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Sustainability analysis in integrated inventory control and transportation systems

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SUSTAINABILITY ANALYSIS IN INTEGRATED INVENTORY CONTROL AND TRANSPORTATION SYSTEMS

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ABSTRACT

Due to the importance of costs as well as environmental effects of logistical activities throughout supply chains, such as inventory holding, freight transportation, and warehousing activities, this dissertation models and analyzes four integrated inventory control and transportation problems that account for economic and environmental aspects of a supply chain agents related decisions.

The first model presents an integrated inventory control and transportation problem in a single item deterministic demand setting. A supply chain agents inventory control and transportation mode selection problem is solved under carbon cap, carbon cap and trade, carbon cap and offset, and carbon tax regulations. The second model focuses on an integrated inventory control and transportation problem in a single item stochastic demand setting integrating environmental objectives into a continuous review inventory control system with considerations of two different transportation modes.

The third model studies an integrated inventory control and transportation problem in a multi-item deterministic demand setting, in which, a decision making method is developed considering the economic and environmental objectives. In the fourth model, a multi-item stochastic demand consolidation policy is analyzed with the consideration of heterogeneous freight trucks for transportation. It is shown that the consolidation policy suggested can result in substantial economic as well as environmental benefits for the supply chain agents.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

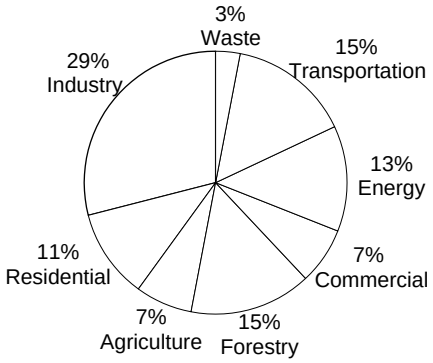
1. INTRODUCTION, MOTIVATIONS, AND LITERATURE REVIEW

In the past decade, the public's concerns for the environment have been increasing (Borgstede et al., 2013). Their concerns have been rising due to the increases in the generation of greenhouse gases which, unless the rate of greenhouse gas generation is controlled, may become a major threat to the human race and the current way of life (Hua et al., 2011; Benjaafar et al., 2013). Figure 1.1a documents the global greenhouse gas emissions by economic sector in 2010 (ECOFYS, 2010). Figure 1.1b compares the 2011 U.S. greenhouse gas emissions by economic sector (EPA, 2013a)[1]. The United States (US) Environmental Protection Agency (EPA) notes that the contribution of the transportation sector to national greenhouse gas emissions in 2011 was around 27% (EPA, 2013a). As seen in Figure 1.1c, when different transportation modes are compared, it is noted that trucking constitutes the second largest greenhouse gas emission generator following passenger transportation (EPA, 2013a). This implies that freight transportation by trucks dominates the greenhouse emissions compared to other freight transportation modes such as rail, air, and marine transportation. It is also noted that over 75% of greenhouse gas emissions from domestic freight transportation in the U.S. are due to trucking activities (FHWA, 2011).

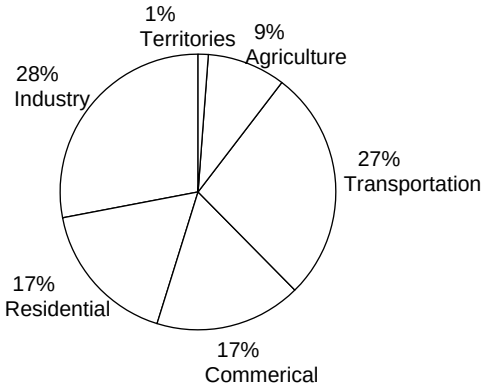
In terms of monetary value, the total U.S. business logistics costs in 2011 was $1.33 trillion, 8.5% of the U.S. gross domestic product (Wilson, 2013). This was also an increase of 6.6% from the year before. A report by the U.S. Federal Highway Adminstration (FHWA, 2005) specifies that transportation and inventory holding costs account for 96% of the logistics costs in the U.S. It is, therefore, important in any logistics research question to consider both the transportation and inventory holding components of the logistics costs. Moon et al. (2011) notes that many companies have looked into strategies to optimize their inventory control and delivery policies in recent years and have found significant cost savings.

These statistics are not surprising as trucks are the most common transportation mode used for freight transportation. According to the FHWA, over 68% of freight tonnage is shipped by trucks and the FHWA further notes that, "By 2040, long

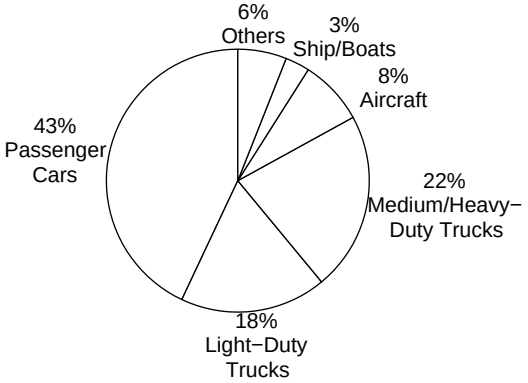
[1]Due to rounding, the totals may not sum up to 100% (EPA, 2013a).



(a) Global GHG emissions by economic sector in 2010



(b) US GHG emissions by economic sector in 2011



(c) US GHG emissions by transportation mode in 2011

Figure 1.1. Greenhouse gas (GHG) emissions statistics

haul freight truck traffic in the United States is expected to increase dramatically on interstate highways and other arterials throughout the nation" and truck travel is predicted to reach 662 million miles per day (FHWA, 2008). Similar observations are noted for the European Union countries. Forecasted growth of freight transportation

from 2000 to 2020 in European countries is noted to be 50% (Toptal and Bingol, 2011). In the European Union, approximately 20% of total greenhouse gas emissions were due to transportation in 2010 (EEA, 2013).

Due to the aforementioned global climate change awareness in recent decades, both government officials and private corporations are looking into ways to reduce the global carbon footprint. The Kyoto Protocol (UNFCCC, 1998) was introduced by the United Nations and was one of the first major pushes towards reducing greenhouse gas emissions. It was originally signed in 1997 by the European Union and 37 United Nations states. The protocol has since been ratified by 191 United Nations states, and continues to be in use in the European Union (UNFCCC, 2014). In 1995, the European Commission began an emissions trading system that covers 45% of the total greenhouse gas emissions from the 27 European Union countries (ECCA, 2013). The European Union's cap and trade system imposes a cap on emissions but allows companies to sell or buy excess carbon credits, as necessary.

Another method legislatures are using is placing a tax on emissions. This method is considered efficient for emissions reduction and it was first used by Denmark, Finland, Sweden, Netherlands, and Norway (Lin and Li, 2011). The New Zealand Emissions Trading Scheme, and the Regional Greenhouse Gas Initiative are some examples of government programs established to help companies reduce their carbon emissions. Voluntary programs such as the Chicago Climate Exchange, the Montreal Climate Exchange, and many carbon offset companies also serve to this end (Toptal et al., 2014).

The aforementioned regulations directly force companies to update their operational strategies and become more sustainable. Nevertheless, these regulations are not the only motivation for companies to become more sustainable. According to a 2011 survey of over 4,000 managers from 113 countries, the changing public opinion on the environment has encouraged 70% of the surveyed companies to permanently place sustainability in their management agendas (Haanaes et al., 2012). The same survey also demonstrated an increase from 55% in 2010 to 67% in 2011 of respondents saying that sustainability practices are necessary for being competitive. For instance, Bouchery et al. (2012) note that companies choosing to become more sustainable are not only improving their public image but are also getting a competitive

advantage as a result of selling greener products. Another survey study, conducted among 582 European companies, notes that while environmental regulations were the top motivation for companies to implement green actions in 2008, brand image improvement and executive board decisions became the top motivation to becoming more sustainable in 2010 (Loebich et al., 2011).

Either due to environmental regulations or the pressure from customers, the companies, as parts of supply chains, are replanning their operations toward becoming more sustainable. Recent review papers on sustainable supply chain management document the necessity and importance of integrating sustainability with supply chain and operations management (see, e.g., Corbett and Kleindorfer, 2001, Linton et al., 2007, Dekker et al., 2012). The focus in this dissertation is on a company's inventory control and transportation decisions with environmental considerations. In particular, inventory holding, freight transportation, and logistics and warehousing operations are the main emissions generators throughout supply chains along with the manufacturing processes. The inventory control policy of a company derives the levels of inventory and transportation, and logistics and warehousing activities; hence, it is the key determinant of the emissions generated. Furthermore, inventory control is an important activity that appears in almost any type of organization (Tsou, 2008). Because of these observations, the research in this dissertation is needed and it applies to a wide audience. Throughout the research, environmental regulations and objectives were integrated, among other contributions, into four practical inventory control models.

Environmental considerations have been recently integrated into mostly single item inventory control models with both deterministic and stochastic demand scenarios. In particular, environmental considerations are modeled within inventory control models via either associating costs with the environmental hazard of the logistics activities, reformulating the models under environmental regulations or regarding environmental objectives along with the classical economical objectives.

The classical single item deterministic inventory control model, the economic order quantity (EOQ) model, has been revisited with environmental considerations. Bonney and Jaber (2011) reformulate the cost function of the EOQ model by regarding the costs associated with transportation emissions and waste disposal to the

environment in addition to the classical EOQ cost components. In a similar study, Ritha and Martin (2012) revisit the EOQ model by defining additional cost terms for packaging, transportation and packaging emissions, and waste disposal. Digiesi et al. (2012) extend the EOQ model with transportation emissions costs such that the transportation emissions generation rate depends on the delivery speed, which is defined as a decision variable. Recently, Battini et al. (2014) define a sustainable EOQ model through associating costs with warehousing, inventory holding, and transportation emissions.

The EOQ model has also been analyzed with environmental regulations. Specifically, carbon regulations[2] such as carbon cap, carbon tax, and cap and trade are integrated into the EOQ model. Hua et al. (2011) study the EOQ model with a carbon cap and trade regulation, where a retailer is subject to a cap on its carbon emissions level and carbon emissions are tradable through a trading mechanism such as the European Trading System or New Zealand Trading System. They derive an expression for the optimal order quantity and investigate how costs and carbon emissions change with carbon trading price. Chen et al. (2013), on the other hand, examines the EOQ model with a carbon cap regulation. They discuss how sensitive the costs and emissions are to the carbon cap and extend their model for carbon tax regulation. Similar to Hua et al. (2011) and Chen et al. (2013), Arslan and Turkay (2013) revisit the EOQ model with carbon cap, tax, and cap and trade regulations as well as carbon offset regulation. In a recent study, Toptal et al. (2014) combines the EOQ model with carbon emissions reduction investment decisions under cap, tax, and cap and trade policies. They show how carbon emissions regulations and emission reduction investment opportunities affect costs and carbon emissions. In this dissertation, Section 2 studies the EOQ model with four different carbon emissions regulations. Furthermore, the EOQ model has been extended to account for two types of carriers: less-than-truckload (LTL) and truckload (TL) carriers. Section 2, therefore, contributes to the EOQ models with environmental regulations by integrating different transportation modes into the model, which enables comparison of different transportation modes in terms of cost and environmental performance.

[2]Generally, carbon emissions are considered as the environmental performance as other greenhouse gas emissions can be measured in terms of equivalent carbon emissions (see, e.g., EPA, 2013b)

Additionally, a discussion is presented on how the carbon emissions regulations can affect the transportation mode selected.

Finally, the environmental considerations are included into the EOQ model with consideration of environmental objectives. In the study by Bouchery et al. (2012), a sustainable EOQ model is formulated as a multi-objective optimization model, in which a set of sustainability criteria is minimized along with the costs. Sections 3 and 4 also integrate environmental considerations into two different inventory control models by modeling environmental objectives.

The studies cited so far assume single item deterministic demand in a single echelon supply chain. It should be noted that inventory control models within two echelon supply chains under the settings of the EOQ model have also been analyzed with environmental considerations. Particularly, the buyer-vendor coordination problem with deterministic demand has been analyzed in recent studies. Saadany et al. (2011) focus on the single buyer, single vendor coordination problem in the case that the environmental performance of the single item considered affects its demand. Similarly, the single buyer, single vendor coordination problem has been analyzed by Swami and Shah (2013) and Zavanella et al. (2013) such that the demand of the single item depends on its price as well as environmental quality. Wahab et al. (2011) integrate environmental considerations into a single buyer, single vendor coordination problem by associating costs with carbon emissions and Jaber et al. (2013) revisit the single buyer, single vendor coordination problem under environmental regulations. In a recent study, Chan et al. (2013) formulate a multi-objective multi-buyer, single vendor coordination problem. Specifically, they use utility functions for different environmental criteria and use a weighted approach to solve the resulting multi-objective model.

The above studies look at inventory control with environmental considerations under deterministic demand. There is a limited number of studies that consider inventory control models with stochastic demand. Specially, the single period stochastic demand inventory control model, i.e., the classical newsvendor model has been revisited with environmental considerations. For instance, Song and Leng (2012) and Liu et al. (2013) focus on the newsvendor model under carbon regulations. Zhang and Xu (2013) formulate a multi-item newsvendor model in a production planning setting

under a carbon cap and trade regulation. Rosic and Jammernegg (2013) revisit the newsvendor model with dual sourcing under carbon tax and cap and trade regulations and Hoen et al. (2014) study the transportation mode selection problem in the newsvendor model with carbon emissions costs as well as carbon cap, tax, and cap and trade regulations. Choi (2013a,b) study sourcing and supplier selection models with stochastic demand in fashion and apparel industries under carbon taxing. In a recent study, while they do not directly consider environmental costs, objectives, or regulations, Arikan et al. (2013) discuss the effects of lead time variability on environmental performance in a continuous review inventory control model. Section 3 of this dissertation contributes to the state of the art and the state of the practice on inventory control models with environmental considerations by modeling a continuous review inventory control model as a multi-objective optimization problem. Furthermore, similar to Section 2, transportation decisions are explicitly integrated with inventory control decisions.

Particularly, Section 2 analyzes an integrated inventory control and transportation planning problem with carbon emissions regulations. The EOQ model with LTL and TL transportation under carbon cap, cap and trade, cap and offset, and taxing policies has been investigated. Section 2 provides methods to find the retailer's optimal order quantity under each regulation when a LTL or a TL carrier is used for inbound shipment. The tools provided enable analyzing the effects of the regulations on the retailer's costs and emissions with each carrier. A numerical study illustrates how the retailer's preference for carriers depends on the specifications of the regulation.

Section 3 takes sustainability into account in an integrated continuous review inventory control and transportation model. Particularly, similar to Section 2, the retailer can use a LTL or a TL carrier for their inbound shipment. For each case, a bi-objective order quantity (Q) reorder point (R) model, known as a (Q, R) model is formulated with the objectives of expected costs and expected carbon emissions minimization. This bi-objective (Q, R) model is referred to as the sustainable (Q, R) model. Solution methods to approximate the Pareto frontiers of the sustainable (Q, R) model with LTL and TL transportation are proposed. Numerical studies are presented to illustrate the effects of demand variance and lead time on expected

costs and carbon emissions for each case as well as the changes in expected costs and carbon emissions due to sustainability considerations. The methods discussed in Section 3 can be used by a retailer to compare different LTL carriers, LTL carriers to TL carriers, and different TL carriers in terms of not only cost but also environmental considerations. Examples are discussed to illustrate that a retailer's preferences for transportation choice vary depending on their cost and environmental goals.

It should be noted that most of the studies in the intersection of environmental considerations and inventory control models focus on single item inventory systems. Sections 2 and 3 also analyze single item inventory control models. However, in Sections 2 and 3, inventory control models with environmental considerations are analyzed with integrated transportation decisions under deterministic and stochastic demand, respectively. In many practical cases, multiple items are present and their inventory control and transportation decisions are jointly analyzed. Therefore, Sections 4 and 5 focus on multi-item inventory control models integrated with transportation decisions and environmental considerations under deterministic and stochastic demand, respectively.

In particular, Section 4 analyzes a well known multi-item inventory control problem, namely, the joint replenishment problem. Specifically, Section 4 proposes a bi-objective joint replenishment problem, where the costs and carbon emissions generated from inventory operations are minimized simultaneously. This bi-objective model is referred to as the sustainable joint replenishment problem. The sustainable joint replenishment problem is formulated considering two common grouping strategies: indirect and direct grouping. Under each grouping strategy, a method is developed to generate a set of Pareto efficient solutions for the sustainable joint replenishment problem. Specifically, the analytical properties of each grouping strategy are utilized in developing genetic algorithms to approximate the Pareto fronts. A set of numerical studies are conducted in Section 4 to compare different grouping strategies that can be adopted by a retailer not only in terms of costs but also environmental aspects. It is illustrated that, depending on the cost and green goals, a retailer can select different grouping strategies.

Section 5 studies a multi-item stochastic inventory control model. The environmental considerations are not directly integrated; however, an inventory control and

transportation policy is considered to not only reduce costs but also transportation emissions. In particular, Section 5 analyzes a time based shipment consolidation policy in a multi-item stochastic inventory system with heterogeneous freight trucks. A shipment consolidation policy determines which items should be shipped together. In case of explicit truckload transportation considerations, transportation capacity can be utilized better through shipment consolidation. Section 5 proposes a time based order-up-to-level inventory control policy for a set of consolidated items with heterogeneous freight trucks. Then a set partitioning problem is formulated to find the best shipment consolidation policy. Heuristic solution approaches are provided to solve the resulting set partitioning problem. Results of a simulation study are presented to illustrate the efficiency of the proposed heuristic methods as well as the cost and environmental benefits of the proposed time based shipment consolidation policy.

Throughout the research, environmental regulations and objectives were integrated, among other contributions, to four practical inventory control models. Furthermore, the transportation decisions are explicitly considered in these four models: single item deterministic (Section 2), single item stochastic (Section 3), multi-item deterministic (Section 4), and multi-item stochastic (Section 5). In Section 6, the overall research is summarized and some potentials for future research are highlighted.

2. INTEGRATED INVENTORY CONTROL AND TRUCKLOAD TRANSPORTATION UNDER CARBON EMISSION REGULATIONS

This section focuses on four common carbon emissions regulation policies: carbon cap, carbon cap and trade, carbon cap and offset, and carbon taxing. Under the carbon cap policy, a company plans its operations such that a predefined level of carbon emissions, referred to as the carbon cap, is not exceeded. It should be noted that the carbon cap can be determined by a company's own green goals as well as government agencies (Chen et al., 2013). Under the carbon cap and trade policy, on the other hand, a company can sell its excess carbon emissions if its carbon emissions level is lower than the carbon cap or buy carbon emission permits if its carbon emissions level is higher than the carbon cap. That is, carbon emissions are tradable through a trading system such as the European Union's Emissions Trading System, and it is assumed that there are sufficient demand and supply for selling and buying carbon emissions, respectively. Under the carbon cap and offset policy, a company is subject to a carbon cap; however, the company can invest in carbon offset projects to increase its carbon cap. Carbon offset projects abate carbon emissions by compensating a company's emissions. Under the carbon taxing policy, a company is charged in taxes for its carbon emissions.

As mentioned in Section 1, inventory control models have been analyzed with environmental considerations. This section, similar to Chen et al. (2013), Hua et al. (2011), Arslan and Turkay (2013), and Toptal et al. (2014), considers the classical EOQ model; however, different than these studies, the classical EOQ model is extended to consider two common practices of trucking: LTL and TL carriers. In particular, in the previous studies discussed, it is assumed that a single truck has sufficient capacity to transport any amount of shipment, i.e., it is assumed that a LTL carrier is used for inbound shipment. As noted by Hua et al. (2011) and Benjaafar et al. (2013), warehousing and transportation are considered to be the major drivers of carbon emissions in supply chains; and companies do not only use LTL carriers for their deliveries. It is, therefore, crucial to integrate inventory control with explicit transportation mode selection when carbon emissions regulation policies are in place.

In particular, it is assumed that the supply chain agent (a retailer) can select a LTL or TL carrier for their inbound shipment. In the case where a LTL carrier is used, the retailer is subject to per unit transportation costs and a specific amount of carbon emissions are generated for each unit shipped. In the case where a TL carrier is used, it is assumed that a single truck type is available for deliveries. Furthermore, each truck has a fixed capacity and a per truck cost. The number of trucks used by the retailer determines their transportation costs in addition to procurement, inventory holding, and inventory replenishment costs. One may also note that TL transportation costs are similarly modeled in supply chain and logistics literature (see, e.g., Aucamp, 1982, Lee, 1986, Toptal et al., 2003, Toptal and Çetinkaya, 2006, Toptal, 2009, Toptal and Bingol, 2011, Konur and Toptal, 2012). In addition, TL transportation modeling also applies to the calculation of carbon emissions due to transportation. For instance, Rizet et al. (2012) note that carbon emissions can be effectively reduced by changing vehicle efficiency or vehicle design. In the case where a TL carrier is used, each empty truck generates a fixed amount of carbon emissions and total emissions generated by a truck increase with its load. Thus, the number of trucks used along with their loads determine the retailer's carbon emissions due to transportation. Hoen et al. (2014) and Pan et al. (2013) define similar carbon emissions functions. In order to minimize carbon emissions within a supply chain network, Pan et al. (2013) formulate a transportation problem with two modes of transportation (rail and trucks). Specifically, Pan et al. (2013) note that the same structure for the carbon emissions function is also observed in rail transportation.

This section formulates a retailer's integrated inventory control and transportation problem under the aforementioned four carbon emissions regulation policies with LTL and TL carriers. The difference between a LTL carrier and a TL carrier is explicitly accounted for in transportation costs as well as transportation emissions. An exact method to find the retailer's optimal order quantity with any of the carriers is proposed for each regulation policy. The differences are also analyzed in the retailer's costs and carbon emissions due to preferring a LTL over a TL carrier, or vice versa, under each carbon emissions regulation policy. Furthermore, through numerical examples, it is demonstrated that under any carbon emissions regulation policy, the retailer's preference for a carrier varies depending on the settings of the carbon

emissions regulation policy in place. The tools provided in this section can therefore be utilized by a retailer in comparing LTL carriers to one another, LTL carriers to TL carriers, and TL carriers to one another under carbon emissions regulation policies. A set of numerical studies is conducted to analyze the effects of the settings of the carbon emissions regulation policies on the retailer's costs and carbon emissions with a LTL and TL carrier. Another set of numerical studies is conducted to analyze the effects of the transportation costs and transportation emissions parameters of the LTL and TL carriers on the retailer's costs and carbon emissions. Counterintuitive examples are also presented on how a TL carrier's transportation cost and transportation emissions influence the retailer's costs and carbon emissions.

2.1. PROBLEM FORMULATION AND PRELIMINARIES

Consider a retailer who controls the inventory and inbound transportation for an item. The retailer assumes the basic EOQ settings, that is, the demand rate (λ, items per unit time) for the item is deterministic and constant over time, the lead time is fixed, and a long planning horizon is considered. Under the basic EOQ model, the retailer is subject to procurement costs p (cost per unit), inventory holding costs h (cost per unit per unit time), and order setup costs K (cost for each order placed). In this section, the retailer is also subject to additional inbound transportation costs. It is assumed that the retailer will only use one of the two road transportation carriers available: a LTL carrier or a TL carrier. It should be noted that the retailer might simultaneously use LTL and TL carriers for the inbound shipment of an order, i.e., order splitting between two carriers is possible. This case is posted as a future research direction at the end of this section. The analysis provided in this section can be used to study the setting with order splitting.

In the case where a LTL carrier is used for inbound transportation, the retailer is charged based on the number of items transported. Particularly, it is assumed that the retailer is subject to transportation cost of t (cost per unit transported with the LTL carrier). The retailer's objective is to minimize the total inventory and transportation related costs per unit time by determining the optimal order quantity. Under the basic EOQ model with the LTL carrier, the total cost per unit time as a function of the order quantity, Q, reads

$$H^{LTL}(Q) = (p+t)\lambda + \frac{K\lambda}{Q} + \frac{hQ}{2}, \tag{1}$$

where the first component is the total procurement and transportation cost per unit time, the second component is the order setup cost per unit time, and the last component is the inventory holding cost per unit time. It is easy to show that $H^{LTL}(Q)$ is strictly convex in Q; thus, the unique order quantity that minimizes $H^{LTL}(Q)$, which is also referred to as the *economic order quantity*, is

$$Q^{LTL} = \sqrt{\frac{2K\lambda}{h}}. \tag{2}$$

It should be noted that a LTL carrier is assumed to provide sufficient capacity to carry any order size of the retailer. In practice, it might be the case that a retailer can prefer full truck shipments for larger orders. In such a case, the retailer will use a TL carrier. Specifically, as noted by Toptal and Bingol (2011), depending on the per truck cost of the TL carrier and the per unit transportation cost of the LTL carrier, a retailer may select a carrier based on the order size. Therefore, the following discussion is focused on formulating the retailer's cost function with a TL carrier. That is, instead of modeling the order quantity and carrier selection decisions simultaneously, order quantity decisions are modeled for each carrier separately; and, the carrier with the lower minimum costs is assumed to be selected by the retailer.

In the case where a TL carrier is used for inbound transportation, the retailer is charged based on the number of trucks used for transportation. Particularly, it is assumed that a TL carrier offers a single truck type with a capacity of P units and per truck cost of R money units per shipment. The total transportation cost paid for shipping an order of Q units then amounts to $\lceil \frac{Q}{P} \rceil R$. It then follows that the retailer's total cost per unit time with a TL carrier equals to

$$H^{TL}(Q) = p\lambda + \frac{K\lambda}{Q} + \frac{hQ}{2} + \left\lceil \frac{Q}{P} \right\rceil \frac{R\lambda}{Q}, \tag{3}$$

where the first component is the procurement cost per unit time, the second component is the order setup cost per unit time, the third component is the inventory holding cost per unit time, and the last component is the transportation cost per unit

time. Note that, unlike $H^{LTL}(Q)$, $H^{TL}(Q)$ is a discontinuous function; hence, one cannot use the first order derivative to determine the order quantity that minimizes $H^{TL}(Q)$. Nevertheless, through careful investigation of the properties of $H^{TL}(Q)$, an expression for the minimizer(s) of $H^{TL}(Q)$ is stated in the literature (see, e.g., Aucamp, 1982, Toptal et al., 2003). Specifically, $H^{TL}(Q)$ is a piecewise continuous function such that each piece is in the form of Equation (1) over a given quantity range of length P. The following property, stated without proof (one may refer to Toptal et al., 2003 for the proof), leads to the expression for the minimizer(s) of $H^{TL}(Q)$, denoted by Q^{TL}.

Property 1. *Let $Q_i^* = \sqrt{2(K+iR)\lambda/h}$, for some nonnegative integer i and define k to be the unique integer such that $kP < \sqrt{\frac{2K\lambda}{h}} \leq (k+1)P$. Then*

- *$H^{TL}(Q)$ is decreasing over $(i-1)P < Q \leq iP$, $\forall i \leq k$,*
- *If $i \geq k+1$, then $H^{TL}(iP) \leq H^{TL}(Q)$ for $Q \geq iP$,*
- *If $Q_{k+1}^* \geq (k+1)P$, then $H^{TL}(Q)$ is decreasing over $kP < Q \leq (k+1)P$; if $Q_{k+1}^* < (k+1)P$, then $H^{TL}(Q)$ is decreasing over $kP < Q \leq Q_{k+1}^*$ and increasing over $Q_{k+1}^* \leq Q \leq (k+1)P$.*

It then follows from Property 1 that

$$Q^{TL} = \arg\min\{H^{TL}(\min\{Q_{k+1}^*, (k+1)P\}), H^{TL}(kP)\}. \tag{4}$$

Inventory operations generate a significant amount of carbon emissions. Specifically, the carbon emissions are generated by the inventory holding, inventory replenishment, and transportation. Similar to Hua et al. (2011) and Chen et al. (2013), this model defines a linear relation between carbon emissions and holding Q units of inventory. In particular, $\frac{\widehat{K}\lambda}{Q} + \frac{\widehat{h}Q}{2}$ is the level of carbon emissions per unit time when inventory is replenished in orders of Q units, where $\widehat{K}$ is the fixed carbon emissions amount generated by replenishing the inventory and $\widehat{h}$ denotes the carbon emissions generated for holding one item in inventory per unit time. The carbon emissions due to transportation of an order is defined to be fixed by Chen et al. (2013) whereas Hua et al. (2011) define carbon emissions generated by transportation as the sum of a

fixed value and a component linearly increasing with the order quantity. Particularly, Hua et al. (2011) consider the carbon emissions generated by an empty truck and the variable carbon emissions factor per item loaded. The underlying assumption of Hua et al. (2011) is that a single truck has sufficient capacity to carry any order size, i.e., LTL transportation is assumed. This sections considers both LTL and TL transportation.

In the case where a LTL carrier is used for inbound transportation, similar to Hua et al. (2011) and Chen et al. (2013), it is assumed that each unit transported generates $\widehat{t}$ units of carbon emissions. Under the basic EOQ model with a LTL carrier, the total carbon emissions per unit time as a function of the order quantity, Q, reads

$$E^{LTL}(Q) = \widehat{t}\lambda + \frac{\widehat{K}\lambda}{Q} + \frac{\widehat{h}Q}{2}, \tag{5}$$

where the first component is the transportation emissions per unit time, the second component is the inventory replenishment emissions per unit time, and the last component is the inventory holding emissions per unit time.

In the case a TL carrier is used for inbound transportation, it is assumed that the retailer is subject to the emissions from empty truck weights and the loads of the trucks used for deliveries. In particular, let $\widehat{w}$ and $\widehat{e}$ denote the carbon emissions generated by an empty truck and the carbon emissions generated due to unit load of a truck, respectively. When Q units are shipped using trucks, the carbon emissions generated amount to $\left\lfloor \frac{Q}{P} \right\rfloor (\widehat{w}+\widehat{e}P)$ (the carbon emissions from full truckloads) plus $\widehat{w}+e\left(Q - \left\lfloor \frac{Q}{P} \right\rfloor P\right)$ (the carbon emissions from the LTL). Therefore, the carbon emissions generated per unit time by shipping an order of Q units are defined as follows:

$$\frac{\lambda}{Q}\left\{\left\lfloor \frac{Q}{P} \right\rfloor (\widehat{w}+\widehat{e}P) + \widehat{w} + \widehat{e}\left(Q - \left\lfloor \frac{Q}{P} \right\rfloor P\right)\right\} = \widehat{e}\lambda + \left\lceil \frac{Q}{P} \right\rceil \frac{\widehat{w}\lambda}{Q}.$$

Similar transportation emission functions are defined in the literature (see, e.g., Hoen et al., 2014, Pan et al., 2013). Under the basic EOQ model with a TL carrier, the total carbon emissions per unit time as a function of the order quantity Q then amounts to

$$E^{TL}(Q) = \left(\widehat{e}\lambda + \left\lceil \frac{Q}{P} \right\rceil \frac{\widehat{w}\lambda}{Q}\right) + \frac{\widehat{K}\lambda}{Q} + \frac{\widehat{h}Q}{2}, \tag{6}$$

where the first component is the transportation emissions (including empty truck and truck load emissions) per unit time, the second component is the inventory replenishment emissions per unit time, and the last component is the inventory holding emissions per unit time.

Next, the retailer's integrated inventory control and transportation problem are analysed with LTL and TL carriers under four different carbon emissions regulation policies: carbon cap, carbon cap and trade, carbon cap and offset, and carbon taxing policies. For reference, notation is summarized in Appendix A.1 with possible metric values. Additional notation will be defined as needed. Furthermore, Appendix A includes the proofs of the properties discussed in Sections 2.2 and 2.3.

2.2. ANALYSIS WITH CARBON EMISSIONS REGULATIONS

When there is no carbon emissions regulation policy, the solution to the retailer's cost minimization problems are given by Q^{LTL} defined in Equation (2) and Q^{TL} defined in Equation (4) when a LTL or a TL carrier is used for inbound shipment, respectively. In what follows, the retailer's problem is formulated and solved under the aforementioned carbon emissions regulation policies. A general model for the retailer's optimization problem with a carbon emissions regulation can be formulated as follows:

$$\begin{aligned} \text{(M0)}: \quad \min \quad & H(Q) = f_1(Q) + f_2(Q) + f_3(Q) \\ \text{s.t.} \quad & E(Q) = z_1(Q) + z_2(Q) \leq C \\ & Q \geq 0. \end{aligned}$$

where $f_1(Q)$, $f_2(Q)$, and $f_3(Q)$ are the functions of Q in general forms defining the inventory related costs per unit time, transportation costs per unit time, and emissions penalty costs per unit time, respectively. Similarly, $z_1(Q)$ and $z_2(Q)$ are generalized forms of the inventory related carbon emissions per unit time and transportation related carbon emissions per unit time, respectively. Finally, C defines an upper bound on the carbon emissions per unit time. In all of the following models, $f_1(Q) = p\lambda + K\lambda/Q + hQ/2$ and $z_1(Q) = \widehat{K}\lambda/Q + \widehat{h}Q/2$; however, this section investigates different forms of $f_2(Q)$ and $z_2(Q)$ considering LTL and TL carriers; and, assumes different forms for $f_3(Q)$ and C considering different carbon emissions regulations. Particularly, index j is used to define each carbon emissions regulation policy such

that $j = 1$, $j = 2$, $j = 3$, and $j = 4$ identify carbon cap, carbon cap and trade, carbon cap and offset, and carbon taxing policies, respectively.

2.2.1. Analysis under Carbon Cap. Under the carbon cap model (M1), the retailer's objective is to minimize the total inventory and transportation costs per unit time while the carbon emissions rate does not exceed a targeted level, i.e., the carbon cap. Let $C > 0$ denote the carbon cap per unit time.

2.2.1.1. Carbon cap model with a LTL carrier. Considering Equations (1) and (5), M1 with a LTL carrier reads

$$\begin{aligned} \text{(M1-LTL)}: \quad \min \quad & H_1^{LTL}(Q) = (p+t)\lambda + \tfrac{K\lambda}{Q} + \tfrac{hQ}{2} \\ \text{s.t.} \quad & E^{LTL}(Q) = \widehat{t}\lambda + \tfrac{\widehat{K}\lambda}{Q} + \tfrac{\widehat{h}Q}{2} \leq C \\ & Q \geq 0. \end{aligned}$$

Note that M1-LTL has been solved by Chen et al. (2013). Specifically, due to convexity of functions $H_1^{LTL}(Q)$ and $E^{LTL}(Q)$, and $E^{LTL}(Q)$ being a quadratic function, the optimum solution to M1-LTL, denoted by Q_1^{LTL}, can be explicitly characterized. Particularly, let $q_l^{LTL} = \frac{C-\widehat{t}\lambda-\sqrt{(C-\widehat{t}\lambda)^2-2\widehat{K}\widehat{h}\lambda}}{\widehat{h}}$ and $q_u^{LTL} = \frac{C-\widehat{t}\lambda+\sqrt{(C-\widehat{t}\lambda)^2-2\widehat{K}\widehat{h}\lambda}}{\widehat{h}}$. Note that $E^{LTL}(Q) \leq C$ for $q_l^{LTL} \leq Q \leq q_u^{LTL}$. It is assumed that both q_l^{LTL} and q_u^{LTL} are real numbers, that is, the carbon cap is sufficiently large such that there exist a feasible order quantity for M1-LTL. The following corollary then defines Q_1^{LTL}.

Corollary 1. *Suppose that M1-LTL is feasible. Then, if $Q^{LTL} < q_l^{LTL}$, $Q_1^{LTL} = q_l^{LTL}$; if $q_l^{LTL} \leq Q^{LTL} \leq q_u^{LTL}$, $Q_1^{LTL} = Q^{LTL}$; and if $q_u^{LTL} < Q^{LTL}$, $Q_1^{LTL} = q_u^{LTL}$.*

2.2.1.2. Carbon cap model with a TL carrier. Considering Equations (3) and (6), M1 with a TL carrier reads

$$\begin{aligned} \text{(M1-TL)}: \quad \min \quad & H_1^{TL}(Q) = p\lambda + \tfrac{K\lambda}{Q} + \tfrac{hQ}{2} + \left\lceil \tfrac{Q}{P} \right\rceil \tfrac{R\lambda}{Q} \\ \text{s.t.} \quad & E^{TL}(Q) = \left(\widehat{e}\lambda + \left\lceil \tfrac{Q}{P} \right\rceil \tfrac{\widehat{w}\lambda}{Q}\right) + \tfrac{\widehat{K}\lambda}{Q} + \tfrac{\widehat{h}Q}{2} \leq C \\ & Q \geq 0. \end{aligned}$$

It is assumed that the carbon cap is sufficiently large that there exist feasible order quantities for M1-TL. It should be noted that finding the optimum solution to M1-TL, denoted by Q_1^{TL}, requires detailed analysis of $H_1^{TL}(Q)$ and $E^{TL}(Q)$ simultaneously. In what follows, the piecewise structures of these functions are utilized to solve M1-TL.

In particular, recall that each piece of $H_1^{TL}(Q)$ is an EOQ type of function. Similarly, each piece of the $E^{TL}(Q)$ function is an EOQ type of function. Now,

consider the range $((i-1)P, iP]$. In this range, $E^{TL}(Q)$ is defined by $E_i^{TL}(Q) = \widehat{e}\lambda + \frac{(\widehat{K}+i\widehat{w})\lambda}{Q} + \frac{\widehat{h}Q}{2}$. It is easy to verify that $E_i^{TL}(Q) \leq C$ for $Q \in [q_l^{TL(i)}, q_u^{TL(i)}]$ where

$$q_l^{TL(i)} = \frac{C - \widehat{e}\lambda - \sqrt{(C-\widehat{e}\lambda)^2 - 2\widehat{h}\lambda(\widehat{K}+i\widehat{w})}}{\widehat{h}}, \tag{7}$$

$$q_u^{TL(i)} = \frac{C - \widehat{e}\lambda + \sqrt{(C-\widehat{e}\lambda)^2 - 2\widehat{h}\lambda(\widehat{K}+i\widehat{w})}}{\widehat{h}}. \tag{8}$$

Note that Equations (7) and (8) imply that $q_l^{TL(i)} < q_l^{TL(i+1)} \leq q_u^{TL(i+1)} < q_u^{TL(i)}$. It then follows that $Q_1^{TL} \leq q_u^{TL(1)}$. The following property derives the range of Q, where $(i-1)P < Q \leq iP$, such that $E^{TL}(Q) \leq C$.

Property 2. *Suppose that* $((i-1)P, iP] \cap [q_l^{TL(i)}, q_u^{TL(i)}] \neq \emptyset$. *Let* $Q_l^{TL(i)} = \max\{q_l^{TL(i)}, (i-1)P\}$ *and* $Q_u^{TL(i)} = \min\{q_u^{TL(i)}, iP\}$. *Then* $E^{TL}(Q) \leq C$ *for* $Q \in [Q_l^{TL(i)}, Q_u^{TL(i)}]$.

Now, let t_1 be defined as the minimum integer such that $((t_1-1)P, t_1P] \cap [q_l^{TL(t_1)}, q_u^{TL(t_1)}] \neq \emptyset$ and let t_2 be defined as the maximum integer such that $((t_2-1)P, t_2P] \cap [q_l^{TL(t_2)}, q_u^{TL(t_2)}] \neq \emptyset$. Note that both t_1 and t_2 are defined as M1-TL is assumed to be feasible. By definitions of t_1 and t_2, it follows that $Q_l^{TL(t_1)} \leq Q_1^{TL} \leq Q_u^{TL(t_2)}$.

Property 3. *If* $t_1 \neq t_2$, *then* $Q_u^{TL(i)} = iP$ *for* $t_1 \leq i \leq t_2 - 1$.

Property 3 implies that, when $t_1 \neq t_2$, i.e., there exist more than one non-overlapping regions of feasible order quantities and the upper limits of these regions correspond to full truckload quantities except the last region. Property 3 is utilized in the next properties, where the optimal solution for M1-TL is characterized for different values of t_1 and t_2.

Recall from Equation (2) that $Q^{LTL} = \sqrt{\frac{2K\lambda}{h}}$ and from Property 1 that $Q_i^* = \sqrt{\frac{2(K+iR)\lambda}{h}}$ and k is the unique integer such that $kP < Q^{LTL} \leq (k+1)P$. In the following property, Q_1^{TL} is characterized when the retailer will not decrease their order quantity due to the carbon cap constraint.

Property 4. *If $t_1 \geq k+1$, then*

$$Q_1^{TL} = \begin{cases} Q_l^{TL(t_1)} & \text{if } Q_{t_1}^* < Q_l^{TL(t_1)}, \\ Q_{t_1}^* & \text{if } Q_l^{TL(t_1)} \leq Q_{t_1}^* \leq Q_u^{TL(t_1)}, \\ Q_u^{TL(t_1)} & \text{if } Q_u^{TL(t_1)} < Q_{t_1}^*. \end{cases}$$

Observe from Equation (4) that kP is the lower limit on Q^{TL}. Property 4 then captures the case where the feasible order quantity ranges are larger than kP.

Property 5. *If $t_2 \leq k$, then*

$$Q_1^{TL} = \begin{cases} Q_u^{TL(t_1)} & \text{if } t_1 = t_2, \\ \arg\min\{H_1^{TL}(Q_u^{TL(t_2-1)}), H_1^{TL}(Q_u^{TL(t_2)})\} & \text{if } t_1 \neq t_2. \end{cases}$$

Unlike Property 4, Property 5 implies that the retailer will decrease their order quantity due to the carbon cap constraint. Finally, in the next property, the case when $t_1 \leq k < k+1 \leq t_2$ is captured.

Property 6. *If $t_1 \leq k < k+1 \leq t_2$,*
then $Q_1^{TL} = \arg\min\{H_1^{TL}(kP), H_1^{TL}(\min\{Q_{k+1}^*, Q_u^{TL(k+1)}\})\}$.

Based on Properties 4-6, the following corollary summarizes the optimal solution for M1-TL.

Corollary 2. *Suppose that M1-TL is feasible. Then,*

- *If $t_1 \geq k+1$, then $Q_1^{TL} = Q_l^{TL(t_1)}$ if $Q_{t_1}^* < Q_l^{TL(t_1)}$; $Q_1^{TL} = Q_{t_1}^*$ if $Q_l^{TL(t_1)} \leq Q_{t_1}^* \leq Q_u^{TL(t_1)}$; and $Q_1^{TL} = Q_u^{TL(t_1)}$ if $Q_u^{TL(t_1)} < Q_{t_1}^*$.*
- *If $t_2 \leq k$, then $Q_1^{TL} = Q_u^{TL(t_1)}$ if $t_1 = t_2$; and*
  $Q_1^{TL} = \arg\min\{H_1^{TL}(Q_u^{TL(t_2-1)}), H_1^{TL}(Q_u^{TL(t_2)})\}$ if $t_1 \neq t_2$.
- *If $t_1 \leq k < k+1 \leq t_2$,*
  then $Q_1^{TL} = \arg\min\{H_1^{TL}(kP), H_1^{TL}(\min\{Q_{k+1}^*, Q_u^{TL(k+1)}\})\}$.

It should be noted that t_1 and t_2 can be easily determined using the relation $q_l^{TL(i)} < q_l^{TL(i+1)} \leq q_u^{TL(i+1)} < q_u^{TL(i)}$ implied by Equations (7) and (8).

2.2.2. Analysis under Carbon Cap and Trade. Under the carbon cap and trade model (M2), the retailer is subject to a carbon emissions cap C per unit time; however, a carbon emissions trading system is available for buying carbon emission permits or selling extra carbon emissions. The retailer's objective is to minimize the total inventory and transportation costs along with the additional costs or revenues gained through carbon emissions trading. In particular, let X denote the traded carbon emissions amount per unit time; if $X > 0$, additional carbon emissions capacity is purchased and if $X < 0$, excess carbon emissions capacity is sold. Similar to Hua et al. (2011), it is assumed that the market price (selling or buying) for per unit carbon emissions is fixed at α (cost per unit) and there is sufficient supply and sufficient demand for buying and selling carbon emissions capacity, respectively.

2.2.2.1. Carbon cap and trade model with a LTL carrier. When a LTL carrier is used for inbound transportation, the retailer's total cost per unit time is $H^{LTL}(Q) + \alpha X$. Furthermore, the retailer's traded carbon emissions amount to $X = E^{LTL}(Q) - C$. Considering Equations (1) and (5), M2 with a LTL carrier reads

$$\begin{aligned} \text{(M2-LTL)}: \quad & \min \quad H_2^{LTL}(Q) = (p + t + \alpha\widehat{t})\lambda + \tfrac{(K+\alpha\widehat{K})\lambda}{Q} + \tfrac{(h+\alpha\widehat{h})Q}{2} - \alpha C \\ & \text{s.t.} \quad Q \geq 0. \end{aligned}$$

It is straightforward to show that $H_2^{LTL}(Q)$ is strictly convex in Q; hence, the following corollary states the optimum solution of M2-LTL, denoted by Q_2^{LTL} and X^{LTL}, using the first order optimality conditions (see, also, Hua et al. (2011)).

Corollary 3. $Q_2^{LTL} = \sqrt{\frac{2(K+\alpha\widehat{K})\lambda}{h+\alpha\widehat{h}}}$ *and* $X^{LTL} = E^{LTL}(Q_2^{LTL}) - C$.

2.2.2.2. Carbon cap and trade model with a TL carrier. When a TL carrier is used for inbound transportation, the retailer's total cost per unit time is $H^{TL}(Q) + \alpha X$. Furthermore, the retailer's traded carbon emissions amount to $X = E^{TL}(Q) - C$. Considering Equations (3) and (6), M2 with a TL carrier reads

$$\begin{aligned} \text{(M2-TL)}: \quad & \min \quad H_2^{TL}(Q) = (p + \alpha\widehat{e})\lambda + \tfrac{(K+\alpha\widehat{K})\lambda}{Q} + \tfrac{(h+\alpha\widehat{h})Q}{2} + \left\lceil \tfrac{Q}{P} \right\rceil \tfrac{(R+\alpha\widehat{w})\lambda}{Q} - \alpha C \\ & \text{s.t.} \quad Q \geq 0. \end{aligned}$$

Let Q_2^{TL} and X^{TL} denote the optimum solution of M2-TL. Note that $H_2^{TL}(Q)$ follows a similar functional form with $H^{TL}(Q)$; hence, Property 1 can be utilized in determining Q_2^{TL} as noted in the following corollary.

Corollary 4. *Let* $Q_2^{TL(i)} = \sqrt{\frac{2\lambda(K+\alpha\widehat{K}+i(R+\alpha\widehat{w}))}{h+\alpha\widehat{h}}}$ *and* m *be the unique integer such that* $mP < \sqrt{\frac{2\lambda(K+\alpha\widehat{K})}{h+\alpha\widehat{h}}} \leq (m+1)P$. *Then,* $Q_2^{TL} = \arg\min\{H_2^{TL}(\min\{Q_2^{TL(m+1)}, (m+1)P\}), H_2^{TL}(mP)\}$ *and* $X^{TL} = E^{TL}(Q_2^{TL}) - C$.

2.2.3. Analysis under Carbon Cap and Offset. Under the cap and offset model (M3), similar to the cap and trade model, the retailer is subject to carbon emissions cap C per unit time; however, a carbon trading system is not available. On the other hand, carbon offset projects can be used for carbon emissions abatement when the retailer's carbon emissions level from inventory holding and transportation exceeds the carbon cap. It is assumed that carbon emissions can be offset per unit at a cost of r. Let S denote the amount of carbon emissions per unit time that the retailer decides to compensate by investing in carbon offset projects. Then the retailer needs to invest rS money units per unit time for offsetting S level of carbon emissions per unit time. The retailer's objective is to minimize total inventory and transportation costs plus the carbon emissions abatement investment costs such that the carbon emissions level from inventory holding and transportation does not exceed the carbon cap plus the carbon allowances achieved through investing in carbon offset projects.

2.2.3.1. Carbon cap and offset model with a LTL carrier. Considering Equations (1) and (5) and the above discussion, M3 with a LTL carrier reads

$$\begin{aligned} \text{(M3-LTL)}: \quad \min \quad & H_3^{LTL}(Q,S) = (p+t)\lambda + \tfrac{K\lambda}{Q} + \tfrac{hQ}{2} + rS \\ \text{s.t.} \quad & E^{LTL}(Q) = \widehat{t}\lambda + \tfrac{\widehat{K}\lambda}{Q} + \tfrac{\widehat{h}Q}{2} \leq C + S \\ & Q \geq 0 \\ & S \geq 0. \end{aligned}$$

Observe that both Q and S are the retailer's decision variables. One should note that a model similar to M3-LTL is formulated by Arslan and Turkay (2013); however, they do not provide a solution method for the model. The explicit characterization of the optimal solution of M3-LTL follows, denoted by Q_3^{LTL} and S^{LTL}.

In particular, for any given Q, the optimum S value, $S^{LTL}(Q)$ is given by the following equation:

$$S^{LTL}(Q) = \begin{cases} 0 & \text{if } E^{LTL}(Q) \leq C, \\ E^{LTL}(Q) - C & \text{if } E^{LTL}(Q) \geq C. \end{cases}$$

This follows from the fact that the retailer will not invest in extra carbon emissions abatement. Then, by definition, $S^{LTL} = S^{LTL}(Q_3^{LTL})$ and M3-LTL can be investigated by separating it into the following two optimization problems:

$$\begin{aligned}
\text{(M3-LTL-a)}: \quad \min \quad & H_{3a}^{LTL}(Q) = (p+t)\lambda + \tfrac{K\lambda}{Q} + \tfrac{hQ}{2} \\
\text{s.t.} \quad & E^{LTL}(Q) = \widehat{t}\lambda + \tfrac{\widehat{K}\lambda}{Q} + \tfrac{\widehat{h}Q}{2} \leq C \\
& Q \geq 0.
\end{aligned}$$

$$\begin{aligned}
\text{(M3-LTL-b)}: \quad \min \quad & H_{3b}^{LTL}(Q) = (p+t+r\widehat{t})\lambda + \tfrac{(K+r\widehat{K})\lambda}{Q} + \tfrac{(h+r\widehat{h})Q}{2} - rC \\
\text{s.t.} \quad & E^{LTL}(Q) = \widehat{t}\lambda + \tfrac{\widehat{K}\lambda}{Q} + \tfrac{\widehat{h}Q}{2} \geq C \\
& Q \geq 0.
\end{aligned}$$

Let Q_{3a}^{LTL} and Q_{3b}^{LTL} denote the optimum solutions of M3-LTL-a and M3-LTL-b, respectively. Observe that M3-LTL-a is identical to M1-LTL; hence, $Q_{3a}^{LTL} = Q_1^{LTL}$ where Q_1^{LTL} is defined in Corollary 1. The next property identifies Q_{3b}^{LTL}. Recall that $E^{LTL}(Q) \leq C$ for $q_l^{LTL} \leq Q \leq q_u^{LTL}$ where $q_l^{LTL} = \frac{C-\widehat{t}\lambda-\sqrt{(C-\widehat{t}\lambda)^2-2\widehat{K}\widehat{h}\lambda}}{\widehat{h}}$ and $q_u^{LTL} = \frac{C-\widehat{t}\lambda+\sqrt{(C-\widehat{t}\lambda)^2-2\widehat{K}\widehat{h}\lambda}}{\widehat{h}}$.

Property 7. *Let* $q_{3b}^{LTL} = \sqrt{\frac{2\lambda(K+r\widehat{K})}{h+r\widehat{h}}}$. *Then, if* $q_{3b}^{LTL} \leq q_l^{LTL}$, $Q_{3b}^{LTL} = q_{3b}^{LTL}$; *if* $q_l^{LTL} < q_{3b}^{LTL} < q_u^{LTL}$, $Q_{3b}^{LTL} = \arg\min\{H_{3b}^{LTL}(q_l^{LTL}), H_{3b}^{LTL}(q_u^{LTL})\}$; *and if* $q_u^{LTL} \leq q_{3b}^{LTL}$, $Q_{3b}^{LTL} = q_{3b}^{LTL}$.

The next corollary, which follows from the definitions of M3-LTL-a and M3-LTL-b, defines Q_3^{LTL} and S^{LTL}.

Corollary 5. *If* $H_{3a}^{LTL}(Q_{3a}^{LTL}) \leq H_{3b}^{LTL}(Q_{3b}^{LTL})$, *then* $Q_3^{LTL} = Q_{3a}^{LTL}$ *and* $S^{LTL} = 0$; *if* $H_{3a}^{LTL}(Q_{3a}^{LTL}) \geq H_{3b}^{LTL}(Q_{3b}^{LTL})$, *then* $Q_3^{LTL} = Q_{3b}^{LTL}$ *and* $S^{LTL} = E^{LTL}(Q_{3b}^{LTL}) - C$.

2.2.3.2. Carbon cap and offset model with a TL carrier. Considering Equations (3) and (6), M3 with a TL carrier reads

$$\begin{aligned}
\text{(M3-TL)}: \quad \min \quad & H_3^{TL}(Q,S) = p\lambda + \tfrac{K\lambda}{Q} + \tfrac{hQ}{2} + \left\lceil \tfrac{Q}{P} \right\rceil \tfrac{R\lambda}{Q} + rS \\
\text{s.t.} \quad & E^{TL}(Q) = \left(\widehat{e}\lambda + \left\lceil \tfrac{Q}{P} \right\rceil \tfrac{\widehat{w}\lambda}{Q}\right) + \tfrac{\widehat{K}\lambda}{Q} + \tfrac{\widehat{h}Q}{2} \leq C + S \\
& Q \geq 0 \\
& S \geq 0.
\end{aligned}$$

Let Q_3^{TL} and S^{TL} denote the optimum solution of M3-TL. Similar to the analysis of M3-LTL, this section characterizes Q_3^{TL} and S^{TL} by separating M3-TL into two subproblems. Specifically, remark that for any given Q, the optimum S value, $S^{TL}(Q)$ will be

$$S^{TL}(Q) = \begin{cases} 0 & \text{if } E^{TL}(Q) \leq C, \\ E^{TL}(Q) - C & \text{if } E^{TL}(Q) \geq C. \end{cases}$$

Then, the solution of M3-TL will be defined by one of the solutions of the following two optimization problems:

$$\begin{aligned} \text{(M3-TL-a)}: \quad \min \quad & H_{3a}^{TL}(Q) = p\lambda + \tfrac{K\lambda}{Q} + \tfrac{hQ}{2} + \left\lceil \tfrac{Q}{P} \right\rceil \tfrac{R\lambda}{Q} \\ \text{s.t.} \quad & E^{TL}(Q) = \left(\widehat{e}\lambda + \left\lceil \tfrac{Q}{P} \right\rceil \tfrac{\widehat{w}\lambda}{Q}\right) + \tfrac{\widehat{K}\lambda}{Q} + \tfrac{\widehat{h}Q}{2} \leq C \\ & Q \geq 0. \end{aligned}$$

$$\begin{aligned} \text{(M3-TL-b)}: \quad \min \quad & H_{3b}^{TL}(Q) = (p + r\widehat{e})\lambda + \tfrac{(K+r\widehat{K})\lambda}{Q} + \tfrac{(h+r\widehat{h})Q}{2} + \left\lceil \tfrac{Q}{P} \right\rceil \tfrac{(R+r\widehat{w})\lambda}{Q} - rC \\ \text{s.t.} \quad & E^{TL}(Q) = \left(\widehat{e}\lambda + \left\lceil \tfrac{Q}{P} \right\rceil \tfrac{\widehat{w}\lambda}{Q}\right) + \tfrac{\widehat{K}\lambda}{Q} + \tfrac{\widehat{h}Q}{2} \geq C \\ & Q \geq 0. \end{aligned}$$

Let Q_{3a}^{TL} and Q_{3b}^{TL} denote the optimum solutions of M3-TL-a and M3-TL-b, respectively. Notice that M3-TL-a is identical to M1-TL; hence, $Q_{3a}^{TL} = Q_1^{TL}$ where Q_1^{TL} is defined in Corollary 2. In what follows, the focus is on solving M3-TL-b.

Recall from Property 2 that $E^{TL}(Q) \leq C$ when $Q \in [Q_l^{TL(i)}, Q_u^{TL(i)}]$, where $Q_l^{TL(i)} = \max\{q_l^{TL(i)}, (i-1)P\}$ and $Q_u^{TL(i)} = \min\{q_u^{TL(i)}, iP\}$ such that $q_l^{TL(i)}$ and $q_u^{TL(i)}$ are defined in Equations (7) and (8). It then follows that $E^{TL}(Q) \geq C$ for $Q \in (((i-1)P, iP] \backslash (Q_l^{TL(i)}, Q_u^{TL(i)}))$. Observe that $(((i-1)P, iP] \backslash (Q_l^{TL(i)}, Q_u^{TL(i)}))$ can correspond to at most two separate ranges of feasible order quantities. Suppose that one of these ranges is given and let it be denoted by $(\widehat{Q}_l^{TL(i)}, \widehat{Q}_u^{TL(i)})$ (one can utilize Property 2 to determine the ranges of feasible order quantities). In the following property, the minimizer of $H_{3b}^{TL}(Q)$ over $Q \in (\widehat{Q}_l^{TL(i)}, \widehat{Q}_u^{TL(i)})$, denoted by $Q_{3b}^{TL(i)}$, is characterized.

Property 8. *Suppose that $E^{TL}(Q) \geq C$ for $Q \in (\widehat{Q}_l^{TL(i)}, \widehat{Q}_u^{TL(i)})$, where $(\widehat{Q}_l^{TL(i)}, \widehat{Q}_u^{TL(i)}) \subseteq ((i-1)P, iP]$. Let $q_{3b}^{TL(i)} = \sqrt{2(K + r\widehat{K} + i(R + r\widehat{w}))\lambda/(h + r\widehat{h})}$.*

Then

$$Q_{3b}^{TL(i)} = \begin{cases} \lim\limits_{Q\to+} \widehat{Q}_l^{TL(i)} & \text{if } q_{3b}^{TL(i)} \le \widehat{Q}_l^{TL(i)}, \\ q_{3b}^{TL(i)} & \text{if } \widehat{Q}_l^{TL(i)} < q_{3b}^{TL(i)} < \widehat{Q}_u^{TL(i)}, \\ \lim\limits_{Q\to-} \widehat{Q}_u^{TL(i)} & \text{if } \widehat{Q}_u^{TL(i)} \le q_{3b}^{TL(i)}. \end{cases}$$

Recall that $((i-1)P, iP] \setminus (Q_l^{TL(i)}, Q_u^{TL(i)})$ can define two separate ranges of feasible order quantities. In such a case, one can follow Property 8 for both of these regions. Finally, it should be noted that $((i-1)P, iP] \setminus (Q_l^{TL(i)}, Q_u^{TL(i)})$ can correspond to $(\widehat{Q}_l^{TL(i)}, \widehat{Q}_u^{TL(i)})$ or $(\widehat{Q}_l^{TL(i)}, \widehat{Q}_u^{TL(i)}]$ and in the latter case, $\lim_{Q\to-} \widehat{Q}_u^{TL(i)} = \widehat{Q}_u^{TL(i)}$. Property 8 finds the minimizer of M3-TL-b over $Q \in ((i-1)P, iP]$ for any given number of trucks i. Nevertheless, the retailer can use as many trucks as possible. In the following property, an upper bound on i is proposed for M3-TL-b.

Property 9. *Let z be the unique integer such that $zP < \sqrt{2(K + r\widehat{K})\lambda/(h + r\widehat{h})} \le (z+1)P$. Furthermore, let x be the first integer such that $q_u^{TL(x)} \le (x-1)P$. Then $Q_{3b}^{TL} \le (\max\{z, x\} + 1)P$.*

Property 9 indicates that the retailer will use at most $\max\{z, x\} + 1$ trucks in the optimal solution of M3-TL-b. Let $y = \max\{z, x\} + 1$. Properties 8 and 9, then, readily imply that

$$Q_{3b}^{TL} = \arg\min\{H_{3b}^{TL}(Q_{3b}^{TL(1)}), H_{3b}^{TL}(Q_{3b}^{TL(2)}), \ldots, H_{3b}^{TL}(Q_{3b}^{TL(y)})\}. \tag{9}$$

The next corollary defines Q_3^{TL} and S^{TL} based on the definitions of Q_{3a}^{TL} and Q_{3b}^{TL}.

Corollary 6. *If $H_{3a}^{TL}(Q_{3a}^{TL}) \le H_{3b}^{TL}(Q_{3b}^{TL})$, then $Q_3^{TL} = Q_{3a}^{TL}$ and $S^{TL} = 0$; if $H_{3a}^{TL}(Q_{3a}^{TL}) \ge H_{3b}^{TL}(Q_{3b}^{TL})$, then $Q_3^{TL} = Q_{3b}^{TL}$ and $S^{TL} = E^{TL}(Q_{3b}^{TL}) - C$.*

2.2.4. Analysis under Carbon Taxing. Under the carbon taxing model (M4), the retailer's objective is to minimize total inventory and transportation costs along with the additional costs paid in taxes for carbon emissions. In particular, let γ money units be charged as tax, per unit carbon emission per unit time.

2.2.4.1. Carbon taxing model with a LTL carrier. Under M4 with a LTL carrier, the retailer is charged $\gamma E^{LTL}(Q)$ in taxes per unit time for their carbon

emissions as a result of ordering decisions. Considering Equations (1) and (5), M4 with a LTL carrier then reads

$$\text{(M4-LTL)}: \quad \min \quad H_4^{LTL}(Q) = (p + t + \gamma\widehat{t})\lambda + \frac{(K+\gamma\widehat{K})\lambda}{Q} + \frac{(h+\gamma\widehat{h})Q}{2}$$
$$\text{s.t.} \quad Q \geq 0.$$

Similar to M2-LTL, it can be easily seen that $H_4^{LTL}(Q)$ is strictly convex in Q; therefore, the optimum solution of M4-LTL, denoted by Q_4^{LTL} can be determined by the first order conditions as stated in the following corollary.

Corollary 7. $Q_4^{LTL} = \sqrt{\frac{2(K+\gamma\widehat{K})\lambda}{h+\gamma\widehat{h}}}$.

2.2.4.2. Carbon taxing model with a TL carrier. In the case where a TL carrier is used for inbound transportation, the retailer's total cost per unit time including carbon emissions taxes amounts to $H^{TL}(Q) + \gamma E^{TL}(Q)$. Considering Equations (3) and (6), M4 with a TL carrier reads

$$\text{(M4-TL)}: \quad \min \quad H_4^{TL}(Q) = (p + \gamma\widehat{e})\lambda + \frac{(K+\gamma\widehat{K})\lambda}{Q} + \frac{(h+\gamma\widehat{h})Q}{2} + \left\lceil \frac{Q}{P} \right\rceil \frac{(R+\gamma\widehat{w})\lambda}{Q}$$
$$\text{s.t.} \quad Q \geq 0.$$

Let Q_4^{TL} denote the optimum solution of M4-TL. $H_4^{TL}(Q)$ is defined similar to $H^{TL}(Q)$; hence, Property 1 can be be used to find Q_4^{TL} as noted in the following corollary.

Corollary 8. *Let* $Q_4^{TL(i)} = \sqrt{\frac{2\lambda(K+\gamma\widehat{K}+i(R+\gamma\widehat{w}))}{h+\gamma\widehat{h}}}$ *and* n *be the unique integer such that* $nP < \sqrt{\frac{2\lambda(K+\gamma\widehat{K})}{h+\gamma\widehat{h}}} \leq (n+1)P$. *Then,* $Q_4^{TL} = \arg\min\{H_4^{TL}(\min\{Q_4^{TL(n+1)}, (n+1)P\}), H_4^{TL}(nP)\}$.

2.3. LTL VS. TL CARRIER UNDER EMISSIONS REGULATIONS

Note that it is possible that both the costs per unit time and the carbon emissions per unit time of a retailer are lower with a specific transportation mode under any carbon emissions regulation. It might be the case that a LTL carrier has significant cost and environmental advantages over a TL carrier or vice versa. In this section, the focus is to illustrate how the retailer's choice of transportation mode, i.e., LTL carrier vs. TL carrier, depends on the carbon emissions regulation policy in place. Particularly, the tools presented in Section 2.2 can be used by a retailer to compare LTL carriers or a LTL carrier to a TL carrier or TL carriers under any of the carbon emissions regulation policies considered. In what follows, analytical

results are presented on comparing a LTL carrier to a TL carrier and four examples are discussed, each of which corresponds to a carbon emissions regulation policy.

2.3.1. LTL vs. TL Carrier under Carbon Cap. Consider that a retailer is subject to a carbon cap regulation with carbon cap value C. In the next property, the cases when a TL carrier results in better environmental performance is characterized.

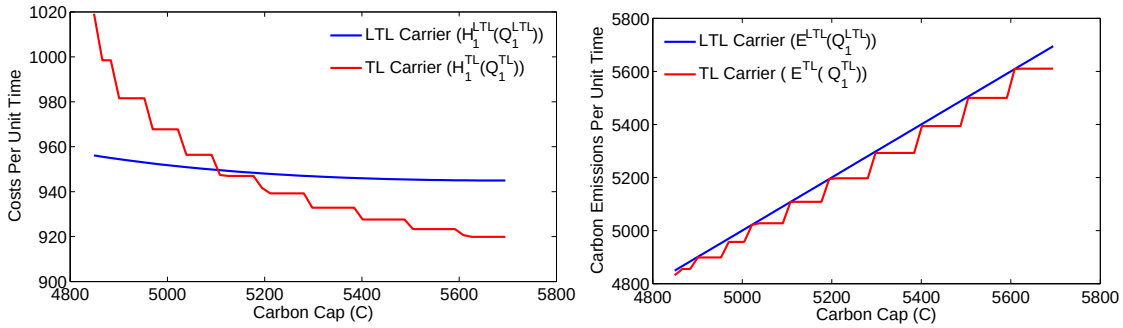
Property 10. *If* $E^{LTL}(Q^{LTL}) \geq C$ *then* $E^{TL}(Q_1^{TL}) \leq E^{LTL}(Q_1^{LTL})$.

It is known that if the carbon cap C is restrictive when a LTL carrier is preferred, the retailer will increase their total costs per unit time to decrease their emissions to the level imposed by the carbon cap (see, e.g., Chen et al., 2013). Property 10 states that in such a case, the retailer can decrease carbon emissions further with a TL carrier. Moreover, this suggests that under a restricting carbon cap regulation, it is possible that the retailer can have lower costs as well as lower carbon emissions per unit time with a TL carrier. This case is illustrated in the following example (please refer to Section 2.4 for a discussion on the selection of the values for the parameters).

Example 1. *Suppose that a retailer is subject to a carbon cap regulation and they can use a LTL or a TL carrier for their inbound shipment. The retailer has the following specifications:* $\lambda = 2000$, $p = 0$, $h = 0.3$, $K = 50$, $\widehat{h} = 10$, *and* $\widehat{K} = 250$. *The LTL carrier has the following specifications:* $t = 0.35$ *and* $\widehat{t} = 0.5$. *The TL carrier has the following specifications:* $R = 10$, $P = 30$, $\widehat{w} = 10$, *and* $\widehat{e} = 0.5$.

Example 1 considers the carbon cap values varying between the maximum of the minimum carbon emissions possible with LTL and TL carriers and the minimum of the maximum carbon emissions possible with LTL and TL carriers (i.e., the C values considered are feasible and binding for both cases when the LTL or the TL carrier is used for inbound transportation). The costs and carbon emissions per unit time with each carrier as C changes are illustrated in Figure 2.1.

As it can be seen from Figure 2.1a, depending on the carbon cap value, the retailer can prefer a LTL over a TL carrier or vice versa. Corollaries 1 and 2 can be used for such comparison. Particularly, in Example 1, for smaller values of C, the retailer would prefer the LTL carrier as it results in lower costs per unit time; however, for larger values of C, the retailer would prefer the TL carrier. Figure 2.1b



(a) Carbon cap vs. costs (b) Carbon cap vs. carbon emissions

Figure 2.1. LTL vs. TL carrier under carbon cap regulation

illustrates the carbon emissions as C increases. As noted above, all of the C values are restrictive for both models M1-LTL and M1-TL. As suggested by Property 10, the TL carrier results in less carbon emissions per unit time. This then implies that for larger values of restricting C, the TL carrier not only reduces costs but also carbon emissions per unit time.

2.3.2. LTL vs. TL Carrier under Carbon Cap and Trade. Consider that a retailer is subject to carbon cap and trade regulation with a carbon cap value C and carbon trading price α. The next property characterizes a case when a LTL (TL) carrier will be preferred over a TL (LTL) carrier under a carbon cap and trade regulation. Prior to stating the property, the retailer's marginal shipment opportunity cost first needs to be defined for LTL and TL carriers under a carbon cap and trade regulation. Marginal shipment opportunity cost refers to the marginal shipment cost plus the retailer's sunk opportunity cost that would be achieved by selling the emissions generated due the shipment. With a LTL carrier, the retailer's total shipment opportunity cost for an order of Q units amounts to $(t+\alpha\widehat{t})Q$; therefore, the retailer's marginal shipment opportunity cost with a LTL carrier, denoted by θ^{LTL}, is defined as $\theta^{LTL} = t + \alpha\widehat{t}$. With a TL carrier, the retailer's shipment opportunity cost for an order of Q units amounts to $\alpha\widehat{e}Q + \left\lceil \frac{Q}{P} \right\rceil (R + \alpha\widehat{w})$; therefore, the retailer's marginal shipment opportunity cost with a TL carrier, denoted by θ^{TL}, is defined as $\theta^{TL} = \alpha\widehat{e} + \left\lceil \frac{Q}{P} \right\rceil \frac{(R+\alpha\widehat{w})}{Q}$.

Property 11. *(i) If* $t + \alpha\widehat{t} < \alpha\widehat{e}$ *then* $H_2^{LTL}(Q_2^{LTL}) < H_2^{TL}(Q_2^{TL})$. *(ii) If* $\lceil \frac{Q}{P} \rceil \leq Q$ *for any* Q *and* $t + \alpha\widehat{t} > R + \alpha\widehat{w} + \alpha\widehat{e}$ *then* $H_2^{LTL}(Q_2^{LTL}) > H_2^{TL}(Q_2^{TL})$.

Note that for any order size Q, $\alpha\widehat{e} < \theta^{TL}$. Part (i) of Property 11 indicates that if the LTL carrier's marginal shipment opportunity cost is less than the TL carrier's minimum marginal shipment opportunity cost, the retailer will prefer the LTL carrier over the TL carrier. The first condition in part (ii) of Property 11 states that the TL carrier will charge for at most Q trucks to ship an order of Q units, which is practical assuming that both Q and P are or can be defined discretely. This suggests that $R + \alpha\widehat{w} + \alpha\widehat{e} > \theta^{TL}$. Property 11 (ii) then implies that if the LTL carrier's marginal shipment opportunity cost is greater than the TL carrier's maximum marginal shipment opportunity cost, the retailer will prefer the TL carrier over the LTL carrier.

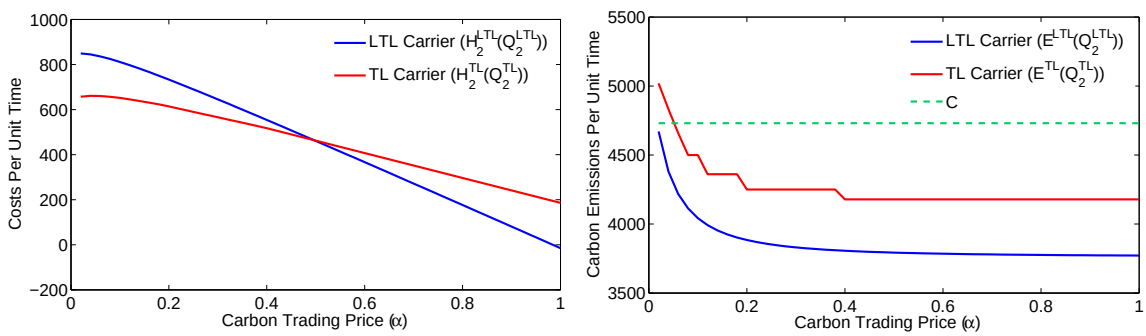
Property 11 (i) further implies that when $\widehat{e} > \widehat{t}$, $H_2^{LTL}(Q_2^{LTL}) < H_2^{TL}(Q_2^{TL})$ for $\alpha > \frac{t}{\widehat{e}-\widehat{t}}$ That is, the retailer will prefer the LTL carrier over the TL carrier for carbon trading prices higher than a specific level when the TL carrier's unit carbon emissions for each item loaded to a truck, $\widehat{e}$, is greater than the LTL carrier's unit carbon emissions for each item transported, $\widehat{t}$. Property 11 (ii) further implies that when $\widehat{t} > \widehat{e} + \widehat{w}$, $H_2^{LTL}(Q_2^{LTL}) > H_2^{TL}(Q_2^{TL})$ for $\alpha > \frac{R-t}{\widehat{t}-\widehat{e}-\widehat{w}}$. That is, the retailer will prefer the TL carrier over the LTL carrier for carbon trading prices higher than a specific level when the LTL carrier's unit carbon emissions for each item transported is greater than the TL carrier's carbon emissions generated by shipping one item with one truck. These observations suggest that a LTL or a TL carrier can impact the retailer's preference not only by the costs they charge but also by the environmental benefits they offer. For instance, for a given carbon trading price λ, a LTL carrier can become the retailer's preference by decreasing the unit transportation cost or the unit transportation emissions. Similarly, a TL carrier can become the retailer's preference by decreasing the emissions generated per unit load in their truck (for instance, by changing the fuel type) or the emissions generated by empty truck weight (for instance, by having greener trucks).

One should note that Property 11 gives sufficient conditions for preferring a LTL (TL) over a TL (LTL) carrier. For other cases, the retailer's preference can be determined using Corollaries 3 and 4. The next example illustrates how the retailer's

choice of transportation mode changes with carbon trading price under a carbon cap and trade regulation.

Example 2. *Suppose that the retailer of Example 1 is subject to a carbon cap and trade regulation and they can use a LTL or a TL carrier for their inbound shipment. The LTL carrier has the following specifications:* $t = 0.3$ *and* $\widehat{t} = 0.3$*. The TL carrier has the following specifications:* $R = 10$*,* $P = 50$*,* $\widehat{w} = 10$*, and* $\widehat{e} = 0.3$*.*

Example 2 considers carbon trading prices varying between 0 and 1. The carbon cap value is considered to be fixed as the mid-point of the carbon cap range defined similar to Example 1. The costs and carbon emissions per unit time with each carrier as α changes are illustrated in Figure 2.2.



(a) Carbon trading price vs. costs (b) Carbon trading price vs. carbon emissions

Figure 2.2. LTL vs. TL carrier under carbon cap and trade regulation

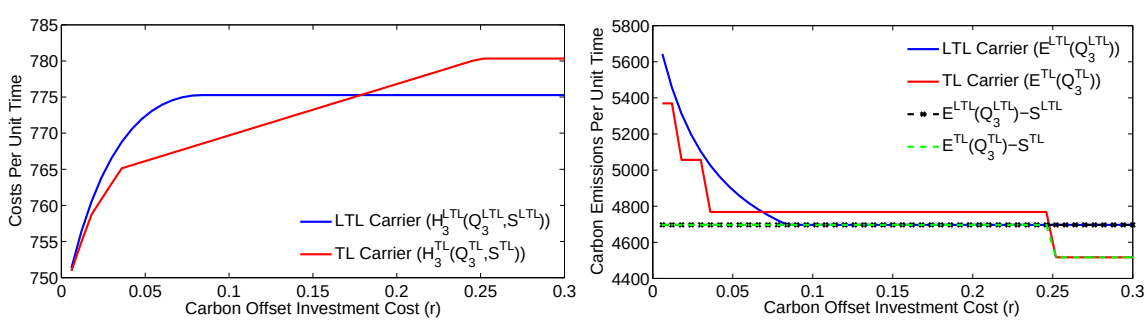
As it can be seen from Figure 2.2a, depending on the carbon trading price, the retailer can prefer a LTL over a TL carrier or vice versa. Corollaries 3 and 4 can be used for such comparison. Particularly, for Example 2, for smaller values of α, the retailer would prefer the TL carrier as it results in lower costs per unit time; however, for larger values of α, the retailer would prefer the LTL carrier. Figure 2.2b illustrates the carbon emissions as α increases. It can be observed that carbon emissions per unit time decrease with α. Moreover, in Example 2, for larger values of α, the LTL carrier not only reduces costs but also carbon emissions.

2.3.3. LTL vs. TL Carrier under Carbon Cap and Offset. Consider that a retailer is subject to carbon cap and offset regulations with a carbon cap value C and a unit offset cost r. Similar to Property 10, one can show that $E^{TL}(Q_1^{TL}) - S^{TL} \leq E^{LTL}(Q_1^{LTL}) - S^{LTL}$ when $E^{LTL}(Q^{LTL}) \geq C$. Furthermore, when $E^{LTL}(Q^{LTL}) \geq C$ and $S^{TL} > 0$, it can be shown that $E^{TL}(Q_1^{TL}) - S^{TL} = E^{LTL}(Q_1^{LTL}) - S^{LTL} = C$. That is, if using a TL carrier requires carbon offsetting, the carbon emissions after offsetting, i.e., carbon emissions minus the carbon offset, are the same with a LTL and a TL carrier under a carbon cap and offset regulation with restricting carbon cap value. This is also observed in the following example, where the unit carbon offset cost affects the retailer's choice of transportation mode.

Example 3. *Suppose that the retailer of Examples 1-2 is subject to a carbon cap and offset regulation and they can use a LTL or a TL carrier for their inbound shipment. The LTL carrier has the following specifications:* $t = 0.25$ *and* $\widehat{t} = 0.6$. *The TL carrier has the following specifications:* $R = 20$, $P = 80$, $\widehat{w} = 15$, *and* $\widehat{e} = 0.35$.

Example 3 considers carbon offset investment costs varying between 0 and 0.3. The carbon cap value is considered to be fixed as the mid-point of the carbon cap range defined similar to Example 1. The costs and carbon emissions per unit time with each carrier as r changes are illustrated in Figure 2.3.

As it can be seen from Figure 2.3a, depending on the carbon offset investment cost, the retailer can prefer a LTL over a TL carrier or vice versa. Corollaries 5 and 6 can be used for such comparison. Particularly, for Example 3, for smaller values of



(a) Carbon offset cost vs. costs (b) Carbon offset cost vs. carbon emissions

Figure 2.3. LTL vs. TL carrier under carbon cap and offset regulation

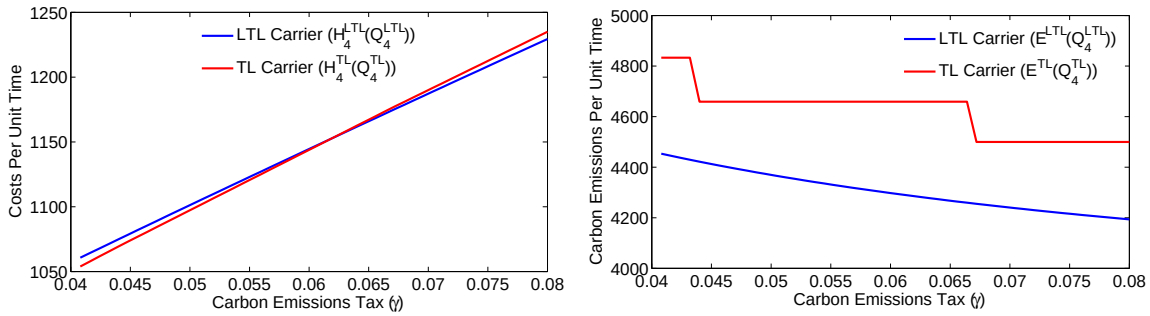
r, the retailer would prefer the TL carrier as it results in lower costs per unit time; however, for larger values of r, the retailer would prefer the LTL carrier. Figure 2.3b illustrates the carbon emissions as r increases. It can be observed from Figure 2.3b that, for small r values, for instance $r \leq 0.05$, the TL carrier results in lower costs as well as lower carbon emissions per unit time. On the other hand, for $r \in (0.2, 0.25)$, the LTL carrier results in lower costs as well as lower carbon emissions per unit time. Finally, one can observe that $E^{TL}(Q_1^{TL}) - S^{TL} = E^{LTL}(Q_1^{LTL}) - S^{LTL}$ for $r \lesssim 0.25$ (where $S^{TL} > 0$ actually) and $E^{TL}(Q_1^{TL}) - S^{TL} < E^{LTL}(Q_1^{LTL}) - S^{LTL}$ for $r \gtrsim 0.25$.

2.3.4. LTL vs. TL Carrier under Carbon Taxing. Consider that a retailer is subject to carbon taxing regulation with carbon cap tax γ. Similar to Property 11, one can show that if $t + \gamma\widehat{t} < \gamma\widehat{e}$ then $H_4^{LTL}(Q_4^{LTL}) < H_4^{TL}(Q_4^{TL})$; and, if $\lceil \frac{Q}{P} \rceil \leq Q$ for any Q and $t + \gamma\widehat{t} > R + \gamma\widehat{w} + \gamma\widehat{e}$ then $H_4^{LTL}(Q_4^{LTL}) > H_4^{TL}(Q_4^{TL})$. That is, the carbon tax in place affects the retailer's choice of transportation mode as illustrated in the following example.

Example 4. *Suppose that the retailer of Examples 1-3 is subject to a carbon taxing regulation and they can use a LTL or a TL carrier for their inbound shipment. The LTL carrier has the following specifications:* $t = 0.31$ *and* $\widehat{t} = 0.34$*. The TL carrier has the following specifications:* $R = 15$*,* $P = 50$*,* $\widehat{w} = 10$*, and* $\widehat{e} = 0.3$*.*

Example 4 considers carbon emissions taxes varying between 0.04 and 0.08. The costs and carbon emissions per unit time with each carrier as γ changes are illustrated in Figure 2.4.

As it can be seen from Figure 2.4a, depending on the carbon emissions tax, the retailer can prefer a LTL over a TL carrier or vice versa. Corollaries 7 and 8 can be used for such comparison. Particularly, for Example 4, for smaller values of γ, the retailer would prefer the TL carrier as it results in lower costs per unit time; however, for larger values of γ, the retailer would prefer the LTL carrier. Figure 2.4b illustrates the carbon emissions as γ increases. It can be observed that carbon emissions per unit time increases with γ for both LTL and TL carriers. Also, in Example 4, for larger values of γ, the LTL carrier not only reduces costs but also carbon emissions.



(a) Carbon tax vs. costs (b) Carbon tax vs. carbon emissions

Figure 2.4. LTL vs. TL carrier under carbon taxing regulation

2.4. NUMERICAL ANALYSES

In this section, the focus is on two sets of numerical analyses. The first set of numerical analyses demonstrates the effects of the carbon emissions regulation policy parameters on the retailer's costs and carbon emissions with LTL and TL carriers. In the second set of numerical analyses, the effects of transportation costs and emissions parameters of LTL and TL carriers are illustrated on the retailer's costs and carbon emissions under each carbon emissions regulation policy.

For the numerical analyses discussed in this section, the problem instances are generated as follows. In all of the problem instances solved, assume that $p = 0$ and $\lambda = 2,000$ units (note that total purchase cost per unit time is a constant; hence, it is not effective in the optimum order quantity decisions). The retailer's cost parameters are randomly generated assuming that $h \sim U[1, 5]$ and $K \sim U[50, 250]$, where $U[a, b]$ denotes a uniform distribution with bounds a and b (studies focusing on inventory control and inventory control with carbon emissions assume similar values, see, e.g., Benjaafar et al., 2013, Hua et al., 2011, Chen et al., 2013, Toptal et al., 2014). The retailer's emissions parameters are randomly generated assuming that $\widehat{h} \sim U[2, 8]$ and $\widehat{K} \sim U[50, 300]$ (again, similar values are used in the literature, see, e.g., Benjaafar et al., 2013, Hua et al., 2011, Chen et al., 2013, and some of these studies are real life applications, see, e.g., Arikan et al., 2013). The details of the transportation related cost and carbon emissions parameters are as follows.

TL Transportation: In defining per truck costs and per truck capacities, assume that $R \sim U[150, 450]$ and $P \sim U[100, 300]$ (integrated inventory control and truckload transportation studies assume similar values, see, e.g., Toptal et al., 2003, Toptal and Çetinkaya, 2006, Toptal, 2009, Konur and Toptal, 2012). For practical purposes, P is rounded up to the nearest multiplier of 10. Since emissions generated from freight trucks are calculated using empty truck and full truck emissions per unit distance (see, e.g., Pan et al., 2013, Reed et al., 2010, Hoen et al., 2014), empty truck and full truck emissions per unit distance are defined first. Particularly, define $\widehat{w}_e$ and $\widehat{w}_f$ as the empty truck and full truck carbon emissions per unit distance. In applied studies from literature (see, e.g., Pan et al., 2013 and Reed et al., 2010), it is observed that $\widehat{w}_f$ is approximately $1.5\widehat{w}_e$ even for different truck types. Furthermore, it can be observed from the values given in those studies that $\widehat{w}_e$ is generally between 1 and 1.5 kg CO_2/km (a simulation study provided by Daccarett-Garcia, 2009 also assumes similar values). Therefore, in generating $\widehat{w}_e$ and $\widehat{w}_f$, this section assumes that $\widehat{w}_e \sim U[1, 1.5]$ and $\widehat{w}_f = \beta\widehat{w}_e$, where β is the ratio of $\widehat{w}_f/\widehat{w}_e$ and it is assumed that $\beta \sim U[1.2, 1.8]$. Using $\widehat{w}_e$, $\widehat{w}_f$ and P, one can define carbon emissions generated per unit load of a truck as $\widehat{e} = d\frac{\widehat{w}_f - \widehat{w}_e}{P}$ and carbon emissions generated by an empty truck as $\widehat{w} = d\widehat{w}_e$, where d is the distance from point of supply to the retailer. Assume that $d \sim U[100, 500]$. That is, $\widehat{e}$ and $\widehat{w}$ are randomly generated by randomly generating $\widehat{w}_f$, β, and d for a given P.

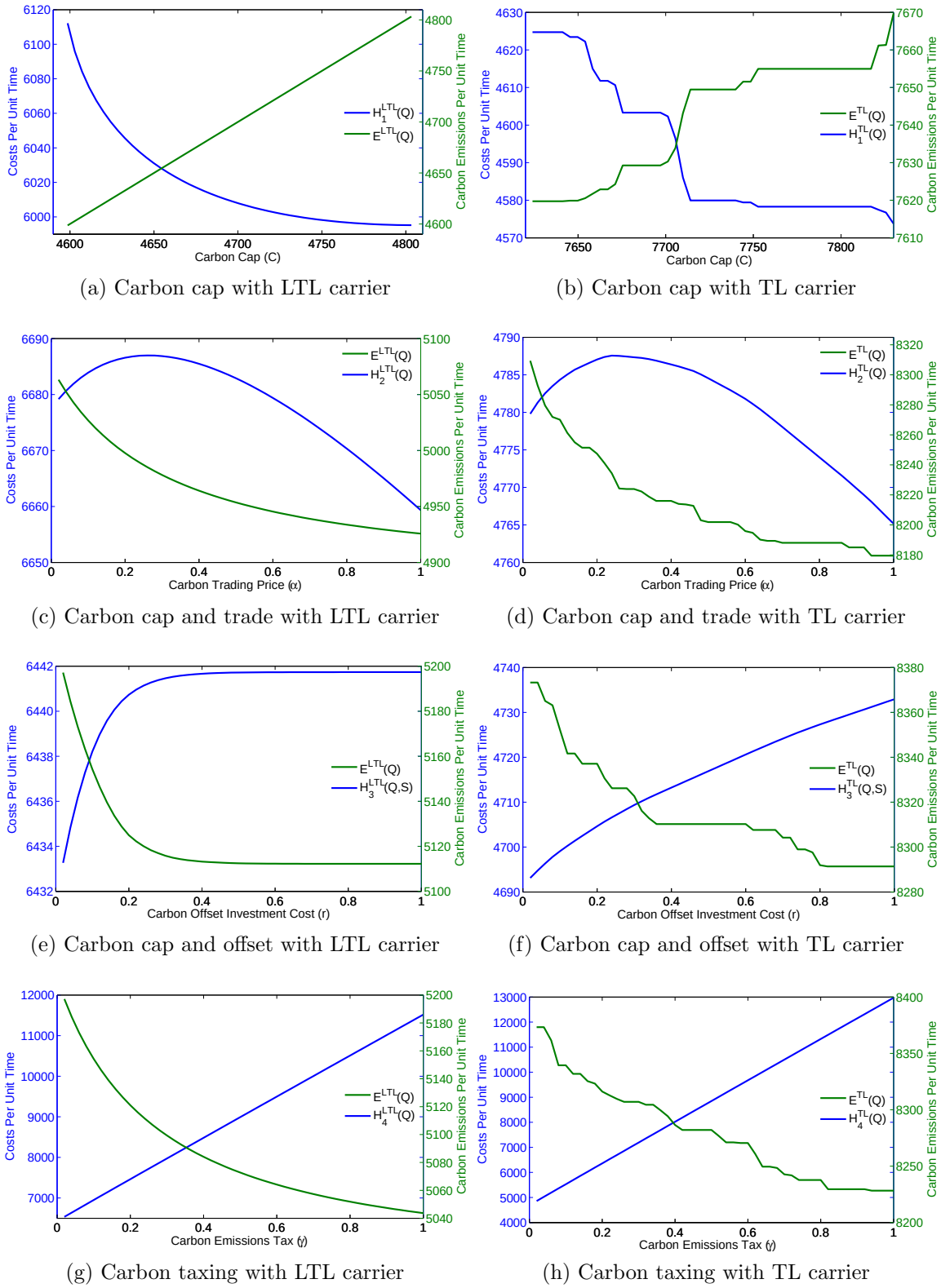
LTL Transportation: Toptal and Bingol (2011) note that $\frac{R}{P} < t < P$; hence, this section assumes that $t \sim U[\frac{R}{P}, 2\frac{R}{P}]$ (the cases where $t \to \frac{R}{P}$ are not practical as per unit transportation cost of a LTL carrier would be very close to per truck cost charged by a TL carrier). In defining $\widehat{t}$, in order to account for cases where a LTL carrier's per unit carbon emissions can be both lower and higher compared to the TL carrier's carbon emissions from per unit load in a truck, this section assumes that $\widehat{t} = \phi\widehat{e}$ such that $\phi \sim U[0.5, 2]$.

2.4.1. Effects of Carbon Emissions Regulations. In order to analyze the effects of the parameters of the carbon emissions regulations on the retailer's costs and carbon emissions per unit time, numerical studies were conducted for both LTL

and TL carriers. Particularly, for a given policy parameter under each carrier option, 100 problem instances were randomly generated. Figure 2.5 illustrates the changes in the average costs and carbon emissions over 100 problem instances for 50 different policy parameters considered.

2.4.1.1. Effects of carbon cap under carbon cap. To analyze the carbon cap, for any problem instance, this section considers 50 different C values increasing from the minimum carbon emissions possible (i.e., carbon emissions of the order quantity minimizing carbon emissions function) to the maximum carbon emissions possible (i.e., carbon emissions of the order quantity minimizing cost function) in equal increments. It can be observed from Figures 2.5a and 2.5b that as C increases, carbon emissions per unit time increase while costs per unit time decrease with both LTL and TL carriers. These observations are expected as the retailer's set of feasible order quantities enlarges as C increases in the cases where a LTL or a TL carrier is used. Note that in the case of TL carriers, the piecewise structures of the cost and emissions functions lead to piecewise increasing and decreasing cost and emissions, respectively. It should be noted that while the carbon cap constraint is going to be tight for the case with a LTL carrier, this is not necessarily true for the case with a TL carrier due to the integer definition of the number of trucks used for shipment.

2.4.1.2. Effects of trading price under carbon cap and trade. Under a cap and trade policy, the focus is to analyze the effects of the carbon trading price, α. To do so, this section assumes that the carbon cap is the mid-point between the maximum and minimum carbon emissions possible defined above. For any problem instance, this section considers 50 different α values increasing from 0 to 1 in equal increments. It can be observed from Figures 2.5c and 2.5d that as α increases, carbon emissions per unit time decrease since it is either more expensive to buy additional carbon permits or more profitable to sell extra carbon allowances. On the other hand, as α increases, costs per unit time first increase then decrease. This is due to the fact that up to a point of carbon trading price, the retailer continues to purchase carbon permits but after a point they prefer to sell carbon allowances. Similar results are observed in Hua et al. (2011). The results generalize these observations for TL transportation as well.



(a) Carbon cap with LTL carrier

(b) Carbon cap with TL carrier

(c) Carbon cap and trade with LTL carrier

(d) Carbon cap and trade with TL carrier

(e) Carbon cap and offset with LTL carrier

(f) Carbon cap and offset with TL carrier

(g) Carbon taxing with LTL carrier

(h) Carbon taxing with TL carrier

Figure 2.5. Costs and carbon emissions vs. carbon regulation parameters

2.4.1.3. Effects of offset cost under carbon cap and offset. Under a cap and offset policy, the focus is to analyze the effects of carbon offset investment cost, r. To do so, this section defines the carbon cap similar to the analysis of cap and trade. For any problem instance, this section considers 50 different r values increasing from 0 to 1 in equal increments. It can be observed from Figures 2.5e and 2.5f that as r increases, carbon emissions per unit time decrease while costs per unit time increase with both LTL and TL carriers. This is expected since higher r values encourage the retailer to decrease their emissions so that high carbon emissions abatement investment costs are avoided. Nevertheless, when r is higher, the costs will increase due to higher emissions abatement investment costs and/or preferring order quantities to reduce carbon emissions instead of cost decreasing order quantities.

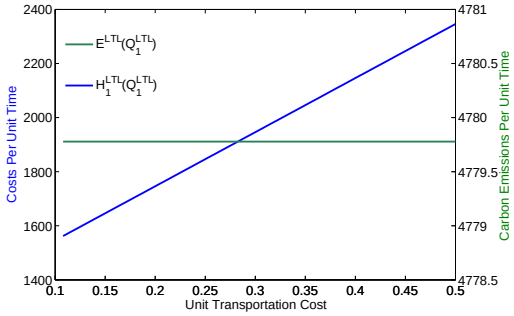
2.4.1.4. Effects of carbon tax under carbon taxing. For a carbon taxing policy, the focus is to analyze the effects of carbon emissions tax, γ. For any problem instance, 50 different γ values increasing from 0 to 1 in equal increments were considered. As expected and can be observed from Figures 2.5g and 2.5h, as γ increases, the carbon emissions per unit time decrease and the costs per unit time increase for both LTL and TL carriers.

2.4.2. Effects of Transportation Parameters. In order to analyze the effects of the parameters of the LTL and TL carriers on the costs and carbon emissions under each carbon emissions regulation policy, numerical studies for both LTL and TL carriers' transportation costs and emissions were conducted. Particularly, the effects of unit transportation cost t and unit transportation emissions $\widehat{t}$ of a LTL carrier were analyzed; and, the effects of the per truck cost R and the empty truck emissions $\widehat{w}$ of a TL carrier were also analyzed. Under each regulation policy, 100 problem instances were randomly generated and each problem instance was solved with 50 different values of the parameters under consideration. Figure 2.6 illustrates the changes in the average costs and carbon emissions per unit time over 100 problem instances solved for 50 different values of t and $\widehat{t}$. Similarly, Figure 2.7 illustrates the changes in the average costs and carbon emissions per unit time over 100 problem instances for 50 different values of R and $\widehat{w}$.

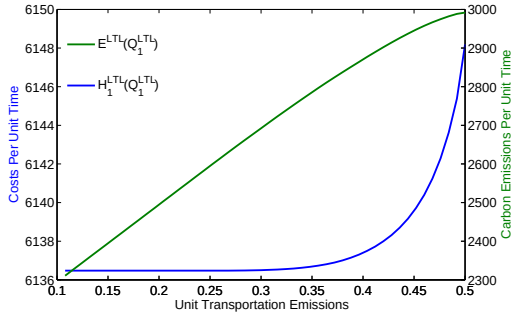
2.4.2.1. Effects of LTL transportation. It is easy to analytically show that as t increases, the retailer's total costs per unit time increase and carbon emissions per

unit time remain the same under each of the carbon emissions regulations considered when a LTL carrier is used for inbound transportation. That is, it can be shown that $\frac{dH_1^{LTL}(Q_1^{LTL})}{dt} = \lambda > 0$, $\frac{dH_2^{LTL}(Q_2^{LTL})}{dt} = \lambda > 0$, $\frac{dH_3^{LTL}(Q_3^{LTL},S^{LTL})}{dt} = \lambda > 0$, and $\frac{dH_4^{LTL}(Q_4^{LTL})}{dt} = \lambda > 0$; and, $\frac{dE^{LTL}(Q_1^{LTL})}{dt} = 0$, $\frac{dE^{LTL}(Q_2^{LTL})}{dt} = 0$, $\frac{dE^{LTL}(Q_3^{LTL})}{dt} = 0$, and $\frac{dHE^{LTL}(Q_4^{LTL})}{dt} = 0$. These observations can be noted in Figures 2.6a, 2.6c, 2.6e, and 2.6g. Particularly, it can be seen that as unit transportation cost t of a LTL carrier increases, the retailer's costs per unit time increases while carbon emissions per unit time do not change under any carbon emissions regulation policy. This is due to the fact that t is not effective in the solution of models M1-LTL, M2-LTL, M3-LTL, and M4-LTL. That is, the optimal order quantity does not depend on t in any of the LTL models.

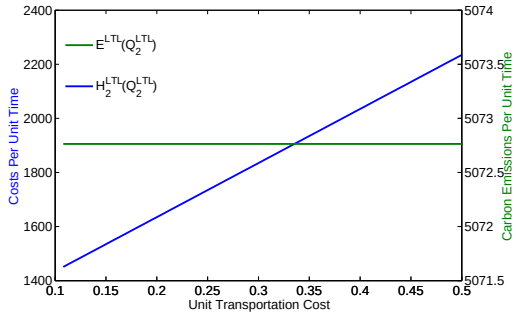
Moreover, one can prove that, as unit transportation emissions $\widehat{t}$ of a LTL carrier increases, the retailer's costs and carbon emissions per unit time linearly increase under carbon cap and trade and carbon taxing regulations as observed in Figures 2.6d and 2.6h. That is, one can show that $\frac{dH_2^{LTL}(Q_2^{LTL})}{d\widehat{t}} = \alpha\lambda > 0$ and $\frac{dH_4^{LTL}(Q_4^{LTL})}{d\widehat{t}} = \gamma\lambda > 0$; and, $\frac{dE^{LTL}(Q_2^{LTL})}{d\widehat{t}} = \lambda > 0$ and $\frac{dE^{LTL}(Q_4^{LTL})}{d\widehat{t}} = \lambda > 0$. These simply follow from the fact that, in M2-LTL and M4-LTL, unit transportation emissions have direct costs and total costs of transportation emissions is a constant; therefore, the optimal order quantities of models M2-LTL and M4-LTL are not affected by $\widehat{t}$. On the other hand, the retailer's costs per unit time remain fixed then start to increase while the retailer's carbon emissions per unit time increase then remain fixed as unit transportation emissions $\widehat{t}$ of a LTL carrier increases under carbon cap and carbon cap and offset regulations. That is, $\frac{dH_1^{LTL}(Q_1^{LTL})}{d\widehat{t}} = 0$ and $\frac{dE_1^{LTL}(Q_1^{LTL})}{d\widehat{t}} > 0$ for $\widehat{t} < \tau$; and, $\frac{dH_1^{LTL}(Q_1^{LTL})}{d\widehat{t}} > 0$ and $\frac{dE_1^{LTL}(Q_1^{LTL})}{d\widehat{t}} = 0$ for $\widehat{t} > \tau$. Similarly, $\frac{dH_3^{LTL}(Q_3^{LTL},S^{LTL})}{d\widehat{t}} = 0$ and $\frac{dE_3^{LTL}(Q_3^{LTL})}{d\widehat{t}} > 0$ for $\widehat{t} < \psi$; and, $\frac{dH_3^{LTL}(Q_3^{LTL},S^{TL})}{d\widehat{t}} > 0$ and $\frac{dE_3^{LTL}(Q_3^{LTL})}{d\widehat{t}} = 0$ for $\widehat{t} > \psi$. These follow from the fact that, for smaller values of $\widehat{t}$, the retailer will order Q^{LTL} if the carbon cap is not restrictive. Therefore, increases in $\widehat{t}$ up to a point will not change the optimal order quantity as long as the carbon cap is not restrictive. This, in return, implies no increase in costs but a linear increase in carbon emissions. After a value of $\widehat{t}$, on the other hand, the carbon cap will become restrictive; hence, the retailer's carbon emissions will not change. However, the retailer's set of feasible order quantities will get smaller at higher values of $\widehat{t}$, which then increases costs per



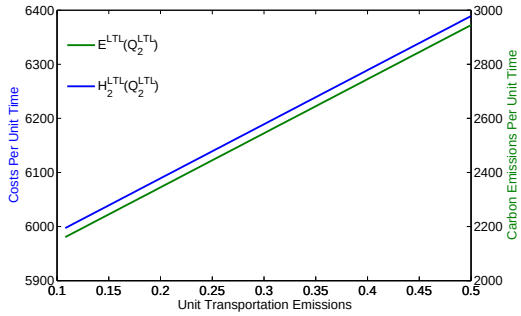
(a) t vs. $H_1^{LTL}(Q_1^{LTL})$ and $E^{LTL}(Q_1^{LTL})$



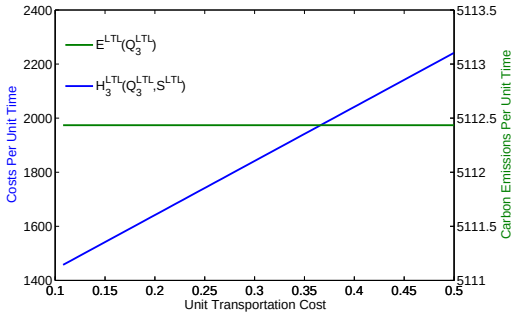
(b) $\widehat{t}$ vs. $H_1^{LTL}(Q_1^{LTL})$ and $E^{LTL}(Q_1^{LTL})$



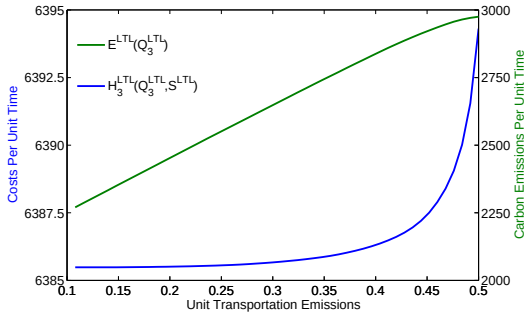
(c) t vs. $H_2^{LTL}(Q_2^{LTL})$ and $E^{LTL}(Q_2^{LTL})$



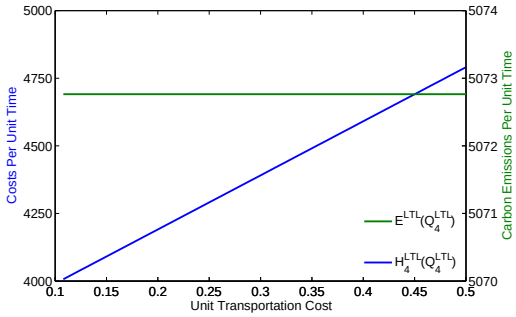
(d) $\widehat{t}$ vs. $H_2^{LTL}(Q_2^{LTL})$ and $E^{LTL}(Q_2^{LTL})$



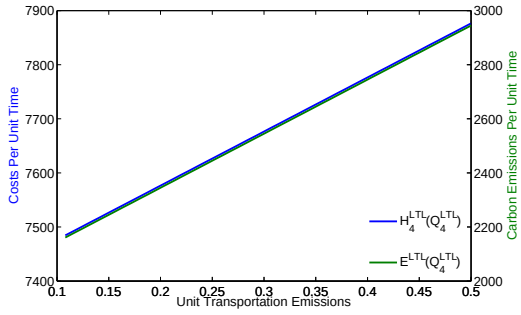
(e) t vs. $H_3^{LTL}(Q_3^{LTL}, S^{LTL})$ and $E^{LTL}(Q_3^{LTL})$



(f) $\widehat{t}$ vs. $H_3^{LTL}(Q_3^{LTL}, S^{LTL})$ and $E^{LTL}(Q_3^{LTL})$



(g) t vs. $H_4^{LTL}(Q_4^{LTL})$ and $E^{LTL}(Q_4^{LTL})$



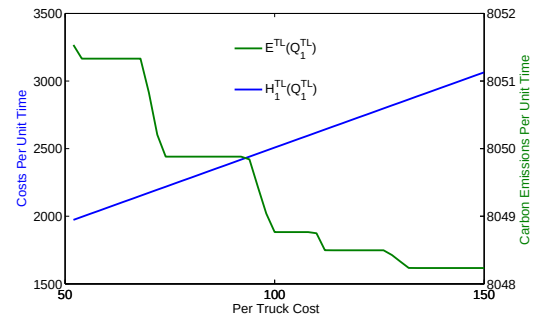
(h) $\widehat{t}$ vs. $H_4^{LTL}(Q_4^{LTL})$ and $E^{LTL}(Q_4^{LTL})$

Figure 2.6. Costs and carbon emissions vs. LTL transportation

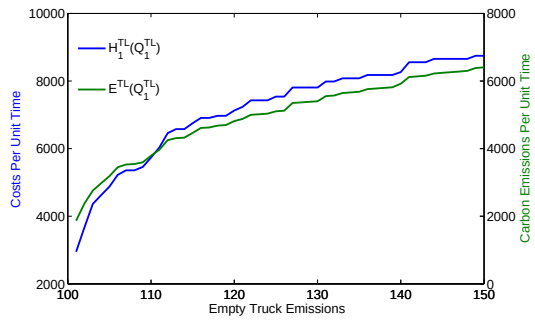
unit time, which can be observed in Figures 2.6b and 2.6f.

2.4.2.2. Effects of TL transportation. The retailer's costs per unit time can be analytically shown to increase with the per truck cost R charged by a TL carrier under any carbon emissions regulation policy. That is, $\frac{dH_1^{TL}(Q_1^{TL})}{dR} > 0$, $\frac{dH_2^{TL}(Q_2^{TL})}{dR} > 0$, $\frac{dH_3^{TL}(Q_3^{LTL},S^{LTL})}{dR} > 0$, and $\frac{dH_4^{LTL}(Q_4^{LTL})}{dR} > 0$. On the other hand, while one can expect the retailer's carbon emissions per unit time to decrease with an increase in R since the retailer is expected to use fewer trucks for inbound shipment, the carbon emissions per unit time can both increase or decrease with an increase in R. This is specifically due to the fact that the retailer's order quantity can increase or decrease with a change in R. These are observed in Figures 2.7a, 2.7c, 2.7e, and 2.7g. For instance, when the order quantity decreases with an increase in R, which implies that the retailer prefers to use fewer trucks for each order, this increases the shipment frequency, which can increase carbon emissions as increased carbon emissions from order setups can be significantly higher than the reduced carbon emissions from inbound shipment with fewer trucks. Similarly, when the order quantity increases with an increase in R so that the retailer avoids paying high setup costs by replenishing their inventory less frequently, this might increase carbon emissions per unit time as increased emissions from inbound shipment with one more truck can be significantly higher than the reduced carbon emissions from order setups. The tools provided in Section 2.2 can be used to evaluate the effects of the changes in R for specific cases.

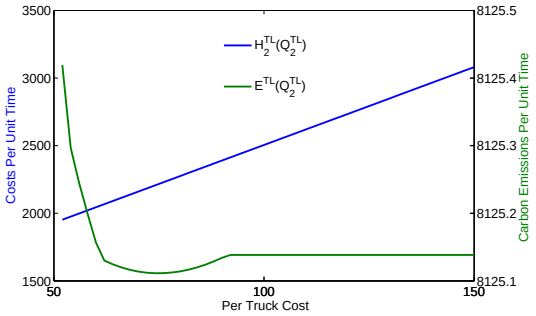
As empty truck emissions $\widehat{w}$ increase, similar to the increases in R, it can be analytically shown that the retailer's costs per unit time are non-decreasing for carbon cap and carbon cap and offset regulations and increasing for carbon cap and trade and carbon taxing regulations. That is, $\frac{dH_1^{TL}(Q_1^{TL})}{d\widehat{w}} \geq 0$, $\frac{dH_2^{TL}(Q_2^{TL})}{d\widehat{w}} > 0$, $\frac{dH_3^{TL}(Q_3^{LTL},S^{LTL})}{d\widehat{w}} \geq 0$, and $\frac{dH_4^{LTL}(Q_4^{LTL})}{d\widehat{w}} > 0$. Furthermore, one can analytically show that, when the retailer's optimal order quantity is unique, $\frac{dH_2^{TL}(Q_2^{TL})}{d\widehat{w}} \geq 0$ and $\frac{dH_4^{TL}(Q_4^{TL})}{d\widehat{w}} \geq 0$, i.e., the carbon emissions per unit time increase with $\widehat{w}$ under carbon cap and trade and carbon taxing regulations (different than the effects of R on models M2-TL and M4-TL, $\widehat{w}$ not only affects costs but also emissions). These are observed in Figures 2.7b, 2.7d, 2.7f, and 2.7h. On the other hand, while average carbon emissions per unit time over the problem instances solved for models M1-TL and M3-TL tend to increase with $\widehat{w}$ as observed in Figures 2.7b and 2.7f, it is possible that carbon emissions per unit



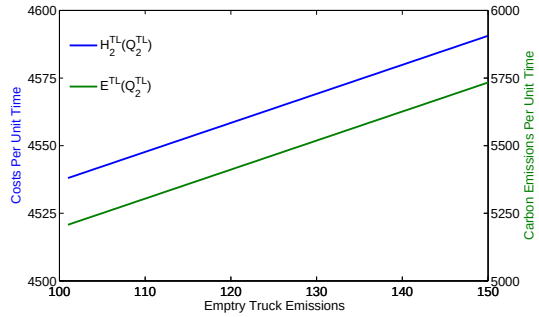
(a) R vs. $H_1^{TL}(Q_1^{TL})$ and $E^{TL}(Q_1^{TL})$



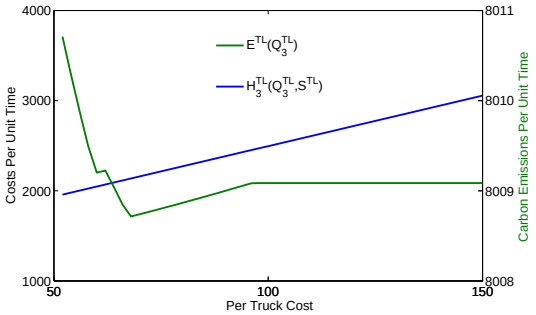
(b) $\widehat{w}$ vs. $H_1^{TL}(Q_1^{TL})$ and $E^{TL}(Q_1^{TL})$



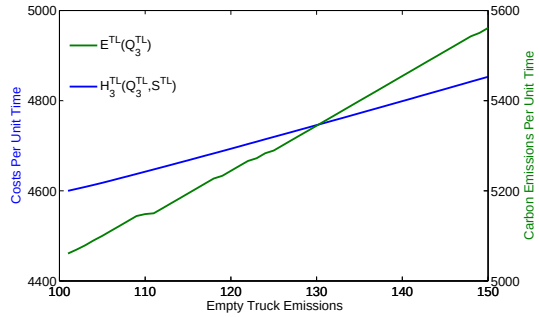
(c) R vs. $H_2^{TL}(Q_2^{TL})$ and $E^{TL}(Q_2^{TL})$



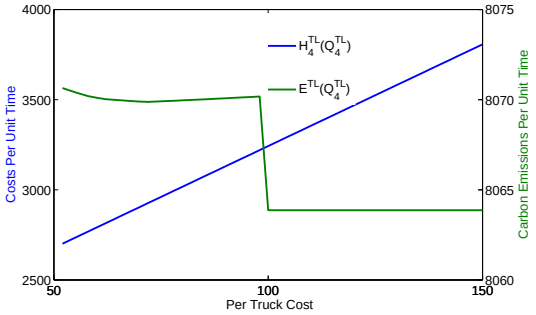
(d) $\widehat{w}$ vs. $H_2^{TL}(Q_2^{TL})$ and $E^{TL}(Q_2^{TL})$



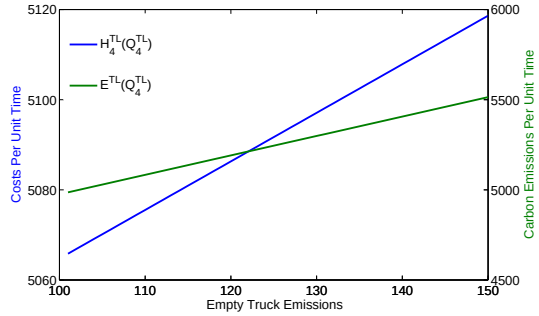
(e) R vs. $H_3^{TL}(Q_3^{TL}, S^{TL})$ and $E^{TL}(Q_3^{TL})$



(f) $\widehat{w}$ vs. $H_3^{TL}(Q_3^{TL}, S^{TL})$ and $E^{LTL}(Q_3^{TL})$



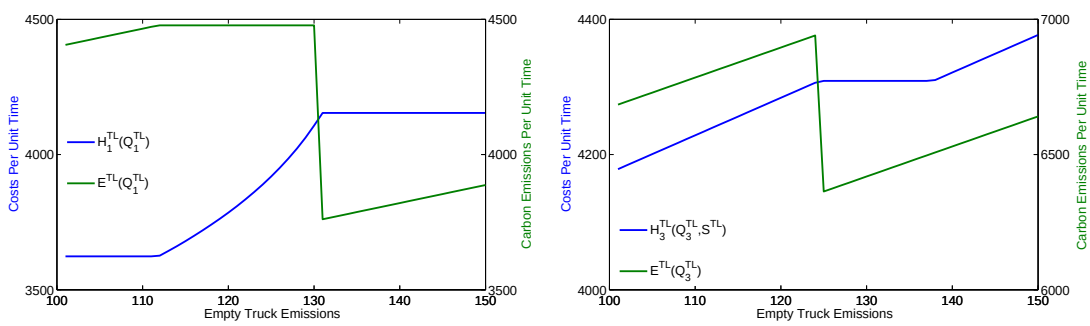
(g) R vs. $H_4^{TL}(Q_4^{TL})$ and $E^{TL}(Q_4^{TL})$



(h) $\widehat{w}$ vs. $H_4^{TL}(Q_4^{TL})$ and $E^{TL}(Q_4^{TL})$

Figure 2.7. Costs and carbon emissions vs. TL transportation

time can decrease with $\widehat{w}$ under carbon cap and carbon cap and offset regulations. Again, this can be due to the fact that the change in the order quantity as a result of a change in $\widehat{w}$ can lower emissions similar to the aforementioned discussion for R. Therefore, it is important to analyze the tradeoff between transportation emissions and other emissions; otherwise, as noted by Browne et al. (2005) and Rizet et al. (2012), the emissions reduced from transportation activities can be lower compared to the increased carbon emissions due to preferring a greener transportation option. Specifically, Figures 2.8a and 2.8b illustrate two examples of M1-TL and M3-TL where carbon emissions per unit time both increase and decrease with increasing $\widehat{w}$. That is, a decrease in $\widehat{w}$, using a TL carrier with a greener truck for instance, can increase the retailer's carbon emissions per unit time.



(a) M1-TL: carbon cap regulation (b) M3-TL: carbon cap and offset regulation

Figure 2.8. Costs and carbon emissions vs. empty truck emissions

2.5. CONCLUSIONS AND FUTURE RESEARCH

Considering the fact that emissions generated by trucking constitute the majority of the emissions of the transportation industry, a substantial amount of carbon emissions can be reduced through explicitly accounting for transportation emissions in cases of LTL and TL carriers. The sustainability of supply chains are getting more important everyday and comparing LTL to TL carriers is crucial for cost efficient as well as sustainable supply chains. Nevertheless, the current studies in the literature have not explicitly considered these two common practices of road transportation.

The models presented in this section pioneer sustainability analyses of integrated inventory control and explicit transportation modeling in supply chains. Different carbon emissions regulation policies are modeled with LTL and TL transportation. The properties of these models are analyzed to determine the retailer's inventory control and transportation decisions.

Particularly, the integrated inventory control and transportation decisions of a retailer under four different carbon emissions regulation policies were analyzed. In this setting, the retailer assumes the basic EOQ model in controlling their inventory and two carrier options available for inbound shipment are LTL and TL carriers. In the case where a TL carrier is preferred by the retailer, to accurately account for truck costs and emissions, truckload transportation and truckload carbon emissions are explicitly modeled regarding per truck costs, per truck capacities, and the emissions generated from the truck itself in addition to the load it carries. The retailer's problem is formulated and optimally solved under carbon cap, carbon cap and trade, carbon cap and offset, and carbon taxing policies for both carrier options.

Analytical results were discussed when a LTL carrier is preferable over a TL carrier under each carbon emissions regulation. Specifically, it is observed that transportation costs are not the only factor affecting a retailer's preference. Transportation emissions of the carriers are important for the retailer's transportation mode selection. Furthermore, a set of sample scenarios are studied to support these results and illustrate the practical use of the tools discussed. Specifically, these tools can be used by a retailer to compare LTL carriers, a LTL carrier to a TL carrier, and TL carriers under carbon emissions regulation policies. The examples studied show that a retailer's preference for a LTL over a TL carrier or vice versa also depends on the specifications of the carbon emissions regulation policy in place.

A set of numerical studies documents the effects of carbon emissions regulation policies on a retailer's costs and carbon emissions with LTL and TL transportation. Furthermore, in another set of numerical studies, the effects of transportation costs and transportation emissions on a retailer's costs and carbon emissions are investigated. Specifically, for a TL carrier's transportation emissions, counterintuitive cases are observed. It is possible that an increase in truck emissions can decrease a retailer's overall carbon emissions under carbon cap and carbon cap and offset regulations.

This research contributes to the literature on environmentally sensitive supply chain and logistics models by integrating inventory control and transportation decisions. The tools provided here give cost efficient decision making under carbon regulations by modeling carbon emissions, specifically, transportation emissions explicitly. Furthermore, the effects of transportation emissions on costs and carbon emissions are analyzed. The formulations provided herein can be applied to many other inventory control models with deterministic or stochastic demand. For instance, as mentioned previously, the settings of this section can be easily extended to analyze the case where an order splitting between LTL and TL carriers is possible.

Specifically, as noted in Section 1, there are a limited number of studies on stochastic inventory control problems with environmental considerations. Furthermore, explicit transportation modeling is not considered in most of these studies. Stochastic inventory control models such as the newsvendor model, continuous review inventory systems, and periodic review inventory systems can be analyzed with integrated transportation decisions under environmental considerations. In Section 3, a continuous review inventory control model is investigated with environmental objectives through explicitly integrating transportation decisions. Particularly, similar to this section, LTL and TL carriers are simultaneously modeled and trucking activities and their effects on costs and carbon emissions are explicitly formulated.

Moreover, analyses of multi-item inventory systems with environmental considerations are limited. The settings of this section can be extended for joint control of multiple items' inventory and transportation processes. For instance, in Section 4, a sustainable joint replenishment problem is formulated and analyzed with an environmental objective. As noted by Ülkü (2012), shipment consolidation not only reduces costs but also environmental damage. In Section 5, considering explicit truck modeling, a multi-item stochastic inventory system is analyzed with shipment consolidation. It is discussed how shipment consolidation can improve economical as well as environmental performance.

3. A SUSTAINABLE CONTINUOUS REVIEW INVENTORY MODEL WITH INTEGRATED TRANSPORTATION DECISIONS

This section analyzes sustainability in a continuous review inventory control system with integrated transportation decisions. Under the typical EOQ model, and as done in Section 2, the demand for the product of interest was assumed to be deterministic. Nevertheless, a deterministic demand assumption can be restrictive. Inventory control with environmental considerations in stochastic demand scenarios has been investigated for single period decisions (see, e.g. Song and Leng, 2012, Liu et al., 2013, Hoen et al., 2014 and Zhang and Xu, 2013). This section integrated the continuous review inventory control model with integrated transportation decisions and environmental objectives over a long planning horizon under stochastic demand. Specifically, this section analyzes the (Q, R) policy, in which a retailer orders Q units whenever their inventory level is R. In the classic (Q, R) model, the retailer's objective is to minimize expected costs due to inventory holding, order setups, and shortages. However, as noted by Dekker et al. (2012), profit maximization (or cost minimization) is not the only objective for companies. Many studies on sustainable supply chains, therefore, consider not only economic objectives such as cost minimization or profit maximization but also consider environmental objectives such as emission minimization (see, e.g., Li et al., 2008, Kim et al., 2009, Ramudhin et al., 2010, Wang et al., 2011, Bouchery et al., 2012, Chaabane et al., 2012).

In particular, Bouchery et al. (2012) integrate sustainability into the classical EOQ model by formulating a multi-objective EOQ model, in which costs as well as a set of social and environmental criteria are minimized. Similar to Bouchery et al. (2012), this section formulates a sustainable continuous review inventory control model by considering two objectives: cost minimization and emission minimization. Multi-objective continuous review inventory control models have been analyzed in the literature for the classical (Q, R) settings (see, e.g., Agrell, 1995, Puerto and Fernandez, 1998, Tsou, 2008, Tsou, 2009). Nevertheless, this section is the first that introduces an environmental objective into a continuous review inventory control model. Furthermore, this research contributes to the sustainable inventory control

models by analyses of multi-period stochastic demand inventory systems with integrated transportation decisions.

The aforementioned sustainable inventory control studies generally model emissions generated due to inventory control decisions by considering inventory holding emissions linearly proportional to the inventory level and replenishment emissions as being a fixed amount per replenishment. The studies, nevertheless, fail to model transportation decisions explicitly. To this end, this section considers the classical (Q, R) model with two different types of road transportation: LTL transportation and TL transportation. In LTL transportation, the retailer is charged on the number of units (or volume or weight units) shipped. This assumption is considered realistic as containers are usually standardized for a truck, or the effective capacity can usually be estimated accurately (Ben-Khedher and Yano, 1994). The settings of the (Q, R) model with LTL transportation are therefore parallel to the classical (Q, R) model. On the other hand, in TL transportation the retailer is charged on the number of trucks used for transportation, which requires explicit transportation modeling. As is done in the previous section, TL transportation costs and emissions are explicitly modeled by taking per truck costs and per truck capacities into account.

This section presents two bi-objective (Q, R) models: one for LTL transportation and one for TL transportation. For each of these models, the solution analyses are towards approximating a set of Pareto efficient (Q, R) policies, i.e., a Pareto front, among which the retailer can select a policy regarding their sensitivity to the environment and/or how much they are willing to pay to be more sustainable. In approximating the Pareto front for the sustainable (Q, R) model with LTL transportation, this section proposes a method that adopts a normalized weighted approach, a common approach used for multi-objective optimization problems. This method is then utilized in approximating the Pareto front for the sustainable (Q, R) model with TL transportation. Particularly, given the number of trucks to be used, the normalized weighted approach can be used to generate a set of Pareto efficient (Q, R) policies considering the given transportation capacity. Then, a dominance relation between two sets of Pareto efficient solutions is used, each for different transportation capacities, to approximate the Pareto front of the sustainable (Q, R) model with TL transportation.

The contribution in this section lies in including sustainability in multi-period stochastic inventory control models with explicit transportation decisions. The effects of freight trucks are explicitly modeled in cost as well as emission calculations. The methods presented in this section can be used by a retailer to compare different LTL carriers, LTL carriers to TL carriers, and different TL carriers in terms of not only cost but also environmental aspects.

Numerical studies are presented to analyze the effects of demand variance and lead time on the retailer's costs and emissions. It has been demonstrated for deterministic demand inventory control models that carbon emissions can be significantly reduced with low cost increases (see, e.g., Chen et al., 2013). This research further generalizes these results for a continuous review inventory control model under stochastic demand with LTL and TL transportation. Finally, this section demonstrates how the retailer can utilize the tools presented in this section to adopt a policy and select a carrier considering their cost and environmental goals.

3.1. SUSTAINABLE (Q,R) MODEL

This model considers a retailer's continuous review inventory control policy for a single product. In particular, the demand per unit time for the product is a random variable with mean λ and standard deviation ϑ. This section assumes that the demand per unit time is normally distributed (the methods discussed in the rest of the section are also valid for uniform and exponential demand distributions). The retailer adopts a (Q, R) policy such that an order of Q units is placed whenever R units are left in the inventory. That is, Q and R denote the order quantity and the re-order point, respectively. This section assumes that there is a fixed lead time, τ time units, for order delivery. Let $f(D)$ an $F(D)$ denote the probability density and cumulative distribution functions of the lead time demand, D, respectively. Furthermore, let μ and σ denote the expected lead time demand and the standard deviation of the lead time demand. As the retailers prefer to hold positive safety stock in most practical cases, it is assumed that $R \geq \mu + k\sigma$, where $k \geq 0$ denotes a preferred safety factor.

In this setting, the retailer is subject to procurement costs, inventory holding costs, order setup costs, penalty costs associated with shortages, and transportation costs. It is assumed that shortages are backordered. Particularly, let c denote the unit procurement cost, h denote the inventory holding cost per unit per unit time, K

denote the set-up cost per order, and p denote the penalty cost per unit shortage. It is well known that under a classical (Q, R) model, the retailer's expected purchase cost, inventory holding cost, order set-up cost, and penalty cost per unit time amount to $c\lambda$, $h\left(R-\mu+\frac{Q}{2}\right)$, $\frac{K\lambda}{Q}$, and $\frac{p\lambda n(R)}{Q}$, respectively, where $n(R)$ is the expected number of shortages backordered between two consecutive deliveries (Nahmias, 2009).

In cases when the retailer uses a LTL carrier, there is a unit transportation cost. Let t denote the per unit transportation cost under LTL transportation. Then, unit transportation cost can be included within the unit purchase cost and the retailer's expected cost per unit time under LTL transportation reads as

$$C^1(Q,R) = (c+t)\lambda + h\left(R-\mu+\frac{Q}{2}\right) + \frac{K\lambda}{Q} + \frac{p\lambda n(R)}{Q}. \tag{10}$$

On the other hand, if the retailer uses a TL carrier, they are charged based on the number of trucks used for inbound shipment. Particularly, let v denote the capacity of one truck and w the cost charged by a TL carrier for one truck. Then, if the retailer decides to use m trucks, they can ship at most mv units and will pay mw transportation cost at each shipment. Then, the retailer's expected cost per unit time under TL transportation reads as

$$C^2(Q,R,m) = c\lambda + h\left(R-\mu+\frac{Q}{2}\right) + \frac{(K+mw)\lambda}{Q} + \frac{p\lambda n(R)}{Q}. \tag{11}$$

As discussed previously, companies adjust their operations to curb carbon emissions. Particularly, similar to Li et al. (2008), Kim et al. (2009), Ramudhin et al. (2010), Wang et al. (2011), Chaabane et al. (2012), and Bouchery et al. (2012), it is considered that the retailer wishes to minimize not only costs but also carbon emissions. It is noted that emissions are generated due to procurement from the energy used in purchasing or processing a product or material handling required, inventory holding from the energy used for heating and refrigeration or warehousing activities, and order placement from the energy used in transportation or order initiation (see, e.g., Benjaafar et al., 2013, Chen et al., 2013). Following the same line with the literature, this research lets $\widehat{c}$, $\widehat{h}$, and $\widehat{K}$ denote the emissions generated from unit procurement, inventory holding per unit per unit time, and order set-up per order, respectively. Furthermore, it is assumed that additional carbon emissions are generated

due to backordered shortages. The carbon emissions generated from backorders can be due to the fact that the retailer ships the items to the backordered customers (see, e.g., Anderson et al., 2012) or the customer may need to return to the retailer's store to pickup their backordered item. Let $\widehat{p}$ denote the carbon emissions generated due to unit backorder. Similar to the cost components, one can observe that the retailer's expected carbon emissions per unit time from procurement, inventory holding, order set-up, and shortages amount to $\widehat{c}\lambda$, $\widehat{h}\left(R-\mu+\frac{Q}{2}\right)$, $\frac{\widehat{K}\lambda}{Q}$, and $\frac{\widehat{p}\lambda n(R)}{Q}$, respectively.

In cases where the retailer uses a LTL carrier, it is assumed that transportation emissions are proportional to the quantity shipped. Let $\widehat{t}$ denote the per unit transportation emission under LTL transportation. Then, similar to the cost function, unit transportation emissions can be included within the unit procurement emissions and the retailer's expected carbon emissions per unit time under LTL transportation reads as

$$E^1(Q,R) = (\widehat{c}+\widehat{t})\lambda + \widehat{h}\left(R-\mu+\frac{Q}{2}\right) + \frac{\widehat{K}\lambda}{Q} + \frac{\widehat{p}\lambda n(R)}{Q}. \tag{12}$$

On the other hand, if the retailer uses a TL carrier, they are responsible for the emissions generated from the trucks used. Recall that $\widehat{K}$ is used to denote the amount of emissions generated with each inventory replenishment. Particularly, Hua et al. (2011) attribute $\widehat{K}$ to the transportation emissions generated for shipping an order. The underlying assumption in their modeling approach is that a single truck has the sufficient capacity to deliver any order amount. Nevertheless, in practice, the retailer may have to use multiple trucks for shipping their order. Therefore, in what follows, TL transportation emissions are explicitly modeled. Ligterink et al. (2012) note that truck characteristics such as fuel type, engine type, build year, and vehicle mass influence emission generation of a truck. Particularly, sustainable supply chain and logistics studies that explicitly account for such truck characteristics mostly focus on vehicle routing models (see, e.g., Bektas and Laporte, 2011, Suzuki, 2011, Jabali et al., 2012, Erdogan and Miller-Hooks, 2012, and Demir et al., 2012). As noted by Ligterink et al. (2012), a truck's empty weight is effective in the amount of carbon emissions generated by that truck, thus, $\widehat{w}$ is defined as the carbon emissions generated by an empty truck, i.e., the truck's weight. Furthermore, depending on the aforementioned characteristics of the truck, each unit loaded into the truck will result in additional emission generation. Let $\widehat{e}$ denote the emissions generated per

unit loaded onto the truck. Then, the retailer's expected carbon emissions per unit time under TL transportation reads as

$$E^2(Q,R,m) = (\widehat{c}+\widehat{e})\lambda + \widehat{h}\left(R-\mu+\frac{Q}{2}\right) + \frac{(\widehat{K}+m\widehat{w})\lambda}{Q} + \frac{\widehat{p}\lambda n(R)}{Q}. \tag{13}$$

As noted previously, minimization of inventory related costs is not necessarily the only objective of a company (Bouchery et al., 2012) and an assumption that the company will only focus on emissions minimization is not realistic. Therefore, a bi-objective optimization model is presented in which the retailer aims to minimize both expected costs and expected carbon emissions per unit time.

The *sustainable* (Q,R) *model with less-than-truckload transportation* (S-(Q,R)-LTL) is stated as follows:

$$\begin{array}{lll} \text{S-(Q,R)-LTL}: & \min & C^1(Q,R) = (c+t)\lambda + h\left(R-\mu+\frac{Q}{2}\right) + \frac{K\lambda}{Q} + \frac{p\lambda n(R)}{Q} \\ & \min & E^1(Q,R) = (\widehat{c}+\widehat{t})\lambda + \widehat{h}\left(R-\mu+\frac{Q}{2}\right) + \frac{\widehat{K}\lambda}{Q} + \frac{\widehat{p}\lambda n(R)}{Q} \\ & \text{s.t.} & R \geq \mu + k\sigma \\ & & Q \geq 0, \end{array}$$

where the first constraint ensures that the safety stock is greater than or equal to the desired level and the second constraint is the non-negativity of the order quantity.

The *sustainable* (Q,R) *model with truckload transportation* (S-(Q,R)-TL) is stated as follows:

$$\begin{array}{lll} \text{S-(Q,R)-TL}: & \min & C^2(Q,R,m) = c\lambda + h\left(R-\mu+\frac{Q}{2}\right) + \frac{(K+mw)\lambda}{Q} + \frac{p\lambda n(R)}{Q} \\ & \min & E^2(Q,R,m) = (\widehat{c}+\widehat{e})\lambda + \widehat{h}\left(R-\mu+\frac{Q}{2}\right) + \frac{(\widehat{K}+m\widehat{w})\lambda}{Q} + \frac{\widehat{p}\lambda n(R)}{Q} \\ & \text{s.t.} & Q \leq mv \\ & & R \geq \mu + k\sigma \\ & & Q \geq 0 \\ & & m \in \{0,1,2,\ldots\}, \end{array}$$

where the first constraint ensures that the order quantity is less than or equal to the total transportation capacity, the second and the third constraints are defined as in the first and the second constraints of S-(Q,R)-LTL, respectively. The last constraint defines the number of trucks used for shipment to be integer.

Two common methods used for multi-objective optimization problems are Pareto front generation/approximation and reduction to a single objective formulation. A

Pareto front consists of non-dominated solutions, also known as Pareto efficient solutions, and it provides a set of alternative solutions to the decision maker. The decision maker then can select a solution from the Pareto front to adopt. On the other hand, reduction to a single objective formulation (for instance, by assigning weights to the objective functions or focusing on finding a solution to minimize the deviations from the optimum solutions of each individual objective function) results in a single solution and pre-models the decision maker's preferences. In solving S-(Q,R)-LTL and S-(Q,R)-TL, the focus is on generating a set of Pareto efficient solutions for each model. This enables the decision maker to compare different (Q, R) policies and adopt one regarding costs and carbon emissions.

The next section discusses the methods to approximate the Pareto fronts of S-(Q,R)-LTL and S-(Q,R)-TL. The notation used throughout the models and possible metrics for each notation are summarized in Appendix B.1. Additional notation will be defined as needed. The super-/sub-scripts 1 and 2 are associated with LTL and TL transportation respectively.

3.2. SOLUTION ANALYSIS

Weighted approaches are one of the most common methods used to solve multi-objective optimization problems by reducing the problems of interest to single objective models (Marler and Arora, 2004). Moreover, these approaches can also be used to approximate the Pareto front under certain convexity assumptions. In this section, a normalized weighted approach is first proposed for approximating the Pareto front of S-(Q,R)-LTL. Then, utilizing the analysis of S-(Q,R)-LTL, a method to approximate the Pareto front of S-(Q,R)-TL is proposed.

3.2.1. Pareto Front Approximation for S-(Q,R)-LTL. Let the Pareto front of S-(Q,R)-LTL be denoted by PF^1. To approximate PF^1, one should first focus on generating a set of Pareto efficient (Q, R) solutions. For normally distributed demand, PF^1 is convex as both of the objective functions are convex for $R \geq \mu$ (see, e.g., Brooks and Lu, 1969, Hariga, 2010) and the feasible region is convex (Ehrgott, 2005). In the case of convex Pareto fronts, weighted sum approaches can be used to generate the full Pareto front (see, e.g., Das and Dennis, 1997, Marler and Arora, 2010). Specifically, the normalized weighted approach is used to approximate PF^1. The normalized weighted approach for multi-objective optimization models associates

weights to the normalized objective functions[3]. The cost of the cost minimizing solution under LTL transportation and carbon emissions of the carbon emission minimizing solution under LTL transportation are used to normalize $C^1(Q,R)$ and $E^1(Q,R)$, respectively. Particularly, let (Q_1^C, R_1^C) and (Q_1^E, R_1^E) denote the cost minimizing and emission minimizing (Q,R) policies under LTL transportation. Then, for a given weight ω such that $\omega \in [0,1]$, the solution of the following optimization model will be in PF^1:

$$\begin{aligned} \text{S-(Q,R)-LTL}(\omega): \quad \min \quad & M^1(Q,R|\omega) = \omega \frac{C^1(Q,R)}{C^1(Q_1^C, R_1^C)} + (1-\omega)\frac{E^1(Q,R)}{E(Q_1^E, R_1^E)} \\ \text{s.t.} \quad & R \geq \mu + k\sigma \\ & Q \geq 0. \end{aligned}$$

Let (Q_1^ω, R_1^ω) be the solution of S-(Q,R)-LTL(ω). Given (Q_1^C, R_1^C) and (Q_1^E, R_1^E), one can derive that

$$M^1(Q,R|\omega) = \widetilde{c}(\omega)\lambda + \widetilde{h}(\omega)\left(R - \mu + \frac{Q}{2}\right) + \frac{\widetilde{K}(\omega)\lambda}{Q} + \frac{\widetilde{p}(\omega)\lambda n(R)}{Q}, \tag{14}$$

where $\widetilde{c}(\omega) = \omega(c+t)/C^1(Q_1^C, R_1^C) + (1-\omega)(\widehat{c}+\widehat{t})/E^1(Q_1^C, R_1^C)$, $\widetilde{h}(\omega) = \omega h/C^1(Q_1^C, R_1^C) + (1-\omega)\widehat{h}/E^1(Q_1^C, R_1^C)$, $\widetilde{K}(\omega) = \omega K/C^1(Q_1^C, R_1^C) + (1-\omega)\widehat{K}/E^1(Q_1^C, R_1^C)$, and $\widetilde{p}(\omega) = \omega p/C^1(Q_1^C, R_1^C) + (1-\omega)\widehat{p}/E^1(Q_1^C, R_1^C)$. Note that $M^1(Q,R|\omega)$ has a very similar functional form with Equations (10) and (12). An efficient method to heuristically find the minimizer of the expected cost per unit time of the classical (Q,R) model, i.e., $C^1(Q,R)$, is stated by Hadley and Whitin (1963). This method is used in solving S-(Q,R)-LTL(ω) as follows. Particularly, a minimizer is first found for $M^1(Q,R|\omega)$ using the method of Hadley and Whitin (1963). This method iteratively solves the following two equations, implied by the first order conditions, until a pre-determined

[3]The constrained approach, introduced by Lin (1976), is another method that can be used to approximate the Pareto front of multi-objective optimization problems. Particularly, this approach reformulates the multi-objective optimization problem to a single objective problem such that one of the objective functions is used as the single objective function and the other objective functions are included in the constraints with bounds on their values. Compared to the normalized weighted approach, this approach does not require convexity assumptions; however, solving the constrained subproblems can be challenging. In a set of preliminary numerical studies conducted, it was observed that the weighted approach is computationally more efficient for the problem of interest in this section.

precision is reached between two consecutive iterations:

$$Q = \sqrt{\frac{2\lambda(\widetilde{K}(\omega) + \widetilde{p}(\omega)n(R))}{\widetilde{h}(\omega)}} \tag{15}$$

$$1 - F(R) = \frac{Q\widetilde{h}(\omega)}{\widetilde{p}(\omega)\lambda}. \tag{16}$$

Then, if the resulting solution is feasible for S-(Q,R)-LTL(ω), it is accepted as the solution of S-(Q,R)-LTL(ω). On the other hand, the resulting solution can be infeasible if $R < \mu + k\sigma$ (note that Equation (15) implies $Q \geq 0$). In this case, set $R = \mu + k\sigma$ and solve for Q using Equation (15). Notice that given R, the Q minimizing $M^1(Q, R|\omega)$ will be given by Equation (15) due to the convexity of $M^1(Q, R|\omega)$ with respect to Q. This routine to solve S-(Q,R)-LTL(ω) is summarized as follows:

Routine 1: Solving S-(Q,R)-LTL(ω)

0: Let ω, $C^1(Q_1^C, R_1^C)$ and $E^1(Q_1^E, R_1^E)$ be given:

1: Determine (Q_1^ω, R_1^ω) using the iterative method of Hadley and Whitin (1963)

2: If $R_1^\omega < \mu + k\sigma$

3: Set $R_1^\omega = \mu + k\sigma$ and determine Q_1^ω using Equation (15)

4: Return (Q_1^ω, R_1^ω).

Routine 1 takes $C^1(Q_1^C, R_1^C)$ and $E^1(Q_1^E, R_1^E)$ values as input data. To determine (Q_1^C, R_1^C), Routine 1 is executed with $\omega = 1$, $C^1(Q_1^C, R_1^C) = 1$, and $E^1(Q_1^E, R_1^E) > 0$. The resulting (Q_1^ω, R_1^ω) is taken as (Q_1^C, R_1^C). Similarly, (Q_1^E, R_1^E) can be estimated by executing Routine 1 with $\omega = 0$, $E^1(Q_1^E, R_1^E) = 1$, and $C^1(Q_1^C, R_1^C) > 0$.

Notice that when $\omega = 1$, $(Q_1^\omega, R_1^\omega) = (Q_1^C, R_1^C)$ and when $\omega = 0$, $(Q_1^\omega, R_1^\omega) = (Q_1^E, R_1^E)$. This implies that both of the cost minimizing and carbon emission minimizing (Q, R) policies under LTL transportation are in PF^1. Through solving S-(Q,R)-LTL(ω) with different ω values, an approximation for the PF^1 can be achieved. The following procedure determines $\ell + 1$ number of solutions within PF^1 including (Q_1^C, R_1^C) and (Q_1^E, R_1^E).

Algorithm 1: Approximating PF^1

0: Given problem parameters and ℓ, set $PF^1 = \emptyset$:

1: Execute Routine 1 with $\omega = 1$, $C^1(Q_1^C, R_1^C) = 1$, and $E^1(Q_1^E, R_1^E) > 0$

2: Set $(Q_1^\omega, R_1^\omega) = (Q_1^C, R_1^C)$

3: Execute Routine 1 with $\omega = 0$, $E^1(Q_1^C, R_1^C) = 1$, and $C^1(Q_1^E, R_1^E) > 0$

4: Set $(Q_1^\omega, R_1^\omega) = (Q_1^E, R_1^E)$

5: For $j = 1 : \ell + 1$

6: Execute Routine 1 with $\omega = \frac{j-1}{\ell}$

7: Set $PF^1 := PF^1 \cup \{(Q_1^\omega, R_1^\omega)\}$

8: End

9: Return PF^1.

3.2.2. Pareto Front Approximation for S-(Q,R)-TL. Let the Pareto front of S-(Q,R)-TL be denoted by PF^2. To approximate PF^2, the focus is on generating a set of Pareto efficient (Q, R, m) solutions. To do so, the Pareto front of S-(Q,R)-TL is first analyzed given that $m = m^0$, that is, the number of trucks used for inbound transportation is fixed. Given $m = m^0$, S-(Q,R)-TL reduces to

$$\begin{aligned}
\text{S-(Q,R)-TL}(m^0): \quad & \min \quad C^2(Q, R|m^0) = c\lambda + h\left(R - \mu + \tfrac{Q}{2}\right) + \tfrac{(K+m^0 w)\lambda}{Q} + \tfrac{p\lambda n(R)}{Q} \\
& \min \quad E^2(Q, R|m^0) = (\widehat{c} + \widehat{e})\lambda + \widehat{h}\left(R - \mu + \tfrac{Q}{2}\right) + \tfrac{(\widehat{K}+m^0\widehat{w})\lambda}{Q} + \\
& \qquad \tfrac{\widehat{p}\lambda n(R)}{Q} \\
& \text{s.t.} \quad Q \leq m^0 v \\
& \qquad R \geq \mu + k\sigma \\
& \qquad Q \geq 0,
\end{aligned}$$

Let $PF^2(m^0)$ denote the Pareto front of S-(Q,R)-TL(m^0). It should be remarked that $PF^2 \subseteq \bigcup_{m=1}^{\infty} PF^2(m)$. Note that S-(Q,R)-TL($m^0$) is very similar to S-(Q,R)-LTL: the only difference is the additional upper bound constraint on the order quantity due to the fixed transportation capacity. Therefore, in approximating $PF^2(m^0)$, an approach similar to the one used to approximate PF^1 is used.

In particular, similar to PF^1, one can observe that $PF^2(m^0)$ is convex as both $C^2(Q,R|m^0)$ and $E^2(Q,R|m^0)$ are convex for $R \geq \mu$ and the feasible region is convex. Therefore, the normalized weighted approach can be used to approximate $PF^2(m^0)$. Now, let $(Q_2^C(m^0), R_2^C(m^0))$ and $(Q_2^E(m^0), R_2^E(m^0))$ be the cost and emission minimizing solutions of S-(Q,R)-TL(m^0). Then, for a given weight θ such that $\theta \in [0,1]$, the solution of the following optimization model will be in $PF^2(m^0)$:

$$\begin{aligned}
\text{S-(Q,R)-TL}(m^0,\theta): \quad \min \quad & M^2(Q,R|m^0,\theta) = \theta \frac{C^2(Q,R|m^0)}{C^2(Q_2^C(m^0), R_2^C(m^0), m^0)} + \\
& (1-\theta)\frac{E^2(Q,R|m^0)}{E^2(Q_2^E(m^0), R_2^E(m^0), m^0)} \\
\text{s.t.} \quad & Q \leq m^0 v \\
& R \geq \mu + k\sigma \\
& Q \geq 0.
\end{aligned}$$

Let $(Q_2^\theta(m^0), R_2^\theta(m^0))$ be the solution of S-(Q,R)-TL(m^0,θ). Notice that, given $(Q_2^C(m^0), R_2^C(m^0))$ and $(Q_2^E(m^0), R_2^E(m^0))$, $M^2(Q,R|m^0,\theta)$ has a very similar functional form with $M^1(Q,R|\omega)$. Therefore, in solving S-(Q,R)-TL(m^0,θ), the iterative method of Hadley and Whitin (1963) is applied to find the minimizer of $M^2(Q,R|m^0,\theta)$. Equivalent versions of Equations 15 and 16 for $M^2(Q,R|m^0,\theta)$ can be derived to be:

$$Q = \sqrt{\frac{2\lambda(\widetilde{K}(m^0,\theta) + \widetilde{p}(m^0,\theta)n(R))}{\widetilde{h}(m^0,\theta)}} \tag{17}$$

$$1 - F(R) = \frac{Q\widetilde{h}(m^0,\theta)}{\widetilde{p}(m^0,\theta)\lambda}, \tag{18}$$

where $\widetilde{h}(m^0,\theta) = h\theta/C^2(Q_2^C(m^0), R_2^C(m^0), m^0) + \widehat{h}(1-\theta)/E^2(Q_2^E(m^0), R_2^E(m^0), m^0)$, $\widetilde{K}(m^0,\theta) = (K + m^0 w)\theta/C^2(Q_2^C(m^0), R_2^C(m^0), m^0) + (\widehat{K} + m^0\widehat{w})(1-\theta)/E^2(Q_2^E(m^0), R_2^E(m^0), m^0)$, and $\widetilde{p}(\theta) = p\theta/C^2(Q_2^C(m^0), R_2^C(m^0), m^0) + \widehat{p}(1-\theta)/E^2(Q_2^E(m^0), R_2^E(m^0), m^0)$.

Then, if the resulting solution is feasible for S-(Q,R)-TL(m^0,θ), it is accepted as the solution of S-(Q,R)-TL(m^0,θ). On the other hand, the resulting solution can be infeasible in three cases: (i) the order quantity is greater than the truck capacity available, i.e., $Q > m^0 v$, (ii) the safety stock constraint is not satisfied, i.e., $R < \mu + k\sigma$, and (iii) both $Q > m^0 v$ and $R < \mu + k\sigma$. In cases (i) and (iii), the order quantity is set to be equal to the full capacity available, i.e., $Q = m^0 v$, and solve for R using

Equation (18). Note that given Q, the R minimizing $M^2(Q, R|m^0, \theta)$ will be given by Equation (18) due to convexity of $M^2(Q, R|m^0, \theta)$ with respect to R. Finally, if the updated R value through Equation (18) does not satisfy the safety stock constraint, let $R = \mu + k\sigma$. In case (ii), set $R = \mu + k\sigma$ and solve for Q using Equation (17) for $M^2(Q, R|m^0, \theta)$. Note that given R, the Q minimizing $M^2(Q, R|m^0, \theta)$ will be given by Equation (17) due to the convexity of $M^2(Q, R|m^0, \theta)$ with respect to Q. Finally, if the updated Q value through Equation (17) is over the truck capacity available, let $Q = m^0 v$. This routine to solve S-(Q,R)-TL(m^0, θ) is summarized as follows:

Routine 2: Solving S-(Q,R)-TL(m^0, θ)

Let θ, $C^2(Q_2^C(m^0), R_2^C(m^0), m^0)$, and $E^2(Q_2^E(m^0), R_2^E(m^0), m^0)$ be given:
Determine $(Q_2^\theta(m^0), R_2^\theta(m^0))$ using the iterative method of Hadley and Whitin (1963)
If $Q_2^\theta(m^0) > m^0 v$
    Set $Q_2^\theta(m^0) = m^0 v$ and determine $R_2^\theta(m^0)$ using Equation (18)
    If $R_2^\theta(m^0) < \mu + k\sigma$
        Set $R_2^\theta(m^0) = \mu + k\sigma$
If $R_2^\theta(m^0) < \mu + k\sigma$
    Set $R_2^\theta(m^0) = \mu + k\sigma$ and determine $Q_2^\theta(m^0)$ using Equation (17)
    If $Q_2^\theta(m^0) > m^0 v$
        Set $Q_2^\theta(m^0) = m^0 v$
Return $(Q_2^\theta(m^0), R_2^\theta(m^0))$.

Note that, similar to Routine 1, $C^2(Q_2^C(m^0), R_2^C(m^0), m^0)$ and $E^2(Q_2^E(m^0), R_2^E(m^0), m^0)$ are input for Routine 2. To determine $(Q_2^C(m^0), R_2^C(m^0))$, execute Routine 2 with $\theta = 1$, $C^2(Q_2^C(m^0), R_2^C(m^0), m^0) = 1$, and $E^2(Q_2^E(m^0), R_2^E(m^0), m^0) > 0$. The resulting $(Q_2^\theta(m^0), R_2^\theta(m^0))$ is taken as $(Q_2^C(m^0), R_2^C(m^0))$. Similarly, $(Q_2^E(m^0), R_2^E(m^0))$ can be estimated by executing Routine 2 with $\theta = 0$, $E^2(Q_2^E(m^0), R_2^E(m^0), m^0) = 1$, and $C^2(Q_2^C(m^0), R_2^C(m^0), m^0) > 0$.

$PF^2(m^0)$ can be approximated via executing Routine 2 with different weight values such that $\theta \in [0, 1]$. Specifically, to generate $\ell + 1$ solutions in $PF^2(m^0)$, similar to Algorithm 1, Routine 2 can be run with θ values increasing from 0 to 1 in increments of $1/\ell$. Note that both $(Q_2^C(m^0), R_2^C(m^0))$ and $(Q_2^E(m^0), R_2^E(m^0))$ will be in $PF^2(m^0)$.

Ideally, the purpose is to approximate the Pareto front of S-(Q,R)-TL, PF^2. Prior to analysis of PF^2, note that Routine 2 is a heuristic approach for solving S-(Q,R)-TL(m^0, θ). S-(Q,R)-TL(m^0, θ) is a nonlinear optimization problem with inequality constraints. Interior point methods are commonly used to solve nonlinear optimization problems (see, e.g., Forsgren et al., 2002). However, in a set of numerical studies conducted to analyze the efficiency of Routine 2 compared to the interior point method, it was observed that Routine 2 finds the same solutions or very close solutions (sometimes better) with the interior point method solutions in less computational time. Appendix B.2 gives the details of the numerical studies comparing Routine 2 to the interior point method. Therefore, Routine 2 is used in approximating PF^2.

In generating PF^2, $PF^2(m)$ is compared for different m values. First, the definition of the dominance between two $PF^2(m)$ sets is needed:

Definition 1. *$PF^2(m^a)$ dominates $PF^2(m^b)$ if any $(Q, R) \in PF^2(m^a)$ is Pareto efficient compared to every $(Q, R) \in PF^2(m^b)$.*

The dominance relation between two Pareto fronts is represented as $PF^2(m^a) \prec PF^2(m^b)$ as the minimization of both objective functions is considered. In the following algorithm, $PF^2(m)$ sets are generated until the next Pareto front, $PF^2(m + 1)$ is dominated by $PF^2(m)$. Then, the final set of Pareto efficient solutions is selected from the $\bigcup_{j=1}^{m} PF^2(j)$.

Algorithm 2, simply starting with one truck, generates a set of Pareto efficient solutions using Routine 2, then checks whether making one more truck available for inbound transportation can result in new Pareto efficient solutions. If not, adding one more truck is not considered and Algorithm 2 terminates. Step 13 of Algorithm 2 finally compares all of the Pareto efficient solutions with the given number of trucks to generate a set of Pareto efficient (Q, R, m) solutions.

Algorithm 2: Approximating PF^2

0: Given problem parameters and ℓ, set $PF^2(0) = \emptyset$, $\widehat{PF}^2 = \emptyset$, and $m = 1$:

1: Let $\widehat{PF}^2 := \widehat{PF}^2 \cup PF^2(m-1)$

2: Execute Routine 2 with $\theta = 1$, $C^2(Q_2^C(m), R_2^C(m), m) = 1$, and $E^2(Q_2^E(m), R_2^E(m), m) > 0$

3: Set $(Q_2^\theta(m), R_2^\theta(m)) = (Q_2^C(m), R_2^C(m))$

4: Execute Routine 2 with $\theta = 0$, $C^2(Q_2^C(m), R_2^C(m), m) > 0$, and $E^2(Q_2^E(m), R_2^E(m), m) = 1$

5: Set $(Q_2^\theta(m), R_2^\theta(m)) = (Q_2^E(m), R_2^E(m))$

6: For $j = 1 : \ell + 1$

7: Execute Routine 2 with $\theta = \frac{j-1}{\ell}$

8: Set $PF^2(m) := PF^2(m) \cup \{(Q_2^\theta(m), R_2^\theta(m))\}$

9: End

10: Return $PF^2(m)$

11: If $PF^2(m-1) \prec PF^2(m)$, go to step 13

12: Else, set $m = m + 1$, go to step 1

13: Set PF^2 as the Pareto efficient solutions in $\widehat{PF}^2$.

In comparing two sets of Pareto fronts with specific number of trucks in Step 11 of Algorithm 2, i.e., comparing $PF^2(m-1)$ to $PF^2(m)$, the Pareto efficient solutions in $PF^2(m-1) \cup PF^2(m)$ are found using Routine 3 detailed below. If the resulting set of Pareto efficient solutions is equal to $PF^2(m-1)$, this implies that $PF^2(m-1) \prec PF^2(m)$. Furthermore, Routine 3 is also used in Step 13 to find the Pareto efficient solutions in $\widehat{PF}^2$. The details of Routine 3 are as follows:

Routine 3: Finding the Pareto efficient solutions in a set S

```
Let C_i and E_i denote the cost and emissions of the i^th member of S:
For i = 1 : |S|
    For j = i + 1 : |S|
        If C_i < C_j and E_i < E_j
            Set S := S − {(C_j, E_j)}
        Else, if C_i > C_j and E_i > E_j
            Set S := S − {(C_i, E_i)}
    End
End
Return PF = S.
```

3.3. NUMERICAL STUDIES

In this section, a set of numerical studies is conducted to provide insights on the two models discussed. Particularly, the focus is on three sets of numerical analyses: (i) effects of demand variability and lead time duration, (ii) sustainability analysis of different policies, and (iii) comparison of different carriers.

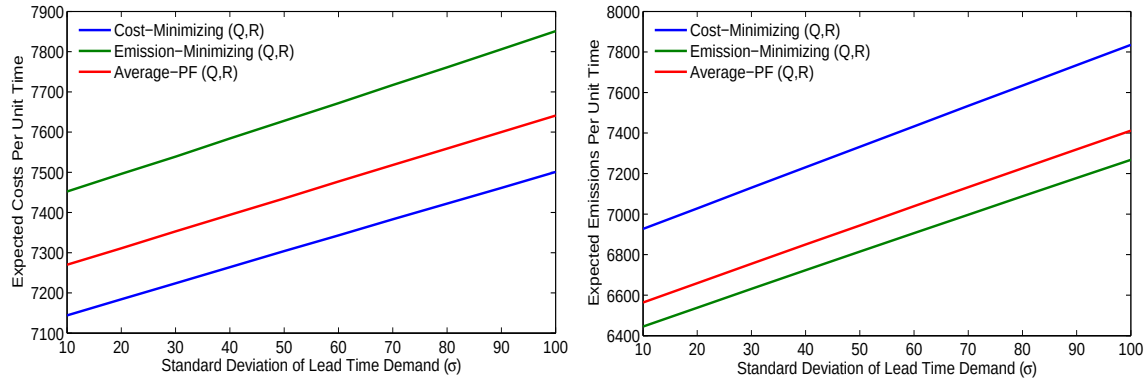
The routines and the algorithms discussed in Section 3.2 are coded in Matlab 2013 and all problem instances are solved using a personal computer with a 2.80 GHz processer and 10 GB RAM. Throughout the numerical analyses, it is assumed that the demand per unit time is normally distributed with mean λ and standard deviation ϑ. This then suggests that the lead time demand is also normally distributed with mean $\mu = \tau\lambda$ and standard deviation $\sigma = \sqrt{\tau}\vartheta$. For analyses (i)-(ii), the details of the design of the problem instance generation and definitions of parameter values are explained in Appendix B.3. The tables discussed in Sections 3.3.1 and 3.3.2 are described and presented in Appendix B.4.

3.3.1. Effects of Demand Variability and Lead Time. This set of numerical studies focuses on demonstrating the changes in the retailer's expected costs and carbon emissions per unit time with both LTL and TL transportation as lead time demand variability and lead time duration change. In particular, for

each transportation mode, the observed changes are compared in the cost minimizing (Q, R) policy, emission minimizing (Q, R) policy, and average of the (Q, R) policies in the Pareto front. The average of the (Q, R) policies in a Pareto front is calculated by taking the average of the expected costs and carbon emission per unit over all the Pareto-efficient (Q, R) policies that lay on the approximated Pareto front.

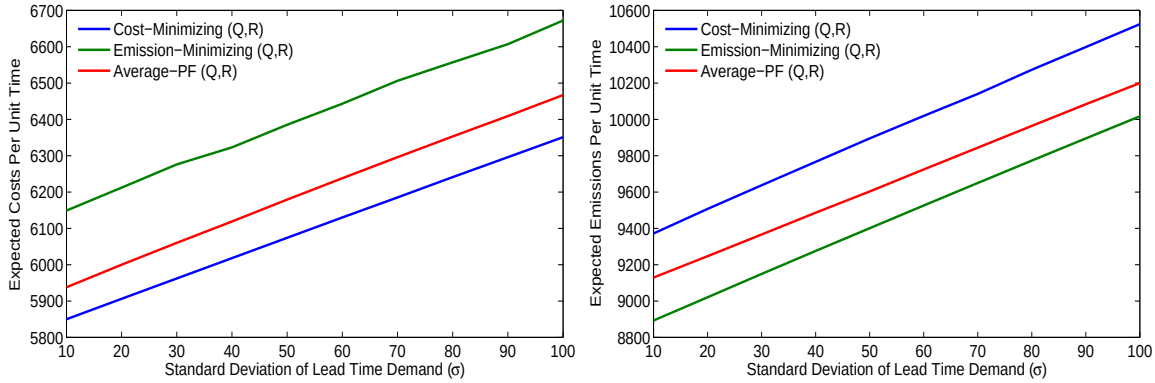
For investigating the effects of demand variability, 250 problem instances are randomly generated for each of the 10 different values of lead time demand standard deviation, σ, starting from 10 increasing up to 100 in increments of 10. Similarly, for investigating the effects of lead time duration, 250 problem instances are randomly generated for each of the 10 different values of lead time duration, τ, starting from 0.1 increasing up to 1 in increments of 0.1. Tables B.5 and B.6 summarize the average changes for each σ value in expected costs and emissions for the cost minimizing (Q, R) policy, the emission minimizing (Q, R) policy, and the average of the (Q, R) policies in the Pareto front, for both LTL and TL transportation, respectively. Tables B.7 and B.8 are constructed in the same manner, but showing the effects of each τ value.

Figures 3.1 and 3.2, constructed from Tables B.5 and B.6 respectively, illustrate the changes in expected costs and carbon emissions per unit time as σ increases under LTL and TL transportation, respectively. It can be observed from Figures 3.1 and 3.2 that, as the standard deviation of the lead time demand increases, expected costs and carbon emissions per unit time both increase. Similar results are observed through simulation of real life cases by Daccarett-Garcia (2009) and Arikan et al. (2013). Note that these results are not surprising as cost and emission functions have similar forms and it has been discussed in the literature that expected costs increase with increasing demand variability. Nevertheless, these observations have important implications about green technology investment. Particularly, in recent studies, inventory control and transportation models have been analyzed with green technology investment decisions (see, e.g., Bae et al., 2011, Swami and Shah, 2013, Toptal et al., 2014). The observations in Figures 3.1 and 3.2 suggest that an investment in demand variance reduction is actually a green investment. Furthermore, if demand variance can be reduced by investment such that the cost of investment is compensated by the decrease in expected costs, it is possible to reduce carbon emissions without additional costs.



(a) σ vs. expected costs (b) σ vs. expected emissions

Figure 3.1. σ vs. costs and emissions under different policies with LTL transportation

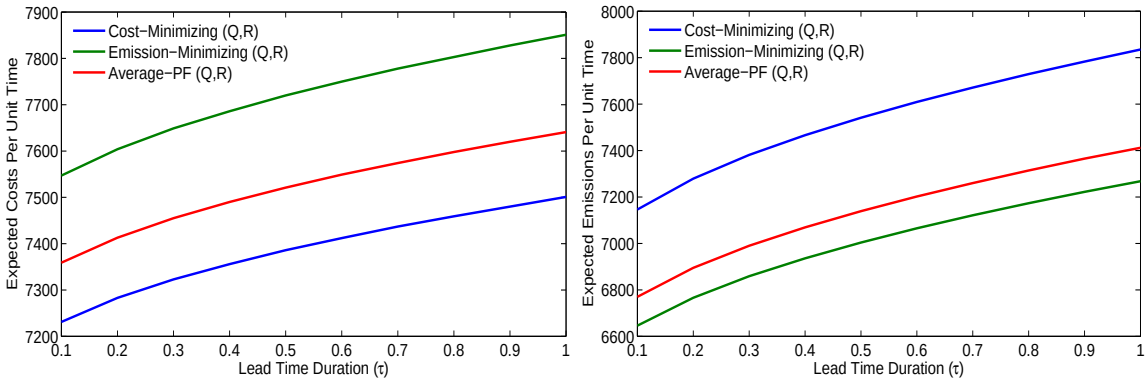


(a) σ vs. expected costs (b) σ vs. expected emissions

Figure 3.2. σ vs. costs and emissions under different policies with TL transportation

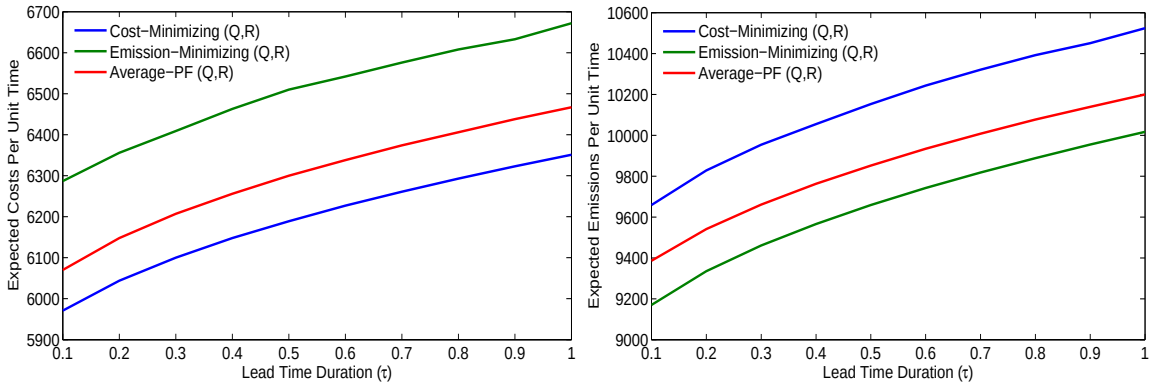
Figures 3.3 and 3.4, constructed from Tables B.7 and B.8 respectively, illustrate the changes in expected costs and carbon emissions per unit time as τ increases under LTL and TL transportation, respectively. It can be observed from Figures 3.3 and 3.4 that, as the lead time increases, expected costs and carbon emissions per unit time both increase under any policy. As expected, similar observations have been made with the increase of the standard deviation of the lead time demand; hence, the observations in Figures 3.3 and 3.4 suggest that if an investment for reducing lead time is compensated by the decrease in costs, it will provide additional benefits by reducing

expected carbon emissions. Note that, specially for TL transportation, the lead time can be changed by controlling the speed of trucks; however, the speed of the truck affects the emission generation rate ($\widehat{e}$ in this setting). The study by Jabali et al. (2012), for instance, analyzes a green vehicle routing problem, where truck speed is a decision variable and emission generation rate of a truck is a function of the speed (furthermore, this function is a convex function with decreasing and increasing sections). Depending on the range of the speed, an increase in speed (or decrease in lead time) can increase or decrease the emission generation rate. The authors believe that the current observations and the models presented in this section will be helpful in analyzing sustainable stochastic demand inventory systems with controllable lead time (where delivery speed is a decision variable). This problem is posed as a future research direction in Section 3.4.



(a) τ vs. expected costs (b) τ vs. expected emissions

Figure 3.3. τ vs. costs and emissions under different policies with LTL transportation



(a) τ vs. expected costs (b) τ vs. expected emissions

Figure 3.4. τ vs. costs and emissions under different policies with TL transportation

3.3.2. Sustainability Analysis. This set of numerical studies focuses on demonstrating the changes in the retailer's expected costs and carbon emissions when the retailer adopts a (Q, R) policy from the set of policies in the Pareto front. Recall that cost minimizing and emission minimizing policies are in the Pareto front both with LTL and TL transportation. Denote any (Q, R), (Q, R, m), and $(Q, R, \mathbf{x})$ policy in PF^1, PF^2, and PF^3 respectively, other than the cost minimizing policy, as sustainable (Q, R) and sustainable (Q, R, m) policies. In the following analysis, the focus is specifically on the percent changes in expected costs and carbon emissions due to preferring a sustainable policy instead of the cost minimizing policy.

For LTL transportation, the following two measures are defined:

$$\Delta C^1 = \frac{C^1(Q^S, R^S) - C^1(Q^C, R^C)}{C^1(Q^C, R^C)} \times 100\%, \tag{19}$$

$$\Delta E^1 = \frac{E^1(Q^S, R^S) - E^1(Q^C, R^C)}{E^1(Q^C, R^C)} \times 100\%, \tag{20}$$

where (Q^C, R^C) and (Q^S, R^S) denote the cost minimizing and a sustainable (Q, R) policy, respectively. That is, ΔC^1 and ΔE^1 define the percent changes in expected costs and expected emissions due to preferring a sustainable (Q, R) policy from PF^1 over the cost minimizing (Q, R) policy under LTL transportation. Similarly, for TL

transportation, the following two measures are defined:

$$\Delta C^2 = \frac{C^2(Q^S, R^S, m^S) - C^2(Q^C, R^C, m^C)}{C^2(Q^C, R^C, m^C)} \times 100\%, \tag{21}$$

$$\Delta E^2 = \frac{E^2(Q^S, R^S, m^S) - E^2(Q^C, R^C, m^C)}{E^2(Q^C, R^C, m^C)} \times 100\%, \tag{22}$$

where (Q^C, R^C, m^C) and (Q^S, R^S, m^S) denote the cost minimizing and a sustainable (Q, R, m) policy, respectively. That is, ΔC^2 and ΔE^2 define the percent changes in expected costs and expected emissions due to preferring a sustainable (Q, R, m) policy from PF^2 over the cost minimizing (Q, R, m) policy under TL transportation.

In Tables 3.1 and 3.2, for each problem instance solved, the percent changes due to switching from the cost minimizing policy to any sustainable policy in the Pareto front are calculated, i.e., ΔC^1, ΔE^1, ΔC^2, and ΔE^2 values for each policy in the Pareto front other than the cost minimizing policy, and then determine the average percent changes for that problem instance by taking the average of ΔC^1, ΔE^1, ΔC^2, ΔE^2, ΔC^3, and ΔE^3 values over the sustainable policies in the Pareto front. Then, the averages are calculated of the average percent changes over 250 problem instances solved for LTL and TL carriers for each σ value, respectively. Tables 3.3 and 3.4 are constructed in the same manner for the LTL and TL carriers, respectively, but for each τ value. Note that a positive (negative) value for percent changes in Tables 3.1–3.4 indicate an increase (decrease). It can be observed from Tables 3.1–3.4 that when the retailer prefers a (Q^S, R^S) policy over (Q^C, R^C) and a (Q^S, R^S, m^S) policy over (Q^C, R^C, m^C), the percent increase in expected costs is less than the percent decrease in expected carbon emissions. These observations suggest that the retailer can significantly reduce carbon emissions with relatively less increase in costs by adopting a sustainable policy over the cost minimizing policy. Note that similar results are given by Chen et al. (2013). Furthermore, one can observe that as the standard deviation of lead time demand and the lead time duration increase, the percent changes in both costs and emissions tend to increase; thus a conclusion cannot be made that as the standard deviation of lead time demand or the lead time duration increase, the retailer will observe less or more increase in costs per unit decrease in emissions.

Nevertheless, in addition to defining the percent changes in expected costs and

carbon emissions due to switching from the cost minimizing policy, the cost of emission reduction is defined similar to Chen et al. (2013) and Toptal et al. (2014) that as the increase in expected costs per unit decrease in expected carbon emissions, denoted by CoR^1 and CoR^2 for LTL and TL transportation, respectively, as follows:

$$CoR^1 = \frac{C^1(Q^S, R^S) - C^1(Q^C, R^C)}{E^1(Q^C, R^C) - E^1(Q^S, R^S)}, \tag{23}$$

$$CoR^2 = \frac{C^2(Q^S, R^S, m^S) - C^2(Q^C, R^C, m^C)}{E^2(Q^C, R^C, m^C) - E^2(Q^S, R^S, m^S)}, \tag{24}$$

Tables 3.1–3.4 document the averages of minimum, maximum, and average cost of reduction observed in the Pareto fronts under LTL and TL transportation. It can be observed that from Tables 3.1 and 3.3 that, under LTL transportation, minimum, maximum, and average cost of reduction is increasing with increasing demand variability and lead time duration. These suggest that, when demand variance is higher or the lead time is longer, it would cost more to reduce emissions for the retailer if they use LTL transportation for inbound shipment. On the other hand, for TL transportation, a strictly increasing or decreasing pattern is not observed for minimum, maximum, and average of the cost of reduction; as shown in Tables 3.2 and 3.4.

In Table 3.1, for a single problem instance solved, ΔC^1 and ΔE^1 are calculated for any sustainable policy in PF^1, for each σ value. The average values of ΔC^1 and ΔE^1 for the problem instance are denoted by $\overline{\Delta C}^1$ and $\overline{\Delta E}^1$, respectively. Similarly, CoR^1 for any sustainable policy in the Pareto front of the single problem instance is calculated. Then, the minimum, maximum, and the average of these values is determined, denoted by CoR_{min}, CoR_{max}, and CoR_{avg}, respectively. Table 3.2 is constructed similarly to Table 3.1 for a TL carrier. Tables 3.3 and 3.4 are constructed similarly to Tables 3.1 and 3.2, but for each τ value.

Table 3.1. Percent changes and cost of reduction with LTL transportation as σ changes

	Percent Changes		CoR^1		
σ	$\overline{\Delta C}^1$	$\overline{\Delta E}^1$	CoR^1_{min}	CoR^1_{avg}	CoR^1_{max}
10	1.89%	-5.40%	0.02223	0.30102	0.65106
20	1.90%	-5.41%	0.02224	0.30248	0.65473
30	1.91%	-5.42%	0.02225	0.30409	0.65904
40	1.92%	-5.44%	0.02227	0.30574	0.66356
50	1.94%	-5.46%	0.02229	0.30742	0.66825
60	1.95%	-5.48%	0.02232	0.30912	0.67298
70	1.97%	-5.50%	0.02235	0.31081	0.67773
80	1.99%	-5.52%	0.02238	0.31248	0.68247
90	2.01%	-5.54%	0.02242	0.31415	0.68720
100	2.03%	-5.57%	0.02245	0.31580	0.69193
avg.	1.95%	-5.47%	0.02232	0.30831	0.67090

Table 3.2. Percent changes and cost of reduction with TL transportation as σ changes

	Percent Changes		CoR^2		
σ	$\overline{\Delta C}^2$	$\overline{\Delta E}^2$	CoR^2_{min}	CoR^2_{avg}	CoR^2_{max}
10	1.59%	-2.65%	0.28478	1.01158	4.61865
20	1.67%	-2.78%	0.26732	0.86926	2.45863
30	1.73%	-2.85%	0.30418	0.81770	1.88143
40	1.76%	-2.90%	0.29625	0.76999	1.63953
50	1.81%	-3.00%	0.29815	0.75223	1.69695
60	1.84%	-3.00%	0.30667	0.75546	1.63063
70	1.88%	-2.99%	0.31121	0.75325	1.52936
80	1.89%	-3.09%	0.29755	0.73754	1.41684
90	1.89%	-3.12%	0.30640	0.75331	1.56077
100	1.92%	-3.17%	0.36255	0.81322	1.59399
avg.	1.80%	-2.95%	0.30351	0.80335	2.00268

Table 3.3. Percent changes and cost of reduction with LTL transportation as τ changes

	Percent Changes		CoR^1		
τ	$\overline{\Delta C}^1$	$\overline{\Delta E}^1$	CoR^1_{min}	CoR^1_{avg}	CoR^1_{max}
0.1	1.91%	-5.43%	0.02225	0.30435	0.65976
0.2	1.93%	-5.45%	0.02228	0.30653	0.66576
0.3	1.94%	-5.47%	0.02230	0.30823	0.67050
0.4	1.96%	-5.48%	0.02233	0.30967	0.67452
0.5	1.97%	-5.50%	0.02235	0.31093	0.67807
0.6	1.98%	-5.51%	0.02237	0.31206	0.68127
0.7	1.99%	-5.53%	0.02239	0.31310	0.68420
0.8	2.00%	-5.54%	0.02241	0.31406	0.68694
0.9	2.02%	-5.55%	0.02243	0.31495	0.68950
1.0	2.03%	-5.57%	0.02245	0.31580	0.69193
avg.	1.97%	-5.50%	0.02236	0.31097	0.67824

Table 3.4. Percent changes and cost of reduction with TL transportation as τ changes

	Percent Changes		CoR^2		
τ	$\overline{\Delta C}^2$	$\overline{\Delta E}^2$	CoR^2_{min}	CoR^2_{avg}	CoR^2_{max}
0.1	1.74%	-2.86%	0.30274	0.81512	1.83553
0.2	1.79%	-2.96%	0.29797	0.75004	1.48228
0.3	1.82%	-3.01%	0.30051	0.78220	2.85438
0.4	1.85%	-2.97%	0.32612	0.77277	1.56077
0.5	1.88%	-3.03%	0.30643	0.74799	1.53125
0.6	1.88%	-3.09%	0.30099	0.73926	1.45681
0.7	1.90%	-3.11%	0.30268	0.74558	1.46904
0.8	1.89%	-3.12%	0.30465	0.75027	1.53601
0.9	1.91%	-3.07%	0.39815	0.85775	1.59170
1.0	1.92%	-3.17%	0.36255	0.81322	1.59399
avg.	1.86%	-3.04%	0.32028	0.77742	1.69118

3.3.3. Transportation Mode Comparison. In this set of numerical studies, examples are presented on how the models and solution methods proposed in this paper can be used by a retailer for comparing two LTL carriers, a LTL carrier and a TL carrier, and two TL carriers. The comparison may be made not only in terms of costs but also from an environmental point of view. Prior to discussing examples, in the case of a single carrier option, the retailer can adopt a (Q, R) policy from the Pareto front depending on their sustainability and cost goals. Furthermore, in the case of two carriers of any type, i.e., LTL or TL, the retailer can approximate the Pareto front with each carrier and compare the Pareto fronts. If one of the Pareto fronts dominates the other Pareto front, the carrier with the dominating Pareto front would be preferred as it enables (Q, R) or (Q, R, m) policies with lower expected costs and as well as lower carbon emissions per unit time. In the case there is no dominance relation between the two Pareto fronts, as is the case in the following examples, the preference will depend on the retailer's sustainability and cost goals.

The settings of the following examples are given in Appendix B.5[4].

Example 5. *Consider that a retailer is planning to adopt a (Q, R) policy for a single product. Suppose that there are two LTL carriers available for the retailer's inbound shipment: LTL carrier A and LTL carrier B. LTL carriers have different per unit transportation costs and per unit emission generation rates (see Table B.10). Figure 3.5a shows the retailer's set of Pareto efficient (Q, R) policies when the retailer contracts with LTL carrier A and LTL carrier B for their inbound transportation. The intersection point of the two Pareto fronts is when expected costs amount to 20,374 and expected emissions amount to 123,174 per unit time.*

- If the retailer does not have environmental considerations (i.e., they only want to minimize expected costs per unit time), the retailer would prefer to contract with LTL carrier A as LTL carrier A would result in lower expected costs per unit time. That is, the cost minimizing policy with LTL carrier A has lower expected costs per unit time compared to the cost minimizing policy with LTL carrier B.

[4]For Examples 5-7, the point where two different Pareto fronts intersect (see, e.g., Figure 3.5) is estimated by assuming a straight line between the two points of each Pareto front, where these two points are the first points to dominate the points of the other Pareto front.

- If the retailer does not have cost considerations (i.e., they only want to minimize expected carbon emissions per unit time), the retailer would prefer to contract with LTL carrier B as LTL carrier B would result in lower expected carbon emissions per unit time. That is, the emission minimizing policy with LTL carrier B has lower expected carbon emissions per unit time compared to the emission minimizing policy with LTL carrier A.

- If the retailer does have both cost and environmental considerations, depending on the level of their sustainability or cost goals, the retailer can prefer LTL carrier A or LTL carrier B:

  - If the retailer targets their expected carbon emissions per unit time to be less than E such that $E < 123,174$, they would prefer LTL carrier B because LTL carrier B results in lower expected costs per unit time at any expected carbon emissions target E per unit time if $E < 123,174$. For expected carbon emissions target E per unit time such that $E > 123,174$, the retailer would prefer LTL carrier A because LTL carrier A results in lower expected costs per unit time at any expected carbon emissions target E per unit time if $E > 123,174$.
  - If the retailer, on the other hand, targets their expected costs to be less than C such that $C < 20,374$, they would prefer LTL carrier A because LTL carrier A results in lower expected emissions at any expected cost target C per unit time if $C < 20,374$. For expected costs target C per unit time such that $C > 20,374$, the retailer would prefer Carrier B because LTL carrier B results in lower expected carbon emissions per unit time at any expected cost target C per unit time if $C > 20,374$.

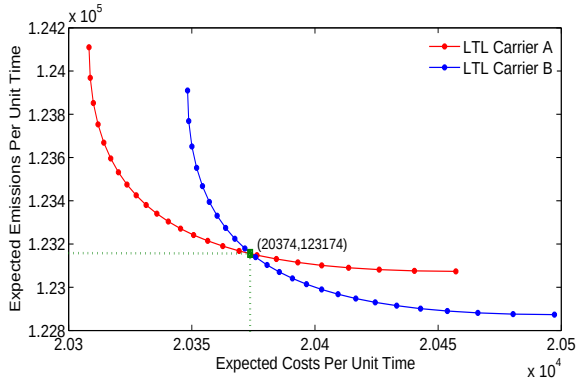
Example 6. *Consider the same retailer of Example 5 and suppose that there are two carriers available for the retailer's inbound shipment: a LTL carrier and a TL carrier. Figure 3.5b shows the retailer's set of Pareto efficient* (Q, R) *and* (Q, R, m) *policies when the retailer contracts with the LTL carrier and TL carrier, respectively, for their inbound transportation. The intersection point of the two Pareto fronts is when expected costs amount to 20,377 and expected emissions amount to 123,346 per unit time.*

Similar to Example 5, the retailer's preference of carrier will depend on their sustainability or cost considerations. It can be observed that when the retailer's objective is only to minimize expected costs (expected carbon emissions) per unit time, they would prefer to contract with the LTL carrier (TL carrier). If the retailer targets their expected carbon emissions at a level less than (greater than) 123,346 per unit time, they would prefer to contract with the TL carrier (LTL carrier). On the other hand, if the retailer targets their expected costs at a level less than (greater than) 20,377 per unit time, they would prefer to contract with the LTL carrier (TL carrier).

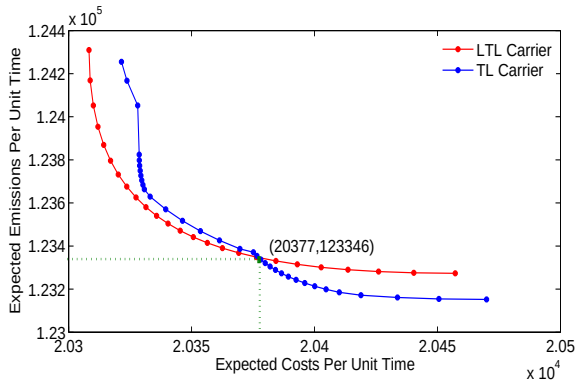
Example 7. *Consider the same retailer of Examples 5 and 6, and suppose that there are two TL carriers available for the retailer's inbound shipment: TL carrier A and TL carrier B. Figure 3.5c shows the retailer's set of Pareto efficient* (Q, R, m) *policies when the retailer contracts with the TL carrier A and TL carrier B for their inbound transportation. The intersection point of the two Pareto fronts is when expected costs amount to 20,328 and expected emissions amount to 123,805 per unit time.*

Similar to Examples 5 and 6, when the retailer's objective is only to minimize expected costs (expected carbon emissions) per unit time, they would prefer to contract with TL carrier A (TL carrier B). If the retailer targets their expected carbon emissions at a level less than (greater than) 123,805 per unit time, they would prefer to contract with TL carrier B (TL carrier A). On the other hand, if the retailer targets their expected costs at a level less than (greater than) 20,328 per unit time, they would prefer to contract with TL carrier A (TL carrier B).

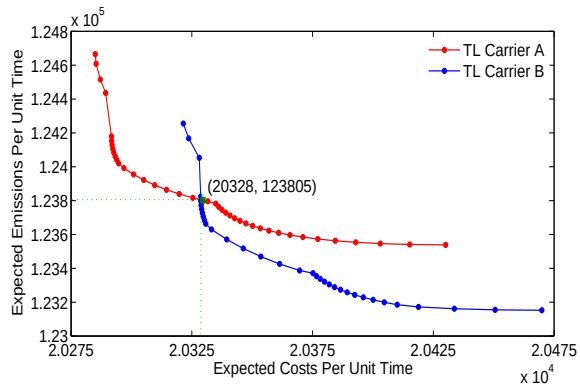
Note that while Examples 5–7 compare two carriers, similar analyses can be done when the retailer has more than two carrier options. Furthermore, note that similar analyses can be used when the retailer is subject to carbon cap constraints. The targeted carbon emissions level per unit time can be considered as the carbon cap regulated by governmental agencies. Nevertheless, as noted by Benjaafar et al. (2013) and Chen et al. (2013), companies not only have carbon caps because of governmental regulations but also because of the green goals they set as mentioned in Examples 5–7.



(a) LTL carrier vs. LTL carrier



(b) LTL carrier vs. TL carrier



(c) TL carrier vs. TL carrier

Figure 3.5. Comparison of different carriers

3.4. CONCLUSIONS AND FUTURE RESEARCH

In this section, sustainability was integrated into continuous review inventory control systems by formulating a bi-objective (Q, R) model with expected costs and expected carbon emissions minimization. This is the first study that introduces

sustainability in continuous review inventory systems. The sustainable (Q, R) model was analysed with two different transportation modes: LTL and TL transportation. For each case, a method was proposed to approximate the Pareto front by determining a set of Pareto efficient (Q, R) policies. Particularly, for the sustainable (Q, R) model with LTL transportation, a normalized weighted approach based method has been proposed for approximating the Pareto front. For the sustainable (Q, R) model with TL transportation, utilizing the method of LTL transportation, a method is proposed that compares Pareto fronts with given transportation capacities and then generates a set of Pareto efficient policies with different numbers of trucks used.

Defining a sustainable (Q, R) model also enabled analyzing the effects of demand variance and lead time duration on expected costs as well as expected carbon emissions. As expected, it is observed that both expected costs and carbon emissions increase as demand variance and lead time duration increase with both LTL and TL transportation. The managerial insight of these observations is that an investment opportunity to reduce demand variance or lead time duration can be a free or low cost green action if the investment spending is fully or partially compensated by the reduction in expected costs because expected carbon emissions will also be reduced. Through a set of numerical analyses, it is further shown that adopting a sustainable (Q, R) policy instead of a cost minimizing (Q, R) policy for a continuous review inventory control system with LTL or TL transportation can reduce carbon emissions without significant cost increases. These observations generalize the results of Chen et al. (2013) for deterministic inventory control to stochastic continuous review inventory control with both LTL and TL transportation. Finally, how the methods proposed in this section can be used by a retailer to select a carrier from a set of available carriers is discussed, considering not only the retailer's cost but also their environmental goals.

An immediate future research direction would be to analyze continuous review inventory control systems under carbon emission regulation policies. A (Q, R) model with carbon taxing, carbon trading, carbon cap, and carbon offset policies can be studied. The authors believe that the emissions function defined in this section will be utilized in such future research studies. Furthermore, analyses of integrated investment decisions on lead time and demand variance reduction and inventory control

decisions is a promising research area. Especially, as mentioned in Section 3.3, the sustainable (Q, R) model under TL transportation with lead time flexibility due to controllable truck speed is an important and practical future research direction. The sustainable (Q, R) model with TL transportation can also be studied by considering availability of different truck types for the inbound shipments. For instance, Section 5 considers heterogeneous freight trucks in a multi-item inventory control and transportation model under stochastic demand. Finally, the sustainable continuous review inventory control model can be extended to multi-item inventory systems, similar to the settings considered in Sections 4 and 5, and/or multi-echelon supply chains, similar to the settings analyzed by Saadany et al. (2011); Swami and Shah (2013); Zavanella et al. (2013); Wahab et al. (2011); Jaber et al. (2013).

4. COORDINATED MULTI-ITEM INVENTORY SYSTEMS: ECONOMICAL AND ENVIRONMENTAL IMPLICATIONS OF GROUPING STRATEGIES

In this section, a multi-item inventory control system is investigated with environmental considerations. In many practical scenarios, the companies need to jointly control inventories of different items to reduce costs by utilizing the shared resources better. A joint replenishment problem analyzes how to group replenishment of different items together to minimize total costs. Grouping items together enables efficient use of the transportation capacity; and, hence, can decrease transportation costs. Other inventory related costs such as inventory holding costs or items' individual order setup costs, on the other hand, may increase due to enforcing group replenishment. The main motivation of the joint replenishment problem is to find the best grouping policy to balance this trade-off in order to minimize costs. In this section, the joint replenishment problem is extended by considering not only an economical objective but also an environmental objective. Dekker et al. (2012) note that economical objectives are not the only objectives for companies any longer and it should be noted that economical as well as environmental objectives are simultaneously considered in supply chain design models (see, e.g., Li et al., 2008, Kim et al., 2009, Ramudhin et al., 2010, Wang et al., 2011, Chaabane et al., 2012) as well as in inventory control models (see, e.g., Bouchery et al., 2012, Chan et al., 2013). This is the first study to integrate environmental considerations in coordinated multi-item inventory systems and one of the limited studies that analyzes a multi-objective joint replenishment problem.

While the single objective JRPs, in both deterministic and stochastic demand settings, have been well studied in the literature (see, e.g. the review by Khouja and Goyal, 2008), there are limited studies on multi-objective JRPs. Particularly, Wee et al. (2009) model a fuzzy bi-objective joint replenishment problem for deteriorating items such that total average profit and the return on inventory investment are maximized. They consider the indirect grouping strategy in the two models presented, where the first model has fuzzy shortage costs and the second model has

fuzzy demand values. Fuzzy programming approaches are used to solve the resulting models. In another study, Yousefi et al. (2012) formulate a bi-objective joint replenishment problem problem where total costs (including inventory holding and order setup costs) as well as transportation costs are simultaneously minimized. They also consider the indirect grouping strategy and develop three different genetic algorithms to solve the resulting model. However, the model presented by Yousefi et al. (2012) is problematic as a retailer is unlikely to minimize different cost terms separately, i.e., formulating a multi-objective model where different cost terms are minimized is not realistic as a retailer is likely to minimize the total costs in practice. In a recent study, Wang et al. (2013) analyze a multi-objective stochastic joint replenishment problem with indirect grouping strategy. Particularly, due to difficulty of shortage cost estimations, they formulate the problem of interest as a multi-objective model where the costs (inventory holding plus setup costs) and total shortage quantity of the items are simultaneously minimized. A set of heuristic methods are proposed for the resulting bi-objective model.

Particularly, a sustainable joint replenishment problem is modeled as a bi-objective joint replenishment problem, where one of the objectives is cost minimization and the other is carbon emissions minimization. Two common grouping strategies in coordinated multi-item inventory systems are the indirect grouping strategy and the direct grouping strategy (Khouja and Goyal, 2008). In the indirect grouping strategy, a base replenishment cycle length is specified to place an order and it is determined how often each item is going to be included in an order. Therefore, the groups of items replenished in the same order vary over time. On the other hand, in the direct grouping strategy, the groups of items to be always replenished together are determined and for each such group, a replenishment cycle length is specified. The sustainable joint replenishment problem is formulated with both of these grouping strategies. Furthermore, using the properties of the costs and carbon emissions functions within the formulations of each grouping strategy, an evolutionary heuristic method is developed for the sustainable joint replenishment problem under each grouping strategy. For each sustainable joint replenishment problem, these heuristic methods give the decision maker a set of alternative solutions. A numerical study is conducted to demonstrate the convergence of these evolutionary heuristic methods.

Note that genetic algorithms are successfully used to solve single objective JRPs. In particular, Khouja et al. (2000) compare genetic algorithms to RAND (proposed by Kaspi and Rosenblatt, 1991), a commonly used heuristic method for solving deterministic JRPs with indirect grouping. Through an extensive numerical experiment, they note that genetic algorithms can find better solutions than RAND and did not find solutions with more than 1% increased cost. Furthermore, it is noted that genetic algorithms can easily be modified to account for different constraints and practical settings; thus, different variations of JRPs have been solved with genetic algorithms in the literature.

Chan et al. (2003), for instance, propose a genetic algorithm to solve a multi-buyer joint replenishment problem with indirect grouping and discuss its efficiency. Olsen (2005) develop a genetic algorithm for the joint replenishment problem with direct grouping and compare its solution to the joint replenishment problem with indirect grouping solved by RAND. In a similar study, Olsen (2008) use genetic algorithms to solve a joint replenishment problem with indirect grouping, where the minor setup costs depend on the items included within an order. A genetic algorithm for the joint replenishment problem with indirect grouping under a resource restriction is studied by Moon and Cha (2006). Hong and Kim (2009) construct a genetic algorithm for the joint replenishment problem with indirect grouping, where a base replenishment cycle assumption is relaxed. JRPs with other modifications are also solved with genetic algorithms (see, e.g., Yao, 2007, Wang et al., 2012b, Wang et al., 2012a) and genetic algorithms are used for bi-objective JRPs (Yousefi et al., 2012, Wang et al., 2013).

In this section, the two genetic algorithms developed have similarities in their chromosome representations and mutation operations; however, they have differences in the fitness evaluations. Specifically, for the indirect grouping strategy, the integer decision variables (the multiplier of an item determining the replenishment cycle length for the item) are used to represent the chromosomes and exact lower and heuristic upper bounds are developed on the genes of the chromosomes, i.e., the integer decision variables, for a Pareto efficient solution. These bounds are utilized within the mutation operations of the genetic algorithm proposed for the indirect grouping strategy. For the direct grouping strategy, the binary decision variables (defining

which item should be included in which group) are represented by an integer chromosome such that each gene defines the group an item belongs to. Since the minimum and maximum number of groups are defined, the genes of a chromosome have exact lower and upper bounds. Note that along with integer and binary decision variables, JRPs have continuous decision variables for each grouping strategy. In fitness evaluation of each genetic algorithm, a Pareto front of these continuous decision variables is therefore first generated for a given chromosome. While the joint replenishment problem with the indirect grouping strategy has a single continuous decision variable, the joint replenishment problem with the direct grouping strategy has multiple continuous decision variables. Taking this into account, different routines are discussed to approximate the Pareto front of the continuous decision variables for a given chromosome. However, a common dominance relation is used between two Pareto fronts for determining the parent chromosomes in each genetic algorithm.

Specifically, JRPs consider a major order setup cost with each replenishment and including an item's order within a replenishment has a minor setup cost. The major order setup cost generally accounts for the cost of the transportation capacity such as the freight truck used for inbound shipment. It is discussed in the joint replenishment problem literature that the ratio of the major setup cost to minor setup costs is an important factor for comparing indirect and direct grouping strategies in terms of cost performance (van Eijs et al., 1992). It is demonstrated that this ratio is also important for comparing the grouping strategies in terms of their environmental performance. Furthermore, the ratio of carbon emissions from each replenishment to the carbon emissions due to individual items' orders is crucial for the cost and environmental performances of grouping strategies.

In this section, the contributions are as follows. First, sustainability is integrated into a coordinated multi-item inventory control model by formulating a bi-objective joint replenishment problem, referred to as the sustainable joint replenishment problem, which is analyzed under two common practical grouping strategies. An efficient evolutionary heuristic method is developed for each grouping strategy. Then, it is demonstrated that the grouping strategy adopted is important for not only cost performance but also environmental performance. This suggests that, depending on a company's cost and environmental targets, a grouping strategy may be preferred over

the other. Furthermore, a set of sensitivity analyses is noted to illustrate in which scenarios a grouping strategy can be preferred over the other.

4.1. SUSTAINABLE JOINT REPLENISHMENT PROBLEM

Consider a retailer who needs to control the inventory of a set of n products. Let the products be indexed by i such that $i = \{1, 2, \ldots, n\}$. It is assumed that each product operates under the assumptions of the EOQ model. That is, any product i has a constant demand rate denoted by λ_i (units/unit time). Let p_i be the per unit purchase cost for product i. Inventory holding cost of h_i is charged for carrying one unit of product i in inventory per unit time. The retailer is subject to a major setup cost of A money units for placing an order. If product i is included within an order, a minor setup cost of a_i money units is charged additional to the major setup cost.

Under the current operations, a significant level of carbon emissions are generated. The carbon emissions are generated from inventory holding, warehousing, packaging, logistics, and transportation operations (Hua et al., 2011, Benjaafar et al., 2013). Particularly, let $\widehat{h}_i$ denote the carbon emissions generated due to keeping one unit of product i in inventory per unit time (this can be considered as the carbon emissions generated by electricity and warehousing activities, see, e.g., Chen et al., 2013) and let $\widehat{A}$ be the carbon emissions amount generated by placing an order (this can be considered as the carbon emissions generated by the weight of the empty truck, see, e.g., Hua et al., 2011). Furthermore, assume that replenishment of any product i generates a fixed amount of carbon emissions denoted by $\widehat{a}_i$ (this can be considered as the carbon emissions generated by packaging and warehouse activities required for product i, see, e.g., Ülkü, 2012, Toptal et al., 2014).

The main idea of the joint replenishment problem is to order products in groups to avoid paying high major setup costs. This can be done with two strategies (Khouja and Goyal, 2008): indirect grouping and direct grouping. The retailer's total costs and total emissions per unit time will depend on the grouping strategy adopted. Similar to Bouchery et al. (2012), this section assumes that the retailer not only minimizes costs but also emissions. This bi-objective optimization problem will be referred to as the *sustainable joint replenishment problem* (SJRP). In what follows, the retailer's sustainable joint replenishment problem is formulated under indirect and direct grouping strategies.

4.1.1. Indirect Grouping Strategy: Formulation and Preliminaries. When the indirect grouping strategy is adopted, the retailer determines a base replenishment cycle length t, and an integer number m_i for each product i such that product i's replenishment cycle length is $m_i t$ (i.e., product i is included in every m_i^{th} order). Under indirect grouping, the groups of products ordered vary depending on the order timing, that is, there is no fixed group of products that are always being ordered together. The retailer is subject to purchase, inventory holding, and order setup costs. In particular, under the indirect grouping strategy, the retailer's total costs per unit time read as

$$C^1(t, \mathbf{m}) = \sum_{i=1}^{n} p_i \lambda_i + \frac{1}{t}\left(A + \sum_{i=1}^{n} a_i/m_i\right) + \frac{t}{2}\sum_{i=1}^{n} h_i \lambda_i m_i, \tag{25}$$

where $\mathbf{m} = [m_1, m_2, \ldots, m_n]^t$, i.e., $\mathbf{m}$ is the n-vector of m_i values. The first term of Equation (25) is the total procurement cost, the second term determines the total order setup cost, and the last term is the total inventory holding cost per unit time.

One can similarly derive the carbon emissions function per unit time. Recall that the carbon emissions are generated from procurement, inventory holding, and order setups. Then the retailer's carbon emissions per unit time under indirect grouping strategy amount to

$$E^1(t, \mathbf{m}) = \sum_{i=1}^{n} \widehat{p}_i \lambda_i + \frac{1}{t}\left(\widehat{A} + \sum_{i=1}^{n} \widehat{a}_i/m_i\right) + \frac{t}{2}\sum_{i=1}^{n} \widehat{h}_i \lambda_i m_i, \tag{26}$$

where the first, second, and third terms of Equation (26) define the carbon emissions due to procurement, order setups, and inventory holding per unit time, respectively.

The sustainable joint replenishment problem with indirect group strategy (SJRP-IGS) then reads as

$$\begin{array}{lll} \text{(SJRP-IGS)}: & \min & C^1(t, \mathbf{m}) = \sum_{i=1}^{n} p_i \lambda_i + \frac{1}{t}\left(A + \sum_{i=1}^{n} a_i/m_i\right) + \frac{t}{2}\sum_{i=1}^{n} h_i \lambda_i m_i \\ & \min & E^1(t, \mathbf{m}) = \sum_{i=1}^{n} \widehat{p}_i \lambda_i + \frac{1}{t}\left(\widehat{A} + \sum_{i=1}^{n} \widehat{a}_i/m_i\right) + \frac{t}{2}\sum_{i=1}^{n} \widehat{h}_i \lambda_i m_i \\ & \text{s.t.} & t \geq 0 \\ & & m_i \in \{1, 2, \ldots\}. \end{array}$$

It should be noted that even the single objective joint replenishment problem with indirect grouping is a NP-hard problem (Arkin et al. (1989)); therefore, the focus is on developing a heuristic solution method for the bi-objective joint replenishment problem defined in SJRP-IGS. The following properties will be utilized in Section 4.2 for constructing the heuristic solution method.

Let $(t^C, \mathbf{m}^C)$ be the solution minimizing $C^1(t, \mathbf{m})$ and $(t^E, \mathbf{m}^E)$ be the solution minimizing $E^1(t, \mathbf{m})$. Note that, for a given $\mathbf{m}$, both $C^1(t, \mathbf{m}|\mathbf{m})$ and $E^1(t, \mathbf{m}|\mathbf{m})$ are strictly convex with respect to t; hence, for the given $\mathbf{m}$, the t value minimizing $C^1(t, \mathbf{m}|\mathbf{m})$, denoted by $t^C(\mathbf{m})$, and the t value minimizing $E^1(t, \mathbf{m}|\mathbf{m})$, denoted by $t^E(\mathbf{m})$, can be determined via the first order conditions. In particular, one can show that

$$t^C(\mathbf{m}) = \sqrt{\frac{2(A + \sum_{i=1}^n a_i/m_i)}{\sum_{i=1}^n h_i \lambda_i m_i}}, \tag{27}$$

$$t^E(\mathbf{m}) = \sqrt{\frac{2(\widehat{A} + \sum_{i=1}^n \widehat{a}_i/m_i)}{\sum_{i=1}^n \widehat{h}_i \lambda_i m_i}}. \tag{28}$$

It then follows that $t^C = t^C(\mathbf{m}^C)$ and $t^E = t^E(\mathbf{m}^E)$.

Similarly, it can be observed that for a given t, both $C^1(t, \mathbf{m}|t)$ and $E^1(t, \mathbf{m}|t)$ are the summations of n strictly convex functions of m_i values. Particularly, $C^1(t, \mathbf{m}|t) = \sum_{i=1}^n p_i\lambda_i + A/t + \sum_{i=1}^n C_i^1(m_i|t)$ where $C_i^1(m_i) = (1/t)\sum_{i=1}^n a_i/m_i + (t/2)\sum_{i=1}^n h_i\lambda_i m_i$. Thus, for the given t, the $\mathbf{m}$ value minimizing $C^1(t, \mathbf{m}|t)$ will be $\mathbf{m}^C(t) = [m_1^C(t), m_2^C(t), \ldots, m_n^C(t)]^t$ such that

$$m_i^C(t) = \arg\min\{C_i^1(\lfloor \widetilde{m}_i^C(t) \rfloor | t), C_i^1(\lceil \widetilde{m}_i^C(t) \rceil | t)\} \tag{29}$$

where

$$\widetilde{m}_i^C(t) = \frac{1}{t}\sqrt{\frac{2a_i}{h_i\lambda_i}}. \tag{30}$$

Moreover, $E^1(t, \mathbf{m}|t) = \sum_{i=1}^n \widehat{p}_i\lambda_i + \widehat{A}/t + \sum_{i=1}^n E_i^1(m_i|t)$ where $E_i^1(m_i) = 1/t\sum_{i=1}^n \widehat{a}_i/m_i + (t/2)\sum_{i=1}^n \widehat{h}_i\lambda_i m_i$. Therefore, for the given t, the $\mathbf{m}$ value minimizing $E^1(t, \mathbf{m}|t)$ will be $\mathbf{m}^E(t) = [m_1^E(t), m_2^E(t), \ldots, m_n^E(t)]^t$ such that

$$m_i^E(t) = \arg\min\{E_i^1(\lfloor \widetilde{m}_i^E(t) \rfloor | t), E_i^1(\lceil \widetilde{m}_i^E(t) \rceil | t)\}, \tag{31}$$

where

$$\widetilde{m}_i^E(t) = \frac{1}{t}\sqrt{\frac{2\widehat{a}_i}{\widehat{h}_i\lambda_i}}. \tag{32}$$

It then follows that $\mathbf{m}^C = \mathbf{m}^C(t^C)$ and $\mathbf{m}^E = \mathbf{m}^E(t^E)$.

4.1.2. Direct Grouping Strategy: Formulation and Preliminaries. When direct grouping strategy is adopted, the retailer determines the products that are grouped together and the replenishment cycle lengths for the individual groups of products. Under direct grouping, each product is included within a single group and the products in the same group are ordered simultaneously. Note that the retailer can have at most n of such groups. Therefore, let G_j for $j = 1, 2, \ldots, n$ define a possible group and let

$$x_{ij} = \begin{cases} 1 & \text{if product } i \text{ is in } G_j, \\ 0 & \text{otherwise.} \end{cases}$$

Once x_{ij} values are known, the groups will be defined. The retailer further needs to determine the replenishment cycle length for each group formed. Let t_j define the replenishment cycle length for the jth group. Similar to indirect grouping, the retailer is subject to purchase, inventory holding, and order setup costs. In particular, under the direct grouping strategy, the retailer's total costs per unit time read as

$$C^2(\mathbf{T}, \mathbf{X}) = \sum_{i=1}^{n} p_i\lambda_i + \sum_{j=1}^{n} \frac{[A + \sum_{i=1}^{n} a_i x_{ij}]}{t_j} + \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n} t_j h_i \lambda_i x_{ij}, \tag{33}$$

where $\mathbf{T} = [t_1, t_2, \ldots, t_n]^t$, i.e., $\mathbf{T}$ is the n-vector of t_j values and $\mathbf{X}$ is the $n \times n$ matrix of x_{ij} values. The first, second, and third terms of Equation (33) define the total procurement cost, total order setup cost, and total inventory cost per unit time, respectively.

Carbon emissions function per unit time has a similar form. Particularly, the retailer's carbon emissions per unit time under the direct grouping strategy amount to

$$E^2(\mathbf{T}, \mathbf{X}) = \sum_{i=1}^{n} \widehat{p}_i\lambda_i + \sum_{j=1}^{n} \frac{\left[\widehat{A} + \sum_{i=1}^{n} \widehat{a}_i x_{ij}\right]}{t_j} + \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n} t_j \widehat{h}_i \lambda_i x_{ij}, \tag{34}$$

where the first, second, and third terms of Equation (34) define the carbon emissions due to procurement, order setups, and inventory holding per unit time, respectively.

The sustainable joint replenishment problem with direct group strategy (SJRP-DGS) then reads as

$$
\begin{aligned}
(\text{SJRP-DGS}): \quad \min \quad & C^2(\mathbf{T},\mathbf{X}) = \textstyle\sum_{i=1}^n p_i\lambda_i + \sum_{j=1}^n \frac{\left[A+\sum_{i=1}^n a_i x_{ij}\right]}{t_j} + \\
& \tfrac{1}{2}\textstyle\sum_{j=1}^n \sum_{i=1}^n t_j h_i \lambda_i x_{ij} \\
\min \quad & E^2(\mathbf{T},\mathbf{X}) = \textstyle\sum_{i=1}^n \widehat{p}_i\lambda_i + \sum_{j=1}^n \frac{\left[\widehat{A}+\sum_{i=1}^n \widehat{a}_i x_{ij}\right]}{t_j} + \\
& \tfrac{1}{2}\textstyle\sum_{j=1}^n \sum_{i=1}^n t_j \widehat{h}_i \lambda_i x_{ij} \\
\text{s.t.} \quad & \textstyle\sum_{j=1}^n x_{ij} = 1 \ \forall i = 1,2,\ldots,n \\
& t_j \geq 0 \ \forall j = 1,2,\ldots,n \\
& x_{ij} \in \{0,1\} \ \forall i = 1,2,\ldots,n, \ \forall j = 1,2,\ldots,n.
\end{aligned}
$$

Note that the single objective joint replenishment problem with direct grouping is a set partitioning problem, which is a NP-hard problem (Garey and Johnson, 1979); hence, focus is needed on developing a heuristic solution method for the bi-objective joint replenishment problem defined in SJRP-DGS as well. The following properties will be utilized in Section 4.2 for constructing the heuristic solution method.

Let $(\mathbf{T}^C, \mathbf{X}^C)$ be the solution minimizing $C^2(\mathbf{T},\mathbf{X})$ and $(\mathbf{T}^E, \mathbf{X}^E)$ be the solution minimizing $E^2(\mathbf{T},\mathbf{X})$. It can be easily shown that, for a given feasible $\mathbf{X}$, both $C^2(\mathbf{T},\mathbf{X}|\mathbf{X})$ and $E^2(\mathbf{T},\mathbf{X}|\mathbf{X})$ are the summations of n independent convex functions of t_j for $j = 1,2,\ldots,n$. Therefore, the $\mathbf{T}$ minimizing $C^2(\mathbf{T},\mathbf{X}|\mathbf{X})$ for any given feasible $\mathbf{X}$, denoted by $\mathbf{T}^C(\mathbf{X})$, and the $\mathbf{T}$ minimizing $E^2(\mathbf{T},\mathbf{X}|\mathbf{X})$ for any given feasible $\mathbf{X}$, denoted by $\mathbf{T}^E(\mathbf{X})$, can be determined by finding $t_j^C(\mathbf{X})$ and $t_j^E(\mathbf{X})$ values for $j = 1,2,\ldots,n$ using the first order conditions. Particularly, one can derive that

$$t_j^C(\mathbf{X}) = \sqrt{\frac{2(A + \sum_{i=1}^n a_i x_{ij})}{\sum_{i=1}^n h_i \lambda_i x_{ij}}}, \tag{35}$$

$$t_j^E(\mathbf{X}) = \sqrt{\frac{2(\widehat{A} + \sum_{i=1}^n \widehat{a}_i x_{ij})}{\sum_{i=1}^n \widehat{h}_i \lambda_i x_{ij}}}. \tag{36}$$

Note that when $\sum_{i=1}^n x_{ij} = 0$ for some j, it means that the jth column of $\mathbf{X}$ consists of zeros, i.e., $G_j = \emptyset$. In this case, Equations (35) and (36) imply that $t_j^C(\mathbf{X}) = t_j^E(\mathbf{X}) \to \infty$, which makes the total costs and carbon emissions associated with G_j equal to zero. It then follows that $\mathbf{T}^C = [t_1^C(\mathbf{X}^C), t_2^C(\mathbf{X}^C), \ldots, t_n^C(\mathbf{X}^C)]^t$ and $\mathbf{T}^E = [t_1^E(\mathbf{X}^E), t_2^E(\mathbf{X}^E), \ldots, t_n^E(\mathbf{X}^E)]^t$.

Furthermore, given $\mathbf{T}$, one can show that the feasible $\mathbf{X}$ minimizing $C^2(\mathbf{T}, \mathbf{X}|\mathbf{T})$, denoted by $\mathbf{X}^C(\mathbf{T})$, is defined by $x_{ij}^C(\mathbf{T})$ values such that

$$x_{ij}^C(\mathbf{T}) = \begin{cases} 1 & \text{if } j = j^{C(i)}, \\ 0 & \text{otherwise,} \end{cases} \tag{37}$$

where

$$j^{C(i)} = \arg\min_j \{t_j h_i \lambda_i\}. \tag{38}$$

Similarly, it can be shown that, given $\mathbf{T}$, the feasible $\mathbf{X}$ minimizing $E^2(\mathbf{T}, \mathbf{X}|\mathbf{T})$, denoted by $\mathbf{X}^E(\mathbf{T})$, is defined by $x_{ij}^E(\mathbf{T})$ values such that

$$x_{ij}^E(\mathbf{T}) = \begin{cases} 1 & \text{if } j = j^{E(i)}, \\ 0 & \text{otherwise,} \end{cases} \tag{39}$$

where

$$j^{E(i)} = \arg\min_j \{t_j \widehat{h}_i \lambda_i\}. \tag{40}$$

It then follows that $\mathbf{X}^C$ and $\mathbf{X}^E$ are defined by $x_{ij}^C(\mathbf{T}^C)$ and $x_{ij}^E(\mathbf{T}^E)$ values.

4.2. SOLUTION ANALYSIS

Reducing the multi-objective model into a single objective model via weighted approaches or a min-max deviation and generating a set of Pareto efficient solutions are the two commonly used solution approaches for multi-objective optimization problems. Reduction to single objective, nevertheless, can be problematic as it pre-models the decision maker's preferences. Furthermore, it generates a single solution. On the other hand, when a set of Pareto efficient solutions are generated, the decision maker can then make a selection among the alternative solutions. In this section, considering that a retailer can have different environmental and economical targets, the focus is on approximating the Pareto front of the problems SJRP-IGS and SJRP-DGS. Approximating the Pareto front further enables a retailer to see how costly would it be to improve environmental performance at different operational levels.

As is discussed in the previous section, SJRP-IGS and SJRP-DGS are both complex problems even with the consideration of the single objective. Therefore, focus is needed on developing heuristic solution methods for these problems. Specifically,

a genetic algorithm (GA) is structured for each problem. As previously noted, genetic algorithms are successfully used for JRPs. In this section, the details of the genetic algorithms proposed are explained for SJRP-IGS and SJRP-DGS. Each of the genetic algorithms have the following main steps: (i) chromosome representation and initialization, (ii) fitness evaluation, (iii) genetic operations, and (iv) termination. The genetic algorithms proposed differ in steps (i) and (iii) due to the different formulations of the different grouping strategies but share a common dominance relation between two distinct sets of solutions in step (ii) and a common stopping criteria in step (iv). Prior to describing the details of each GA, the definitions and procedures used in both of the genetic algorithms are first discussed.

Let Φ denote a feasible solution to either SJRP-IGS or SJRP-DGS and $C(\Phi)$ and $E(\Phi)$ be the values of the first and second objective functions, respectively (note that when $\Phi = (t, \mathbf{m})$, $C(\Phi) = C^1(t, \mathbf{m})$ and $E(\Phi) = E^1(t, \mathbf{m})$; and, when $\Phi = (\mathbf{T}, \mathbf{X})$, $C(\Phi) = C^2(\mathbf{T}, \mathbf{X})$ and $E(\Phi) = E^2(\mathbf{T}, \mathbf{X})$).

Definition 2. *A solution $\Phi \in \mathcal{S}$ is Pareto efficient in $\mathcal{S}$ if and only if $\nexists \Phi' \in \mathcal{S}$ such that $C(\Phi') < C(\Phi)$ and $E(\Phi') < E(\Phi)$ (Berube et al., 2009).*

Note that by definition, $(t^C, \mathbf{m}^C)$ and $(t^E, \mathbf{m}^E)$ are Pareto efficient for SJRP-IGS and, $(\mathbf{T}^C, \mathbf{X}^C)$ and $(\mathbf{T}^E, \mathbf{X}^E)$ are Pareto efficient for SJRP-DGS. Ideally, the retailer will want to generate the set of all Pareto efficient solutions in $\mathcal{F}^{IGS}$ and $\mathcal{F}^{DGS}$, where $\mathcal{F}^{IGS}$ and $\mathcal{F}^{DGS}$ denote the set of all feasible solutions for SJRP-IGS and SJRP-DGS, respectively. However, due to the complexity of the problems, the focus was placed on approximating the Pareto fronts, i.e., the set of Pareto efficient solutions, for SJRP-IGS and SJRP-DGS. The following procedure determines the set of Pareto efficient solutions within any given set of solutions $\mathcal{S}$, denoted by $PE(\mathcal{S})$.

Routine 4: Determining $PE(\mathcal{S})$ for set $\mathcal{S}$

```
Let Φ^ℓ denote the ℓ^th solution of S. Set PE(S) = S.
For ℓ = 1 : |S|
    For r = ℓ + 1 : |S|
        If C(Φ^ℓ) < C(Φ^r) and E(Φ^ℓ) < E(Φ^r)
            Set PE(S) := PE(S)\{Φ^r}
        Else, if C(Φ^ℓ) > C(Φ^r) and E(Φ^ℓ) > E(Φ^r)
            Set PE(S) := PE(S)\{Φ^ℓ}
    End
End
Return PE(S).
```

Next, the Pareto dominance relation is defined between two sets of solutions, namely $\mathcal{S}^1$ and $\mathcal{S}^2$.

Definition 3. *$\mathcal{S}^1$ Pareto dominates $\mathcal{S}^2$ if and only if any solution $\Phi^1 \in \mathcal{S}^1$ is Pareto superior compared to any solution $\Phi^2 \in \mathcal{S}^1$, i.e., $C(\Phi^1) < C(\Phi^2)$ and $E(\Phi^1) < E(\Phi^2)$.*

Pareto dominance between two sets of solutions will be used in the fitness evaluation step of the genetic algorithms developed. The notation used is $\mathcal{S}^1 \prec \mathcal{S}^2$ when $\mathcal{S}^1$ Pareto dominates $\mathcal{S}^2$. Particularly, one can determine Pareto dominance between $\mathcal{S}^1$ and $\mathcal{S}^2$ as follows. Let $\mathcal{S} = \mathcal{S}^1 \bigcup \mathcal{S}^2$. If $PE(S) \bigcap \mathcal{S}^2 = \emptyset$, $\mathcal{S}^1 \prec \mathcal{S}^2$; and if $PE(S) \bigcap \mathcal{S}^1 = \emptyset$, $\mathcal{S}^2 \prec \mathcal{S}^1$.

4.2.1. Pareto Front Approximation for SJRP-IGS. Here, the details of the genetic algorithm that approximates the Pareto front for the sustainable joint replenishment problem under indirect grouping strategy (GA-I) are explained step by step. Prior to describing the details of each step of the GA-I, some properties of the SJRP-IGS need to be discussed that are utilized in GA-I.

4.2.1.1. Properties of SJRP-IGS. Let the Pareto front of SJRP-IGS be denoted by PF^I, that is, PF^I consists of Pareto efficient $(t, \mathbf{m})$ pairs. Furthermore, let $PF^I(\mathbf{m})$ denote the Pareto front of SJRP-IGS for a given $\mathbf{m}$, that is, $PF^I(\mathbf{m})$ consists of Pareto efficient $(t(\mathbf{m}), \mathbf{m})$ solutions, where $t(\mathbf{m})$ denotes a t value for any given $\mathbf{m}$.

Similarly, let $PF^I(t)$ denote the Pareto front of SJRP-IGS for a given t, that is, $PF^I(t)$ consists of Pareto efficient $(t, \mathbf{m}(t))$ solutions, where $\mathbf{m}(t) = [m_1(t), m_2(t), \ldots, m_n(t)]^t$ denotes an $\mathbf{m}$ vector for any given t. Note that $PF^I \subseteq \bigcup_{\mathbf{m} \in \mathbb{Z}_+^n} PF^I(\mathbf{m})$ and $PF^I \subseteq \bigcup_{t:t>0} PF^I(t)$. In the following property, the range of Pareto efficient t values is defined for a given $\mathbf{m}$ and the range for $m_i(t)$ values in Pareto efficient $\mathbf{m}$ vectors for a given t.

Property 12. *Given* $\boldsymbol{m}$, $(t(\boldsymbol{m}), \boldsymbol{m}) \in PF^I(\boldsymbol{m})$ *if* $t(\boldsymbol{m}) \in [\min\{t^C(\boldsymbol{m}), t^E(\boldsymbol{m})\}, \max\{t^C(\boldsymbol{m}), t^E(\boldsymbol{m})\}]$. *Given* t, *if* $(t, \boldsymbol{m}(t)) \in PF^I(t)$ *then* $m_i(t) \in [\min\{m_i^C(t), m_i^E(t)\}, \max\{m_i^C(t), m_i^E(t)\}]$ $\forall i = 1, 2, \ldots, n$.

An approximation of $PF^I(\mathbf{m})$ can be generated by several approaches such as the normalized weighted sum method and the constrained method (see, e.g., Marler and Arora, 2010 and Lin, 1976). In the normalized weighted sum method, weights of ω and $(1 - \omega)$ are assigned to the normalized objective functions and a single objective optimization problem is solved for different ω values such that $\omega \in [0, 1]$. The t value minimizing $f(t|\omega) = \omega C^1(t, \mathbf{m}|\mathbf{m})/C^1(t^C(\mathbf{m}), \mathbf{m}|\mathbf{m}) + (1 - \omega)E^1(t, \mathbf{m}|\mathbf{m})/E^1(t^E(\mathbf{m}), \mathbf{m}|\mathbf{m})$ yields a solution, i.e., $(t, \mathbf{m})$ such that $(t, \mathbf{m}) \in PF^I(\mathbf{m})$ (Marler and Arora, 2010). It can be easily shown that $f(t|\omega)$ is strictly convex in t and the minimizer of $f(t|\omega)$ can be explicitly determined by the first order conditions (similar to Equations (27) and (28)). In the constrained method, a single objective optimization problem is formulated by including an upper bound constraint on one of the objective functions (Lin, 1976). Suppose that $E^1(t, \mathbf{m}|\mathbf{m})$ is taken as the constraint. Then, the t value solving $\min_t\{C^1(t, \mathbf{m}|\mathbf{m}) : E^1(t, \mathbf{m}|\mathbf{m}) \leq U, t > 0\}$ for any upper bound value U such that $U \geq E^1(t^E(\mathbf{m}), \mathbf{m}|\mathbf{m})$ yields a solution $(t, \mathbf{m})$ such that $(t, \mathbf{m}) \in PF^I(\mathbf{m})$. Further note that an explicit expression for the solution of $\min_t\{C^1(t, \mathbf{m}|\mathbf{m}) : E^1(t, \mathbf{m}|\mathbf{m}) \leq U\}$ can be derived as $C^1(t, \mathbf{m}|\mathbf{m})$ is a convex function and $E^1(t, \mathbf{m}|\mathbf{m})$ is a quadratic convex function of t.

Property 12 notes that any $(t(\mathbf{m}), \mathbf{m})$ such that $t(\mathbf{m}) \in [\min\{t^C(\mathbf{m}), t^E(\mathbf{m})\}, \max\{t^C(\mathbf{m}), t^E(\mathbf{m})\}]$ is in $PF^I(\mathbf{m})$. On the other hand, while Property 12 implies that an $\mathbf{m}(t)$ such that $(t, \mathbf{m}(t)) \in PF^I(t)$ will be a combination of $m_i(t)$ values such that $m_i(t) \in \left[\min\{m_i^C(t), m_i^E(t)\}, \max\{m_i^C(t), m_i^E(t)\}\right]$ $\forall i = 1, 2, \ldots, n$, any $(t, \mathbf{m}(t))$ where $\mathbf{m}(t)$ is such a combination, is not necessarily in $PF^I(t)$. This suggests that

one needs to generate all $\mathbf{m}(t)$ vectors corresponding to combinations of $m_i(t)$ values and evaluate the resulting set using Routine 4 to generate $PF^I(t)$. Since the number of such combinations would be exponential, it is more tractable to evolve $\mathbf{m}$ vectors and generate $PF^I(\mathbf{m})$ for given $\mathbf{m}$ vectors in the fitness evaluation of GA-I.

As mentioned above, both the normalized weighted approach and the constrained approach can be used to approximate $PF^I(\mathbf{m})$. Instead of these methods, $PF^I(\mathbf{m})$ is approximated for a given $\mathbf{m}$ using Routine 5, as described below.

Routine 5: Approximating $PF^I(\mathbf{m})$ for a given $\mathbf{m}$

0: Let $\mathbf{m}$ and ℓ be given. Set $\widehat{t} = (\max\{t^C(\mathbf{m}), t^E(\mathbf{m})\} - \min\{t^C(\mathbf{m}), t^E(\mathbf{m})\})/\ell$ and $PF^I(\mathbf{m}) = \emptyset$.

1: For $y = 1 : \ell + 1$

2: $\quad t = \min\{t^C(\mathbf{m}), t^E(\mathbf{m})\} + (y-1)\widehat{t}$ and set $PF^I(\mathbf{m}) := PF^I(\mathbf{m}) \bigcup \{t, \mathbf{m}\}$

3: End

4: Return $PF^I(\mathbf{m})$.

Routine 5 generates $\ell + 1$ $(t(\mathbf{m}), \mathbf{m})$ solutions in $PF^I(\mathbf{m})$ by starting with $t(\mathbf{m}) = \min\{t^C(\mathbf{m}), t^E(\mathbf{m})\}$ and increasing for ℓ equal increments up to $t(\mathbf{m}) = \max\{t^C(\mathbf{m}), t^E(\mathbf{m})\}$. Note that, considering that t can take discrete values in practice (such as days or weeks), the approach this section adopts can be used to generate all discrete Pareto efficient $(t(\mathbf{m}), \mathbf{m})$ solutions such that $t(\mathbf{m})$ is between $\min\{t^C(\mathbf{m}), t^E(\mathbf{m})\}$ and $\max\{t^C(\mathbf{m}), t^E(\mathbf{m})\}$, while this would not be guaranteed by the normalized weighted approach or the constrained approach.

Property 12 provides the range of t values in $PF^I(\mathbf{m})$ for a given $\mathbf{m}$. Nevertheless, $(t, \mathbf{m})$ does not necessarily belong to PF^I given that $t \in [\min\{t^C(\mathbf{m}), t^E(\mathbf{m})\}, \max\{t^C(\mathbf{m}), t^E(\mathbf{m})\}]$. In the following property, the focus is on providing bounds on t and $\mathbf{m}$ for any $(t, \mathbf{m}) \in PF^I$. To do so, the following equations are first defined:

$$t^{UB} = \max\left\{\sqrt{\frac{2\left(A + \sum_{i=1}^{n} a_i\right)}{\sum_{i=1}^{n} \lambda_i h_i}}, \sqrt{\frac{2\left(\widehat{A} + \sum_{i=1}^{n} \widehat{a}_i\right)}{\sum_{i=1}^{n} \lambda_i \widehat{h}_i}}\right\}, \tag{41}$$

$$\widetilde{m}_i^{LB} = \min\left\{\frac{1}{t^{UB}}\sqrt{\frac{2a_i}{h_i\lambda_i}}, \frac{1}{t^{UB}}\sqrt{\frac{2\widehat{a}_i}{\widehat{h}_i\lambda_i}}\right\}. \tag{42}$$

Property 13. *If* $(t, \boldsymbol{m}) \in PF^I$*, then* $t \leq t^{UB}$ *and* $m_i \geq \lfloor \widetilde{m}_i^{LB} \rfloor$ $\forall i, i = 1, 2, \ldots, n$.

Property 13 provides an upper bound for t and a lower bound for m_i $\forall i, i = 1, 2, \ldots, n$. Specifically, an upper bound for an m_i value cannot be determined because the lower bound for t is 0. On the other hand, a commonly used lower bound value for the t value in minimization of $C^1(t, \mathbf{m})$ is $\min_i \left\{\sqrt{2a_i/\lambda_i h_i}\right\}$ independent of $\mathbf{m}$ (see, e.g., Goyal, 1974, Moon and Cha, 2006, Khouja and Goyal, 2008). Similarly, one can consider $t \geq \min_i \left\{\sqrt{2\widehat{a}_i/\lambda_i \widehat{h}_i}\right\}$ for minimizing $E^1(t, \mathbf{m})$. Therefore, for SJRP-IGS, it is assumed that $t \geq t^{LB}$, where

$$t^{LB} = \min\left\{\min_i \left\{\sqrt{2a_i/\lambda_i h_i}\right\}, \min_i \left\{\sqrt{2\widehat{a}_i/\lambda_i \widehat{h}_i}\right\}\right\}. \tag{43}$$

Then, following the argument in the proof of Property 13, one can show that $m_i \leq \lceil \widetilde{m}_i^{UB} \rceil$ $\forall i, i = 1, 2, \ldots, n$, where

$$\widetilde{m}_i^{UB} = \min\left\{\frac{1}{t^{LB}}\sqrt{\frac{2a_i}{h_i\lambda_i}}, \frac{1}{t^{LB}}\sqrt{\frac{2\widehat{a}_i}{\widehat{h}_i\lambda_i}}\right\}. \tag{44}$$

Therefore, in GA-I, for which the details are explained next, assume that $t^{LB} \leq t \leq t^{UB}$ and $\lfloor \widetilde{m}_i^{LB} \rfloor \leq m_i \leq \lceil \widetilde{m}_i^{UB} \rceil$ $\forall i, i = 1, 2, \ldots, n$ for any $(t, \mathbf{m}) \in PF^I$.

4.2.1.2. Genetic algorithm for SJRP-IGS. GA-I consists of the four aforementioned steps. The details for each step are as follows.

(i) Chromosome Representation and Initialization: As noted above, approximating $PF^I(\mathbf{m})$ for a given $\mathbf{m}$ is relatively easier than approximating $PF^I(t)$ for a given t. Therefore, in GA-I, each chromosome is defined as an n-vector of integer m_i values. To initiate the GA-I, a set of $2n$ chromosomes are randomly generated as follows. For each chromosome, m_i $\forall i, i = 1, 2, \ldots, n$ is randomly selected such that $m_i \in [\lfloor \widetilde{m}_i^{LB} \rfloor, \lceil \widetilde{m}_i^{UB} \rceil]$. Let S^r be the set of chromosomes in the rth population and let $\mathbf{m}^{rk}$ define the kth chromosome in the rth population such that $k \in \{1, 2, \ldots, |S^r|\}$.

(ii) Fitness Evaluation: Given the rth population of chromosomes, i.e., S^r, first generate $PF^I(\mathbf{m}^{rk})$ $\forall k \in \{1, 2, \ldots, |S^r|\}$ using Routine 5. If $PF^I(\mathbf{m}^{rk_1}) \prec$

$PF^I(\mathbf{m}^{rk_2})$ for any k_1 and k_2 such that $k_1, k_2 \in \{1, 2, \ldots, |S^r|\}$, $\mathbf{m}^{rk_2}$ is not considered in generating the next population. To find the dominance relations between any pair of chromosomes within the current population, instead of making pairwise comparisons, the following approach is adopted. First, the set $PF_r^I = PF^I(\mathbf{m}^{r1}) \bigcup PF^I(\mathbf{m}^{r2}) \bigcup \cdots \bigcup PF^I(\mathbf{m}^{r|S^r|})$ is defined. Then, $PE(PF_r^I)$ is generated using Routine 4. Note that $PE(PF_r^I)$ will consist of a set of $(t, \mathbf{m})$ pairs. The distinct $\mathbf{m}$ vectors in $PE(PF_r^I)$ are taken as the set of parent chromosomes for the next generation. Figure 4.1 illustrates this process for a given population r with 8 chromosomes in it such that each $PF^I(\mathbf{m}^{rk})$ (the blue points) for $1 \leq k \leq 8$ has 10 $(t, \mathbf{m}^{rk})$ solutions. $PE(PF_r^I)$ consists of the blue points with red circles around them. The parent set of chromosomes for population $r+1$ will be $\{\mathbf{m}^{r2}, \mathbf{m}^{r3}, \mathbf{m}^{r4}, \mathbf{m}^{r7}\}$.

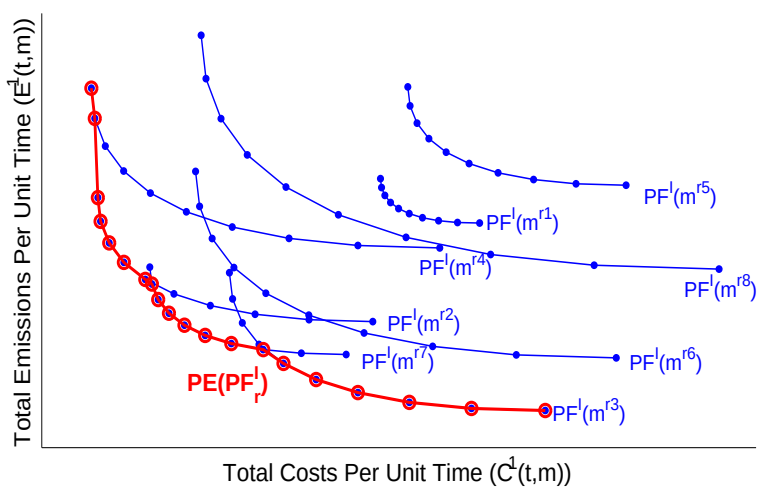


Figure 4.1. Illustration of fitness evaluation of GA-I

(iii) Genetic Operations: Given a set of parent chromosomes, three different mutation operators are used to generate the new population of chromosomes. The new population of chromosomes will consist of the current set of parent chromosomes plus the newly generated chromosomes. The current set of parent chromosomes are used in the next population to ensure that the next set of parent chromosomes is not Pareto dominated by the current set of parent chromosomes. This guarantees that

GA-I will not find worsening solutions. The new chromosomes are generated with the following mutation operators: neighbor search, random mutation, and crossover.

- The neighbor search operator generates at most $2n$ new chromosomes from a given parent chromosome. For each i, $i = 1, 2, \ldots, n$, neighbor search simply generates 2 new chromosomes by increasing and decreasing m_i by 1 if possible.

- The random mutation operator generates one new chromosome from each given parent chromosome by replacing each m_i value of the given parent chromosome with a m_i value that is randomly generated from $[\lfloor \widetilde{m}_i^{LB} \rfloor, \lceil \widetilde{m}_i^{UB} \rceil]$ with a pre-specified probability (in case the m_i value is not replaced, it is kept the same in the new chromosome).

- The crossover operator randomly selects two parent chromosomes and performs a single point crossover at a randomly selected m_i value. The crossed over parent chromosomes are not considered for further crossover. The crossover operator is repeated until there are no new pair of parent chromosomes available for crossover. Through crossover, the number of the newly generated chromosomes is equal to half of the number of the parent chromosomes.

(iv) Termination: The GA-I is terminated when there is no change in the set of parent chromosomes for a pre-specified number of consecutive populations. That is, if $PE(PF_r^I)$ remains the same for a pre-specified number of populations, GA-I stops. The $PE(PF_r^I)$ at termination is accepted as PF^I.

4.2.2. Pareto Front Approximation for SJRP-DGS. Here, the details of the genetic algorithm that approximates the Pareto front for the sustainable joint replenishment problem under the direct grouping strategy (GA-D) are explained step by step. Prior to describing the details of each step of the GA-D, some properties of the SJRP-DGS are discussed first that are utilized in GA-D.

4.2.2.1. Properties of SJRP-DGS. Let the Pareto front of SJRP-DGS be denoted by PF^D, that is, PF^D consists of Pareto efficient $(\mathbf{T}, \mathbf{X})$ pairs. Furthermore, let $PF^D(\mathbf{X})$ denote the Pareto front of SJRP-DGS for a given $\mathbf{X}$, that is, $PF^D(\mathbf{X})$ consists of Pareto efficient $(\mathbf{T}(\mathbf{X}), \mathbf{X})$ solutions, where $\mathbf{T}(\mathbf{X})$ denotes a $\mathbf{T}$ vector for any given $\mathbf{X}$. Similarly, let $PF^D(\mathbf{T})$ denote the Pareto front of SJRP-DGS for a

given $\mathbf{T}$, that is, $PF^D(\mathbf{T})$ consists of Pareto efficient $(\mathbf{T}, \mathbf{X}(\mathbf{T}))$ solutions, where $\mathbf{X}(\mathbf{T})$ denotes a $\mathbf{X}$ matrix for any given $\mathbf{T}$ vector. Note that $PF^D \subseteq \bigcup_{\mathbf{X}\in\chi} PF^D(\mathbf{X})$ and $PF^D \subseteq \bigcup_{\mathbf{T}\in\mathbb{R}^n_+} PF^D(\mathbf{T})$, where χ is the set of $n \times n$ binary $\mathbf{X}$ matrices such that $\sum_{j=1}^n x_{ij} = 1 \ \forall i = 1, 2, \ldots, n$.

Recall that given a $\mathbf{T}$, $\mathbf{X}^C(\mathbf{T})$ and $\mathbf{X}^E(\mathbf{T})$ can be found using Equations (37)-(38) and Equations (39)-(40), respectively, which requires sorting at most n values. On the other hand, as given in Equations (35) and (36), given an $\mathbf{X}$, $\mathbf{T}^C(\mathbf{X})$ and $\mathbf{T}^E(\mathbf{X})$ have explicit solutions. Furthermore, due to the binary nature of $\mathbf{X}$, representing it as a chromosome is more tractable. Therefore, in what follows, focus is placed on characterizing $PF^D(\mathbf{X})$. The next property defines the range for $\mathbf{T}(\mathbf{X})$ such that $(\mathbf{T}(\mathbf{X}), \mathbf{X}) \in PF^D(\mathbf{X})$.

Property 14. *Given $\boldsymbol{X} \in \chi$, if $(\boldsymbol{T}(\boldsymbol{X}), \boldsymbol{X}) \in PF^D(\boldsymbol{X})$, then $t_j(\boldsymbol{X}) \in [\min\{t_j^C(\boldsymbol{X}), t_j^E(\boldsymbol{X})\}, \max\{t_j^C(\boldsymbol{X}), t_j^E(\boldsymbol{X})\}] \ \forall j = 1, 2, \ldots, n$.*

It follows from Property 14 that both $(\mathbf{T}^C(\mathbf{X}), \mathbf{X})$ and $(\mathbf{T}^E(\mathbf{X}), \mathbf{X})$ are in $PF^D(\mathbf{X})$. However, any $(\mathbf{T}(\mathbf{X}), \mathbf{X})$ such that $\mathbf{T}(\mathbf{X})$ consists of $t_j(\mathbf{X})$ values that are randomly generated from $[\min\{t_j^C(\mathbf{X}), t_j^E(\mathbf{X})\}, \max\{t_j^C(\mathbf{X}), t_j^E(\mathbf{X})\}] \ \forall j = 1, 2, \ldots, n$ is not necessarily in $PF^D(\mathbf{X})$. To approximate $PF^D(\mathbf{X})$, the normalized weighted approach is utilized as detailed in the next property.

Property 15. *Given $\boldsymbol{X} \in \chi$ and $\omega \in [0, 1]$, $(\boldsymbol{T}^\omega(\boldsymbol{X}), \boldsymbol{X}) \in PF^D(\boldsymbol{X})$ such that*

$$t_j^\omega(\boldsymbol{X}) = \sqrt{\frac{2((w_1 A + w_2 \widehat{A}) + \sum_{i=1}^n (w_1 a_i + w_2 \widehat{a}_i) x_{ij})}{\sum_{i=1}^n (w_1 h_i + w_2 \widehat{h}_i) \lambda_i x_{ij}}}, \tag{45}$$

where $w_1 = \omega / C^2(\boldsymbol{T}^C(\boldsymbol{X}), \boldsymbol{X}|\boldsymbol{X})$ and $w_2 = (1 - \omega)/E^2(\boldsymbol{T}^E(\boldsymbol{X}), \boldsymbol{X}|\boldsymbol{X})$.

Property 15 indicates that one can approximate $PF^D(\mathbf{X})$ by generating different $\mathbf{T}^\omega(\mathbf{X})$ vectors for different ω values as described in Routine 6.

Routine 6: Approximating $PF^D(\mathbf{X})$ for a given $\mathbf{X}$

0: Let $\mathbf{X}$ and ℓ be given. Set $PF^D(\mathbf{X}) = \emptyset$.

1: For $y = 1 : \ell + 1$

2: $\quad w = (y-1)/\ell$ and set $PF^D(\mathbf{X}) := PF^D(\mathbf{X}) \bigcup \{(\mathbf{T}^\omega(\mathbf{X}), \mathbf{X})\}$

3: End

4: Return $PF^D(\mathbf{X})$.

Routine 6 generates $\ell+1$ $(\mathbf{T}(\mathbf{X}), \mathbf{X})$ solutions in $PF^D(\mathbf{X})$ by starting with $\omega = 0$ and increasing ω in ℓ equal increments up to $\omega = 1$. At each ω value, Equation (45) is used to find $\mathbf{T}^\omega(\mathbf{X})$. Routine 6 is used in GA-D, for which the details are explained next.

4.2.2.2. Genetic algorithm for SJRP-DGS. Similar to GA-I, GA-D consists of the four main steps, for which the details are explained next.

(i) Chromosome Representation and Initialization: As mentioned previously, the GA-D evolves with $\mathbf{X}$ matrices. To represent $\mathbf{X}$, similar to Olsen (2005) and Wang et al. (2012b), $\mathbf{v} = [v_1, v_2, \ldots, v_n]^t$ is defined as the n-vector of v_i values, where v_i is an integer number denoting the group that item i belongs to. In this sense, $\mathbf{v}$ actually defines a vector similar to $\mathbf{m}$. Note that given a $\mathbf{v}$, the $\mathbf{X}$ matrix can be constructed such that $\mathbf{X} \in \chi$. Therefore, $\mathbf{v}$ is used as the chromosomes of the GA-D. Furthermore, note that $1 \leq v_i \leq n$ $\forall i = 1, 2, \ldots, n$. Similar to the initialization of GA-I, $2n$ chromosomes are generated by randomly selecting integer v_i values such that $v_i \in [1, n]$ $\forall i = 1, 2, \ldots, n$. Let S^r be the set of chromosomes in the rth population and let $\mathbf{v}^{rk}$ define the kth chromosome in the rth population such that $k \in \{1, 2, \ldots, |S^r|\}$ and $\mathbf{X}^{rk}$ denote the $\mathbf{X}$ matrix constructed using $\mathbf{v}^{rk}$.

(ii) Fitness Evaluation: Given S^r, first generate $PF^D(\mathbf{X}^{rk})$ $\forall k \in \{1, 2, \ldots, |S^r|\}$ using the normalized weighted approach given in Routine 6. If $PF^D(\mathbf{X}^{rk_1}) \prec PF^D(\mathbf{X}^{rk_2})$ for any k_1 and k_2 such that $k_1, k_2 \in \{1, 2, \ldots, |S^r|\}$, $\mathbf{v}^{rk_2}$ is not considered in generating the next population. Similar to GA-I, one can use Routine 4 to find the set of parent chromosomes for the next generation, which will consist of the $\mathbf{v}^{rk}$ vectors that resulted in non-dominated $PF^D(\mathbf{X}^{rk})$ sets and $PE(PF_r^D)$ is defined similar to $PE(PF_r^I)$.

(iii) Genetic Operations: To generate a new population, the mutation operators of GA-I are used, i.e., neighbor search, random mutation, and crossover, in GA-D as well since the chromosomes in both genetic algorithms are n-vectors of integers with upper and lower bounds. The parent set of chromosomes are also included in the new generation to assure generating non-worsening parent chromosomes.

(iv) Termination: The GA-D is terminated if $PE(PF_r^D)$ remains the same for a pre-specified number of populations.

4.3. NUMERICAL STUDIES

In this section, the focus is on two sets of numerical studies: convergence of the genetic algorithms, and comparison of the indirect and direct grouping strategies in terms of cost and environmental performance. The test data used in the analyses is similar to the data used for JRPs (see, e.g., Olsen, 2005, Goyal and Deshmukh, 1993, Kaspi and Rosenblatt, 1991). As the procurement costs and procurement emissions per unit time are constants and not effective in the search of the Pareto efficient solutions, $p_i = \widehat{p}_i = 1 \; \forall i = 1, 2, \ldots, n$. For any problem instance, the demand of item i is randomly generated from a uniform distribution such that $\lambda_i \sim U[1000, 2000]$.

For any item i, the cost parameters used to randomly generate problem instances assume uniform distributions with the following ranges: $a_i \sim U[1, 10]$ and $h_i \sim U[0.2, 10]$. For any item i, the carbon emissions parameters used to randomly generate problem instances assume uniform distributions with the following ranges: $\widehat{a}_i \sim U[1, 10]$ and $\widehat{h}_i \sim U[2, 22]$ (note that similar carbon emissions parameter values are used in inventory control models with carbon emissions considerations, see, e.g., Hua et al., 2011, Chen et al., 2013, and Toptal et al., 2014).

In the single objective JRPs, a major factor for comparing the indirect grouping strategy to direct grouping strategy in terms of cost performance is the ratio of the major setup costs to the minor setup costs (van Eijs et al., 1992). For the bi-objective JRPs defined in SJRP-IGS and SJRP-DGS, this section uses both the ratio of major setup costs (A) to the minor setup costs (a_i) and the ratio of carbon emissions from an order ($\widehat{A}$) to the carbon emissions due to including an item within the order ($\widehat{a}_i$). To do so, the major setup costs are assumed to take values $A \in \{2.75, 5.5, 11, 55, 550\}$ and the carbon emissions from an order are assumed to take values $\widehat{A} \in \{4, 8, 16, 80, 800\}$. Using these values for A and $\widehat{A}$ indicate that the possible values for the average A/a_i

and $\widehat{A}/\widehat{a}_i$ ratios are $\{0.5, 1, 2, 10, 100\}$.

In the following studies, 4 different problem sizes are considered: $n \in \{5, 10, 15, 20\}$. For each problem size, 25 different problem classes are considered, each of which corresponds to a combination of A/a_i and $\widehat{A}/\widehat{a}_i$ ratios. Routines 4, 5, and 6 and GA-I an GA-D are implemented in MATLAB 2012. The problem instances generated are solved on a desktop PC with 2.8 GHz processor and 10 GB of RAM. For each problem size and for each problem class, 10 problem instances are generated and solved with GA-I and GA-D (i.e., 1000 problem instances are solved with GA-I and GA-D).

4.3.1. Convergence of GA-I and GA-D. To evaluate the convergence of GA-I and GA-D, the following statistics are considered assuming that z^I and z^D are the population numbers at termination: number of solutions returned at termination (i.e., $|PF^I|$ and $|PF^D|$), population size of the last population (i.e., the number of $\mathbf{m}$ vectors and $\mathbf{X}$ matrices) denoted as $|S^{z^I}|$ and $|S^{z^D}|$, average population size denoted as $\overline{|S^I|}$ and $\overline{|S^D|}$, population number at termination (i.e., z^I and z^D), and the computational time, in seconds (CPU). In both of the genetic algorithms, the algorithm terminates if there is no improvement in 20 consecutive populations and Routines 5 and 6 generate 15 solutions.

Table 4.1 shows the average result over 250 problem instances solved for each n. Tables 4.2 and 4.3 shows the average computation time over 10 problem instances solved for each 25 combinations of A/a_i and $\widehat{A}/\widehat{a}_i$ ratios. As expected, it can be observed from Table 4.1 that computational time is increasing as the problem size increases with both indirect and direct grouping. Furthermore, as n increases, the number of Pareto efficient solutions returned also increases. It can be observed from Tables 4.2 and 4.3 that the smaller the A/a_i and $\widehat{A}/\widehat{a}_i$ ratios, the longer the computational time. This is also expected as the smaller the ratio is, the denser the Pareto front gets. Finally, in Figures 4.2 and 4.3, the changes of the Pareto fronts over populations of the GA-I and GA-D can be observed for an example with 20 items. As noted previously, both of the genetic algorithms guarantee non-worsening Pareto fronts over populations, as observed in Figures 4.2 and 4.3.

4.3.2. Comparison of Grouping Strategies. Comparing different grouping strategies in the case of single objective JRPs is relatively easier as one can just compare the minimum costs achieved with each grouping strategy. On the other

Table 4.1. Genetic algorithm statistics for different problem sizes

	GA-I					GA-D				
n	$\lvert PF^I \rvert$	$\lvert S^{z^I} \rvert$	$\lvert S^I \rvert$	z^I	CPU	$\lvert PF^D \rvert$	$\lvert S^{z^D} \rvert$	$\lvert S^D \rvert$	z^D	CPU
5	25.8	3.5	12.8	26.4	0.6	17.4	1.6	7.3	22.8	0.6
10	40.3	9.4	41.9	36.3	2.7	21.2	3.1	18.0	25.9	2.0
15	84.4	25.2	88.0	65.3	11.4	33.2	7.1	40.0	30.3	5.4
20	164.7	59.5	141.9	126.0	46.2	52.4	14.6	69.7	38.5	13.0
Average	78.8	24.4	71.1	63.5	15.2	31.0	6.6	33.7	29.4	5.3

Table 4.2. CPU of GA-I for different A/a_i and $\widehat{A}/\widehat{a}_i$ ratios

		$\widehat{A}/\widehat{a}_i$				
		0.5	1	2	10	100
A/a_i	0.5	85.3	44.5	30.6	9.5	9.0
	1	36.5	27.9	15.5	7.3	7.1
	2	19.1	15.4	10.5	6.0	5.7
	10	8.1	6.3	5.4	3.1	2.8
	100	9.1	6.9	5.3	3.1	1.0

Table 4.3. CPU of GA-D for different A/a_i and $\widehat{A}/\widehat{a}_i$ ratios

		$\widehat{A}/\widehat{a}_i$				
		0.5	1	2	10	100
A/a_i	0.5	16.6	13.2	11.9	6.6	6.4
	1	11.7	8.5	6.9	4.4	4.4
	2	7.9	5.0	3.4	2.5	2.1
	10	4.7	2.4	1.1	0.8	0.8
	100	4.7	2.5	1.2	0.8	0.8

hand, in the case of multi-objective models, comparing different strategies requires accounting for all the objectives considered. To compare indirect and direct grouping strategies for the SJRP, the comparison is made between PF^I and PF^D returned by GA-I and GA-D, respectively. Specifically, if one of these approximated Pareto fronts dominates the other, one can conclude that the corresponding grouping strategy is better. On the other hand, if there is no dominant Pareto front, then both grouping

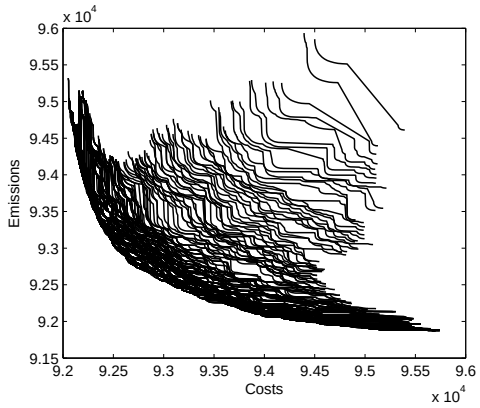


Figure 4.2. Convergence of GA-I

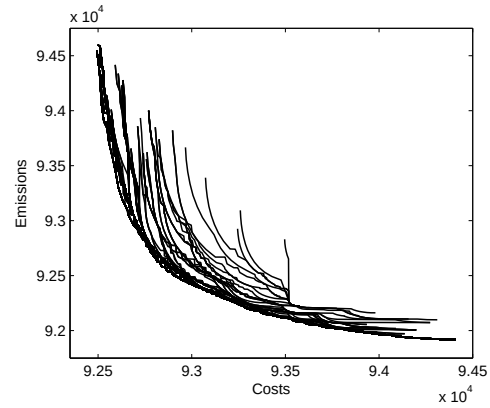


Figure 4.3. Convergence of GA-D

strategies may have advantages over one another.

Table 4.4 documents the percentages of problem instances where the set of Pareto efficient solutions of indirect grouping dominates the set of Pareto efficient solutions of direct grouping, i.e., $PF^I \prec PF^D$, for different A/a_i and $\widehat{A}/\widehat{a}_i$ ratios. It can be observed that as A/a_i or $\widehat{A}/\widehat{a}_i$ ratio is increasing, it is less likely that $PF^I \prec PF^D$. Therefore, one can conclude that, specially in scenarios where A/a_i or $\widehat{A}/\widehat{a}_i$ ratio is lower, indirect grouping is often preferred over direct grouping. In all of the problem instances solved, it is not observed that $PF^D \prec PF^I$; hence, one cannot say that direct grouping is always better than indirect grouping strategy. Particularly, Table 4.5 shows the percentages of the problem instances where no dominance relation is observed between PF^I and PF^D, denoted as $PF^I \gtrless PF^D$. Similarly, it can be concluded that if A/a_i or $\widehat{A}/\widehat{a}_i$ ratio is higher, the direct grouping strategy may be preferred over the indirect grouping strategy depending on the economical and environmental goals. For instance, a sample problem instance is illustrated in Figure 4.4, where PF^I and PF^D are given. As it can be seen, for a given environmental goal, it is possible that direct grouping will result in lower costs. Similarly, it is also possible that for a given cost target, direct grouping will result in less emissions.

Table 4.4. Percentage of problem instances where $PF^I \prec PF^D$ for different A/a_i and $\widehat{A}/\widehat{a}_i$ ratios

		$\widehat{A}/\widehat{a}_i$				
		0.5	1	2	10	100
A/a_i	0.5	55.0%	62.5%	70.0%	37.5%	0.0%
	1	65.0%	70.0%	70.0%	35.0%	0.0%
	2	55.0%	55.0%	55.0%	32.5%	0.0%
	10	12.5%	12.5%	15.0%	10.0%	0.0%
	100	0.0%	0.0%	0.0%	0.0%	0.0%

Table 4.5. Percentage of problem instances where $PF^I \gtrless PF^D$ for different A/a_i and $\widehat{A}/\widehat{a}_i$ ratios

		$\widehat{A}/\widehat{a}_i$				
		0.5	1	2	10	100
A/a_i	0.5	45.0%	37.5%	30.0%	62.5%	100.0%
	1	35.0%	30.0%	30.0%	65.0%	100.0%
	2	45.0%	45.0%	45.0%	67.5%	100.0%
	10	87.5%	87.5%	85.0%	90.0%	100.0%
	100	100.0%	100.0%	100.0%	100.0%	100.0%

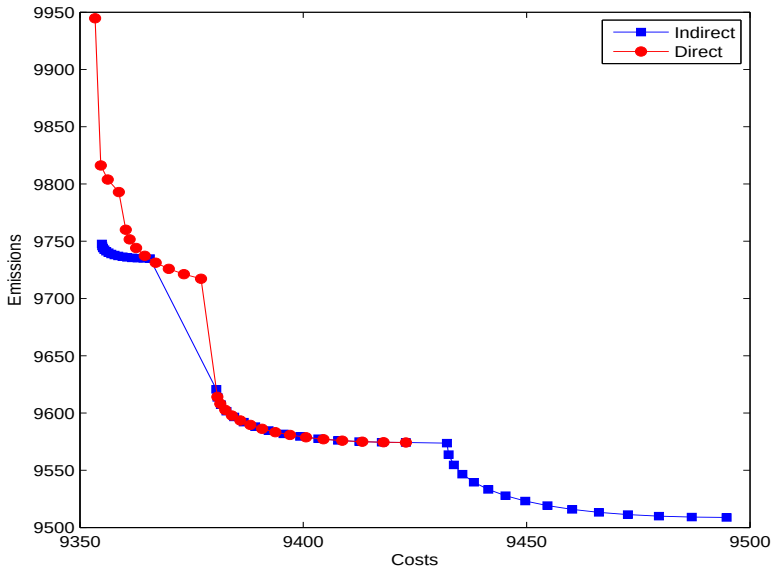


Figure 4.4. Costs and emission results for varying cap levels

4.4. CONCLUSIONS AND FUTURE RESEARCH

In this section, a bi-objective deterministic joint replenishment problem is analyzed, where a retailer's costs and carbon emissions are minimized. This problem is referred to as the sustainable joint replenishment problem. Two common practical grouping strategies are considered for the problem of interest: indirect grouping and direct grouping. For each sustainable joint replenishment problem with different grouping strategy, a genetic algorithm is developed utilizing the properties of the bi-objective optimization method. A set of numerical studies is documented to analyze the efficiency of the heuristic methods. Furthermore, a set of numerical analyses is conducted to compare the indirect grouping strategy to the direct grouping strategy not only in terms of costs but also carbon emissions. It is observed that the major setup to minor setup ratio is important for preferring one strategy over the other as well as the ratio of emissions from order setup to emissions due to including an item within an order. Specifically, it is observed that when these ratios are lower, the indirect grouping strategy can perform better both with regards to costs and emissions. On the other hand, in scenarios where these ratios are higher, it is retailer's economical and environmental targets that will determine the grouping strategy to adopt.

This section contributes to inventory control models with environmental considerations by modeling and developing solution methods for a multi-item coordinated inventory control model with environmental objective in addition to the classical economical objectives. Furthermore, analysis of multi-objective joint replenishment problems is rather limited in the literature and the solution methods discussed here give some properties and develop approaches for the bi-objective joint replenishment problems of interest, which can be used for different settings. Future research directions include to analyze stochastic joint replenishment problem with environmental considerations. Furthermore, coordinated multi-echelon inventory control models can be studied with environmental considerations. For instance, Section 5 focuses on an integrated inventory control and transportation problem in a multi-item stochastic inventory system. While environmental considerations are not directly formulated, environmental performance of coordination, specifically, consolidation of the deliveries of different items, is evaluated.

5. TIME BASED SHIPMENT CONSOLIDATION IN MULTI-ITEM STOCHASTIC INVENTORY SYSTEMS WITH HETEROGENEOUS FREIGHT TRUCKS

Integrated inventory control and transportation problems can be challenging considering the nonlinear nature of inventory related costs, jointly controlled inventories of multiple items, and demand uncertainties. This section focuses on a retailer's integrated inventory control and transportation problem for multiple items, each of which has its own stochastic demand. Inbound transportation costs are also explicitly modeled by taking into account that a retailer can use different freight truck types to ship an order. Furthermore, to utilize transportation capacity better, the retailer can consolidate shipments of different items. To avail consolidation, it is assumed that a retailer adopts a time based order-up-to-level inventory control policy, where the retailer replenishes each consolidated set of items in equal time intervals which enables joint use of the transportation capacity by the consolidated items. The retailer's problem is to find the cost minimizing consolidation strategy, i.e., of which items' orders are replenished together, the time interval between two consecutive orders of a set of consolidated items, and the order-up-to-level for each item within a consolidation.

Due to the stochastic demand environment, the retailer's objective is to minimize the expected costs. While the expected inventory holding costs, order setup costs, and penalty costs associated with shortages are well defined, the derivation of the expected inbound transportation costs is cumbersome due to the fact that freight truck choices for each order of a set of consolidated items are dynamic in nature. That is, the retailer can determine how many trucks of each truck type to be used for each order at order initiation depending on the order quantities of the individual items in the consolidation. This, in turn, makes the retailer's problem of expected cost minimization a bi-level optimization model with infinitely many lower level problems (each one is corresponding to a combination of the demands of the items within a given consolidation).

In this section, the retailer's problem is first formulated for a given consolidation of items. Here, a bi-level mixed integer nonlinear optimization problem is modeled, where the retailer decides on the common replenishment cycle length for the consolidated items and the order-up-to-level for each item within the given consolidation. Then, a set partitioning problem is presented to find the best consolidation strategy. As a solution approach, an approximation formulation is provided for a given consolidation and solves the approximated formulation with a neighborhood search heuristic. Then, an evolutionary heuristic method is discussed for the set partitioning problem of interest. A set of numerical studies are conducted to justify the approximation formulation and use of heuristic methods. Furthermore, a set of numerical studies demonstrate the cost savings and environmental benefits of the proposed time based order-up-to-level inventory control with shipment consolidation and explicit freight trucks modeling in multi-item stochastic inventory systems. This section contributes to the inventory control literature and practice in the following fields: explicit transportation modeling, shipment consolidation, and stochastic joint replenishment problems.

This section assumes TL transportation with the availability of heterogeneous freight trucks for inbound shipment. In multi-item inventory settings, there are a limited number of studies assuming TL transportation. Ben-Khedher and Yano (1994) analyze a multi-item deterministic joint replenishment problem with trucking costs as well as capacity constraints. They propose a heuristic method to solve the resulting NP-hard problem. In a similar setting, Kiesmuller (2009) analyzes a multi-item stochastic inventory system with periodic review and they account for TL transportation costs. Specifically, they propose a periodic order-up-to inventory policy where the trucks used for shipment have to be fully loaded; nevertheless, it is noted that a full truckloads policy can be suboptimal for a retailer as it might lead to increased holding costs at such levels that a decrease in shipping costs cannot counterbalance. A similar observation has been made by Toptal et al. (2003) in a single item model; they note that it might be beneficial to have one of the trucks to be partially loaded.

In the aforementioned studies, only a single truck type is considered. TL transportation modeling is further generalized by taking different freight trucks into consideration, as was done in the previous sections. In cases where a retailer uses

second or third party logistics for inbound transportation, there might be different TL carriers available, each of which has distinct truck fleets. Even in the case of a single TL carrier, it might be the case that the retailer can be forced to select among a set of different freight trucks for their inbound transportation. In such a case, the retailer needs to dynamically determine how many trucks of each truck type to use for the inbound shipment of each order. This section contributes to the multi-item inventory control models by providing generalized formulation for TL transportation with heterogeneous freight trucks. Specifically, different per truck capacities and per truck costs are considered for distinct truck types available for inbound shipment. Furthermore, the aforementioned studies define truck capacity in terms of the number of items that can be carried. The truck capacity definition is extended by jointly regarding the weight and volume capacities for different truck types.

As mentioned previously, transportation costs constitute a significant part of total costs in many industries; therefore, utilization of transportation capacity can substantially save costs. The practice of shipment consolidation targets better utilization of the transportation capacity by combining shipments of small quantities to achieve a shipment with a larger quantity that utilizes the transportation capacity better. This, in turn, reduces costs due to economies of scale in the transportation costs (Mutlu et al., 2010).

Three common shipment consolidation policies considered are quantity-based, time based, and time-and-quantity-based consolidation (Çetinkaya et al., 2006). In the quantity-based shipment consolidation, the customer demands are accumulated until a specified quantity is achieved; and, then a shipment is released. On the other hand, in the time based shipment consolidation, the customer demands are accumulated for a specified time period; and, then a shipment is released. In the time-and-quantity-based consolidation, the customer demands are accumulated until a specified quantity is achieved or a specified time period has ended; and, then a shipment is released. Çetinkaya (2005) provides a detailed review of coordinated inventory control models with shipment consolidation. This section assumes a time based shipment consolidation policy, that is, an order is placed in equal time intervals. However, note that the decisions on which items to consolidate is also formulated.

The joint replenishment problem considers how to jointly replenish a set of

different products in a multi-item inventory system. The main motivation for jointly replenishing the different products are the economies of scale of the order setup costs. Generally, order setup costs are defined by the transportation costs of a shipment. The reader is referred to a review of joint replenishment problems by Khouja and Goyal (2008) for different settings, models, and solution approaches studied in the literature for joint replenishment problems. In stochastic joint replenishment problems, each product has its own stochastic demand.

Balintfy (1964) proposes a can-order policy for a stochastic joint replenishment problem, where each item has a must-order level s, a can-order level c, and an order-up-to-level S. In a can-order policy, denoted by (s, c, S), an item is ordered when its inventory level reaches the must-order level, and any other item, whose inventory level is below the can-order level, is then ordered with it such that the order quantities for the ordered items build their inventory levels to the specified order-up-to-levels. While Balintfy (1964) assumes continuous inventory review, Johansen and Melchiors (2003) analyze the can-order policy under periodic review noting that replenishment opportunities may only come once or twice a day and; therefore, a periodic review model can be superior for some customers.

Atkins and Iyogun (1988) analyze joint replenishment problem strategies where the items are ordered up to an order-up-to-level R every time period of length T. These policies are referred to as (R, T) polices and Atkins and Iyogun (1988) investigate two (R, T) polices: a periodic policy, where all items are ordered with each replenishment and a modified periodic policy, where a base set of items is ordered with each replenishment and the remaining items are ordered at each specified consecutive replenishment. In this section, a (R, T) type of policy is adopted for a given set of consolidated items: the inventories of the items in the consolidation are replenished every T time units up to their individual order-up-to-levels. Atkins and Iyogun (1988) conclude that the periodic (R, T) type policies show more promise than the (s, c, S) type policies. However, Pantumsinchai (1992) notes that different policies can be superior to the others depending on the specific problem parameters.

Viswanathan (1997) introduces a new class of policies known as the $P(s, S)$ policy. The $P(s, S)$ policy is a periodic review policy where the amount of items on hand are reviewed at intervals of time T. If the amount of items on hand is less than

s then items are ordered to bring the inventory up to S. They test their algorithm against the same problems in Atkins and Iyogun (1988) and find that their proposed policy generally gives dominating solutions and that the extra computational requirement is nominal. Nielsen and Larsen (2005) use Markov decision theory and find an analytical solution to the $Q(s, S)$ policy, which was listed as a future research direction by Viswanathan (1997). In the $Q(s, S)$ policy, the total number of items are reviewed continuously but the items themselves are only reviewed once the total demand reaches Q. Nielsen and Larsen (2005) find the $Q(s, S)$ model to be superior to the periodic review $P(s, S)$ models. Ozkaya et al. (2006) propose a new hybrid (Q, S, T) policy. The policy is considered to be both continuous and periodic as orders are placed to the order-up-to level S whenever total demand level Q is reached or time T has elapsed since the last order. Using the same problem settings with Atkins and Iyogun (1988) and Viswanathan (1997) as a benchmark, Ozkaya et al. (2006) find their proposed method to be better 72% of the time.

All of the above models are unconstrained and Zhao et al. (2012) state that "Inventory systems with limited and sharable-common resource exist widely in the real logistics field, yet studies on such systems are limited." Minner and Silver (2005) develop a multi-product inventory replenishment problem where the inventory level at any time is constrained by budget or space limitations. They assume a Poisson demand, zero lead time, and no backorders and formulate the problem as a semi-Markov decision process. Zhao et al. (2012) also study a constrained policy, specifically, the (r, Q) policy with a limited sharable common resource. In the (r, Q) policy, when an item's inventory drops below r then Q units of that item are ordered. Betts and Johnston (2005) study a similar model with a constraint on the investment capital available. In this section, the resource commonly shared is the transportation capacity, which is also a decision variable of the retailer at each replenishment.

5.1. PROBLEM FORMULATION

Consider a set of n items indexed by i, $i \in I$, where $I = \{1, 2, \ldots, n\}$, such that each item has a stochastic demand. Let $f^i(D_i)$ and $F^i(D_i)$ denote the probability density function and cumulative distribution function of item i's demand, D_i, over unit time. This section assumes that the unit time demand for any item i is normally distributed with mean λ_i and standard deviation σ_i. Thus, item i's demand over a

period of t time units is normally distributed with mean $\lambda_i t$ and standard deviation $\sigma_i\sqrt{t}$ (see, e.g., Nahmias, 2009). Denote $f_i(D_i^{(t)})$ as the probability density function of item i's demand over a period of t time units, where $D_i^{(t)}$ is the random variable defining item i's demand over t time units[5].

Under the current settings, the retailer is subject to inventory holding, order setup, and shortage costs. In particular, let h_i denote the inventory holding cost per unit per unit time, a_i denote the order setup cost per each order, and p_i denote the penalty cost per unit shortage for item i. In addition to these costs, the retailer is subject to explicit transportation costs associated with each order. This section assumes that the retailer can use m different truck types for inbound shipment. Let different truck types be indexed by j, $j \in J$, where $J = \{1, 2, \ldots, m\}$ such that a single truck of type j has a weight capacity of W_j, volume capacity of V_j, and cost of R_j. Furthermore, let each unit of item i have weight w_i and volume v_i.

The retailer is assumed to adopt a time based order-up-to-level inventory control policy. That is, for a single item or a set of consolidated items, the retailer will place an order at identical time intervals such that each item's order quantity is determined to increase the inventory level of that item to a specific point. This section assumes that delivery lead time is negligible[6]. If the retailer plans to manage item i individually, their decision variables would be order-up-to-level for item i, denoted by s_i, and the replenishment cycle length t_i. Figure 5.1 illustrates the expected inventory level over time for a single item with replenishment cycle length t, order-up-to-level s, and λ demand per unit time.

5.1.1. Single Item Time Based Order-up-to-level Inventory. Consider that item i is individually replenished. As noted previously, the retailer is subject to inventory holding, order setup, shortage, and inbound transportation costs. Due to the stochastic demand, the retailer's objective is to minimize the total expected costs per unit time associated with item i. Expected inventory holding cost per unit time amounts to $h_i(s_i - \frac{\lambda_i t_i}{2})$. Order setup cost per unit time is a deterministic variable depending on t_i and it amounts to a_i/t_i. Now, let $n_i(s_i, t_i)$ be the expected

[5]The problem formulation and the solution methods presented can be easily modified for other demand distributions.

[6]It should be noted that the problem formulation provided can be modified to handle constant lead times. Specifically, once the time interval for consecutive orders is determined, a retailer can initiate the order accordingly regarding the delivery lead time.

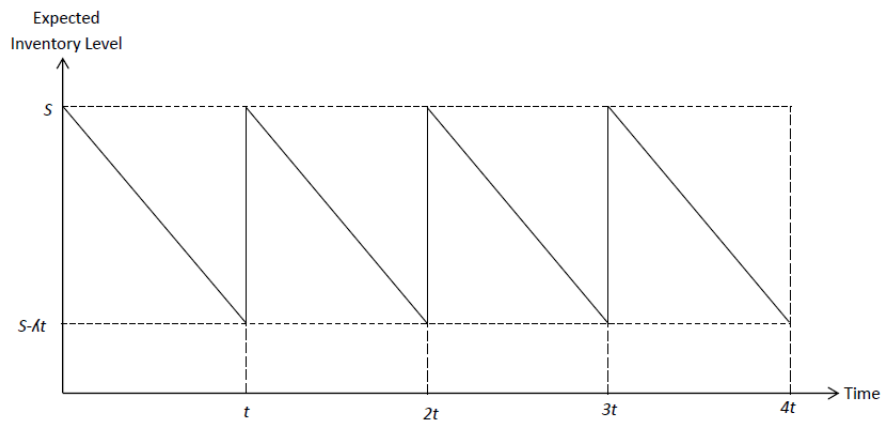


Figure 5.1. Inventory level in time based order-up-to-level control for single item

number of shortages within one replenishment cycle as a function of s_i and t_i. Then, expected shortage cost per unit time amounts to $p_i n_i(s_i, t_i)/t_i$. Note that the number of shortages within a replenishment cycle depends on both the replenishment cycle length t_i and the order-up-to-level s_i; hence, $n_i(s_i, t_i)$ is a function of s_i and t_i. One can show that $n_i(s_i, t_i) = \int_{s_i}^{\infty}(D_i^{(t_i)} - s_i) f_i(D_i^{(t_i)}) dD_i^{(t_i)}$. Therefore, expected shortage per unit time is $p_i/t_i \int_{s_i}^{\infty}(D_i^{(t_i)} - s_i) f_i(D_i^{(t_i)}) dD_i^{(t_i)}$.

The only remaining cost term is the expected inbound transportation costs. Recall that the retailer can use m different truck types for inbound transportation. At each order replenishment, the retailer needs to decide on how many of each truck type should be used. Let x_j be the integer number of type j trucks to be used for inbound transportation of an order and $\mathbf{x} = [x_1, x_2, \ldots, x_m]$. The order quantity to be shipped will be equal to the demand realized during the replenishment cycle, i.e., $D_i^{(t_i)}$. In this case, the retailer will determine the truck configuration $\mathbf{x}$ that will minimize inbound transportation costs to ship $D_i^{(t_i)}$ units. Therefore, the following problem needs to be solved at each replenishment:

$$\begin{aligned} ITC_i(D_i^{(t_i)}) \quad = \quad & \min_{\mathbf{x}} \quad \textstyle\sum_{j \in J} x_j R_j \\ & \text{s.t.} \quad \textstyle\sum_{j \in J} x_j W_j \geq w_i D_i^{(t_i)} \\ & \qquad \textstyle\sum_{j \in J} x_j V_j \geq v_i D_i^{(t_i)} \\ & \qquad x_j \in \{0, 1, 2, \ldots\} \ \forall j \in J. \end{aligned} \tag{46}$$

The objective function in the definition of $ITC_i(D_i^{(t_i)})$ given in Equation (46) is the total trucking cost. The first and second constraints assure that the selected trucks cumulatively have the sufficient weight and volume capacity to ship $D_i^{(t_i)}$ units, respectively. The third set of constraints is the integer definition for the x_j values. (Note that if $D_i^{(t_i)} \leq 0$, $x_j = 0\ \forall j = 1, 2, \ldots, m$; hence, $ITC_i(D_i^{(t_i)}) = 0$ for $D_i^{(t_i)} \leq 0$.) Then, expected inbound transportation cost per unit time amounts to $1/t_i \int_0^\infty ITC_i(D_i^{(t_i)}) f_i(D_i^{(t_i)}) dD_i^{(t_i)}$.

The retailer's total expected costs per unit time when item i is individually replenished, denoted by $g_i(s_i, t_i)$, amount to

$$\begin{aligned} g_i(s_i, t_i) \quad = \quad & h_i \left(s_i - \frac{\lambda_i t_i}{2} \right) + \frac{a_i}{t_i} + \frac{p_i}{t_i} \int_{s_i}^{\infty} \left(D_i^{(t_i)} - s_i \right) f_i(D_i^{(t_i)}) dD_i^{(t_i)} \\ & + \frac{1}{t_i} \int_0^\infty ITC_i(D_i^{(t_i)}) f_i(D_i^{(t_i)}) dD_i^{(t_i)} \end{aligned} \tag{47}$$

where the first, second, third, and forth terms of Equation (47) are the expected inventory holding, order setup, shortage, and inbound transportation costs per unit time. The retailer's optimization problem for individually replenished item i then reads as

$$\begin{aligned} (\mathbf{P}^i) \quad & \min_{(s_i, t_i)} \quad g_i(s_i, t_i) \\ & \text{s.t.} \quad t_i \geq 0 \\ & \qquad\ \ s_i \geq 0 \\ & \qquad\ \ ITC_i(D_i^{(t_i)}) = \quad \min_{\mathbf{x}} \quad \sum_{j \in J} x_j R_j \\ & \qquad\qquad\qquad\qquad\quad \text{s.t.} \quad \sum_{j \in J} x_j W_j \geq w_i D_i^{(t_i)} \\ & \qquad\qquad\qquad\qquad\qquad\quad \sum_{j \in J} x_j V_j \geq v_i D_i^{(t_i)} \\ & \qquad\qquad\qquad\qquad\qquad\quad x_j \in \{0, 1, 2, \ldots\} \qquad \forall j \in J. \end{aligned}$$

5.1.2. Consolidated Time Based Order-up-to-level Inventory. Now suppose that a set of items are ordered together, that is, their shipments are consolidated. The retailer's objective is to determine the order-up-to-level for each item in the consolidation and the replenishment cycle length for the consolidation so that the total expected costs per unit time for the items in the consolidation are minimized.

Any subset of the set of items I is a possible consolidation; thus, there are $2^n - 1$ subsets of items that can be consolidated. Let each possible subset of items be indexed by k, $k \in K$ where $K = \{1, 2, \ldots, 2^n - 1\}$ and Ω_k denote a subset. Furthermore, let T_k denote the common replenishment cycle when Ω_k is selected as a consolidation, i.e., $t_i = T_k \ \forall i \in \Omega_k$.

Similar to the single item case, a consolidated set of items has inventory holding, order setup, shortage, and inbound transportation costs. Note that inventory holding, order setup, and shortage costs of the items in a consolidation are individual cost terms; therefore, total expected holding, order setup, and shortage costs per unit time for the consolidation will be equal to the sum of the expected holding, order setup, and shortage cost per unit time of each item in the consolidation. That is, the total expected holding cost per unit time of consolidation Ω_k $k \in K$ is equal to the sum of the expected holding costs per unit time of the consolidated items. The total expected holding cost per unit time of the consolidation is, therefore, equal to $\sum_{i \in \Omega_k} h_i s_i - \frac{T_k}{2} \sum_{i \in \Omega_k} h_i \lambda_i$. Similarly, it follows that the total order setup cost per unit time for Ω_k amounts to $\frac{1}{T_k} \sum_{i \in \Omega_k} a_i$, and the total shortage cost per unit time for Ω_k is equal to $\frac{1}{T_k} \sum_{i \in \Omega_k} p_i (\int_{s_i}^{\infty} (D_i^{(T_k)} - s_i) f_i(D_i^{(T_k)}) dD_i^{(T_k)})$.

Unlike the inventory holding, order setup, and shortage costs for Ω_k, the inbound transportation costs will not be equal to the sum of the individual items' transportation costs as different items can share truck capacities due to being replenished simultaneously. In particular, at each replenishment, the retailer needs to decide on the number of trucks of each type to ship the realized demands of the items in the consolidation. Let $\mathbf{D}_{\Omega_k}^{(T_k)}$ be the $|\Omega_k|$-vector of $D_i^{(T_k)}$ values for $i \in \Omega_k$. The following problem then should be solved at each replenishment to determine the inbound transportation cost of consolidation Ω_k:

$$
\begin{aligned}
ITC_{\Omega_k}(\mathbf{D}_{\Omega_k}^{(T_k)}) \quad = \quad & \min_{\mathbf{x}} \quad \sum_{j \in J} x_j R_j \\
& \text{s.t.} \quad \sum_{j \in J} x_j W_j \geq \sum_{i \in \Omega_k} w_i D_i^{(T_k)} \\
& \qquad \sum_{j \in J} x_j V_j \geq \sum_{i \in \Omega_k} v_i D_i^{(T_k)} \\
& \qquad x_j \in \{0, 1, 2, \ldots\} \ \forall j \in J.
\end{aligned} \tag{48}
$$

Similar to Equation (46), the objective function in the definition of $ITC_{\Omega_k}(\mathbf{D}_{\Omega_k}^{(T_k)})$ given in Equation (48) is the total trucking cost. The first and second constraints guarantee that the selected trucks cumulatively have the sufficient weight and volume capacity to ship $D_i^{(t_i)}$ $\forall i \in \Omega_k$, respectively. The third set of constraints is the integer definition for the x_j values. Now, this section assumes that $\Omega_k = \{1, 2, \ldots, \ell\}$ such that $\ell \leq n$. Then, expected inbound transportation cost per unit time amounts to $\frac{1}{T_k}\int_0^\infty ITC_{\Omega_k}(\mathbf{D}_{\Omega_k}^{(T_k)}) f(\mathbf{D}_{\Omega_k}^{(T_k)}) d\mathbf{D}_{\Omega_k}^{(T_k)} = \frac{1}{T_k}\int_0^\infty \int_0^\infty \ldots \int_0^\infty ITC_{\Omega_k}(\mathbf{D}_{\Omega_k}^{(T_k)}) f_1(D_1^{(T_k)})$ $f_2(D_2^{(T_k)}) \ldots f_\ell(D_\ell^{(T_k)}) dD_1^{(T_k)} dD_2^{(T_k)} \ldots dD_\ell^{(T_k)}$.

The retailer's total expected costs per unit time when items in Ω_k are consolidated, denoted by $G_k(\mathbf{S}_k, T_k)$, amount to

$$\begin{aligned} G_k(\mathbf{S}_k, T_k) \quad = \quad & \sum_{i\in\Omega_k} h_i s_i - \frac{T_k}{2}\sum_{i\in\Omega_k} h_i \lambda_i + \frac{1}{T_k}\sum_{i\in\Omega_k} a_i \\ & + \tfrac{1}{T_k}\textstyle\sum_{i\in\Omega_k} p_i \left(\int_{s_i}^\infty \left(D_i^{(T_k)} - s_i\right) f_i(D_i^{(T_k)}) dD_i^{(T_k)}\right) \\ & + \frac{1}{T_k}\int_0^\infty ITC_{\Omega_k}(\mathbf{D}_{\Omega_k}^{(T_k)}) f(\mathbf{D}_{\Omega_k}^{(T_k)}) d\mathbf{D}_{\Omega_k}^{(T_k)} \end{aligned} \tag{49}$$

where $\mathbf{S}_k$ is a $|\Omega_k|$-vector of s_i values for $\forall i \in \Omega_k$. The first, second, third, and forth terms of Equation (49) are the expected inventory holding, order setup, shortage, and inbound transportation costs per unit time for the consolidation Ω_k. The retailer's optimization problem for consolidation Ω_k then reads as

$$\begin{aligned} (\mathbf{P}^{\Omega_k}) \quad & \min_{(\mathbf{S}_k, T_k)} \quad G_k(\mathbf{S}_k, T_k) \\ & \text{s.t.} \quad T_k \geq 0 \\ & \qquad\quad s_i \geq 0 \\ & \qquad\quad ITC_{\Omega_k}(\mathbf{D}_{\Omega_k}^{(T_k)}) = \min_{\mathbf{x}} \quad \sum_{j\in J} x_j R_j \\ & \qquad\qquad\qquad\qquad\qquad\quad \text{s.t.} \quad \sum_{j\in J} x_j W_j \geq \sum_{i\in\Omega_k} w_i D_i^{(T_k)} \\ & \qquad\qquad\qquad\qquad\qquad\qquad\quad \sum_{j\in J} x_j V_j \geq \sum_{i\in\Omega_k} v_i D_i^{(T_k)} \\ & \qquad\qquad\qquad\qquad\qquad\qquad\quad x_j \in \{0, 1, 2, \ldots\} \forall j \in J. \end{aligned}$$

Let $\mathbf{S}_k^*$ and T_k^* denote an optimum solution of $(\mathbf{P}^{\Omega_k})$.

5.1.3. Consolidation Decisions. Ultimately, the retailer's goal is to determine which items will be consolidated and what will be the common replenishment

cycle length for each consolidation and order-up-to-level for each set of items in the consolidations. Therefore, the retailer needs to select which subsets of items will be consolidated such that each item will be replenished within a single consolidation. A given consolidation Ω_k can be defined by c_{ik} values such that

$$c_{ik} = \begin{cases} 1 & \text{if item } i \text{ is in consolidation } \Omega_k, \\ 0 & \text{otherwise.} \end{cases}$$

Let

$$y_k = \begin{cases} 1 & \text{if consolidation } \Omega_k \text{ is selected,} \\ 0 & \text{otherwise.} \end{cases}$$

Assuming that the retailer will adopt the optimum common replenishment cycle length and order-up-to-levels for any consolidation Ω_k, i.e., $\mathbf{S}_k^*$ and T_k^*, the retailer's consolidation problem reads as

$$\begin{array}{llll} (\mathbf{P}) & \min\limits_{(\mathbf{y})} & C(\mathbf{y}) = \sum\limits_{k \in K} y_k G_k(\mathbf{S}_k^*, T_k^*) & \\ & \text{s.t.} & \sum\limits_{k \in K} c_{ik} y_k = 1 & \forall i \in I \\ & & y_k \in \{0, 1\} & \forall k \in K. \end{array}$$

where $\mathbf{y}$ is the binary $(2^n - 1)$-vector of y_k values. The objective function of $(\mathbf{P})$ minimizes the total expected costs per unit time. The first set of constraints ensures that each item is included within one of the selected consolidations. The second set of constraints are the binary definitions for the decision variables. Note that $(\mathbf{P})$ is a set partitioning problem, which is known to be NP-hard (see, e.g., Garey and Johnson, 1979). Furthermore, definitions of $\mathbf{S}_k^*$ and T_k^* require a bi-level mixed integer nonlinear optimization problem to be solved.

5.2. SOLUTION ANALYSIS

In this section, a genetic algorithm is proposed based on the meta-heuristic approach for solving problem $(\mathbf{P})$, denoted by GA-P. GA-P has the following four main steps: (i) chromosome representation and initialization, (ii) fitness evaluation, (iii) mutation, and (iv) termination. The details of each step are discussed in what follows.

5.2.1. Chromosome Representation and Initialization. Note that the retailer can select at most n consolidations (when each item is individually replenished), that is, $\sum_{k \in K} y_k \leq n$. Therefore, a solution to $(\mathbf{P})$ can be presented by an integer n-vector $chrom = [c_1, c_2, \ldots, c_n]$, where c_i denotes the consolidation number that item i belongs to. Note that one should have $1 \leq c_i \leq n \ \forall i \in I$. The important point about defining a solution for $(\mathbf{P})$ as a *chrom* vector is that the corresponding consolidation decisions are feasible for $(\mathbf{P})$ as each item is guaranteed to be within one consolidation. For instance, for a problem instance with $n = 5$ items, let $chrom = [3, 1, 2, 3, 2]$; then, items 1 and 4 form one consolidation, items 3 and 5 form one consolidation, and item 2 forms one consolidation. That is, $\{1, 4\}, \{3, 5\}, \{2\}$ are the three consolidations selected. Furthermore, *chrom* representation enables mutation operations to be simply executed. As an initialization, nm number of *chrom* vectors are randomly generated by randomly generating c_i values such that $1 \leq c_i \leq n$ $\forall i \in I$.

5.2.2. Fitness Evaluation. Now suppose that a set of chromosomes are given. For each chromosome, one can determine the number of consolidations and the items in each consolidation as explained above. The fitness value for a chromosome is the total expected costs of the consolidations in the chromosome. Therefore, one needs to find the total expected costs per unit time for each consolidation of a given chromosome and calculate the summation to find the fitness value of the chromosome. To do so, problem $(\mathbf{P}^{\Omega_k})$ should be solved for each consolidation associated with the chromosome. Note that $(\mathbf{P}^{\Omega_k})$ is a bi-level mixed integer nonlinear optimization problem due to the calculation of expected inbound transportation costs present in the objective function, i.e., Equation (48). Even the simplest bi-level optimization problems, when optimization problems at both levels are linear, are shown to be NP-hard (see, e.g., Hansen et al., 1992). Furthermore, one needs to solve $(\mathbf{P}^{\Omega_k})$ at least once and at most n times for each chromosome to be evaluated. Therefore, an efficient method to solve $(\mathbf{P}^{\Omega_k})$ is required. In what follows, an approximated reformulation is discussed for $(\mathbf{P}^{\Omega_k})$, which gives a single level mixed integer nonlinear optimization problem; then, a local search algorithm is proposed to solve the resulting single level mixed integer nonlinear optimization problem.

5.2.3. Approximated Reformulation for A Consolidation. In determining $\mathbf{S}_k^*$ and T_k^* for a given consolidation Ω_k, the retailer should consider how much inbound transportation costs on average will be paid. However, inbound transportation decisions, i.e. $\mathbf{x}$, are dynamic in the sense that the retailer will find their optimal truck choices with every replenishment. Nevertheless, since $\mathbf{S}_k$ and T_k heavily affect the replenishment quantities, problem $(\mathbf{P}^{\Omega_k})$, therefore, explicitly includes the expected inbound transportation costs in finding $\mathbf{S}_k^*$ and T_k^*. This, in turn, results in the bi-level optimization problem given by $(\mathbf{P}^{\Omega_k})$. Specifically, the lower level of $(\mathbf{P}^{\Omega_k})$ is required in order to find the exact expected inbound transportation costs per unit time. As aforementioned, bi-level optimization problems are complex, therefore this section approximates $(\mathbf{P}^{\Omega_k})$ with a single level optimization problem as follows.

Note that the expected order quantity for each item in Ω_k will be equal to the expected demand during one replenishment cycle, i.e., $\lambda_i T_k \ \forall i \in \Omega_k$. Then, Equation (48) is approximated by defining the expected number of trucks of type j used for consolidation Ω_k, denoted by $\widetilde{x}_{jk}$. That is, Equation (48) is defined assuming that, on average, the retailer decides to use $\widetilde{x}_{jk}$ number of type j trucks in each replenishment of the items in Ω_k. Let $\widetilde{\mathbf{x}}^k$ be the m-vector of $\widetilde{x}_{jk}$ values. Using this approximation, average shipment cost per replenishment of Ω_k amounts to $ITC_{\Omega_k}(\widetilde{\mathbf{x}}^k) = \sum_{j\in J} \widetilde{x}_{kj} R_j$. Then, the retailer's approximated total expected costs per unit time when items in Ω_k are consolidated, denoted by $\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k)$, are equal to

$$\begin{aligned}\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k) = \sum_{i\in\Omega_k} h_i s_i - \frac{T_k}{2}\sum_{i\in\Omega_k} h_i\lambda_i + \frac{1}{T_k}\sum_{i\in\Omega_k} a_i + \\ \frac{1}{T_k}\sum_{i\in\Omega_k} p_i n(s_i, T_k) + \frac{1}{T_k}\sum_{j\in J} \widetilde{x}_{kj} R_j.\end{aligned} \tag{50}$$

The only difference between Equation (50) and Equation (49) is that Equation (50) uses $ITC_{\Omega_k}(\widetilde{\mathbf{x}}^k)$ while Equation (49) requires the solution of Equation (48) for any combinations of demand realizations of the items in Ω_k. Using Equation (50), the retailer's optimization problem for consolidation with approximated total expected costs per unit time reads as

$$
\begin{aligned}
(\widetilde{\mathbf{P}}^{\Omega_k}) \quad & \min_{(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k)} && \widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k) \\
& \text{s.t.} && T_k \geq 0 \\
& && s_i \geq 0 && \forall i \in \Omega_k \\
& && \sum_{j \in J} \widetilde{x}_{jk} W_j \geq \sum_{i \in \Omega_k} w_i \lambda_i T_k \\
& && \sum_{j \in J} \widetilde{x}_{jk} V_j \geq \sum_{i \in \Omega_k} v_i \lambda_i T_k \\
& && x_j \in \{0, 1, 2, \ldots\} && \forall j \in J.
\end{aligned}
$$

$(\widetilde{\mathbf{P}}^{\Omega_k})$ is a single level mixed integer nonlinear optimization problem. Note that $(\widetilde{\mathbf{P}}^{\Omega_k})$ is NP-hard as a special case of $(\widetilde{\mathbf{P}}^{\Omega_k})$ when $w_i = 0 \ \forall i \in I$ (or $W_j \to \infty$) is an integer knapsack problem for given $\mathbf{S}_k$ and T_k. Therefore, a heuristic method is next developed to solve $(\widetilde{\mathbf{P}}^{\Omega_k})$.

5.2.4. Local Search Heuristic for Consolidation Approximation. A local search heuristic is proposed for solving $(\widetilde{\mathbf{P}}^{\Omega_k})$, denoted by LSH-k. Particularly, LSH-k works as follows. Given $\widetilde{\mathbf{x}}^k$, $\mathbf{S}_k$ and T_k are first determined by solving $(\widetilde{\mathbf{P}}^{\Omega_k})$ with the given $\widetilde{\mathbf{x}}^k$. Given $\widetilde{\mathbf{x}}^k$, $(\widetilde{\mathbf{P}}^{\Omega_k})$ reduces to the following optimization problem:

$$
\begin{aligned}
(\widetilde{\mathbf{P}}^{\widetilde{\mathbf{x}}^k}) \quad & \min_{(\mathbf{S}_k, T_k)} && \widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k | \widetilde{\mathbf{x}}^k) \\
& \text{s.t.} && T_k \leq \min \left\{ \frac{\sum_{j \in J} \widetilde{x}_{jk} W_j}{\sum_{i \in \Omega_k} w_i \lambda_i}, \frac{\sum_{j \in J} \widetilde{x}_{jk} V_j}{\sum_{i \in \Omega_k} v_i \lambda_i} \right\} \\
& && T_k \geq 0 \\
& && s_i \geq 0 \ \forall i \in \Omega_k
\end{aligned}
$$

$(\widetilde{\mathbf{P}}^{\widetilde{\mathbf{x}}^k})$ is a nonlinear optimization problem. A common method to solve such nonlinear models is the interior point method. Since $(\widetilde{\mathbf{P}}^{\widetilde{\mathbf{x}}^k})$ needs to be solved many times within LSH-k (which is also needed to be executed many times within GA-P), developing an efficient method to find solutions for $(\widetilde{\mathbf{P}}^{\widetilde{\mathbf{x}}^k})$ in less computational time is studied. In particular, given $\mathbf{S}_k$, if the number of expected shortages is overestimated for any item within one replenishment cycle and assume it is equal to the expected demand for that item within one replenishment cycle, i.e., $n_i(s_i, T_k) \cong \lambda_i T_k$; then, one can easily show that $T_k = \min \left\{ \frac{\sum_{j \in J} \widetilde{x}_{jk} W_j}{\sum_{i \in \Omega_k} w_i \lambda_i}, \frac{\sum_{j \in J} \widetilde{x}_{jk} V_j}{\sum_{i \in \Omega_k} v_i \lambda_i} \right\}$ minimizes $\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k | \widetilde{\mathbf{x}}^k, \mathbf{S}_k)$ over the feasible T_k values of $(\widetilde{\mathbf{P}}^{\widetilde{\mathbf{x}}^k})$. Furthermore, given T_k, $\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k | \widetilde{\mathbf{x}}^k, T_k)$ is separable in and convex with respect to each $s_i \forall i \in \Omega_k$. Thus, it follows from the first order condition that the s_i that minimizes $\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k | \widetilde{\mathbf{x}}^k, T_k)$ will be the

solution of $F_i^{(T_k)}(s_i) = 1 - \frac{h_i T_k}{p_i}$, where $F_i^{(T_k)}(\cdot)$ is the cumulative distribution function of item i's demand over T_k time units (i.e., cumulative distribution of the normal random variable, $D_i^{(T_k)}$, with mean $\lambda_i T_k$ and standard deviation $\sigma_i\sqrt{T_k}$). Therefore, the solution of $(\widetilde{\mathbf{P}}^{\widetilde{\mathbf{x}}^k})$ is accepted, denoted by $\widetilde{\mathbf{S}}_k$ and $\widetilde{T}_k$, as given in the following equations:

$$\widetilde{T}_k = \min\left\{\frac{\sum_{j\in J}\widetilde{x}_{jk}W_j}{\sum_{i\in\Omega_k} w_i\lambda_i}, \frac{\sum_{j\in J}\widetilde{x}_{jk}V_j}{\sum_{i\in\Omega_k} v_i\lambda_i}\right\}, \tag{51}$$

$$F_i^{(T_k)}(\widetilde{s}_i) = 1 - \frac{h_i\widetilde{T}_k}{p_i}. \tag{52}$$

In Section 5.3, Equations (51) and (52) are compared to the interior point method and it can be seen from Table 5.2 that Equations (51) and (52) are computationally very efficient compared to the interior point method. Furthermore, the solution qualities are very close over the problem instances solved. Therefore, Equations (51) and (52) are used to solve $(\widetilde{\mathbf{P}}^{\widetilde{\mathbf{x}}^k})$.

Once $(\widetilde{\mathbf{P}}^{\widetilde{\mathbf{x}}^k})$ is solved, $\widetilde{G}_k(\widetilde{\mathbf{S}}_k, \widetilde{T}_k, \widetilde{\mathbf{x}}^k)$ is calculated as the cost value of $\widetilde{\mathbf{x}}^k$. After that, all neighbors of $\widetilde{\mathbf{x}}^k$ are checked. To do so, the number of trucks of each type are increased and decreased (if possible) by 1. That is, $\widetilde{x}_{jk}$ is increased by 1 and $\widetilde{x}_{jk}$ decreased by 1 (if $\widetilde{x}_{jk} \geq 1$) for each j. This generates all neighbors of $\widetilde{\mathbf{x}}^k$. If there is a neighbor with a lower cost value, the neighbor with the lowest cost is taken as the new solution and the neighbor search is repeated with this solution. This process is repeated until no neighbor with a lower cost value is determined. At termination, a local minimum is guaranteed.

To avoid getting a high cost local minimum, the LSH-k is started with multiple $\widetilde{\mathbf{x}}^k$. Initially, m $\widetilde{\mathbf{x}}^k$ vectors are randomly generated such that $0 \leq \widetilde{x}_{jk} \leq u_k$ where $u^k = \max_{j\in J}\left\{\left\lceil\frac{\sum_{i\in\Omega_k} w_i\lambda_i t^{max}}{W_j}\right\rceil, \left\lceil\frac{\sum_{i\in\Omega_k} v_i\lambda_i t^{max}}{V_j}\right\rceil\right\}$ and $t^{max} = \max_{i\in\Omega_k}\{\sqrt{\frac{2a_i}{h_i\lambda_i}}\}$ (note that $\sqrt{\frac{2a_i}{h_i\lambda_i}}$ is the replenishment cycle length of item i assuming that $\sigma_i = 0$, i.e., the economic order quantity model); thus, u_k is the maximum number of trucks needed to ship the total order quantity of the items in the consolidation assuming that each item's order quantity is given by the economic order quantity and a single truck type is used. The details of LSH-k for a given starting solution are explained below.

Local Search Heuristic for ($\widetilde{\mathbf{P}}^{\widetilde{\mathbf{x}}^k}$) (LSH-k)

Step 0: Let $\widetilde{\mathbf{x}}^k$ be given for a consolidation Ω_k.

Step 1: Calculate $\mathbf{S}_k$ and T_k using Equations (51) and (52) and determine $\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k|\widetilde{\mathbf{x}}^k)$

Step 2: For $j = 1 : m$

Step 3: Let $\widetilde{\mathbf{x}}^{k[-j]} = \widetilde{x}_{jk}^{[+j]} = \widetilde{\mathbf{x}}^k$. If $\widetilde{x}_{jk}^{[-j]} > 0$, let $\widetilde{x}_{jk}^{[-j]} = \widetilde{x}_{jk}^{[-j]} - 1$; and, let $\widetilde{x}_{jk}^{[+j]} = \widetilde{x}_{jk}^{[+j]} + 1$

Step 4: Calculate $\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^{k[-j]}|\widetilde{\mathbf{x}}^{k[-j]})$ and $\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^{k[+j]}|\widetilde{\mathbf{x}}^{k[+j]})$ using Equations (51) and (52)

Step 5: End

Step 6: If $\min_{j\in J}\{\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^{k[-j]}|\widetilde{\mathbf{x}}^{k[-j]}), \widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^{k[+j]}|\widetilde{\mathbf{x}}^{k[+j]})\} < \widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k|\widetilde{\mathbf{x}}^k)$

Step 7: Set $\widetilde{\mathbf{x}}^k = \arg\min_{j\in J}\{\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^{k[-j]}|\widetilde{\mathbf{x}}^{k[-j]})\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^{k[+j]}|\widetilde{\mathbf{x}}^{k[+j]})\}$, go to Step 2

Step 8: Else, terminate and return $\widetilde{\mathbf{x}}^k$

5.2.5. Mutation. Now suppose that there is a population of evaluated chromosomes, that is, the total approximated expected cost per unit time for each chromosome is known. Let $chrom^{dl}$ be the dth $d \in \{1, 2, \ldots, pop^l\}$ chromosome in the lth population, where pop^l is the number of chromosomes in the lth population. Furthermore, let $\widetilde{C}(chrom^{dl})$ be the total approximated expected cost per unit time of $chrom^{dl}$. Without loss of generality, let $\widetilde{C}(chrom^{1l}) < \widetilde{C}(chrom^{2l}) < \ldots < \widetilde{C}(chrom^{pop^l l})$. To generate the $(l+1)^{st}$ population, the following three mutation operations are executed:

(i) Local Mutation: A local mutation is applied to the chromosomes that are randomly selected from the first 45% of the pop^l chromosomes within the lth population, i.e., the best 45% of the population. Local search mutation randomly picks an item i from a selected chromosome and randomly increases or decreases c_i of the chromosome by 1. For a given population of evaluated chromosomes, $\lceil 0.45 pop^l \rceil$ new chromosomes are generated at the end of local mutation operations.

(ii) Crossover: Crossover mutation is applied to the chromosomes in the best 50% of the population. Pairs of chromosomes are randomly selected from the best 50% of the population and have the random single point crossover mutation applied to them. Each pair of chromosomes crossover mutated generates two new chromosomes, one from each chromosome within the pair. For a given population of evaluated chromosomes, $\lceil 0.5pop^l \rceil$ new chromosomes are generated at the end of the crossover operations.

(iii) Random Mutation: Random mutation is applied to create a number of chromosomes so that the new population has the same population size with the current population. First, the number of chromosomes needed after local mutation and crossover operations is determined. Then, chromosomes are randomly selected from the best 50% of the population and random mutation is applied. A random mutation on a selected chromosome randomly generates a c_i value such that $1 \leq c_i \leq n$ for a randomly selected item i.

At the end of mutation operations, the newly generated population has the same number of chromosomes with the previous population.

5.2.6. Termination. If there is no improvement in $\widetilde{C}(chrom^{1l})$ for L consecutive populations or O populations are evaluated, the GA-P terminates.

5.3. NUMERICAL ANALYSES

In this section, the focus is on two sets of numerical analyses. In the first set of numerical analyses, the subroutine defined by Equations (51) and (52) is compared to the interior point method and the approximated reformulation of a consolidation is tested with a simulation study. In the second set of numerical analyses, the cost and environmental benefits of consolidating items and using multiple truck types for shipment are illustrated. In both of the numerical analyses, the demand per unit time for any item i is assumed to be normally distributed with mean λ_i and standard deviation σ_i. The problem instances are randomly generated using uniform distributions with the given ranges in Table 5.1. Similar numerical values are assumed for these parameters in the literature on integrated inventory control and transportation (see, e.g., Toptal et al., 2003, Toptal and Çetinkaya, 2006, Toptal, 2009, Konur and Toptal, 2012). In all of the following analysis, 15 different problem classes are considered, each of which corresponds to a combination of $n = \{5, 10, 15, 20, 25\}$ and

Table 5.1. Problem parameters

λ	$\sim$	$U[1750, 2250]$	w_i	$\sim$	$U[1, 4]$
σ	$\sim$	$U[150, 250]$	v_i	$\sim$	$U[0.5, 2]$
h_i	$\sim$	$U[1, 5]$	W_j	$\sim$	$U[200, 600]$
a_i	$\sim$	$U[50, 250]$	V_j	$\sim$	$U[100, 300]$
p_i	$\sim$	$U[2, 10]$	R_j	$\sim$	$U[150, 450]$

$m = \{5, 10, 15\}$. For each problem class, 10 problem instances are generated. The values shown in the tables of this section for a given problem class are the average values over all 10 problem instances solved within that problem class.

Equations (51) and (52) are first compared to the interior point method. Here, it is assumed that all of the items are consolidated in one single group and the approximated truck choices for the consolidation is given as the two alternative solution methods are being compared for problem ($\widetilde{\mathbf{P}}^{\widetilde{\mathbf{x}}^k}$). That is, $\widetilde{\mathbf{x}}^k$ is given for Ω_k such that $\Omega_k = I$. Given the number of truck types, $\widetilde{\mathbf{x}}^k$ is randomly generated such that $\widetilde{x}_{jk} \in [0, 5]$. For each problem class, Table 5.2 shows average values, over the 10 randomly generated problem instances, for T_k and corresponding $\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k | \widetilde{\mathbf{x}}^k)$ values along with the computation times in seconds (CPU) for Equations (51) and (52) and the interior point method. Furthermore, the cost difference column gives the average difference in $\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k | \widetilde{\mathbf{x}}^k)$ values between Equations (51) and (52) and the interior point method.

As it can be seen from Table 5.2, the average computational time (CPU) with Equations (51) and (52) is significantly lower than the average computational time with the interior point method. Moreover, while the interior point method method results in lower approximated costs, i.e., $\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k | \widetilde{\mathbf{x}}^k)$ values, Equations (51) and (52) were able to find good quality solutions; the increase in costs is less than 4% on average. Finally, T_k values returned by each alternative method are very close on average. Therefore, one can conclude that Equations (51) and (52) are efficient for solving ($\widetilde{\mathbf{P}}^{\widetilde{\mathbf{x}}^k}$) and they are used in GA-P.

Next, the approximated reformulation of a given consolidation is evaluated. Recall that truck choice decisions are dynamic as the retailer can select the number of trucks of each type to ship each order. However, the calculation of the expected

Table 5.2. Comparing solution methods for $(\widetilde{\mathbf{P}}^{\widetilde{\mathbf{x}}^k})$

		Equations (51) and (52)			Interior Point			Cost
n	m	$\widetilde{T}_k$	$\widetilde{G}_k$	CPU	$\widetilde{T}_k$	$\widetilde{G}_k$	CPU	Difference
5	5	0.93	36,307	0.001	0.78	35,927	0.272	0.90%
	10	1.91	48,520	0.001	1.64	47,238	0.275	2.63%
	15	2.70	57,123	0.001	2.02	53,281	0.260	6.74%
10	5	0.88	74,563	0.001	0.81	74,044	0.702	0.68%
	10	1.93	91,088	0.001	1.50	87,296	0.757	4.35%
	15	2.65	112,154	0.001	2.00	104,929	0.746	6.89%
15	5	0.80	106,073	0.001	0.76	105,705	1.270	0.35%
	10	1.65	135,966	0.001	1.44	132,389	1.390	2.66%
	15	2.62	164,629	0.001	1.90	154,024	1.290	6.70%
20	5	0.90	146,863	0.001	0.82	146,040	1.367	0.58%
	10	1.72	183,057	0.001	1.47	179,075	1.365	2.25%
	15	2.75	217,362	0.001	1.93	203,171	1.370	6.78%
25	5	0.87	183,909	0.001	0.79	183,602	1.362	0.21%
	10	1.69	229,604	0.001	1.37	223,512	1.356	2.70%
	15	2.62	274,293	0.001	1.87	256,578	1.358	6.79%
Average		1.78	137,434	0.001	1.41	132,454	1.009	3.41%

transportation costs resulted in a bi-level optimization problem $(\mathbf{P}^{\Omega_k})$, which has been approximated by problem $(\widetilde{\mathbf{P}}^{\Omega_k})$. Particularly, in $(\widetilde{\mathbf{P}}^{\Omega_k})$, $\widetilde{\mathbf{x}}^k$ defines the approximated number of trucks of each truck type to be used by the retailer for a given consolidation. To see how close $G_k(\mathbf{S}_k, T_k)$ and $\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k)$ are to one another, the truck choice decisions are simulated as well as the order quantity decisions for a given consolidation. Particularly, given a problem instance, it is assumed that all of the items are consolidated in one single group. Then, $\mathbf{S}_k$, T_k, and $\widetilde{\mathbf{x}}^k$ values are determined using LSH-k. After that, with the determined $\mathbf{S}_k$ and T_k values, 1,000 replenishment cycles are simulated for the problem instance (to do so, for each item $i \in I$, 1,000 demand realizations, i.e., $D_i^{(T_k)}$ values, are generated using normal distribution with mean $\lambda_i T_k$ and standard deviation $\sigma_i\sqrt{T_k}$). At each replenishment of the simulation, the best truck choices for the order are determined by solving Equation (48) with CPLEX (as the number of decision variables are 15 maximum, it was not very time consuming to solve Equation (48) at each of the 1,000 replenishments). As a result of simulation, the mean value of the cost per cycle is found and then one can determine

the mean value of the cost per unit time, denoted by $\overline{G}_k(\mathbf{S}_k, T_k)$. Furthermore, the mean number of trucks of each type used is found, denoted by $\overline{x}_{jk}$. Note that, in the approximated formulation, $\overline{x}_{jk}$ is assumed to be given by $\widetilde{x}_{jk}$ values.

Table 5.3. Comparing approximated and simulated results for a given consolidation

			Approximation		Simulation	
n	m	T_k	$\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k)$	$\sum_{j\in J} \widetilde{x}_{jk}$	$\overline{G}_k(\mathbf{S}_k, T_k)$	$\sum_{j\in J} \overline{x}_{jk}$
5	5	0.090	26,198	5.2	26,390	5.5
	10	0.121	25,574	7.1	23,793	7.0
	15	0.126	24,504	7.1	22,996	7.1
10	5	0.044	77,365	6.0	70,302	6.2
	10	0.077	60,447	10.0	50,552	9.5
	15	0.101	58,872	13.2	47,650	12.2
15	5	0.030	142,823	5.9	129,527	6.0
	10	0.053	116,794	10.7	92,551	9.5
	15	0.073	96,696	14.6	74,720	13.3
20	5	0.022	221,195	5.9	205,388	6.1
	10	0.041	164,768	11.0	135,303	10.7
	15	0.056	146,727	15.2	111,474	13.5
25	5	0.018	327,658	6.0	300,110	6.1
	10	0.035	212,767	10.5	176,580	9.6
	15	0.049	183,637	15.9	145,397	13.8
Average		0.062	125,735	9.6	107,516	9.1

In Table 5.3, the average values over the 10 randomly generated problem instances for T_k, $\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k)$, $\sum_{j\in J} \widetilde{x}_{jk}$, $\overline{G}_k(\mathbf{S}_k, T_k)$, and $\sum_{j\in J} \overline{x}_{jk}$ are documented for each problem class. It can be observed that from Table 5.3 that $\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k)$ overestimates $\overline{G}_k(\mathbf{S}_k, T_k)$ for most of the problem classes (and this was the case in most of the problem instances solved). This result was expected since the approximation reformulation does not define the minimum transportation costs in each replenishment. Specifically, $\widetilde{G}_k(\mathbf{S}_k, T_k, \widetilde{\mathbf{x}}^k)$ over estimated $\overline{G}_k(\mathbf{S}_k, T_k)$ by approximately 17% on average. Nevertheless, $\widetilde{x}_{jk}$ can over or under estimate $\overline{x}_{jk}$ values and the same observation holds for the total number of trucks used for inbound shipment; however, the difference between $\sum_{j\in J} \widetilde{x}_{jk}$ and $\sum_{j\in J} \overline{x}_{jk}$ is within $\pm 15\%$ and is 6% on average.

These observations demonstrate that the approximation reformulation of a consolidation is sufficiently well reflecting the actual costs; hence, can be naively used to evaluate the cost performance of a given consolidation and find good $\mathbf{S}_k$ and T_k values for a given consolidation Ω_k.

The following numerical analyses document the cost and environmental benefits of consolidation. Specifically, a comparison is made between the three consolidation policies: (i) consolidation policy, the consolidations adopted as the solution of problem ($\mathbf{P}$) via GA-P, (ii) no-consolidation policy, when all of the items are individually replenished, and (iii) single-consolidation policy, when all of the items are consolidated in a single group. For each policy, the approximated expected costs are determined (denoted by $\widetilde{C}$) and truck density (denoted by ϕ). Truck density is defined as the average number of trucks used per unit time. Particularly, for a given consolidation of items, Ω_k, truck density, ϕ_k, is defined as follows:

$$\phi_k = \frac{\sum_{j \in J} \widetilde{x}_{jk}}{T_k}.$$

Then, truck density of a consolidation policy, is equal to the sum of the truck densities of the consolidations suggested by the policy. Tables 5.4 and 5.5 give the average values over 10 problem instances solved within each problem class for $\widetilde{C}$ and ϕ for the no-consolidation policy and the single consolidation policy, respectively. Furthermore, the average values of the percent increases of $\widetilde{C}$ and ϕ due to adopting no-consolidation and single-consolidation policies over the consolidation policy are given as $\Delta\widetilde{C}$ and $\Delta\phi$, respectively.

As it can be seen from Tables 5.4 and 5.5, consolidation policies heavily affect the costs and truck density. Specifically, a retailer can save in costs by efficiently determining which items will be consolidated. Note that both single-consolidation and no-consolidation policies are suboptimal for problem ($\mathbf{P}$); therefore, as expected, consolidation results in lower costs than no-consolidation and single-consolidation policies. Compared to the no-consolidation policy, consolidation can save costs over 50% on average; and, compared to the single-consolidation policy, consolidation can save costs over 75% on average over the problem instances solved. Furthermore, efficient consolidation can reduce truck density. As expected, truck density is the

Table 5.4. Comparing consolidation to no consolidation for (**P**)

		Consolidation		No Consolidation			
n	m	$\widetilde{C}$	ϕ	$\widetilde{C}$	ϕ	$\Delta\widetilde{C}$	$\Delta\phi$
5	5	21,650	65.6	32,266	79.2	51.1%	34.0%
	10	20,896	58.1	28,099	63.3	35.9%	23.6%
	15	19,814	55.8	30,439	68.3	53.7%	19.5%
10	5	45,067	135.0	61,120	138.1	37.5%	8.4%
	10	42,020	118.5	62,761	150.3	50.5%	38.2%
	15	39,230	111.3	63,512	144.6	61.8%	25.5%
15	5	76,210	200.1	109,856	241.0	44.0%	23.4%
	10	60,394	179.3	99,554	230.3	64.8%	22.9%
	15	56,535	168.4	96,132	206.4	70.0%	12.5%
20	5	84,902	244.3	127,190	297.1	50.2%	20.4%
	10	78,609	233.1	131,619	278.5	68.2%	40.9%
	15	74,479	213.5	126,769	276.1	70.0%	33.3%
25	5	106,471	291.5	163,163	345.5	54.0%	26.0%
	10	106,574	288.7	182,291	395.7	72.3%	49.0%
	15	102,087	288.9	165,087	398.0	62.4%	24.6%
Average		62,329	176.8	98,657	220.8	56.4%	26.8%

Table 5.5. Comparing consolidation against single consolidation for (**P**)

		Consolidation		Single Consolidation			
n	m	$\widetilde{C}$	ϕ	$\widetilde{C}$	ϕ	$\Delta\widetilde{C}$	$\Delta\phi$
5	5	21,650	65.6	25,322	64.5	14.7%	9.1%
	10	20,896	58.1	25,422	57.2	17.1%	6.3%
	15	19,814	55.8	25,707	57.9	27.3%	4.3%
10	5	45,067	135.0	69,559	128.2	53.6%	3.4%
	10	42,020	118.5	63,063	131.6	53.1%	14.0%
	15	39,230	111.3	56,851	126.6	46.6%	14.1%
15	5	76,210	200.1	164,045	222.2	119.9%	23.4%
	10	60,394	179.3	104,445	194.0	77.1%	4.3%
	15	56,535	168.4	93,314	186.9	68.5%	10.3%
20	5	84,902	244.3	205,520	260.1	138.5%	11.3%
	10	78,609	233.1	156,509	260.6	103.4%	29.8%
	15	74,479	213.5	134,975	257.2	77.1%	23.5%
25	5	106,471	291.5	287,827	312.3	161.0%	7.1%
	10	106,574	288.7	235,357	347.1	126.2%	28.1%
	15	102,087	288.9	184,080	323.1	89.2%	9.1%
Average		62,329	176.8	122,133	195.3	78.2%	13.2%

highest on average for the no-consolidation policy as the utilization of truck capacities is minimum in the no-consolidation policy. Compared to the no-consolidation policy, consolidation can decrease truck density over 25% on average; and, compared to the single-consolidation policy, consolidation can decrease truck density over 10% on average. These observations suggest that efficient consolidation in multi-item inventory systems can save costs and result in environmental benefits significantly.

Finally, the cases of when the retailer uses a single truck type instead of multiple truck types are compared for inbound transportation. Specifically, it is assumed that the retailer will select the truck type which minimizes the total of the approximated expected costs of the consolidations selected, i.e., sum of the costs defined in Equation (50) over the consolidations. To find the single truck type to be used, the consolidation policy assuming a single truck type is first found via GA-P for each truck type, and selects the one which gives lower approximated expected costs. Table 5.6 gives the average values over 10 problem instances solved within each problem class for $\widetilde{C}$ and ϕ for inbound transportation with consideration of multiple truck types and a single truck type. Furthermore, the average values of the percent increases in $\widetilde{C}$ and ϕ are given (denoted by $\Delta\widetilde{C}$ and $\Delta\phi$, respectively) due to adopting the restricting single truck type for inbound shipment.

As expected and can be observed in Table 5.6, restricting the model to a single truck type for inbound shipment increases costs. On average, a single truck type inbound shipment increases costs by 2.3% compared to allowing use of different truck types for inbound shipment. Furthermore, the single truck type restriction increases the truck density by 4.9% on average over the problem instances solved. Therefore, one can conclude that consideration of different truck types simultaneously for inbound shipment can have cost savings as well as environmental benefits.

5.4. CONCLUSIONS AND FUTURE RESEARCH

The models given in this section study a multi-item inventory system with shipment consolidation and explicit TL transportation in a stochastic demand environment. A time based order-up-to-level inventory policy is proposed for a set of consolidated items. Furthermore, a retailer's consolidation decisions are formulated as a set partitioning problem. Due to the complexity of the problem, heuristic methods are developed. First, for a given consolidation, an approximated reformulation of

Table 5.6. Comparing consolidation with multiple truck types to single truck type

		Multiple-Truck		Single-Truck			
n	m	$\widetilde{C}$	ϕ	$\widetilde{C}$	ϕ	$\Delta\widetilde{C}$	$\Delta\phi$
5	5	21,650	65.6	21,991	69	1.6%	4.8%
	10	20,896	58.1	21,108	59	0.8%	0.8%
	15	19,814	55.8	20,490	62	3.3%	9.7%
10	5	45,067	135.0	45,713	139	1.3%	3.5%
	10	42,020	118.5	42,785	123	1.8%	3.1%
	15	39,230	111.3	39,714	113	1.2%	1.7%
15	5	76,210	200.1	78,276	216	2.6%	6.3%
	10	60,394	179.3	62,054	189	2.8%	5.4%
	15	56,535	168.4	58,435	177	3.2%	4.7%
20	5	84,902	244.3	90,581	281	5.8%	15.3%
	10	78,609	233.1	79,519	240	1.1%	3.2%
	15	74,479	213.5	75,105	214	0.9%	0.3%
25	5	106,471	291.5	107,900	298	1.4%	2.0%
	10	106,574	288.7	110,478	311	3.7%	6.7%
	15	102,087	288.9	104,624	308	2.4%	6.2%
Average		62,329	176.8	63,918	186.6	2.3%	4.9%

the time based order-up-to-level inventory policy with heterogeneous freight trucks is provided. A local search heuristic is proposed for the approximated reformulation. This search heuristic is utilized in a genetic algorithm to find good quality consolidation strategies for the retailer's consolidation problem.

This section contributes to the literature on multi-item inventory systems by explicitly accounting for transportation costs when heterogeneous freight trucks can be used for inbound shipment, proposing a practical inventory control policy for a set of consolidated items with distinct characteristics, and developing a solution method for determining consolidation strategies.

With a set of numerical studies, the accuracy of the approximated reformulation of a consolidation is presented. Furthermore, a set of numerical studies is conducted to illustrate the economical as well as environmental benefits of shipment consolidation with heterogeneous freight trucks. Specifically, it is observed that shipment consolidation not only saves costs but also reduces truck density. Reduced truck density implies less transportation emissions and less truck congestion.

A future research direction would be to analyze different inventory control policies for a given set of consolidated items. For instance, a quantity based order-up-to-level policy can be studied and compared to the time based order-up-to-level policy examined in this section. Furthermore, the joint replenishment problem with explicit transportation costs considering the availability of different truck types is a remaining problem to be investigated.

6. CONCLUSIONS

Greenhouse gas emissions are becoming increasingly high and the public is becoming motivated to reverse this trend. Everyone from regulators, stakeholders, and the end consumers are changing their habits in response to the increasing carbon emissions. As a major percentage of carbon emissions comes from trucking, logistics, and inventory holding, it is therefore important for companies to reevaluate their policies regarding these topics. This dissertation introduces four new models for retailers to consider that take into account both their costs and carbon emissions from inventory holding and its associated transportation activities. Also, since these models are targeted towards retailers, they can be used by virtually all companies. These models can be used to help reduce their carbon footprint as well as to save costs.

This dissertation considered two carrier options available for inbound shipment, the LTL and the TL carrier. Both transportation costs and emissions were explicitly taken into account. In Section 2, a retailer's problem was formulated assuming the basic EOQ model with both LTL and TL carriers under different carbon emission regulations. The model was optimally solved given carbon cap, carbon cap and trade, carbon cap and offset, and carbon taxing regulations. The tools provided would give a retailer their optimal ordering quantity under each carbon regulation with each transportation carrier. Results were presented showing that under a given carbon regulation, a retailer may prefer a different carrier (LTL or TL) depending on the parameters of the retailer, the carrier, and the regulation.

While Section 2 assumed a deterministic demand, Section 3 considered a stochastic demand environment, which can be more representative of a retailer's demands. In Section 3, it was not carbon regulations that motivated the retailer, but the retailer's own green goals. The (Q, R) model presented in Section 3 gives the retailer tools to select both an ordering quantity and a reorder point minimizing both costs and emissions based on their own goals. This bi-objective model was introduced as the sustainable (Q, R) model and, again, accounted for both LTL and TL carriers. A discussion was presented on the effects of demand variance and lead time on the expected costs and carbon emissions for each case. The tools provided in this section

allow retailers to choose between various LTL and TL carriers depending on their green goals.

The previous models accounted for a single item and it was acknowledged that many retailers must consider the replenishment of multiple items simultaneously. Therefore, in Sections 4 and 5, multi-item models were presented that considered deterministic and stochastic demand, respectively. A bi-objective model was introduced in Section 4 that enables a retailer to jointly replenish different items while minimizing their costs and emissions simultaneously. This model was introduced as the sustainable joint replenishment problem, an extension of the popular joint replenishment problem. The model considered two common grouping strategies, indirect and direct grouping. Results demonstrated that a retailer may want to choose one grouping strategy over the other depending on their environmental goals.

A multi-item stochastic inventory control model was introduced in Section 5. Environment considerations were not directly integrated, however, inventory control models with a transportation policy are considered to not only reduce costs but also transportation emissions. The model proposed in this section considers the explicit costs and emissions from heterogenous freight trucks. The policy determines which items are to be shipped together, how much should be ordered, the order cycle length, and how many of each truck type to use. A heuristic method was developed for the model and results of a numerical study showed the efficiency of the proposed heuristic. Furthermore, the savings in costs and reduction in carbon emissions due to the adoption of the proposed consolidation strategy are documented with a set of numerical studies.

APPENDIX A

PROOFS AND DETAILS FOR THE CARBON EMISSIONS REGULATIONS MODELS

A.1. NOTATION AND POSSIBLE METRICS

Table A.1 lists the notation and possible metrics for the carbon emissions regulations models.

Table A.1. Notation and possible metrics for the carbon emissions regulation models

Notation	Description	Metric
Retailer Parameters		
λ:	Demand rate	units/year
p:	Per unit procurement cost	\$/unit
K:	Fixed order setup cost	\$/order
h:	Inventory holding cost per unit per unit time	\$/unit/year
$\widehat{K}$:	Emissions due to order placement	lbs CO_2/order
$\widehat{h}$:	Emissions due to inventory holding per unit per unit time	lbs CO_2/unit/year
Transportation Parameters		
t:	Transportation cost per unit by LTL carrier	\$/unit
$\widehat{t}$:	Emissions due to per unit transportation with LTL carrier	lbs CO_2/unit
R:	Transportation cost per truck by TL carrier	\$/truck
P:	Transportation capacity per truck by TL carrier	units/truck
$\widehat{w}$:	Emissions due to per empty truck transportation with TL carrier	lbs CO_2/truck
$\widehat{e}$:	Emissions due to per unit transportation with TL carrier	lbs CO_2/unit
Carbon Emissions Regulation Parameters		
C:	Carbon cap	lbs CO_2/year
α:	Carbon emissions trading price	\$/lbs CO_2
r:	Carbon emissions offset investment cost	\$/lbs CO_2
γ:	Carbon emissions tax	\$/lbs CO_2

A.2. PROOF OF PROPERTY 2

Note that when $((i-1)P, iP] \cap [q_l^{TL(i)}, q_u^{TL(i)}] = \emptyset$, it follows from Equations (7) and (8) and definition of $E_i^{TL}(Q)$ that $E^{TL}(Q) > C$ for $Q \in ((i-1)P, iP]$, i.e., order quantities within the range $((i-1)P, iP]$ are not feasible for M1-TL. In case $(i-1)P < iP < q_l^{TL(i)} < q_u^{TL(i)}$ and $q_l^{TL(i)} < q_u^{TL(i)} < (i-1)P < iP$, it is easy to verify that $((i-1)P, iP] \cap [q_l^{TL(i)}, q_u^{TL(i)}] = \emptyset$. Therefore, the following 4 possible cases, where $((i-1)P, iP] \cap [q_l^{TL(i)}, q_u^{TL(i)}] \neq \emptyset$, are considered.

Case 1: $(i-1)P \leq q_l^{TL(i)} \leq iP \leq q_u^{TL(i)}$

In this case, if $(i-1)P < q_l^{TL(i)}$, then $((i-1)P, iP] \cap [q_l^{TL(i)}, q_u^{TL(i)}] = [\max\{q_l^{TL(i)}, (i-1)P\}, \min\{q_u^{TL(i)}, iP\}]$ and $E_i^{TL}(Q) \leq C$ for $Q \in [\max\{q_l^{TL(i)}, (i-1)P\}, \min\{q_u^{TL(i)}, iP\}]$, which means that $E^{TL}(Q) \leq C$ for $Q \in [\max\{q_l^{TL(i)}, (i-1)P\}, \min\{q_u^{TL(i)}, iP\}]$. On the other hand, when $(i-1)P = q_l^{TL(i)}$, since $E_{(i-1)}^{TL}(Q) < E_i^{TL}(Q)$ for any $Q \geq 0$ and $i \geq 2$, it follows that $E^{TL}(Q) \leq C$ for $Q \in [\max\{q_l^{TL(i)}, (i-1)P\}, \min\{q_u^{TL(i)}, iP\}]$.

Case 2: $(i-1)P \leq q_l^{TL(i)} \leq q_u^{TL(i)} \leq iP$

Similar to Case 2, in this case, if $(i-1)P < q_l^{TL(i)}$, then $((i-1)P, iP] \cap [q_l^{TL(i)}, q_u^{TL(i)}] = [\max\{q_l^{TL(i)}, (i-1)P\}, \min\{q_u^{TL(i)}, iP\}]$ and $E_i^{TL}(Q) \leq C$ for $Q \in [\max\{q_l^{TL(i)}, (i-1)P\}, \min\{q_u^{TL(i)}, iP\}]$, which means that $E^{TL}(Q) \leq C$ for $Q \in [\max\{q_l^{TL(i)}, (i-1)P\}, \min\{q_u^{TL(i)}, iP\}]$. On the other hand, when $(i-1)P = q_l^{TL(i)}$, since $E_{(i-1)}^{TL}(Q) < E_i^{TL}(Q)$ for any $Q \geq 0$ and $i \geq 2$, it follows that $E^{TL}(Q) \leq C$ for $Q \in [\max\{q_l^{TL(i)}, (i-1)P\}, \min\{q_u^{TL(i)}, iP\}]$.

Case 3: $q_l^{TL(i)} \leq (i-1)P \leq iP \leq q_u^{TL(i)}$

In this case, $((i-1)P, iP] \cap [q_l^{TL(i)}, q_u^{TL(i)}] = (\max\{q_l^{TL(i)}, (i-1)P\}, \min\{q_u^{TL(i)}, iP\}]$ and $E_i^{TL}(Q) \leq C$ for $Q \in (\max\{q_l^{TL(i)}, (i-1)P\}, \min\{q_u^{TL(i)}, iP\}]$, which means that $E^{TL}(Q) \leq C$ for $Q \in (\max\{q_l^{TL(i)}, (i-1)P\}, \min\{q_u^{TL(i)}, iP\}]$. Furthermore, since $E_{(i-1)}^{TL}(Q) < E_i^{TL}(Q)$ for any $Q \geq 0$ and $i \geq 2$, it follows that $E^{TL}(Q) \leq C$ for $Q \in [\max\{q_l^{TL(i)}, (i-1)P\}, \min\{q_u^{TL(i)}, iP\}]$.

Case 4: $q_l^{TL(i)} \leq (i-1)P \leq q_u^{TL(i)} \leq iP$

Similar to Case 4, in this case, $((i-1)P, iP] \cap [q_l^{TL(i)}, q_u^{TL(i)}] = (\max\{q_l^{TL(i)},$

$(i-1)P\}, \min\{q_u^{TL(i)}, iP\}]$ and $E_i^{TL}(Q) \leq C$ for $Q \in (\max\{q_l^{TL(i)}, (i-1)P\}, \min\{q_u^{TL(i)}, iP\}]$, which means that $E^{TL}(Q) \leq C$ for $Q \in (\max\{q_l^{TL(i)}, (i-1)P\}, \min\{q_u^{TL(i)}, iP\}]$. Furthermore, since $E_{(i-1)}^{TL}(Q) < E_i^{TL}(Q)$ for any $Q \geq 0$ and $i \geq 2$, it follows that $E^{TL}(Q) \leq C$ for $Q \in [\max\{q_l^{TL(i)}, (i-1)P\}, \min\{q_u^{TL(i)}, iP\}]$.

The result then follows from Cases 1-4. □

A.3. PROOF OF PROPERTY 3

Suppose that $t_1 \neq t_2$. To get a contradiction, let $Q_u^{TL(i)} = q_u^{TL(i)}$ for some i such that $t_1 \leq i \leq t_2 - 1$. It then follows from Property 2 that $(i-1)P < q_u^{TL(i)} \leq iP$. By definition of $q_u^{TL(i)}$ given in Equation (8), one can show that $q_u^{TL(i+1)} < q_u^{TL(i)}$; therefore, $q_u^{TL(t_2)} < q_u^{TL(i)}$ for $i \leq t_2 - 1$. This, then means that $q_u^{TL(t_2)} < (t_2 - 1)P$ as it is assumed that there exists some i such that $i \leq t_2 - 1$ and $q_u^{TL(i)} \leq iP$. This further implies that $((t_2 - 1)P, t_2P] \cap [q_l^{TL(t_2)}, q_u^{TL(t_2)}] = \emptyset$ as $q_l^{TL(t_2)} < q_u^{TL(t_2)}$, which is a contradiction as $((t_2 - 1)P, t_2P] \cap [q_l^{TL(t_2)}, q_u^{TL(t_2)}] \neq \emptyset$ by definition. Therefore, when $t_1 \neq t_2$, there does not exist any i such that $t_1 \leq i \leq t_2 - 1$ and $Q_u^{TL(i)} \neq iP$. □

A.4. PROOF OF PROPERTY 4

Suppose that $t_1 \geq k + 1$. Consider the following two cases:

Case (i): $t_1 = t_2$

In this case, $E^{TL}(Q) \leq C$ only for $Q \in [Q_l^{TL(t_1)}, Q_u^{TL(t_1)}]$. Now, when $Q_{t_1}^* < Q_l^{TL(t_1)}$, $H_1^{TL}(Q)$ is increasing over $[Q_l^{TL(t_1)}, Q_u^{TL(t_1)}]$; hence, $Q_1^{TL} = Q_l^{TL(t_1)}$. When $Q_l^{TL(t_1)} \leq Q_{t_1}^* \leq Q_u^{TL(t_1)}$, $H_1^{TL}(Q)$ is minimized at $Q_{t_1}^*$ by the definition of $Q_{t_1}^*$. When, $Q_u^{TL(t_1)} < Q_{t_1}^*$, $H_1^{TL}(Q)$ is decreasing over $[Q_l^{TL(t_1)}, Q_u^{TL(t_1)}]$; hence, $Q_1^{TL} = Q_u^{TL(t_1)}$.

Case (ii): $t_1 \neq t_2$

In this case, it follows from Property 3 that $Q_u^{TL(t_1)} = t_1P$. From Property 1, it is known that $H_1^{TL}(iP) \leq H_1^{TL}(Q)$ for $i \geq k + 1$, therefore, it follows that $H_1^{TL}(t_1P) \leq H_1^{TL}(Q)$ for $Q \geq t_1P$. It then leads that $Q_l^{TL(t_1)} \leq Q_1^{TL} \leq Q_u^{TL(t_1)}$. Following the same discussion in Case (i), one then can conclude that $Q_1^{TL} = Q_l^{TL(t_1)}$ if $Q_{t_1}^* < Q_l^{TL(t_1)}$; $Q_1^{TL} = Q_{t_1}^*$ if $Q_l^{TL(t_1)} \leq Q_{t_1}^* \leq Q_u^{TL(t_1)}$; and $Q_1^{TL} = Q_u^{TL(t_1)}$ if $Q_u^{TL(t_1)} < Q_{t_1}^*$.

The result then follows from Cases (i) and (ii). □

A.5. PROOF OF PROPERTY 5

Suppose that $t_2 \leq k$. Note that, from Property 1, it is known that $H_1^{TL}(Q)$ is decreasing over $(i-1)P < Q \leq iP$ for $i \leq k$. Consider the following two cases:

Case (i): $t_1 = t_2$
In this case, $E^{TL}(Q) \leq C$ only for $Q \in [Q_l^{TL(t_1)}, Q_u^{TL(t_1)}]$. Furthermore, since $t_2 = t_1 \leq k$, it is known from Property 1 that $H_1^{TL}(Q)$ is decreasing over $Q \in [Q_l^{TL(t_1)}, Q_u^{TL(t_1)}]$. Therefore, $Q^1 = Q_u^{TL(t_1)}$.

Case (ii): $t_1 \neq t_2$
In this case, it follows from Property 3 that $Q_u^{TL(t_2-1)} = (t_2-1)P$, which implies that $Q_1^{TL} \geq (t_2-1)P$. Furthermore, since $t_2 \leq k$, it is known from Property 1 that $H_1^{TL}(Q_u^{TL(t_2)}) < H_1^{TL}(Q)$ for $Q \in [Q_l^{TL(t_2)}, Q_u^{TL(t_2)}]$. Thus, it follows that $Q_1^{TL} = \arg\min\{H_1^{TL}(Q_u^{TL(t_2-1)}), H_1^{TL}(Q_u^{TL(t_2)})\}$.

The result then follows from Cases (i) and (ii). □

A.6. PROOF OF PROPERTY 6

Suppose that $t_1 \leq k < k+1 \leq t_2$. In this case, $t_1 \neq t_2$. Property 3 implies that kP is feasible, i.e., $E^{TL}(kP) \leq C$. Furthermore, it is known from Property 1 that $Q_1^{TL} \geq kP$ when $t_1 \leq k < k+1 \leq t_2$. Then one can show that $Q_1^{TL} = \arg\min\{H_1^{TL}(kP), H_1^{TL}(\min\{Q_{k+1}^*, Q_u^{TL(k+1)}\})\}$. □

A.7. PROOF OF PROPERTY 7

Observe that $H_{3b}^{LTL}(Q)$ is a strictly convex function with respect to Q and $q_{3b}^{LTL} = \sqrt{\frac{2\lambda(K+r\widehat{K})}{h+r\widehat{h}}}$ minimizes $H_{3b}^{LTL}(Q)$. Furthermore, by definition of q_l^{LTL} and q_u^{LTL}, any $Q \leq q_l^{LTL}$ or $Q \geq q_u^{LTL}$ is feasible for M3-LTL-b. Therefore, if $q_{3b}^{LTL} \leq q_l^{LTL}$ or $q_{3b}^{LTL} \geq q_u^{LTL}$, $E^{LTL}(q_{3b}^{LTL}) \geq C$, which means that $Q_{3b}^{LTL} = q_{3b}^{LTL}$. On the other hand, if $q_l^{LTL} < q_{3b}^{LTL} < q_u^{LTL}$, $E^{LTL}(q_{3b}^{LTL}) < C$. Moreover, $H_{3b}^{LTL}(Q)$ is decreasing over $0 \leq Q \leq q_l^{LTL}$ and $H_{3b}^{LTL}(Q)$ is increasing over $Q \geq q_u^{LTL}$. It then follows that $Q_{3b}^{LTL} = \arg\min\{H_{3b}^{LTL}(q_l^{LTL}), H_{3b}^{LTL}(q_u^{LTL})\}$ if $q_l^{LTL} < q_{3b}^{LTL} < q_u^{LTL}$. □

A.8. PROOF OF PROPERTY 8

First note that $q_{3b}^{TL(i)} = \sqrt{2(K + r\widehat{K} + i(R + r\widehat{w}))\lambda/(h + r\widehat{h})}$ is the minimizer of $H_{3b}^{TL}(Q)$ when i trucks are used for transportation. Furthermore, by definition of $(\widehat{Q}_l^{TL(i)}, \widehat{Q}_u^{TL(i)})$, $(\widehat{Q}_l^{TL(i)}, \widehat{Q}_u^{TL(i)}) \subset ((i-1)P, iP]$ and $E^{TL}(Q) \geq C$ for $Q \in (\widehat{Q}_l^{TL(i)},$

$\widehat{Q}_u^{TL(i)}$). Now, if $q_{3b}^{TL(i)} \leq \widehat{Q}_l^{TL(i)}$, it follows from convexity of $H_{3b}^{TL}(Q)$ over $((i-1)P, iP]$ that $H_{3b}^{TL}(Q)$ is increasing over $Q \in (\widehat{Q}_l^{TL(i)}, \widehat{Q}_u^{TL(i)})$; thus, $Q_{3b}^{TL(i)} = \lim_{Q\to+} \widehat{Q}_l^{TL(i)}$. If, $\widehat{Q}_l^{TL(i)} < q_{3b}^{TL(i)} < \widehat{Q}_u^{TL(i)}$, it then follows that $Q_{3b}^{TL(i)} = q_{3b}^{TL(i)}$ as $q_{3b}^{TL(i)}$ minimizes $H_{3b}^{TL}(Q)$ for $Q \in ((i-1)P, iP]$. Finally, if $\widehat{Q}_u^{TL(i)} \leq q_{3b}^{TL(i)}$, it then follows from convexity of $H_{3b}^{TL}(Q)$ over $((i-1)P, iP]$ that $H_{3b}^{TL}(Q)$ is decreasing over $Q \in (\widehat{Q}_l^{TL(i)}, \widehat{Q}_u^{TL(i)})$; thus, $Q_{3b}^{TL(i)} = \lim_{Q\to-} \widehat{Q}_u^{TL(i)}$. □

A.9. PROOF OF PROPERTY 9

First note that $H_{3b}^{TL}(Q)$ has a similar form with $H^{TL}(Q)$; thus, one can apply this to find the minimizer of $H_{3b}^{TL}(Q)$. Particularly, it follows from Property 1 that the minimizer of $H_{3b}^{TL}(Q)$ is less than or equal to $(z+1)P$ where z is the unique integer such that $zP < \sqrt{2(K + r\widehat{K})\lambda/(h + r\widehat{h})} \leq (z+1)P$ (see Equation (4)). Therefore, if the minimizer of $H_{3b}^{TL}(Q)$ is feasible to M3-TL-b, it will be the optimum solution of M3-TL-b. However, it is possible that the minimizer of $H_{3b}^{TL}(Q)$ is not feasible to M3-TL-b. Let x be the first integer such that $q_u^{TL(x)} \leq (x-1)P$. Note that if $q_u^{TL(x)} \leq (x-1)P$ then $(((i-1)P, iP] \setminus (Q_l^{TL(i)}, Q_u^{TL(i)})) = ((i-1)P, iP]$ for $i \geq x$ as $q_u^{TL(x)} < q_u^{TL(x+1)}$ by definition of $q_u^{TL(x)}$ given in Equation (8). It then follows that $Q \geq (x-1)P$ is feasible for M3-TL-b. Now, if $(z+1)P \leq xP$, $Q_{3b}^{TL} \leq xP$ as known from Property 1 that $H_{3b}^{TL}(iP) \leq H_{3b}^{TL}((i+1)P)$ for $i \geq (z+1)P$. If, $(z+1)P \geq (x-1)P$, it is already known from Equation (4) that $Q_{3b}^{TL} \leq (z+1)P$. Therefore, one can conclude that $Q_{3b}^{TL} \leq (\max\{z, x\} + 1)P$. □

A.10. PROOF OF PROPERTY 10

When $E^{LTL}(Q^{LTL}) \geq C$, by definitions of q_l^{LTL} and q_u^{LTL}, one will have either $Q^{LTL} \leq q_l^{LTL}$ or $Q^{LTL} \geq q_u^{LTL}$. Corollary 1 the implies that either $Q_1^{LTL} = q_l^{LTL}$ or $Q_1^{LTL} = q_u^{LTL}$. In both cases, $E^{LTL}(Q_1^{LTL}) = C$. Since $E^{TL}(Q_1^{TL}) \leq C$, it follows that $E^{TL}(Q_1^{TL}) \leq E^{LTL}(Q_1^{LTL})$. □

A.11. PROOF OF PROPERTY 11

Part (i) is proven first. Suppose that $t + \alpha\widehat{t} < \alpha\widehat{e}$. $H_2^{LTL}(Q) - H_2^{TL}(Q) = (t + \alpha\widehat{t})\lambda - \alpha\widehat{e}\lambda - \left\lceil \frac{Q}{P} \right\rceil \frac{(R+\alpha\widehat{w})\lambda}{Q}$; thus, $H_2^{LTL}(Q) < H_2^{TL}(Q)$ when $(t + \alpha\widehat{t}) < \alpha\widehat{e} + \left\lceil \frac{Q}{P} \right\rceil \frac{(R+\alpha\widehat{w})}{Q}$ for any Q. Since $\left\lceil \frac{Q}{P} \right\rceil \frac{(R+\alpha\widehat{w})}{Q} \geq 0$ for any Q and $t + \alpha\widehat{t} < \alpha\widehat{e}$, it follows that $H_2^{LTL}(Q) < H_2^{TL}(Q)$ for any Q. This implies that $H_2^{LTL}(Q_2^{TL}) < H_2^{TL}(Q_2^{TL})$. By definition of Q_2^{LTL}, $H_2^{LTL}(Q_2^{LTL}) \leq H_2^{LTL}(Q_2^{TL})$. It then follows that, if $t + \alpha\widehat{t} < \alpha\widehat{e}$, $H_2^{LTL}(Q_2^{LTL}) < H_2^{TL}(Q_2^{TL})$. Proof of part (ii) is similar. $H_2^{LTL}(Q) > H_2^{TL}(Q)$ when

$(t + \alpha\widehat{t}) > \alpha\widehat{e} + \left\lceil \frac{Q}{P} \right\rceil \frac{(R+\alpha\widehat{w})}{Q}$. Since $\left\lceil \frac{Q}{P} \right\rceil \leq Q$ for any Q and $t + \alpha\widehat{t} > R + \alpha\widehat{w} + \alpha\widehat{e}$, it follows that $H_2^{LTL}(Q) > H_2^{TL}(Q)$ for any Q. This implies that $H_2^{LTL}(Q_2^{LTL}) > H_2^{TL}(Q_2^{LTL})$. By definition of Q_2^{TL}, $H_2^{TL}(Q_2^{TL}) \leq H_2^{TL}(Q_2^{LTL})$. It then follows that, if $t + \alpha\widehat{t} > R + \alpha\widehat{w} + \alpha\widehat{e}$, $H_2^{LTL}(Q_2^{LTL}) > H_2^{TL}(Q_2^{TL})$. $\square$

APPENDIX B

DETAILS FOR THE SUSTAINABLE (Q,R) MODELS

B.1. NOTATION AND POSSIBLE METRICS

Table B.1 lists the demand, cost, and emissions notation along with some example metrics for use in the sustainable (Q, R) models. Table B.2 lists the notation and possible metrics for the transportation and the retailer's parameters and decision variables for use in the sustainable (Q, R) models.

Table B.1. Demand, cost, and emissions notation and possible metrics for the sustainable (Q,R) models

Notation	Description	Metric
Demand Parameters		
λ:	Expected demand rate	units/year
ϑ:	Standard deviation of demand rate	units
τ:	Lead time duration	year
D:	Random variable defining lead time demand	units
$f(D)$:	Probability density function of D	
$F(D)$:	Cumulative distribution function of D	
μ:	Expected lead time demand	units
σ:	Standard deviation of lead time demand	units
Cost Parameters		
c:	Per unit procurement cost	\$/unit
K:	Fixed order setup cost	\$/order
h:	Inventory holding cost per unit per unit time	\$/unit/year
p:	Unit backorder cost	\$/unit
Emission Parameters		
$\widehat{c}$:	Emissions due to per unit procurement	CO_2 lbs/unit
$\widehat{K}$:	Emissions due to order placement	CO_2 lbs/order
$\widehat{h}$:	Emissions due to inventory holding per unit per unit time	CO_2 lbs/unit/year
$\widehat{p}$:	Emissions due to per unit backorder	CO_2 lbs/unit

It should be noted that emissions are given in terms of carbon emissions as other greenhouse gas emissions can be measured in terms of equivalent CO_2 emissions (see, e.g., EPA, 2013a).

Table B.2. Transportation and retailer's notation and possible metrics for the sustainable (Q,R) models

Notation	Description	Metric
Transportation Parameters		
t:	Transportation cost per unit by LTL carrier	\$/unit
w:	Transportation cost per truck by TL carrier	\$/truck
v:	Transportation capacity per truck by TL carrier	units/truck
$\widehat{t}$:	Emissions per unit due to transportation with a LTL carrier	CO_2 lbs/unit
$\widehat{w}$:	Emissions per empty truck due to transportation with a TL carrier	CO_2 lbs/truck
$\widehat{e}$:	Emissions per unit due to transportation with a TL carrier	CO_2 lbs/unit
Retailer Parameters and Decision Variables		
k:	Retailer's safety factor	$k \geq 0$
Q:	Retailer's order quantity per order	units
R:	Retailer's re-order quantity	units
m:	Number of trucks of TL carrier used in each order	$m \in \{0, 1, 2, \ldots\}$
x:	Number of trucks of certain type of TL carrier used in each order	$x \in \{0, 1, 2, \ldots\}$

B.2. COMPARISON OF ROUTINE 2 TO INTERIOR POINT METHOD

Recall that Routine 2 is proposed to solve S-(Q,R)-TL(m, θ) and the interior point method is another method available to solve S-(Q,R)-TL(m, θ). To compare Routine 2 to the interior point method, problem instances are considered for different m values increasing from 1 to 10 in increments of 1. For each m value, 250 problem instances are generated using the design in Appendix B.3 and each problem is then solved with 50 different values of θ. That is, for each m value, 12500 different problem instances are solved with Routine 2 and the interior point method. Note that for a given problem instance with a specific value m, solving the problem instance with different θ values approximates the $PF^2(m)$; hence, 50 Pareto efficient solutions are generated for each problem instance with the given number of trucks. Once $PF^2(m)$ is approximated with 50 points for a given problem instance and number of trucks,

the standard deviation of the costs and emissions are calculated for the solutions in $PF^2(m)$.

For each m value, the "mean" columns in Tables B.3 and B.4 summarize the averages of costs and emissions over the problem instances solved (i.e., averages over 12500 problem instances), averages of the standard deviations of costs and emissions of the solutions in the Pareto fronts under are shown under the "standard deviation" columns (i.e., averages of standard deviations over 250 problem instances), and the average time to solve a problem instance (i.e., the time to generate a Pareto efficient solution, PE) and average time to generate a Pareto front with 50 Pareto efficient solutions are shown under the "time" columns, for Routine 2 and the interior point method, respectively.

Table B.3. Routine 2 statistics

	Mean		Standard Deviation		Time (in secs)	
m	Costs	Emissions	Costs	Emissions	PP	PF
1	7,394.42	11,251.49	2,040.17	4,231.04	0.00070	0.03481
2	6,829.90	10,748.95	1,671.42	3,839.48	0.00069	0.03401
3	6,825.01	10,886.86	1,546.62	3,682.06	0.00068	0.03330
4	6,965.34	11,184.68	1,487.48	3,590.37	0.00067	0.03352
5	7,162.50	11,539.83	1,463.06	3,536.94	0.00066	0.03355
6	7,384.53	11,916.82	1,461.71	3,511.53	0.00066	0.03294
7	7,616.64	12,299.64	1,476.46	3,507.52	0.00066	0.03330
8	7,851.47	12,679.97	1,502.22	3,520.12	0.00066	0.03362
9	8,085.09	13,053.62	1,535.8	3,545.23	0.00066	0.03336
10	8,315.72	13,418.71	1,574.72	3,579.77	0.00067	0.03322
avg	7,443.06	11,898.06	1,575.96	3,654.40	0.00067	0.03356

It can be observed from Tables B.3 and B.4 that Routine 2 and the interior point methods find very close solutions. Furthermore, the Pareto fronts generated with each method have very close standard deviations. Nevertheless, Routine 2 is more efficient compared to the interior point method in terms of computational time to find a Pareto efficient solution and to approximate the Pareto front for a problem instance.

Table B.4. Interior point method statistics

	Mean		Standard Deviation		Time (in secs)	
m	Costs	Emissions	Costs	Emissions	PP	PF
1	7,394.42	11,251.49	2,040.17	4,231.04	0.08299	4.24734
2	6,829.90	10,748.95	1,671.42	3,839.48	0.07840	4.02386
3	6,825.02	10,886.86	1,546.62	3,682.06	0.07898	4.05503
4	6,965.36	11,184.68	1,487.47	3,590.37	0.07696	3.95535
5	7,162.51	11,539.84	1,463.06	3,536.96	0.07661	3.93904
6	7,384.58	11,916.85	1,461.71	3,511.58	0.07700	3.95904
7	7,616.72	12,299.7	1,476.47	3,507.63	0.07760	3.99030
8	7,851.60	12,679.97	1,502.26	3,520.35	0.07767	3.99374
9	8,085.29	13,053.58	1,535.85	3,545.63	0.07771	3.99659
10	8,316.04	13,418.53	1,574.81	3,580.36	0.07783	4.00281
avg	7,443.14	11,898.05	1,575.98	3,654.55	0.07818	4.01631

B.3. DESIGN DETAILS FOR THE NUMERICAL STUDIES OF SECTION 3.3

In all of the problem instances solved, it is assumed that $k = 0$, $\lambda = 2,000$ units, and $\vartheta = 200$. Note that in analyses (i) and (ii) of Section 3.3, the standard deviation of lead time demand, and lead time duration will vary; thus, different demand characteristics will be captured. Furthermore, this section assumes that the retailer orders from a single supplier; hence, their procurement costs and procurement emissions are fixed per unit time. This further suggests that they are not effective in decision making. Therefore, this section simply assumes that $c = \widehat{c} = 1$.

In generating cost and emission parameters that are not related to transportation (except c and $\widehat{c}$), a lower and an upper bound is defined for each parameter. The parameter value in a problem instance is then determined by randomly generating a value from a uniform distribution defined within the lower and upper bounds of the parameter. $U[a, b]$ denotes a uniform distribution with bounds a and b.

- The uniform distribution for each cost parameter is designed as follows: $h \sim U[1, 5]$, $K \sim U[50, 250]$, and $p \sim U[2, 10]$. Note that similar values are assumed in many inventory control studies as well as in numerical analysis of the studies focusing on inventory control models with carbon emission considerations (see,

e.g., Benjaafar et al., 2013, Hua et al., 2011, Chen et al., 2013, Toptal et al., 2014).

- The uniform distribution for each emission parameter is designed as follows: Following the similar values in related literature (see, e.g., Benjaafar et al., 2013, Hua et al., 2011, Chen et al., 2013, Arikan et al., 2013), this section sets $\widehat{h} \sim U[2, 8]$ and $\widehat{K} \sim U[50, 300]$. In defining the range for $\widehat{p}$, it is assumed that it is defined similar to the relation between h and p, therefore, it is assumed that $\widehat{p} \sim U[5, 15]$.

In generating transportation parameters related to emissions, values are adopted from integrated inventory control and truckload transportation studies. Specifically, it is assumed that $w \sim U[150, 450]$ and $v \sim U[100, 300]$ (similar values are defined in integrated inventory control and truckload transportation, see, e.g., Toptal et al., 2003, Toptal and Çetinkaya, 2006, Toptal, 2009, Konur and Toptal, 2012). v is rounded to the nearest multiplier of 10 for practical purposes. Furthermore, it is noted by Toptal and Bingol (2011) that $\frac{w}{v} < t < w$. However, assuming that $t \to \frac{w}{v}$ is not practical as unit transportation cost would be very close to per truck cost. Therefore, this section assumes that $t \sim U[\frac{w}{v}, 2\frac{w}{v}]$. In generating transportation parameters related to emissions, this section focuses on the following observations from the literature. Generally, emission characteristics for trucks are given for empty truck and full truck per mile or kilometer (km)(see, e.g., Pan et al., 2013, Reed et al., 2010). Let $\widehat{w}_e$ and $\widehat{w}_f$ denote the carbon emissions generated per unit distance by an empty and full truck, respectively. It is observed from the values given by Pan et al. (2013) and Reed et al. (2010) that $\widehat{w}_f \approx 1.5\widehat{w}_e$ for different truck types and $\widehat{w}_e$ varies between 1 and 1.5 kg CO_2/km (similar numbers can also be deducted from a simulation study provided by Daccarett-Garcia, 2009). Therefore, problem instances with $\widehat{w}_e \sim U[1, 1.5]$ and $\widehat{w}_f = \beta\widehat{w}_e$ are considered where $\beta \sim U[1.2, 1.8]$. Then, the emissions generated from a unit load per unit distance is determined as $\frac{\widehat{w}_f - \widehat{w}_e}{v}$ for a truck with capacity of v units. Given the distance between supplier and the retailer store, say g units, one can estimate $\widehat{w} = g\widehat{w}_e$ and $\widehat{e} = g\frac{\widehat{w}_f - \widehat{w}_e}{v}$. Furthermore, problem instances are considered with $g \sim U[100, 500]$. Therefore, $\widehat{w}$ and $\widehat{e}$ are randomly generated by randomly generating $\widehat{w}_e$ (empty truck emissions per unit distance), β

(the ratio of full truck emissions per unit distance to empty truck emissions per unit distance), and g (the distance between the retailer and the supplier). In generating $\widehat{t}$, this section assumes that $\widehat{t} = \varphi\widehat{e}$, where $\varphi \sim U[0.5, 2]$. This enables this model to capture cases where unit transportation emissions of a LTL carrier can be higher and lower than the unit transportation emissions of the TL carrier.

In all of the problem instances solved, Routine 1 and Routine 2 generated 25 points on the PFs.

B.4. TABLES OF SECTION 3.3

For each σ value, Table B.5 summarizes the changes in averages over 250 problem instances under LTL transportation in expected costs (C^1) and emissions (E^1) for the cost minimizing (Q, R) policy ($(Q^C, R^C)^1$), emission minimizing (Q, R) policy ($(Q^E, R^E)^1$), and the average of the (Q, R) policies in PF^1 ($(Q^S, R^S)^1$). Table B.6 summarizes the changes in averages over 250 problem instances under TL transportation in expected costs (C^2) and emissions (E^2) for the cost minimizing (Q, R, m) policy ($(Q^C, R^C, m^C)^2$), emission minimizing policy (Q, R, m) policy ($(Q^E, R^E, m^E)^2$), and the average of the (Q, R, m) policies in PF^2 ($(Q^S, R^S, m^S)^2$). Tables B.7 and B.8 are constructed similar to Tables B.5 and B.6 but for the τ values.

Table B.5. Expected costs and emissions with LTL transportation as σ changes

	$(Q^C, R^C)^1$		$(Q^E, R^E)^1$		$(Q^S, R^S)^1$	
σ	C^1	E^1	C^1	E^1	C^1	E^1
10	7144	6927	7452	6445	7270	6564
20	7184	7028	7496	6538	7311	6659
30	7224	7130	7539	6631	7353	6755
40	7264	7231	7584	6723	7394	6850
50	7304	7332	7628	6815	7435	6944
60	7343	7433	7672	6906	7477	7039
70	7383	7534	7717	6997	7518	7133
80	7422	7634	7761	7088	7559	7226
90	7461	7735	7806	7178	7600	7319
100	7501	7835	7851	7268	7641	7412
avg.	7323	7382	7650	6859	7456	6990

Table B.6. Expected costs and emissions with TL transportation as σ changes

	$(Q^C, R^C, m^C)^2$		$(Q^E, R^E, m^E)^2$		$(Q^S, R^S, m^S)^2$	
σ	C^2	E^2	C^2	E^2	C^2	E^2
10	5850	9372	6149	8893	5938	9129
20	5906	9507	6212	9021	6000	9247
30	5962	9637	6276	9149	6060	9366
40	6018	9766	6323	9276	6119	9486
50	6074	9895	6385	9401	6179	9603
60	6130	10019	6443	9526	6238	9724
70	6185	10140	6506	9650	6296	9844
80	6241	10273	6557	9773	6353	9964
90	6296	10398	6607	9895	6409	10083
100	6351	10524	6672	10017	6467	10200
avg.	6101	9953	6413	9460	6206	9665

Table B.7. Expected costs and emissions with LTL transportation as τ changes

	$(Q^C, R^C)^1$		$(Q^E, R^E)^1$		$(Q^S, R^S)^1$	
τ	C^1	E^1	C^1	E^1	C^1	E^1
0.1	7231	7146	7547	6646	7359	6770
0.2	7283	7279	7604	6766	7413	6895
0.3	7323	7381	7649	6859	7455	6990
0.4	7356	7466	7686	6936	7490	7069
0.5	7386	7541	7720	7004	7521	7139
0.6	7412	7609	7750	7065	7549	7202
0.7	7437	7671	7778	7121	7574	7260
0.8	7459	7729	7803	7173	7598	7314
0.9	7480	7783	7828	7222	7620	7365
1.0	7501	7835	7851	7268	7641	7412
avg.	7387	7544	7722	7006	7522	7142

B.5. EXAMPLES OF SECTION 3.3.3

In Examples 5–7, the same retailer has been considered to control inventory and transportation of a single product such that the demand per unit time for the product is normally distributed with $\lambda = 2,000$ units and $\vartheta = 200$. The lead time is assumed to be fixed at $\tau = 0.25$; hence, the lead time demand is normally distributed with $\mu = 500$ and $\sigma = 100$. The safety factor is assumed to be fixed at $k = 0$. The

Table B.8. Expected costs and emissions with TL transportation as τ changes

	$(Q^C, R^C, m^C)^2$		$(Q^E, R^E, m^E)^2$		$(Q^S, R^S, m^S)^2$	
τ	C^2	E^2	C^2	E^2	C^2	E^2
0.1	5971	9659	6287	9170	6070	9386
0.2	6044	9828	6356	9335	6148	9541
0.3	6100	9954	6409	9461	6207	9661
0.4	6148	10055	6463	9566	6256	9763
0.5	6189	10153	6510	9659	6300	9852
0.6	6227	10243	6542	9742	6338	9934
0.7	6261	10321	6576	9818	6374	10008
0.8	6293	10392	6608	9888	6406	10077
0.9	6323	10451	6633	9955	6438	10139
1.0	6351	10524	6672	10017	6467	10200
avg.	6191	10158	6506	9661	6300	9856

retailer has the cost and emission parameters given in Table B.9.

LTL carriers A and B in Example 5 and the LTL carrier in Example 6 have the parameter values given in Table B.10 below.

The TL carrier in Example 6 is the TL carrier B of Example 7 and TL carriers A and B in Example 7 have the parameter values given in Table B.11 below.

Table B.9. Retailer's cost and emission parameters

Cost Parameters		Emission Parameters	
Parameter	Value	Parameter	Value
c	10	$\widehat{c}$	10
h	0.2	$\widehat{h}$	5
K	50	$\widehat{K}$	250
p	5	$\widehat{p}$	10

Table B.10. Cost and emission parameters for LTL carriers

	LTL carrier A	LTL carrier B	LTL carrier
t	0.03	0.05	0.03
$\widehat{t}$	50	49.9	50.01

Table B.11. Cost and emission parameters for TL carriers A and B

	TL carrier A	TL carrier B
w	5	10
v	280	280
$\widehat{e}$	50	50
$\widehat{w}$	60	10

APPENDIX C

DETAILS FOR THE SUSTAINABLE JOINT REPLENISHMENT MODELS

C.1. PROOF OF PROPERTY 12

Recall that for a given $\mathbf{m}$, both $C^1(t, \mathbf{m}|\mathbf{m})$ and $E^1(t, \mathbf{m}|\mathbf{m})$ are strictly convex with respect to t. This implies that $PF^I(\mathbf{m})$ is a convex set (see, e.g., Marler and Arora, 2004). Along with the continuity of these functions, this further indicates that any $(t(\mathbf{m}), \mathbf{m})$ such that $t(\mathbf{m}) \in [\min\{t^C(\mathbf{m}), t^E(\mathbf{m})\}, \max\{t^C(\mathbf{m}), t^E(\mathbf{m})\}]$ is in $PF^I(\mathbf{m})$.

Recall that for a given t, $C^1(t, \mathbf{m}|t)$ and $E^1(t, \mathbf{m}|t)$ are the summation of n separable convex functions denoted by $C_i^1(m_i)$ and $E_i^1(m_i)$ for each i, $i = 1, 2, \ldots, n$. Thus, if $(t, \mathbf{m}(t)) \in PF^I(t)$, $m_i(t)$ has to be in the Pareto front of the bi-objective optimization model which minimizes $C_i^1(m_i)$ and $E_i^1(m_i)$. Due to convexity of $C_i^1(m_i)$ and $E_i^1(m_i)$, it then follows similar to above that $m_i(t) \in [\min\{m_i^C(t), m_i^E(t)\}, \max\{m_i^C(t), m_i^E(t)\}]$ $\forall i = 1, 2, \ldots, n$. $\square$

C.2. PROOF OF PROPERTY 13

Recall that $PF^I \subseteq \bigcup_{\mathbf{m} \in \mathbb{Z}_+^n} PF^I(\mathbf{m})$. This means that if $(t, \mathbf{m}) \in PF^I$, $(t, \mathbf{m}) \in PF^I(\mathbf{m})$. It then follows from Property 12 that $\min\{t^C(\mathbf{m}), t^E(\mathbf{m})\} \leq t \leq \max\{t^C(\mathbf{m}), t^E(\mathbf{m})\}$. Therefore, for any $(t, \mathbf{m})$ such that $(t, \mathbf{m}) \in PF^I$, one should have $\min_{\mathbf{m} \in \mathbb{Z}_+^n} \left\{\min\{t^C(\mathbf{m}), t^E(\mathbf{m})\}\right\} \leq t \leq \max_{\mathbf{m} \in \mathbb{Z}_+^n} \left\{\max\{t^C(\mathbf{m}), t^E(\mathbf{m})\}\right\}$, where $t^C(\mathbf{m})$ and $t^E(\mathbf{m})$ are defined in Equations (27) and (28), respectively. This implies that $\min\left\{\min_{\mathbf{m} \in \mathbb{Z}_+^n} \left\{t^C(\mathbf{m})\right\}, \min_{\mathbf{m} \in \mathbb{Z}_+^n} \left\{t^E(\mathbf{m})\right\}\right\} \leq t \leq \max\{\max_{\mathbf{m} \in \mathbb{Z}_+^n} \left\{t^C(\mathbf{m})\right\}, \max_{\mathbf{m} \in \mathbb{Z}_+^n} \left\{t^E(\mathbf{m})\right\}\}$. Note that both $t^C(\mathbf{m})$ and $t^E(\mathbf{m})$ are decreasing with $\mathbf{m}$ (see, e.g., Goyal, 1974); thus, one can show that $\min_{\mathbf{m} \in \mathbb{Z}_+^n} \left\{t^C(\mathbf{m})\right\} = \min_{\mathbf{m} \in \mathbb{Z}_+^n} \left\{t^E(\mathbf{m})\right\} = 0$ by having $m_i \to \infty$ $\forall i = 1, 2, \ldots, n$. Therefore, there does not exist a positive lower bound for t. Furthermore, one can show that $\max_{\mathbf{m} \in \mathbb{Z}_+^n} \left\{t^C(\mathbf{m})\right\} = \sqrt{2\left(A + \sum_{i=1}^n a_i\right) / \sum_{i=1}^n \lambda_i h_i}$ and $\max_{\mathbf{m} \in \mathbb{Z}_+^n} \left\{t^E(\mathbf{m})\right\} = \sqrt{2(\widehat{A} + \sum_{i=1}^n \widehat{a}_i) / \sum_{i=1}^n \lambda_i \widehat{h}_i}$ by having $m_i = 1$ $\forall i = 1, 2, \ldots, n$. These imply that, for any $(t, \mathbf{m}) \in PF^I$, one should have $0 \leq t \leq \max\left\{\sqrt{\frac{2\left(A + \sum_{i=1}^n a_i\right)}{\sum_{i=1}^n \lambda_i h_i}}, \sqrt{\frac{2\left(\widehat{A} + \sum_{i=1}^n \widehat{a}_i\right)}{\sum_{i=1}^n \lambda_i \widehat{h}_i}}\right\}$.

Recall that $PF^I \subseteq \bigcup_{t: t > 0} PF^I(t)$. This means that if $(t, \mathbf{m}) \in PF^I$, $(t, \mathbf{m}) \in PF^I(t)$. It then follows from Property 12 that $\min\{m_i^C(t), m_i^E(t)\} \leq m_i \leq \max\{m_i^C(t), m_i^E(t)\} \forall i = 1, 2, \ldots, n$, where $m_i^C(t)$ and $m_i^E(t)$ are defined in Equations (29) and (31), respectively. Therefore, for any $(t, \mathbf{m})$ such that $(t, \mathbf{m}) \in PF^I$, one should

have $\min_{t:t>0}\left\{\min\{m_i^C(t), m_i^E(t)\}\right\} \leq m_i \leq \max_{t:t>0}\left\{\max\{m_i^C(t), m_i^E(t)\}\right\}$. Considering definitions of $m_i^C(t)$ and $m_i^E(t)$ in Equations (29) and (31), respectively, and referring to Equations (30) and (32), it then follows that $\lfloor\min\{\min_{t:t>0}\{\widetilde{m}_i^C(t)\}, \min_{t:t>0}\{\widetilde{m}_i^E(t)\}\}\rfloor \leq m_i \leq \left\lceil\max\left\{\max_{t:t>0}\{\widetilde{m}_i^C(t)\}, \max_{t:t>0}\{\widetilde{m}_i^E(t)\}\right\}\right\rceil$. Note that both $\widetilde{m}_i^C(t)$ and $\widetilde{m}_i^E(t)$ are decreasing with t. Since, $0 < t \leq t^{UB}$, one can show that t^{UB} is the solution of $\min_{t:t>0}\{\widetilde{m}_i^C(t)\}$ and $\min_{t:t>0}\{\widetilde{m}_i^E(t)\}$; hence, $m_i \geq \left\lfloor\min\left\{\frac{1}{t^{UB}}\sqrt{\frac{2a_i}{h_i\lambda_i}}, \frac{1}{t^{UB}}\sqrt{\frac{2\widehat{a}_i}{\widehat{h}_i\lambda_i}}\right\}\right\rfloor$. Furthermore, since $\max_{t:t>0}\{\widetilde{m}_i^C(t)\} \to \infty$ and $\max_{t:t>0}\{\widetilde{m}_i^E(t)\} \to \infty$ as $t \to 0$, there does not exist an upper bound for m_i. □

C.3. PROOF OF PROPERTY 14

Suppose that $\mathbf{X} \in \chi$ and $\mathbf{T}(\mathbf{X}) \in PF^D(\mathbf{X})$. To establish a contradiction, one should assume that there exists at least one j, $j \in \{1, 2, \ldots, n\}$ such that $t_j(\mathbf{X}) \notin [\min\{t_j^C(\mathbf{X}), t_j^E(\mathbf{X})\}, \max\{t_j^C(\mathbf{X}), t_j^E(\mathbf{X})\}]$. Without loss of generality, assume that $t_j(\mathbf{X}) < \min\{t_j^C(\mathbf{X}), t_j^E(\mathbf{X})\}$. Then, by definition of $t_j^C(\mathbf{X})$ and $t_j^E(\mathbf{X})$, it follows that both $C^2(\mathbf{T}(\mathbf{X}), \mathbf{X}|\mathbf{X})$ and $E^2(\mathbf{T}(\mathbf{X}), \mathbf{X}|\mathbf{X})$ can be reduced by increasing $t_j(\mathbf{X})$. This contradicts that $\mathbf{T}(\mathbf{X}) \in PF^D(\mathbf{X})$ as it is Pareto dominated by another $\mathbf{T}$ for the given $\mathbf{X}$. □

C.4. PROOF OF PROPERTY 15

First note that, given $\mathbf{X} \in \chi$, $PF^D(\mathbf{X})$ is convex as both $C^2(\mathbf{T}, \mathbf{X}|\mathbf{X})$ and $E^2(\mathbf{T}, \mathbf{X}|\mathbf{X})$ are convex in $\mathbf{T}$. In case of convex Pareto fronts, the normalized weighted approach can be used to generate the full Pareto front (see, e.g., Marler and Arora, 2010). Specifically, it follows that, for a given ω such that $\omega \in [0, 1]$, the solution of the following optimization problem will be in $PF^D(\mathbf{X})$

$$\min \ f(\mathbf{T}|\omega) = \omega\frac{C^2(\mathbf{T}, \mathbf{X}|\mathbf{X})}{C^2(\mathbf{T}^C(\mathbf{X}), \mathbf{X}|\mathbf{X})} + (1-\omega)\frac{E^2(\mathbf{T}, \mathbf{X}|\mathbf{X})}{E^2(\mathbf{T}^E(\mathbf{X}), \mathbf{X}|\mathbf{X})}$$

One can observe that $f(\mathbf{T}|\omega)$ is summation of n convex functions of t_j $\forall j = 1, 2, \ldots, n$. Similar to Equations (35) and (36), one can show, using first order conditions, that $\mathbf{T}$ minimizing $f(\mathbf{T}|\omega)$ is equal to $\mathbf{T}^\omega = [t_1^\omega, t_2^\omega, \ldots, t_n^\omega]$ such that

$$t_j^\omega = \sqrt{\frac{2((w_1 A + w_2\widehat{A}) + \sum_{i=1}^n (w_1 a_i + w_2\widehat{a}_i)x_{ij})}{\sum_{i=1}^n (w_1 h_i + w_2\widehat{h}_i)\lambda_i x_{ij}}}$$

where $w_1 = \frac{\omega}{C^2(\mathbf{T}^C(\mathbf{X}), \mathbf{X}|\mathbf{X})}$ and $w_2 = \frac{1-\omega}{E^2(\mathbf{T}^E(\mathbf{X}), \mathbf{X}|\mathbf{X})}$. □

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