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A Note on Nonoscillatory Solutions for Higher Dimensional Time Scale Systems

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Abstract

In this paper, we focus on nonoscillatory solutions of two (2D) and three (3D) dimensional time scale systems and discuss nonexistence of such solutions.

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Introduction

The motivation of studying dynamic equations on time scales is to unify continuous and discrete analysis and harmonize them in one comprehensive theory and eliminate obscurity from both. A *time scale* T is an arbitrary nonempty closed subset of the real numbers R. The most well known examples for time scales are R (leading to dierential equations, [10]), Z (leading to dierence equations, [11]), and $q^{\mathbb{N}_0} := \{1, q, q^2, \ldots\}$, $q > 1$ (leading to q-dierence equations, [9]). In 1988, the theory of time scales was initiated by Stefan Hilger in his Ph.D. thesis, [8]. Since the calculus of time scales has been recently developed, we give a brief introduction to time scales calculus. Nevertheless, an excellent introduction can be found in [5, 6] by Bohner and Peterson.

There are two jump operators: For t ε T, the *forward jump operator* σ: T → T is given by σ(t):=inf {s ε T: s>T} for all t ε T while the *backward jump operator* ρ: T→T is dened by ρ(t):= sup{s ε T : s < t} for all t ε T. Finally, the *graininess function*: μ: T → [0,∞) is given by μ(t):= σ(t) - t for all t ε T.

We dene inf ∅ = sup T. There are four types of points in T. If σ(t)>t, hen t is called right - scattered, while if ρ(t)<t, t is called *left - scattered.* If *t* is right and left - scattered at the same time, then we say that *t* is *isolated*. If t < supT and σ(t)=t, then *t* is called *right - dense*, while if t > inf T and ρ(t)=t, we say *t* is left - dense. Also, if *t* is right and left - dense at the same time, then we say that *t* is *dense*.

If supT < ∞, then T^κ=T\(ρ(sup T), sup T], and T^κ=T if sup T=∞. Suppose that f : T→R is a function. Then f^σ: T→R is dened by $f^\sigma(t)=f(\sigma(t))$ for all t ε T. For any ε > 0, if there exists a δ > 0 such that

$$|f^\sigma(t) - f(s) - f^\Delta(t)(\sigma(t)-s)| \le \varepsilon |\sigma(t)-s| \text{ for all } \quad s \in (t-\delta,\ t+\delta) \cap \mathrm{T},$$

then *f* is called *delta (or Hilger) dierentiable* on T^k and f^Δ is called *delta derivative* of *f*. Let *f* : T→R be a function with t ε T^κ. If *f* is dierentiable at *t*, *f* is continuous at *t*. If *f* is continuous at *t* and *t* is right-scattered, then *f* is dierentiable at *t* and

$$f^\Delta(t) = \frac{f^\sigma(t) - f(t)}{\mu(t)}$$

If *t* is right dense, then *f* is dierentiable at *t* if

$$f^\Delta(t) = \lim_{s \to t} \frac{f^\sigma(t) - f(s)}{t - s}$$

exists as a fnite number. If *f* is dierentiable at *t*, then $f^\sigma(t)=f(t)+\mu(t)f^\Delta(t)$ holds for all type of points in T. Let f, g : T→R be dierentiable at t ε T^κ. Then we have the product and quotient rules for derivatives as follows:

1. $(fg)^\Delta(t) = f^\Delta(t)g(t) + f^\sigma(t)g^\Delta(t) + f^\Delta(t)g^\sigma(t)$
2. If g(t)g(σ(t))≠0, then $\frac{f}{g}$ is dierentaible at t with

$$\left(\frac{f}{g}\right)^\Delta(t) = \frac{f^\Delta(t)g(t) - f(t)g^\Delta(t)}{g(t)g^\sigma(t)}$$

f : T→R is called *right dense continuous* (rd-continuous), denoted by C_{rd},C_{rd}(T); or C_{rd}(T,R), if it is continuous at right dense points in T and its left sided limits exist as a nite number at left dense points in T. Throughout we denote continuous functions by *C*. Also, the Cauchy integral is dened by

$$\int_a^b f(t)\Delta t = F(b) - F(a) \text{ for all } a, b \in \mathrm{T}$$

Every rd-continuous function has an antiderivative. Moreover, F given by

$$F(t) = \int_{t_0}^t f(s)\Delta s \text{ for } t \in \mathrm{T}$$

is an antiderivative of *f*.

Higher Dimensional Time Scale Systems

The study of higher dimensional time scale systems in nature and society has been motivated by their applications such as population dynamics, genomic and neuron dynamics and epidemiology in biological sciences, [1,16] and in astrophysics, gas dynamics and fuid mechanics, relativistic mechanics, nuclear physics, and chemically reacting systems, [4,7,12,17].

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Recently, nonoscillatory solutions of 2D and 3D time scale systems have been considered in [13-15] and [2,3], respectively. One of our main goals in these articles is to nd integral conditions which eliminate nonoscillatory solutions of given systems. Depending on which dimension we deal with one naturally attempt to nd single, double or triple integral conditions. A question we have here is whether we can eliminate all types of nonoscillatory solutions or not. In this contribution, we only satisfy single integral conditions for both time scale systems and other integral conditions can be found in references above. Our approach is based on the sign of components of nonoscillatory solutions of systems and we assume without loss of generality that the rst component of such solutions is always positive.

We rst start with the following 2D time scale system

$$\begin{cases} x^{\Delta}(t) = a(t)f(y(t)) \\ y^{\Delta}(t) = -b(t)g(x(t)), \end{cases} \qquad (2.1)$$

where a, b ε C_{rd} ($[t_0,\infty)_T$, R^+) and f, g ε C(R,R) satisfying *uf(u) > 0, ug(u) > 0* for u≠0 and

$$a, b \in \mathrm{C}_{\mathrm{rd}}([t_0,\infty)_{\mathrm{T}}, \mathrm{R}^+) \qquad (2.2)$$

where *F* and *G* are positive constants and Φ_α, and Φ_β are old power functions, i.e., $\Phi_p(u)=|u|^p$sgn u, p > 0 and p ε {α, β}.

Throughout this article, we assume that T is unbounded above. We call *(x, y)* a *proper solution* if it is dened on $[t_0, \infty)_T$ and sup{|x(s)|, |y(s)| : s ε $[t, \infty)_T$} > 0 for t ≥t_0. By t ≥t_1, we mean t ε $[t_1, \infty)_T := [t_1, \infty)\cap T$. A solution *(x, y)* of (2.1) is said to be *nonoscillatory* if the component functions *x* and *y* are both nonoscillatory, i.e., either eventually positive or eventually negative. Otherwise, it is said to be *oscillatory*. Denitions above can be modied for higher dimensional time scale systems.

Assume that *(x, y)* is a nonoscillatory solution of system (2.1) such that *x* oscillates but *y* is eventually positive. Then the rst equation of system (2.1) yields $x^{\Delta}(t)=a(t)f(y(t))>0$ eventually one sign for all large t ≥ t_0, a contradiction. The case where y is eventually negative is similar. Therefore, we have that the component functions x and y are themselves nonoscillatory. In other words, any nonoscillatory solution (x, y) of system (2.1) is one of the following types:

Type (a): sgn x(t) = sgn y(t);
Type (b): sgn x(t) ≠ sgn y(t).

For convenience, let us set

$$A(t_0) = \int_{t_0}^{\infty} a(t)\Delta t \text{ and } B(t_0) = \int_{t_0}^{\infty} b(t)\Delta t, \ t_0 \in \mathrm{T} \qquad (2.3)$$

Theorem

Any nonoscillatory solution of system (2.1) cannot be of

1. *Type (a) if $A(t_0) < 1$ and $B(t_0) = 1$.*
2. *Type (b) if $A(t_0) = 1$ and $B(t_0) < 1$.*

Secondly, let's consider 3D time scale systems of the form

$$\begin{cases} x^{\Delta}(t) = a(t)f(y(t)) \\ y^{\Delta}(t) = b(t)g(z(t)) \\ z^{\Delta}(t) = -c(t)h(x(t)) \end{cases} \qquad (2.4)$$

where a, b, c ε $C_{rd}([t_0,\infty)_T,R^+$ amd f, g, h ε C(R, R) satisfying uf(u) > 0, ug(u) > 0, and uh(u) > 0 for u ≠ 0 and

$$\frac{f(u)}{\Phi_\alpha(u)} \geq F, \ \frac{g(u)}{\Phi_\beta(u)} \geq G, \ \frac{h(u)}{\Phi_\gamma(u)} \geq H \text{ for all } u \neq 0, \qquad (2.5)$$

where F, G, and H are positive constants and $\Phi_p(u)=|u|^p$sgn u, p>0 and p ε {α, β, γ}. By a similar argument, we can conclude that any nonoscillatory solution (x, y, z) of (2.4) is one of following types:

Type (a): sgn *x(t)* = sgn *y(t)* = sgn *z(t),*
Type (b): sgn *x(t)* ≠ sgn *y(t)* = sgn *z(t),*
Type (c): sgn *x(t)* = sgn *y(t)* ≠ sgn *z(t),*
Type (d): sgn *x(t)* = sgn *z(t)* ≠ sgn *y(t).*

For convenience let's set

$$c(t_0) = \int_{t_0}^{\infty} c(t)\Delta t, \ t_0 \in \mathrm{T}$$

Lemma

Any nonoscillatory solution of system (2.4) can not be of

9. Type (a) if $C(t_0) = 1$.
10. Type (c) if $A(t_0) = 1$;
11. Type (d) if $B(t_0) = 1$.

Not only from Lemma 2.2 but also from other results in above references one can observe that Type (b) solution of system (2.1) is not eliminated. In fact, it is worth to emphasize that components of Type (b) solutions have nite limits. Note that when we deal with 2D time scale systems such type does not occur. Here is the question comes to our mind: Is it related with dimensions of given systems? When we focus on four dimensional (4D) time scale systems, we realize that nonoscillatory solutions whose components have nite limits do not occur either. Moreover, we have already known that all types of nonoscillatory solutions of 4D time scale systems can be eliminated by some integral conditions. Therefore, we would like to emphasize that this is a matter of considering odd or even dimensional time scale systems.

Competing Interests

The authors declare that there is no competing interest regarding the publication of this article.

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