Proceedings of the Interdisciplinary STEM Teaching and Learning Conference

Volume 2   Article 4

2018

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This paper supports grades 3-5 mathematics teachers and considers how technology in the classroom can be used to support "low threshold, high ceiling" tasks and productive discussion. We present a description of a card-sorting task to support the "5 Practices of Productive Mathematics Discussions" focused on an online task designed to: be open to multiple levels of strategies, reveal misconceptions, and support students in developing more sophisticated conceptual understandings of area measurement. We present a sampling of strategies created by teachers (who were pretending to be elementary students) in past activities. We discuss approaches to connecting strategies for deeper understanding of area measurement.

Recommended Citation
Stehr, Eryn Michelle; He, Jia; and Nguyen, Ha (2018) "Selecting, Sequencing, and Connecting: Using Technology to Support Area Measurement through Tasks, Strategies, and Discussion," *Proceedings of the Interdisciplinary STEM Teaching and Learning Conference:* Vol. 2 , Article 4.

Introduction

Teachers at all levels are expected ever more frequently to integrate use of emergent digital technologies (e.g., calculators, software, online tools, device applets) in mathematics teaching and learning. Too often technology is used for technology's sake, rather than in intentional ways to support mathematical reasoning, sense- making, and understanding. Online offerings of mathematical tools, tasks, and experiences span pedagogical spectra similar to those recognized in traditional curricula: procedural/ conceptual, simple/rich, disconnected/connected, and so on. Applying long-tested educational quality frameworks to online tasks may help teachers choose and implement technology in ways that help students' reasoning, sense-making, and understanding. One such set of strategies is the *Five Practices for Facilitating Productive Mathematical Discussion.*

In this paper we describe how teachers can be supported in planning their use of the "Five Practices" productively. We use the context of an open, online area measurement task. That is, we describe two tasks: (a) an online area measurement task and (b) a sorting card task to be used to support teachers in developing strategies for using the Five Practices to support learning goals. The sorting task was developed based on preservice and practicing teachers' strategies and discussion after encountering the area task. The area task was designed by a mathematics education research group focused on spatial measurement in K-8 curricula. The area task asks students to measure the area of an irregular shape using rectangles. The team designed the area task to support multiple grade level learning goals, to allow multiple student strategies, to allow access from a wide range of levels of sophistication, and to potentially reveal student misconceptions about important ideas surrounding area measurement. Because of its openness, the area task can be used to support productive discussion. The sorting task was developed, based on the area task, to support future or practicing teachers to discuss the affordances and limitations of different selections and sequences of student responses. The goal of the sorting task is to support teachers in thinking through a practical application of the Five Practices *without* the chaos of a live classroom to support their classroom implementation and decision-making.

Through our discussion of these tasks, we hope teacher educators and teachers will be supported in developing practical and critical considerations for integrating open, online tasks and productive mathematics discussions into their pedagogies.

Literature Review

In this review of relevant literature, we describe what has been said in the field of mathematics education about how beginning teachers and practicing teachers can be supported to use technology to support mathematical reasoning and sense- making. We propose that not all online tasks are designed for such mathematical practice. Hence, we describe perspectives on mathematical tasks from mathematics education literature that may help teachers and educators recognize particular characteristics of online tasks that are more likely to support these practices. Providing opportunities to students to interact with such tasks does not reliably result in deep understanding. Stein and Smith (2011) have developed and tested a set of strategies that teachers can use to support students in productive discussion (i.e., discussion that supports students in actively reflecting, analyzing strategies, and making mathematical connections).

Technology for Mathematical Reasoning and Sense-Making

In the following paragraphs, we describe recommendations for use of technology to support mathematical reasoning and sense-making. First, we explain what is meant by mathematical reasoning and sense-making and how digital technologies can allow students to encounter mathematical consequences of their actions. Next, we give examples of how online applets have helped students engage in these tasks.

The Association of Mathematics Teacher Educators (AMTE) developed a set of recommendations and standards for preparing beginning K-12 mathematics teachers. AMTE standards recommend that teachers are able to use technology in ways that support "mathematical reasoning and sense-making" and allow students to encounter mathematical consequences of actions (p. 11). Beginning teachers may have such opportunities in teacher preparation programs, but practicing teachers may have fewer opportunities to search for, choose, and implement such tools in their classrooms. In searching for online mathematical tasks and activities, teachers may find a wide variety of types of tasks. Some tasks are targeted to support teachers' evaluation of isolated skills, others provide interactions with procedures meant to support conceptual understanding, and still others are developed based on recognized needs for students to engage in rich, messy, or open mathematical tasks.

To provide background on these tasks, we provide some examples here. Many online applets and tasks have been developed by for-profit companies (e.g., IXL Math ixl.com/math/grade-8/graph-a-line-using-slope) to connect directly with content standards from the *Common Core State Standards in Mathematics (CCSSM, 2010)*. The mathematical tasks in such applets are intended as targeted assessments that provide feedback to teachers and students about students' performance. Many other online applets and tasks have been developed by educational researchers and mathematics educators (e.g., teacher.desmos.com/waterline). These applets and tasks are designed to allow students to encounter and experiment with mathematical consequences of their actions, to support mathematical thinking and conceptual understanding. Along with other types, these two types of applets and tasks--assessment/feedback and exploration/consequences--can be used in balance to support student growth. When student work is confined to only assessment/feedback tasks, however, they lose opportunities to develop deeper mathematical understanding and to gain expertise in using online tools and technologies to support their reasoning.

In this paper, we propose building on research and professional development opportunities about use of open, rich tasks to support productive discussion of mathematics to support teachers in similar use of open, rich online tasks.

Open and Rich Mathematical Tasks

Open, rich tasks are described from multiple perspectives. In this paper, we focus on five of these perspectives, although there are others that could be included. We focus on complex instruction and group-worthy tasks, cognitive demand of tasks, tasks that support mathematical connections, open tasks with multiple entry points, and authentic and relevant tasks. We briefly discuss these perspectives here to support later discussion about the use of these perspectives in categorizing online mathematical tasks.

One perspective is on complex instruction and group-worthy tasks (e.g., Featherstone et al., 2011). Group-worthy tasks include complex problems open to and

requiring multiple smartnesses. In engaging with such tasks, each student is a valued member of the team and students can learn from each other's strengths, especially when those strengths are not typically privileged.

A second perspective is on high cognitive demand tasks and tasks that require critical thinking (e.g., Stein, Grover, & Henningsen, 1996). Such tasks include problems that make students think and develop their own strategies and solutions. One goal of such tasks is to support students in developing and owning the ideas and strategies that emerge. High cognitive demand tasks are defined in a framework with lower cognitive demand tasks (e.g., memorization, procedures without connections) compared to higher cognitive demand tasks (e.g., procedures with connections, doing mathematics through complex, non-procedural thinking).

Mathematical tasks that support integrating funds of knowledge, past knowledge or understanding, and connecting to different types of knowledge (e.g., mathematical, other subject areas, lived experiences) are a third perspective (e.g., Aguirre et al., 2013). With a problem that integrates multiple topics, strategies, and mathematical understandings, students continue to make connections between classroom mathematics and previously learned knowledge from mathematics or other subject areas and from their lived experiences outside of school.

A fourth perspective focuses on multiple access or entry points (e.g., Boaler, 1998; Turner et al., 2012). Problems that allow students access at their own level of mathematical sophistication, bring challenges and growth to all students whether they are normally identified as struggling, average, or advanced. This type of task may be called "low threshold/high ceiling." Then, through discussion, students learn from each other's strategies and consider the mathematical ideas at a higher level, asking which strategies (and in which situations) are more efficient, more straightforward, or more valid.

A fifth perspective focuses on authentic and relevant tasks with meaningful contexts (e.g., Aguirre et al., 2013, Turner et al., 2012). With a problem that connects to students' real-world experiences in a way that is interesting and motivating, they can bring their prior knowledge, experiences, intuition, and previously developed problem-solving strategies into the classroom.

Five Practices for Productive Discussion of Mathematics

Stein and Smith (2011) described how teachers can build on tasks that meet the five criteria in the previous section; they explained that the tasks alone are not as effective in supporting learning as incorporating discussion that helps students put together their ideas and develop more sophisticated strategies. They proposed five practices for supporting productive mathematics discussions: (1) anticipating likely student responses, (2) monitoring students' actual responses, (3) selecting student strategies to be shared in the discussion, (4) sequencing shared strategies to support a learning goal, and (5) connecting mathematical ideas across strategies and to larger mathematical concepts. We describe each practice in more depth in the next paragraph.

A rich, open task should allow students to develop individual strategies, resulting in many student solution strategies emerging in one classroom (Stein & Smith, 2011). The student solution strategies will not all contribute to productive discussion; indeed, using more than a few strategies may be overwhelming to both teacher and students (Stein & Smith). Hence, the authors explain that teachers must develop their expertise in

selecting and sequencing strategies that support a particular learning goal. The strategies and discussion can result in very different stories depending on which strategies are selected and how they are sequenced (Stein & Smith). A teacher must plan a task by generating possible strategies that she might see and focusing on a handful that she hopes to see (Stein & Smith). This step is *anticipating student strategies* (Stein & Smith). During the implementation of the task, the teacher must engage in everyday classroom management along with *monitoring actual strategies*; she asks students questions, looks at their work, and marks down on her planning sheet to track which student groups have developed which strategies (Stein & Smith). As she monitors, she draws on what she had planned for the selecting and sequencing (Stein & Smith). She asks particular groups to share their strategies in the discussion (Stein & Smith). Finally, she orchestrates the discussion by asking students to share their strategies, and asking their classmates questions to keep them involved in analyzing the strategies (Stein & Smith). Each step is non-trivial in terms of effort and necessity.

Open Online Area Task

The *Strengthening Tomorrow's Education in Measurement (STEM)*, a multi-stage project funded by *National Science Foundation* with principal investigator Dr. John P. Smith, III, found that elementary mathematics curricula lack tasks that target student development of conceptual understanding of area measurement (Smith et al., 2008) As a response to this finding, the *STEM* project developed and adapted measurement tasks that could be accessed through physical manipulatives or online applets. One such area task is the "area of a puddle" task (see http://tinyurl.com/STEM-puddle) shown in Figure 1.

Figure 1. Open online area measurement task: Measuring the area of a puddle.
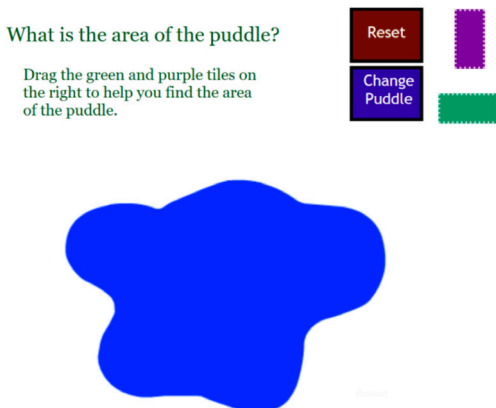
In the puddle task, the user is asked to drag green and purple tiles to the puddle (a blue, irregular shape) as units to measure the area of the puddle. The purple tiles and green tiles have the same measurements, but the purple has a vertical orientation and the green has horizontal. For each, one length is twice the measure of the other length. Neither type of tile exists in sufficient quantity to entirely cover the irregular shape. The designers chose elements of the manipulative and task deliberately to allow students

to experiment with different strategies, to make mistakes and reveal misconceptions. For example, because puddle is an irregular shape, and because the measuring units are rectangles, it is not possible for a student to find one valid solution. Rather, students must devise strategies to using the tools to estimate. The measuring units are rectangles rather than squares to support student thinking about filling an area, the meaning of area units, and why square units can be useful. The rectangles can be placed anywhere, so students might overlap the units or leave gaps. There are not enough rectangular units to cover the puddle with only one orientation, so students must use both orientations or develop other strategies for measuring. The designers intended to push students to use two orientations to support discussion about the meaning of the area formula; that the units are rectangles rather than squares supports useful discussion, even if the rectangles are all one direction.

Based on examination of the *Georgia Standards of Excellence in Mathematics: K-5* (an adaptation for Georgia students of *Common Core State Standards in Mathematics*), we created six potential learning goals. That is, by the end of a lesson, students should show ability to do one of the following: (a) give reasons to not leave gaps/overlaps (MGSE3.MD.5b, MGSE3.MD.6), (b) divide a whole area into equal area parts (MGSE3.G.2), (c) find strategies to partition a shape and add areas (MGSE3.MD.7c/MGSE4.MD.8), (d) building to area formula: Find strategies to count number of rows & columns (MGSE3.MD.7a), (e) reason about the meaning of the area formula for rectangles, specifically: describe reasoning about whether or not the area formula changes if rectangular units are used to measure a rectangle instead of square units or describe reasoning about whether or not area formula changes if rectangle units are two different orientations (MGSE3.MD.5a /MGSE5.NF.4), or (f) develop strategies that use over- and under- estimates to approximate a more accurate measure (MP5). Even though elements of each of the learning goals above could emerge through a productive discussion about a task, focusing the selection and sequence of strategies on one learning goal may make the discussion more manageable for both teachers and students.

## Strategies Card Sorting Task

The sorting task was originally developed in a mathematics methods course to support senior-level preservice teachers who were preparing to design and teach their first lesson. As a part of the lesson, the preservice teachers would use the Five Practices to lead a productive discussion after the students worked on a high-level task. Future teachers were divided in two groups: one group to engage with the task (as themselves first and then pretending to be third grade students) and the other group to monitor, select, and sequence strategies to support discussion. Based on their strategies, we created 21 strategy cards shown below, with brief descriptions. In the following sections we show selected strategies to illustrate their use in supporting productive discussions.

In Figure 2, six sorting strategies are shown. The strategies are chosen to illustrate *covering the space*, a method of measuring area that has a lower level of sophistication. In Strategy 1, the student has use 40 rectangles in two orientations to cover the space. The student left gaps and overlaps which reveals potential misconceptions about the meaning of area and the need for tessellation to ensure consistent results. Strategy 2 illustrates covering the space in a more systematic way. The rectangles are tessellated. Some rectangles hang off the irregular shape, while other parts of the shape are left uncovered.

Strategy 3 is more systematic than Strategy 1. There are no overlapping rectangles, and there are only a few overhanging rectangles, but there are many gaps left across the shape. Because each solution is different (40, 33, and 28 rectangles, respectively) these may support productive discussion about the need to avoid gaps and overlaps in order to obtain consistent results.

Strategies 18-20 are very similar and result in measures of 34, 32, and 33, respectively. They are systematically created with no gaps or overlaps in the central portion of the irregular shape. The rectangular units are shifted to different locations to cover as much of the enclosed space as possible, while leaving as little overhang as possible. These strategies could be used to discuss consistent results, in comparison with Strategies 1-3. They could also be used to discuss precision and limitations of measuring tools.

Figure 2. Low sophistication strategies: covering through tessellation or leaving gaps and overlaps.
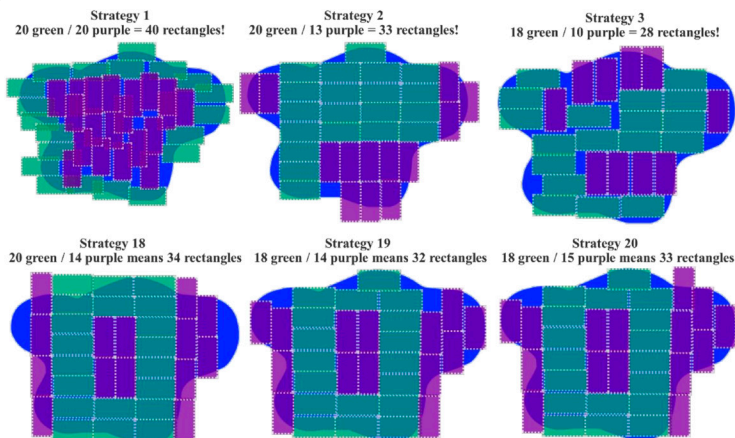
In Figure 3, Strategies 4-6 are shown with Strategies 17 and 21. These strategies show a slightly higher level of sophistication because the student covers only half of the irregular shape and then multiplies by two. The students seem to have attempted to tessellate the rectangles, and have different strategies for covering the space that lead to different solutions in Strategy 4, compared to the other two. Strategies 5 and 6 may be used to compare the same strategy with the same solution, but differently oriented rectangles. Strategies 17 and 21 also use the strategy of covering half and multiplying by two, but the lower and upper halves are covered rather than the left and right. The right and left side are less clearly different sizes, while the bottom side is clearly smaller than the upper side. In Strategy 17, some attempt is made to address this inequality by cutting the half along a diagonal rather than straight across. In Strategy 21, the rectangles trespass slightly into the upper portion to address the inequality between upper and lower sides. The solutions are similar, despite using different orientations of rectangles and measuring different portions of the irregular shape. Considering the five strategies together could support good discussion about the meaning of half of an irregular shape as well as the ways to estimate measures of half. Students can discuss whether the orientation of the rectangles matters in these estimates and how the two

orientations may be used strategically for better estimates. For example, in Strategy 6, two purple rectangles seem to be used to fill the space precisely.

Figure 3. Slightly higher sophistication strategies: covering half and multiplying by two.
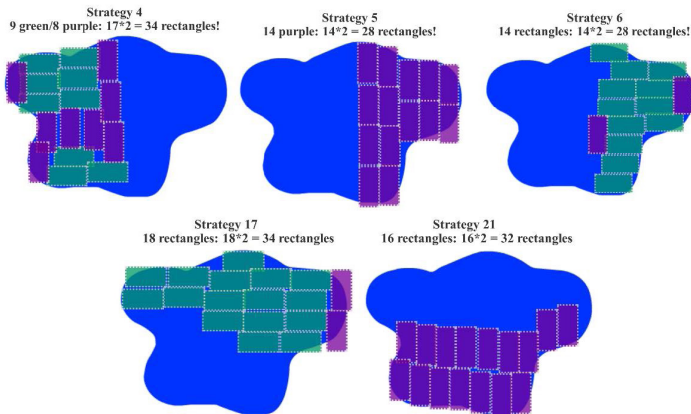
In Figure 4, we show Strategies 7-10. These strategies illustrate the use of the area formula for rectangles. They build on the strategy of dividing the irregular shape into two "rectangles." Rather than simply measuring and multiplying by two, however, they measure two regions and add to find the overall area. Adding the measures of two areas in this way seem to create a reasonably accurate estimate of the overall measure. Strategies 7 and 8 can be compared because the measure is the same for both, despite different orientations of rectangles. Students may discuss the meaning of multiplying to find area when rectangular units are used rather than square units. The area formula for rectangles can be used to find the number of rectangular units that cover a larger rectangle, because it is simply counting the number of objects in an array (number of rows multiplied by number of columns). Strategy 9 reveals an important misconception about the meaning of the area formula and its validity. In Strategy 9, the number of rows and columns loses meaning because two orientations of rectangles are used. This mismatch may result in questions about the imagined array: Are rectangles in the horizontal or vertical orientation in its rows and columns? Can it be both, or must it be only one? The solution is much lower than other solutions which indicates the strategy is invalid.

Figure 4. Sophisticated strategies: using tiles to find heights and lengths; subdividing the irregular shape in different ways for more accurate estimates.
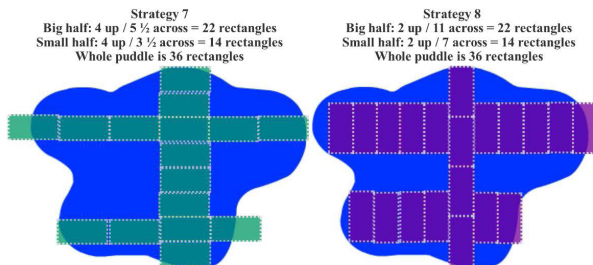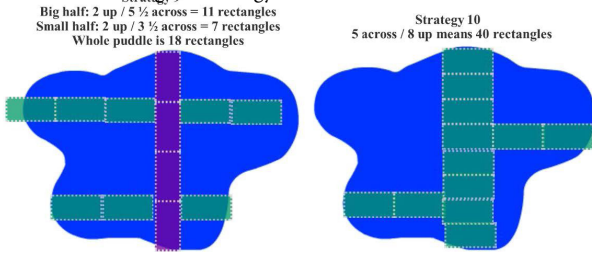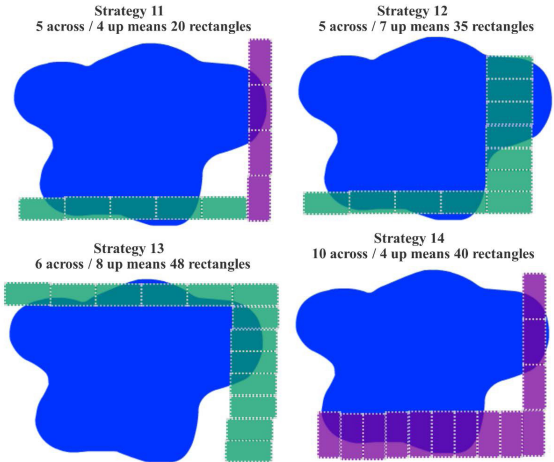
Strategy 9
Big half: 2 up / 5 ½ across = 11 rectangles
Small half: 2 up / 3 ½ across = 7 rectangles
Whole puddle is 18 rectangles

Strategy 10
5 across / 8 up means 40 rectangles

Strategies 11-14 are shown in Figure 5. These strategies illustrate a higher sophistication. Similar to Strategy 9 above, Strategy 11 can be used to question the meaning of the area formula when it is used with rectangular objects in arrays rather than square units found by multiplying lengths and widths. The resulting solution is similar to Strategy 9, and solutions from Strategies 9 and 11 are quite a bit lower than other solutions which may indicate to students that something is amiss. Strategies 12 and 13 further illustrate under-estimating and over-estimating the area. Students may visualize creating a box based on the placement of the green rectangles. Strategy 14 has a solution almost midway between those of Strategies 12 and 13. Students may discuss the accuracy of each estimate.

Figure 5. Sophisticated strategies involving the meaning of area and estimates.

Strategy 11
5 across / 4 up means 20 rectangles

Strategy 12
5 across / 7 up means 35 rectangles

Strategy 13
6 across / 8 up means 48 rectangles

Strategy 14
10 across / 4 up means 40 rectangles

Sorting the Cards to Tell a Story

There are many valid ways of selecting and sequencing student strategies when using the Five Practices for productive mathematics discussion (Stein & Smith, 2011). Teachers may choose student strategies to ensure that all students participate. At times, tracking participation and ensuring all students have a chance to show their strategies can be overwhelming for the teacher and the students. Another method is selecting students' strategies that illustrate particular conceptions and misconceptions to support student thinking. Our method presented here is to choose strategies to tell a story that supports the lesson learning goal; that is, to select and sequence student strategies allow comparison and analysis and that build on each other along levels of sophistication or complexity toward a natural conclusion, the lesson learning goal.

Selecting and sequencing to tell a story to students through student-centered discussion is challenging for teachers to do, with many complexities in implementation. This card-sorting task can help teachers think through possibilities without the pressure of the classroom. We provide two examples of selecting and sequencing the cards to tell a story through discussion to support a learning goal. In the first example, we present a potential selection and sequence to support two of the learning goals listed above: (c) students are able to find strategies to partition a shape and add areas (MGSE3.MD.7c/ MGSE4.MD.8) and (d) students build to area formula and are able to find strategies to count number of rows & columns (MGSE3.MD.7a).

Strategies for Partitioning Shapes and Adding Areas

Several of the student strategies might support discussion about partitioning shapes and adding areas. As one example, we select strategies 5, 21, 8, and 7 (shown and described in more detail above). We show their sequencing in Figure 6 below.

Figure 6. Strategies selected and sequenced for (c) strategies to partition a shape and add areas.
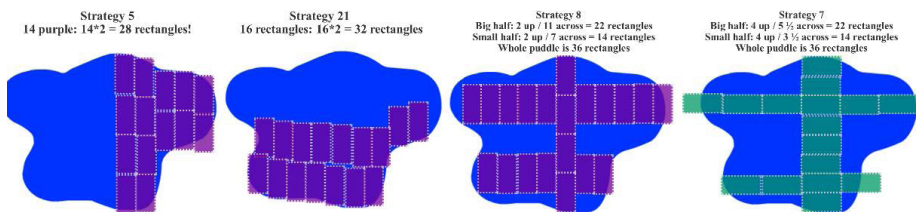
We selected strategies 5 and 21 to support students in thinking about two ways that the irregular shape (the puddle) can be partitioned into two parts. In these strategies, discussion might focus on the meaning of *half* for an irregular shape; that is, that dividing the shape into two equal parts is difficult in this situation. Students can discuss why dividing the shape into two parts vertically results in a fairly different solution than dividing the shape in half horizontally. We chose the two strategies because they both use the same orientation of rectangles. Moving from Strategies 5 and 21 to Strategy 8, may help students connect to a more sophisticated approach. In Strategy 8, the student divided the irregular shape into two parts, but noticed that the two parts are different in size. The student used the area formula to measure the area of each part and then to add the areas. This strategy leads to Strategy 7 where the two parts are measured using the area formula, but with rectangles in a different orientation. Although the strategies are different, the solutions in Strategies 8 and 7 are the same which could be surprising and might support discussion about the way the area formula works when rectangles are used rather than squares.

This sequence of strategies then can lead discussion that focuses on making connections between strategies for partitioning shapes using the covering method to using area formulas. The discussion may support analysis of strategies for efficiency and validity. That is, students can discuss efficiency of strategies: as shapes grow larger, time and materials become more important; covering the space uses more time and more materials than measuring the length and width. Students can also discuss validity of strategies: covering one part of an irregular shape leads to less precision than measuring

the length and width of both parts. Student discussion can support connections between efficiency, validity and precision, and meanings of *half* and of units and the area formula.

Strategies for Building to the Area Formula

In the previous example, the strategies could support thinking about the meaning of the area formula. In this example, we select strategies to specifically target this learning goal. As in the previous example, many strategies might be chosen; we choose Strategies 2, 11, 10, and 7 shown in Figure 7 that could support this story. (These strategies are shown and described in more detail above.)

Figure 7. Strategies to build to area formula and to count number of rows & columns.
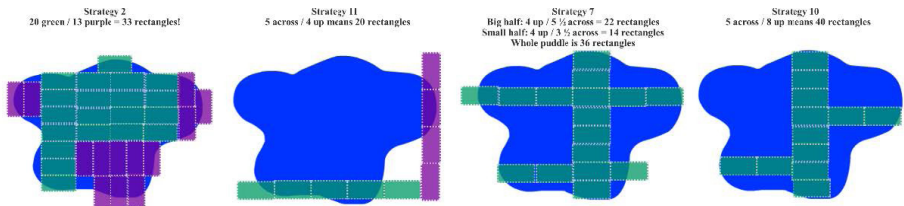
We selected these strategies to tell a story, moving from (2) covering the entire irregular shape with rectangles (both orientations must be used because the students run out of rectangles if they try to use only one orientation), to (11) measuring length and width with two orientations (which is problematic for the area formula and results in a much smaller measure), to (10) using one orientation and measuring the longest length and longest width, to (7) using one orientation, taking two measures of width, and using fractional parts of a rectangle in the solution.

Similar to the previous example, strategies can be analyzed for efficiency of time and materials, validity, and precision. In Strategy 11, some discussion can explore the meaning of the area formula. Students may notice that this measure is much smaller than measures resulting from other strategies (including their own strategy and comparing to Strategy 2). Students may discuss the consequences of multiplying rectangles in two orientations. Students could discuss the differences in units: rectangles, green rectangles, purple rectangles, squares. One consequence can be shown by comparing the results when adding all of the green and purple rectangles and then multiplying the height and width; this comparison could support a discussion about the result of multiplying green and purple rectangles (does the multiplication result in rectangular units or square units?). A second consequence that can be discussed is that this measure actually can be accurate and meaningful if the units are considered as squares rather than rectangles. In moving from Strategy 2 to Strategies 10 and 7, students can discuss the precision of the area formula when estimating the area of an irregular shape.

Conclusion

Teachers need support in developing expertise over time and through community and collaboration with other teachers. In the first year they use a particular task, they may not know what strategies will emerge from their own students. Sharing strategies from other teachers' classrooms can help them anticipate. Over time, as they gather

strategies from students through particular tasks, they can develop their use of those strategies to support more rich and complex discussions. In this paper, we discuss one open task and how strategies from the task might support six learning goals across third, fourth, and fifth grade standards.

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