstract law (Armstrong 1993, 438; Davidson 1967, 701–02). In Armstrong's words, 'It is true that there appears to be no *a priori* argument that takes one from singular causation to law' (1993, 438). However, '[i]t may be noted that the unity of the space-time world is not constituted by the mere conjunction of the state of affairs [...] [t]he real *unity* is given by the fact that all the particulars are directly or recursively linked to each other by real, that is external, relations' (1993, 435). Davidson emphasizes that we are often limited in our epistemic access to external causal law—but notes: 'very often, I think, our justification for accepting a singular causal statement is that we have reason to believe an appropriate causal law exists, though we do not know what it is' (1967, 701).

Armstrong presents an explicit argument for his subscription to the separate existence of abstract law as a metaphysical basis for correlation. We either see correlation as instantiation of a universal and abstract entity, or instantiation of a mere regularity. The abstract entity in the former view acts as a basis for *explanation*. Mere regularity does not—any attempt at explanation would be circular, explaining correlation in terms of correlation. Armstrong believes the former view is inherently more desirable, simply because genuine explanation is desirable (Armstrong 1983, 40–41).

For logical consequence we have further reason to prefer the former view, even if we do not share Armstrong's belief in the intrinsic appeal of explanation. Genuine explanation implies a degree of modal transparency. A genuine explanation can describe why a particular consequence relation *must*, or *should* hold. Thus, by maintaining an anti-sceptic position on the existence of abstract metaphysical nature for logical consequence, we automatically ease modal problems associated with a purely model-theoretic view. These problems have been examined at length by Prawitz (2005).

Furthermore, we can appeal to an intuition raised by Prawitz. As he points out, purely model-theoretic consequence would be intuitively back-to-front. On such a pure view, 'we cannot really say that we infer the truth of the conclusion by the use of a valid inference. It is, rather, the other way around: we can conclude that the inference is valid after having established for all inferences of the same form that the conclusion is true in all cases where the premises are' (2005, 675). This is a crucial and telling point to acknowledge. Intuitively, we want some part

of logical consequence to be a fundamental property of relationships between abstract ideas. We lose this if the validity of an inference depends purely on running through all possible interpretations or states of the world. In some sense we are demoting logical consequence, from fundamental to dependent—in particular, to dependent on possible discrete states of the world. On the view I have proposed, logical consequence is instead metaphysically dependent on rules for argument and proof, from which possible discrete states are derived. Here we see a key benefit of my view, for solving Prawitz's intuitive problem with model-theory: namely, the ability to push logical consequence back to its intuitive status, without denying Humean limits on our *epistemic* access to *a priori* reasoning.

Thus, we have established that we do have grounds to assert that logical consequence has non-trivial metaphysical content. I still need to explain why that metaphysical nature should plausibly be seen as proof-theoretic. I argue that the past few paragraphs serve as a perfect explanation, in their association of metaphysical nature with *law-like* concepts. Implicitly, in my argument for the non-triviality of metaphysical nature for logical consequence, I have shown that this metaphysical nature is associated with some kind of law-like system of relationships. Proof-theory fits this description perfectly. It is a deductive system, based on rules that determine the relationship between premises and a conclusion. Therefore, given that we should not neglect the metaphysical nature of logical consequence, that metaphysical nature is plausibly proof-theoretic.

Overall, then, my proposal amounts to the idea that we never observe the underlying abstract proofs that exist, just like we never directly observe causal laws. We observe logical regularities that we take to be epistemically accessible instantiations of more abstract and general deductive relationships, just like we take physical observations to be epistemically accessible instantiations of more abstract and general causal laws. You might object to this by claiming that you *do* feel as if you reason using proof-theory, and that you *do* think you picture your knowledge in terms of proof theory. The response to this is clear: proof-theoretic reasoning is useful as a tool, just like reasoning in terms of general causal laws is useful for understanding. Nevertheless, we do not strictly have epistemic access to the underlying proof-theoretic relationships themselves, just like we do not strictly have epistemic access to causal laws.

5 Conclusions

It may seem as if the essence of logical consequence is model-theoretic, if we subscribe to a model-theoretic program when we come to know or believe consequence relationships. But it is possible to see logical consequence as metaphysically proof-theoretic even if we endorse a model-theoretic program for its epistemic nature. We can examine the potential for existence or dominance of fundamental a priori causal law as a basis for analogy. We have seen that on this issue, the historic rivalry between empirical versus *a priori* views is not in fact a strict rivalry. Limits on our epistemic access to abstract nature might apply, given a model-theoretic epistemic nature. But if such limits do exist, they seem to imply approximate epistemic representation of

some non-trivial exact metaphysical entity. Non-trivial abstract nature might still exist, and metaphysically dominate. And intuitively we understand logical consequence as a *fundamental* relation, from which cases should derive, not follow. I have proposed that we can adopt a hybrid view, one that will satisfy each of these appeals to analogy and intuition. By allowing for a distinction between epistemic and metaphysical nature, we can characterise logical consequence as epistemically model-theoretic but metaphysically proof-theoretic.

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