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Application of a finite element model update technique to detect damage in a beam structure

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APPLICATION OF A FINITE ELEMENT MODEL UPDATE TECHNIQUE TO DETECT DAMAGE IN A BEAM STRUCTURE

by

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ABSTRACT

The interest in the ability to monitor a structure and detect damage at the earliest possible stage is pervasive throughout the civil, mechanical and aerospace engineering communities. The thesis focuses on the application of a finite element model updating technique to monitor and detect damage in beams. A Sensitivity Based Element-by-Element (SBEBE) methodology is chosen as the finite element model updating technique. In this method, damage is detected by updating a finite element model with test data obtained from "healthy" and "damaged" structures and observing the relative changes in the updated finite element models. The performance, efficiency and sensitivity of the SBEBE algorithm in detecting the damage location and severity are studied through numerical and experimental test cases on a cantilever beam. The location and extent of damage are successfully predicted with all numerical cases. The presence of noise in the numerical data and its effects on the damage detection process are examined. The SBEBE algorithm is capable of detecting the presence, location and extent of damage for noise levels in the numerically generated data up to 5% of the signal amplitude. Also experimental studies are carried out on a cantilever beam with modal data measured using a laser doppler vibrometer. A small section of the cantilever beam is mechanically removed, and the SBEBE algorithm is used successfully to detect the damage location and severity.

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NOMENCLATURE

Symbol	Description
i	Mode number
λ_i	Eigenvalue for mode i
Φ_i	Eigenvector for mode i
$\Phi_{m_{iE}}$	Modeshape at measured degrees of freedom
$\Phi_{o_{iE}}$	Modeshape at unmeasured degrees of freedom
ω_{iE}	Experimentally measured frequency (rad/s) for mode i
$\{\Phi_{iE}\}$	Experimentally measured mode shape vector for mode i
$\Delta\omega_i$	Difference between the measured and estimated modal data
ΔP^i	Unknown vector of design parameters
$[V_{ee}]$	Weighting matrix
$f(t)$	Forcing function
$\{x\}$	Column matrix of displacements
p	Vector of parameters to be updated
Δp	Change in parameter vector p
$[B_i]$	Parameter sensitivities for mode i
$\{R_i\}$	Dynamic residual (modal force) error vector for mode i
$\{Z_i\}$	Undamped Impedance matrix for mode i
$[\ ]_m, [\ ]_o$	Measured and unmeasured partitions of [ ]
$[M],[C],[K]$	Nominal mass, damping, and stiffness matrices
J	Objective function
g	Linearised gradient
G	Hessian matrix

1. INTRODUCTION

1.1. BACKGROUND

On April 28, 1988, Aloha B-737-200 (N73711) lost a major section of its upper-forward fuselage (Figure 1.1) over the Hawaiian Islands. In the report given by the National Transportation and Safety Board (NTSB) for the cause of the accident, it was mentioned that disbonding and fatigue damage led to the failure of the lap joint causing the separation of the fuselage upper skin. This incident brought attention and awareness of the structural health-monitoring problem for aging aircraft and other mechanical systems into the public arena.

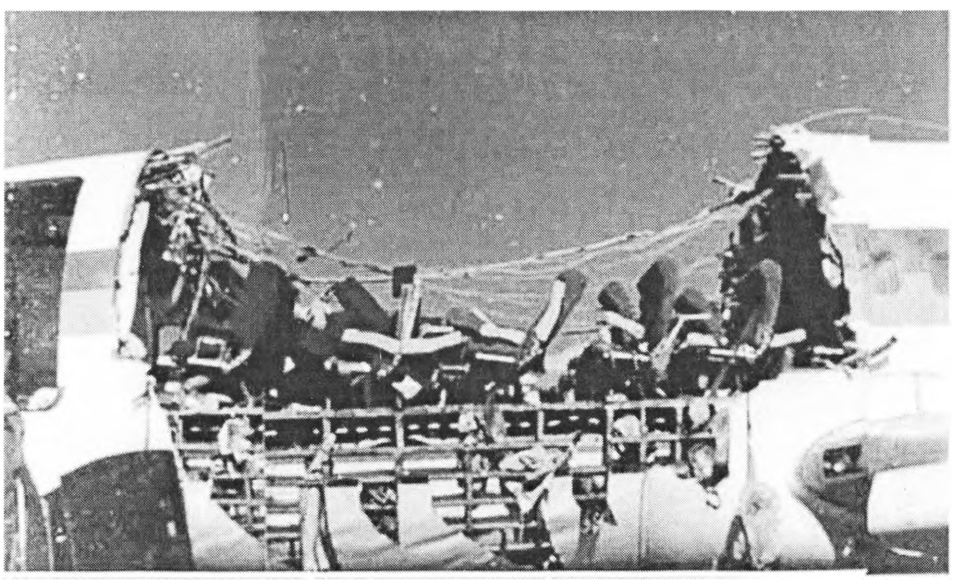

Figure 1.1. Side View of Damaged Aloha B-737

The process of implementing a damage detection strategy is referred to as structural health monitoring. Nearly all in-service structures require some form of maintenance for monitoring their integrity and to prolong their life span and prevent catastrophic failure of these structures. The ultimate objective for the end users, maintenance crews and manufacturers is to have access to the knowledge of the integrity of in-service structures on a continuous real time basis. With such knowledge, they can count with confidence on the optimal use of the structures, minimize downtime and increase productivity. Thus, the indirect benefits from the development of the technology for the society as a whole can be very significant in many sectors of the industry.

Structural health monitoring processes involve the observation of the system over a period of time using periodically spaced measurements and the analysis of these features to determine the current state of health of the system. The output of this process is updated information regarding the ability of the system to continue to perform its desired function in light of the inevitable aging and degradation resulting from the operational environments. For these processes, new methods of structural health monitoring are being explored to better determine the functional safety of structures.

Methods which determine the condition or health of a structure without altering the performance or integrity of the structure are referred to as Non-Destructive Evaluation (NDE) techniques. A good overview of NDE techniques can be found in Witherell [1]. Some NDE approaches consist of visual inspection, sometimes preceded by application of a penetrating dye, which highlights cracks, deterioration, or other blemishes. This category also includes x-ray imaging, ultrasonic, and radiography. Other methods are based on local variations in the electro-mechanical properties of the structure and include eddy-current and magnetic particle inspections, among others. A third type of NDE depends on changes in the dynamic response characteristics of the structure. This thesis addresses NDE methodologies of this type. Systems of NDE testing which depend on changes in the dynamic response characteristics of the structure are often called modal based damage detection methods. These methods are typically based on vibration testing of the structure and measurement of the mechanical response of the structure to a specified excitation input.

Damage detection methods are based on changes in dynamic response characteristics, and this area is a subject that has been receiving considerable attention in recent times. This approach is based on the idea that damage will significantly alter the physical properties of the structure, such as stiffness, mass or energy dissipation properties of a system. This change in physical or material properties in turn alters the measured dynamic response in the form of modal parameters, such as natural frequencies, mode shapes, and damping of that system. By observing these changes in the measured vibration response, damage detection can be performed.

Damage detection algorithms can be broken down into the following categories: non-model-based schemes and model-based schemes. Here, "model" refers to a set of parameters used to describe the structure in a mathematical representation, viz., a finite element model. Non-model-based schemes determine direct changes in the sensor output signal to locate damage in the structure. Model-based schemes depend on the finite element model and the data from the sensor output signal. One of the popular model based methods up to date is the model updating method. Model updating can be defined as the adjustment of an existing finite element model using the measured vibration data from an experimental-derived model. After adjustment, the updated model is expected to represent the dynamic behavior of the structure more accurately. This feature of model adjustment is used in detecting the damage. An excellent review of model-based damage detection methods has recently been compiled by Doebling et al. [2].

Classification of damage identification methods, as presented by Rytter [3], defines four levels of damage identification:

Level 1: Determination that damage is present in the structure

Level 2: Determination of the geometric location of damage

Level 3: Quantification of the severity of the damage

Level 4: Prediction of the remaining service life of the structure

Ideally, a robust damage detection scheme will be able to perform all the above four functions and be well suited to automation. To the greatest extent the method should not rely on the engineering judgment of the user or an analytical model of the structure. A less ambitious but more attainable goal would be to develop a method that possesses all the features described above, but one that uses an initial measurement of an undamaged structure as the baseline for future comparisons of measured response. Also, the methods should take into account any operational constraints. For example, a common assumption with most damage identification methods developed in the technical literature to date is that the mass of the structure does not change appreciably as a result of the damage.

"Damage" can be defined as the process when the structure undergoes a non-reversible change in composition. Examples of non-reversible changes are corrosion in metals, plastic deformation, cracks, delamination, brittle damage, etc. Damage in mechanisms typically results in a decrease in mechanical properties (such as elastic modulus) or in the physical properties (thickness of a plate or other structural component), which is then manifested as a decrease in strength or life expectancy. In this thesis, damage from any source is assumed to affect parameter models as a decrease in stiffness based on a linear dynamic model. This corresponds to the effects of cracks, delamination between composite plies, and necking, among other sources.

Damage due to fatigue (Figure 1.2) is a major cause of crack formation. These cracks are initiated from regions of high stress. These cracks, when left unattended, can grow at alarmingly fast rates and can cause catastrophic failure. From safety considerations, repairs are done to arrest/retard further crack growth. The two popularly used repair techniques in the aerospace industry are riveting a metallic doubler with or without removing the damaged portion and bonding a composite material patch over the damaged portion.

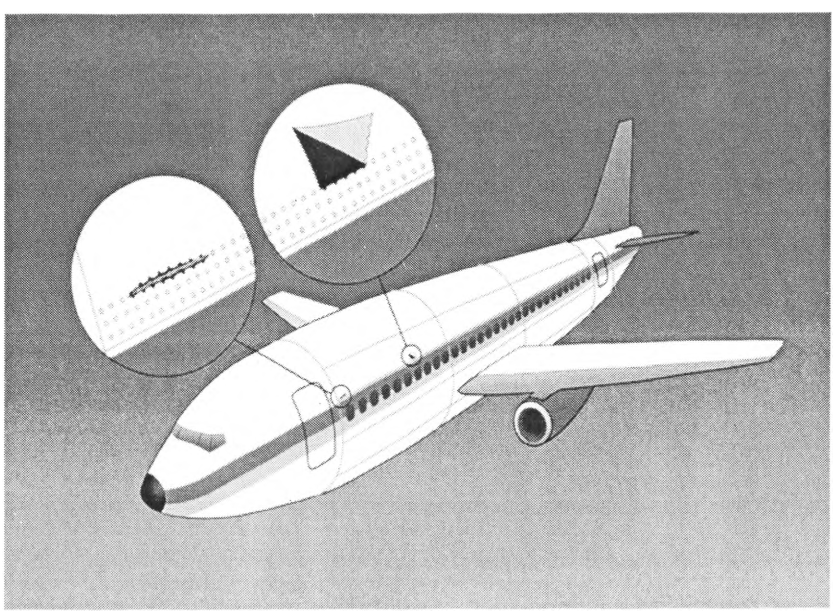

Figure 1.2. Examples of Fatigue Damage

The former technique is often not effective due to the introduction of fresh sources of stress concentrations, additions of weight, stress corrosion and stress altercation problems. The later one, bonded composite material patch repair, provides a highly efficient and cost effective method for repairing metallic aircraft components subjected to crack or delaminations. The bonded composite patch repair (Figure 1.3) enhances the fatigue resistance of the structure and restores the stiffness and strength of the damaged structure.

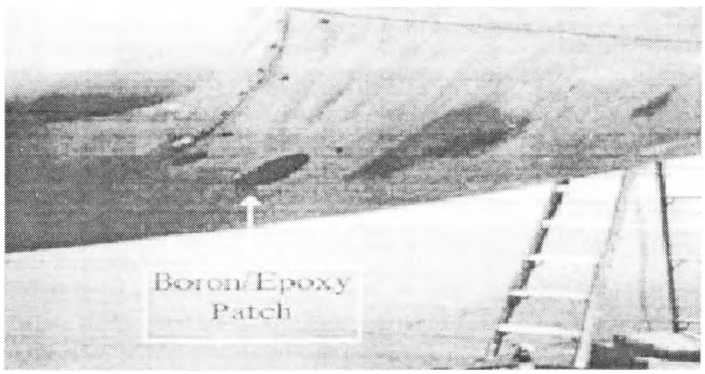

Figure 1.3. Bonded Composite Material Patch Repair

1.2. OBJECTIVE OF THESIS

The objective of the current study is to examine the performance of a sensitivity based element-by-element (SBEBE) model updating methodology in predicting the Level 1 and Level 2 types of damage detection, namely the presence and location of damage. In this thesis an attempt is made to use experimentally measured modal parameters (i.e., natural frequencies and modeshapes) to improve the finite element models and then by comparing the updated finite element models from the healthy and damaged structures, to detect the presence and location of damage. In order to assess the performance of the method, a numerical study followed by an experimental study on one specimen is has been carried out. Study has been carried out to determine the method's effectiveness to detect damage with noisy numerical data.

1.3. OVERVIEW OF THESIS

The thesis is organized as follows. Chapter 2 describes the literature behind the model updating methods and provides information regarding direct and iterative modal updating methods. Chapter 3 provides detailed theory and mathematical formulation of SBEBE method. The performance of the algorithm in detecting damage to a cantilever beam is studied using numerical data with out noise and noisy data in chapter 4. Chapter 5 discusses the experimental setup, test specimens, finite element modeling of cantilever beam, correlation of experimental to FEM modal data and model updating procedure applied to the experimental modal data. Finally, conclusions about the performance of the algorithm are discussed in Chapter 6.

2. LITERATURE SURVEY

2.1. INTRODUCTION

Finite Element Models of structures are not exact representations of real structures because discrepancies exist due to uncertainty in the governing physical relations (for example, modeling non-linear elastic behavior with linear FEM theory), the use of inappropriate boundary conditions or elemental material and geometrical property assumptions and modeling using too coarse of a mesh. In practice these 'errors' are rather due to lack of information than modeling errors. Their effects on the FE analysis results should be analyzed, and improvements must be made to reduce errors associated with the FEM. Model updating has become the popular name for using measured structural data to correct the errors in a FEM. Model updating can be defined as the adjustment of an existing analytical model (FEM) in the light of measured vibration test data. After adjustment, the updated model is expected to represent the dynamic behavior of the structure more accurately. This chapter is aimed to provide a review of the relevant literature related to model updating techniques.

2.2. MODEL UPDATING METHODS USING MODAL DATA

This section is a review of existing model updating methods using modal test data. Two categories, namely direct and iterative methods, are considered.

2.3. DIRECT METHODS USING MODAL DATA

Direct methods update the complete structural stiffness and mass matrices so that the updated matrices are those closest to the initial analytical matrices that reproduce the measured modal data. Direct model updating methods have the great advantage of not requiring iteration, thus, the possibilities of divergence and excessive computation are eliminated. These methods are representational, meaning they reproduce the measured data exactly. The main advantages of these methods are:

- Assured convergence
- No iterations required
- Minimal CPU time is required compared to an iterative method

- Measured data is reproduced exactly
- The disadvantages of direct model updating methods are:
- Connectivity of nodes is not ensured
- Updated matrices are fully populated
- Updated stiffness and mass matrices are not guaranteed to be positive definite (Non-singular)

2.3.1. Lagrange Multiplier Methods. Lagrange multiplier methods require two quantities as crucial to the updating process: the measured modal data and the finite elemental global mass and stiffness matrices. The Lagrangian multiplier method involves minimizing an objective function subject to some constraints on the independent variables (stiffness and mass matrices). Baruch [4] considered these methods as reference basis methods because one of the three quantities (the measured modal data, the analytical mass or stiffness matrix) is assumed to be exact, or the reference, and the remaining two quantities are updated. Baruch and Bar Itzhac [5] considered the mass matrix to be reference and developed a technique that minimizes the weighted Euclidean norm of the eigenvectors. The measured eigenvectors are corrected so that they are orthogonal with respect to the mass matrix, and then an updated stiffness matrix is computed which is closest to the analytical mass matrix but reproduces the measured data. Berman [6] assumed that the measured modes were correct and therefore, applies the updating procedure to the mass matrix. Berman and Nagy [7] used the same assumptions and updated the stiffness and mass matrices sequentially. Caesar [8] suggested a range of methods that updated the mass and the stiffness matrices using different cost functions and constraints. To improve the physical meaning of the updated results, he also introduced additional constraints from rigid body considerations, such as the position of the center of gravity, total mass and moments of inertia. Wei [9] updated the mass and stiffness matrices simultaneously using the measured eigenvectors. He used constraints of mass orthogonality, the equations of motion, and the symmetry of the updated matrices. Fuh and Chen [10] developed a reference basis method for representational updating of structural systems with non-proportional damping. A detailed review of Lagrange multiplier methods is given by Heylen and Sas [11].

2.3.2. Matrix Mixing Methods. The matrix mixing method were originally developed by Thoren [12], and further developments have been introduced by Caesar, et al., [13]. If all vibration modes are measured at all degrees of freedom, the mass and stiffness matrices can be directly constructed using a mass orthogonality concept. Often the number of measured modes is far fewer than the order of the analytical model. The matrix mixing approach works around this problem by using the data from the finite element model to fill in the gaps in the measured data.

2.3.3. Error Matrix Methods. Error matrix methods are a group of technique that directly estimate the error in the mass and stiffness matrices. One of the earliest papers in this subject is by Sidhu and Ewins [14], who obtained a flexibility matrix by considering the first order terms of the Taylor series expansion of the error matrix. Lieven and Ewins [15] proposed a modified version of the error matrix method [16] by using singular value decomposition. The advantage of this approach is that the analytical system matrices are no longer required. Lieven and Ewins [17] discussed the effects of incompleteness and noise on the quality of the results obtained from the error matrix method.

2.3.4. Eigen Structure Assignment Methods. The eigenstructure assignment method from control theory has also been used to update finite element models. As the name suggests, the method reproduces the measured eigenvalues and eigenvectors (natural frequencies, damping ratios and modeshapes). If only the eigenvalues are used in the process of updating finite element models, then the method is called pole placement [18]. Using state feedback, Moore [19] formulated the necessary and sufficient conditions for simultaneous eigenvalue and eigenvector assignment for the case of distinct eigenvalues. Srinathkumar [20] addressed the problem of pole-assignment in linear time-invariant, multi-variable systems using output feedback. Andry and Chung [21] were among the first apply the technique to a linear mechanical system for the purpose of parameter identification. The method is very powerful in the control system design context. A system will have given output variables, which are measured, and some input variables, which are able to supply excitation to the system. The problem is then to reproduce a linear combination of the output variables which gives the required input excitation signal and yields a satisfactory closed loop response. Thus, unstable poles, or eigenvalues, of the open loop system are transformed into stable poles in the closed loop system. In the application of these methods to model updating, these input and output variables are not given, but their number and form are chosen at will. The 'controller' is then designed to reproduce the measured eigenvalues and eigenvectors. Shulz and Inman [22] used the eigenstructure assignment technique with a number of constraints that could be related to the physical properties of the system to be updated. The constraints were built into a non-linear optimization procedure that preserved the desired properties of the updated model. They considered small-order systems that were symmetric, banded and bounded. Ziaei and Imregun [23] modified this formulation to accommodate large systems by developing a quadratic linear optimization procedure, which is unconditionally stable. They also considered the updating of damping matrices. The advantages of eigenstructure assignment methods are:

1. The measured eigenvalues and modeshapes are reproduced exactly.
2. The updated damping matrix can be explicitly calculated, something the Lagrange multiplier methods cannot do.

The four main disadvantages of these methods are:

1. A large amount of computation is required because it is a non-linear optimization problem.
2. Some or all of the input and output matrices have to be specified.
3. No physical insight is provided into what is being minimized to obtain the updated matrices.
4. There is no guarantee that the updated matrices will be positive semi-definite.

2.4. ITERATIVE METHODS

The basic approach of iterative updating methods using modal test data is to improve the correlation between the experimental and analytical models via a penalty function. Penalty function method requires iterative optimization and linearization of the analytical model parameters (FEM). Iterative methods have two main advantages. First, a wide range of parameters can be updated simultaneously, and second, both measured and analytical data can be weighted.

2.4.1. Penalty Function Methods. Penalty function methods are generally based on the use of a Taylor series of the modal data expanded as a function of the unknown updating parameters. This series is often truncated to produce a linear set of equations involving modeshapes, natural frequencies from the modal data and design parameters (physical parameter such as modulus of elasticity, thickness, etc. from the FEM).

A sensitivity matrix is defined as the first derivative of the stiffness or mass matrix with respect to the design parameters. The sensitivity methods differ in the choice of design parameters and the definition of optimization constraints (orthogonality constraint). Individual elements of the mass and stiffness matrices, sub-matrices, geometric or material properties can be used as design parameters to be updated or corrected with these methods. Constraints are usually imposed on natural frequencies and mode shapes.

Fox and Kapoor [25] calculated the sensitivities of the eigenvalues with respect to the design parameters. They have also suggested two methods for calculating the sensitivities of the eigenvectors to the design parameters. Lim [26] suggested an approximate method for calculating the sensitivities of the eigenvectors, which is only valid for the low frequency modes. Other methods for calculating mode shape sensitivities have been suggested by Chu and Rudisill [27] , Ojalvo [28] and Tan and Andrew [29].

Usually, the number of design parameters and the number of measurement locations are not equal, and, hence, the sensitivity matrix is not square. The case in which there are more design parameters than measurements was considered by Chen and Garba [30]. They found the solution to the problem by seeking a set of design parameters that minimizes the norm of the residual obtained from stiffness and mass matrices and the eigenvalues and eigenvectors from modal data. Similarly, the singular value decomposition technique was used by Hart and Yao [31] and Ojalvo, et al., [32] for a case with less design parameters than measurements.

In practical situations, all measured data do not have the same accuracy. Usually, mode shape data are less accurate than natural frequency data. The accuracy of measured data can be incorporated into the updating process by including a positive definite weighting matrix. Another approach [33] is to add an extra term to minimize the change of the design parameters. Many researchers have used this method with different sets of unknown parameters. Thomas [34] and Dascotte and Vanhonacker [35] used the approach to update the elements of the mass and stiffness matrices. Dascotte [36] demonstrated and discussed the relative merit of combining analytical and experimental modal data on a practical application. Physical parameters (modulus of elasticity, thickness, density, etc.) were also chosen by many authors. Such parameters allow an easier interpretation of the updated model. Wei [37] selected moments of inertia as design parameters to update a simple 3D beam structure. They compared the results with that of using a penalty function method, whereby each elemental matrix is corrected on a non-physical basis. Dascotte [38] updated a composite structure, selecting the material constant as design parameters. A second-order sensitivity method has been tried by Kuo and Wada [39], who produced correction terms to improve the convergence properties compared to that of the linearized algorithm. Ojalvo and Pilon [40] used second-order natural frequency sensitivities to update the system mass and stiffness matrices.

The present algorithm discussed in this thesis is an application of the Sensitivity Based Element-By Element (SBEBE) methodology to damage detection. There is a large body of literature available on the subject of the SBEBE model updating method. Friswell and Mottershead [41] provide a comprehensive overview that illustrates many of the different techniques and issues involved in the SBEBE updating procedures. The authors divide the SBEBE model update technique into two groups based on the form of the experimental data that they utilize: 1.) those that use modal data and 2.) those that use Frequency Response Function (FRF) data. The authors also discuss the selection of physical parameters to be updated during the procedure and several recommendations are made. The parameters should be chosen to correct a recognized uncertainty in the model, and the modal test data should be sensitive to the parameters chosen so that it effectively predicts the uncertainties in the FEM model and produces an improved match between model and test data.

Dascotte [42] provides details regarding the SBEBE updating methodology, illustrations of its efficiency and a comparison of performance with commercially available updating software. The research presented in this thesis brings together the frameworks presented by Dascotte [42], Friswell and Mottershead [43] and Alvin [44].

3. DAMAGE DETECTION BASED ON SENSITIVITY BASED ELEMENT-BY ELEMENT METHOD

3.1. INTRODUCTION

The objective of this section is to discuss the application of the Sensitivity Based Element-By-Element (SBEBE) method in detecting damage to structures. A detailed description of the theory and mathematical formulation is discussed.

3.2. SBEBE METHOD

The SBEBE method is an iterative based model updating method. The method works by modifying the mass, stiffness, and damping parameters of the finite element model until an improved agreement between modal data predicted by the finite element model and the test data is achieved. Thus the goal of the method is to achieve an improved match between the finite element model and the test data by making physically meaningful changes to the model at the elemental level.

In comparison to other model updating methods, the SBEBE method has a unique feature of updating the model properties at the elemental level. Model properties, such as modulus of elasticity, thickness, density etc. can be selected. Model properties should be chosen so that they are sensitive to the changes in the structure properties. For example, if damage occurs in the structure the model property should be sensitive enough to represent it when updated with test data from the damaged structure.

To perform model updating using the SBEBE method, a finite element model and two sets of modal data from the test structure are required. In this thesis damping is ignored and only the stiffness and mass matrices are included in the analysis. Modal test data from the test structure is obtained using a Laser Doppler Vibrometer (LDV). Section 5.4 describes in detail the method of collecting the modal test data from LDV.

The SBEBE method is a multi-step procedure requiring a finite element model and experimental vibration data from the test structure both before (when it is "healthy") and after damage is presumed to occur. The first step is to update the original finite element model with modal test data from the "healthy" structure. This updated FEM represents a mathematical model of the healthy structure. It acts as a reference standard to which future measurements can be compared after the structure has endured an extended period of service. At any time when the structure has to be tested for damage, subsequent vibrational test data is captured, and then the SBEBE algorithm is applied to the reference finite element model using the new experimental data to yield a refined finite element model of the structure. A comparison of the refined finite element model to the reference finite element model can reveal the damage, if any, (at the elemental level) that has occurred. This was the principal logic employed in the thesis for the damage detection process. Figure 1.5 depicts the flow chart of the SBEBE algorithm in damage location and quantification.

The damage detection process occurs in two steps. In step 1, a refined finite element model of the healthy structure is obtained from updating the elemental parameters of the FEM with modal test data from the healthy specimen. In step 2, the refined finite element model of the healthy structure is used as the base for updating with the modal test data from the supposedly damaged structure. The parameters that change between the two updating procedures can be used to predict the onset of damage as well as its location and severity. Severity of damage is assessed by comparing the relative change in the parameters selected before and after updating.

3.3. THEORY AND PROBLEM FORMULATION

The equations of motion for a structured modeled with *n*-degrees of freedom (neglecting damping, c=0) can be written as

$$[M]\{\ddot{x}\}+[K]\{x\}=\{f(t)\} \tag{1}$$

where $[M]$ is a *n* x *n* mass matrix and $[K]$ is a *n* x *n* stiffness matrix, $\{x\}$ is a column vector of n generalized coordinate variables corresponding to the degrees of freedom in the structure and $\{f(t)\}$ is a column vector of *n* generalized forcing functions.

Considering a homogeneous form of Eq. (1) ($f(t)$=0) and substituting the general solution $x(t)=\Phi_i e^{i\omega t}$ into Eq. (1) leads to the eigenvalue problem,

$$[K]\{\Phi_i\}=\lambda_i[M]\{\Phi_i\} \tag{2}$$

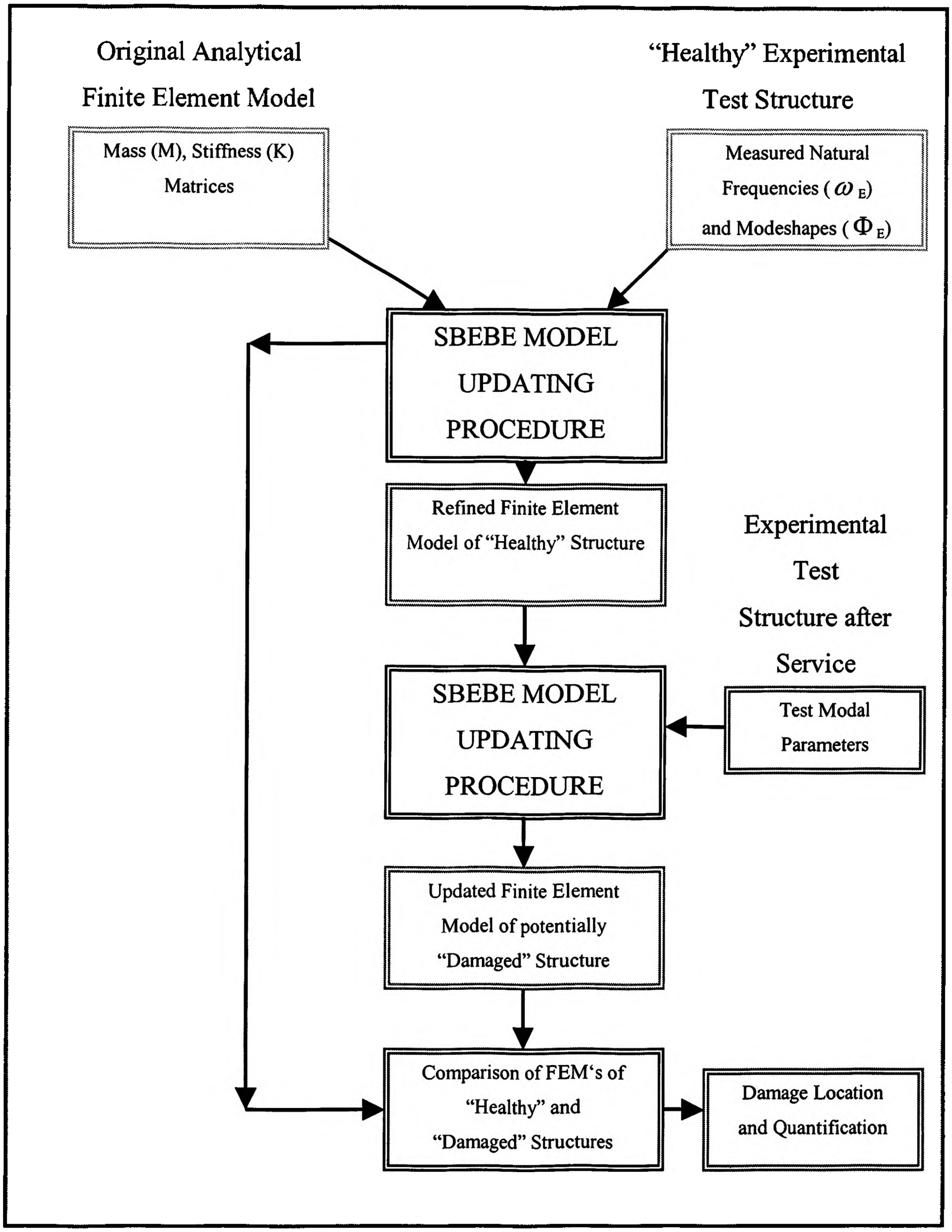

Figure 3.1 Flow Chart of SBEBE Method in Damage Location and Quantification

$$\lambda_i = \omega_i^2 \tag{3}$$

In modal updating literature, λ_i and Φ_i are referred to as modal parameters or modal data. The modal parameters, are numerically computed to satisfy Eq. (3); they represent the modal parameters predicted by the FEM. However, these numerically computed modal parameters are inevitably different from the corresponding modal properties experimentally measured from the test structure. The discrepancies result from inherent modeling errors, errors in the experimental data (noise and measurement error), and uncertainty in boundary conditions from the experimental set up. Therefore, substituting the modal properties measured experimentally from the test structure into Eq. (2) yields a residual, referred as the dynamic force residual, given by

$$R_i = \left(K - \lambda_i M\right)\Phi_i \tag{4}$$

The purpose of the SBEBE model updating procedure is to alter physically meaningful parameters that define the FE model (e.g., elastic modulus, thickness, etc.) with a goal toward minimizing the dynamic force residual

3.3.1. Modal Expansion Algorithm. From Eq. (4), the dynamic force residual depends on the natural frequencies and modeshapes measured from the structure. The number of degrees of freedom experimentally measured is typically much smaller than the number of degrees of freedom in the finite element model. The measured set is a subset of the complete set of modeshapes. Therefore, to apply Eq. (4), either the model must be reduced to the measured degrees of freedom, or the measured portion of the modeshapes must be expanded to the displacement basis, or the size of the finite element model. Reducing the model to the measured degrees of freedom destroys the connectivity between the elements of the finite element model and, hence, is not recommended for iterative-based model updating procedures [44]. In the SBEBE method proposed in this thesis, a dynamic modal expansion method is used. This method is also referred to as "mode shape projection", as discussed by Alvin [44].

The first step in the modal expansion algorithm is to partition the mode shape, Φ_{iE}, into its measured and unmeasured components, and also to partition the associated columns of the mass and stiffness matrices.

The first step in the modal expansion algorithm is to partition the mode shape, Φ_{iE}, into its measured and unmeasured components, and also to partition the associated columns of the mass and stiffness matrices.

$$R_i = \left([K_m \;\; K_O] - \lambda_i [M_m \;\; M_O]\right) \begin{Bmatrix} \Phi_{m_{iE}} \\ \Phi_{o_{iE}} \end{Bmatrix} \tag{5}$$

where Φ_{m_i} is the modeshape for mode i at the measured degrees of freedom, Φ_{o_i} is the unmeasured portion of the same modeshape, and the m and o subscripts correspond to the column sets of the degrees of freedom. The mode shape projection, or modal expansion algorithm, works by minimizing the dynamic residual with respect to Φ_{o_i}, assuming no change in the model parameters, viz.,

$$\min_{\Phi_o} \sum_i R_i^T R_i \tag{6}$$

Defining $Z_i = K_i - \lambda_i M_i$ as the dynamic stiffness for mode i and partitioning Z_i into sets m and o, the residual R_i can be written as

$$R_i = \left[Z_{m_i} \;\; Z_{o_i}\right] \begin{Bmatrix} \Phi_{m_{iE}} \\ \Phi_{o_{iE}} \end{Bmatrix} \tag{7}$$

Substituting Eq. (7) into Eq. (6) and expanding yields the following minimization problem

$$\min_{\Phi_{o_{iE}}} \left(\Phi_{m_{iE}}{}^T Z_{m_i}{}^T Z_{m_i} \Phi_{m_{iE}} + 2\Phi_{o_{iE}}{}^T Z_{o_i}{}^T Z_{m_i} \Phi_{m_{iE}} + \Phi_{o_{iE}}{}^T Z_{o_i}{}^T Z_{o_i} \Phi_{o_{iE}} \right) \tag{8}$$

which leads to the mode shape projection

$$\Phi_{o_{iE}} = -\left(Z_{o_i}{}^T Z_{o_i}\right)^{-1} Z_{o_i}{}^T Z_{m_i} \Phi_{m_{iE}} \tag{9}$$

Therefore,

$$\Phi_{iE} = \begin{Bmatrix} \Phi_{m_{iE}} \\ \Phi_{o_{iE}} \end{Bmatrix} = \begin{bmatrix} I \\ -\left(Z_{o_i}{}^T Z_{o_i}\right)^{-1} Z_{o_i}{}^T Z_{m_i} \end{bmatrix} \Phi_{m_{iE}} \tag{10}$$

In Eq. (10), Φ_{iE} represents the complete eigenvector matrix formed from the mass and stiffness matrices of the finite element model and the measured modal data from the experiment. Thus, employing a modal expansion algorithm leads to the approximations for the degrees of freedom from each eigenvector, Φ_{iE}, that are not measured from the experiment.

3.4. SBEBE MATHEMATICS

Consider an idealized, correct FE model of the test specimen that has no errors due to modeling. The stiffness and mass matrices for such a model will be designated as K_c and M_c, respectively. These idealized stiffness and mass matrices can be separated into two components. One component is the portion of each matrix that is realistically feasible to develop with standard finite element methodology. The second part, ΔK or ΔM, represents the error between the exact stiffness and mass matrices, K_c and M_c, and the realistic finite element model matrices, *K* and *M*, respectively, giving

$$K_c = K + \Delta K$$
$$M_c = M + \Delta M \tag{11}$$

The correct model also satisfies the eigenvalue problem in Eq. (2), given by

$$K_c \Phi_{iE} = \lambda_i M_c \Phi_{iE} \tag{12}$$

Substituting for *K* and *M* in Eq. (11) in Eq. (12) leads to

$$R_i = \left(\{K_c - \Delta K\} - \lambda_i \{M_c - \Delta M\}\right)\Phi_{iE} \tag{13}$$

Making use of Eq. (12) reduces Eq. (13) to

$$R_i = -\left(\Delta K - \lambda_i \Delta M\right)\Phi_{iE} \tag{14}$$

The stiffness and mass matrices depend on physical parameters, such as the elastic modulus, density, thickness, etc. A small set of these parameters will be altered with the goal of matching the finite element model to the experimentally measured modal data using the SBEBE methodology. The set of physical parameters to be updated will be generally designated by the vector *p*. The changes in these parameters are defined by Δp. The SBEBE method determines the changes, Δp, to a set of physical parameters of the model, that minimize the norm of the dynamic force residual, viz.,

$$\min_{\Delta p}\left(\sum_i \|R_i\|_2^2\right) \tag{15}$$

Deciding which parameters to be updated is an important step in model updating. The parameters selected to update should be sensitive to the changes expected to occur in the structure due to damage. In general, if a parameter cannot reflect a change due to damage, it should not be considered in the set of physical parameters for updating.

The next step after selecting parameters to update is computing the updated parameter values (Δp) which satisfy Eq. (15). To facilitate this process, an estimate E is defined.

$$E = \min_{\Delta p} \sum_i R_i^T R_i \tag{16}$$

The estimator E is determined by expanding R_i in a first-order Taylor series with respect to the parameter variations, Δp,

$$R_i + \delta R_i = Z_i \Phi_i + \sum_j \left(\frac{\partial Z_i}{\partial p_j} \Phi_i \right) \left(\Delta p_j \right) \tag{17}$$

$$= R_i + B_i \Delta p \tag{18}$$

where the subscript i refers to the mode number j refers to the j^{th} element of the p parameter vector being updated and B_{ij} represents the sensitivity of R_i to the j^{th} parameter being updated.

where
$$B_{ij} = \frac{\partial Z_i}{\partial p_j} \Phi_i \tag{19}$$

and

$$B_i = \left[B_{i1}\, B_{i2}\, B_{i3} \cdots\cdots B_{in} \right] \tag{20}$$

also,

$$Z_i = K - \lambda_i M \tag{21}$$

and

$$\frac{\partial Z_i}{\partial p_j} = \frac{\partial K}{\partial p_j} - \lambda_i \frac{\partial M}{\partial p_j} \tag{22}$$

Plugging Eq. (18) into Eq. (16) and minimizing with respect to the parameter variations Δp, gives

$$E = \min_{\Delta p} \sum_i (R_i + B_i \Delta p)^T (R_i + B_i \Delta p) \tag{23}$$

The estimator is minimized when the slope reaches a critical point, i.e.,

$$\frac{\partial}{\partial \Delta p}\left[\sum_i (R_i + B_i \Delta p)^T (R_i + B_i \Delta p) \right] = 0 \tag{24}$$

Equation (23) reduces to

$$\sum_i B_i^T B_i \Delta p + B_i^T R_i = 0 \tag{25}$$

or simply

$$G\Delta p = -g \tag{26}$$

where $$G = \sum_i B_i^T B_i \quad \text{and} \quad g = \sum_i B_i^T R_i \tag{27}$$

The solution to Eq. (28) is

$$\Delta p = -G^{-1} g \tag{28}$$

From Eq. (27), the perturbed global stiffness (ΔK) and mass (ΔM) matrices are given by

$$\Delta K = \sum_j \left(\frac{\partial K_i}{\partial p_j} \right) (\Delta p_j) \quad \text{and} \quad \Delta M = \sum_j \left(\frac{\partial M_i}{\partial p_j} \right) (\Delta p_j) \tag{29}$$

The corrected stiffness and mass matrices are obtained by substituting the value of ΔK, ΔM from Eq. (28) into Eq. (11)

$$K_c = K + \sum_j \left(\frac{\partial K}{\partial p_j} \right) (\Delta p_j) \qquad M_c = M + \sum_j \left(\frac{\partial M}{\partial p_j} \right) (\Delta p_j) \tag{30}$$

Eq. (30) represents the updated global mass and stiffness matrices of the finite element model. Until equation (30), it is a one step procedure. The procedure of correcting the elemental stiffness and mass matrices with the modeshapes and frequencies obtained from test, should be continued until a maximum relative change in the selected parameter reaches a maximum. In the case considered the iterations continue until the change in the selected parameter vector (Δp) to the original parameter vector p reaches to a value of 10^{-6}, as given by Eq. (31),

$$\left|\frac{\Delta p_j}{p_j}\right| \leq 10^{-6} \tag{31}$$

When the iteration process converges, the residual R_i approaches its minimum value, thus yielding a finite element model that more closely matches or represents the true test structure. When damage is present in the structure, the finite element model parameters at the elemental level are updated in such a fashion to represent the true natural changes that occurred as a result of the damage. Elements where the largest changes in the elemental properties have occurred are the most probable elements where damage has occurred. Later sections discuss the application of the SBEBE method to different structures.

4. NUMERICAL RESULTS

4.1. INTRODUCTION

The aim of this section is to assess the performance of the Sensitivity Based Element-by-Element (SBEBE) algorithm to numerical experiments. Noisy and noise free numerical modal data are the two sub cases considered. The cases where noise is added to the numerical data are considered in order to mimic the physical experiment situation as close as possible. Numerical results are presented for a cantilever beam.

4.2. CANTILEVER BEAM

The finite element model of the cantilever beam will be developed using Euler Bernoulli beam theory. Consider the cantilever beam shown in Figure. 4.1, in which each node has two degrees of freedom.

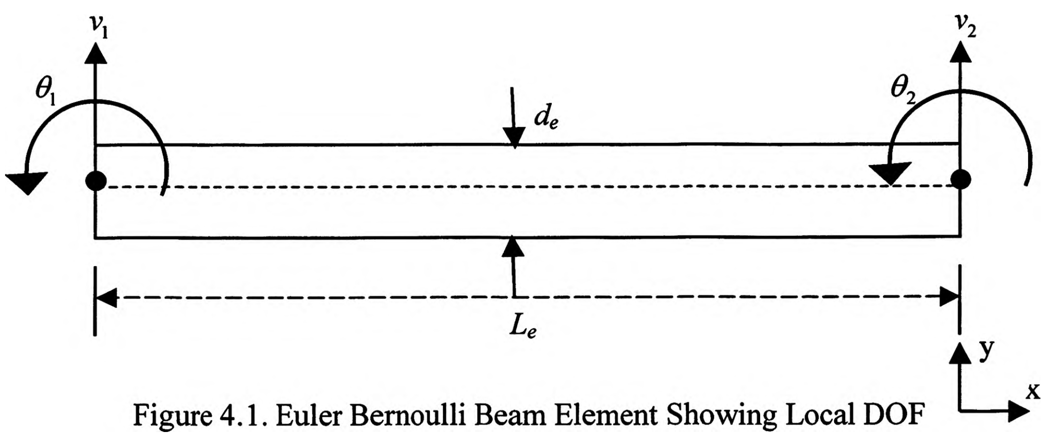

Figure 4.1. Euler Bernoulli Beam Element Showing Local DOF

The subscripts *1* and *2* refer to the two end nodes of the element. Equations (20) and (21) define the elemental stiffness (K_e) and mass matrices (M_e) for the two degree of freedom beam element.

$$[K_e] = \frac{2EI}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h^2 \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix} \tag{33}$$

$$[M_e] = \frac{\rho A h}{420}\begin{bmatrix} 156 & -22h & 54 & 13h \\ -22h & 4h^2 & -13h & -3h^2 \\ 54 & -13h & 156 & 22h \\ 13h & -3h^2 & 22h & 4h^2 \end{bmatrix} \tag{34}$$

$$[M_e] = 0.05\rho A h\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{35}$$

where E is the modulus of elasticity, I is the moment of inertia, h is the element thickness, ρ is the mass density of beam material, and A is the cross sectional area of the element in the yz plane shown in the Fig. 4.1. Equations (34) and (35) represent the consistent and lumped mass matrices, respectively.

4.2.1. Damage Detection in Beam Using Numerical Modal Data. The objective of this section is to test the performance of the SBEBE algorithm to numerical modal data obtained for a cantilever beam. In order to achieve this, a cantilever beam (Figure 4.2) is divided into a number of finite elements. At each element the stiffness matrix is generated using equation (33). The mass matrix is either generated using equations (34) or (35), depending upon the choice of mass matrix. Every element in the beam is connected to neighboring element by a node, the complete set of such connecting node numbers is defined to be the connectivity matrix and is used to develop the global stiffness matrix. The global stiffness and mass matrices are obtained by assembling the elemental stiffness and mass matrices respectively, using the connectivity matrix. In equation form elemental assembly can be written as:

$$GK = \sum_{n=1}^{N} K_e \qquad GM = \sum_{n=1}^{N} M_e \tag{36}$$

where GK and GM represent the global stiffness and mass matrices, respectively. Typically, damage to a structure e.g., crack growth, fiber breaking, corrosion etc., manifests as an area of localized weakness. This physical damage is simulated by reducing the modulus of elasticity for an element or small group of elements at the specific location of damage. The elemental stiffness and mass matrices for the damaged structure are generated using equations (33), (34) and (35). The corresponding global stiffness and mass matrices are found using equation (36). It should however be remembered that stiffness and mass matrices simulating the "damaged" structure are used only to generate the modal test data and are not to be used as the input matrices for the SBEBE algorithm. Modal test data is then generated numerically by performing eigenvalue analysis using the simulated global stiffness and mass matrices.

The complete set of eigenvectors consists of modeshapes ranging from *1* to *N*, where *N* is the number of degrees of freedom in the finite element model. These eigenvectors are composed of transverse displacements and rotations at each node of the structure. In the numerical results obtained, only the transverse displacement parts of the eigenvector are considered. The rotational degrees of freedom are obtained through a modal expansion algorithm discussed in section 3.3.1. The complete set of eigenvectors for the simulation cases are formed with the rotational DOF calculated using the modal expansion algorithm.

4.2.2. Numerical Modal Data Without Noise. With these simulated, eigenvalues and eigenvectors or modeshapes from the "damaged" structure and the finite element model, the objective of the SBEBE algorithm is to locate the damage and specify its extent. In order to achieve this objective, a FE model of the cantilever beam is generated by dividing the beam into 10 finite elements. Damage is simulated at element number 5 (Figure 4.2) by reducing its elastic modulus by a prescribed percentage of its original value. Elemental stiffness matrices are generated using equations (33). Consistent and lumped mass formulations (Eq's. (34) and (35)) respectively are considered. Global stiffness and mass matrices are generated using equation (36). Eigenvectors and eigenvalues from the simulated "damaged" structure are obtained using an eigenvalue analysis program in MATLAB.

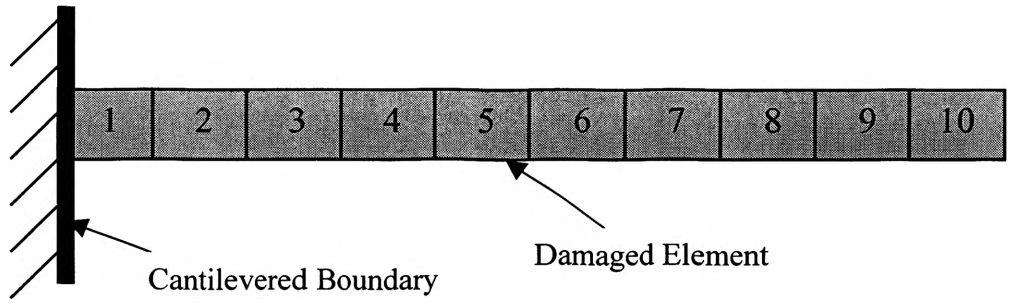

Figure 4.2. Cantilever Beam with reduced Elastic Moduli at Element 5

Only first mode is used in the analysis and modal expansion is used to compute the rotational degree of freedom in the eigenvector. Figure 4.3 shows the relative percentage change in each elemental elastic modulus after updating with the SBEBE method. The bar plot shows a 10 percent relative change in the modulus of elasticity at element number 5 after 87 iterations. This result matches exactly with the induced damage. Employing either a consistent or lumped mass matrix has no affect on the result or on the time of convergence. The following cases were run to test the performance of the algorithm using various number of modes, and the type of mass matrix employed in the finite element model.

Table 4.1 lists the number of iterations needed for convergence when employing lumped and consistent mass matrices. With 10-element beam case the number of iterations needed to converge depended on the number of modes used in the analysis. Increasing the number of modes improved the convergence rate. Employing either a lumped or consistent mass matrix made little difference to the convergence rate.

The location and extent of damage were predicted accurately using the SBEBE method for numerical modal data obtained for a 10-element beam with numerically induced damage. Only one mode from the numerical modal data was sufficient in detecting the damage.

4.2.3. Numerical Modal Data with Noise. The aim of this section is to test the SBEBE algorithm using numerical modal data polluted with noise. The purpose of

Table 4.1 Cantilever Beam Case with 10 Elements and Varying Number of Modes

Number of Modes Used In the Analysis	Number of Iterations necessary Using Lumped Mass Matrix	Number of Iterations necessary Using Consistent Mass Matrix
Mode 1	87	86
Mode 1 and 2	87	86
Mode 1, 2 and 3	36	35
Mode 1, 2, 3 and 4	15	21

assessing the performance of the algorithm to modal data with noise is to mimic a realistic test environment as close as possible. The section will allow a means to quantify the extent to which noise is present in the numerical data and still detect damage. Since the number of modes from an experiment will be limited, the minimum number of modes needed to assess damage detection is also studied in this section.

As before, 10-element cantilever beam case is considered. In this case, however, noise is added to the numerically produced eigenvectors in the following way. An array of random numbers from −1 to +1 is generated and then multiplied by an appropriate scaling factor equal to some small percentage of the maximum value in the eigenvector array. This new array of scaled random numbers is multiplied by the eigenvector array and the product is then added to the eigenvectors to simulate the noise. Noise is added only to the transverse displacements and not to the rotational degrees of freedom. The rotational degrees of freedom in the eigenvectors are approximated from the transverse displacements using the modal expansion algorithm as discussed in section 3.3.1. In Table 4.2, a study has been made by considering different percentages of noise added to and number of modes needed to detect the location and extent of damage. In general, more modes are needed to yield acceptable results as the percentage of noise increases. For example, for noise levels less than 1 percent, (cases 1 and 2) the location and extent of damage is successfully predicted only when the first three modes were used.

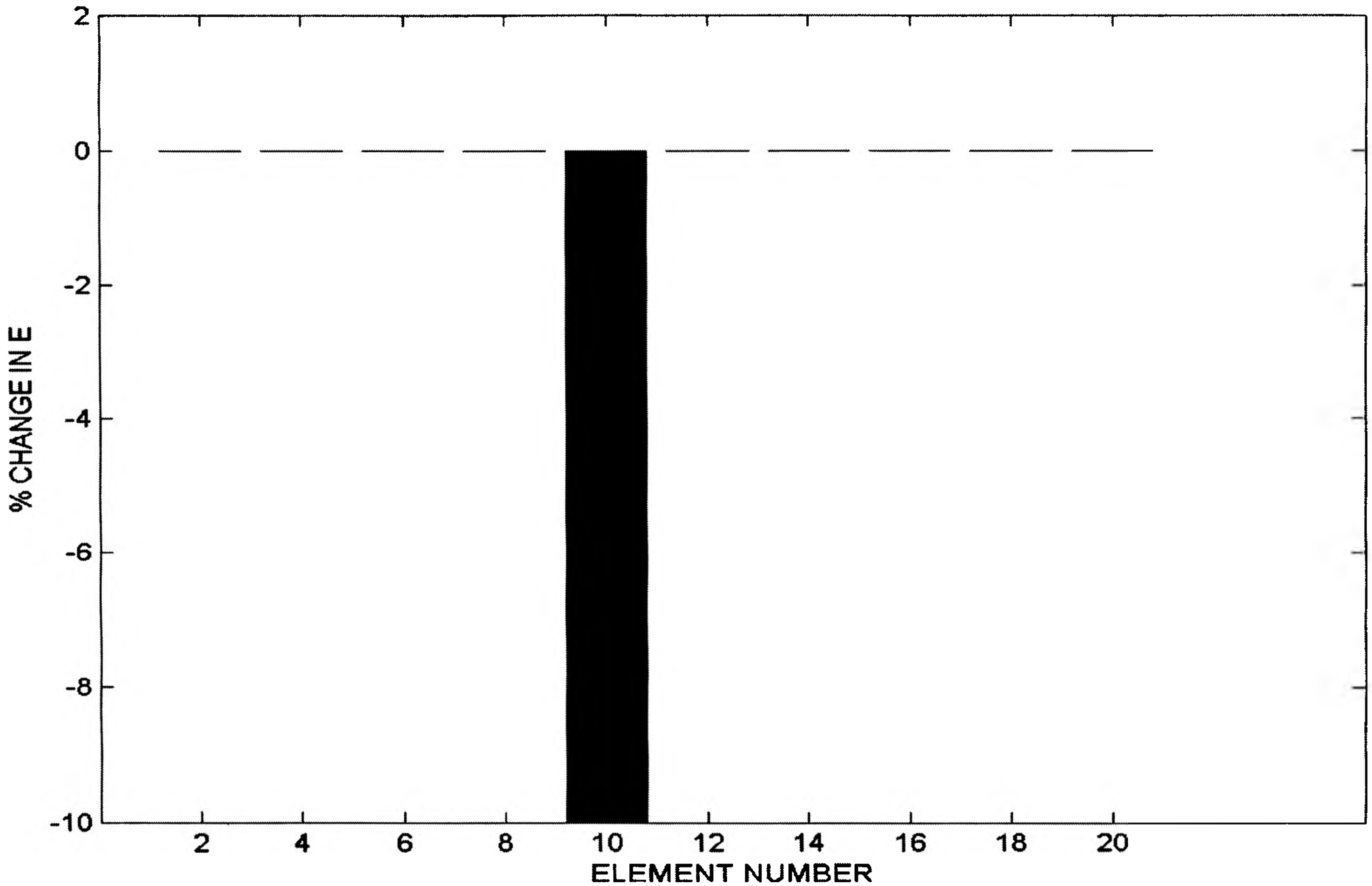

Figure. 4.3. SBEBE Applied to 10 Element Beam Case with One Mode and No Noise

Therefore, only one mode is needed to confidently yield the location and severity of damage. However, in cases 3, 4, 5 (Fig 4.4, 4.5) where the noise percent was more than 1 percent three modes are needed to predict the location and severity. When the percentage of noise reaches to 5 the algorithm fails to predict the location and extent of damage in this case. This result suggests that the SBEBE algorithm is acutely sensitive to noise in the data so much so that it could in some cases inhibit this technique from being used as a, practical non-destructive testing method.

Table 4.2. Modal Updating with Varying Noise Percents and Number of Modes

Case No	Percent Noise	Modes Employed	Number of Iterations Necessary	Extent of Damage Detected	Mass Formulation
1	0.1	1	84	Undetected	Consistent
2	0.1	1, 2, 3	37	0.095	Consistent
3	3.0	1, 2, 3	33	Undetected	Consistent
4	3.0	1, 2, 3, 4	55	0.115	Consistent
5	5.0	1, 2, 3, 4	No Convergence	Undetected	Consistent

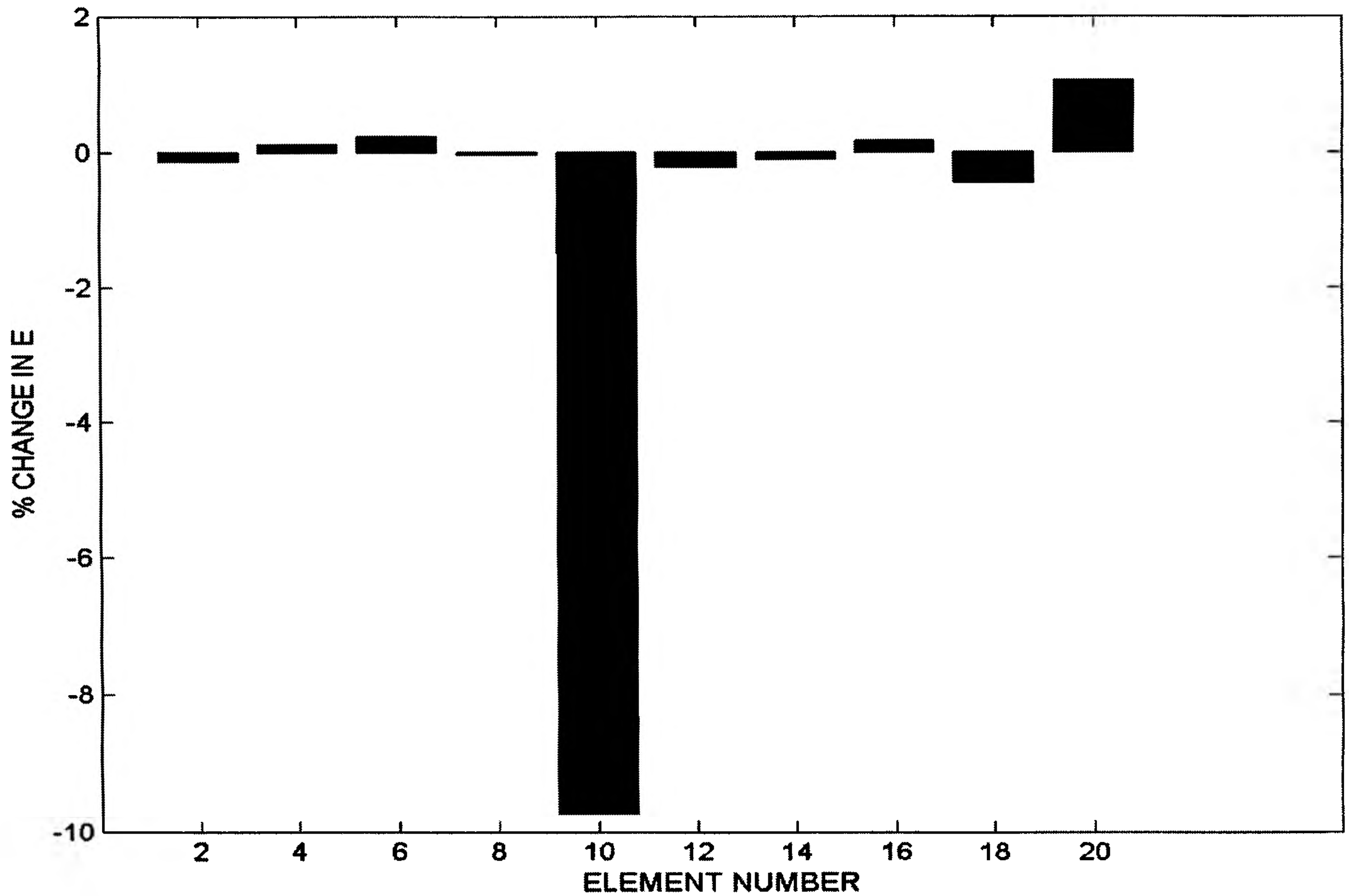

Figure 4.4. SBEBE Applied to a 10 Element Beam Case with 10 Percent Decrease in Elastic Modulus in Element 5 and 0.1 Percent Noise and Three Modes

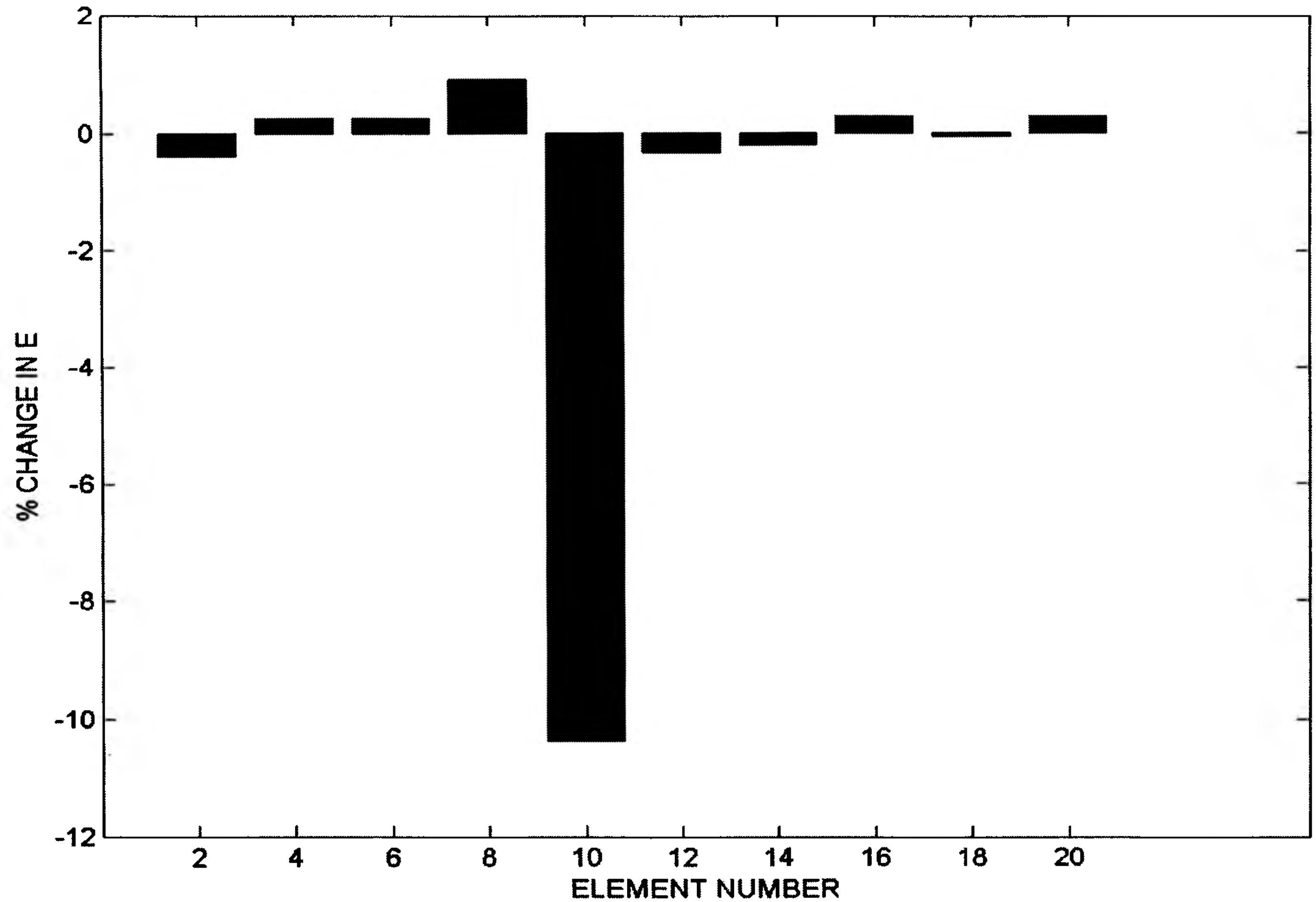

Figure 4.5. SBEBE Applied to a 10 Element Beam Case with 10 Percent Decrease in Elastic Modulus in Element 5 and 3.0 Percent Noise and Four Modes

5. EXPERIMENTAL CASE STUDY

5.1. INTRODUCTION

The overall aim of this chapter is to investigate the applicability of the introduced Sensitivity Based Element-by-Element (SBEBE) technique in experimentally detecting localized damage in a cantilever beam. A cantilever beam as shown in Figure 5.1, is chosen because of its simplicity and convenience in modeling and testing. The nominal dimensions of the beam are 22 x 0.75 x 0.1875 inches (See Fig. 5.1). For the finite element model, the material properties are estimated to be: elastic modulus, $E=10^6$ psi and density, $\rho=0.0975$ lb-force/in^3.

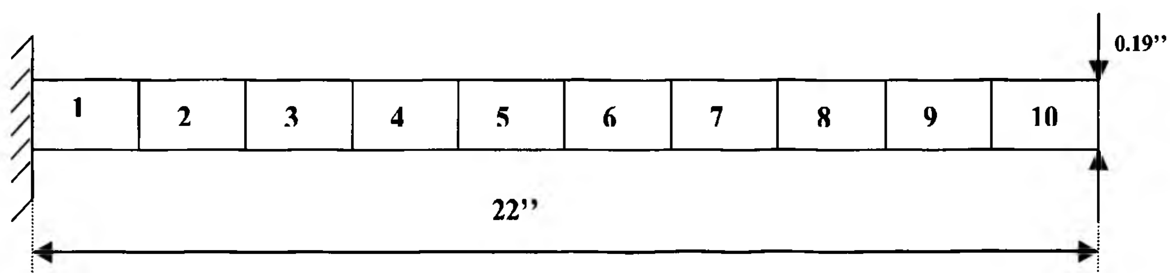

Figure 5.1 Cantilever beam with 10 Finite Elements

5.2. "HEALTHY" AND "DAMAGED" TEST SPECIMENS

Two beam structures were used in assessing the performance of the algorithm using experimentally measured data. These two beams were geometrically the same, except in one beam a deep slot was removed to simulate damage. Approximately half of the thickness of the beam was removed. The beam in which the cut is present is referred to as the "damaged" beam, while the one with no cut is referred to as the "healthy" beam. Figure 5.2 depicts the location of the notch in the beam and gives a photograph of both beams.

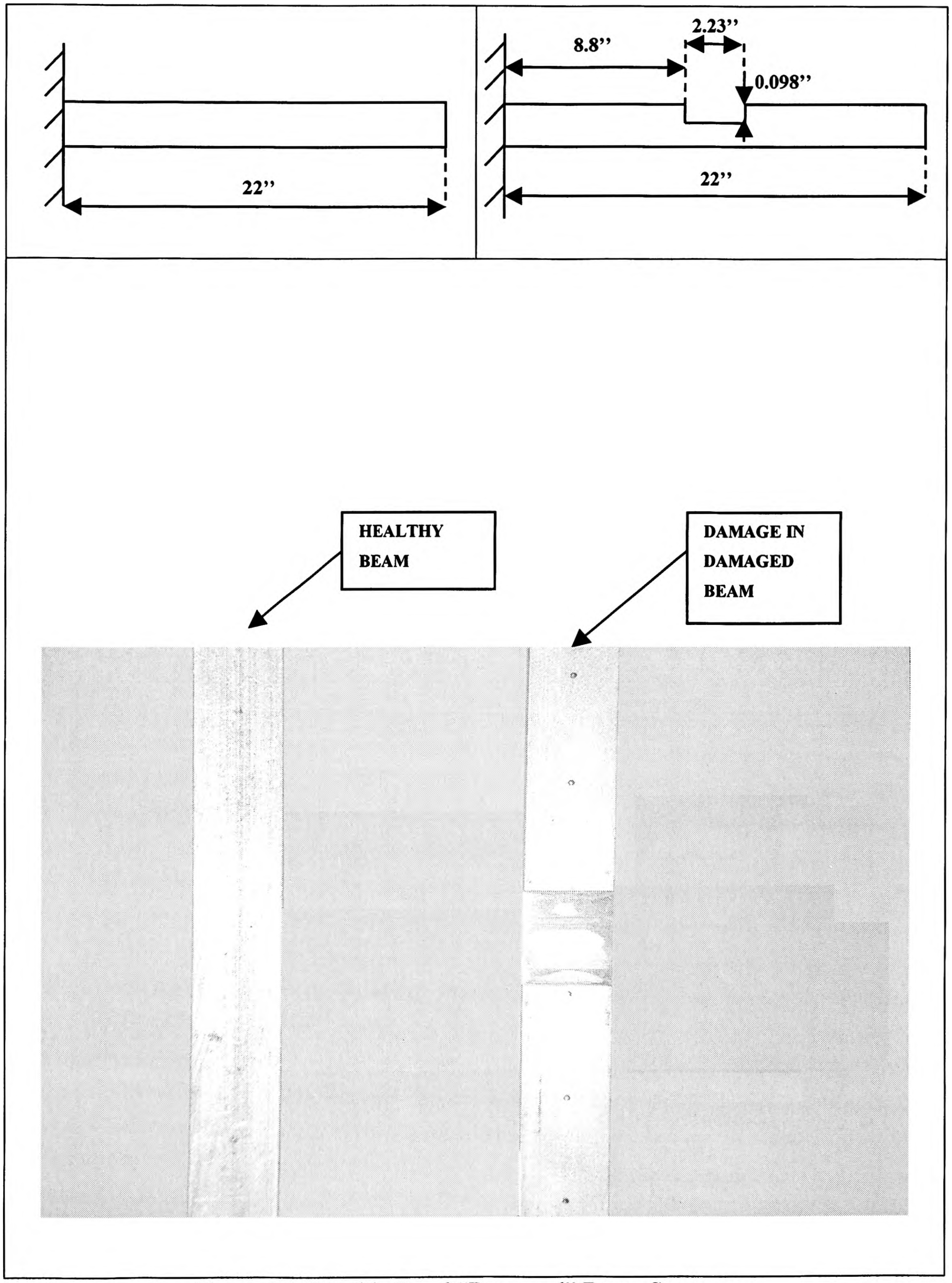

Figure 5.2 “Healthy” and “Damaged” Beam Structures

5.3. FINITE ELEMENT MODELING OF CANTILEVER BEAM

As shown in Fig. 5.1, the cantilever beam is divided into ten finite elements. Euler-Bernoulii beam elements (Eq. (33) and (34)) are used in generating the elemental stiffness and mass matrices. Global stiffness and mass matrices are obtained by assembling the elemental stiffness and mass matrices, respectively, using the connectivity matrix, represented by Eq. (23).

5.4. EXPERIMENTAL TEST SET UP

The experimental data in the form of mode shapes and natural frequencies of the structure were obtained using a scanning laser doppler vibrometer (Model no# OFV 512, Manufacturer: Polytec). The laser vibrometer is a non-contact, full-field system for automated vibration measurement, visualization and analysis. A picture of the laser vibrometer is shown in Figure 5.3

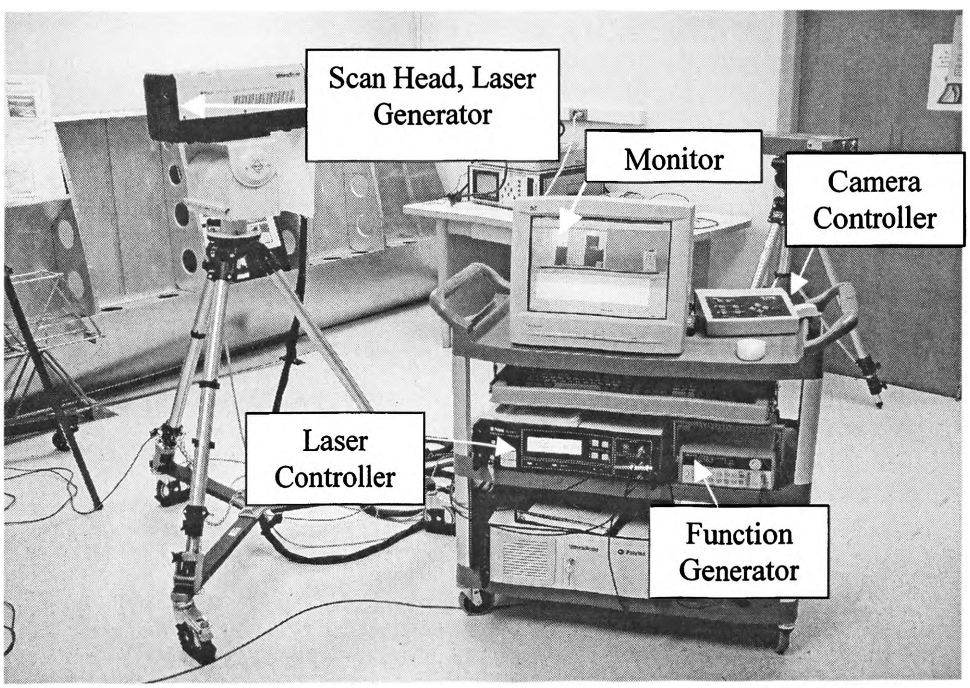

Figure 5.3 Experimental set up for the Collection of Modal Data from the Cantilever Beam Structure

For measuring the vibration of points over an area, the laser is scanned over the test surface. Movement of the laser beam is controlled by a set of mirrors mounted inside the scan head. A digital data management system controls the whole process of positioning the beam, mirror movements and velocity measurement at each scanned point. The analog velocity is digitized, processed and stored. The data acquisition consists of defining the grid with points of measurement on the structure, specifying the test parameters such as type of signal to excite the structure, frequency bandwidth, sampling ratios etc. Once these characteristics are defined, the structure is excited and scanned by the laser beam to measure the vibration signature at each point on the grid. The data is stored and made available for further analysis.

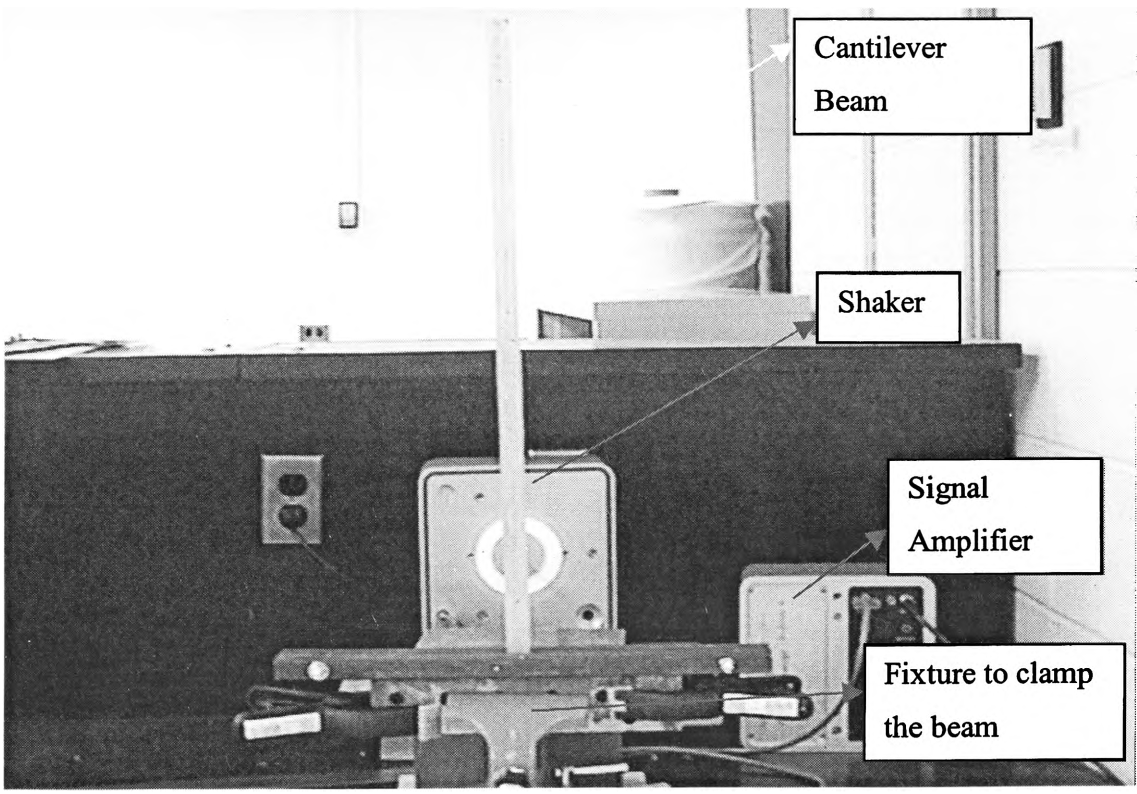

Figure 5.4 Experimental set up to Measure the Modal Properties of a Cantilever Beam

5.4.1. Experimental Set Up of Cantilever Beam. The first three transverse mode shapes of the "healthy" and "damaged" structures are measured by the laser vibrometer. To reduce experimental noise, the experiments are performed 6 times on each structure, and the mode shapes from all the experiments are averaged to obtain the final results. The beam is clamped in a cantilever position (See Fig. 5.4). It is excited using a magnetic stinger near the fixed end. The beam is excited using a periodic chirp function ranging between 0 to 200 Hz generated using an Hewlett Packard 8111A/001 pulse function generator, controlled by the laser vibrometer controller.

5.5. CORRELATION OF EXPERIMENTAL AND PREDICTED RESULTS FROM BEAM STRUCTURE

Prior to the updating the FE model of the beam structure, a comparison of the experimental and FE data was carried out. This involves correlation of modes and frequencies. The modes and frequencies obtained from finite element model and from experiment are matched to observe the quality and closeness of data. Table5.1 and Figures 5.5, 5.6, 5.7, 5.8 represent the comparison of frequencies and first, second, third and fourth modeshape comparison to FEM and experiment. Two specific cases healthy and damaged were considered from experiment. It can be noticed from all the figures that they are close and lie on top of each other. It should be noticed that the numerical data obtained through simulation and experimental data fall close. It shows the assurance that can be kept on the data from the experimental case.

Table 5.1. Natural frequencies of FEM vs Experiment

Mode Number	Natural Frequency from FEM (Hz)	Natural Frequency From Experiment	
		Undamaged	Damaged
Mode One	12.4	13.1	12.1
Mode Two	79.0	79.6	68.6
Mode Three	227.9	224.3	229.3
Mode Four	467.6	460.6	428.8

5.6. SBEBE MODEL UPDATING PROCEDURE APPLIED TO EXPERIMENTAL DATA

In this section, the procedure for damage detection is laid out using experimental data. There are 5 steps involved in the damage detection process:

1. Collect the modal data from the healthy structure.
2. Update the original FEM with the modal data from step 1, which yields a FEM of the healthy structure
3. Collect the modal data from the damaged structure.
4. Update the FEM of the healthy structure with the new modal data measured from the damaged structure.
5. Compare the FEM of healthy and damaged structures for damage location and quantification.

In step 1, the modal data from the healthy beam structure is collected using the laser vibrometer. The modal data consists of natural frequencies and modeshapes. Figure 5.8 depicts the four modeshapes and the associated natural frequencies for the healthy structure. In step 2, the modal data obtained from the healthy beam structure is used as the input for updating the original FEM of the beam structure. The elemental moduli of elasticity (E) are considered as the parameters to be modified. The updating process is continued until the maximum relative change in the selected modulus of elasticity parameter is small, 10e-4 in this case, when the iteration process converges, a finite element model is yielded which closely matches or represents the healthy beam structure. This FEM is referred as the FEM of the healthy structure, as it represents the original FEM, updated with the healthy modal test data. The moduli of elasticity parameters in this step are stored for comparison purposes. The updated frequencies are listed in Table 5.2. The updated frequency does not match to that of the healthy beam test structure, this is due to the fact that, no constraints (orthogonality) were kept on the natural frequencies.

Step 3 is similar to step 1, except that modal data from the damaged structure is collected. Figure 5.9 depicts the modeshapes and natural frequencies from the damaged cantilever beam. The comparison of the damaged beam modal data to the healthy beam modal data is shown in Figures, 5.5, 5.6, 5.7 and 5.8.

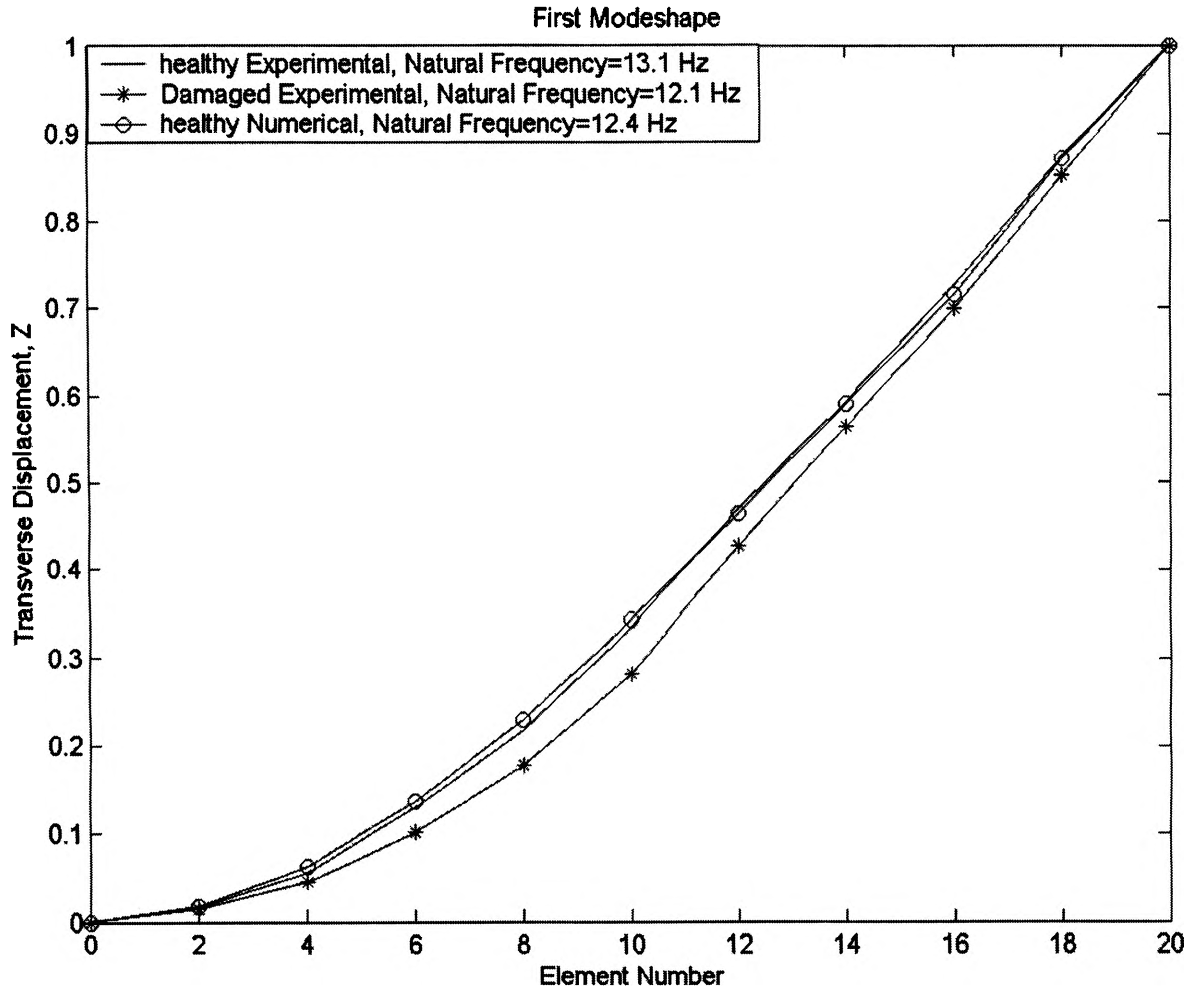

Figure 5.5. First Mode Shape of 10 Element Cantilever Beam

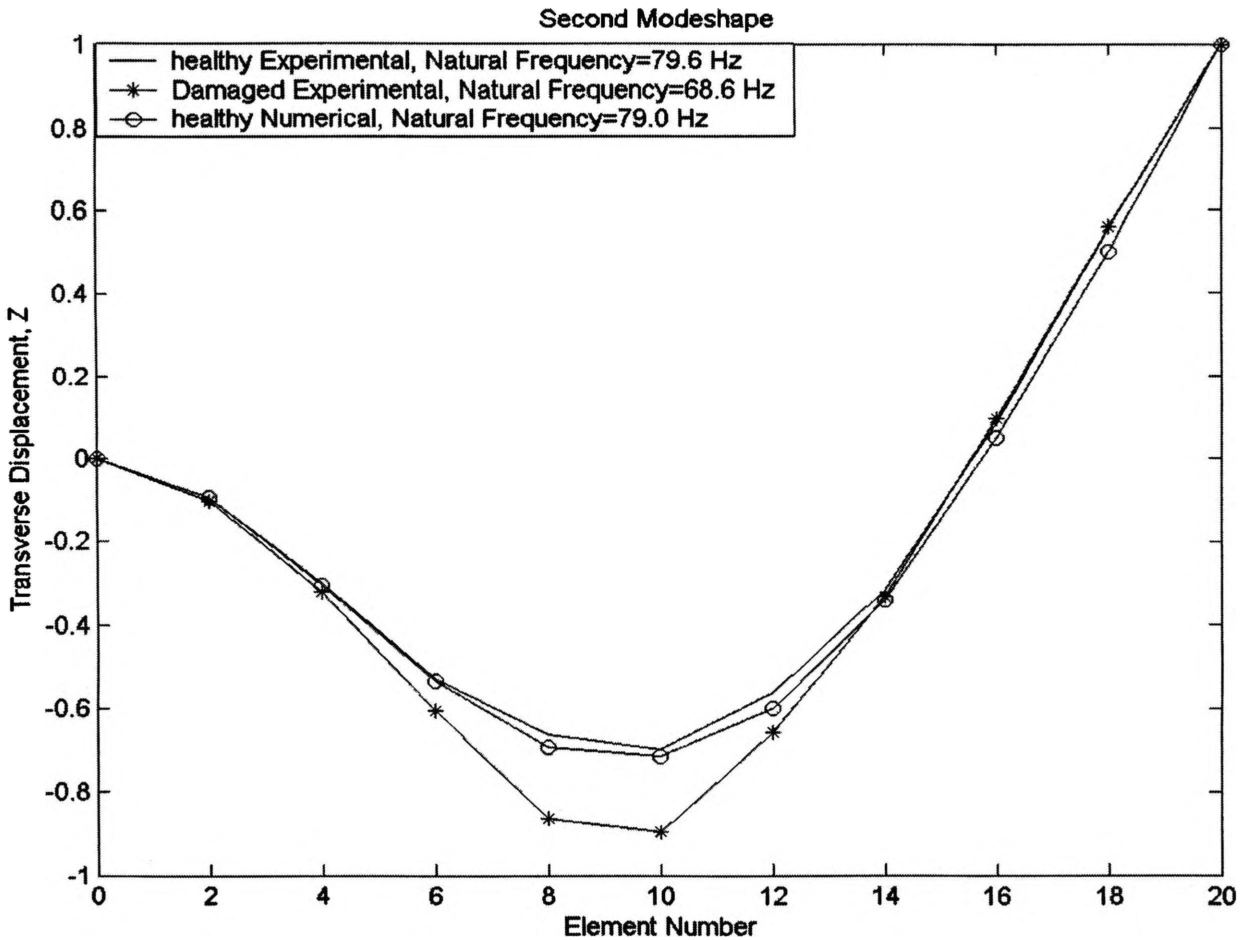

Figure 5.6. Second Mode Shape of 10 Element Cantilever Beam

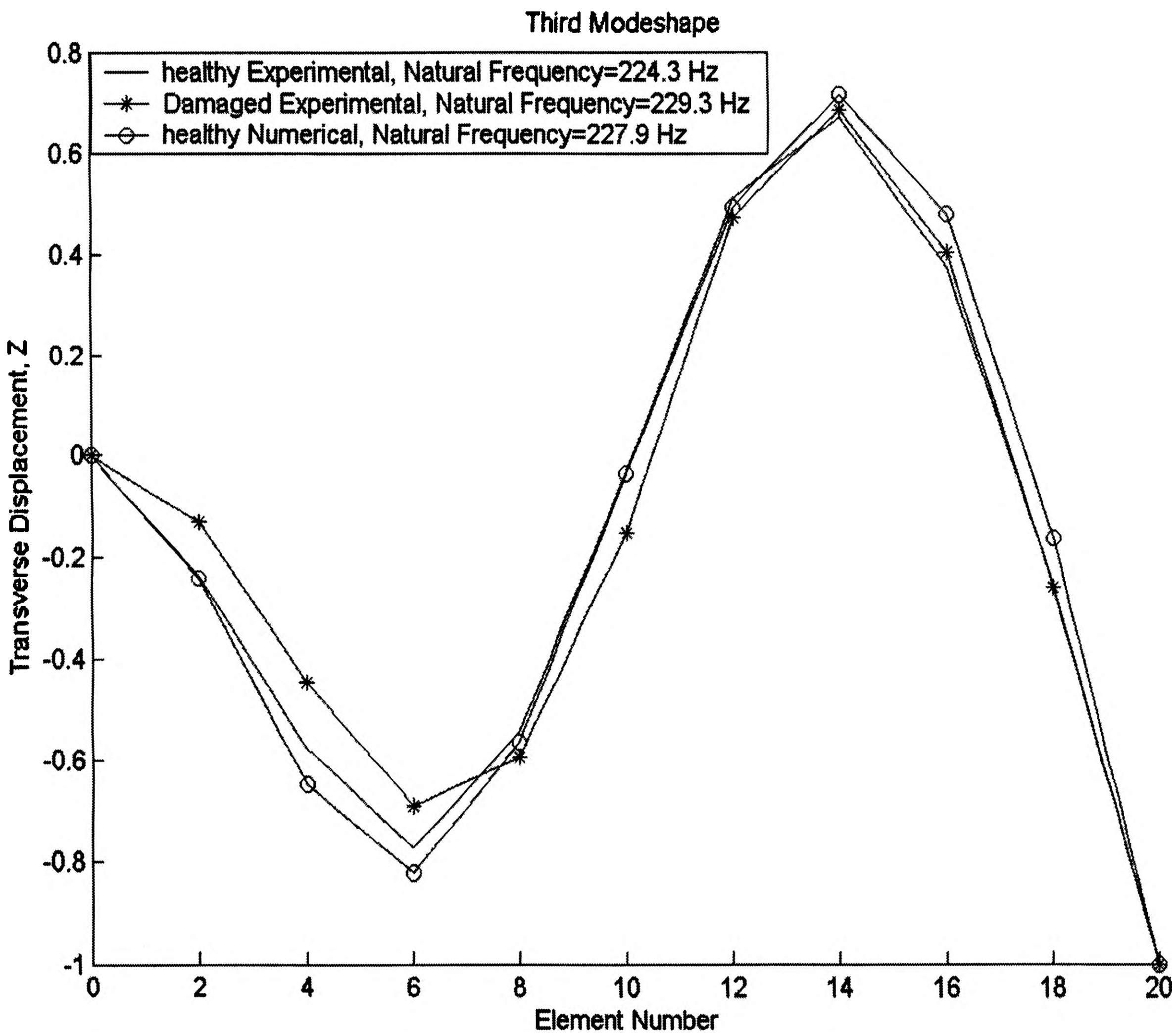

Figure 5.7. Third Mode Shape of 10 Element Cantilever Beam

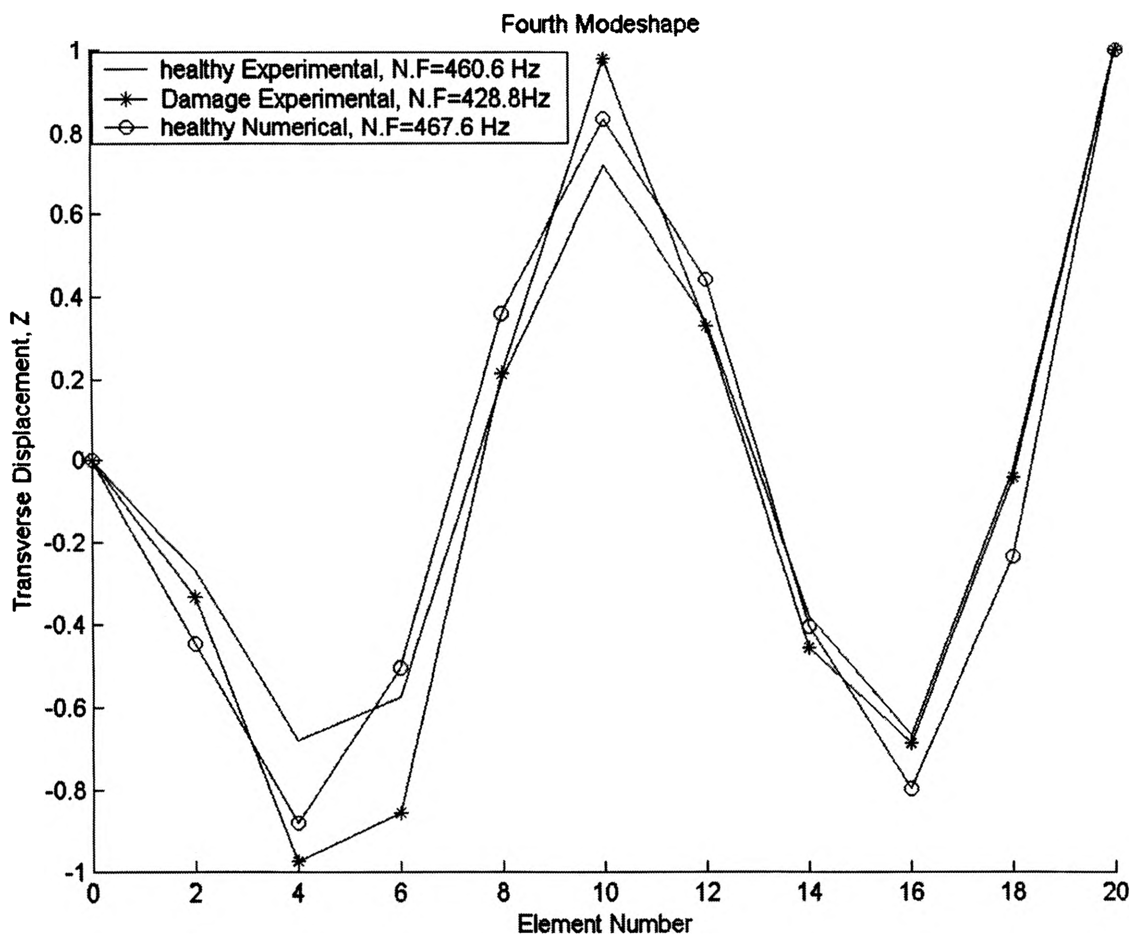

Figure 5.8. Fourth Mode Shape of 10 Element Cantilever Beam

Step 4 is a crucial step in the damage detection process. The modal data obtained from step 3 (modal data from the damaged structure) is used in updating the FEM obtained from step 2 (FEM of the healthy structure). The elemental moduli of elasticity (E) are again chosen to maintain consistency and for comparison purposes. Updating is performed, and the iteration process is continued again until the maximum relative change in each E is small. The updated model now represents FEM of the damaged structure. Table 5.3 displays the comparison of frequencies for healthy, damaged structure and damaged structure after update.

In this final step 5, the presence, location and severity of the damage is predicted based on the modulus of elasticity parameters obtained in step 2 and 4. Table 5.4 displays the modulus of elasticity parameters stored at steps 2 and 4. A relative change in the moduli of elasticity in step 2 and step 4 predicts the presence, location and severity of damage. A thresholding technique was employed to threshold or predict the actual damage location. The thresholding technique works by considering the largest positive magnitude value of the elastic moduli in step 2 and everything below the threshold is assumed to be noise, so it is ignored by setting it to zero. Figure 5.10 shows that maximum reduction in modulus of elasticity occurs at element 5 and is the point where the damage was induced. There is approximately a 70 %(Table 5.5, E5) reduction in modulus of elasticity at element 5, and corresponds to the severity of damage, indicating that there is a 55% reduction in stiffness at that element.

Table 5.2. Natural Frequencies of FEM, Healthy Structure after and before Update

Mode Number	Natural Frequency from FEM (Hz) Before Updating	Natural Frequency Experimentally Measured from the Healthy Structure (Hz)	Natural Frequency from the FEM of the Healthy Structure (Hz)
Mode One	12.4	13.1	12.2
Mode Two	79.0	79.6	78.5
Mode Three	227.9	224.3	223.7
Mode Four	467.6	460.6	460.2

Table 5.3. Natural Frequencies of Healthy, Damaged Structure after and before Update

Mode Number	Natural Frequency from the Healthy Structure After the First Updating Procedure (Hz)	Natural Frequency Experimentally Measured from Damaged Structure (Hz)	Natural Frequency from the FEM of the Damaged Structure After Updating the FEM (Hz)
Mode One	12.2	12.1	11.29
Mode Two	78.5	68.6	66.34
Mode Three	223.7	229.3	218.2
Mode Four	460.2	428.8	417.8

Table 5.4. Modulus of Elasticity before and after Update

Element Number	Modulus of Elasticity from FEM Original 10^8*	Modulus of Elasticity From Healthy Structure 10^8* (X1)	Modulus of Elasticity From Damaged Structure FEM 10^8* (X2)	Relative Difference between Step 2 and Step 4 (X2-X1)/X1
E1	1.0000	1.0209	1.1247	0.1016
E2	1.0000	1.0381	0.8840	-0.1484
E3	1.0000	0.4652	0.4389	-0.0565
E4	1.0000	0.3943	0.3251	-0.1755
E5	1.0000	0.3893	0.1136	**-0.7083**
E6	1.0000	0.4675	0.3277	-0.2989
E7	1.0000	0.4791	0.7936	0.0594
E8	1.0000	0.6544	0.6696	0.0232
E9	1.0000	0.5090	0.4735	-0.0697
E10	1.0000	4.6007	4.2361	-0.0793

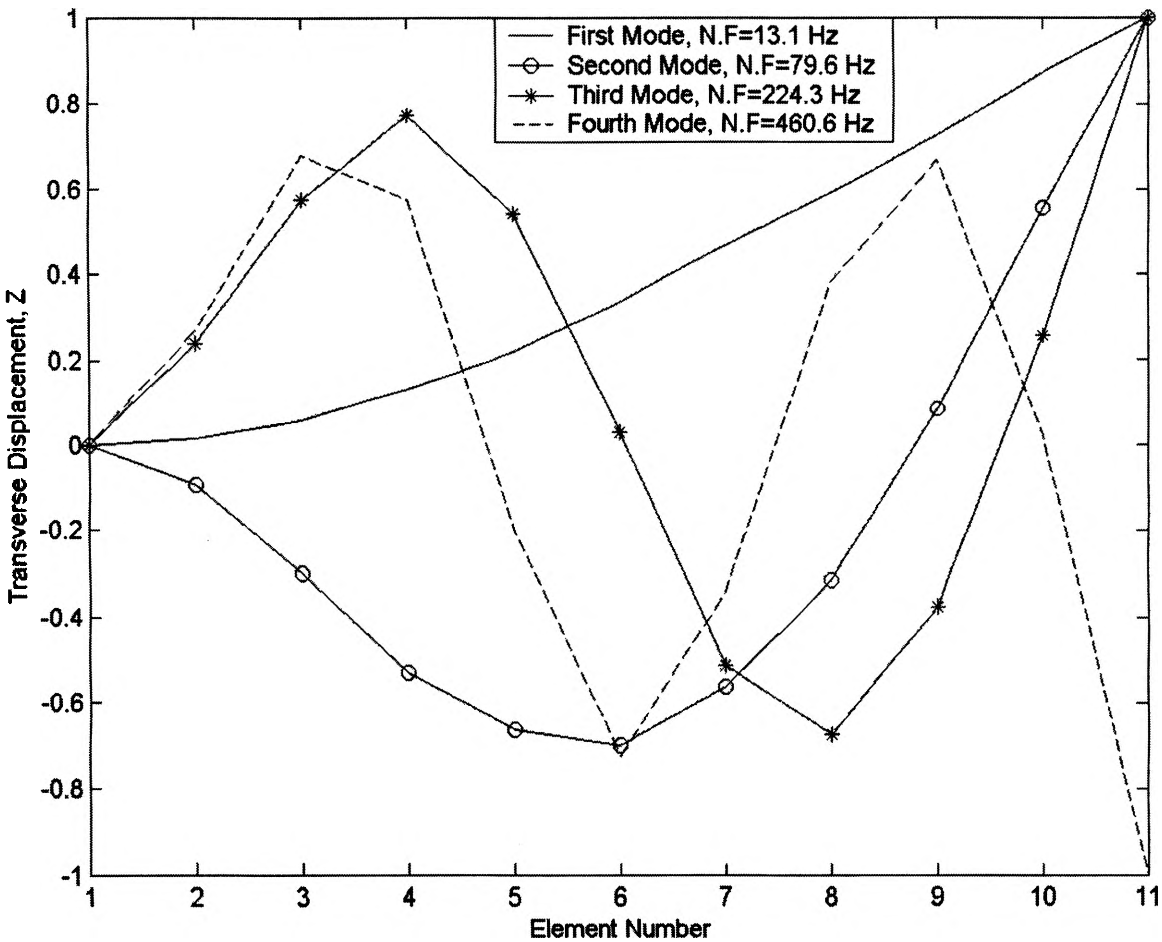

Figure 5.9. Mode Shape Data from Healthy Cantilever Beam Measured Experimentally

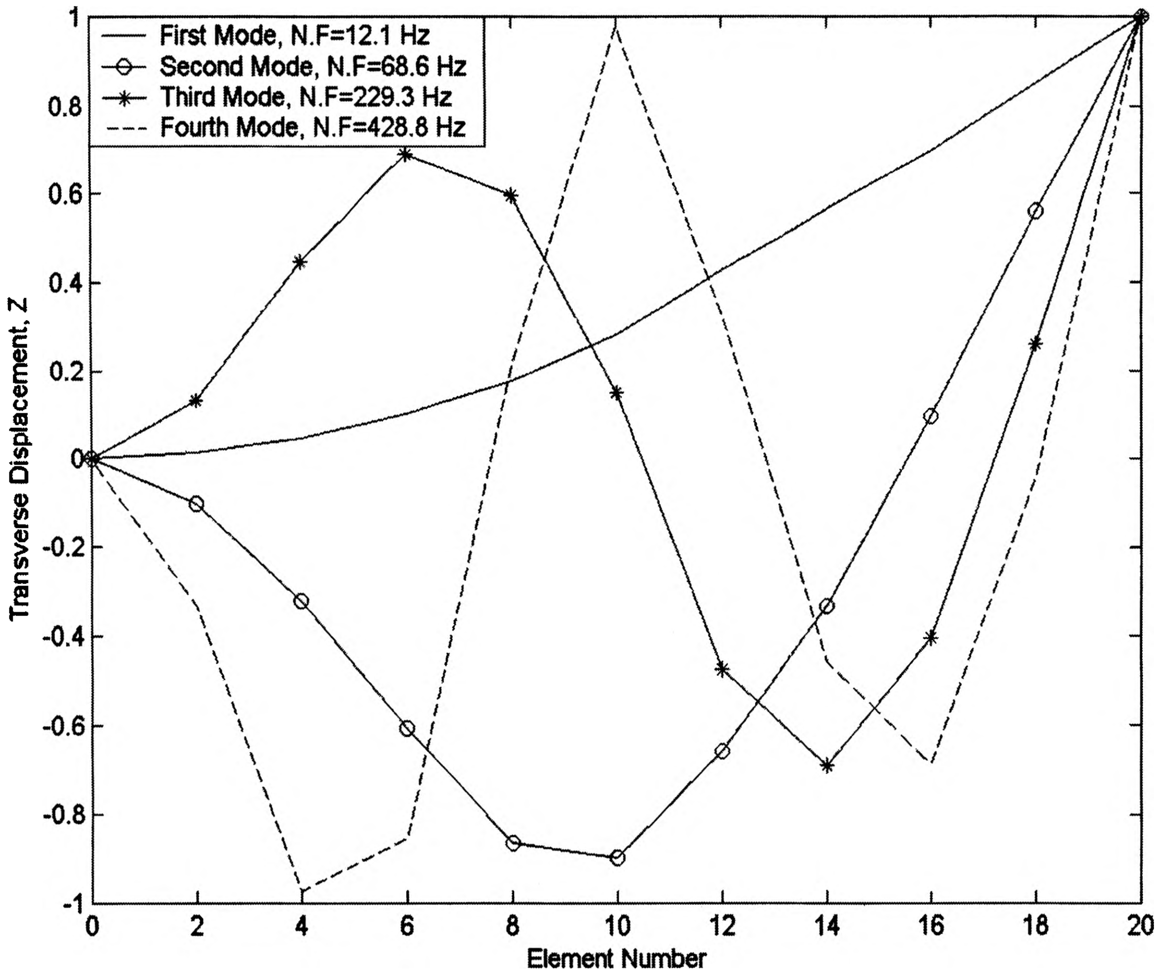

Figure 5.10. Mode Shape Data from Damaged Cantilever Beam Measured Experimentally

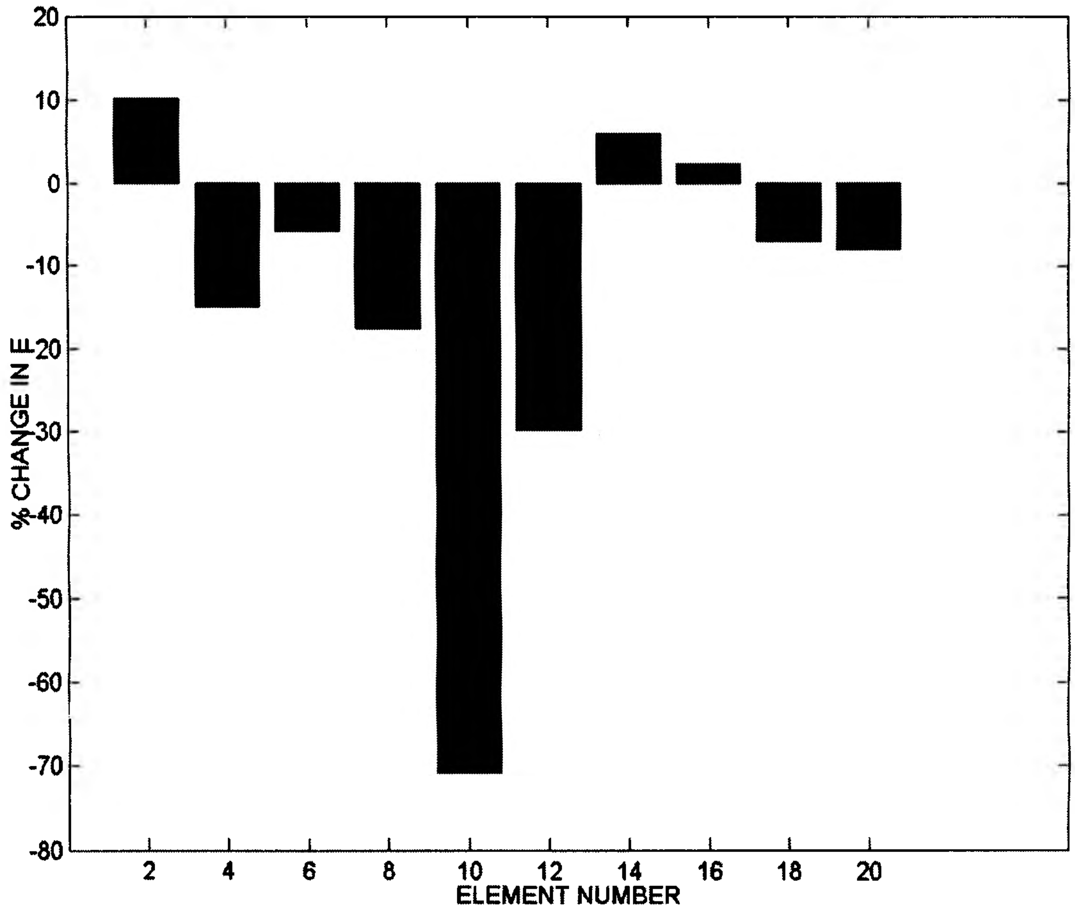

Figure 5.11. Relative Change in the Elastic Modulus after Updating the FEM of the Healthy Beam with Modal Data from the Damaged Beam

6. CONCLUSIONS

The current study was undertaken to investigate the application of the Sensitivity Based Element-by-Element (SBEBE) methodology to detect the presence of damage, the location, if present, and its severity in structures. The method has been tested with numerical experiments with and without noise added as well as with experimentally measured data. Numerical modal data has been generated using Finite Element Analysis, while the experimentally measured data was obtained using a laser vibrometer.

Numerical experiments were performed on a cantilever beam to test the algorithm performance using numerically generated data with and without noise added. The location and extent of damage were predicted accurately for the cases without added noise. Increasing the number of modes used in the algorithm helped to improve the algorithm's convergence rate. The addition of noise to the modal data was found to deteriorate the damage detection process. When noise percentages above 5% of the original modal data amplitudes were added, the damage detection process was completely inhibited. In general, increasing the number of modes used in the SBEBE algorithm helped in detecting damage with the presence of noise. These results show that the SBEBE algorithm is sensitive to noise, so much so that significantly noisy data may hinder the damage detection process using the SBEBE algorithm.

Finally, the performance of the algorithm using experimentally measured data has been studied. A "damaged" beam was constructed by mechanically removing a small section of the beam, and the model updating technique was applied to detect the damage. Damage was detected in the correct location, and its severity was also predicted accurately within 5% of the theoretical value. However, sizeable traces of damage were also predicted in the neighboring material where no damage was located. This error can be attributed to the sensitivity of the algorithm to noise.

Overall, the SBEBE algorithm successfully detected the presence and severity of damage in a cantilever beam using numerically and experimentally generated data. The technique was found to be sensitive to noise, and using more modes in the algorithm tended to increase its effectiveness. Therefore, when using this algorithm, significant effort should be placed toward reducing noise in the data and obtaining the maximum number of modes possible.

7. BIBILIOGRAPHY

1. Witherell, C. E., "Mechanical Failure Avoidance: Strategies and Techniques", McGraw-Hill, New York, 1994.

2. Doebling, S. W., "Damage Identification and Health Monitoring of Structural and Mechanical Systems from Changes in Their Vibration Characteristics: A Literature Review," Los Alamos National Laboratory, Paper No.LA-13070-MS, 1996.

3. Rytter, A., "Vibration Based Inspection of Civil Engineering Structures," Ph.D. Dissertation, Department of Building Technology and structural engineering, Aalborg University, Denmark, 1993.

4. Baruch, M., "Optimal Correction of Mass and Stiffness Matrices using Measured Modes," *AIAA Journal* **20(11)**, 1623-1626, 1982.

5. Baruch, M., and Bar Itzhac, Y., "Optimization Weighted Orthogonalization of Measured Modes," *AIAA Journal* **16(4)**, 346-351, 1978.

6. Berman, A., "Mass Matrix Correction using an Incomplete Set of Measured Modes," *AIAA Journal* **17(1)**, 1147-1148, 1979.

7. Berman, A., and Nagy, E .J., "Improvement of a Large Analytical Model using Test Data," *AAIA Journal* **21(8)**, 1168-1173, 1983.

8. Caesar, B., "Updating System Matrices using Modal Test Data," *Proc. of the 5th IMAC*, 453-459, London, England, 1987.

9. Wei, F. S., "Structural Dynamic Model Improvement using Vibration Test Data," *AIAA Journal* **28(1)**, 1686-1688, 1990.

10. Fuh, J. S., and Chen, S. Y., "System Identification of Analytical Models of Damped Structures," *AIAA Journal* **30(2)**, 384-392, 1984.

11. Heylen, W., and Sas, P., "Review of Model Optimization Techniques," *Proc. of the 5th IMAC*, 1177-1182, London, England, 1987.

12. Ross, R.G., "Synthesis of Stiffness and Mass Matrices from Experimental Vibration Modes," *National Aeronautics and Space Engineering and Manufacturing Meeting Society of Automotive Engineers*, 2627-2635, 1971.

13. Caesar, B., "Updating System Matrices using Modal Test Data," *Proc. of the 5th IMAC*, 453-459, London, England, 1987.

14. Sidhu, J., and Ewins, D. J., "Correlation of Finite Element and Modal Test Studies of a Practical Structure," *Proc. of the 2nd IMAC*, 756-762, Orlando, Florida, 1984.

15. Lieven, N. A. J., and Ewins, D. J., "Error Location and Updating Finite Element Models Using Singular Value Decomposition," *Proc. of the 8th IMAC*, 768-773, Kissimmee, Florida, 1990.

16. Maia, M., "An Introduction to the Singular Value Decomposition Technique," *Proc. of the 7th IMAC*, 335-339, Las Vegas, Nevada, 1989.

17. Lieven, N. A. J., and Ewins, D. J., "Effect of Incompleteness and Noise on Error Matrix Calculations," *Proc. of the 10th IMAC*, 1406-1413, San Diego, California, 1992.

18. Porter, B., and Crossley, R., "Modal Control Theory and Applications," *London: Taylor and Francis*, 1972.

19. Moore, B. C., "On The Flexibility Offered by State Feedback in Multivariable System Beyond Closed Loop Eigenvalue Assignment," *IEEE Transactions on Automatic Control*, 689-692, 1976.

20. Srinathkumar, S., "Eigenvalue/Eigenvector Assignment using Output Feedback," *IEEE Transaction on Automatic Control*, 79-81, 1978.

21. Andry, A. N., and Chung, J. C., "Eigenstructure Assignment for Linear Systems," *IEEE Transactions on Aerospace and Electronic Systems* **19(5)**, 711-729, 1983.

22. Shulz M. J., and Inman, D. J., "Model Updating using Constrained Eigenstructure Assignment," *Journal of Sound and Vibration* **178(1)**, 113-130, 1994.

23. Ziaei Rad, S., and Imregun, M., "A Modified Eigenstructure Assignment Technique for Finite Element Model Updating," *Journal of Shock and Vibration* **3(4)**, 247-258, 1996a.

24. Shulz, M. J., and Inman, D. J., "Model Updating Using Constrained Eigenstructure Assignment," *Journal of Sound and Vibration* **178(1)**, 113-130, 1994.

25. Fox, R. L., and Kapoor, M. P., "Rate of Change of Eigenvalues And Eigenvectors," *AIAA Journal* **12(6)**, 2426-2429, 1968.

26. Lim, B., "Re-Examination of Eigenvector Derivatives," *AIAA Journal of Guidance Control and Dynamics* **10(6)**, 581-587, 1987.

27. Chu, Y., and Rudisill, C. S., "Numerical Methods for Evaluating The Derivatives of Eigenvalues And Eigenvectors," *AIAA Journal*, 834-837, 1975.

28. Ojalvo, I. U., "Efficient Computation of Mode Shape Derivatives for Large Dynamic Systems," *AIAA Journal*, 1386-1390, 1987.

29. Tan, R. C. E., and Andrew, A. L., "Computing Derivatives Of Eigenvalues And Eigenvectors by Simultaneous Iteration," *Institute of Mathematics and its Application, Journal of Numerical Analysis* **9(1)**, 111-122, 1989.

30. Chen, J. C., and Garba, J. A ., "Analytical Model Improvement using Modal Test Results," *AIAA Journal* **18(6)**, 684-690, 1980.

31. Hart, G. C., and Yao, J. P., "System Identification in Structural Dynamics. American Society of Civil Engineers," *Journal of Engineering Mechanics Division*, 1089-1104, 1977.

32. Ojalvo, I. U ., Ting, T., Pilon, D .,and Twomey, W., "Practical Suggestions for Modifying Math Models to Correlate with Actual Modal Test Results," *Proc. of the 7th IMAC,* 347-354, Las Vegas, Nevada, 1989.

33. Natke, H. G., "Updating Computational Models in the Frequency Domain Based on Measured Data: A Survey," *Probabilistic Engineering Mechanics*, 28-35, 1988.

34. Thomas, M., "Identification of System Physical Parameters from Force Appropriation Technique," *Proc. of the 4th IMAC*, 1098-1103, Los Angeles, California, 1986.

35. Dascotte, E., and Vanhonacker, P., "Development of an Automatic Model Updating Program," *Proc. of the 7th IMAC*, 596-602, Las Vegas, Nevada, 1989.

36. Dascotte, E.,"Practical Applications of Finite Element Tuning using Experimental Modal Data," *Proc. of the 8th IMAC,* 1032-1037, Kissimmee, Florida, 1990.

37. Wei, J. C., "Correction of Finite Element Model Via Selected Physical Parameters," *Proc of the 7th IMAC*, 1231-1238, Las Vegas, Nevada, 1989.

38. Dascotte, E., "Material Identification of Composite Structure from Combined Use of Finite Element Analysis and Experimental Modal Analysis," *Proc. of the 10th IMAC*1274-1280, San Diego, California, 1992.

39. Kuo, C. P., and Wada, B. K., "Nonlinear Sensitivity Coefficient and Correction in System Identification," *AIAA Journal*, 1463-1468, 1987.

40. Ojalvo, I. U., and Pilon, D., "A Second Order Iteration Procedure for Correlation of Analysis Frequencies with Test Data," *Proc. of the 9th IMAC*, 499-502, Florence Italy, 1991.

41. Friswell, M. I., and Mottershead, J. E., "Finite Element Model Updating in Structural Dynamics," *Kluwer Academic publishers*, 1995.

42. Dascotte, E., and Vanhonacker, P., "Development of an Automatic Model Updating Program," *Proc. of the 7th IMAC*, 596-602, Las Vegas, Nevada, 1989.

43. Friswell, M. I., and Mottershead, J. E., "Finite Element Model Updating in Structural Dynamics," *Kluwer Academic publishers*, 1995.

44. Alvin, K. F., "Finite Element Model Update via Bayesian Estimation and Minimization of Dynamic Residuals," *AIAA Journal* **35(5)**, 879-886, 1997.

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