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Application of statistical methods to evaluation of mineral deposits

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APPLICATION OF STATISTICAL METHODS TO EVALUATION OF MINERAL DEPOSITS

BY

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ABSTRACT

Only in recent years have statistical methods been applied to evaluation of ore reserves. An attempt is made in this study to prepare an outline to be followed throughout the exploration in order that statistical methods can be applied to valuation problems. Particular emphasis is given to the preliminary requirements to a statistical approach. Representative parameters, sampling pattern, and sample size are discussed in separate chapters.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

Chapter I

INTRODUCTION

Statistical methods have been employed for only a few years by the mining industry in operational control and in mineral exploration. The higher number of variates to be considered, the impossibility of complying exactly with the requirements for random sampling, the long delay between the elaboration of an hypothesis and its verification, may account for the slowness of adoption, compared with other fields of industry.

Almost all of the research has been carried out in the last ten years and it has stagnated in theoretical evaluations. Besides university teachers and some official organizations (e.g. U. S. Bureau of Mines in the U.S.A., Commissariat a l'Energie Atomique in France) only a few mining companies have applied and tested these methods (e.g. Climax Molybdenum, San Manuel, Bear Creek).

The existing bibliography (approximately 250 articles and papers to date as of January 1st, 1959, with exception of the almost unknown Russian literature) covers only articles which are generally concerned with a specific problem. So far no attempt at a general publication has been made. The aim of this study is to prepare an outline for application of statistical methods for the computation of ore reserves of a deposit during the exploration phase. This exploration is assumed to be carried out by drilling, before any underground mining has really taken place.

Numerous other problems in Mineral Engineering can be solved by application of statistical methods: mine sampling (27)*, location of favorable areas for exploration (13), or the probability of finding a given amount of a certain commodity over a certain area (52) are a few examples of applications.

What are the advantages of the statistical methods over the classical or numerical methods used in the computation of ore reserves?

First of all they are better fitted to approach problems on which only dispersed and fragmentary data are present. The few representative parameters to which a set of data is "cooked down" are not only given with their naked numerical values, but also with their level of confidence. Trends in variations can be detected.

Besides these more technical advantages, there is also a financial advantage, as the investor obtains a better knowledge of the risks presented by a mining venture. This feature will probably be more and more prevalent with the development of huge, low grade operations requiring greater capital investments.

A good introduction to the statistics used in this study will be found in Freund (5). More detailed textbooks on the theory are listed in the bibliography.

For the convenience of those not familiar with statistical studies, a brief glossary of terms is included in the appendix.

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* All references are in bibliography.

Chapter II

ANALYSIS OF THE CONDITIONS FOR A STATISTICAL APPROACH TO ORE-RESERVES COMPUTATIONS

In order to be possibly submitted to statistical methods, data, whatever may be their nature, have to fulfill the following conditions:

- sufficiently large number of samples
- similarity of samples
- randomness in sampling and distribution

These conditions can be expressed in terms of drilling and sampling.

A. Number of data

The theory of statistics usually requires that each statistical study be based upon a minimum of 30 homogeneous samples, in order to ascertain valid results. This figure is exceeded in almost every mineral exploration project. Less freedom in the sampling of a deposit, than is offered in fabrication control, and a very small ratio between volumes of samples and deposit justify and require this larger number.

Knowing the optimum number of samples is of importance to the mining engineer exploring a deposit. It can be approximated by different ways:

(1) Theory of statistics

Relations have been established between number of samples, mean and standard deviation of the statistical universe and standard deviation of the sample mean (with respect to the true mean) as well as of the standard deviation of sample (with respect to the true standard deviation). These relations enable us to determine, once a first set of data is obtained, how far the sampling has to go in order to ascertain a certain level of confidence in the final result.

This approach by the theory of statistics has some disadvantages, as it is based on the assumption of normal distribution of sample mean around true mean. This is correct in a normal distribution, but only approximated in other distributions, however, it is satisfactory if the groups of samples are large. Nevertheless this method gives us a minimum, since in the case that the assumption of normal distribution of sample mean is not correct more samples will be required in order to give the same level of confidence.

(2) Limit of the standard deviation

Common sense indicates that the standard deviation will decrease with an increasing number of samples. A cumulative plot of standard deviation vs. number of samples, in their chronologic order of appearance can be expected to approach a limit, which, under ideal conditions, is the standard deviation of the entire population. If a further increase in the number of samples does not decrease appreciably the standard deviation, there is no need in further sampling, and it can be stopped.

In order to obtain a limit for the standard deviation, the samples have to belong to the same population. It can happen that with an increasing number of samples the standard deviation oscillates in a wide range. In that case the superposition of two, or occasionally more populations has to be examined. If the different populations are spatially separated it is of advantage to introduce a stratification, and to make a separate plot for each zone or strate.

The exploration of a zone can then be considered finished if the standard deviation related to it nears the limit.

It may not be possible to relate the various populations to different parts of the deposit. In that case a close geological investigation should reveal if the same commodity can be found in different minerals, with different contents and grades, or if there are several distinct mineralization periods.

(3) Financial considerations

Filippo Falini (27) emphasizes the cost of sampling. The other items he considers are initial plant cost and ratio between the annual working expenses and the depletion and depreciation allowances.

As shown before, there is a technical limit to the accuracy of the information obtained through sampling, and there is a limit beyond which a higher number of samples does not bring more accuracy, only confirmation. Therefore Falini's proposition to increase the number of samples in proportion to initial cost and working expenses may lead to unnecessary expenses. However his concluding statement corrects this fault:

"The number of samples to be taken is limited by the consideration that, beyond a certain number the risks of economic loss in the exploitation (or of loss of profit if it is decided to not exploit the deposit) no longer offsets the costs of further samples."

Practically, the number of samples to be taken can be determined in advance only if the characteristics of the deposit are already known, by comparison with similar neighboring deposits. For instance, in a wildcat exploration the first drillings will give the information necessary to determine the approximate number of samples.

The decision to stop a drilling campaign should be taken as soon as the optimum number of samples has been reached, the optimum being given by the combined Methods No. 2 and No. 3, whichever may give the smallest number. Method No. 3 is usefully checked by the theoretical approach.

It can be noticed that in order to obtain the maximum information of a drill hole the cores should be cut in reasonably short length (1' to 3'). All the samples in a deposit must be of the same length to prevent overweighting or underweighting any hole.

B. Similarity of samples

In mineral exploration it is often difficult to fulfill the requirements of similarity. In order that each sample shall influence the total average grade to the proper degree, it is supposed to have the same weight (influence), which factor can be, for instance, the weight or the volume.

As Filippo Falini (27) pointed out, the degree of dispersion (defined as the ratio of the average deviation of grade to the mean grade) of samples assays taken in a homogeneous deposit is an inverse function of the number of elementary particles in the sampling volume and of the grade (by volume). We prefer to substitute the standard deviation for the degree of dispersion, the former being more commonly used.

In order to have similar samples, the number of particles should be constant. Their total weight (i.e., the weight of the sample) is a function of the densities and proportions of the different minerals present. The volume is independent of densities and proportions and depends essentially on the number of elementary particles, therefore it is the quantity to be kept constant.

An example of lead ore formed by galena (density 7.5) and dolomite (density 2.5) shows convincingly the variation in weight of a given volume of ore as the relative proportions of the two components change.

Volume galena	Volume gangue	Density of ore	Percent galena by weight
1%	99%	2.55	2.94
10%	90%	3.00	25.0

It would be interesting to study the application of the principle of constant volume to channel sampling. It would probably mean that in a vein with varying width, the channels would have to be subdivided in small lengths, 1 foot for instance, so that the excess at the ends of a channel would be closer to a sample length.

Two remarks should be made:

The constancy of volume is hampered by an imperfect core recovery, which has a strong effect on the assays. For instance if the explored deposit contains sulphides which are concentrated in planes an underevaluation is likely to occur. The cores are susceptible to breakage along the sulphide planes, so that the valuable minerals are eroded and pass into the slime.

The assays are given by weight, not by volume. In the case of heavy minerals (about twice the density of the gangue or above) the density contrast introduces an important bias. This problem will be discussed in a Report of Investigations of the U. S.

Bureau of Mines by Leonhard W. Becker and Scott W. Hazen, Jr.

The samples have to come from statistically homogeneous zones in order that representative parameters can be computed for each zone. Such subdivision of a deposit, if any, may have been produced by a difference in:

(1) Process: supergene vs. hypogene, for instance.

(2) Time: different periods of mineralization.

(3) Source of the material.

(4) Host rock: according to physical and chemical behavior.

In a frequency diagram, several populations belonging to different zones are likely to appear as a plurimodal curve. If this is the case a separation of the superposed populations is to be attempted by one of the following methods.

A new zonation procedure for oil fields, with respect to the permeability parameter, has been proposed in 1958 by LeRoy Allan Beghtol (51). This method can be usefully applied to the detection of distinct vertical grade zones within the profile of each hole. Between the different assays obtained from one drilling exist variations, from which a part only, are due to chance alone. Variance tests, worked out by Ronald A. Fisher (4) and others, are used to detect significant differences at any preset level of confidence. Same zones, appearing in neighboring holes are then connected.

A statistical approach to the valuation of gold placers has been proposed by V. Baty (32). Gold in placers is assumed by this author to have three modes of deposition. The first two of which result in homogeneous distributions, following the normal law, whereas the third does not answer to any statistical law. Mean and standard deviation of the entire set of data are computed, and the contents deviating from the mean by more than two times the standard deviation are eliminated. Mean and standard deviation are now computed for the remaining contents, which enables us to divide them (as in the first elimination) in two fractions:

(A) Homogeneous content of the placer

(B) Leftover contents

The latter are plotted on the prospection plan, what gives an indication of probable runs. Therefore (B) is divided in

(C) Heterogeneous content of the placer, not belonging to the run

(D) Run contents

The run contents (D) are distributed in

(E) Normal run contents

(F) Heterogeneous run contents

The second point in Baty's study is concerned with the valuation of reserves. Designating by A the tonnage of the group (A), by t_a the content of this group, and so forth, and assuming that (A) and (C), as well as (E) and (F) have respectively the same content, Baty derives following formula giving T the mean content of the deposit:

$$T = \frac{(A + C)\ t_a + (E + F)\ t_e}{A + C + E + F}$$

The third point of this study is concerned with the graphical determination of placer content, run content and the heterogeneous values. The plottings of his figures 1 and 2 are merely frequency distributions. Through an unfortunate choice in the intervals (too large) the usual skewed curve, which can be fitted to a lognormal curve, does not appear entirely. Only the descending branch is present.

The best usage of this method is to be found in the graphical plotting of the runs on a map. The graphical zonation procedure can be used, with a slight modification, however. The assumption of normal distribution has to be checked in each case. Ordinarily a lognormal distribution will be found, so that in the operating modus the assays have to be replaced by their logarithms.

For valuation purposes this method is objectionable. The author considers a run as a superimposed phenomenon (though homogeneous) on a normal and homogeneous background, which is still present under the higher values of a run. In a placer a run can hardly be considered being epigenetic with respect to the normal mineralized volume, so that there cannot be superposition. (Baty does not specify explicitly that a run is epigenetic, but this is a logical conclusion of his hypothesis on the assay distribution in a placer.) As long as the distribution of assays is normal, there can be no objection to this formula. In the case of a skewed distribution it should be abandoned.

The frequency curve has a continuously varying slope, not a slope in three parts as suggested. A unique solution can therefore not be obtained graphically.

C. Randomness in sampling and distribution

This characteristic may be discussed under two headings:

(1) Randomness of distribution of the assays in the deposit.

This cannot be controlled by the sampler, and the methods of sampling have to submit to imposed conditions. Friedrich Stammberger (43) gives two criteria for a random distribution; the grade of the sample should not interfere with the grade of a neighboring sample, nor should it be influenced by its position in space.

The interdependency of assays coming from two neighboring holes appearsin a regression study. A close relation is to be expected in bedded deposits: minette iron ore (England, France and Germany) or Lake Superior iron ore (United States, Brazil, Canada). These justify a wide spacing of the drill holes. It would be interesting to make a survey of the behavior of different genetic types in that respect. A study of correlation has been carried out at Climax, Colorado. After an oral communication with Mr. Hazen, from the U. S. Bureau of Mines, the coefficient of correlation between samples coming from two holes, three feet apart has been insignificant (0.09, from a set of more than fifty samples). The correlation between successive samples taken along the same hole has been higher, 0.9.

Different commodities may have a different behavior in the same deposit. For instance in a deposit in Aroostook County, Maine, the iron shows a high correlation between neighboring holes, whereas manganese is not correlated at all. A closer geological investigation shows that the iron is still in banded form, but that manganese underwent remobilization and recrystallization in small fissures.

A remobilization will not always have the effect of randomnization. According to the distance of transportation of the remobilized material and sample size, two different results can be produced:

- small ratio: (for instance regrouping in little and close fractures) the distribution will be randomized.
- large ratio: (for instance cementation) the distribution is organized and zonation is produced.

The influence of position in space is difficult to eliminate. A deposit can have limits of two sorts. Geological limits can be found in a vein for instance, where the ore has usually a definite boundary with the wall rock. In a porphyry copper deposit the limit of what is called ore is purely economical, as it is determined by a cutoff grade which varies according to the circumstances. In the last case a good valuation procedure requires a zonation according to the grade.

(2) Randomness of sampling

A random sampling procedure requires that one sample does not have more chances to be selected than another. The human factor does not have much influence in selecting a sample in exploration (whereas in mine sampling great attention has to be given to this bias). Drill sampling cannot be considered completely random. The location may be random, so will be the first length of core, but the location of all the following cores is determined. Therefore each hole has to be considered as one sample.

The method of obtaining a random sample will be discussed extensively in the chapter on sample location.

(3) Tests of randomness

a. Data coming from one hole. In the case of data coming from a certain number of holes it is impossible to use the traditional tests of randomness, all based upon the order of production of the data. However data coming from one hole can be analyzed that way, if they are numerous enough and if trends do not already appear at visual examination. The following four methods are indicated by Hald (7):

Runs above and below median. All the data can be divided in two groups: data smaller than the median, and data larger than the median. A sequence of data of the same kind is called a run. Assuming that all arrangements have the same probability of occurrence, the probability of a run of given type and length can be computed, so checking if the material is random.

Runs up and down. Consider a sequence of observations and the sequence of signs (+) or (-) of the successive differences of the data. A sequence of signs (+) is called a run up, a sequence of signs (-) is called a run down. Here also the probability of occurrence of a run of given type and length can be computed and checked against the observation data.

Mean square successive difference. This figure is divided by the variance. This ratio has a significance level, which is a function of the number of data.

Subgroups. If at least one hundred data are given, they can be divided in subgroups of four data each. Mean and standard deviation of each subgroup are computed and their distribution checked. If the sampled material is randomly distributed, the mean and the standard deviations of the subgroups follow a normal distribution.

And at last, de Wijs' coefficient of variability (48), which will be explicitly described in Chapter IV, can also be considered a test of randomness.

b. Data coming from a set of holes. The average grade for each hole could be computed and preceding tests applied to these average grades in their chronologic order of appearance. There is however less significance to the order of execution of drill holes than to the succession of assays in a hole.

In the already mentioned Report of Investigations, Leonhard W. Becker and Scott W. Hazen indicate the following test which seems more adequate: using v_n to denote the volume of a sample of the class n, s_n the standard deviation of that class, if the distribution is random, following relation is verified:

$$v_n \cdot s_n^2 = \text{constant}$$

Let a hole be subdivided into successive core sections c_1, c_2, c_3, ... of same volume v. Standard deviations are computed for different populations of volume V_1, V_2, V_3, such as:

Volume	Units	Standard deviation
$V_1 = v$	$v_1, v_2 \ldots$	s_1
$V_2 = 2v$	$v_1 + v_2;\ v_3 + v_4 \ldots$	s_2
$V_3 = 3v$	$v_1 + v_3 + v_3;\ v_4 + v_5 + v_6 \ldots$	s_3

The distribution of grades in the deposit can be considered random (and also for the applied sampling procedure) if following relation is checked:

$$V_1 \cdot s_1^2 = v_2 \cdot s_2^2 = v_3 \cdot s_3^2$$

or:

$$s_1^2 = 2\ s_2^2 = 3\ s_3^2$$

If the distribution is not random there is no further advantage in using statistical methods, and classical methods of computing ore reserves should be applied.

Chapter III

DISTRIBUTIONS OF ASSAYS AND GRADES

A. Distribution as function of the geological type of the deposit

So far this problem has not received much attention. Fernand Blondel (18) studied statistics of the copper production in the United States and arrived at the following conclusions:

(1) The grades are fairly constant for 50% of the American copper production.

(2) The data are much less explicit for the remaining 50% and are only given by district, county or state.

(3) Variations in grade are of two types:

a. Sharp increases due to the development of new properties, which begin their output by cementation and oxidation material, that is high grade ores.

b. Slow decrease due to the exhaustion of these enriched zones and extension of the operations to the lower grade protore.

In another article (19), published the same year, Fernand Blondel extends his conclusions to other metals, gold and tin particularly. A cumulative negative frequency curve (i.e. tonnage of ore mined below a certain grade) has a stairlike shape. Over a certain period of time some fluctuations are noticeable around the modal values. What is even more important: the record of reserves follows the same curve, but the modal levels have slightly higher values this time. This is to be explained by an imperfect recovery (mining factor), which lowers the grade.

From these observations Blondel deducts the following hypothesis:

"The grades are not distributed at random. To each type of deposit corresponds an average grade, around which the highest tonnages are concentrated. These types are not numerous and form a series of discrete figures."

The preceding statements call for a few remarks. The hypothesis of the distribution of the grades around a few modal values seems extremely logical. There are no objections to the determination of the grades by the physicochemical conditions under which the deposit was formed, as these conditions are closely related to the geological type. This study has been carried out on copper mainly and it would have been instructive to read actual figures. A recent book on the porphyry coppers (53) supports this hypothesis by the figures which it gives. Annual production of ten copper mines in both Americas:

Tonnage above	1.0%	5.00 millions of tons	5.8%
-- at	1.0%	14.90	17.2%
-- -	0.9%	45.9	53.2%
-- -	0.8%	20.5	23.8%
Total production		86.3	100.0%

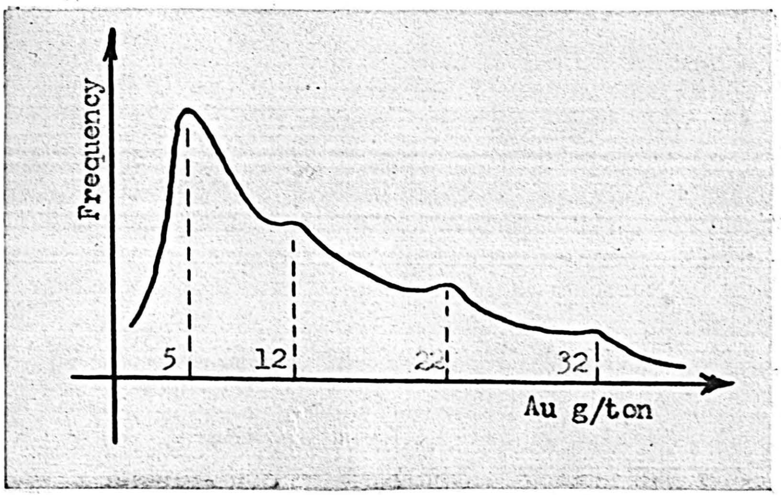
This shows that in the porphyry coppers the grade is fairly constant. Unfortunately it is not possible to plot a frequency curve of grades vs. tonnages for all rocks presenting the characteristics of a porphyry copper, simply because detailed investigations are only carried out where some indices are already present. In other words it is impossible to know if there are metal concentrations below the present cutoff grade, but above the geochemical trace content, and what is their tonnage. This would show whether the indicated modes are significant or not. It can be noticed that the geochemical trace content depends upon the host rock, which is a proof of relation between grade and environment.

In narrow veins obviously the grade will be lowered through wall rock, broken in order to obtain a minimum stoping width. In the case of the porphyry coppers this is less obvious, and the discrepancy can probably be explained like it has been for the Witwatersrand as we will see in the discussion on the arithmetic mean.

It would be useful to carry out a survey of all ore deposits, to check if there is actually a trend toward a few modal values. Such a survey, in order to respect small changes in the physicochemical conditions, would have to follow a very diversified classification, like Schneiderhoehn's (55). Should some results appear the number of types of deposits should be reduced, and the characteristics of more comprehensive groups studied. Not only is the average grade to be considered, but also the dispersion of the grades, which can be characteristic of the constancy (or the absence thereof) of the genetic conditions, as well as of the degree of dependability of the grade toward these conditions.

A part of this program is being carried out by Professor Henri Lepp, of the Geology Department of the University of Minnesota. In order to prove a sedimentary origin for the Lake Superior iron ore deposits, he studied several other districts where the iron is undoubtedly of sedimentary origin. As a temporary conclusion he could state that 80% of the assays coming from such deposits are between 25% and 35% iron.

A plotting of grade frequencies of Canadian gold ores, prepared by Falini (27) after production data taken in the "Canadian Mines Handbook 1954" shows that the grades oscillate around few modal values: 5, 12, 22 and eventually 32 g/ton. With an increasing grade, the frequency decreases, and the higher the grade, the less pronounced is the



frequency peak. No attempt has been made to check if each level belongs to a different geological background. If a law can be established, relating the grade of a deposit to its environment, the plan of exploration can be more carefully devised.

B. <u>Distribution as function of sampling</u>

In the usual case, when the average grade of a deposit is around 5% or less, the distribution curve is skewed to the right. This curve can be fitted to several theoretical curves. The best approximations are, however, given by the lognormal and the binomial distributions.

The lognormal distribution has been described by Ahrens for a limited case (15), but has been extended by the same author to a geochemical law: "The concentration of an element is lognormally distributed in a specific igneous rock" (16).

Mean grade and dispersion vary widely according to the metal and to the host rock. Sometimes the dispersion may be so small as to hide the law of distribution if a prohibitive number of samples is not used. Ahrens mentioned also the likelihood of the lognormal distribution in sedimentary rocks.

As corollary the following law was announced: "The abundance of an element in an igneous rock is always higher than the most prevalent concentration; the difference may be immeasurably small or very large and is determined solely by the magnitude of dispersion of its concentration."

In 1953 Krige (39) showed that in the examination of bore hole values the assays can also fit a lognormal curve.

Working on different data, de Wijs (48) found that the binomial distribution gives a better approximation. It seems that with an increasing grade the skewness of that frequency curve is decreasing, and getting negative above 50%, therefore the lognormal distribution, always positively skewed cannot claim to be the only approximation. Besides, the grades vary only between 0% and 100% (in the case of native metals, and much less, of course in any compound), whereas the lognormal curve is extending to the infinite.

If, instead of plotting individual assays, we plot the means of groups of assays, taken in the same part of the deposit, the distribution curve will be more symmetrical; it is tending toward a normal curve, as a consequence of the central limit theorem.

The use of an estimator as described in Chapter IV, requires the knowledge of the distribution followed by the assays. It is necessary to check in each case which curve gives the best approximation. It is possible that different genetic conditions give different distributions. The lognormal distribution cannot yet be considered as a law, however it gives a very satisfactory approximation for the most low grade deposits.

Chapter IV

REPRESENTATIVE PARAMETERS

A. Introduction

A population can be represented by various parameters. According to the type of the distribution or the purpose of the study, one or the other will be better suited for representation.

A good description of a set of data is given by following characteristics:

- Type of the distribution, or the type which gives the best approximation.
- Parameter of central tendency: locating the center around which the data are distributed.
- Parameter of variation: description of the dispersion of the data around the center.
- Parameter of symmetry: indicating how the data are balanced around the center.
- Parameter of peakedness: indicating how close to the center the data are located.

B. Parameters of central tendencies

(1) Arithmetic mean (In abbreviation: AM)

The AM has been used for a long time for computation of grades, as a straight mean or as a weighted mean. It is very easy to calculate, and gives undoubtedly the true average if the sampling is exhaustive or at least represents an important part of the sampled volume. In the case of a normal distribution, the AM is identical with mode and median, and is therefore the ideal parameter.

Furthermore the AM can be computed from the AM of subgroups of data, weighted by the number of data in each group.

However the assays coming from a deposit do not follow a normal, nor a symmetrical distribution, but a more or less skewed one. It has been demonstrated in many exploration projects that where few samples were taken, the grade of the deposit will appear to be higher than it actually is.

One of the most famous examples of overevaluation is that of the gold mines of the Witwatersrand, where the AM for a long time was employed for computation of grades. A discrepancy was noticed between t_1, the assays of drift sampling, and t_2, the assay of stope sampling of the same panel. The ratio t_2/t_1 is constant and smaller than 1. A second discrepancy was noticed between t_3, the assay of the mill feed, taken at the crusher and t_2. Ratio t_3/t_2 is constant also and equal to 0.9. The first of these ratios is called "block plan factor," the second "mine call factor." The true average is therefore smaller than the grade given by the AM.

A satisfactory mathematical explanation has not yet been worked out. This overevaluation is a well known fact, curiously enough partially compensated by an underevaluation of the tonnages! The underevaluation of grades above 50% is much less known, since iron is almost the only commodity occurring with such a high grade. Two possible reasons for that discrepancy may be the sensitivity of the AM to extreme values, and the high dispersion of the AM as compared to other estimators of the true average, which will be dealt with later.

The aim has always been to determine a method giving

<u>exactly</u> the grade of the mill feed from the sample assays taken in the deposit. This way of attacking the problem is wrong: besides the unavoidable bias and errors, a sampling cannot be expected to give anything else than an approximation of the sampled universe. The dispersion of the results can be shrunk, not supressed. Therefore the search for a representative parameter has to be oriented in another direction: the theory of maximum likelihood, worked out by R. A. Fisher mainly (4), and applied to valuation by Sichel (42). The estimators based on this theory have a smaller variance than the AM; less samples are required in order to obtain the same accuracy.

The weighted AM calls for a remark: the usual weighting factor is the thickness of the sampled bed or vein. In a numerical method this procedure is correct. In statistics, however, there cannot be any weighting.

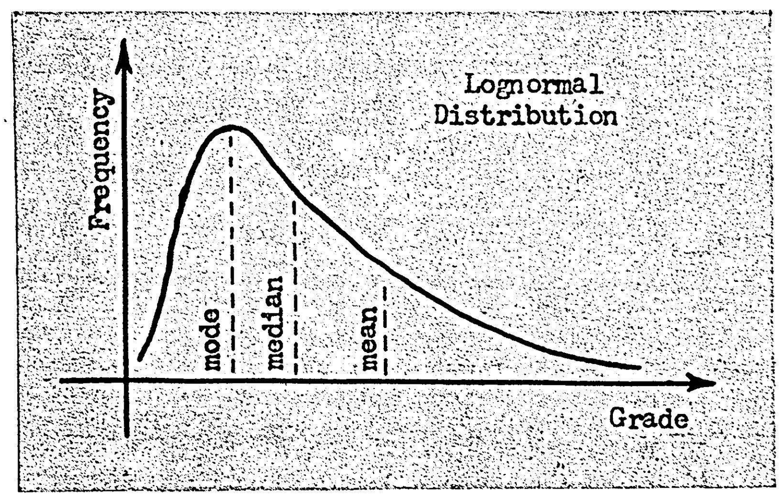
N. W. Wolodomonov (49) conducted an experiment to prove the uselessness of weighting, even in numerical methods. Dividing a gold deposit in blocks, he computed the grades of each block, first by a straight AM, then by a weighted mean. The distribution of the differences between the grades of each block appeared to be symmetrical and is not significant. (The assay distribution in this deposit is not mentioned.)

The AM is a good estimate of the abundance of an element in a deposit, if this element has a symmetrical distribution. If the latter is skewed, the AM can still be used, provided that a large number of samples are present. Its large variance should however restrict its use.

(2) Geometric mean (In abbreviation: GM)

The GM has been considered by various authors as giving the true average grade of a deposit. A first reason may be that it is always smaller than the AM, and so, closer to the mill feed, which is generally below the estimated grade. Hancock (33) tried to give a justification to the use of the GM. He based it on the assumption, taken over from Caddum (6), that the mode of a lognormal distribution is equal to the GM of the data.

To show that this assumption is not true let us plot the frequency curve of a lognormal distribution with a logarithmic scale on abscissa. The curve is symmetrical, the AM of the abscissas represents mean, median, and mode as well as the logarithm of the GM.



If we return to a normal plotting, i.e. with an arithmetic scale on abscissa, the three parameters, mean, median, and mode will be dissociated. The median does not change its position and is still equal to the GM.

The GM cannot take into account the zero values, for which we are obliged to substitute small arbitrary values.

The GM is associative and can be combined from the GM of parts of the universe. In the GM the high values are underweighted, whereas the low values are overemphasized, resulting in a lower average grade than truly exists; and the greater the range of values, the higher the discrepancy between GM and actual grade.

(3) Median and mode

There is no advantage in using one of these two parameters for grade estimate, because they are not very representative. The mode is the value with the highest probability of appearance. In the lognormal and binomial distributions it is equal to the GM, and has therefore been indirectly advocated by the defenders of the GM. It is too sensitive to the choice of the class intervals and cannot give an exact estimate.

The median is not sensitive to extreme figures, and has been recommended for this reason.

The major disadvantage, for which both should be discarded, is their high deviation from true median and mode. In other words, let M and X_o be median and AM of a population, and m and x_o median and AM of the sample set taken from this population, then:

$$M - m > X_o - x_o$$

(4) Estimators based on class frequency

The first attempt to decrease the discrepancy between mill grade and estimated grade has been made by Watermeyer in 1919 (47).

His study is mostly concerned with drift sampling during the development of a new block, and tries to eliminate the fact that a sample can only be taken along the drift. The probability of finding a given assay in the block is the same as its frequency in the drift. Therefore, according to Watermayer, weighting by the frequency is desirable. In fact the frequency will be considered twice, since in computing the AM of a set of data, the frequency is included implicitly. The frequency is determined by the records of the mine.

Truscott (45) adds as weighting factor the assay itself, considering that behind the wall the assay appears itself, not only its probability.

The comparison of the two estimators:

Watermeyer: $g' = g \cdot f_g / f_g$

Truscott: $g'' = g^2 \cdot f_g / g \cdot f_g$

shows that in the first formula the central values are overweighted; whereas, the high grades at least, are rehabilitated in the second formula, through the square.

These formulas have been checked on very large series of mine samples and have been found satisfactory. However their use is not to be recommended, since they are not built upon a sound theory.

(5) Estimators based on the theory of maximum likelihood

This theory has been worked out by Fisher (4). In short: the AM is the unbiased estimate of the average. Other estimators are more likely to be close to the true average than the AM, because they have a smaller variance. Let s_1 be the standard deviation of the AM, and s_2 the standard deviation of any estimator. The efficiency of the latter is defined by the ratio s_1 / s_2, which can be set equal to k. That means that the variance of the AM of k . 100 data is the same as that of an estimator computed from 100 data.

a - Sichel's estimator. The most likely value of the mean of a lognormal population is of the form:

$$m_3 = GM \cdot f(v, N)$$

where:

v: variance of the data (expressed in logarithms)

N: number of data

Particular solutions of the complex function f(v, N) are given by Sichel (41). If the number of data is higher than 100, following approximation can be used:

$$m_1 = GM \cdot e^{v/2}$$

The variance of m_1 is equal to following expression:

$$v = (AM / n) (s^2 + \tfrac{1}{2} S^4)$$

b - de Wijs' estimator. Under the assumption of a binomial distribution, de Wijs proposes following estimator of the mean:

$$m' = X_o \cdot F$$

where: m' is the average grade

X_o median

F a factor given as $F = (1 - d)^{-\frac{1}{2}k}$

d de Wijs coefficient of variability (See next paragraph)

k the binomial coefficient, such as $n = 2^k$

If the number of data is small the GM should be substituted for X_o, and if it is below 30, the correcting factor $(n / n - 1)^{\frac{1}{2}}$

is to be introduced, so that in the last case the estimator will be:

$$m' = (n / n - 1)^{\frac{1}{2}} \cdot (1 - d)^{-\frac{1}{2}k} \cdot GM$$

(6) Conclusions

The law of the large mmbers is the only justification for the AM. It is the easiest way of computing a grade, but other methods will give better results. In following cases it is indispensable to use an estimator:

low grade ores with high coefficient of variation

small number of samples

According to the closer fitting of the assay frequency distribution to a lognormal or binomial type of distribution Sichel's or de Wijs estimator is to be preferred.

If there is still a large discrepancy between estimated and actual average, physical biases should first be investigated.

C. Parameters of variation

(1) Probable error

The probable error is defined as the error which we are 50% sure will not be exceeded by the error of our estimate. It is given by the relation

$$p. e. = 0.6745 \, s$$

where s is the standard deviation of the sample mean.

(2) Coefficient of variation

The standard deviation is the usual measure of variation. In order to compare it with the value of the mean, it can be transformed in Pearson's coefficient of variation c_v:

$$c_v = s / AM$$

(3) Coefficient of variability

De Wijs (48) proposed a new measure of variation, defined as the mean successive difference. By the thought, a deposit of average grade G is divided in two equal parts, of which one will have a grade equal to $(1 + d) \cdot G$, the other $(1 - d) \cdot G$. After k divisions, there will be 2^k parts, and the grade of any one of these parts will be of following form:

$$g = (1 + d)^x \cdot (1 - d)^y$$

with, of course: $x + y = k$

The interest of this parameter is evident for samples taken in a line, as consecutive core sections of a drill hole, because the order of appearance is respected and not affected by a gradient.

D. Conclusion

The estimated average grade of a deposit or of a part thereof should always be followed by a parameter describing the accuracy of this estimate. The traditional classification of ore reserves according to their certainty is very subjective, if applied during exploration. Only the definition of the measured reserves includes the possibility of deviation from the estimated reserves: "The computed tonnage and grade are supposed to be accurate within limits, ... and no such limit is supposed to be different from the computed tonnage or grade by more than 20%."

In presenting a computation of reserves, the traditional classes should be subdivided each time it seems possible into subdivisions characterized by a given mean and standard deviation. The number of classes will be more flexible, adaptable to each deposit, and more descriptive.

Chapter V

SAMPLING PATTERN

" ... to subdivide each of the difficulties under examination into as many parts as possible and as might be necessary for its solution."--Descartes, Rene (Discourse on the Method, 1637)

A. Introduction

The adoption of a given drilling pattern is made under the assumption that it is possible to subdivide a deposit into cells, inside of which the different parameter can be considered constant. At the beginning of an exploration campaign the size of such a cell, as well as the existence of zones are more or less unknown. They will be approximately determined during the first stage of the exploration, by examination of the obtained data. If necessary these cells will be subdivided by new drillings. Thus one criterionof a good pattern is its ability to be further subdivided. The difficulty arises only in terms of statistics, because it should still conform to the conditions given in Chapter II.

Too many holes may have been planned a priori. If at a given moment it appears that no further information can be obtained by continued drilling, the campaign has to be stopped, and this requires a certain flexibility in the drilling pattern.

A third condition is imposed by the statistics, in order to obtain a random sampling: no sample should have more chances to appear than another.

B. Grid pattern or systematic pattern

A grid pattern has long been used for implantation of drill holes and this for practical, rather than theoretical reasons. A nonsystematic location necessitates complicated computations to determine the zone of influence of each hole and to weight the analytical results by the volume of this zone. Different emphasis is given to each hole, according to its location, whereas there is no reason to give more weight to one hole rather than to another. Indeed it is illusory to assign to each hole a zone of influence, since the variations in grade between two holes are randomly distributed, although trends can be observed (a so-called "shifting mean"). Therefore the average obtained that way cannot be expected to be really representative from a theoretical standpoint. In a grid system this inconsistency does not appear any more. Let it be said here that the statistical approach considers each hole to have the same representativity, because it is possible, with a limited number of samples to eliminate these variations at a preset level of confidence.

The volume to be explored is divided in prisms of regular bases: equilateral triangles, squares or hexagons. The extension to this field of Bravais' first law in crystallography shows they are the only three possible unit cells giving an infinite lattice system.

(1) Classical utilization of the results

Each cell is attributed descriptive parameters (thickness, grade, etc.) derived from the holes and supposed to be constant inside of this volume. These parameters are those of a unique hole located at the center, along the axis of the cell, or obtained by combining the data of several holes located at the apices (See Figures 1 and 2, page 34).

(2) Statistical utilization of the results

Instead of combining the results as previously described, they can be submitted to statistical analysis, which will also give figures for the representative parameters.

(3) Discussion of the grid system

The error, i.e. the deviation between true mean and estimated mean is the smallest for this type of sampling, as compared with random sampling. This property has not been demonstrated by theory, but has found a practical affirmation in fabrication control. The variance cannot be defined, so that no estimate of the confidence level can be made, and one of the main advantages of the statistical approach, the knowledge of the accuracy of the results, is lost. Nevertheless a great economical advantage results from this pattern since the random sampling requires more samples in order to ascertain the same error.

A systematic pattern can induce systematic errors. Duval (26) gives several examples.

a. Deposit presenting equidistant parallel runs. Bad luck or the desire to implant one of the rows of holes along a structural direction can introduce an important error. However a second condition must be fulfilled in order to produce this error: the spacing of the holes has to be the same, or approximately, as the interval of the runs.

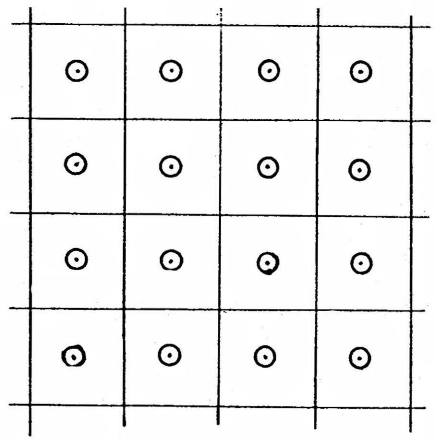


Figure 1 - Grid system with central hole.

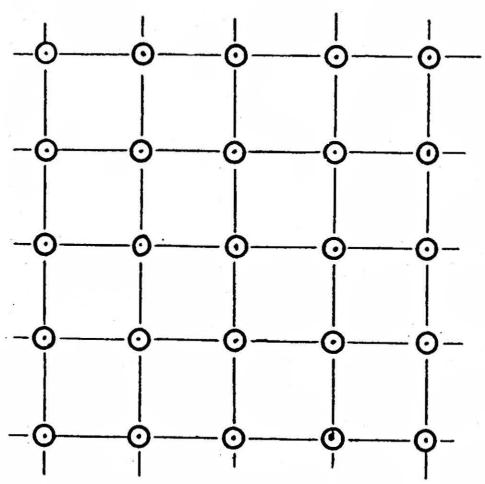


Figure 2 - Grid system with holes at the apices.

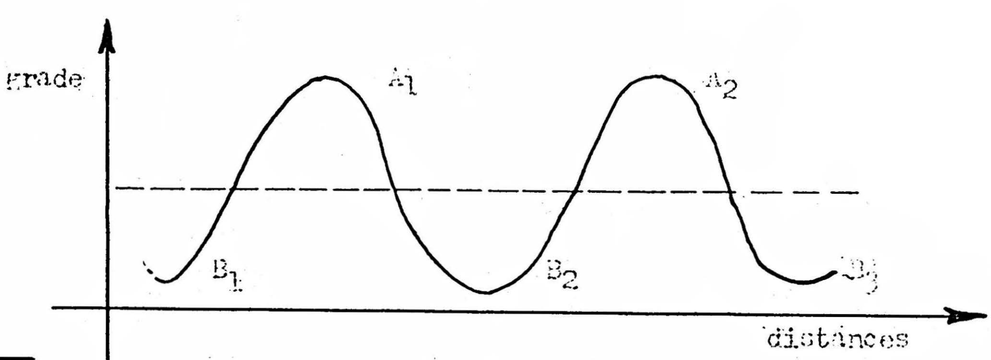


Figure 3 - Periodicity in grade.

A vertical section, normal to the direction of the runs illustrates this possibility (See Figure 3, page 34). If the holes are represented by the points A_1, A_2, A_3, etc. the calculated grade will be too high. At the opposite extreme, should they be represented by the points B_1, B_2, B_3, etc. it will be too low. A small displacement of the grid would give an important variation in the grade.

Alluvial deposits sometimes can present this periodicity, and as a general rule the implantation of prospect pits in placers should always be made at random. Numerous other possibilities can be imagined; regularly folded structures with folds of small amplitude, and "en escalier" fault structures with regularly spaced faults, can present a periodical variation of their properties.

b. Deposit presenting a linear gradient. Figure 4, page 36, represents the variation in grade along a vertical section as a function of the distance. If the implantation of the holes corresponds to the holes 1 to 10, the AM will give an exact estimate of the average grade, but not if the row is slightly displaced to the left or to the right. The higher the gradient, the higher the error. (This is not to be taken as criticism of the AM, as no other method of estimation could give a better approximation, as long as a grid pattern is used.)

c. Irregularly distributed rich spots. Small rich spots can be spread over a large surface of low grade material (See Figure 5, page 36), as in mantos type deposits. An unfortunate implantation, especially if the spacing of holes is very large, may give excessively

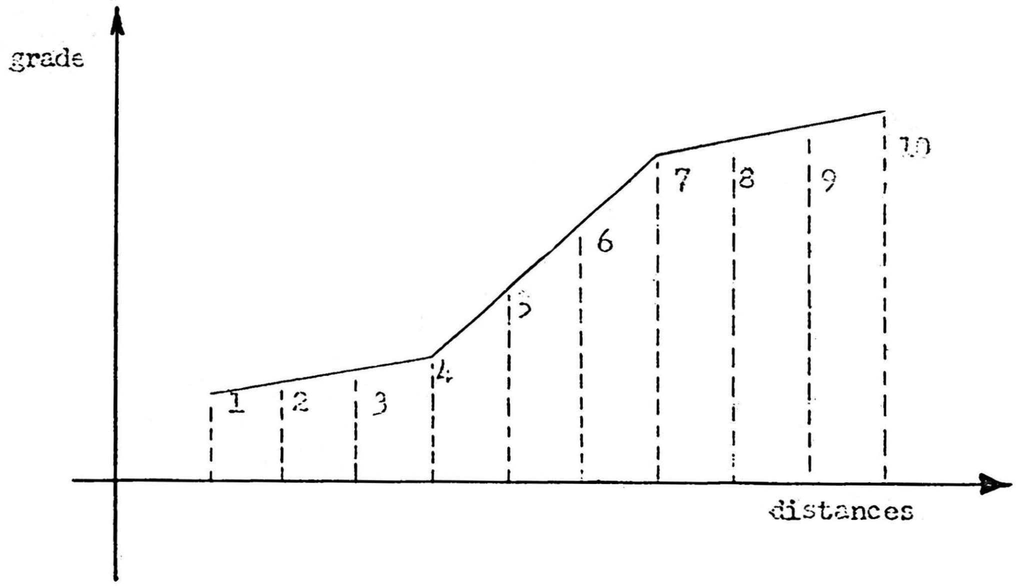


Figure 4 - Linear assay gradient
with broken slope.

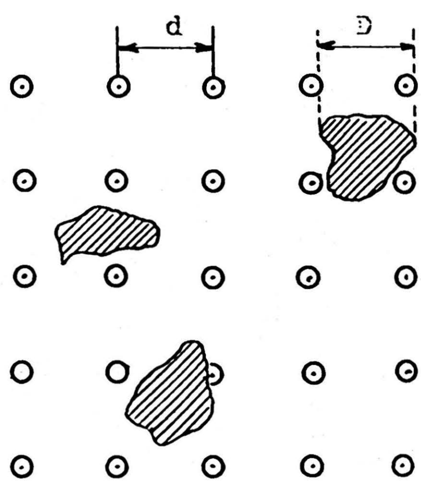


Figure 5 - Rich spots, irregularly
distributed in low grade material.

low or high results. The ratio d/D gives an idea of the accuracy of the pattern, with the ideal value being 1 (As an approximation, D represents the average diameter of the rich spots). Unfortunately this relationship is not known before the exploration.

Errors of type a and b, which require rather exceptional conditions are fortunately not likely to occur. The type c error is more frequent, and a definite answer can only be obtained by mine workings.

A last disadvantage of the grid system is its incomplete agreement with the requirements of random sampling: only the first hole location is random, the following are determined by this first and are not random any more.

(4) Comparison of the bases.

A given number of holes can be arranged in three ways. Here is an attempt to find an advantage to anyone of these three possibilities. Considering a portion of a plan, divided successively in the three grids with always the same number of holes, the three unit cells can be compared:

Parameter	Triangle	Square	Hexagon
Surface	0.5	1	2
Distance apex to center	0.6204	0.7071	0.8774
Distance apex to apex .	1.0746	1	0.8774

The comparison of the three cells shows that in the triangular cell any point is closer to the center than in the two others, thus giving a better coverage of the area to be explored. This advantage is outweighed by the fact that there are three critical directions instead of the two present in the square, thus increasing by 50% the risks of following a periodicity as described previously.

C. Random sampling

The implantation of the bore holes is made at random. The randomness is obtained by two possible means:

a. In drawing lots giving the coordinates of the holes, the abscissae and the ordinates being drawn separately. A lot has to be replaced after drawing in order to not alter the probabilities of appearance.

b. In using tables of random figures, such as given in (8) by A. Hald, the tables are opened at random to a page and line, and the series of consecutive numbers is taken until the required number is reached.

In order to stop the drilling program if necessary, the drilling should be done according to the order of appearance of the coordinates and not to position in the explored area.

(1) Classical utilization of the results

The procedures are classified in two main groups, according to their division of the area in triangles or polygons.

a. Polygon methods. The deposit is divided into a certain number of cells of which each one contains a hole at its center. The parameter given by averaging the results of these holes are extended to the prism.

In the most common method the boundaries of each cell are given by the bisectors passing between two neighboring holes (See Figure 6, page 39).

In the Southeastern Missouri method the area is first divided into triangular cells (See Figure 7, page 39). If point O

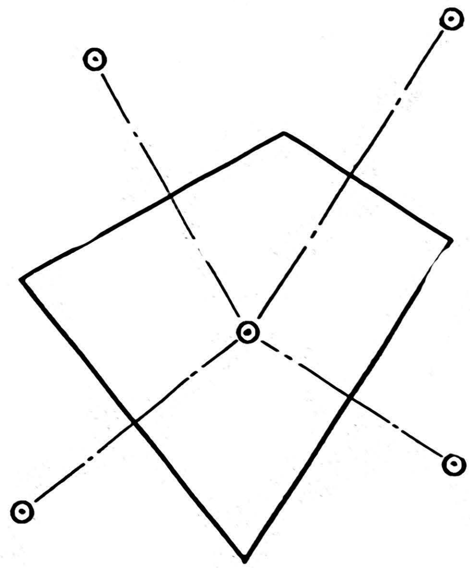


Figure 6 - Polygonal cell, common method.

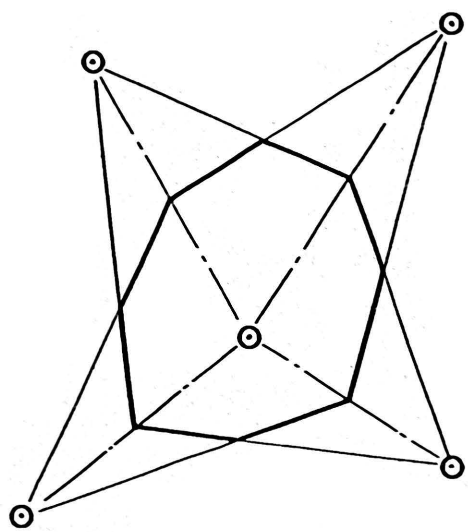


Figure 7 - Polygonal cell,

South Eastern Missouri method.

is the location of the hole whose zone of influence we intend to determine, lines are drawn from apices A and B to the centers of the opposite sides, and the boundaries are obtained as shown on Figure 7. The so obtained area is closer to a circle, and may be more representative, but there are twice as many sides, complicating the computations, and making graphical errors more likely. This method has been described by Poston (54).

b. Triangle methods. The deposit is divided into triangular cells, each hole being located at an apex (See Figure 8, page 41). If the difference in lengths between the three sides is important a weighted mean is usually established. Different formulae has been proposed, of which follow some examples.

One method involves weighting of each hole by the distance from this hole to the center of the cell. Let

d_a, d_b, d_c be the distance from
apex A, B, C, to the center
g_a, g_b, g_c be the grades in A, B, C
g_1 the estimated grade of the cell

Then

$$g_1 = \frac{d_a \cdot g_a + d_b \cdot g_b + d_c \cdot g_c}{d_a + d_b + d_c}$$

A second method weights the AM of assays of each pair of holes by the length of the side connecting these holes. Let

a, b, c be the sides of triangle
ABC, opposite to apices A, B, C
g_2 the estimated grade of the cell

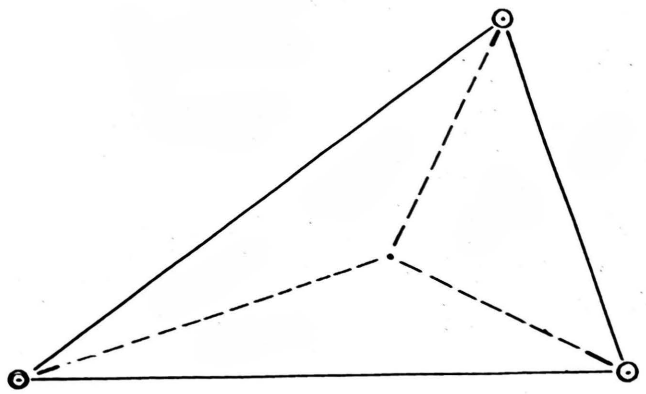


Figure 8 - Assay weighted by the distance from hole to center.

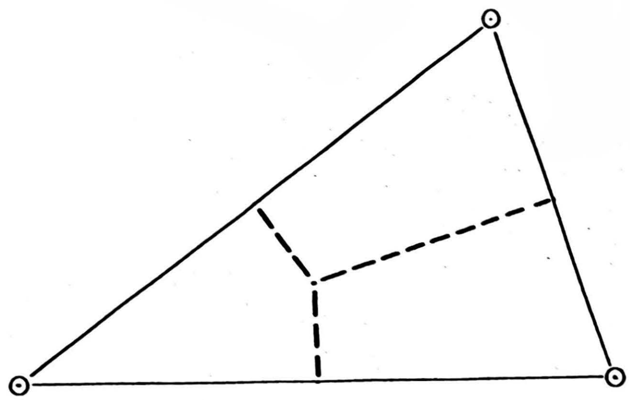


Figure 9 - Assay weighted according to Temperley's method.

Then

$$g_2 = \frac{a(g_b + g_c) + b(g_a + g_c) + c(g_a + g_b)}{2(a+b+c)}$$

A third method weights the assay of each hole by the angle of the adjacent sides. Let

A, B, C be the angles of the adjacent sides of the triangle

g_3 the estimated grade of the cell

Then

$$g_3 = 1/180\ (g_a \cdot A + g_b \cdot B + g_c \cdot C)$$

A fourth method weights the assay of each hole by the partial area of the quadrangle limited by bisectors (See Figure 9, page 41).

Actually this is the usual polygon method with a different grouping of the data, but as they are additive the final result is the same. To avoid complicated measurements of areas a table has been set up by Temperley (57). In this table the weighting factor is expressed in percent, based upon the size of the angles of the triangle. For determining the factor belonging to one of the holes, we place on the horizontal axis the largest of the two other angles and follow this value vertically until the intersection with the curve corresponding to the other angle. We read then the factor in following horizontally to the left.

Comparison of those four methods. The triangle of Figure 10, page 43, has the following characteristics:

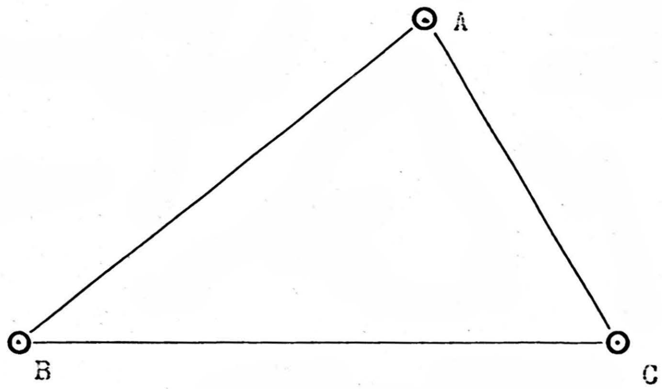


Figure 10 - Triangular cell used in the comparison of different weighting methods.

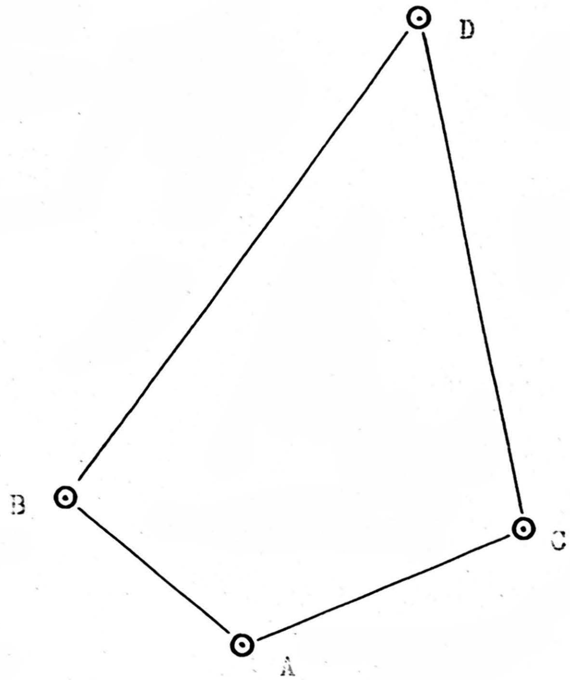


Figure 11 - Quadrangle used to show the importance of hole grouping.

Characteristics	Hole A	Hole B	Hole C
Grade	10	12	13
Distance from apex to center	3.1	4.7	3.8
Length of the side opposite to that hole	8	5	7
Angle of the two adjacent sides at hole	81°	39°	60°
Temperley's coefficient	0.43	0.27	0.30

The results for the grade estimated are as follows:

Method	Grade estimate
I	11.79
II	11.72
III	11.43
IV	11.45

The AM of the three assays is 11.67. Methods I and II give results above the AM and should therefore be discarded. Methods III and IV give very close results, however the weighting by the angle of the adjacent sides is much simpler than the reading of Temperley's coefficients in a table. For this reason only, we prefer Method III.

The deviations from the AM are not important in a cell, therefore it seems very questionable to undergo all these tedious calculations, since the weighting of the holes in itself introduces already some biases. If a large number of holes are present, the straight AM, computed for the total volume, will give certainly as good results as any of previous correction methods if the spacing is reasonably uniform. If the spacing is irregular, the attribution to some of the holes of much larger zones of influence is arbitrary.

c. Discussion of the random implantation used with classical computation methods. As long as these are used, there is not much advantage to the random sampling.

The computations are long and tedious, even if no corrections are applied. The computation sheets are large and difficult to handle.

The graphical errors committed in the determination of the areas may be as large as several percent.

The results are not always unique; there are two ways of combining four holes located at the apices of a quadrangle.

Example: Let us take the quadrangle of Figure 11, page 43, and assume following grades:

Hole A:	6.0	Hole C:	8.5
B:	7.5	D:	10.0

The four triangles will have following surface and grades (grades computed in weighting the assays of each hole by the angle of the two adjacent sides):

Triangle	Areas	Grades
ABC	5.05	7.49
BCD	20.89	11.05
ABD	11.98	8.11
ACD	13.74	8.29

Combining the triangles two by two, and weighting by their area we find following grades for ABC:

ABC and BCD combined together:	10.36
ABD and ACD ....:	8.21

The average grade of ABCD can also be obtained by combining the grades of the holes weighted by the angles of the two adjacent sides, the estimated grade is then 8.97. The AM of the four grades is 8.00.

The following rules can be set up for the subdivision in triangular cells; if several possibilities are present, use the one giving triangles of same or nearly same surface, and place the dividing line parallel to the direction of highest gradient.

d. Conclusion. There is no clear choice between the polygonal and the triangular methods. The first introduces a higher graphical error, but the computations are very easy. The second gives a better accuracy in the determination of the area of influence, but the computations are more complicated.

Among the polygonal methods, the one determining the area of influence by the bisectors of two neighboring holes is definitely to be preferred.

Among the triangular methods, the most advantageous is the one weighting the grade of a hole by the angle of the two adjacent sides. However a straight AM should give sufficiently accurate results. Some attention is to be paid to the manner of subdividing the deposit in triangles.

(2) Statistical utilization of the results

The random sampling allows the computation of a significant variance. However the error is larger than in a systematic pattern. Systematic errors, as described in heading B(3) of this chapter, are avoided here.

D. Stratified random sampling

A deposit can also be explored by means of what statisticians call a random stratified sampling pattern (24, 25). Instead of considering the entire deposit as one unit, it can be subdivided into zones, called strates, each of which can be explored independently. Inside of such a strate the implantation will be made at random, in the same manner as described in preceding paragraph.

(1) Stratification procedure. The stratification can be realized following two different procedures:

a. Uniform and homogeneous deposit. A uniform deposit is divided arbitrarily into cells of equal dimensions, each having the same number of holes distributed at random. In proceeding that way, the distribution of the holes over the deposit will be better balanced, no large area risks being without holes (See Figure 12, page 48).

A particular solution of this type would be to divide the deposit into small and regular cells, to each one of which one unique hole would be assigned at a random location.

b. Irregular deposit. An irregular deposit is divided into zones of same petrographic and mineralogic nature. The number of holes may be proportional to the area of the cell or to the standard deviation of the results in that cell, so giving the "optimum allowance."

The optimum allowance has the advantage of decreasing considerably the variance of the error. In preliminary phase the number of holes will be proportional to the surface of the cell, than according to these first results, the number of holes is extended until it is proportional to the variance. A second adjustment may be necessary if the standard deviation has changed of a significant amount.

(2) Utilization of the results. A classical approach can be used, which will be the same as described in the paragraph on random sampling.

If the stratification in cells or strata of equal size is used, we are in the case of "subsampling with subsamples of equal size" (25). The results of each cell are combined, then these partial

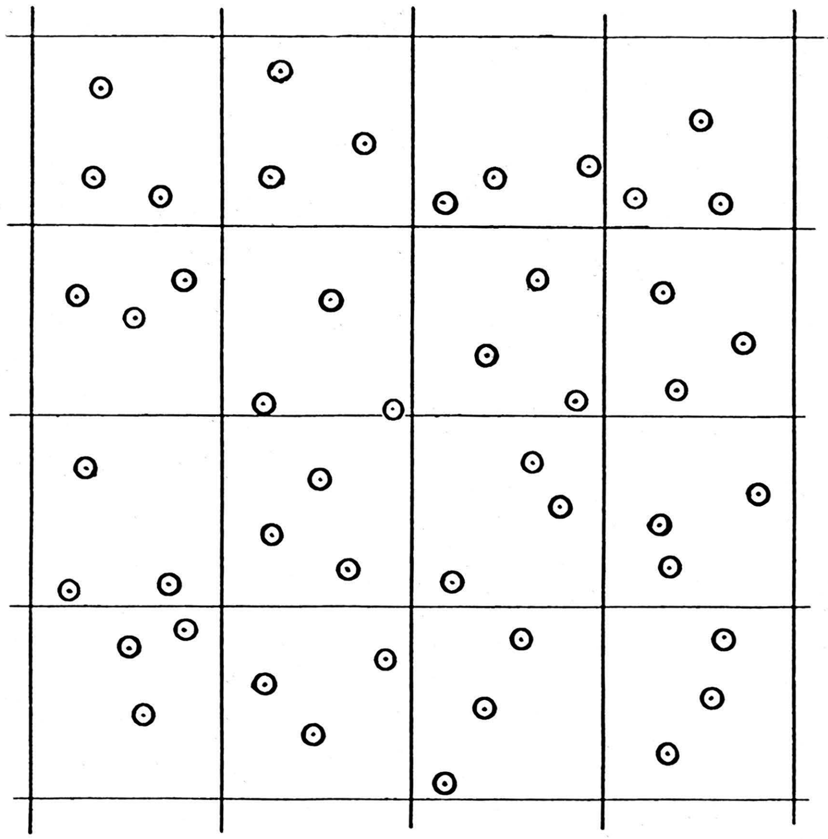


Figure 12 - Stratified random pattern.
Uniform deposit divided in equal cells
having the same number of holes each.

results are combined together in order to determine the parameters for the deposit.

If the stratification has been made with respect to natural conditions, each zone will be studied separately, no general figures for the entire deposit will be computed.

(3) Discussion

The stratified random sampling has several advantages over the random sampling. The holes are better distributed over the explored area. Parts of the deposit, which seem more important or particularly difficult to explore are paid more attention. The error variance is smaller than in the random sampling, especially if the "optimm allowance" is applied. Therefore the main disadvantage of the random sampling, with regard to the systematic sampling disappears in the stratified random pattern.

E. Sectional pattern

If there are good reasons to suppose that an elongated structure is present (e.g., a sedimentary folded structure) a sectional pattern can be used at least during the first stages of exploration (See Figure 13, page 49).

The holes are put down, at equal intervals, along a line perpendicular to the strike of the structure. The next row will be at a distance much larger than that interval. We assume hereby that the variation along a cross section is faster than along a length section.

This pattern has been quoted for completeness, but cannot, strictly speaking, be interpreted statistically. Nevertheless the higher density of implantation along the direction of higher variation

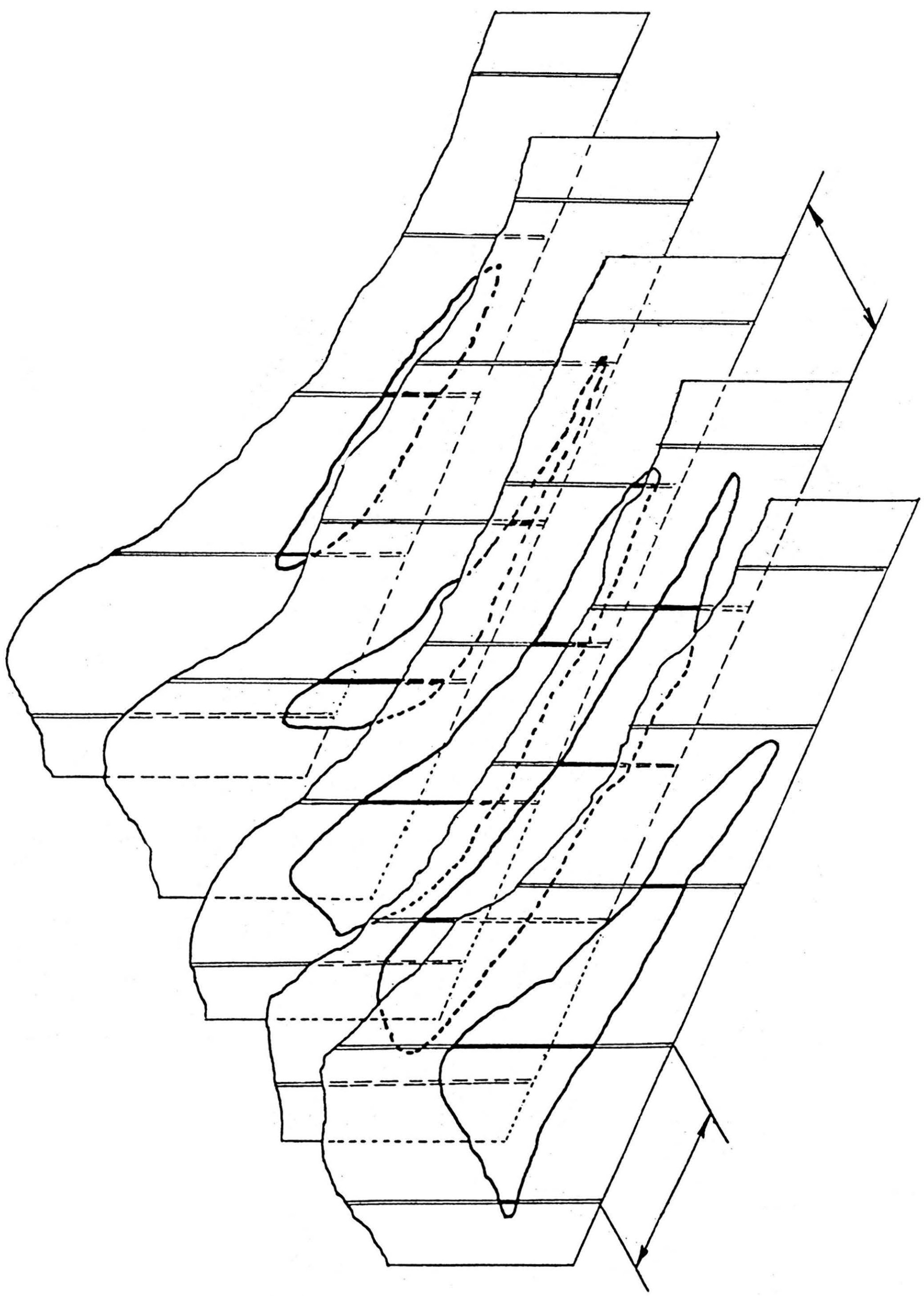


Figure 13 - Sliced view of sectional pattern.

should give the preference of this pattern over the three previously described patterns in an elongated structure. The possibility of transforming the sectional pattern into a systematic pattern or a stratified random pattern (with small cells, having each one hole) should remain open (See Figures 14 and 15, page 51).

The computations will be made according to the so-called section method, as described by Wolff (58).

F. Conclusions

The type of drilling pattern depends upon the method of computation to be adopted. The systematic pattern is to be preferred for its simplicity if a classical method of computation is used; for instance, where statistical methods cannot be applied, such as in flat-lying deposits where a high correlation between neighboring samples does not allow a random sampling.

Should a statistical method be used, the random sampling is definitely to be preferred, though the deviation of the grade estimate from the true grade is greater. But the results are more representative, and a variance can be computed. To ascertain a better accuracy, a stratification can be made, and this is advisable in all cases.

A special mention is to be made for elongated bodies, where at least during the first stages, a sectional pattern can be expected to give more information than a two-dimensional pattern, with the same number of holes.

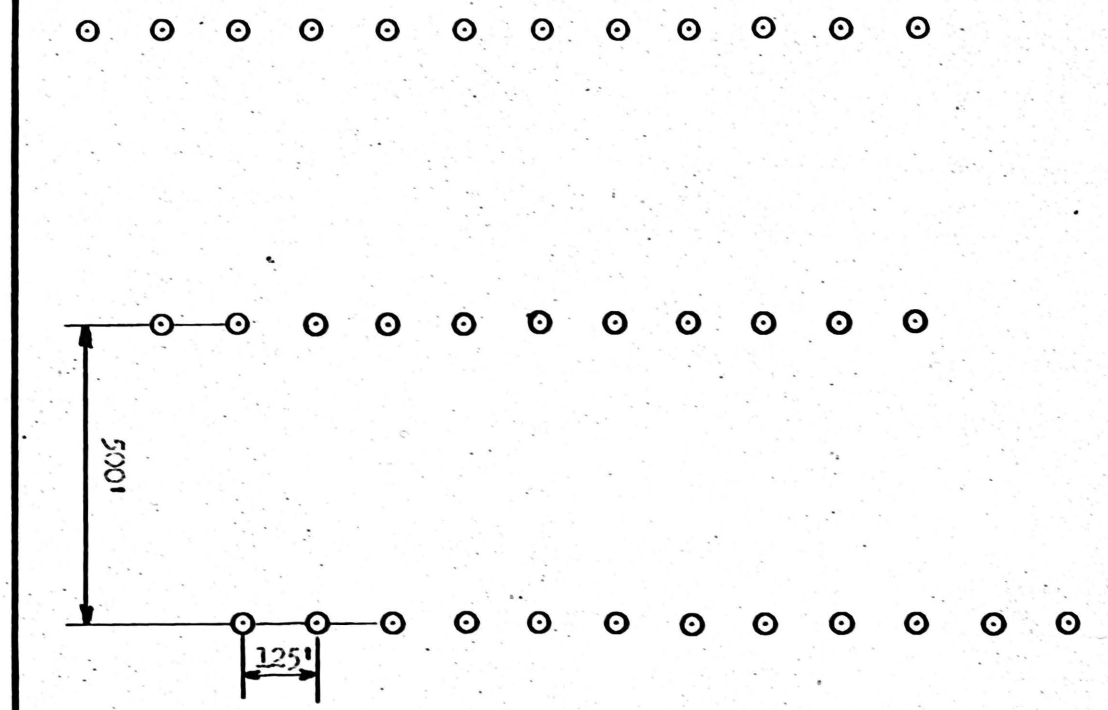


Figure 14 - Sectional pattern with possible extension to systematic pattern.

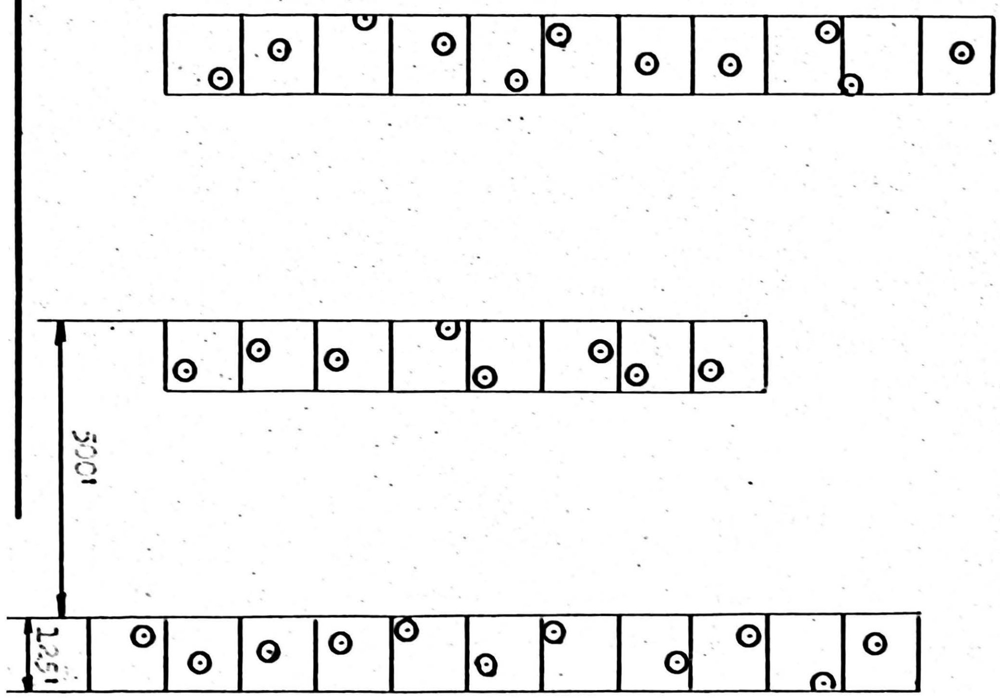


Figure 15 - Sectional pattern with possible extension into random stratified pattern.

Chapter VI

SAMPLE SIZE

" ... accuracy is restricted to practicability." - Baxter and Parks (50) page 77.

In the preceding chapter, sample location has been discussed. It is obvious that there is a large range possible in the weight of samples, mainly if they are taken by churn drilling, rather than by core drilling. For a standard core length of 5 feet, a rock of density 2.75 will give the following weights:

EX core drill (7/8"):	1.62 kg.
Churn drill of 10":	212 kg.

It can be considered that the lower limit of sample weight is determined by the expected representativity, the upper limit by economical reasons only.

Is it possible to give a rule or a formulae, which could be applied to any deposit, to give the optimum sample size, without using a long trial and error method? Unfortunately, so far the answer is no.

The approach to this problem has been made by two different ways: It is possible to determine experimentally the threshold above which the sample is sufficiently representative. Theoretical sampling formula are given, in order to establish the relation between weight and other factors. But if good formula have been worked out for broken ore, there are some difficulties in setting up a rule for the sampling of massive deposits. Let it be noticed that there is no problem for placers.

A. Experimental determination of sample size

A deposit should be sampled in taking samples of constant volume (See Chapter II, heading on similarity). It is this standard volume, which we will try to determine. Should it be too small, the obtained grade will not be representative, on the other hand too large samples are not economical. The homogeneity has an obvious influence on the minimum weight; this characteristic can be represented, for instance, by the standard deviation of the assays.

In order to determine the minimum sample size, we have to go back to Chapter II, where the following relationship was established:

$$v_n \cdot s_n^2 = \text{constant}$$

This constant may be known by previous experience, or will be determined by the first holes, provided that statistical methods can be applied to the deposit in question. The minimum volume is then easy to deduce.

B. Sampling formula

In mines the sampling procedures (the size, for instance) are usually fairly standardized for each operation. For examples, see Baxter and Parks (50), Chapter 6. But in each case it takes a long time to work out the procedure through a trial-and-error method. The determination of the minimum volume by the preceding method is much faster and probably more accurate.

Assuming that a sample has been taken, it will be crushed and ground. What amount of sample will be required in order to give representative results? Numerous attempts have been made to give relation between sample weight and various other quantities, as particle size. Two of these formulae shall be discussed.

(1) Shcherbov. In an article (31) published in 1956, Shcherbov indicates a relation between weight of sample and the size of constituent particles. According to him this relation is currently used in the U.S.S.R. for sampling purposes.

$$Q = K \cdot d^2$$

Where

Q: minimum allowable weight of sample in kg.

d: maximum diameter of particles in mm.

K: coefficient depending upon the deposit

This formula is too rough, the coefficient K does not take in consideration as many factors as the following formula, by P. Gy.

(2) Gy. In two articles (29 and 30), Gy demonstrates following relation giving the minimum weight of sample:

$$P' = C \cdot d^3 / s_1^2$$

Where

P' : weight of sample

d : dimension of the mesh retaining 5% to 10% of the sample

C : constant characteristic of the deposit

s_1^2 : variance of the sampling error

s_1 is the standard deviation of the error committed in cutting the sample. It is not related to the error committed during sampling, whose standard deviation was called s. The variance of the combined error S^2 is given by

$$S^2 = s_1^2 + s^2$$

Constant C is itself a product of four parameters:

$$C = f \cdot g \cdot l \cdot m$$

Where these letters represent following parameters:

f : grain shape (varies from 0 for lamellae to 10 for needles)

g : grain size (the higher, the smaller the range of sizes)

l : liberation (for homogeneous ore it is very small)

m : mineralogical composition

A set of 50 tables were first set up in order to simplify the computations. A nomogram has been published since, giving the results without computations (See Figure 16, page 56). In this nomogram, f and g are supposed constant, l is determined by tables, so that C is obtained by combining l and m.

C. Conclusions

The volume of the core length can only be determined by the relation $v_n \cdot s_n^2 = \text{constant}$. This relation is usually determined for the main commodity. We saw that different metals occurring in the same deposit could have different behaviors. We saw also that the coefficient of variation was higher for the metal with the lower grade. Care has to be given here if the minor constituent is important for penalties or bonuses. In such a case it is the higher volume which is to be taken.

Only an aliquot portion of the sample will be analyzed; the weight of this portion is easily determined with Gy's nomogram.

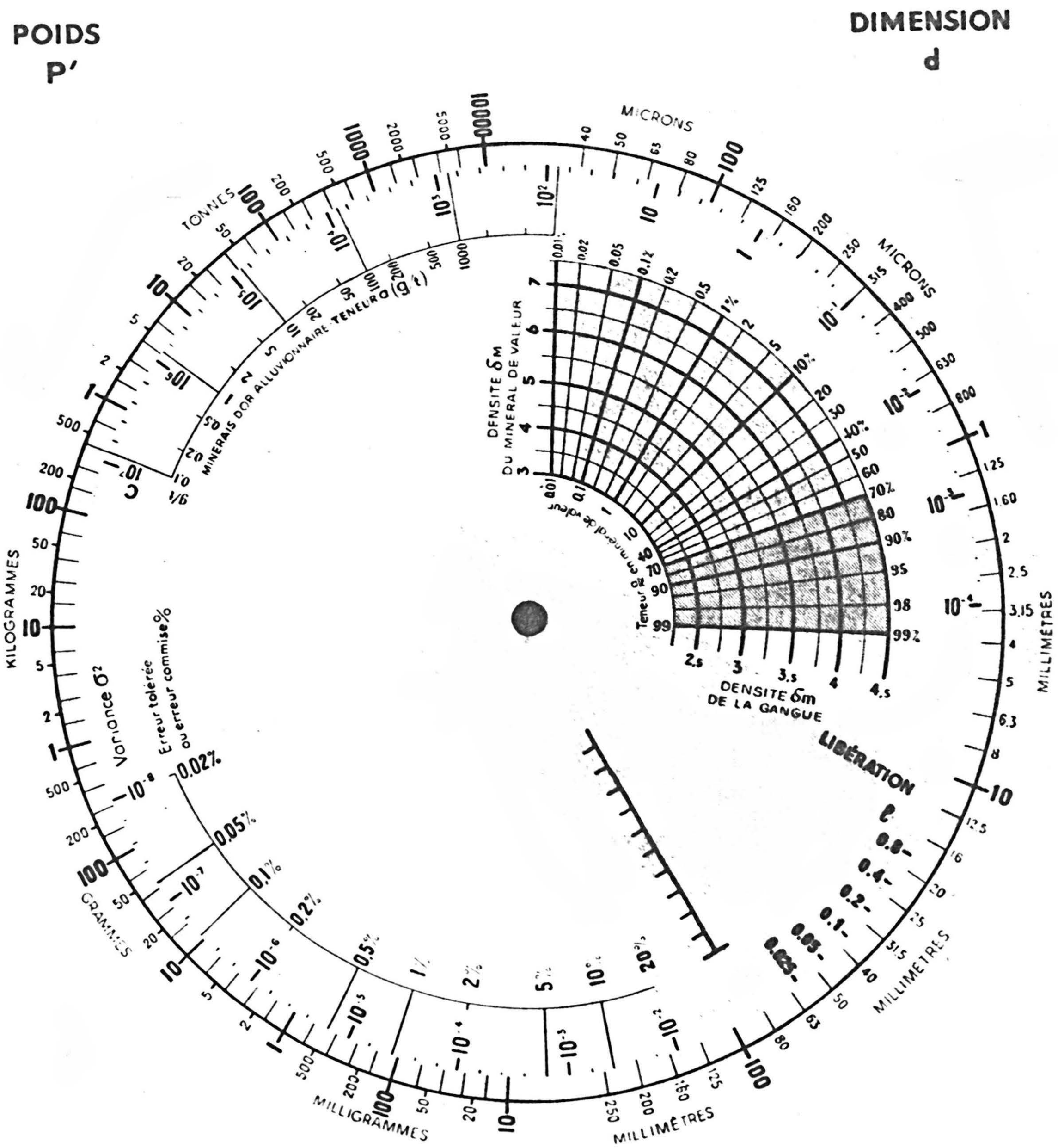


Figure 16 - Sampling Nomogram

by P. Gy.

Chapter VII

OUTLINE FOR THE VALUATION OF RESERVES

A. Introduction

As it appears in the preceding study, no attention whatsoever was given to the density of implantation. This idea may be surprising, but in fact the number of drillings depends only upon:

- expected accuracy
- mean value of grade
- variance, or standard deviation of assays

In relation with the accuracy it can be noticed that a level of confidence of 80% to 90%, i.e., a guess which will be correct 8 or 9 times out of 10, is sufficient, since the possible errors (due to chemical analysis, core losses, etc.) would make a higher accuracy illusory. In no case should it be attempted to exceed a level of confidence of 95%.

B. Planning

(1) Number of holes

The normal distribution is the most favorable case, i.e. the type of distribution requiring the fewest samples for a given accuracy, mean and standard deviation. If we base our determination on the assumption of a normal distribution, we will obtain the minimum number of samples to be taken.

The computation is conducted as follows:

Let m be the AM of the assays

n : number of samples

e : permissible percent error on the determination of the true average

s_2: standard deviation of the sample mean

s : standard deviation of the population

Then following relation can be written:

$$s_a \cdot \frac{s}{\sqrt{n}}$$

$e \cdot m$ = permissible deviation of the sample mean

Consider the inequation:

$$-t < \frac{e \cdot m}{s_a} < t$$

which can be written:

$$\left(\frac{e \cdot m}{s}\right)^2 n < t^2$$

since the distribution is assumed symmetrical. The tables give the level of confidence for each value of t, and vice versa. For example:

$$\begin{aligned} n &= 36 \\ m &= 6 \\ e &= 0.05 = 5\% \\ s &= 2 \\ t &= 0.9 \end{aligned}$$

The table of t values indicates a 63% level of confidence for an average grade laying between 5.7 and 6.3. By the same way there is a 93% level of confidence for the interval 5.4 to 6.6.

In planning an exploration we can take m and s from neighboring deposits, from deposits of same type or merely make a guess. The number of holes can be written as

$$n = \left(\frac{t \cdot s}{e \cdot m}\right)^2$$

If this number should fall below 30, two possibilities exist: either take arbitrarily a number of holes equal to 30, or use the so-called "student-t" distribution to compute the necessary number.

This distribution applies to a small number of samples, but trends toward the normal distribution very fast if that number is above 30.

In its general aspect the formula will be the same, except that (n - 1) is substituted to n, in order to introduce a larger margin of security. Tables give also t as function of the level of confidence and of the number of samples. The best approach will therefore be to write following relation:

$$e = \frac{t \cdot s}{m \cdot n}$$

and try successive values for n until a satisfactory value for e is found. The computation has been made for a certain number of cases, see Table I.

(2) Location of the drill holes

We assume that we know enough of the geology of the deposit in order to decide whether statistical approach can be used or not. If it can be used, we shall decide upon a random sampling pattern.

The approximate boundaries of an ore body are determined in the most cases by a preliminary geophysical work, so that the holes are not implanted blindly.

If lots are used, for example, balls with marked digits, the digits of one coordinate are drawn successively, until the unit. To determine the ordinate along AB, the first draw, giving the hundreds, will be made from an urn containing four balls, marked from 0 to 3, the following two digits will be drawn each from a set of 10 balls. For the abscissa along AC, the first draw will be made from 8 balls, marked from 0 to 7, and so forth. See Figure 17, page 62, where the order of

TABLE 1

NUMBER OF HOLES AS FUNCTION OF AM AND STANDARD DEVIATION

Stand. Dev. \ AM	1	2	5	10	20	30	40	50	60
1	271	68	-	-	-	-	-	-	-
	68	18	-	-	-	-	-	-	-
2	1760	271	44	-	-	-	-	-	-
	271	68	14	-	-	-	-	-	-
5	-	1691	271	68	-	-	-	-	-
	-	423	68	18	-	-	-	-	-
7.5	-	-	609	152	38	-	-	-	-
	-	-	152	38	12	-	-	-	-
10	-	-	1760	271	68	30	-	-	-
	-	-	271	68	18	12	-	-	-
15	-	-	-	609	152	68	38	25	-
	-	-	-	152	38	18	12	9	-
20	-	-	-	1760	271	119	68	44	30
	-	-	-	271	68	30	18	13	10
25	-	-	-	-	422	187	106	68	46
	-	-	-	-	106	47	27	18	14

drawing the abscissa of hole X1 is 3 - 5 - 1 and the ordinate 1 - 6 - 3.

If a table of random figures is used, the coordinates of the holes to be drilled will be plotted in the square, until there is a sufficient number of plots inside of the contour. The holes outside of the contour line will not be drilled. In Figure 18, page 62, the holes Y_1 and Y_3 will be drilled, whereas plot Y_2 will not be drilled, because located outside of the contour.

(3) Execution

a. During the drilling. Once the layout is determined, the operations can begin. It is advisable, as already mentioned, to follow the order of appearance of the location.

Upon completion of a hole, the results will be analyzed in order to determine if the deposit can be zoned around this hole, and to check if the assay deviations are significant.

A continuous plot of standard deviation, as well as of sample mean should be made in order to check if both are tending toward a limit with an increasing number of samples (The plot needs to be made only at the appearance of each fifth or tenth assay). If they are not, the reason has to be discovered probably there is a superposition of several populations. At the completion of the drillings a frequency diagram will be plotted, which will show if different populations can be assumed or not. It will also show the type of distribution and determine which estimator has to be chosen to express the average grade.

C. <u>Adjustments</u>

The results of the campaign can show that our assumptions on

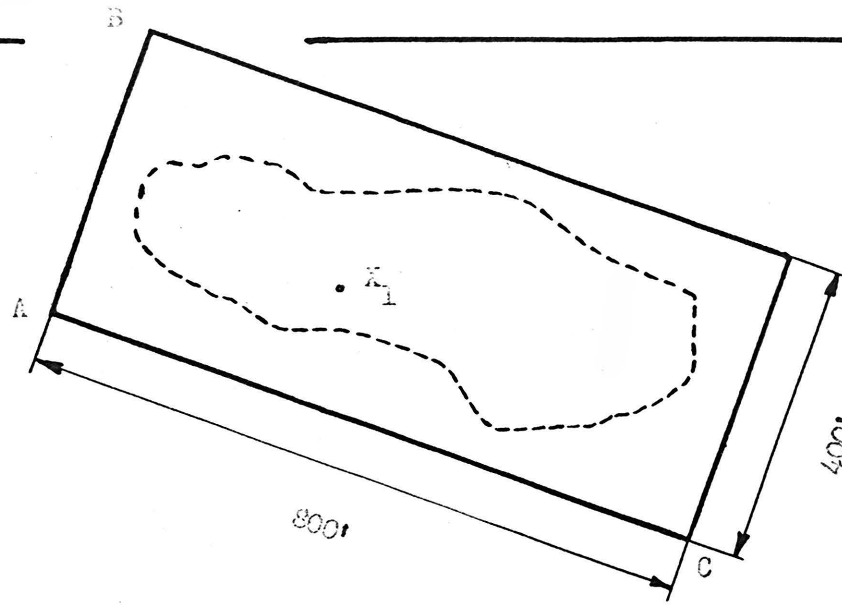


Figure 17 - Location of bore holes
by drawing lots.

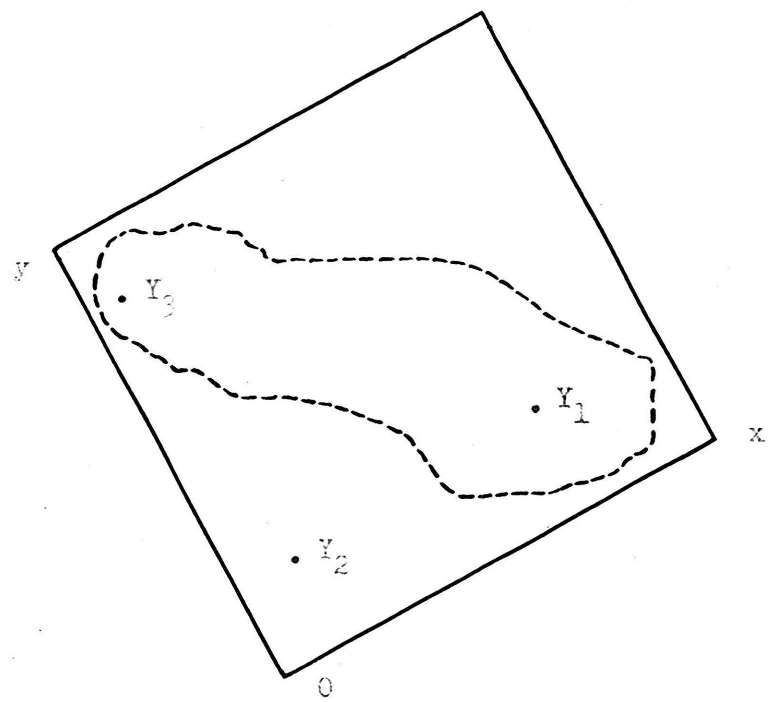


Figure 18 - Location of bore holes

mean and standard deviation were wrong, and that the determined number of samples was too low. In that case the corrected number will be determined and the missing drillings completed. A second adjustment may still be necessary, but is less likely.

A zonation may appear in the deposit, and then one or several zones only, not the entire deposit, may require an additional number of holes. Here the method of "optimum allowance" can be used.

One commodity can be determined with enough accuracy, whereas another would require a higher number of samples. The decision has then to be taken on economical basis.

D. Determination of grade and tonnage

According to the type of distribution the grade will be computed by different ways:

- lognormal distribution: Sichel's estimator

- binomial distribution: de Wijs estimator

- normal distribution: AM or Sichel's estimator, since the latter has a smaller variance. On the basis of the present experience, these are the only three types of distribution we may expect.

If different zones are present, the grades have to be computed separately for each. The knowledge of the average grade of the deposit is not useful in that case.

The volume of a deposit or of a zone will be computed by a classical method.

Chapter VIII

CONCLUSIONS

The basic requirements of a statistical approach are:

- sufficiently large number of samples
- similarity of samples
- randomness in sampling and distribution

If these requirements cannot be satisfied it is more advisable to use traditional methods of publications.

The following conclusions have been drawn:

1. The main advantage of statistical methods is to permit closer estimates of the average grade, and to back the results by their levels of confidence.

2. In two cases at least classical methods should be preferred.

- Bedded deposits with high correlation between neighboring holes. Randomness of distribution is not obtained, and a systematic pattern is to be preferred.

- Elongated deposits. A sectional pattern will yield more information.

3. Care has to be given to zonation any time this is possible.

4. In order to obtain the most beneficial application of statistical methods to the computation of representative parameters of a deposit the following steps must be taken:

a. Planning the exploration

(1) Determination of the number of holes

The necessary data: average grade and its consistency are provided by a neighboring deposit of same type, or by a previous campaign; or they can be assumed.

(2) Determination of the location of holes

The coordinates will be determined by drawing lots or using tables of random figures. The order of drilling the holes should be the same as the order of obtaining the coordinates in order to stop the campaign at any time. An exception can be made for holes which are likely to fall outside of the deposit, which should be drilled only if it is not reasonably certain that they will be blank.

b. Adjustments

(1) Number of holes

Certain holes drilled during the first step will be blank. They will be useful to delimit the deposit, but are useless in the computations. The execution of the first holes may show that the average grade is smaller, or the standard deviation larger than expected. This will require more samples. If the deposit extends beyond the arbitrary limits, if a definite zonation appears, or if the application of the "maximum allowance" is described more drillings must be performed.

(2) Location of the additional holes. Position will be determined as before.

(3) A second adjustment if necessary.

5. The arithmetic mean is advisable for the estimation of the average if the samples are numerous and if the results have to be obtained quickly. However the usage of an estimator is recommended.

6. It is particularly important to recall that the number of holes is not directly dependent upon the size of the deposit, but rather upon its characteristics.

7. The main disadvantage of statistical methods is the rigidity of the sampling pattern with respect to the conditions of relief for the implantation of holes.

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APPENDIX A

GLOSSARY

Coefficient of correlation: indicates the closeness of the relation between two variables, e.g. grades of two metals in the same deposit. For a given number of data and a certain level of confidence, it is significant only above a certain value, given in tables.

Deviation: difference between an experimental value and the expected value.

Discrete: a data which can take only a limited number of values, e.g. only integers.

Distribution:

normal distribution: symmetric distribution around a central value, famous for its bell-shaped curve. It is followed by numerous sets of data.

binomial distribution: discrete distribution denoting the probability that an event will take place x times if n observations are made.

lognormal distribution: similar to the normal distribution. However the logs of the values, not the values themselves are normally distributed.

Estimator: a quantity which may be difficult to compute in using its definition, can be computed by approximation; the latter figure will be the estimator.

Fabrication control: control of quality and its constancy, in mass production.

Frequency curve: plot of the percent frequency of a data vs. value of that data. A cumulative freguency curve is the plot of the percent frequencies of all the values below (negative frequency curve) or above (positive frequency curve) a given value of the data, vs. that data.

Level of confidence: Percent of successes which a method may expect on a long run.

Mean: sum of the values of a set of data, divided by their number.

Median: parameter such that the same numbers of data lay above as below.

Mode: value with the highest frequency of occurrence.

Population: the largest set of possible data from which a sample is taken.

Range: difference between the highest and the lowest value of a set of data.

Regression analysis: study of the interdependence of two quantities, e

Stratification*: subdivision of a population in "strates", of which each will be handled as an independent population.

Universe: see population.

Variance: square of the standard deviation.

Weight: factor by which a value is multiplied, in order to have a balanced representation of all data.

* Standard deviation: square root of the mean squared deviation.

APPENDIX B

EXAMPLE OF VALUATION: ADAM ZONE OF THE SUFFIELD MINE, SHERBROOKE, QUEBEC

A. Introduction

The assay data used in this paragraph are condensed in Table 2. They have been found in a progress report of the Ascot Metals Corporation (now Quebec Ascot Metals Corporation) in date of March 15, 1951. The explored deposit is located near Sherbrooke, in the southernmost part of the Province of Quebec, Canada. At present time it is not operated.

Unfortunately it has not been possible to find a geologic description of the Suffield Mine. However, according to plate I the deposit is flat-lying on an east dipping slope, and imbedded in sedimentary rocks with a NS axis. Therefore it can be admitted that it belongs to the Mississippi Valley type. The northern half only, is surrounded by blank holes, so that a volume estimation, which would be more than a mere guess is not possible.

B. Utilization of the results

(1) Grade estimate

Various figures have been computed for each commodity: AM, GM, Sichel's estimator. The standard deviation and the coefficient of variation (ratio of standard deviation to AM) are also given. This information is condensed in Table 3.

The variations in assays between the different holes are high, thus the use of Sichel's estimator is advisable.

TABLE 2

ASSAY RESULTS AND THICKNESS

Hole No.	Thickness, feet	Zn percent	Pb percent	Cu percent	Ag oz./ton	Au oz./ton
8	14	6.25	1.62	0.40	2.92	-
22	13.3	4.45	1.03	0.87	3.11	0.011
26	11.7	5.04	0.80	0.42	0.73	0.010
23	27	9.27	0.97	2.22	3.89	0.005
20	33.2	5.21	0.59	4.22	8.14	0.033
1	25	17.42	0.46	3.40	7.08	0.062
18	25.5	18.87	0.88	4.32	8.70	0.010
29	17.1	7.60	0.27	2.08	4.46	0.060
61	16	4.44	0.24	0.45	1.68	0.040
57	10.5	4.56	0.49	0.43	4.02	0.025
60	10.5	10.26	1.30	2.62	5.76	0.031
63	7.5	7.31	0.20	2.53	2.38	0.013
67	16.5	6.16	0.89	0.69	0.69	0.004
38	58	2.29	0.20	0.22	-	-
32	23.5	2.30	0.06	0.35	0.04	-
33	13	9.10	0.98	0.80	1.76	-
35	58	2.29	0.24	0.28	0.05	-
86	8	3.54	0.06	0.58	1.39	0.003
88	7	11.49	3.76	0.57	3.30	0.005
41	10	3.49	0.67	0.05	1.26	0.004
42	65	3.56	0.46	0.54	0.86	-
45	21	2.91	0.41	1.05	0.10	0.007
44	76.5	3.02	0.28	0.57	0.63	-
51	15	3.19	0.54	0.34	1.24	0.010
90	4.5	10.45	3.02	1.82	3.19	0.010
58	10.5	11.49	3.76	0.57	3.30	0.031
65	12.5	5.23	0.92	0.60	2.72	0.010
72	9.0	3.04	0.09	0.35	0.37	-
70	14	5.10	0.63	0.60	2.57	0.014
80	15	8.56	1.68	1.98	6.69	0.010
79	4	9.54	1.20	1.99	5.74	0.030
81	14	10.93	1.05	2.03	3.01	0.007
82	20.5	5.27	0.95	0.64	1.56	0.015
85	14	4.03	0.77	0.66	1.25	0.006
93	21	14.98	1.11	1.54	3.04	0.024
91	4	4.36	0.08	0.61	3.11	0.030

frequent association). We suggest therefore a microscopic investigation, followed by experimental work in a pilot concentration plant in order to determine the conditions of separation of these commodities. Since copper and silver show a high correlation with zinc it is normal that their correlation with each other is high; its negative sign could be explained as occurrence of both metals in one mineral, as reciprocal substitution.

Lead shows a lower, though significant coefficient of correlation with zinc than do copper and silver. Correspondingly the relation with copper and silver is less marked.

TABLE 4

COEFFICIENTS OF CORRELATION

Metal	Thickness	Zn	Pb	Cu	Ag
Zn	0.0796	-	-	-	-
Pb	-0.237	.473	-	-	-
Cu	0.649	.595	.0367	-	-
Ag	-0.216	.703	.259	-0.646	-

Levels of significance of the coefficient of correlation in this set of samples with a confidence limit of

95% r = 0.331

99% r = 0.436

The figures satisfying the 95% level of confidence have been underlined once. Those satisfying the 99% level of confidence, twice.

(3) Distribution of assays

Sturge's rule indicates that the optimum number of intervals is 7. (N the optimum number of intervals is given by

$$N = 1 + 3.3 \log n$$

n being the number of samples.) The number of intervals in the different distribution tables has been kept as close as possible to this figure.

Since the number of samples is relatively small a good fit with a theoretical curve cannot be expected. However all the curves are plurimodal. Two modes are well marked for each metal, a third one is less definite.

Tables 5 to 8 show the distribution for each element, Table 9 recapitulates the modes of each metal.

The logarithmic distributions of lead and copper have a unique peak, that of lead is almost symmetric.

C. <u>Adjustments</u>

(1) Is the number of holes adequate?

Table 10 indicates the partial AM and standard deviation computed upon completion of each fifth hole. The standard deviation of zinc assays is the only quantity that reached an asymptote: an

In the hypothesis of a normal distribution, the minimum number of holes would be indicated by the methods given on page 59.

TABLE 5

DISTRIBUTION OF ZINC ASSAYS

(1) Arithmetical distribution: total range

Assay interval, percent	Frequency	Modes
2 - 4	11	#I
4 - 6	10	
6 - 8	4	
8 - 10	4	
10 - 12	5	#II
12 - 14	-	
14 - 16	1	#III
16 - 18	1	
18 - 20	1	

(2) Arithmetical distribution: range from 2 to 12

Assay interval, percent	Frequency	Modes
2 - 3	4	
3 - 4	6	
4 - 5	5	#I
5 - 6	5	
6 - 7	2	
7 - 8	2	
8 - 9	1	
9 - 10	3	#II
10 - 11	3	
11 - 12	2	

(3) Logarithmic distribution

Assay interval (logs)	Frequency	Modes
0.35 - 0.45	3	
0.45 - 0.55	6	x
0.55 - 0.65	5	or x?
0.65 - 0.75	6	x
0.75 - 0.85	2	
0.85 - 0.95	3	
0.95 - 1.05	6	x
1.05 - 1.15	2	
1.15 - 1.25	1	
1.25 - 1.50	2	x ?

TABLE 6

DISTRIBUTION OF LEAD ASSAYS

(1) Arithmetical distribution: total range

Assay interval, percent	Frequency	Modes
0.0 - 0.5	15	I and II
0.5 - 1.0	11	
1.0 - 1.5	5	
1.5 - 2.0	2	
2.0 - 2.5		
2.5 - 3.0		
3.0 - 3.5	1	
3.5 - 4.0	2	III

(2) Arithmetical distribution: range from 0.50% to 1.75%

Assay interval, percent	Frequency	Modes
0.0 - 0.25	8	I
0.25 - 0.50	6	
0.50 - 0.75	4	
0.75 - 1.00	8	II
1.00 - 1.25	4	
1.25 - 1.50	1	
1.50 - 2.00	2	?

(3) Logarithmic distribution

Assay interval, (logs)	Frequency	Modes
$\bar{2}.5$ - $\bar{2}.0$	4	
$\bar{2}.0$ - $\bar{1}.5$	6	
$\bar{1}.5$ - 0.0	16	x
0.0 - 0.5	8	
0.5 - 1.0	2	

TABLE 7

DISTRIBUTION OF COPPER ASSAYS

(1) Arithmetical distribution: total range

Assay interval, percent	Frequency	Modes
0.0 - 0.5	10	
0.5 - 1.0	13	I
1.0 - 1.5	1	
1.5 - 2.0	4	II
2.0 - 2.5	3	
2.5 - 3.0	2	
3.0 - 3.5	1	
3.5 - 4.0	-	
4.0 - 4.5	2	III

(2) Logarithmic distribution

Assay interval, (logs)	Frequency	Modes
$\bar{2}.5 - \bar{1}.0$	1	
$\bar{1}.0 - \bar{1}.5$	2	
$\bar{1}.5 - 0.0$	20	x
0.0 - 0.5	10	
0.5 - 1.0	3	

TABLE 8

DISTRIBUTION OF SILVER ASSAYS

(1) Arithmetical distribution: total range

Assay interval, percent	Frequency	Modes
0 - 1	8	I
1 - 2	7	
2 - 3	4	
3 - 4	8	II
4 - 5	2	
5 - 6	2	
6 - 7	1	
7 - 8	1	
8 - 9	2	III

(2) Logarithmic distribution

Assay interval, (logs)	Frequency	Modes
$\bar{2}.5 - \bar{1}$	2	
$\bar{1}$	1	
$\bar{1}.5$	5	
0	16	
0.5	12	

TABLE 9

MODES OF THE DISTRIBUTIONS OF THE METALLIC ASSAYS

	I	II	III
Zn	3%	11%	17%
Pb	0.25%	0.82%	1.02% (?)
Cu	0.5%	2.0%	4.0%
Ag	1 oz/ton	3 oz/ton	8 oz/ton

With

tolerated error = 20%

level of confidence = 95% (t = 1,645)

we have following

Metal	Zn	Pb	Cu	Ag
Holes	25	67	57	41

The present number of holes is satisfying only for zinc; but the fact that the distribution is not normal requires evan a higher number of holes.

TABLE 10

PARTIAL ARITHMETIC MEANS AND STANDARD DEVIATIONS

Number of samples*		Zn, percent	Pb, percent	Cu, percent	Ag, oz/ton
10	AM	8.31	0.735	1.88	4.47
	Std Dev	5.34	0.437	1.67	3.07
15	AM	7.43	0.667	1.68	3.54
	Std Dev	4.76	0.503	1.50	2.68
20		7.07	0.786	1.375	3.07
		4.52	0.811	1.35	2.51
25		6.58	0.817	1.27	2.70
		4.37	0.862	1.24	2.40
30		6.60	0.916	1.20	2.77
		4.17	0.967	1.17	2.35
36		6.86	0.907	1.20	2.88
		4.18	0.896	1.10	2.23

* In order of appearance.

(2) Zonation

The contents belonging to the modal peaks of same rank for each metal fall together very often. It is however impossible to delineate general trends over the deposit. The assays of one of these modal peaks are spread over the area, and are not limited to a given part of it.

It should be decided after consideration of the financial situation if an increase in the knowledge of the deposit justifies increased expenses for exploration. A study of the holes not as an entire data, but as series of core segment could throw some light.

D. Conclusions

The application of statistics to the Suffield exploration project leads to following conclusions:

- The average grade of the ore has been estimated by Sichel's estimator as follows:

Zinc	6.79%
Lead	0.81%
Copper	1.09%
Silver	2.20%

- The grade distribution of zinc, lead, copper and silver shows three modes, allowing a subdivision of the assays of each metal into three subpopulations. The assays encountered in a hole usually belong to subpopulations of same rank.

- No relationship exists between thickness and content in zinc, lead or silver. A relationship is possible between thickness and grade in copper.

- A connection grades of copper and silver with the grade of zinc is likely.

- The number of holes which has been drilled guarantees a good accuracy for the zinc grade, which is the main commodity. The know ledge of the other metals could be increased by a higher number of samples, but is not worth-while as they are of less importance.

E. Recommendations

- The presence of three subpopulations indicates the possibility of zonation. It has not been possible to zone the deposit with global data for each drill hole. A study of the drill logs with assays of each core section could throw some light on this problem.

- The high correlation between Cu and Zn could be indicative of a close mineralogical association. Problems in ore dressing could arise for the separation of the Cu from Zn, if for instance the copper would partially occur as blebs of chalcopyrite in sphalerite, which is very frequent. The negative correlation between copper and silver may indicate that in one ore mineral both are in reciprocal substitution, which would complicate the separation. A preliminary microscopic study followed by experiences in pilot plant can be expected to solve this problem.

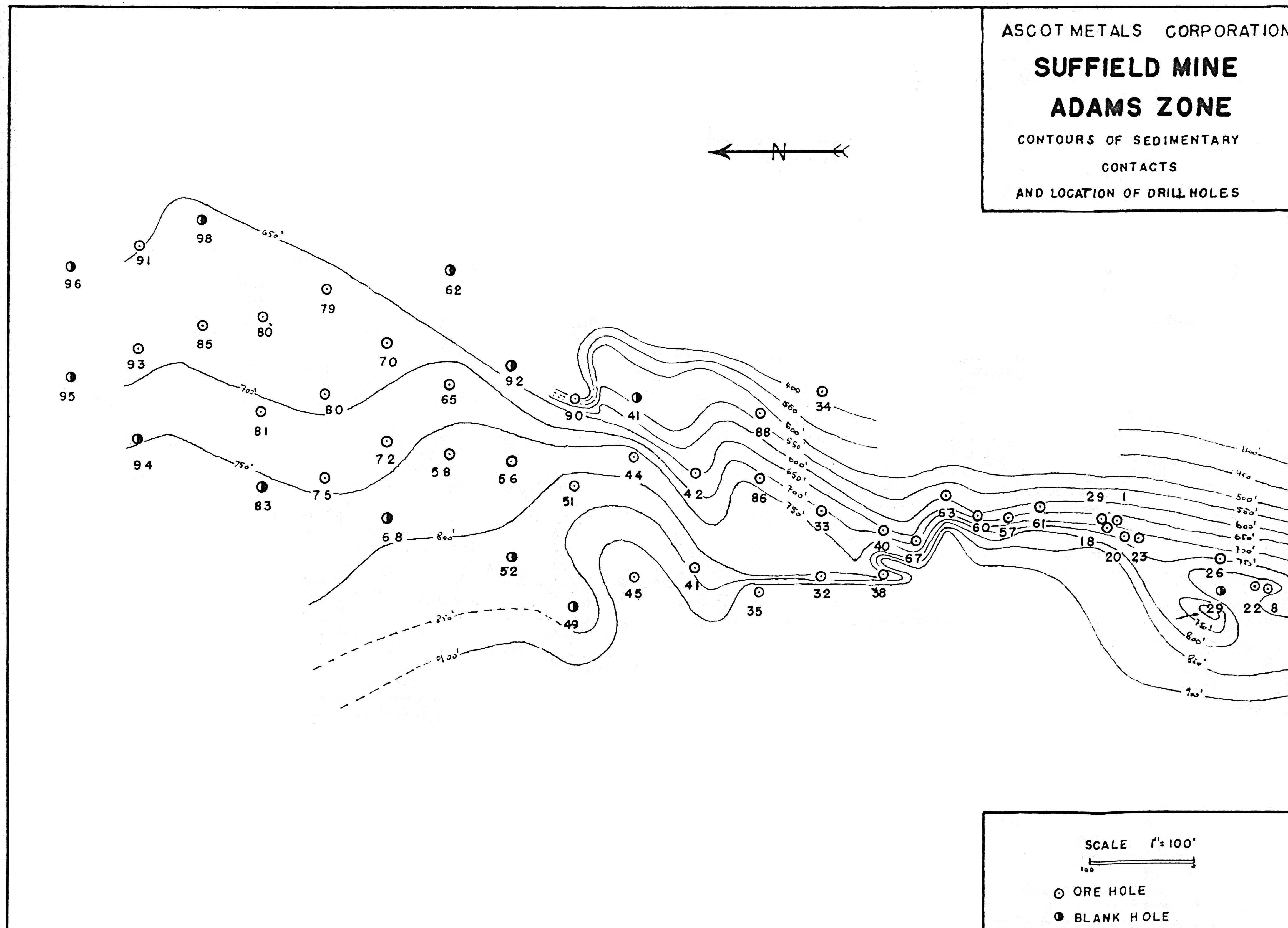
- The present number of holes is satisfactory with respect to the knowledge in grade. However the determination of the volume of reserves requires some peripherical holes in the southern half of the deposit in order to delimit the cutoff.

F. Recommended procedure for exploring this deposit (assuming no previous work had been done)

The historical circumstances concerning the discovery of the Suffield mine are unknown to us, therefore some assumptions have to be made. We can admit that the possible existence of a sulphide body was discovered by geophysical prospecting. The results of such a geophysical campaign, as well as the structural geology would indicate an elongated ore body, oriented north-south, of about 2,000 feet in length and about 500 feet in transverse section.

As it is matter of a definitely elongated ore body, a sectional pattern could seem advisable. However the structure is very irregular and varies in width from south to north. A random pattern can be placed in order to allow an evaluation by statistical methods.

The following layout is suggested on hand of this information: a rectangle of 2,000 x 500 feet will be laid out above the supposed location of the deposit. This has been the first lead and zinc deposit discovered in this area so that no information concerning the characteristics of this deposit can be obtained from similar deposits in the same district. The number of bore holes will be arbitrarily fixed to 30. Upon completion of these first set of holes we will have enough data to decide an adjustment, i.e., the eventual drilling of more holes. An exception to the rule recommending the holes being drilled in the order of appearance of their coordinates can be made here: the holes located at the periphery of the quadrangle can be drilled last in order to avoid too many blank holes. The outer boundary of the deposit is not necessarily included in the rectangle and can on some sides partially leave the rectangle. In that case, another rectangle of same size is to be placed beside the first, and the coordinates of new drill holes determined in order that the number of holes is the same in both rectangles, however only the holes located in the portion where the body protrudes into the second rectangle are drilled. This complicated procedure assures the respect of the condition of random implantation. When the drilling program is completed, the results will be handled as described in the preceding pages.


ASCOT METALS CORPORATION
SUFFIELD MINE
ADAMS ZONE
CONTOURS OF SEDIMENTARY
CONTACTS
AND LOCATION OF DRILL HOLES
N
SCALE 1" = 100'
ORE HOLE
BLANK HOLE

VITA

Francis R. Saupe', son of Paul O. Saupe' and Alphonsine A. M. Saupe', was born May 21, 1934 in Strasbourg, France. His family has resided since 1924 in Schiltigheim, France, where he received his elementary education from 1940 to 1944.

He enrolled at the Lycee Fustel de Coulanges, Strasbourg in 1945, where he stayed until 1950, at which time he changed to the Lycee Kleber. In 1952 he left the latter school, after obtaining the degree of Bachelier es Mathematiques from the Academie of Strasbourg. The two following years he prepared for the entrance examination to the National School of Agriculture, where he was admitted in 1954. Upon resignation, he prepared for the entrance examination to the Ecole Nationale Superieure de Geologie Appliquee et de Prospection Miniere in Nancy, where he was admitted in 1956. He graduated from that school in 1958, with the degree of Ingenieur Geologue and obtained also the degree of Licencie es Sciences from the University of Nancy in the same year.

The summer of 1954 was spent visiting the iron mines of Graengesberg, Norberg and Danemora in central Sweden, through a Zellidja stipend.

Various part-time employments were held during his college and university years, in geological surveying (field work and aerial photography), well logging and interpreting, examination of mineral property, determination of stream sediments, and teaching laboratories in ore-microscopy.

A Fulbright Grant and a Graduate Assistantship made it possible for him to stay at the Missouri School of Mines from September 1958 until July 1959.

Professional societies include the American Institute of Mining Engineers and the Societe' de l'Industrie Minerale. Scientific societies include the Sigma Gamma Epsilon and the Sigma Xi.