1962

Development of a simple bar method thermal conductivity test

V. J. Flanigan

Missouri University of Science and Technology

Department:

Recommended Citation

Flanigan, V. J., "Development of a simple bar method thermal conductivity test" (1962). *Masters Theses*. 2893.
https://scholarsmine.mst.edu/masters_theses/2893

DEVELOPMENT OF A SIMPLE BAR METHOD THERMAL CONDUCTIVITY TEST

BY

VIRGIL J. FLANIGAN

A

THESIS

submitted to the faculty of the

SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI

in partial fulfillment of the work required for the

Degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

Rolla, Missouri

1962

Approved by

Robert V. Wolf (Advisor) G. L. Scofield

Aaron J. Miles John A. Nelson

ABSTRACT

A linear flow bar method type apparatus was set up for finding thermal conductivities by comparison techniques. Comparison techniques consist of using a standard specimen where the thermal conductivity is known and comparing it to the unknown specimens to find their conductivity. Apparatus of this type usually uses thermocouples for the required temperature measurements, but thermistors were used in this experiment. The thermistors used required calibration to measure accurate temperatures and this was accomplished by using the known temperatures of the changes of state of some common materials.

Five alloys were used to evaluate the method: SAE 1020 steel, gray cast iron, 2S Aluminum, a zinc alloy and a magnesium alloy. The SAE 1020 steel was used as the standard and the conductivities of the other alloys were found by comparing to values taken for the steel.

ACKNOWLEDGEMENT

The author wishes to express his sincere appreciation to R. V. Wolf, Associate Professor of Metallurgical Engineering for his guidance and assistance during this study.

Thanks are also due to Mr. John Redman of Redman Manufacturing and Engineering Company who provided the major portion of the equipment for this project.

TABLE OF CONTENTS

Page

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS Cont'd.

Figure Page

LIST OF TABLES

INTRODUCTION

1. Statement of the Problem

The purpose of this experimental investigation was (1) to set up to measure thermal conductivity (2) to evaluate this apparatus using various alloys of known conductivity so that it will be available to be used for making thermal conductivity measurements of metal and alloy systems of interest.

2. Importance of the Study

The thermal conductivity of an alloy depends upon the chemical composition of the alloy, its crystalline structure, the temperature to which it is subjected, and whether or not it is a homogeneous material. The conductivity of many substances are sensitive to such a large number of effects such as impurities, anisotropic properties, and porosity that for a given material it is often necessary to resort to a direct experimental measurement of conductivity rather than reference to published data. The lack of published data on many alloy systems requires that actual measurements be made to determine conductivity.

REVIEW OF LITERATURE

A large number of methods have at one time or another been used for measuring thermal conductivity. Some of these must now be regarded as obsolete, but their theory remains of interest as they are based on solutions of the Fourier equation for simple systems which often occur in practice.

For an example of an obsolete method refer to Preston (1), Theory of Heat who describes the work of Igenhauss (2) in 1789 which demonstrated the difference in thermal conductivity of solid substances by placing bars through holes in a cabinet leaving a portion of the bar exposed to the atomsphere. The exposed portion of the bar was coated with wax and the cabinet filled with hot water. The wax melted according to the rate of heat transfer or thermal conductivity of each bar. Through measuring the different distances at which the wax melted the relative thermal conductivity could be found.

Of all the methods for measuring thermal conductivity the best theoretical method uses the spherical form. This method is performed by machining a spherical shell and placing a heating element inside the shell. If loose homogenous material is to be tested the shell can be filled with this material or if the material to be tested is a solid the sphere must be constructed of the sample. The main advantages of this method is that all the heat supplied must pass through the sample and by proper arrangement of the thermo-elements conductivity at different temperatures can be found.

Referring to Jakob (3) this method was first used by Peclet (4) in 1860 but the steady state was not considered in this experiment.

In 1909 Nusselt (5) succeeded in using this method for finding the thermal conductivities of some insulating materials. At about the same time that Nusselt was using this method Groeber (6) was using the sphere set-up to measure the thermal conductivity at very low temperatures. He accomplished this by submerging the sphere in a bath of liquid air.

Except for the spherical method all other methods for measuring thermal conductivity have the problem of heat loss other then through the sample. To avoid this loss almost all modern apparatus adapt some type of guard apparatus to restrict the flow of heat through the sample under investigation. Berget (7) was the first one to use the guard ring to restrict the heat loss in his study of Mercury. This method wasn't really adopted until Poensgen (8) used it for his apparatus in 1912. This apparatus was called a guarded hot plate and consisted of a flat resistance heater sandwiched between two similar flat slabs of the material of interest. Water coils were attached to the other side of the specimens to act as a heat sink. Another resistance heater enclosed the center one and was separately wired so as to allow for temperature balancing by adjusting the power to the heater. Thermo-elements were attached to the outer face of the specimens, the heating element and the guard ring, by measuring the temperature across the plate and knowing the power input with the use of a wattmeter the thermal conductivity could be found. This method is particularily desirable for measuring the conductivity of insulating materials. The modern version of the guarded hot plate resembles Poensgen's (8) method except for many added extras such as a photo electric cell for balancing the guard heaters and the center heating element. Guarded hot plates have been constructed of almost all sizes. In one form of this apparatus Griffiths (9) used specimens up to a foot in thickness and required a hot plate 3 by 3 feet with a similiarily heated guard ring one foot wide and separated by a narrow air gap. The specimens are 5 feet by 5 feet. Griffiths and Kaye (10) also used an apparatus where the specimen is 45 mm in diameter and .5 mm thick. Because of the size of this apparatus a guard ring was not required.

Christiansen (12) used the guarded hot plate for a comparison method. A plate of the specimen and one of the standard were placed together with thin sheets of copper separating them and on the exposed surfaces. Thermo-elements were embedded in the copper plates and the temperature gradients read. The thermal conductivity is inversely proportional to the two temperature gradients read.

Jakob (3) proposed another method of guarding the heating element by only using one specimen and enclosing it inside a copper cylinder filled with gas. He could balance the temperature of the gas and the heating element and restrict heat flow.

This guarded hot plate method is recommended by American Society for Testing Metals (12) for measuring the thermal conductivity of poor conductors and insulators.

Another common method for measuring thermal conductivity of solid substances is the cylindrical arrangement and seems to have been used first by Niven (13) in 1905 and Clement and Egy (14) in 1909. The material, usually insulation, to be investigated may be held between two concentric tubes and the temperature difference measured between the tubes. There are many ways to guard against the heat loss at the ends for this type of apparatus. One of the ways is to use a very long tube and use only the very center section for temperature measurements. If the design is restricted to rather small lengths thermocouples may be placed along the tube to evaluate the end loss or electrically heated guard coils may be placed at the end of the tube and by equalizing temperature heat flow can be prevented.

Ingersol (15) in his methods of measuring thermal conductivity refers to the work of Kohlrausch (16) who developed a method of determining the ratio of electrical and thermal conductivity. A constant electrical current was sent through the bar whose ends were kept at the same constant temperature by connection with a water bath. From measurements by thermo-elements of the temperature difference between the middle of the rod and two points equidistant on each side, and also of the potential difference between the two outer points, the ratio of electrical and thermal conductivity could be calculated.

Missner (17) also referred to by Ingersol (15) used an application of this method to find conductivities of various materials at 20°C.

The last important method of measuring thermal conductivity is the linear flow bar method. Gray (18) in 1894 was one of the first men to work with this method. He used a 4 to 8 cm long and 2 to 4 mm. diameter bar one end in a copper hot water bath, and the other end screwed into a copper sphere. The sphere served as a calorimeter to measure the heat flow. To prevent excessive heat loss through the bar it was wrapped with tape type insulation. Thermo-elements were embedded in the bar at regular intervals to measure the gradient along the bar.

By enclosing the same type of apparatus in a Dewar Flask, Lee (19) measured thermal conductivity of a number of pure metals and alloys down to liquid air temperatures. Ingersol (15) referred to the work of

Koenigsberger and Weiss (20) who applied the comparison method to this type of test. They compared graphite, silicon etc. to iron by soldering end to end the specimen to the standard and placing thermo-elements in the bar measured the gradient of the total bar. Then the inverse ratio of the temperature gradients will give the ratio of the thermal conductivity.

Modern apparatus of this type has changed from using calorimeters for measuring the heat flow and started using resistance type heating with guard ring heaters and a wattmeter to measure power input. The main advantage of this type of test is that the conductivity can be found at different temperatures with only one run. Another use of this method is for measuring the conductivity of very thin sheets such as mica. Griffiths and Kaye (21) used a copper bar with a space in the middle for the insertion of the specimen. The gradient was measured along the bar as before and by observing the discontinuity the thermal conductivity could be found.

More recent work with the bar method has been in the investigation of alloy systems and their thermal conductivity. Deem (22) worked with Zirconium-tin alloys and plotted the variation of the thermal conductivity with the tin content.

DISCUSSION

1. Description of Apparatus

The entire apparatus is as shown in Figure 1 which includes the thermal conductivity tester, the rheostats for adjusting supply voltage, the Simpson volt-ohm microammeter, and the watt hour meter.

The thermal conductivity tester as shown in Figure 1 was developed and manufactured by Designs for Tomorrow Inc., St. Louis. It is of the linear flow bar method type similiar to that used by R. H. Deem (22) in his studies of the Zirconmium Tin Alloy systems.

A cross section of the apparatus with test specimen in place is shown in Figure 2. The test specimen as shown in Figure 7 is butted up against the heater block Figure 6 and the other end of the specimen is forced into the O-ring seal of the cooling chamber through which tap water is circulated. A thin coating of Dow corning 7 compound, a silicone grease, was placed between the heater block and the specimen to insure good contact. The heat source, a 50 watt cartridge type heater was pressed into the reamed hole provided in the heater block. To insure linear flow the heater block is surrounded by six 50 watt cartridge type heaters inserted in equally spaced holes of the guard ring Figure 3. Thermistors inserts were drilled in the guard ring and the heater block to provide for equalization of the temperatures by adjustment of the rheostats.

The guard ring was cast into the shell of the guard ring heater by positioning it and pouring expanded foam around the guard ring. Two terminal strips were attached to the guard ring shell and thermistors and heater wiring attached to them as shown in Figure 8 and 9 respectively.

The outer shell of the specimen guard ring was prepared from a brass cylinder split longitudinally into two halves and hinged at one side with a magnetic catch at the other for the convenience of inserting specimens and the thermistors leads. Rings, Figure 3, for the assembly were machined from aluminum and used to insure no heat transfer in the longitudinal direction. The average temperature of a ring should be approximately the same as the temperature of the specimen adjacent to the center of each ring. With the air space between each ring there is no possibility of the guard ring being at a higher temperature than the specimen and heat being transferred to the specimen from the guard ring.

The rings were fastened together with epoxy dipped paper and centered in the specimen guard ring shell. Expanded foam was cast around the ring assembly and foam was used to fill the shell. The completed assembly was mounted on the base plate with the cooling head, Figure 5. The cooling head was so designed so a jet of water would impinge upon the butt of the specimen and outlets were larger than the inlet so to remove the cooling water with a very low level maintained in the reservoir. Support posts were also attached to the base plate to act as a slider for the heater block guard ring assembly as shown in Figure 2. The posts were also used for a support for the terminal strip to which the thermistors were wired.

The specimens were prepared by drilling 1/16 inch diameter holes at one inch intervals as shown in Figure 7 for the insertion of the thermistors. All thermistors used were Fenwall bead type GB32J2 and were connected to the terminal board with fine copper wire insulated by dipping in armature lacquer.

The thermal conductivity tester was then bolted to a wooden holding fixture, Figure 1. In which the switch for the thermistors was inserted and wired as shown in Figure 8. A receptacle was also mounted on the back of the fixture for connecting the heating elements to the powerstats and to allow for the removal of the heater block guard ring assembly.

The Simpson ultra high sensitivity volt-ohm-micrometer was used for both the equalizing of the heater block and the heater block guard ring temperatures and the measuring of the temperature gradient along the bar.

2. Calibration

Thermistors are thermal resistors with a high negative temperature coefficient of resistance. As the temperature increases resistance decreases as the temperature decreases the resistance increases; just the opposite of the effect of temperature on metals. Thermistors are semiconductors of ceramic material made by sintering mixtures of metallic oxide such as manganese, nickel, cobalt, iron, and uranium. Various mixtures of these metallic oxides are formed into useful shapes. Their electrical characteristics may be controlled by varying the type of oxide used and the physical size and configuration of the thermistor. Standard forms available are beads, probes, discs, washers, and rods. The bead type was used in this experiment because of their small size. The beads are made by forming small ellipsoids of thermistor material on two fine wires parallel to one another and about 0.010 inches apart. The material is then sintered at a high temperature and the leads become tightly embedded. The thermistors are then coated with a fine glass coating, or mounted in gas filled bulbs. The thermistor

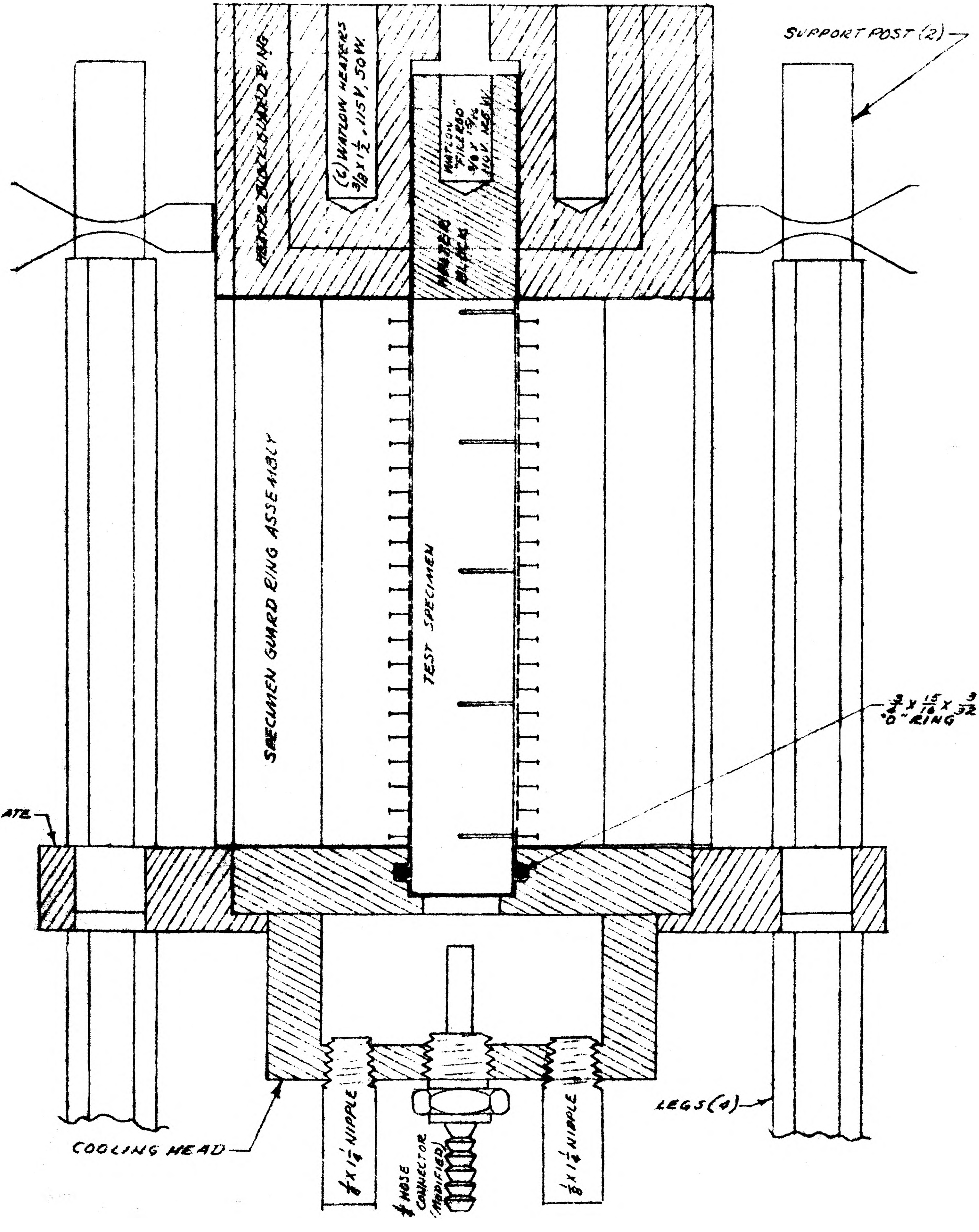
Thermal Conductivity Tester and Auxiliary Equipment

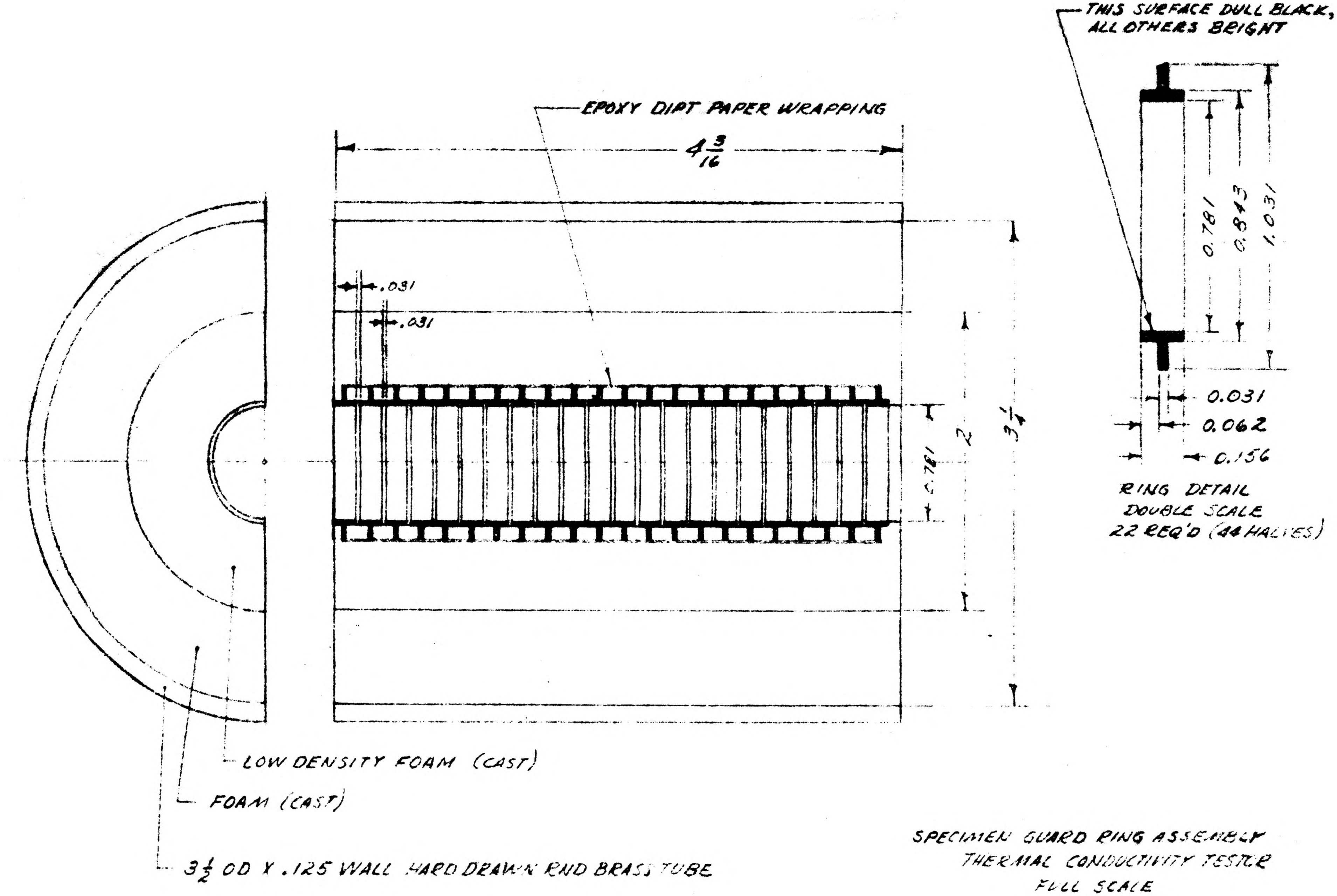
Figure 1

ASSEMBLY
THERMAL CONDUCTIVITY TESTOR
FULL SCALE
SHEET ① OF ○

DESIGNS FOR TOMORROW INC.
ST. LOUIS, MO.

Figure 2

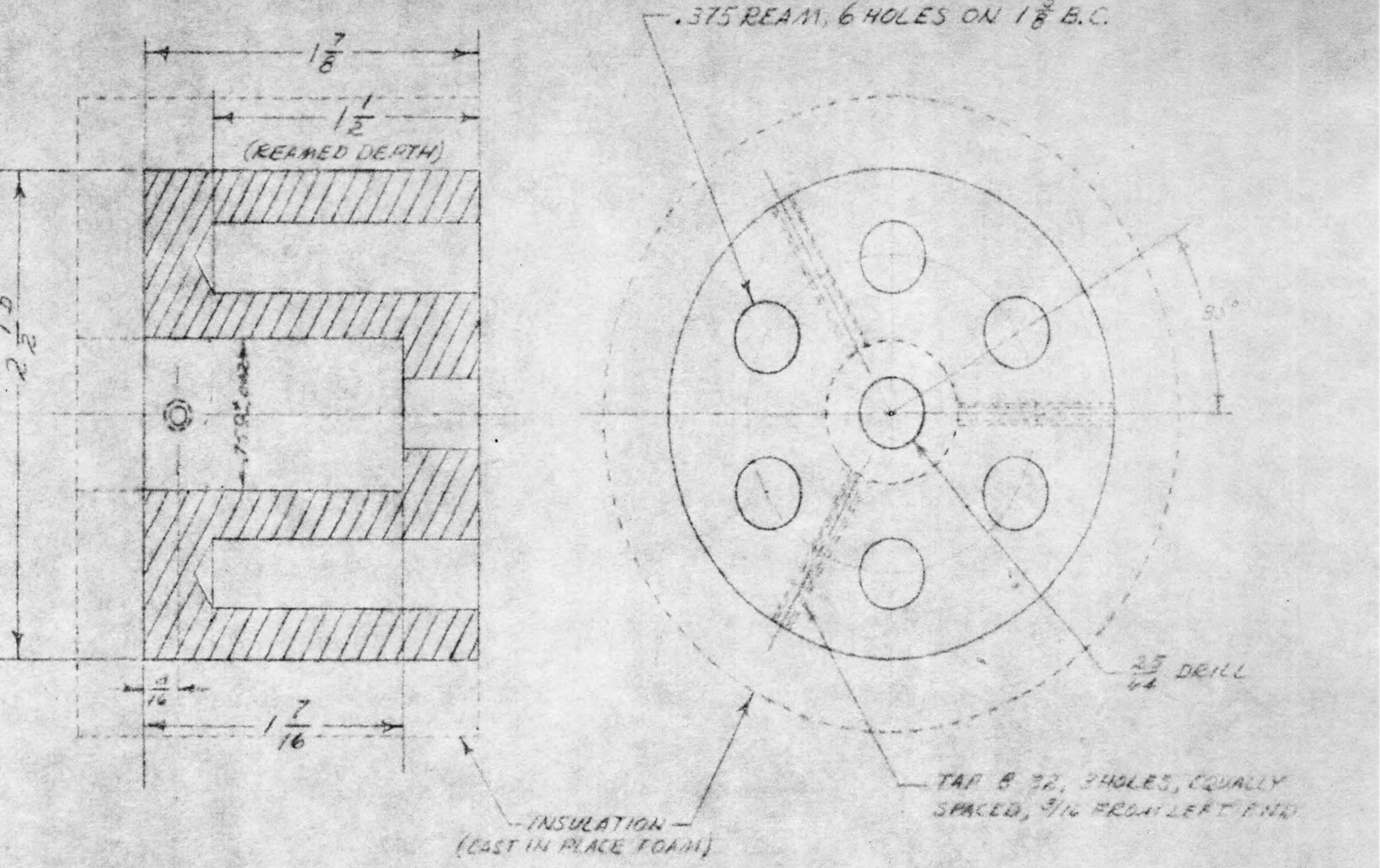




SPECIMEN GUARD RING ASSEMBLY
THERMAL CONDUCTIVITY TESTER
FULL SCALE
SHEET ⑤ OF ○

DESIGNS FOR TOMORROW, INC.
ST LOUIS, MO.

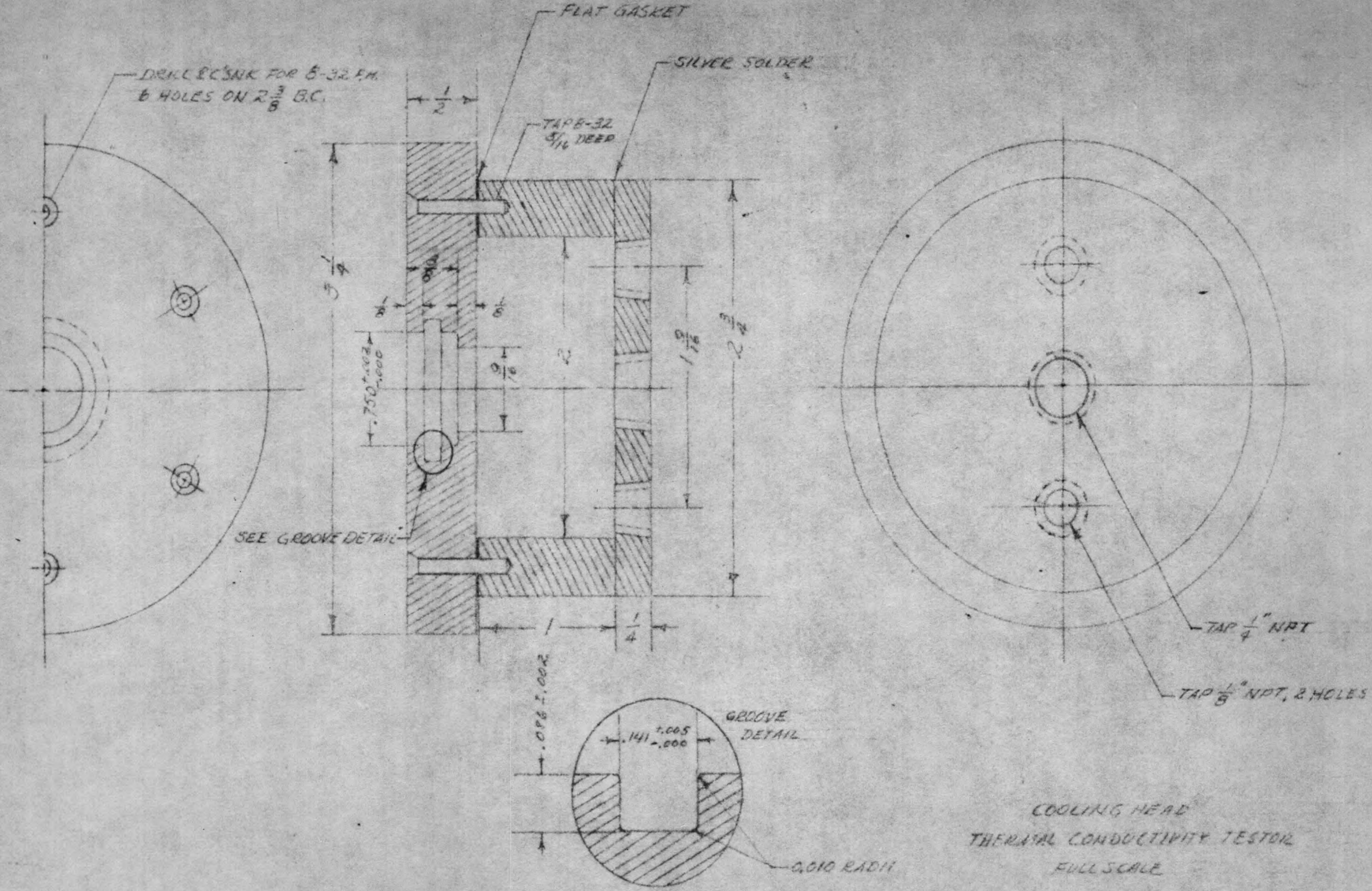
Figure 3



MAT'L: 2024 AL. ROD OR SUBSTITUTE

HEATER BLOCK GUARD RING
THERMAL CONDUCTIVITY TESTOR
FULL SCALE
SHEET ④ OF ○

DESIGNS FOR TOMORROW, INC.
ST. LOUIS, MO.
Figure 4

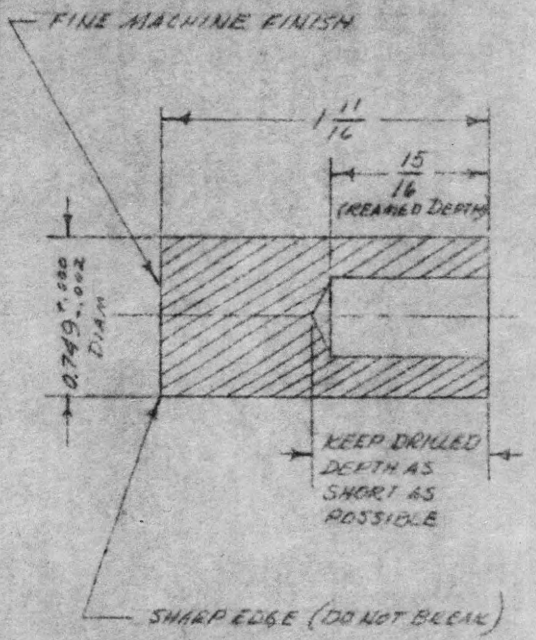


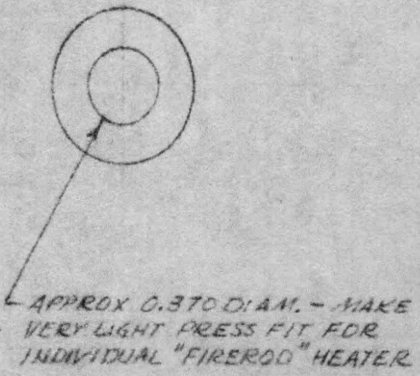
MAT'L: (EXCEPT SCREWS & GASKET)
FREE CUTTING YEL. BRASS

COOLING HEAD
THERMAL CONDUCTIVITY TESTOR
FULL SCALE

SHEET ② OF ○

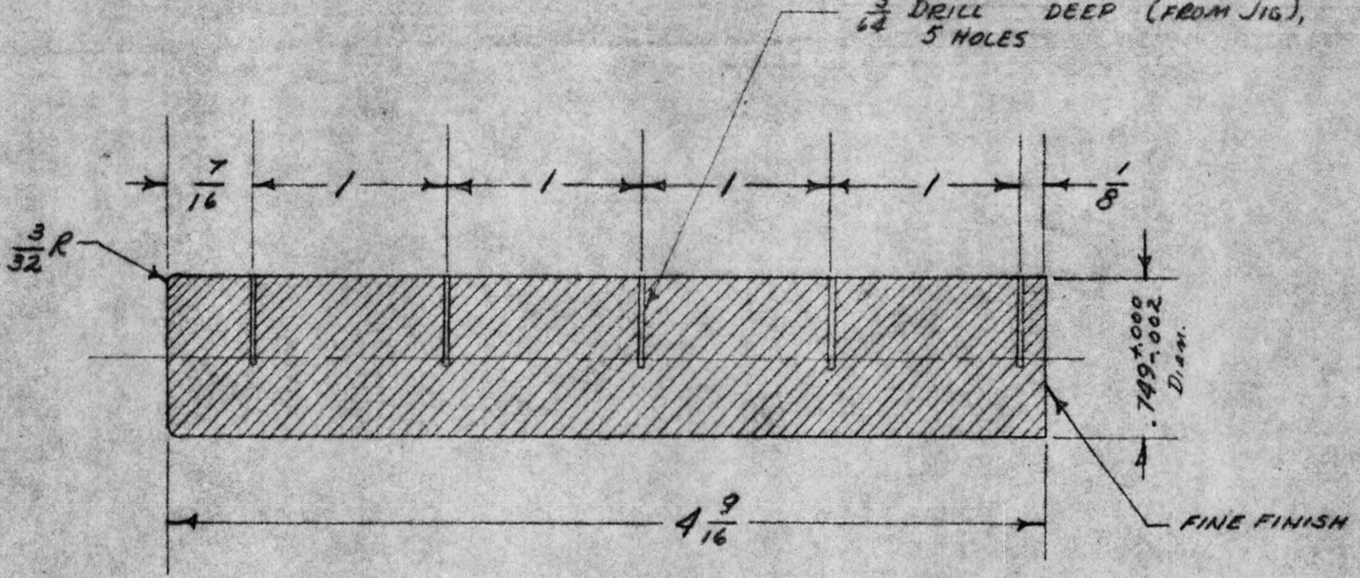
DESIGNS FOR TOMORROW, INC
ST. LOUIS MO
Figure 5





MAT'L. $\frac{13}{16}$ DIAM COPPER ROD, HARD DRAWN, LEADED

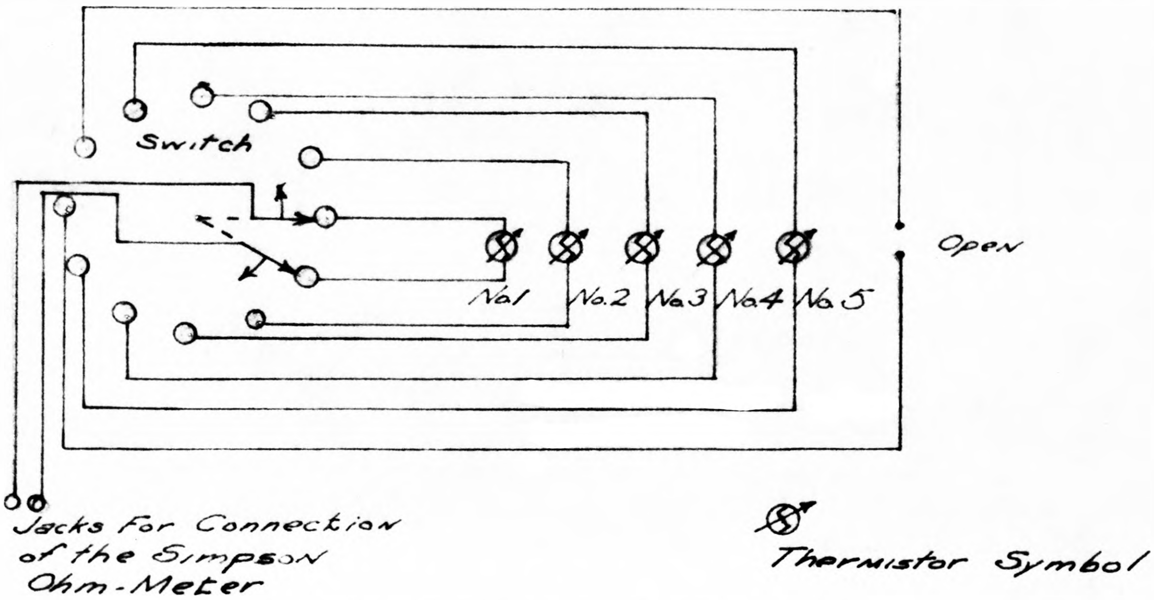
HEATER BLOCK
THERMAL CONDUCTIVITY TESTOR
FULL SCALE
SHEET ③ OF ◯
DESIGNS FOR TOMORROW, INC.
ST. LOUIS, MO.
Figure 6



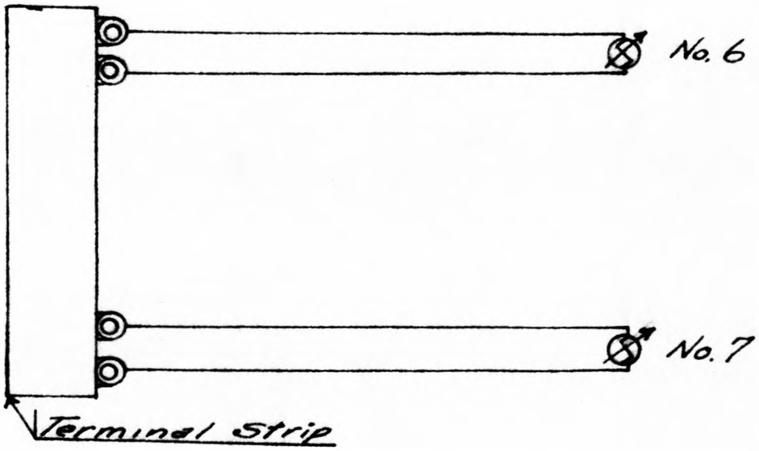
HEAT TRANSFER TEST SPECIMEN

REDMAN TOOL & DIE CO.

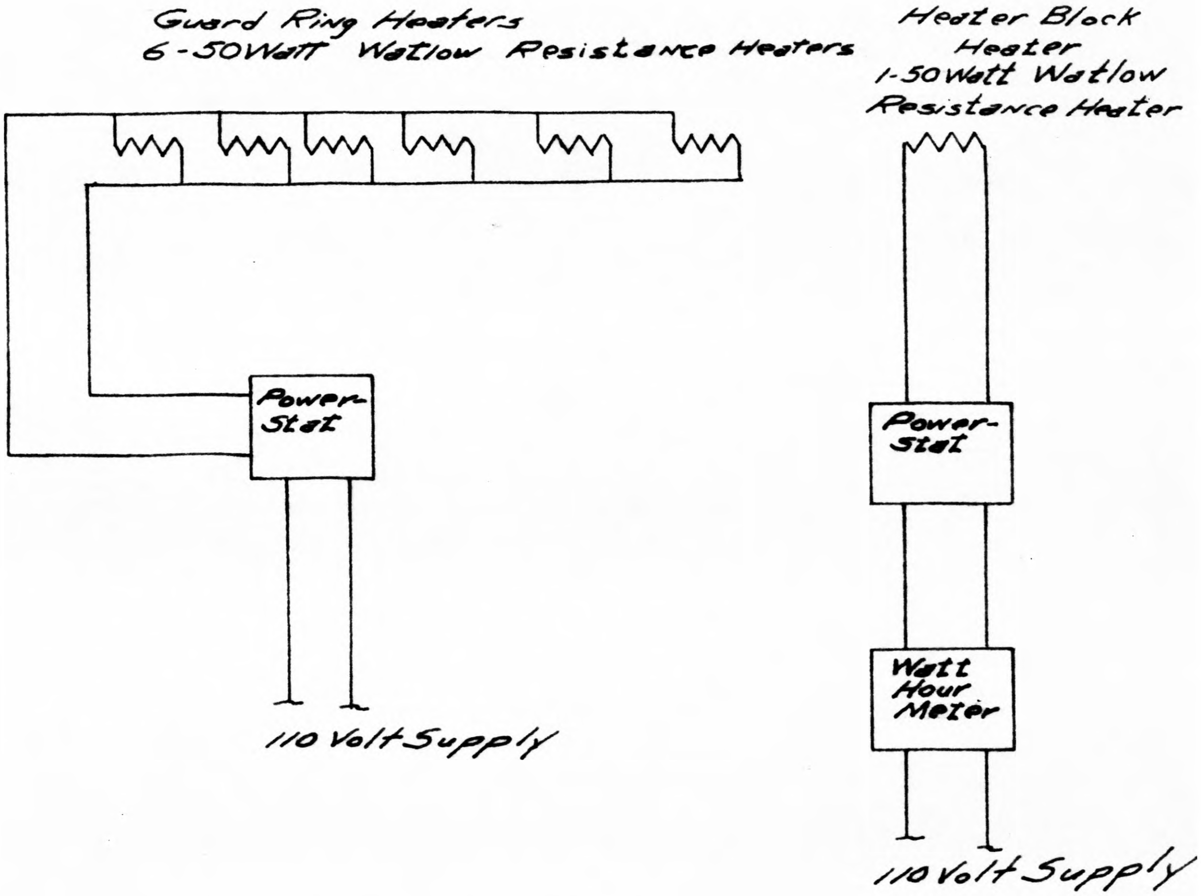
Figure 7



Wiring Diagram For Specimen Thermistors
Fig. 8



Wiring Diagram For Guard Ring Thermistors
Fig. 9



Wiring Diagram For Heating Elements For The Guard Ring Assembly

Fig. 10

used in the experiment were Fenwalls GB32J2 which are glass coated.

Calibration of these thermistors was required because of the large differences in the resistance readings of each thermistor at the same temperature. These errors are caused by small traces of impurities in the thermistors and discontinuities in the wiring. The calibration also takes into account the resistances added by the switch and soldered connections, because the resistance were measured exactly the same way in calibration as in the actual runs.

Five substances were used to find the known values of temperatures. They were melting ice, boiling water, sliver nitrate, napthalene, and napthol (pure beta).

The set-up used for this calibration is shown in Figure 11. A parting cup was used to hold the substance under consideration, a fisher burner for the heat source, the glass tube for insertion of the thermistor to prevent the chemical from attacking the leads, ring stand for support, large block and hollow cylinder of aluminum for a heat sink and the rest of the equipment as described in the description of the apparatus.

The procedure for the silver nitrate and napthalene was to place them in the parting cup and apply heat until they changed state from a solid to liquid. The heat was then removed and the substance allowed to cool, readings of resistance from the Simpson meter were taken every 30 seconds until another change of state was realized. This data was then plotted time versus resistance readings and by observing the float region of the curve the melting point was determined. The high vapor pressure of napthol (pure beta) required that heating curves and not cooling curves be run. So instead of heating to above the melting point and allowing to cool the data was taken as the heat was being applied, up to and through the melting point. The runs using boiling water only required that the water be brought to a boil and taking readings at this condition, but taking several readings to see if there was any noticeable variations. The ice was used in the same way as the boiling water except that no heat was supplied. It was not required to use ice in the calibration of heater block and heater guard ring thermistors because this low temperature was not of interest in this case. This procedure was repeated for each thermistor separately.

Sample curves of the data taken and the curves plotted for thermistor number 3 are shown in Table I and Figures 12 through 14.

In the choice of these substances care was taken to get distilled water and reagent grade chemicals because of the errors caused by impurities in the melting and boiling points. Care was also taken to keep these substances as pure as possible during their use.

From the resistance-time curves and values of resistance for boiling and melting water definite values of resistance were related to definite temperatures for each thermistor Table II. These values were plotted on semi-log paper and curves of the same shape as the standard for these thermistors, Figure 15 was drawn through each point. The curves are shown in Figures 16 through 21.

3. Method of Test

Five samples were used as shown in Figure 22 from left to right respectively: Zinc Alloy, 1125-QQM-44 Magnesium, class 50 gray cast iron, SAE 1020 steel, and 2S Aluminum. The 1020 steel was chosen as the sample because of its low conductivity compared to the other alloys.

TABLE I

RESISTANCE VALUES FROM CALIBRATION RUNS FOR THERMISTOR No. 3

Melting Ice Melting Point 32°F	Napthalene Melting Point 176.4°F	Napthol Pure Beta Melting Point 249.5°F
4,350	266	89
4,350	276	91
4,350	281	93
4,350	282	94
4,350	283	95
4,350	283	95
	284	95
	286	95
	288	95
	291	95
	296	95
		95
		95
		95
		97
		97
		98
		99
		99.5
		101

All Values in ohms.

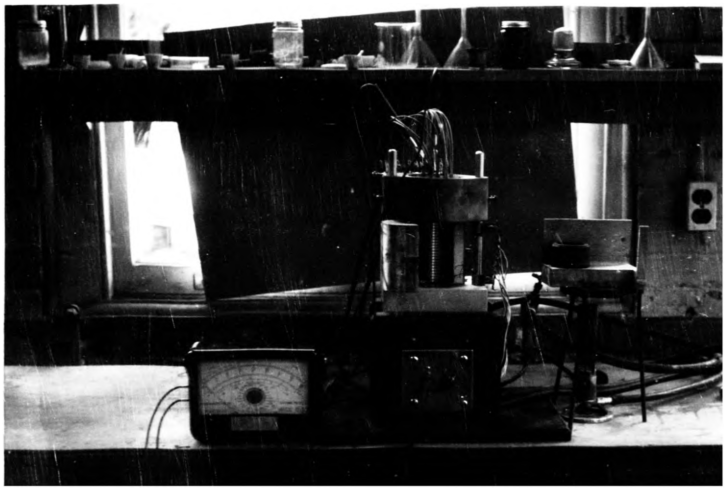
TABLE I Cont'd.

Boiling Water Boiling Point 212°F	Silver Nitrate Melting Point 413.6°F
154	12.4
154	12.9
154	13.4
154	14.0
154	14.6
154	15.1
	15.6
	15.8
	15.8
	15.9

TABLE II

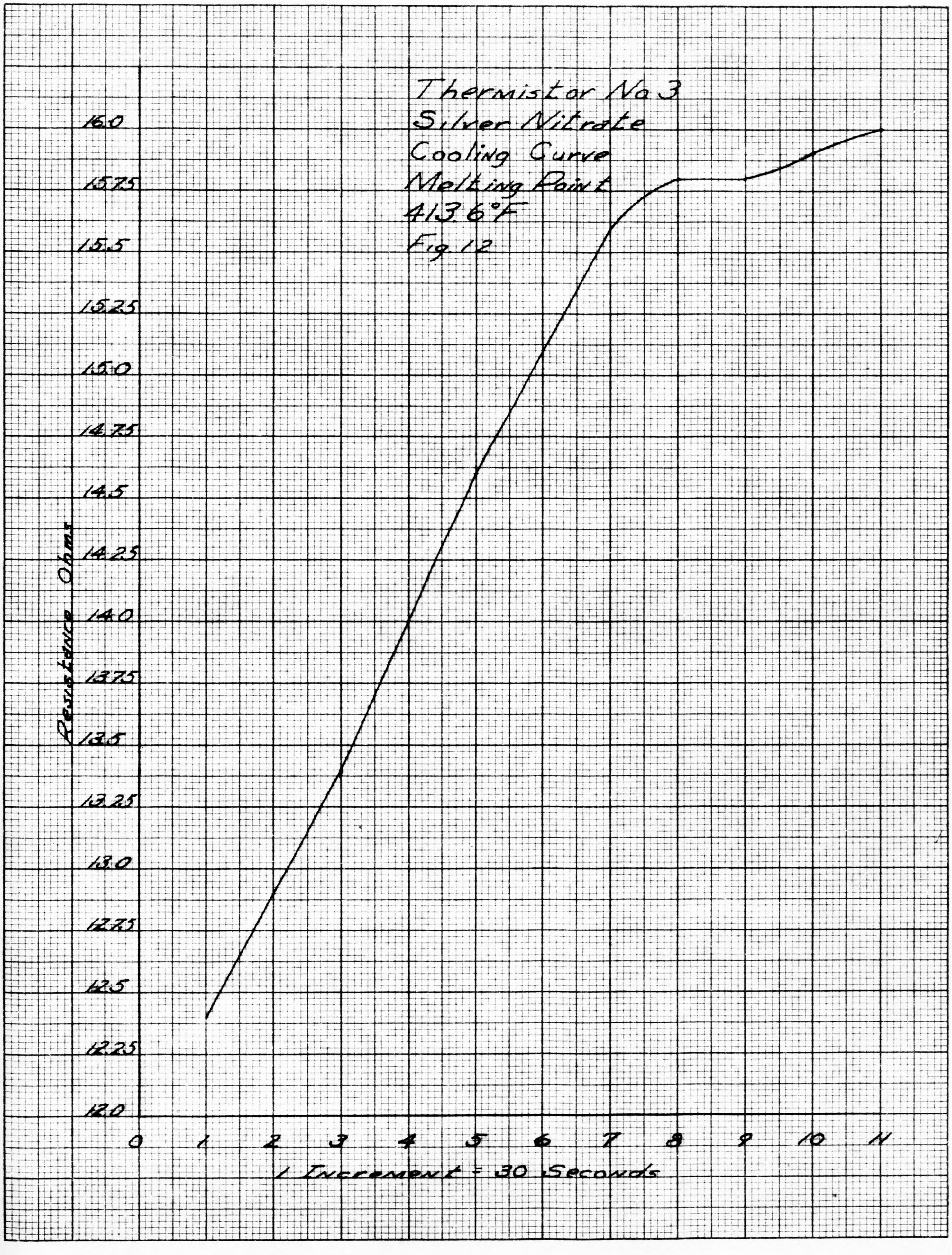
FIXED TEMPERATURES CORRESPONDING TO RESISTANCE FOR CALIBRATION

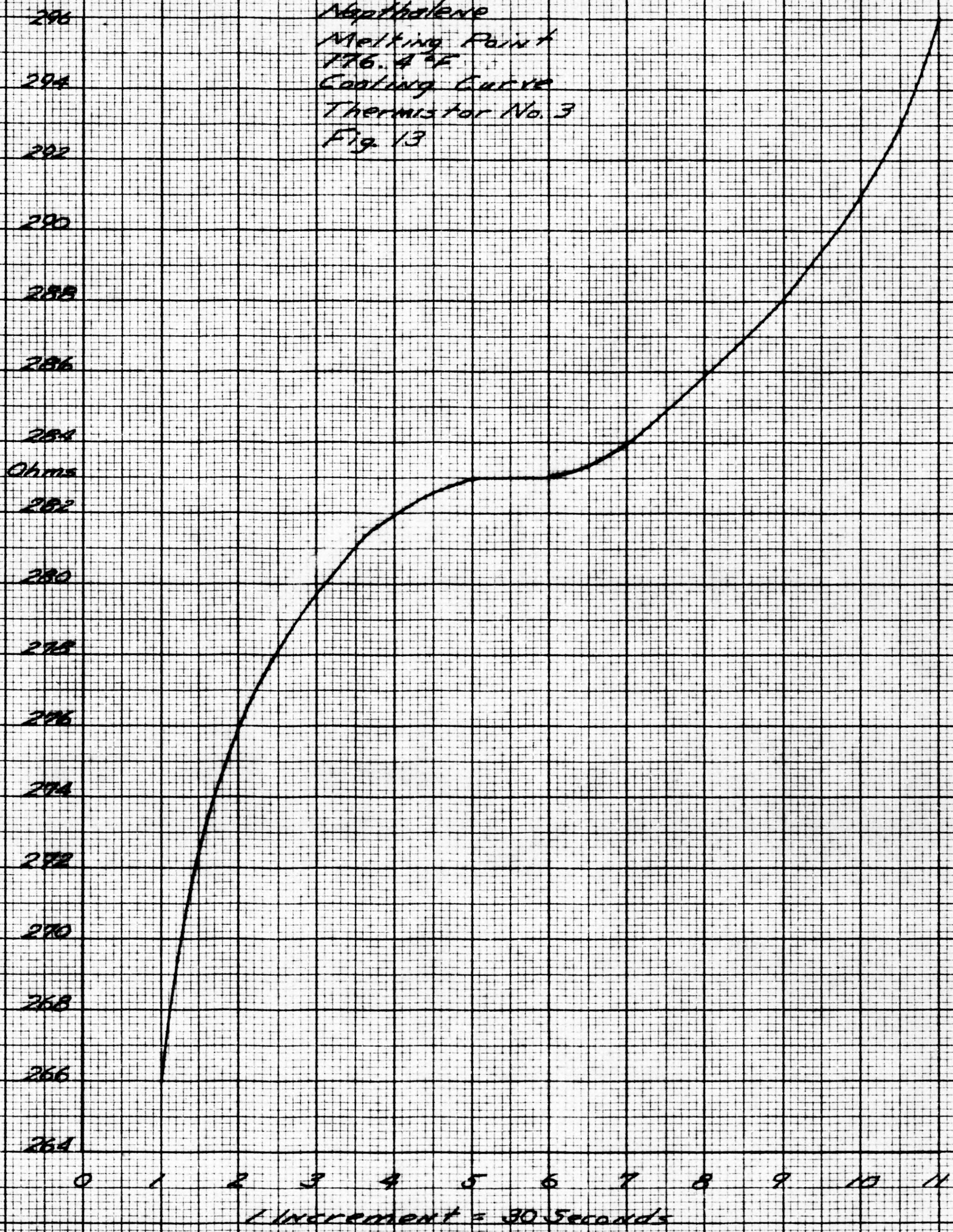
	Ice	Napthalene	Napthol	Water	Silver Nitrate
Therm. 1	14,000	1220	440	760	60.5
Therm. 2	3,970	258	85	153	15.5
Therm. 3	4,350	283	95	154	15.5
Therm. 4	6,000	350	89	220	16.5
Therm. 5	10,000	750	230	550	18.2
Therm. 6		240	96	185	28
Therm. 7		230	86	174	20

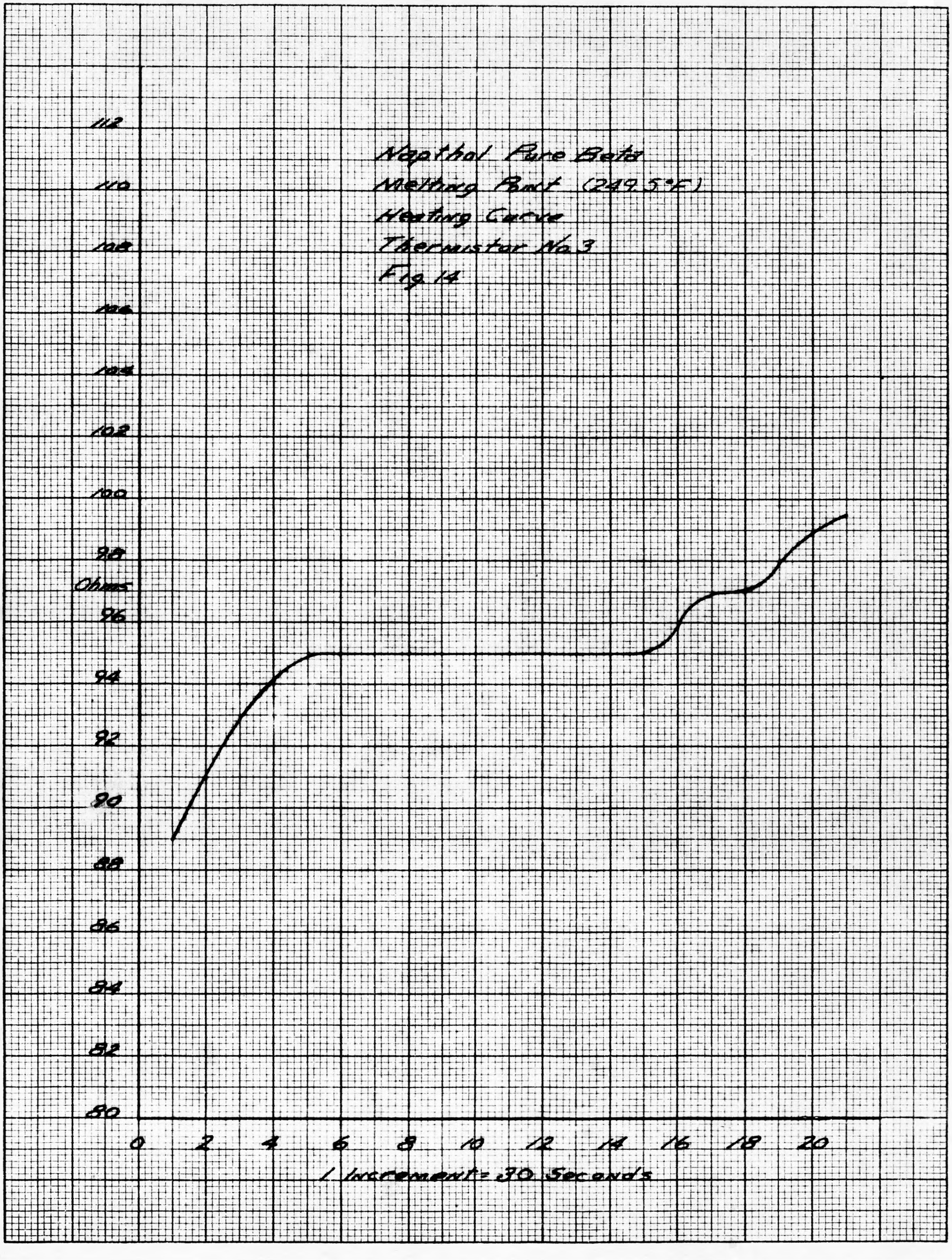


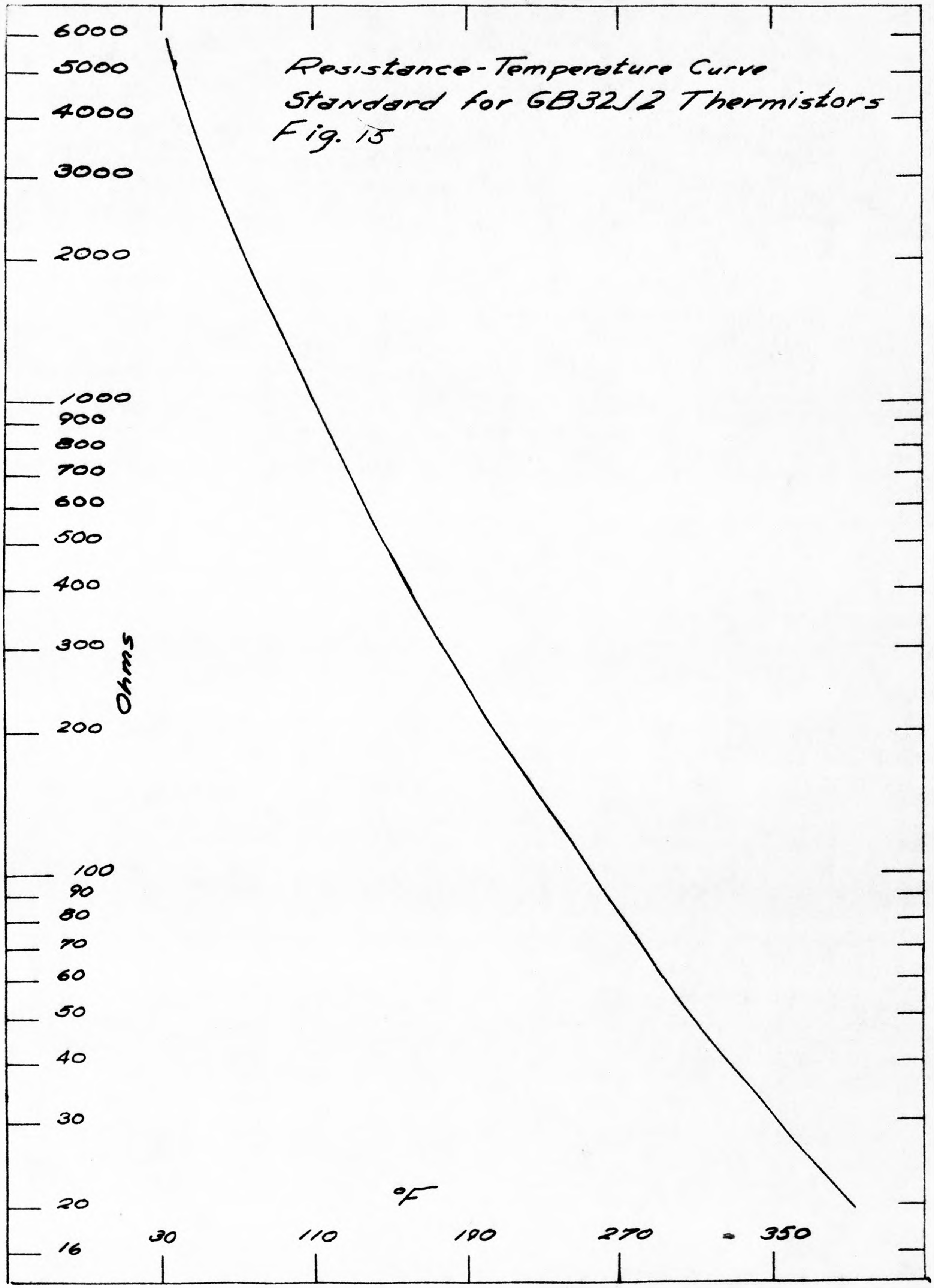
Calibration Apparatus

Figure 11

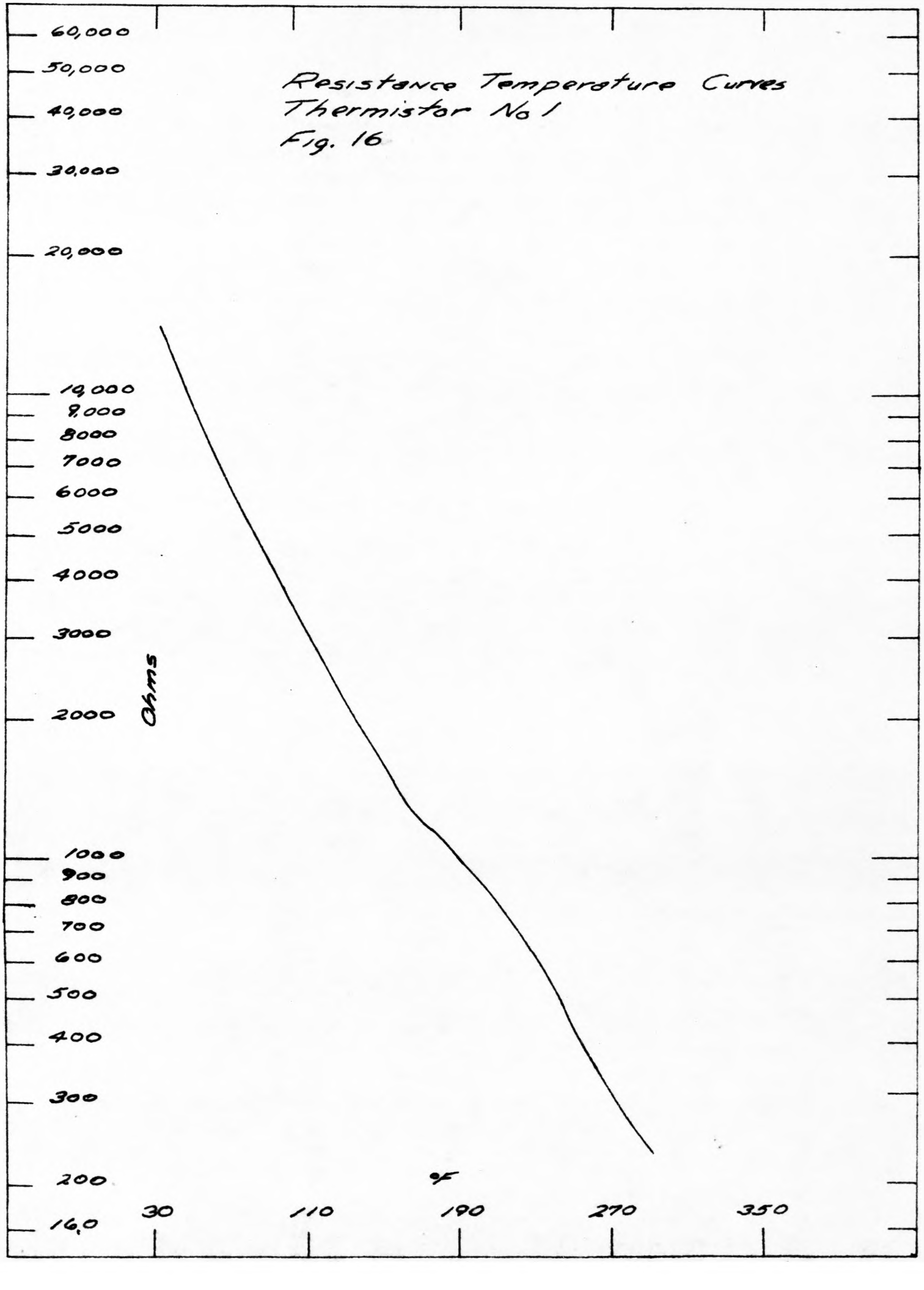

Thermistor No 3
Silver Nitrate
Cooling Curve
Melting Point
413.6°F
Fig. 12
Resistance Ohms
16.0
15.75
15.5
15.25
15.0
14.75
14.5
14.25
14.0
13.75
13.5
13.25
13.0
12.75
12.5
12.25
12.0
0
1
2
3
4
5
6
7
8
9
10
11
1 Increment = 30 Seconds


Napthalene
Melting Point
176.4 °F
Cooling Curve
Thermistor No. 3
Fig. 13
296
294
292
290
288
286
284
Ohms
282
280
278
276
274
272
270
268
266
264
0
1
2
3
4
5
6
7
8
9
10
11
1 Increment = 30 Seconds

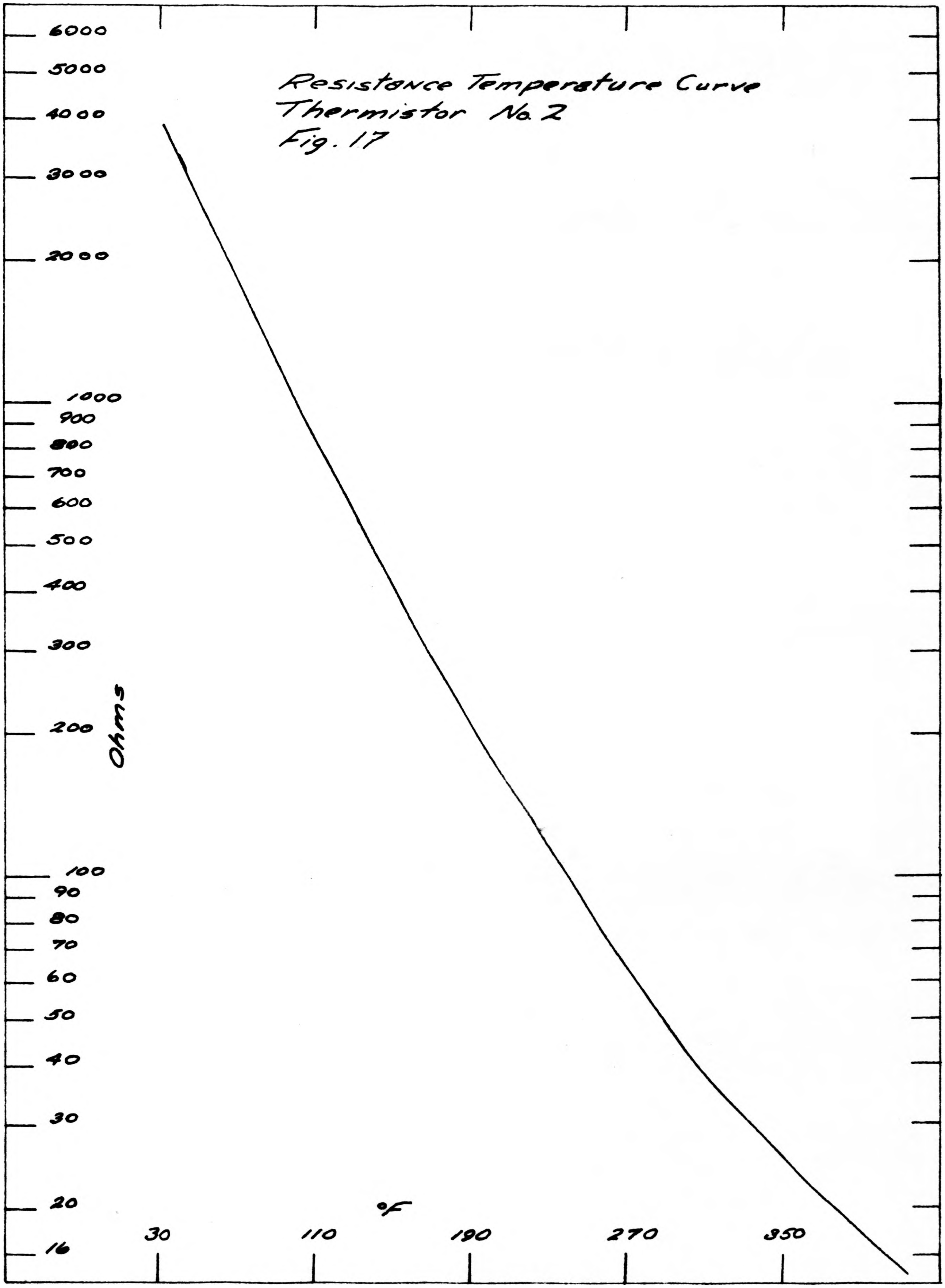

Napthol Pure Beta
Melting Point (249.5°F)
Heating Curve
Thermistor No 3
Fig 14

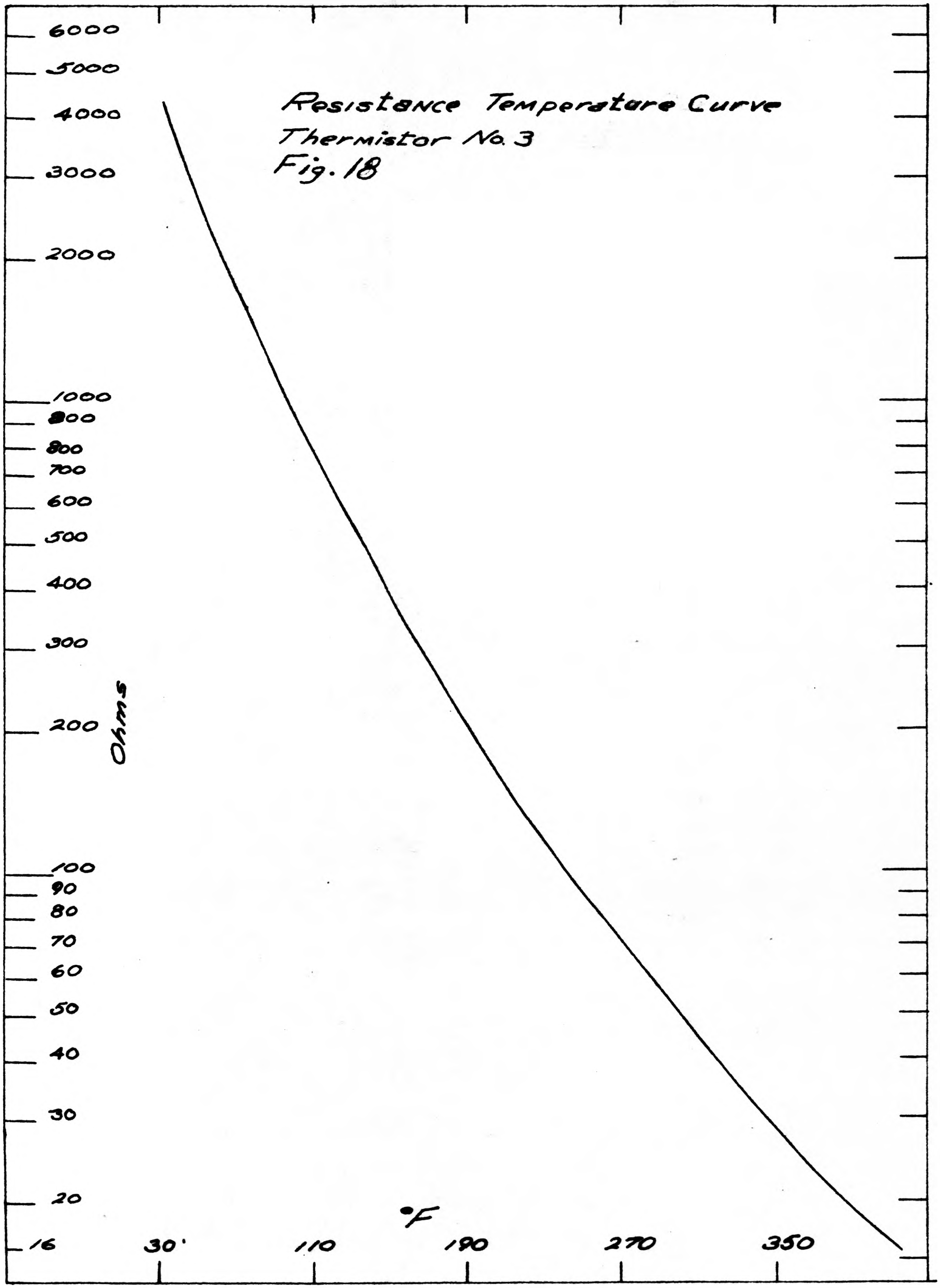


Resistance-Temperature Curve
Standard for GB32J2 Thermistors
Fig. 15

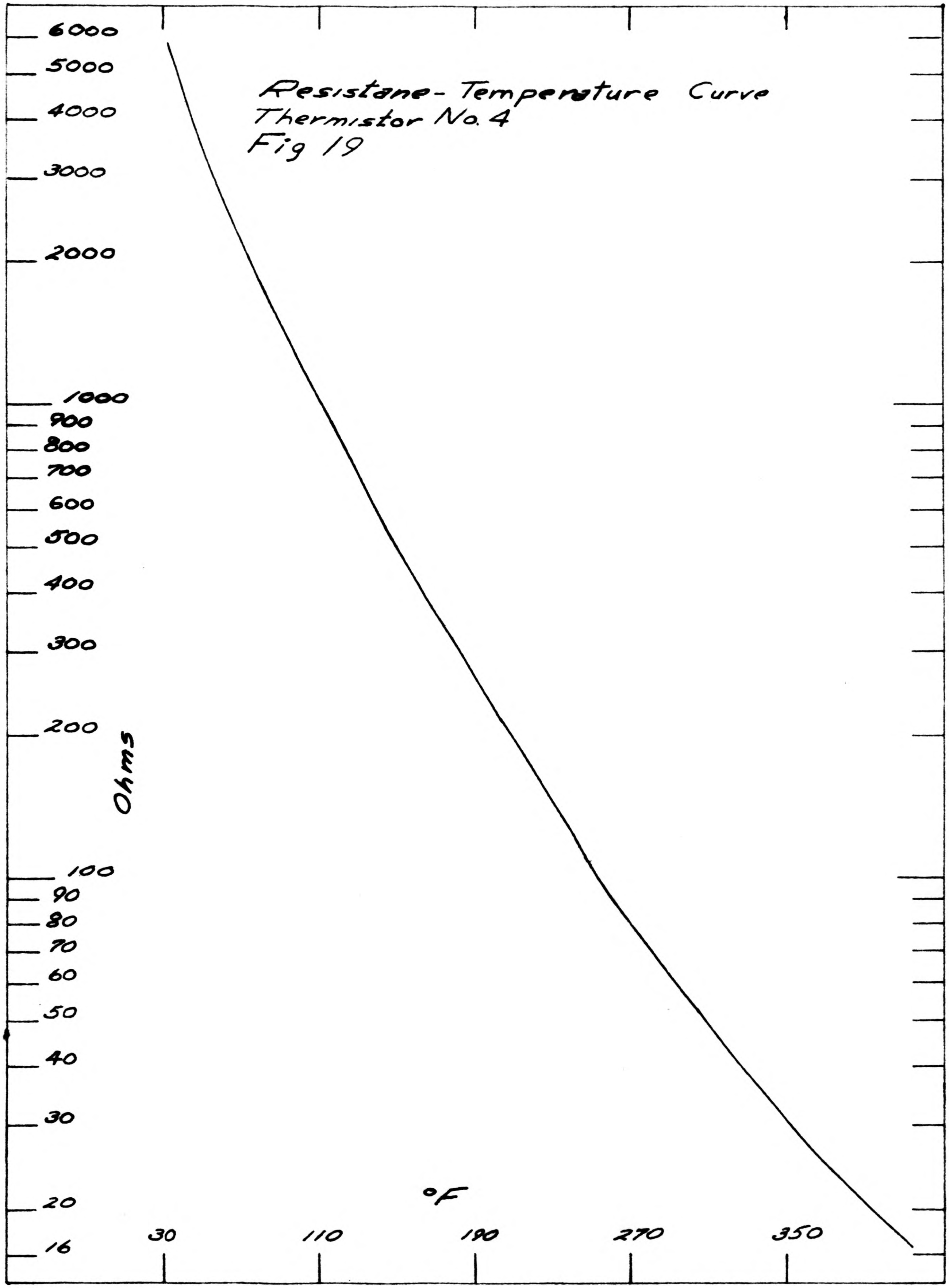


Resistance Temperature Curves
Thermistor No 1
Fig. 16

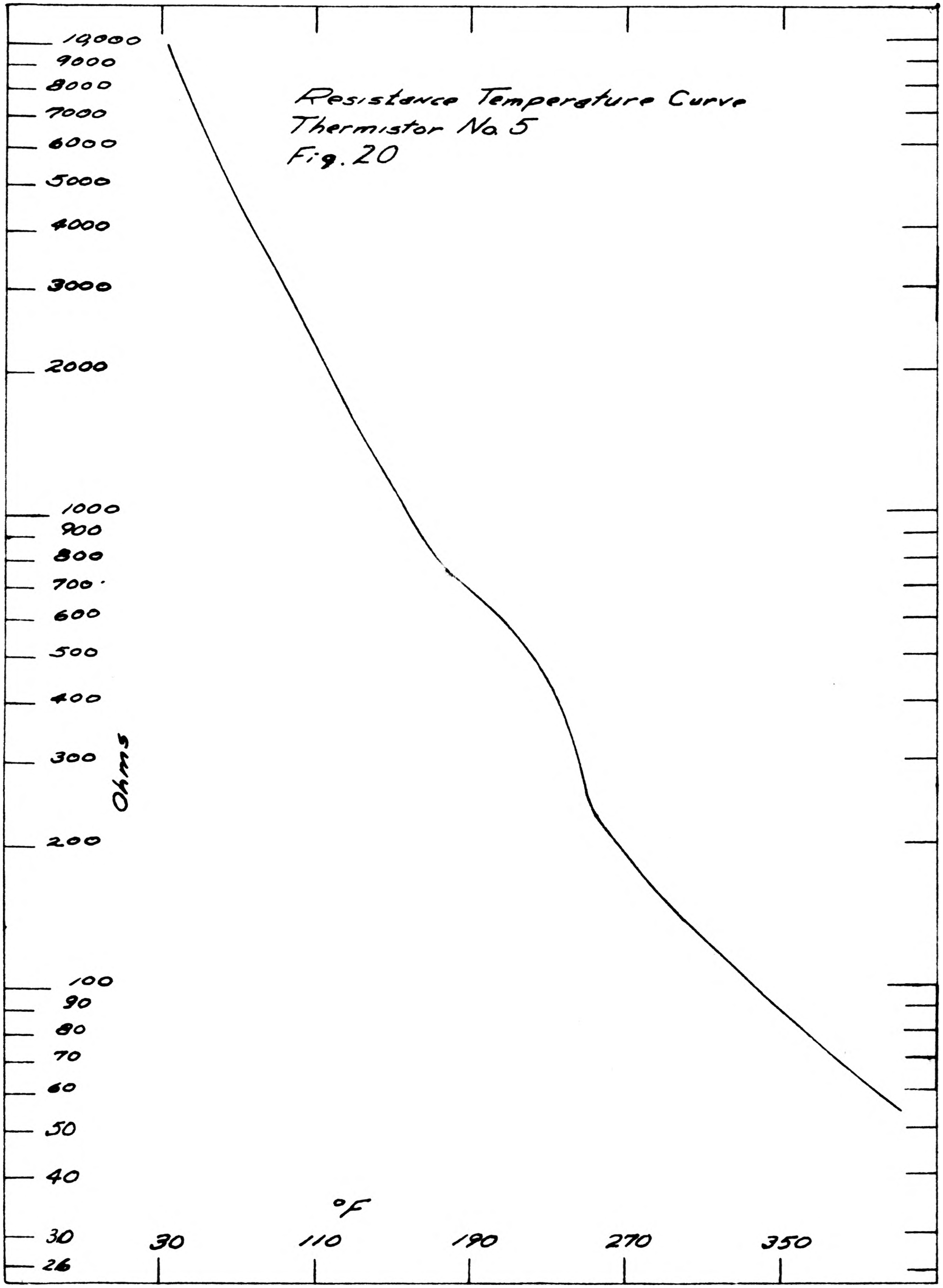

Resistance Temperature Curve
Thermistor No. 2
Fig. 17
6000
5000
4000
3000
2000
1000
900
800
700
600
500
400
300
200
Ohms
100
90
80
70
60
50
40
30
20
16
°F
30
110
190
270
350

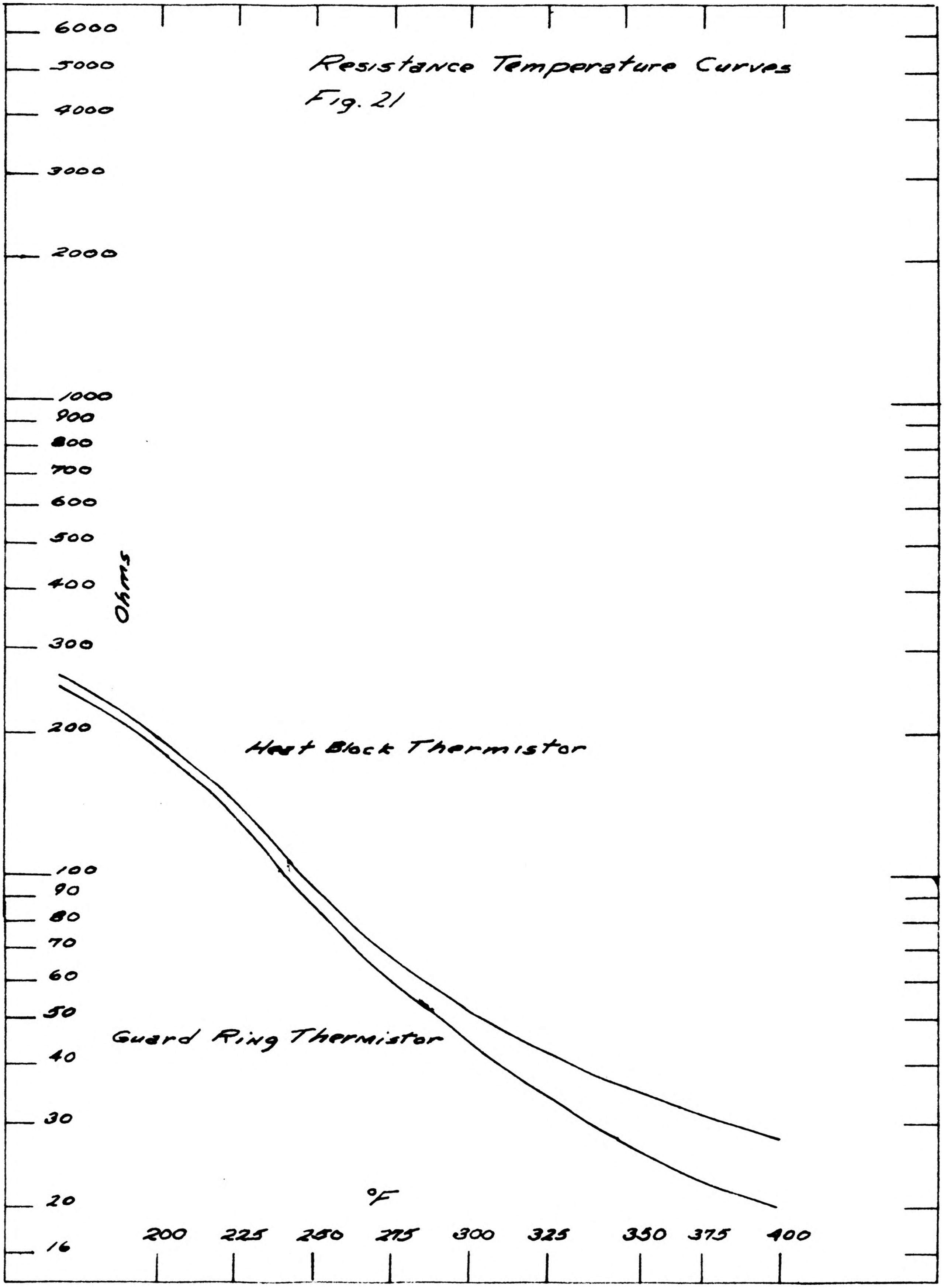


Resistance Temperature Curve
Thermistor No. 3
Fig. 18



Resistane- Temperature Curve
Thermistor No. 4
Fig 19


Resistance Temperature Curve
Thermistor No. 5
Fig. 20
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
26
Ohms
°F
30
110
190
270
350



Resistance Temperature Curves

Fig. 21

The lowest conductivity was used as the standard because it was desired to run the standard first. The apparatus is restricted to a temperature of about 450°F and if one of the high conductivity specimen was chosen as the standard and run around 400°F, it would have put the steel and cast iron out of the safe operating range.

The standard was placed in the thermal conductivity tester and thermistors inserted into the wells made for them. To insure good contact the Dow Corning 7 Compound was placed on the thermistors and the heated end of the specimen.

The heater block guard ring assembly was clamped in place and the power and cooling water turned on. The Simpson meter was hooked up so as to read the heater block temperature and the rheostat controlling the power to the heater block adjusted until a temperature of approximately 450°F was obtained in the heater block. The guard ring heaters were then brought to the same temperature by the powerstat controlling the input power to the guard ring heaters. These adjustments were made in very small steps and after each adjustment, time was required to reach equilbrium. Both the heater block and guard ring temperatures were checked by reading the resistance of both thermistors and comparing to the graphs. Then another adjustment would be made in the direction required until the two temperatures were equal and constant. This procedure usually required about three and one-half hours, but the first two hours were required to heat the apparatus up to operating temperatures and no adjustments were made.

Readings of the resistance of the specimen thermistors were begun, starting with number five at 30 second intervals and reading all five resistance five times as shown by the data in Table III. These five readings were then averaged and the average used as the final value.

This same procedure was then repeated for the other four specimens.

With the completion of the tests on all five alloys the resistance values recorded were averaged and this value used to find temperatures of the corresponding points along the bar, by the use of the resistance temperature curves. The temperatures from the graph corresponding to the resistance measurements are shown in Table III through VIII.

Since all the runs were performed at the same input power, the heat flowing through each sample will be the same. So to find the thermal conductivity all that needs to be done is to set the product of the thermal conductivity of the standard and the temperature difference equal to the product of the unknown conductivity and its temperature difference. The standard conductivity was taken from the Metals Handbook and plotted against temperatures as shown in Figure 23. The value used in the calculations is the mean value of the temperatures from which the difference is found. The area and the length cancel since they are the same in all cases. For example, in the run using cast iron between temperatures 134 and 98°Fahrenheit and the standard between 166 and 134°Fahrenheit, the average temperature for the standard between 166 and 134°Fahrenheit is 150°F a value of conductivity is taken from the graph corresponding to the average temperature and a value is obtained of 29.6 Btu/hr ft^2°F/ft. From this data the conductivity of the cast iron is found as shown

k = coefficient of thermal conductivity

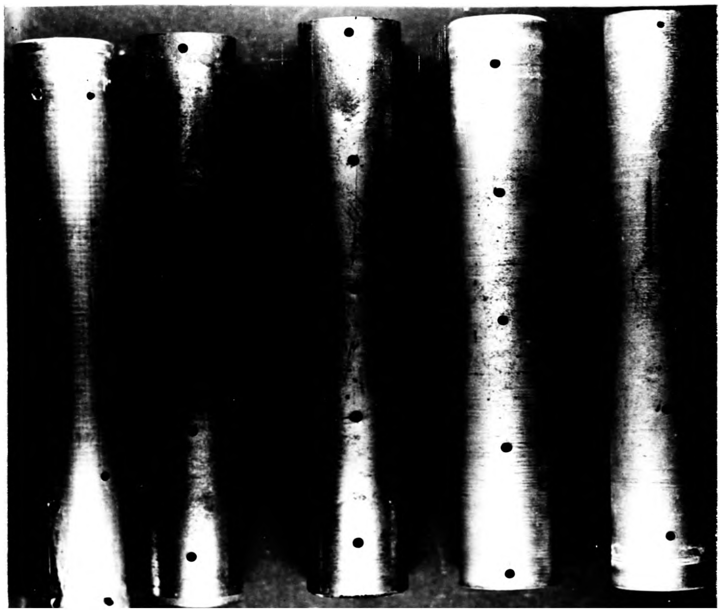
$$k_{\text{unknown}} = \frac{\text{(Temperature gradient of the Sample) Thermal Conductivity}}{\text{Temperature gradient of the specimen}}$$

$$k_{\text{cast iron}} = \frac{(134 - 100^{\circ}\text{F})\ 29.6\ \text{Btu/hr ft}^{2}\,{}^{\circ}\text{F/ft}}{134^{\circ}\text{F} - 98^{\circ}\text{F}}$$

$k = 28.0$ Btu/hr ft^{2}°F/ft

All values of conductivity as shown for the different alloys were found by the same series of calculations. The value found from this calculation is the conductivity for the average temperature of the two temperatures used to find the difference.

The temperature differences used for the standard and the unknown in the calculations was the difference between the same two thermistors, so as to eliminate any constant errors in temperature measurements.



Specimens

Figure 22

TABLE III

THERMISTOR RESULTS

1020 Steel

Thermistor Readings (ohms)

Thermistor No. 1	4400	4400	4400	4400	4400
Thermistor No. 2	540	540	540	540	540
Thermistor No. 3	310	310	310	310	310
Thermistor No. 4	217	216	217	218	217
Thermistor No. 5	415	415	415	415	415

	Average Resistance	Temperature °F	Temperature Gradient °F/in.
Thermistor No. 1	4400	100	
			34
Thermistor No. 2	540	134	
			32
Thermistor No. 3	310	166	
			34
Thermistor No. 4	217	200	
			33
Thermistor No. 5	415	233	

TABLE IV

THERMISTOR RESULTS

Cast Iron

Thermistor Readings (ohms)

Thermistor No. 1	4500	4500	4500	4500	4500
Thermistor No. 2	540	540	540	540	540
Thermistor No. 3	315	315	315	315	315
Thermistor No. 4	220	220	220	220	220
Thermistor No. 5	415	410	410	405	410

	Average Resistance	Temperatures °F	Temperature Gradient °F/in.
Thermistor No. 1	4500	98	
			36
Thermistor No. 2	540	134	
			31
Thermistor No. 3	315	163	
			36
Thermistor No. 4	220	201	
			34
Thermistor No. 5	410	235	

TABLE V

THERMISTOR RESULTS

Zinc Alloy

Thermistor Readings (ohms)

Thermistor No. 1	5100	5100	5100	5100	5100
Thermistor No. 2	900	900	910	900	890
Thermistor No. 3	1075	1080	1080	1075	1080
Thermistor No. 4	890	890	890	890	890
Thermistor No. 5	1660	1660	1660	1660	1660

	Average Resistance	Temperatures °F	Temperature Gradient °F/in.
Thermistor No. 1	5100	80	
			15
Thermistor No. 2	900	95	
			12
Thermistor No. 3	1078	107	
			12
Thermistor No. 4	900	119	
			13
Thermistor No. 5	1660	132	

TABLE VI

THERMISTOR RESULTS

Aluminum Alloy

Thermistor Readings (ohms)

Thermistor No. 1	4400	4400	4400	4400	4400
Thermistor No. 2	1070	1070	1070	1070	1070
Thermistor No. 3	980	975	970	970	970
Thermistor No. 4	1130	1120	1100	1100	1100
Thermistor No. 5	2150	2150	2150	2150	2150

	Average Resistance	Temperature °F	Temperature Gradient °F/in.
Thermistor No. 1	4400	88	
			7
Thermistor No. 2	1070	95	
			6
Thermistor No. 3	973	100	
			6
Thermistor No. 4	1110	106	
			7
Thermistor No. 5	2150	113	

TABLE VII

THERMISTOR RESULTS

Magnesium Alloy

Thermistor Readings (ohms)

Thermistor No. 1	4400	4400	4400	4400	4400
Thermistor No. 2	980	980	980	980	980
Thermistor No. 3	790	795	790	790	790
Thermistor No. 4	820	820	820	820	820
Thermistor No. 5	1650	1650	1650	1650	1650

	Average Resistance	Temperature °F	Temperature Gradient °F/in.
Thermistor No. 1	4400	88	
			14
Thermistor No. 2	980	102	
			9
Thermistor No. 3	791	111	
			10
Thermistor No. 4	820	121	
			12
Thermistor No. 5	1650	133	

4. Results

The first specimen to be run was the 1020 steel since it was to be used as the standard and maximum temperatures desired. The temperature differences found varied from 32 to 34°F as shown in Table III. With these temperature differences and the plot of thermal conductivity, Figure 23, taken from the Metals Handbook the unknown specimens could be run. After the other four runs had been completed and the temperature differences found the conductivities were calculated as previously described using temperature differences found from the same thermistors.

The thermal conductivities of the alloys were then plotted, Figures 24 through 28 against the average temperature corresponding to each value. The points plotted were scattered and no way was apparent to construct a curve through the points representing variation with temperature. If more runs had been made it would have been possible to construct a curve through the average of a series of points. But since the purpose of this study is only to evaluate the method it was decided to use a statistical means to obtain a curve representing the conductivity variation with temperature. The least squares method was used assuming that the conductivities will vary in a linear path with the change in temperature. This assumption is not completely correct, but is a close approximation of the actual case in most conditions. This same assumption was made in the values of conductivity used for the standard. The best straight line representing the change in thermal conductivity with varying temperature for the cast iron was the equation $k = 29.28 - 0.003484\ T$

Where:

k = thermal conductivity

T = Temperature degrees Fahrenheit

Using this the root mean square error was found to be 1.06 Btu/hr. ft^2 °F/ft. and the percent deviation of the experimental values from the same points on the curve varied from -3.06 to 6.05%. This percentage was found by using the values from the curve as the true value. The data obtained from this curve is shown in Table IV and the curve in Figure 24.

The curve found by this method produced conductivities varying from 28.52 to 28.88 Btu/hr. ft^2°F/ft. The published values between the same temperature points varied from 26.3 to 26.8 Btu/hr ft^2°F/ft. The published values were again found in the Metals Handbook and the straight line assumption made again. The percent difference of the values found from the curve to published values varied from 8.43 to 6.42% assuming the published value as the true value. But considering the specimen being run it may be safer to assume that the conductivity of this metal is the value found by the experimental means rather than the values from published work. Cast iron has so many variables in structure and composition that it is very difficult to obtain applicable published data.

The designer would be fairly safe in using these values but the equation cannot be extended over another temperature range because of the straight line assumption.

A set of sample calculations for the least squares method are shown in the Appendix.

The next run was performed on the zinc alloy which was 99.4%

Zinc, 0.3% Magnesium, and 0.3% Copper. In the run itself temperature differences were obtained which varied from 12 to 17°F, Table V. With these temperature differences the conductivities were found and again the scattering of the values were demonstrated. The least squares method was used to pass a linear curve through this data. The equation found from this method was:

$k = 21.18 + 0.516\ T$

Where the symbols have the same meaning as in the previous work on cast iron. With this curve a root mean square error was found to be 5.82 Btu/hr ft^2°F/ft. and the experimental points showed deviations from the same points on the curve of from -10.5 to 8.05%, Table IX.

A published value for thermal conductivity for the zinc alloy could not be located but another zinc alloy containing 0.3% cadnium and 0.3% lead was found in the Metals Handbook. The value of conductivity for this alloy should be close to the value for the alloy used in the experiment. The value of thermal conductivity given was 62.0 Btu/hr.ft^2°F/ft. at 77°F. Comparing this to the value from the curve, Figure 25, at 87.5°F a percent difference was obtained of 6.54%. This would seem to be a good value of conductivity, but if any values were compared at higher temperatures the differences would be continually increasing at a rather rapid rate because of the large slope of the curve. There would be some error here, because the accepted value used should also demonstrate some increase with temperature but not at the rate found by the experimental curve. So it is believed that this curve is not a true picture of the thermal conductivity variation with temperature.

The 2S Aluminum run produced temperature differences of from 6 to 7°F and again the plot of experimental thermal conductivities demonstrated the scattered picture, Figure 26. The curve drawn through these points had the equation:

$k = 208.9 - 0.568\ T$

The root mean square error was 10.31 Btu/hr. ft^2°F/ft. and the % deviation varied from -5.5 to 11.20%, Table X. A published value of conductivity was found in the Metals Handbook to be 128.2 Btu/hr. ft^2°F/ft. at 77°F again no variation with the temperature was available. Comparing this value to the value found at 91.5°F the % difference was found to be 18.4% and at the high temperature the error would be a little less. These values are too far from the standard value to have any real meaning.

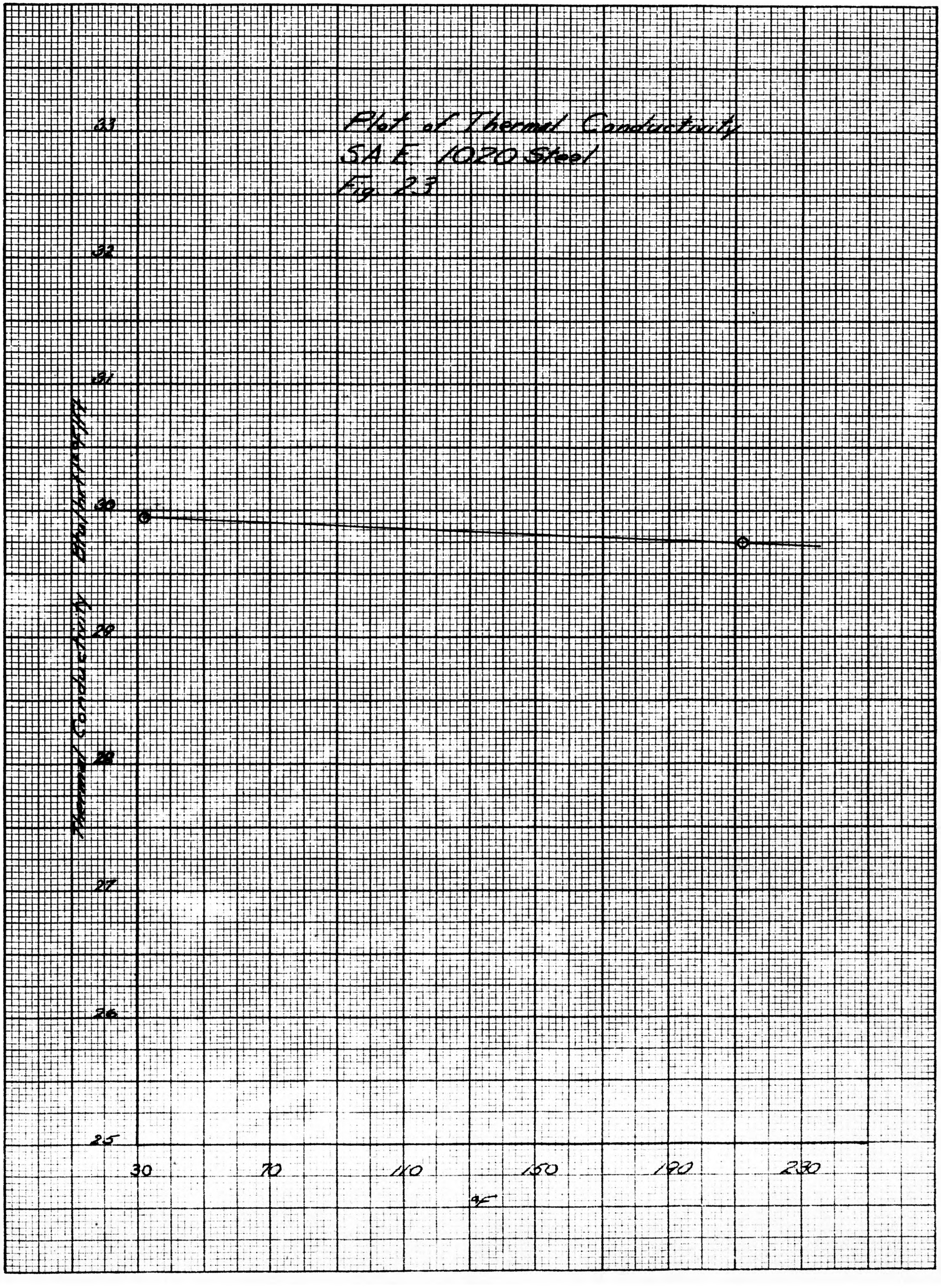
The magnesium alloy could not be identified. The 1125-QQM-44 must be a government specification number and data on it could not be located. A published value of conductivity for pure magnesium was used which was found in Brown and Marco (23). The same method was used for interpreting the data and the equation found was:

$k = 55.15 + 0.317\ T$

The root mean square error was 12.81 and the % deviation varied from -12.5 to 18.0%, Table XI. The published value used was 92.0 Btu/hr. ft^2°F/ft. and the % difference from the 95°F value from the graph, Figure 27, was 7.32% and at the high temperature the value found was 3.69%. These values could possibly be used for actual values but care should be exercised in their use. This is especially true since the alloy could not be identified.

The scattering of the points was the main problem in this study.

The main cause of this was as in any thermal conductivity study the difficulty of accurate temperature measurements. It was very difficult to measure temperatures much closer than one or two degrees. This error was not such a problem where the temperature differences were large as in the cast iron run. The values obtained in this run should be good. To see this more plainly a one degree error in 36 degrees only produces 3.78% error but a one degree error in a 6 degree difference as in the aluminum run produces a 16.6% error. This difficulty is the main factor in this study and limits the apparatus to making runs where the standard and the specimen of interest are of closely related conductivities. Not only can more accuracy be obtained but a wider temperature range can be realized.



Plot of Thermal Conductivity
SAE 1020 Steel
Fig. 23

TABLE VIII

CAST IRON RESULTS

Temperature °F	Thermal Conductivity Experimental Btu/hr.ft^2°F/ft	Thermal Conductivity Curve Btu/hr.ft^2°F/ft
116	28.0	28.88
150	30.5	28.76
183	27.8	28.64
218	28.5	28.52

Deviation Btu/hr.ft^2°F/ft	% Deviation
-0.88	-3.06
+1.74	+6.05
-0.84	-2.93
-0.02	-0.07
0.00	

Squared Error = 1.1271

RMS Error = 1.061

TABLE IX

ZINC ALLOY RESULTS

Temperature °F	Thermal Conductivity Experimental Btu/hr.ft^2°F/ft	Thermal Conductivity Curve Btu/hr.ft^2°F/ft
87.5	67.3	66.33
101	79.2	73.3
113	83.5	79.49
125.5	76.9	85.94

Deviation	% Deviation
-0.97	-1.44
+5.9	+8.05
+4.01	+5.35
-9.04	-10.50
-0.10	

Squared Error = 33.387

RMS Error = 5.82

TABLE X

2S ALUMINUM RESULTS

Temperature °F	Thermal Conductivity Experimental Btu/hr.ft^{2}°F/ft	Thermal Conductivity Curve Btu/hr.ft^{2}°F/ft
91.5	144	151.93
97.5	158	153.52
103	167	150.4
109.5	138.5	146.7

Deviation	% Deviation
- 7.93	- 5.22
+ 4.48	+ 2.92
+16.6	+11.02
- 8.2	- 5.6
+ 4.85	

Squared Error = 106.41

RMS Error = 10.31 Btu/hr.ft^{2}°F/ft

TABLE XI

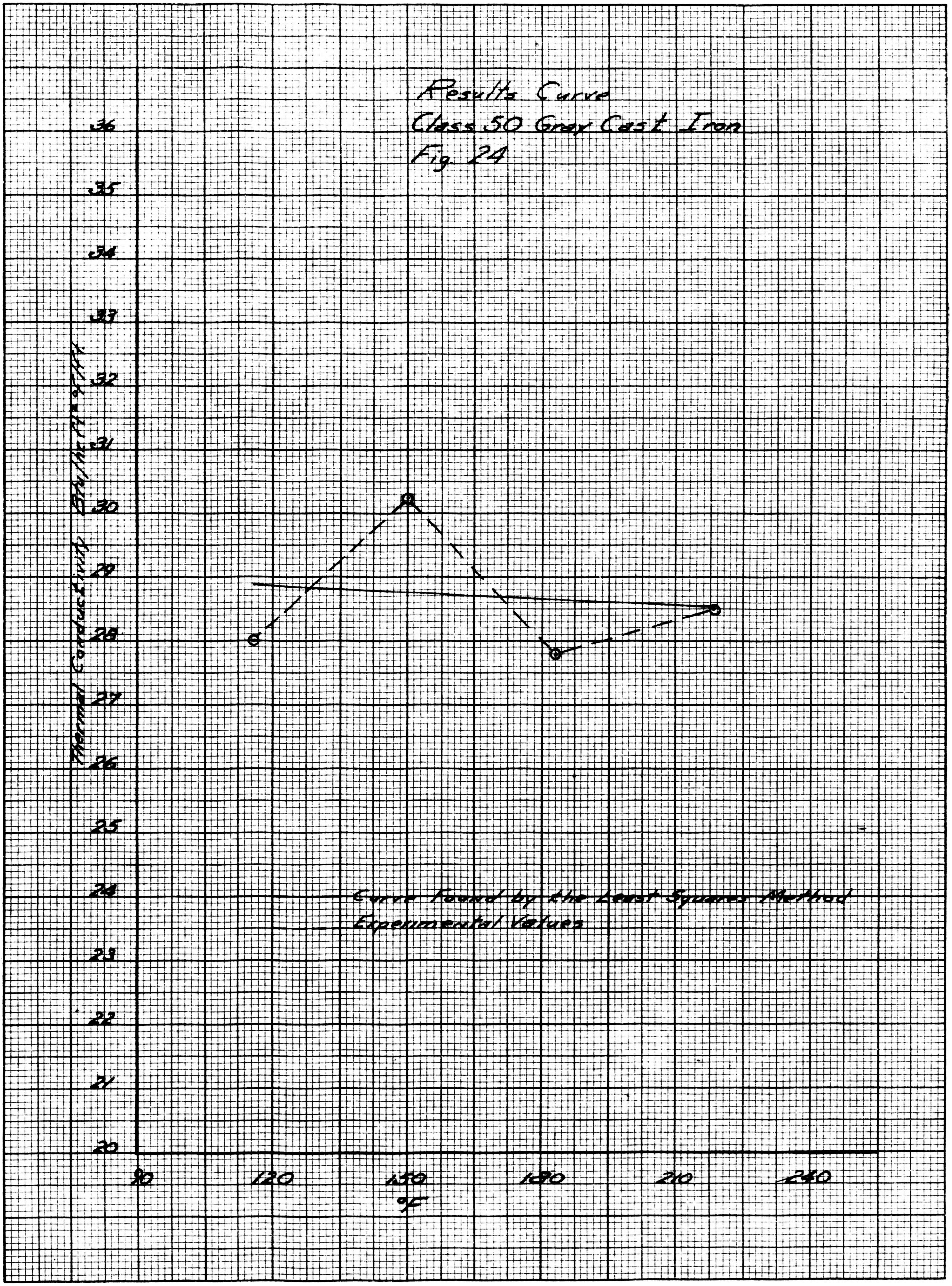
MAGNESIUM RESULTS

Temperature °F	Thermal Conductivity Experimental Btu/hr.ft^2°F/ft	Thermal Conductivity Curve Btu/hr.ft^2°F/ft
95	72.0	85.27
106.5	105.0	88.91
116	101.0	91.92
127	83.5	95.40

Deviation	% Deviation
-13.27	-15.55
+16.09	+18.0
+ 9.08	+ 9.9
-11.90	-12.5
0.00	

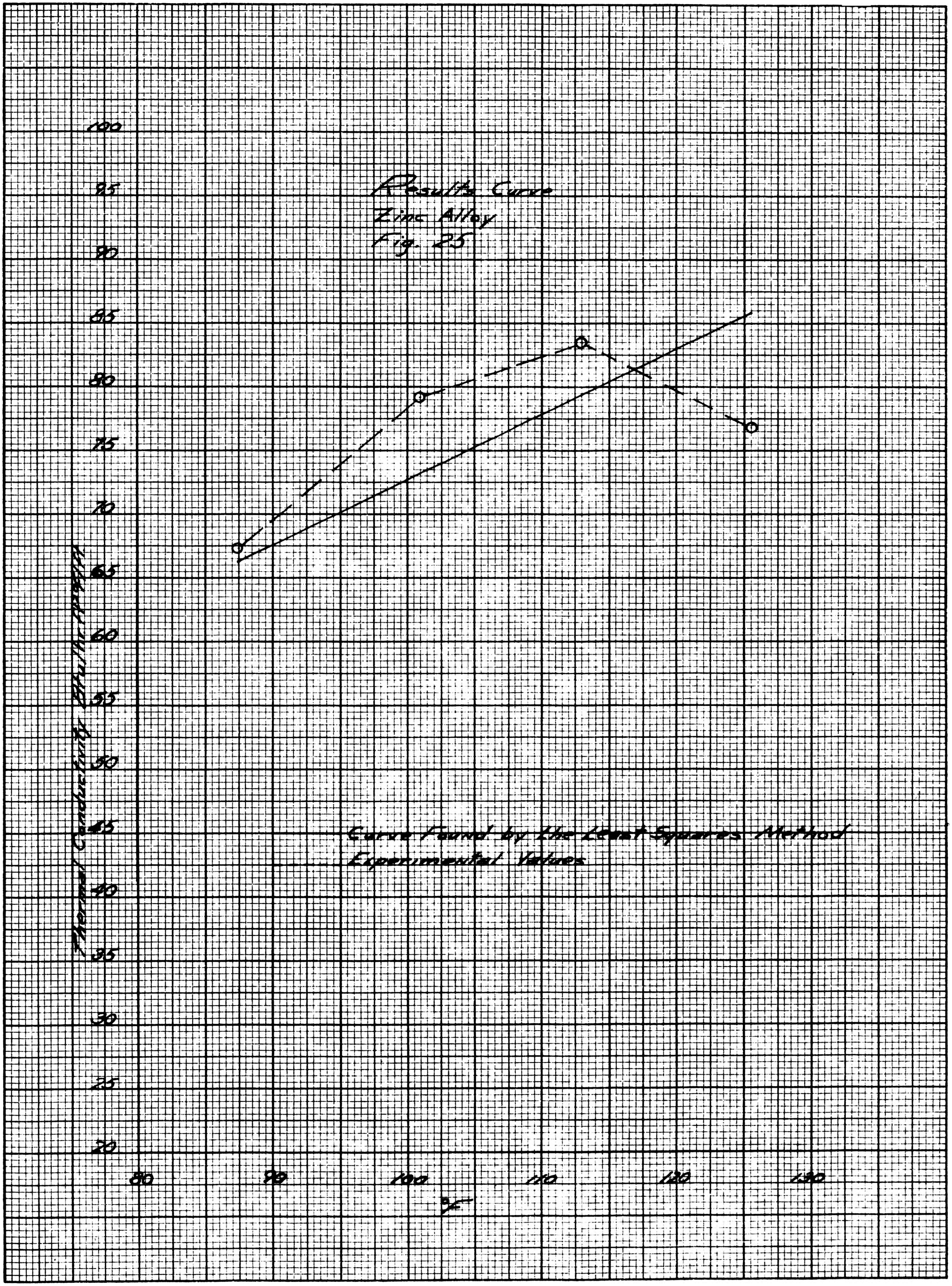
Squared Error = 164.74

RMS Error = 12.81

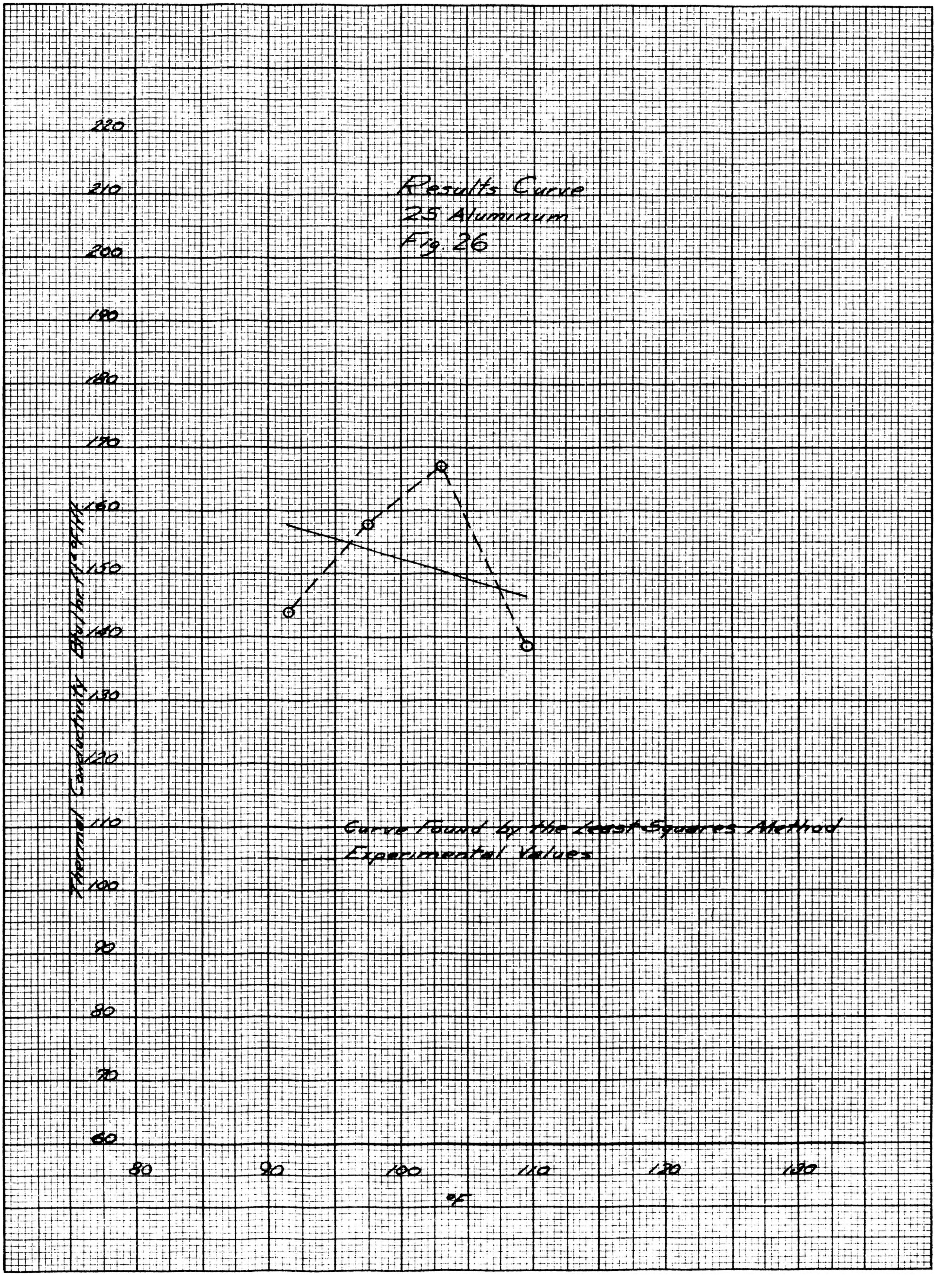


Results Curve
Class 50 Gray Cast Iron
Fig. 24

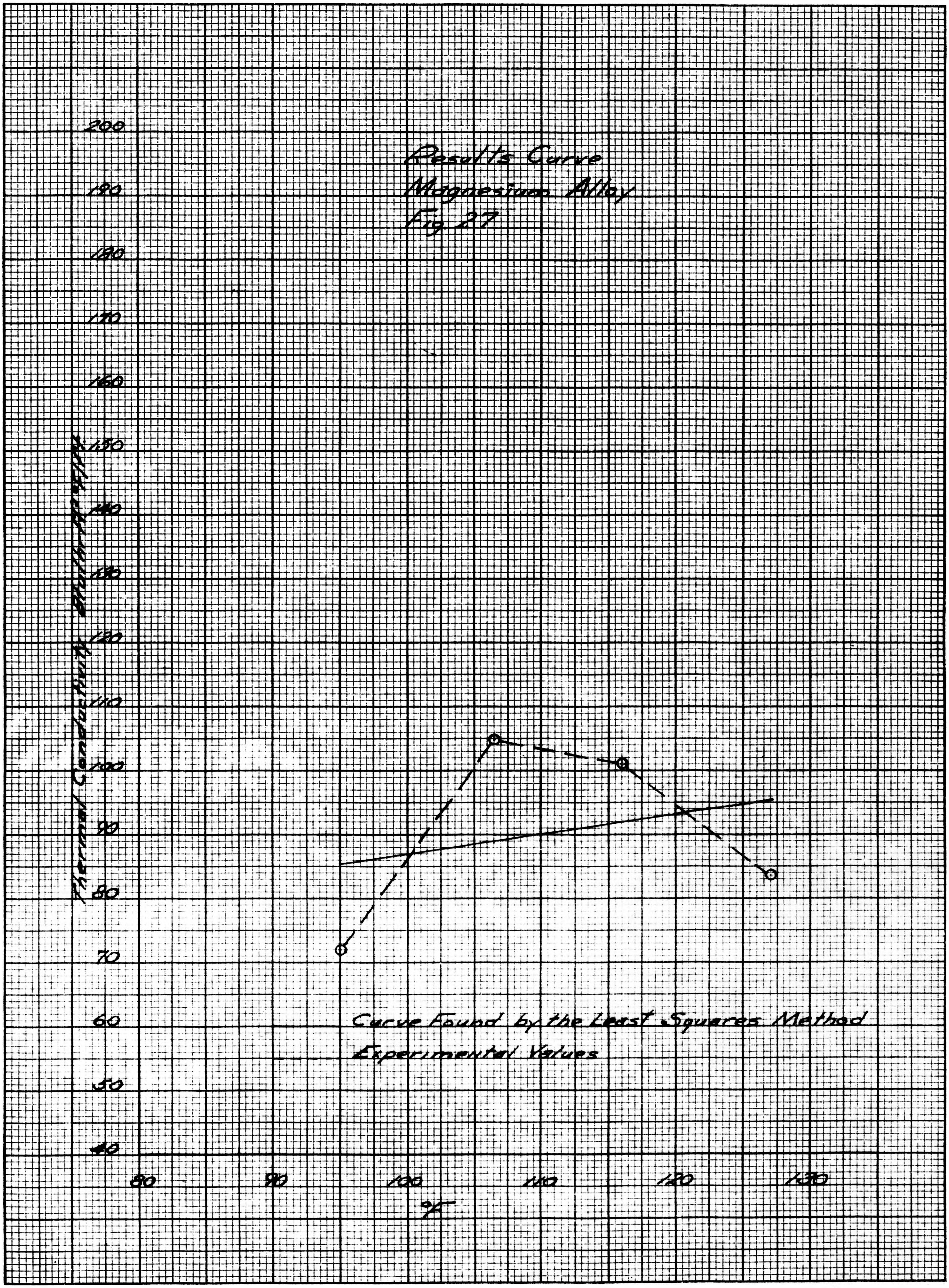
Curve Found by the Least Squares Method
Experimental Values



Results Curve
Zinc Alloy
Fig. 25



Results Curve
25 Aluminum
Fig. 26



Results Curve
Magnesium Alloy
Fig. 27

CONCLUSION

It was the purpose of this problem to set-up and evaluate an apparatus to perform thermal conductivity studies on alloy systems.

These studies would be similar to the runs made on the 1020 steel and cast iron. There would be little variation in thermal conductivity between the specimens and the temperature difference over each sample would be approximately the same, allowing full use of the equipment to have the maximum temperature change across the specimen. Such was not the case in this experiment running such things as the aluminum.

It is believed that this apparatus in its present state is very suitable for this type of measurement, and the purpose of the experiment has been accomplished.

If the apparatus is to be used for making measurements of conductivity on specimens which demonstrate large variations in this value, the apparatus should be converted so the input power can be measured and thermal conductivity found for each separate run with no dependence on standard specimens.

APPENDIX

Example of the Least Squares Method for Cast Iron

The problem is to fit as well as possible a straight line to the points obtained in the measurement of the thermal conductivity of gray cast iron.

The data obtained from the measurements was:

Temperature (°F)	116	150	183	218
Thermal Conductivity (Btu/hr.ft^2°F/ft)	28.0	30.5	27.8	28.5

The equation used is of the form:

$$a_o + a_1 T = k$$

Merely write down the array of the coefficients of a_o and a_1 and the right hand members in the form:

1	116	28.0
1	150	30.5
1	183	27.8
1	218	28.5

Under the assumption that all the data are of equal significance, take all weights equal to unity. The first equation is found by adding the respective columns.

$$4a_o + 667a_1 = 114.8$$

The second equation is found by multiplying the results in each column by the results in column two and suming the product of each column.

$$667a_o + 116969a_1 = 19123.4$$

Now solving the equations simultaneously the values for a_1 and a_o are found.

$a_1 = -0.003484$

$a_o = 29.28$

Yielding the equation:

$k = 29.28 - 0.003584T$

At the temperature listed the values for the thermal conductivity from the curve and the experiment are:

T(°F)	116	150	183	218
k_{curve}	28.88	28.76	28.64	28.52
$k_{experimental}$	28.0	30.5	27.8	28.5

Assuming the curve values as the true values the deviations are found to be:

T(°F)	116	150	183	218
Deviation	-0.88	+1.74	-0.84	-0.02

Squaring and summing these deviations and dividing by 4 gives the squared error which is:

Deviations -0.88^2
$+1.74^2$
-0.84^2
$\underline{-0.02^2}$
1.122 Squared Error

The RMS error is found by taking the square root of the square error and is:

RMS Error = 1.122 = 1.06

The % Deviation was found by using the curve values as the true values and was found to be:

T(°F)	116	150	183	218
% Deviation	-3.06	+6.05	-2.93	-0.0703

BIBLIOGRAPHY

1. Preston, T., The Theory of Heat, London, Mac Millan and Co. Limited, 1919, pp. 616
2. Igenhouss, I., Journ. De Physique, 34, pp. 68 and 380; 1789
3. Jakob, M., Heat Transfer, Vol. I N.Y. Wiley, 1949 pp. 164
4. Peclet, E., Traite dela Chaleur, Vol. I, Paris; 1860
5. Nusselt, W., Forschungsarb a.d. Geb. d. Ingenieurwes, Nos. 63 and 64; 1909
6. Groeber, H., Zeitscher d. Ver deutsch. Ing., 54, 1319, 1910
7. Berget, A., Journal de Physique, (2) 7, 2; 1888
8. Poensgen, R., Forschungsarb a. d. Geb. d. Ingenieurwes, No. 130; 1912
9. Griffiths, E., Proceedings Physics Society, (London), 41, 151-179 (1928-1929)
10. Griffiths, E. and G. W. C. Kaye, Proceedings Royal Society, (London) A, 104
11. Christiansen, C., Wied. Ann., 14, 23 (1881)
12. ASTM Standards, Part 5, pp. 828, 1958
13. Niven, C., Proceedings Royal Society, (London) (A) 76, 34; 1905
14. Clement, J. K., and Egy, W. L., Physics Review, 28, 71, 1909
15. Ingersoll, L. R., Journal Optical Society of America, 9 pp. 495, 1924
16. Kohlrausch, Fr., Annalen d. Physik, (4) 1, p. 145; 1900
17. Meissner, W., Annalen d. Physik, (4) 47, p. 1001; 1915
18. Gray, J. A., Proceedings Royal Society, (London) A, 56, 199 (1894)
19. Lee, C., Phil. Trans. A., 191, p. 399; 1898

20. Koenigsberger, J. and R. Weiss, Annalen d. Physik, 12, 342, 1903

21. Griffith, E. and G. W. C. Kaye, Proceedings Physical Society, 41-42, pp. 160, 1928-1930

22. Deem, H. W., Thermal Conductivity of Zirconium and Zirconium - Tip Alloys, U.S. A.E.C., BMI-849, 1953

23. Brown, A. I. and Marco, S. M., Introduction to Heat Transfer, III Edition, McGraw-Hill, pp. 14, 1958

VITA

Virgil James Flanigan, Jr., son of Mr. and Mrs. Virgil J. Flanigan, was born in Higginsville, Missouri on December 31, 1938.

He attended Higginsville High School and graduated in May 1956. September 1956 he enrolled in Missouri School of Mines and Metallurgy and completed the requirements for a Bachelor of Science Degree in Mechanical Engineering in May of 1960. He spent the next year in industry and the U. S. Army, and in September of 1961 he enrolled at Missouri School of Mines and Metallurgy as a graduate student in Mechanical Engineering. Upon the completion of the requirements for his Master of Science degree he plans to accept a position on the faculty of the Missouri School of Mines and Metallurgy.