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Stress analysis of thin bedded mine roofs subjected to evenly distributed lateral load and supported by barrier pillars

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STRESS ANALYSIS OF THIN BEDDED MINE ROOFS SUBJECTED TO EVENLY DISTRIBUTED LATERAL LOAD AND SUPPORTED BY BARRIER PILLARS

BY

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A

THESIS

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INTRODUCTION

Theoretical stress analysis in mine structures begins with the assumption that the structural elements are homogenous, isotropic and perfectly elastic. It is this writers belief that stresses in a mine roof supported by barrier pillars may be compared to those stresses which may be found in a thin plate supported and restrained in the manner similar to such a mine roof. Therefore the stress and moment equations developed for plates are herein applied to mine roofs of limited thickness.

A thin roof bed which is stressed with evenly distributed lateral load and supported with barrier pillars will be regarded as a large rectangular plate clamped at the two longitudinal ends and with the other edges simply supported.

Figure 1a shows the free body diagram of such a plate. AB and CD are built-in or clamped edges, and AD and BC are simply supported edges. M_y is the bending moment along the built-in edges, and q is the unit load on the mine roof bed. The weight of the bed per unit area may be included into the q. The moment M_y will be considered positive and when it produces compression at the top of the bed.

The most convenient method to analyze the stresses in such a plate will be to find a general equation for the deflection of a rectangular plate which is loaded similar to the mine roof bed.

To determine the general equation of bending for such a plate it is necessary to take two general cases of bending of plates: One of which is the bending of a plate by moments along the edges of the plate as shown in Figure 1c, and the other one is the bending of the plates by evenly distributed load, Figure 1b, and then superimpose them to apply for a plate which is subjected to evenly distributed loads and contains bending moments along two parallel edges.

First, the relations between the bending moments and the curvature in pure bending of plates will be determined.

REVIEW OF LITERATURE

There are numerous articles written on the stress analysis of mine structures, but there are no theoretical studies which may be complimentary to the work done in this thesis. Therefore, the reader, to supplement his knowledge on the subject, should refer to those books listed in the bibliography.

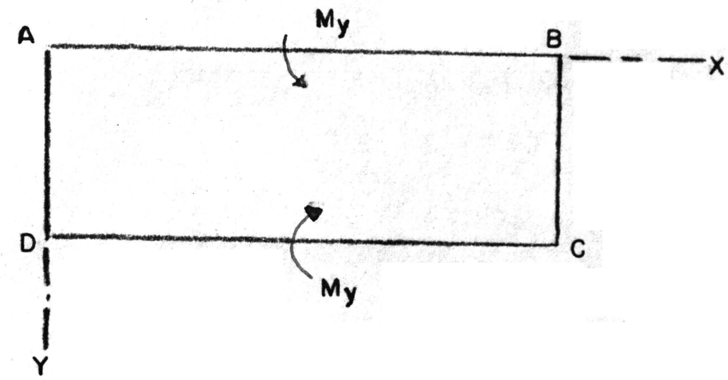

FIGURE_1a Moments in the mine roof.

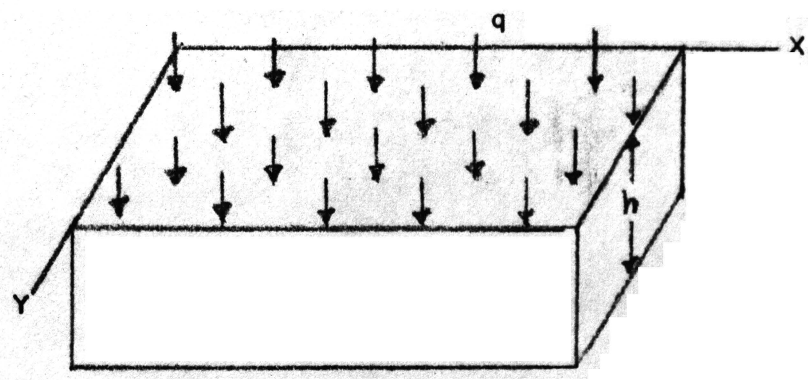

FIGURE_1b Loading of the mine roof.

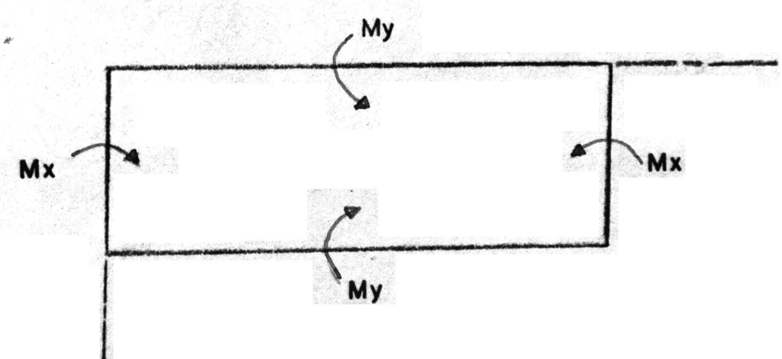

FIGURE_1c Plate under the action of bending moments.

A- RELATIONS BETWEEN BENDING MOMENTS AND CURVATURE IN PURE BENDING OF PLATES

Figure 1c shows a rectangular plate, the bending moments M_x and M_y are uniformly distributed along the edges. The XY plane coincides with the middle plane of the plate before it is deflected. The X- and Y-axis are along the edges of the plate, and Z-axis is perpendicular to the XY plane at point (O), and is taken positive downward. The bending moment M_x acts on the edges parallel to the Y-axis, and M_y also a bending moment-acts on the edges parallel to the X-axis.

Figure 2 shows an element cut out of this plate by two planes parallel to XZ and YZ planes. By assuming that the lateral sides of this element remain plane and rotate about the neutral axis nn so as always to remain normal to the deflected middle surface of the plate, it can be concluded that the middle surface of the plate does not undergo any extention during this bending. The middle surface, nnnn Figure 2, is called the neutral surface.

The curvature of the deflection of a plate, when the plate is bent, can be expressed as $\left[-\frac{\partial^2 \omega}{\partial x^2}\right]$ in the XZ plane, and $\left[-\frac{\partial^2 \omega}{\partial y^2}\right]$ in the YZ plane. Where ω is the deflection of the plate in the Z direction.

The unit elongations ε_x and ε_y of a fiber, along the X- and Y-axis respectively, and at a distance Z from the neutral surface is then

$$\varepsilon_x = -Z\frac{\partial^2 \omega}{\partial x^2}$$
$$\varepsilon_y = -Z\frac{\partial^2 \omega}{\partial y^2} \qquad 1$$

According to Hooke's Law, the unit elongations ε_x and ε_y in terms of normal stresses σ_x and σ_y acting on the element are given as

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$
$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) \qquad 2$$

(1) Timoshenko, S., Theory of elasticity, p.8, New York, McGraw Hill Book Co., 1934.

By using equations (2), the normal stress σ_x and σ_y can be determined in terms of unit elongations ε_x and ε_y.

From the first equation of equations (2),

$$\frac{\sigma_x}{E} = \varepsilon_x + \nu\frac{\sigma_y}{E}$$

and if the right hand side of this equation is substituted in the second equation of equations (2), σ_y can be found as,

$$\sigma_y = -\frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

by the same method σ_x is found as

$$\sigma_x = -\frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

By substituting the values of ε_x and ε_y from equations (1), into these two equations, it will be found that,

$$\sigma_x = \frac{-E}{1-\mu^2} Z \left(\frac{\partial^2 \omega}{\partial x^2} + \mu \frac{\partial^2 \omega}{\partial y^2}\right)$$
$$\sigma_y = \frac{-E}{1-\mu^2} Z \left(\frac{\partial^2 \omega}{\partial y^2} + \mu \frac{\partial^2 \omega}{\partial x^2}\right) \quad 3$$

These normal stresses distributed over the lateral sides of the element in Figure 2, can be reduced to couples, the magnitudes of which per unit length must be equal to the external moments M_x and M_y . In this way the following equations are obtained.

$$\int_{-h/2}^{+h/2} \sigma_x Z \, dz \, dy = M_x \, dy$$
$$\int_{-h/2}^{+h/2} \sigma_y Z \, dz \, dx = M_y \, dx \quad 4$$

(h) is the thickness of plate. By substituting σ_x and σ_y in equations (4) and evaluating between $-\frac{h}{2}$ and $+\frac{h}{2}$ the following equations result. (See Appendix A)

$$M_x = -\frac{Eh^3}{12(1-\mu^2)} \left(\frac{\partial^2 \omega}{\partial x^2} + \mu \frac{\partial^2 \omega}{\partial y^2}\right)$$
$$M_y = -\frac{Eh^3}{12(1-\mu^2)} \left(\frac{\partial^2 \omega}{\partial y^2} + \mu \frac{\partial^2 \omega}{\partial x^2}\right) \quad 4a$$

The term $\left[\frac{Eh^3}{12(1-\mu^2)}\right]$ is called the flectural rigidity of the plate. It takes the place of (EI) in the case of beams. The flectural rigidity is customarily shown with the capital D. By substituting D in equation (4a) in place of $\left[\frac{Eh^3}{12(1-\mu^2)}\right]$,

$$M_x = -D \left(\frac{\partial^2 \omega}{\partial x^2} + \mu \frac{\partial^2 \omega}{\partial y^2}\right)$$
$$M_y = -D \left(\frac{\partial^2 \omega}{\partial y^2} + \mu \frac{\partial^2 \omega}{\partial x^2}\right) \quad 5$$

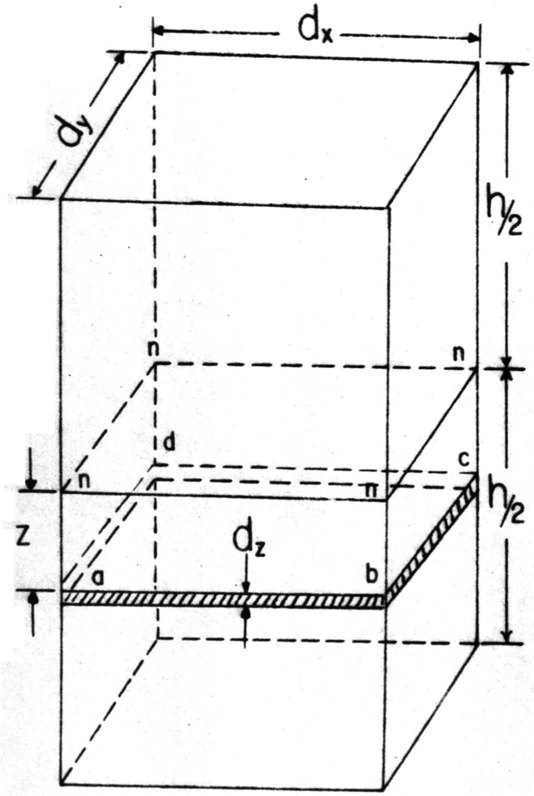

FIGURE _ 2

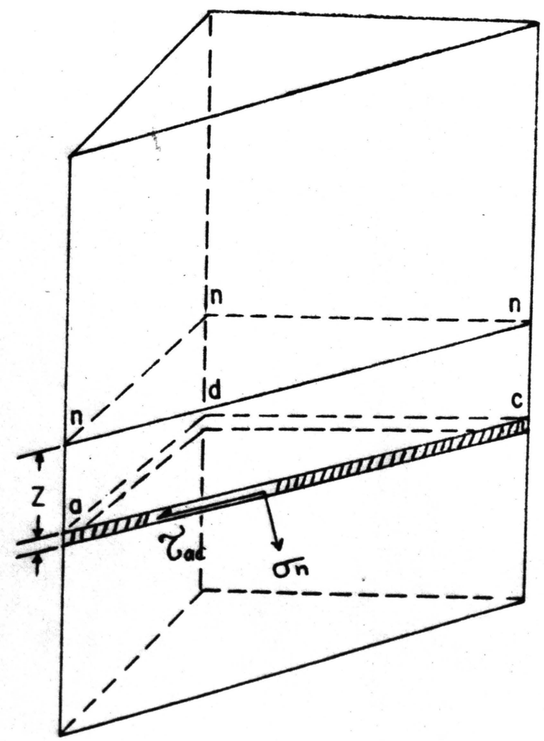

FIGURE _ 2a

Now, consider the stresses acting on a section of the lamina abcd Figure 2. When the plate is bent this lamina undergoes distortion, Figure 3, (O) moves to (O'). The total deformation is equal to the sum of the angles $A'O'X'(\alpha)$ and $B'O'X'(\beta)$. This sum is equal to the shearing strain between the planes XZ and YZ[2].

(2) Ibid. P.6

Therefore,

$$\tan A'O'X' = \frac{\frac{\partial v}{\partial x}dx}{dx} = \frac{\partial v}{\partial x}$$

$$\tan B'O'Y' = \frac{\frac{\partial u}{\partial y}dy}{dy} = \frac{\partial u}{\partial y}$$

These angles are small. By trigonometry, when α is small

$$\tan \alpha = \alpha \quad \text{and} \quad \tan \beta = \beta$$

Then

$$\alpha + \beta = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

The sum of α and β is equal to shearing strain, and it is shown by γ,

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

The subscripts of γ show the plane in which the components of shearing strain act.

The corresponding shearing stress is

$$\tau_{xy} = G\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^{(3)}$$

(3) Ibid. p.9-10

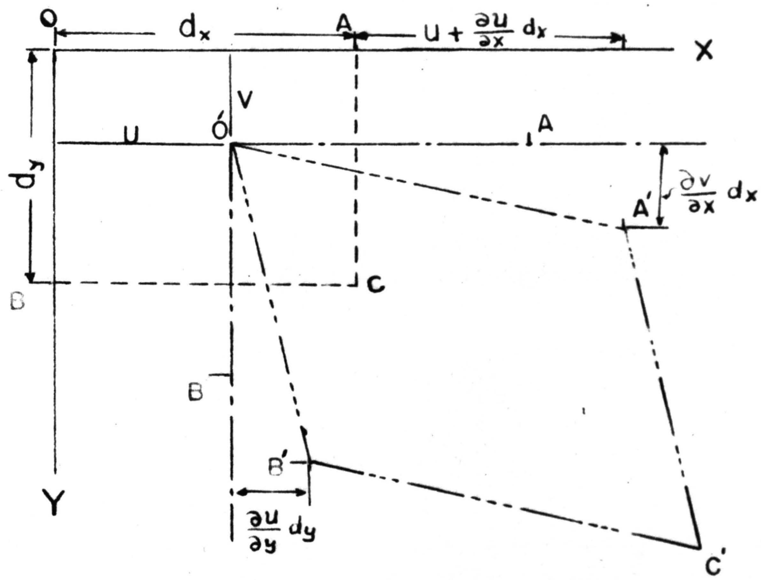

FIGURE _ 3

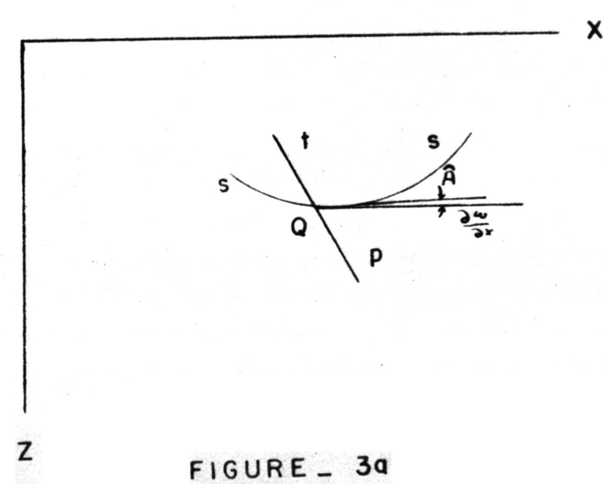

FIGURE _ 3a

Where $\left[G = \frac{E}{2(1+\nu)}\right]$ and is called the modulus of rigidity.

Now, take a section of the deformation curve along the XZ plane, Figure 3a, and consider that the line pt was originally perpendicular to the X-axis or the XY plane. When deformation takes place line pt will rotate in a counterclockwise direction through an angle $\hat{A}$. If this deformation angle is a small angle

$$\hat{A} = \frac{\partial \omega}{\partial x}$$

Owing to the rotation, a point of the element pt at a distance Z from the natural surface has a displacement in the X direction equal to

$$u = -Z\frac{\partial \omega}{\partial x}$$

By considering another section through the YZ plane, it can be shown that the same point has a displacement in the Y direction equal to

$$v = -Z\frac{\partial \omega}{\partial y}$$

Substituting these values of U and V in equation (6), it will be obtained that

$$\tau_{xy} = -2GZ\frac{\partial^2 \omega}{\partial x \partial y} \qquad 6a$$

Considering all laminas, such as adc in Figure 2a, the shearing stresses cause the twisting moment acting on sections ac of the plate. The magnitude of the twisting moment is

$$M_{xy} = \int_{-h/2}^{+h/2} \tau_{xy}\, Z\, dz = \frac{Gh^3}{6} \times \frac{\partial^2 \omega}{\partial x \partial y}$$

If the value of G is substituted in the above expression,

$$M_{xy} = D(1-\nu)\frac{\partial^2 \omega}{\partial x \partial y} \qquad 7$$

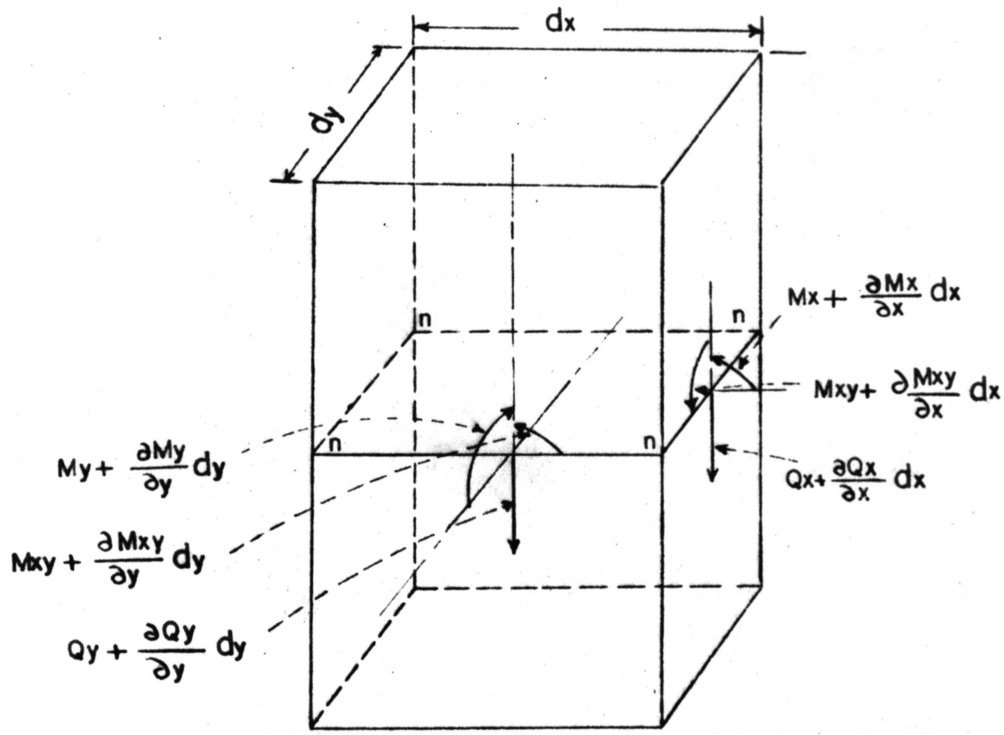

FIGURE _ 4

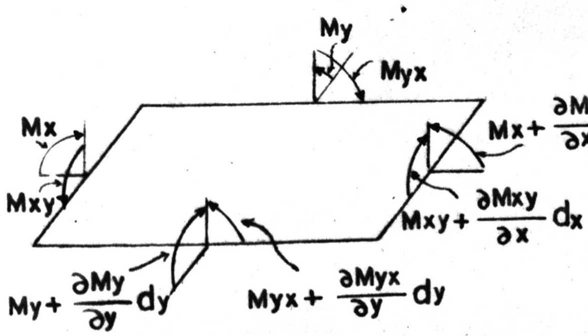

FIGURE _ 4a

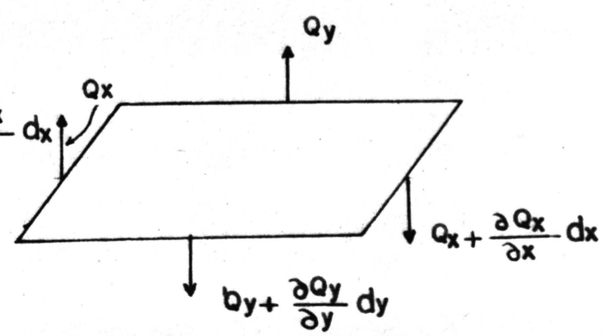

FIGURE _ 4b

B- DEFLECTION DUE TO THE EVENLY DISTRIBUTED LATERAL LOAD

When a plate is bent by an evenly distributed lateral load, in addition to the bending moments M_x and M_y, and the twisting moment M_{xy} which was found when considering the pure bending of a plate with the bending moments along the edges, there are vertical shearing forces acting on the sides of an element which is cut from a laterally loaded plate in the same manner as described in the previous case.(4)

(4) Timoshenko, S., Theory of plates and shells, p. 85, New York, McGraw Hill Book Co., 1940.

The magnitudes of these forces per unit length parallel to the X and Y-axis, will be shown by Q_x and Q_y respectively, Figure 4.

$$Q_x = \int_{-h/2}^{+h/2} \tau_{xz}\, dz \quad , \quad Q_y = \int_{-h/2}^{+h/2} \tau_{yz}\, dz \qquad 8$$

From equations 5, 7, and 8, it can be seen that the moments and the shearing forces are functions of the coordinates X and Y. Therefore, in discussing the equilibrium conditions of the element, the small changes of these quantities will be taken into consideration when the coordinates X and Y change by the small quantities dx and dy.

The neutral surface of the element is shown in Figures 4a and 4b as free body diagrams. Moments are taken positive in the clockwise directions, and negative in counterclockwise direction.

From Figure 4a, by using the fact that the sum of all the forces in the Z direction is equal to zero,

$$-Q_x dy + Q_x dy + \frac{\partial Q_x}{\partial x} \partial x\, dy - Q_y\, dx + Q_y\, dx + \frac{\partial Q_y}{\partial y} dx\, dy + q\, dx\, dy = 0$$

The $(q\, dx\, dy)$ is the load over the element.

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0 \qquad 9$$

The moments of all the forces acting on the element, Figure 4a, which tend to rotate the element about the X-axis, must be equal to zero.

$$M_y\, dx - M_y\, dx - \frac{\partial M_y}{\partial y} dy\, dx - M_{xy}\, dy + M_{xy}\, dy + \frac{\partial M_{xy}}{\partial x} dy\, dx$$

$$+ Q_y\, dy\, dx + \frac{\partial Q_y}{\partial y} d^2y\, dx + q \frac{dx\, d^2y}{2} = 0$$

by simplifying, and neglecting the higher orders of dx and dy the above expression will be reduced to

$$\frac{\partial M_{xy}}{\partial x} - \frac{\partial M_y}{\partial y} + Q_y = 0 \qquad 10$$

Taking the moments about Y-axis in the same manner it will be determined that:

$$\frac{\partial M_{xy}}{\partial y} + \frac{\partial M_x}{\partial x} - Q_x = 0 \qquad 11$$

Equations 9, 10, and 11 completely define the equilibrium of the element, because all the forces acting on the element are taken into consideration. By substituting the values of Q_x and Q_y from equations 10 and 11 into equation 9, it is possible to find a relation between bending twisting moments M_x, M_y, M_{xy}, M_{yx} and q.

$$Q_y = \frac{\partial M_y}{\partial y} - \frac{\partial M_{xy}}{\partial x}$$

$$\frac{\partial Q_y}{\partial y} = \frac{\partial^2 M_y}{\partial y^2} - \frac{\partial^2 M_{xy}}{\partial x \partial y} \qquad 10a$$

and

$$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y}$$

Then,

$$\frac{\partial Q_x}{\partial x} = \frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_{yx}}{\partial x \partial y} \qquad 11a$$

$$\frac{\partial^2 M_{yx}}{\partial x \partial y} + \frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} - \frac{\partial^2 M_{xy}}{\partial x \partial y} + q = 0 \qquad 12a$$

It can be observed from equilibrium of the element that $\tau_{yx} = -\tau_{xy}$, and this gives the condition of $M_{yx} = -M_{xy}$. Substituting this into equation 12a,

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} - 2\frac{\partial^2 M_{xy}}{\partial x \partial y} = -q \qquad 12$$

Now, by substituting the values of M_x and M_y, and M_{xy} from equations 5 and 7, the deflection equation will be obtained.

$$M_x = -D\left(\frac{\partial^2 \omega}{\partial x^2} + \nu \frac{\partial^2 \omega}{\partial y^2}\right)$$

$$M_y = -D\left(\frac{\partial^2 \omega}{\partial y^2} + \nu \frac{\partial^2 \omega}{\partial x^2}\right) \qquad 5$$

$$M_{xy} = -M_{yx} = D(1-\nu)\frac{\partial^2 \omega}{\partial x \partial y} \qquad 7$$

Substituting this expression into equation 12.

$$-D\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 \omega}{\partial x^2}+\nu\frac{\partial^2 \omega}{\partial y^2}\right)-D\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2 \omega}{\partial y^2}+\nu\frac{\partial^2 \omega}{\partial x^2}\right)-2D(1-\nu)\frac{\partial^2}{\partial x\partial y}\cdot\frac{\partial^2 \omega}{\partial x\partial y}=-q$$

$$-D\left[+\frac{\partial^4 \omega}{\partial x^4}+\nu\frac{\partial^4 \omega}{\partial x^2\partial y^2}+\frac{\partial^4 \omega}{\partial y^4}+\nu\frac{\partial^4 \omega}{\partial x^2\partial y^2}+2\frac{\partial^4 \omega}{\partial x^2\partial y^2}-2\nu\frac{\partial^4 \omega}{\partial x^2\partial y^2}\right]=-q$$

and finally

$$\frac{\partial^4 \omega}{\partial x^4}+\frac{\partial^4 \omega}{\partial y^4}+2\frac{\partial^4 \omega}{\partial x^2\partial y^2}=\frac{q}{D} \qquad 13$$

The problem of bending of a mine roof rock subjected to evenly distributed lateral load q is reduced to the integration of equation 13. If for a particular case, a solution of equation 13 is found that satisfies the boundry conditions of the roof; the bending and twisting moments can be calculated from equations 5 and 7, and the corresponding normal stresses can be found from equation 3.

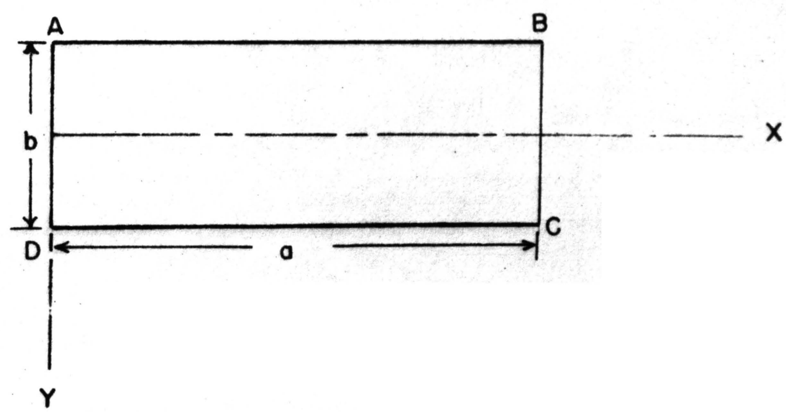

FIGURE_ 5

C-BOUNDARY CONDITIONS IN A MINE ROOF BED SUPPORTED WITH BARRIER PILLARS

Figure 5 shows such a mine roof bed. The edges AB and DC are built-in and edges AD and BC are simply supported.

CONDITIONS AT BUILT-IN EDGES:

The deflection along the built-in edges is zero, and the tangent plane to the deflected middle surface along this edge coincides with the initial position of the middle plane of the roof. Since the X-axis is in the same direction as the built-in edge;

$$\omega = 0 \quad \text{where} \quad y = \pm \frac{b}{2} \qquad \text{a}$$

$$\frac{\partial \omega}{\partial y} = 0 \quad \text{where} \quad y = \pm \frac{b}{2} \qquad \text{b}$$

$$-D \frac{\partial^2 \omega}{\partial y^2} = M_y \quad \text{where} \quad y = \pm \frac{b}{2} \qquad \text{c}$$

CONDITIONS AT THE SIMPLY SUPPORTED EDGES:

The deflection along the simply supported edge is zero. But at the same time this edge can rotate about its own axis; therefore, there are no bending moments along a simply supported edge.

If in Figure 5, the edges $x = 0$ and $x = a$ are simply supported,

$$\omega = 0 \quad \text{where } x = 0,\ x = a$$

$$M_x = 0 \quad \text{where } x = 0,\ x = a$$

in general

$$M_x = -D \left(\frac{\partial^2 \omega}{\partial x^2} + \nu \frac{\partial^2 \omega}{\partial y^2} \right)$$

Then the boundary conditions become

$$\left.\begin{array}{r} \omega = 0 \\ \dfrac{\partial^2 \omega}{\partial x^2} + \nu \dfrac{\partial^2 \omega}{\partial y^2} = 0 \end{array}\right\} \text{where} \left\{\begin{array}{l} x = 0 \\ x = a \end{array}\right.$$

D-DETERMINATION OF GENERAL EQUATION OF DEFLECTION BY SUPERPOSITION.

It was found that the determination of deflection (ω) can be found by integrating the equation

$$\frac{\partial^4 \omega}{\partial x^4} + 2\frac{\partial^4 \omega}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega}{\partial y^4} = \frac{q}{D}$$

and satisfying the existing boundary conditions.

To determine the deflection (ω) that will satisfy the above differential equation of deflection, Nadai[5]

(5) Nadai, A., Elastische platten, Berlin, 1925.

has suggested taking the solution of this equation for uniformly loaded and simply supported plates in the following form:

$$\omega = \omega_1 + \omega_2 \qquad 14$$

and letting,

$$\omega_1 = \frac{q}{24D}(x^4 - 2ax^3 + a^3 x) \qquad 15$$

$$\omega_2 = \sum_{m=1}^{\infty} Y_m \sin\frac{m\pi x}{a} \qquad 16$$

The deflection (ω) in equation 14 represents the deflection of a uniformly loaded and simply supported plate. Therefore to find the deflection of a plate when two parallel edges are clamped, or built-in, it is necessary to superimpose another deflection (ω') upon the deflection (ω) as defined in equation 14. (ω') Is the deflection due to the bending moments at the built-in edges.

1. DEFLECTION OF UNIFORMLY LOADED AND SIMPLY SUPPORTED PLATES

Equation 15 represents the deflection of a uniformly loaded strip parallel to the X-axis. It satisfies the deflection equation

$$\frac{\partial^4 \omega_1}{\partial x^4} + \frac{2\,\partial^4 \omega_1}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega_1}{\partial y^4} = \frac{q}{D}$$

and also the boundary conditions at the edges, x = o and x = a , (Page 13).

for

$$x = o\,, \quad \omega_1 = \frac{q}{24D}(0) = 0\,; \quad x = a\,, \quad \omega_1 = \frac{q}{24D}(a^4 - 2a^4 + a^4) = 0$$

for

$$x = o, \quad \frac{\partial^2 \omega_1}{\partial x^2} + \nu \frac{\partial^2 \omega_1}{\partial y^2} = (2a\,o) = 0; \quad x = a, \quad \frac{\partial^2 \omega_1}{\partial x^2} + \nu \frac{\partial^2 \omega_1}{\partial y^2} = (12a^2 - 12a^2) = 0$$

Since the deflection is composed of ω_1 and ω_2, and ω_1 has already satisfied the right hand member of equation 13, the expression for ω_2, Equation 16, evidently has to satisfy the equation

$$\frac{\partial^4 \omega_2}{\partial x^4} + 2\frac{\partial^4 \omega_2}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega_2}{\partial y^4} = 0 \qquad \text{13a}$$

and it must be so chosen that it will make the right hand side of equation 14 satisfy all boundary conditions of the plate.

From equation 16,

$$\omega_2 = \sum_{m=1}^{\infty} Y_m \sin \frac{m\pi x}{a}$$

here Y_m is a function of y only and from symmetry $m = 1, 3, 5, \ldots$

Since Y_m is a function of Y only, this series readily satisfies the boundary conditions at $x=0$ and $x=a$. This can be shown in the following form;

for,

$$x=0 \qquad \omega_2 = \sum_{m=1}^{\infty} Y_m \sin\frac{m\pi 0}{a} = 0$$

$$x=a \qquad \omega_2 = \sum_{m=1}^{\infty} Y_m \sin\frac{m\pi a}{a} = 0$$

for, $x=0$

$$\frac{\partial^2\omega_2}{\partial x^2} + \nu\frac{\partial^2\omega_2}{\partial y^2} = -\frac{m^2\pi^2}{a^2}\sum_{m=1}^{\infty} Y_m \sin\frac{m\pi 0}{a} + \nu\sum_{m=1}^{\infty} Y_m^{II} \sin\frac{m\pi 0}{a} = 0$$

for, $x=a$

$$\frac{\partial^2\omega_2}{\partial x^2} + \nu\frac{\partial^2\omega_2}{\partial y^2} = -\frac{m^2\pi^2}{a^2}\sum_{m=1}^{\infty} Y_m \sin\frac{m\pi a}{a} + \nu\sum_{m=1}^{\infty} Y_m^{II} \sin\frac{m\pi a}{a} = 0$$

Y_m still remains to be determined. It has to satisfy the boundary conditions at $y = \pm\frac{b}{2}$.

If equation 16 is substituted in equation 13a, the following differential equation will be obtained. (See Appendix B)

$$\sum_{m=1}^{\infty}\left(Y_m^{IV} - 2\frac{m^2\pi^2}{a^2}Y_m^{II} + \frac{m^4\pi^4}{a^4}Y_m\right)\cdot \sin\frac{m\pi x}{a} = 0$$

Since the second factor of this differential equation is a function of X only, all boundary conditions will be satisfied at $x = 0$ and $x = a$, if Y_m satisfies the equation

$$Y_m^{IV} - 2\frac{m^2\pi^2}{a^2} Y_m^{II} + \frac{m^4\pi^4}{a^4} Y_m = 0 \qquad 17$$

Integration of this equation gives (See Appendix C)

$$Y_m = \frac{q a^4}{D}\left(A_m \sinh\frac{m\pi y}{a} + B_m \frac{m\pi y}{a} \sinh\frac{m\pi y}{a} + C_m \cosh\frac{m\pi y}{a} + D_m \frac{m\pi y}{a} \cosh\frac{m\pi y}{a}\right) \qquad 18$$

It can be observed that the deflected surface of the roof bed is symmetrical with respect to the X-axis, Figure 15. Therefore equation 18 contains even functions of y only. Thus it is necessary to take $A_m = D_m = 0$.

Deflection equation 14 is then represented in the following form:

$$\omega = \frac{q}{24D}\left(x^4 - 2ax^3 + a^3x\right) + \frac{qa^4}{D}\sum_{m=1}^{\infty}\left(C_m \cosh\frac{m\pi y}{a} + B_m \frac{m\pi y}{a} \sinh\frac{m\pi y}{a}\right) \sin\frac{m\pi x}{a} \qquad 19$$

This equation of deflection satisfies the equation of the deflected surface, e.i., equation 13, and also the boundary conditions at $x = 0$ and $x = a$. But it is necessary to determine the constants C_m and B_m in such a manner that they will satisfy the boundary conditions at $y = \pm\frac{b}{2}$.

The boundary conditions at $y = \pm\frac{b}{2}$.

$$\left.\begin{aligned} \omega &= 0 \\ \frac{\partial^2\omega}{\partial x^2} + \mu\frac{\partial^2\omega}{\partial y^2} &= 0 \end{aligned}\right\} y = \pm\frac{b}{2}$$

and it has been seen that the deflection equation

$$\omega_2 = \sum_{m=1}^{\infty} Y_m \sin \frac{m\pi x}{a}$$

~~readily~~ satisfies the condition of $\frac{\partial^2 \omega}{\partial x^2} = 0$, there-fore the boundary conditions are reduced to

$$\left.\begin{array}{l} \omega = 0 \\ \frac{\partial^2 \omega}{\partial y^2} = 0 \end{array}\right\} y = \pm \frac{b}{2} \qquad 20$$

To substitute equation 19, in the boundary conditions given by equation 20, it is necessary to develop

$$\omega_1 = \frac{q}{24D}\left(x^4 - 2ax^3 + a^3x\right)$$

into a trigonometric series between (0) and (π) . This gives the following series (See Appendix D)

$$\frac{q}{24D}\left(x^4 - 2ax^3 - a^3x\right) = \frac{4qa^4}{\pi^5 D} \sum_{m=1}^{\infty} \frac{1}{m^5} \sin \frac{m\pi x}{a}$$

where $m = 1, 3, 5, \ldots.$ Equation 19 will now be repre-sented in the following form,

$$\omega = \frac{qa^4}{D} \sum_{m=1}^{\infty} \left(\frac{4}{\pi^5 m^5} + C_m \cosh \frac{m\pi y}{a} + B_m \frac{m\pi y}{a} \sinh \frac{m\pi y}{a} \right) \sin \frac{m\pi x}{a} \qquad 21$$

where $m = 1, 3, 5, \ldots..$ Substituting this expression in the boundary conditions given by equation 20, and using the notation $\frac{m\pi b}{2a} = \alpha_m$, for simplicity, the following equations will be obtained to determine C_m and B_m (See Appendix E)

$$\frac{4}{\pi^5 m^5} + C_m \cosh \alpha_m + \alpha_m B_m \sinh \alpha_m = 0$$

$$(C_m + 2B_m)\cosh\alpha_m + \alpha_m B_m \sinh\alpha_m = 0$$

from these equations C_m and B_m are determined as

$$C_m = -\frac{2(\alpha_m \tanh\alpha_m + 2)}{\pi^5 m^5 \cosh\alpha_m}$$

$$B_m = \frac{2}{\pi^5 m^5 \cosh\alpha_m} \qquad 22$$

By substituting these values of constants C_m and B_m in deflection equation 19, the equation of deflection surface will be obtained in the following form:

$$\omega = \frac{4qa^4}{\pi^5 D}\sum_{m=1}^{\infty}\frac{1}{m^5}\left[1 - \frac{(\alpha_m \tanh\alpha_m + 2)}{2\cosh\alpha_m}\cosh\frac{m\pi y}{a} + \frac{1}{2\cosh\alpha_m}\frac{m\pi y}{a}\sinh\frac{m\pi y}{a}\right]\sin\frac{m\pi x}{a} \qquad 23$$

This equation of the deflected surface satisfies the differential equation 13 and the boundary conditions at $x=0$ and $x=a$, and $y=\pm\frac{b}{2}$, for simply supported plates.

2. DEFLECTION DUE TO BENDING MOMENTS AT $y=\pm\frac{b}{2}$

The deflection ω' due to bending moments at $y=\pm\frac{b}{2}$ remains to be determined.

Taking the solution of ω' in the form of this series

$$\omega' = \sum_{m=1}^{\infty} Y_m \sin\frac{m\pi x}{a} \qquad 24$$

each term of which, as has been seen satisfies the boundary conditions at $x=0$ and $x=a$ and the following differential equation of deflection,

$$\frac{\partial^4 \omega'}{\partial x^4} + 2\frac{\partial^4 \omega'}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega'}{\partial y^4} = 0$$

The functions of Y_m can be found, as before, in the following form, (See Appendix C)

$$Y_m = A_m \sinh\frac{m\pi y}{a} + B_m \frac{m\pi y}{a}\sinh\frac{m\pi y}{a} + C_m \cosh\frac{m\pi y}{a} + D_m \frac{m\pi y}{a}\cosh\frac{m\pi y}{a}$$

In case of symmetry Y_m has to be an even function of Y, therefore it is necessary to put $A_m = D_m = 0$, and equation 24 can be obtained in the following form:

$$\omega' = \sum_{m=1}^{\infty}\left(B_m \frac{m\pi y}{a}\sinh\frac{m\pi y}{a} + C_m \cosh\frac{m\pi y}{a}\right)\sin\frac{m\pi x}{a} \qquad 25$$

To satisfy the boundary conditions (a) for built-in edges given in page 13, it is necessary that

$$B_m \cosh\frac{m\pi b}{2a} + C_m \frac{m\pi b}{2a}\sinh\frac{m\pi b}{2a} = 0$$

and substituting $\alpha_m = \frac{m\pi b}{2a}$ for convenience, it can be found that

$$B_m = -C_m \alpha_m \tanh\alpha_m$$

Substituting this expression in equation 25.

$$\omega' = \sum_{m=1}^{\infty} C_m\left(\frac{m\pi y}{a}\sinh\frac{m\pi y}{a} - \alpha_m \tanh\alpha_m \cosh\frac{m\pi y}{a}\right)\sin\frac{m\pi x}{a} \qquad 26$$

C_m will be determined from boundary conditions(c) on page 13. These boundary conditions are:

$$-D\left(\frac{\partial^2 \omega'}{\partial y^2}\right) = M_y \quad , \quad where \quad y = \pm \frac{b}{2}$$

Timoshenko[6] suggests taking the bending moments along

(6) Op.cit., Timoshenko, Theory of plates and shells, p.205

the edges $y = \pm \frac{b}{2}$, in following series,

$$M_y = \sum_{m=1}^{\infty} E_m \sin \frac{m\pi x}{a} \qquad 27$$

where $m = 1, 3, 5, \ldots$ and the constant E_m will be calculated for each particular case.

Now, substituting equation 27 and the second derivative of equation 26 in the boundary condition given above, it will be obtained that

$$-2D \sum_{m=1}^{\infty} \frac{m^2\pi^2}{a^2} C_m \cosh \alpha_m \sin \frac{m\pi x}{a} = \sum_{m=1}^{\infty} E_m \sin \frac{m\pi x}{a}$$

from which

$$C_m = -\frac{a^2 E_m}{2D m^2 \pi^2 \cosh \alpha_m}$$

and

$$\omega' = \frac{a^2}{2\pi^2 D} \sum_{m=1}^{\infty} \frac{E_m}{m^2 \cosh \alpha_m} \left(\alpha_m \tanh \alpha_m \cosh \frac{m\pi y}{a} - \frac{m\pi y}{a} \sinh \frac{m\pi y}{a}\right) \times \left(\sin \frac{m\pi x}{a}\right) \qquad 28$$

The constant E_m will be determined from the conditions at the built-in edges. The slope at the built-in edge is zero. Therefore to obtain this condition it is necessary that the slopes $\frac{\partial \omega}{\partial y}$ and $\frac{\partial \omega'}{\partial y}$ when superimposed at the edges $y = \pm \frac{b}{2}$ must add up to zero.

Thus

$$\frac{\partial \omega'}{\partial y} + \frac{\partial \omega}{\partial y} = 0, \quad where \quad y = \pm \frac{b}{2}$$

From equations 23 and 28, slopes $\frac{\partial \omega}{\partial y}$ and $\frac{\partial \omega'}{\partial y}$ at $y = \pm \frac{b}{2}$,

$$\frac{\partial \omega}{\partial y} = \frac{2 q a^3}{\pi^4 D} \sum_{m=1}^{\infty} \frac{1}{m^4} \left[\alpha_m - \tanh \alpha_m \left(1 + \alpha_m \tanh \alpha_m\right)\right] \sin \frac{m \pi x}{a} \qquad a$$

$$\frac{\partial \omega'}{\partial y} = \frac{a}{2 \pi D} \sum_{m=1}^{\infty} \frac{E_m}{m} \left[\tanh \alpha_m \left(\alpha_m \tanh \alpha_m - 1\right) - \alpha_m\right] \sin \frac{m \pi x}{a} \qquad b$$

By adding $\frac{\partial \omega}{\partial y}$ and $\frac{\partial \omega'}{\partial y}$ from these values defined in the preceding expressions (a) and (b), E_m will be defined in the following form

$$E_m = \frac{4 q a^2}{\pi^3 m^3} \cdot \frac{\alpha_m - \tanh \alpha_m (1 + \alpha_m \tanh \alpha_m)}{\alpha_m - \tanh \alpha_m (\alpha_m \tanh \alpha_m - 1)}$$

If, E_m is substituted in equation 28, ω' will be defined in the following form:

$$\omega' = \frac{2 q a^4}{\pi^5 D} \sum_{m=1}^{\infty} \frac{\sin \frac{m \pi x}{a}}{m^5 \cosh \alpha_m} \left[\frac{\alpha_m - \tanh \alpha_m (1 + \alpha_m \tanh \alpha_m)}{\alpha_m - \tanh \alpha_m (\alpha_m \tanh \alpha_m - 1)}\right]$$
$$\left[\frac{m \pi y}{a} \sinh \frac{m \pi y}{a} - \alpha_m \tanh \alpha_m \cosh \frac{m \pi y}{a}\right] \qquad 29$$

Since the deflection due to moments at the edges is caused by load (q); deflection W of mine roof beds, two parallel edges of which are built-in and other two edges simply supported becomes as follows

$$W = \omega - \omega'$$

By substituting ω from equation 23 and ω' from equation 29,

$$W = \frac{2 q a^4}{\pi^5 D} \sum_{m=1}^{\infty} \frac{\sin \frac{m \pi x}{a}}{m^5 \cosh \alpha_m} \left\{\left[2 \cosh \alpha_m - (\alpha_m \tanh \alpha_m + 2) \cosh \frac{m \pi y}{a}\right.\right.$$

$$+ \frac{m\pi y}{a} \sinh \frac{m\pi y}{a} + \left[\frac{\alpha_m - \tanh\alpha_m(1+\alpha_m\tanh\alpha_m)}{\alpha_m - \tanh\alpha_m(\alpha_m\tanh\alpha_m - 1)}\right] \left[\alpha_m\tanh\alpha_m \cosh\frac{m\pi y}{a} - \frac{m\pi y}{a}\sinh\frac{m\pi y}{a}\right]\Bigg]\Bigg\} \qquad 30$$

This expression gives the deflection at any point within the roof bed.

The moments M_x and M_y defined by equations 5 will be obtained by substituting $\frac{\partial^2 w}{\partial x^2}$ and $\frac{\partial^2 w}{\partial y^2}$ from equation 30, into equations 5 and they are found to be (See Appendix F)

$$M_x = -D(\beta_x + \nu\beta_y)$$
$$M_y = -D(\beta_y + \nu\beta_x) \qquad 31$$

The twisting moment defined by equation 7 will become (See Appendix G)

$$M_{xy} = D(1-\nu)\beta_{xy} \qquad 32$$

Normal stresses σ_x and σ_y defined by equations 3, now are determined in the following form:

$$\sigma_x = \frac{E}{1-\nu^2} Z(\beta_x + \nu\beta_y)$$
$$\sigma_y = \frac{E}{1-\nu^2} Z(\beta_y + \nu\beta_x) \qquad 33$$

σ_x and σ_y can be determined at upper and lower surfaces of the roof by substituting $Z = \pm \frac{h}{2}$.

The equation of unit shear (equations 6a) becomes,

$$\tau_{xy} = -2GZ\beta_{xy} \qquad 34$$

The general equation of shearing stresses defined by equations 10 and 11 will therefore become

$$Q_x = -D \frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

$$Q_y = -D \frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

Substituting values of $\frac{\partial^2 w}{\partial x^2}$ and $\frac{\partial^2 w}{\partial y^2}$ as given in Appendix F

$$Q_x = -D \frac{\partial}{\partial x}\left(\beta_x + \beta_y\right) \qquad 35$$

$$Q_y = -D \frac{\partial}{\partial y}\left(\beta_x + \beta_y\right)$$

APPENDIX A

Integration of Bending Moments M_x and M_y

$$\int_{-h/2}^{+h/2} \sigma_x z \, dz = M_x$$

$$\sigma_x = -\frac{E z}{1-\nu^2}\left(\frac{\partial^2 \omega}{\partial x^2} + \nu \frac{\partial^2 \omega}{\partial y^2}\right)$$

$$M_x = -\frac{E}{1-\nu^2}\int_{-h/2}^{+h/2} z^2 dz \left(\frac{\partial^2 \omega}{\partial x^2} + \nu \frac{\partial^2 \omega}{\partial y^2}\right) = -\frac{E}{1-\nu^2}\,\frac{z^3}{3}\cdot\left(\frac{\partial^2 \omega}{\partial x^2} + \nu \frac{\partial^2 \omega}{\partial y^2}\right)\Bigg]_{-\frac{h}{2}}^{+\frac{h}{2}}$$

$$M_x = -\frac{E h^3}{12(1-\nu^2)}\left(\frac{\partial^2 \omega}{\partial x^2} + \nu \frac{\partial^2 \omega}{\partial y^2}\right)$$

$$M_y = -\frac{E h^3}{12(1-\nu^2)}\left(\frac{\partial^2 \omega}{\partial y^2} + \nu \frac{\partial^2 \omega}{\partial x^2}\right)$$

APPENDIX B

Development of Differential Equation $Y_m^{IV} - \frac{2m^2\pi^2}{a^2} Y_m^{II} + \frac{m^4\pi^4}{a^4} Y_m = 0$

$$\frac{\partial^4 \omega_2}{\partial x^4} + 2\frac{\partial^4 \omega_2}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega_2}{\partial y^4} = 0$$

$$\omega_2 = \sum_{m=1}^{\infty} Y_m \sin\frac{m\pi x}{a}$$

$$\frac{\partial^4 \omega_2}{\partial x^4} = \frac{\partial^4}{\partial x^4}\sum_{m=1}^{\infty} Y_m \sin\frac{m\pi x}{a} = \frac{m^4\pi^4}{a^4}\sum_{m=1}^{\infty} Y_m \sin\frac{m\pi x}{a}$$

$$2\frac{\partial^4 \omega_2}{\partial x^2 \partial y^2} = \frac{2\partial^4}{\partial x^2 \partial y^2}\sum_{m=1}^{\infty} Y_m \sin\frac{m\pi x}{a} = -\frac{m^2\pi^2}{a^2}(2)\sum_{m=1}^{\infty} Y_m^{II} \sin\frac{m\pi x}{a}$$

$$\frac{\partial^4 \omega_2}{\partial y^4} = \frac{\partial^4}{\partial y^4}\sum_{m=1}^{\infty} Y_m \sin\frac{m\pi x}{a} = \sum_{m=1}^{\infty} Y_m^{IV} \sin\frac{m\pi x}{a}$$

$$\frac{\partial^4 \omega_2}{\partial x^4} + 2\frac{\partial^4 \omega_2}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega_2}{\partial y^4} = \sum_{m=1}^{\infty}\left(Y_m^{IV} - 2\frac{m^2\pi^2}{a^2} Y_m^{II} + \frac{m^4\pi^4}{a^4} Y_m\right)\sin\frac{m\pi x}{a} = 0$$

APPENDIX C

Integration of Differential Equation

$$Y_m^{IV} - 2\frac{m^2\pi^2}{a^2}Y_m^{II} + \frac{m^4\pi^4}{a^4}Y_m = 0$$

$$\Delta = \frac{d}{dy}$$

$$\left(\Delta^4 - 2\frac{m^2\pi^2}{a^2}\Delta^2 + \frac{m^4\pi^4}{a^4}\right)Y_m = 0$$

$$\left[\left(\Delta + \frac{m\pi}{a}\right)^2\left(\Delta - \frac{m\pi}{a}\right)\right]Y_m = 0$$

Roots are : $-\frac{m\pi}{a}, -\frac{m\pi}{a}, \frac{m\pi}{a}, \frac{m\pi}{a}$

$$Y_m = k_1 e^{-\frac{m\pi}{a}y} + k_2 \frac{m\pi}{a}y\, e^{-\frac{m\pi}{a}} + k_3 e^{\frac{m\pi}{a}y} + k_4 \frac{m\pi}{a}y\, e^{\frac{m\pi}{a}y}$$

$$Y_m = \left(k_1 + k_2\frac{m\pi y}{a}\right)e^{-\frac{m\pi y}{a}} + \left(k_3 + k_4\frac{m\pi y}{a}\right)e^{\frac{m\pi y}{a}}$$

$$e^{-\frac{m\pi y}{a}} = \cosh\frac{m\pi y}{a} - \sinh\frac{m\pi y}{a}$$

$$e^{\frac{m\pi y}{a}} = \cosh\frac{m\pi y}{a} + \sinh\frac{m\pi y}{a}$$

$$Y_m = \left(\cosh\frac{m\pi y}{a} - \sinh\frac{m\pi y}{a}\right)\left(k_1 + k_2\frac{m\pi y}{a}\right)$$
$$+ \left(\cosh\frac{m\pi y}{a} + \sinh\frac{m\pi y}{a}\right)\left(k_3 + \frac{m\pi y}{a}k_4\right)$$

$$Y_m = \left[(k_1 + k_3) + (k_2 + k_4) \frac{m\pi y}{a} \right] \cosh \frac{m\pi y}{a} + \left[(k_3 - k_1) + (k_4 - k_2) \frac{m\pi y}{a} \right] \sinh \frac{m\pi y}{a}$$

$$k_3 - k_1 = A_m$$

$$k_4 - k_2 = B_m$$

$$k_1 + k_3 = C_m$$

$$k_2 + k_4 = D_m$$

$$Y_m = A_m \sinh \frac{m\pi y}{a} + B_m \frac{m\pi y}{a} \sinh \frac{m\pi y}{a} + C_m \cosh \frac{m\pi y}{a} + D_m \frac{m\pi y}{a} \cosh \frac{m\pi y}{a}$$

$$Y_m = \frac{q a^4}{D} \left[A_m \sinh \frac{m\pi y}{a} + B_m \frac{m\pi y}{a} \sinh \frac{m\pi y}{a} + C_m \cosh \frac{m\pi y}{a} + D_m \frac{m\pi y}{a} \cosh \frac{m\pi y}{a} \right]$$

APPENDIX D

Development of equation $\frac{q}{24D}(x^4 - 2ax^3 + a^3x)$ into a trigonometric series

$$f(x) = \frac{q}{24D}\left(x^4 - 2ax^3 + a^3x\right) \qquad 0 < x < a$$

Substitute

$$z = \frac{\pi}{a}x \qquad K = \frac{q}{24D}$$

and

$$x = 0 \qquad z = 0$$
$$x = a \qquad z = \pi$$

$$f(z) = K\left(\frac{a^4}{\pi^4}z^4 - 2\frac{a^4}{\pi^3}z^3 + \frac{a^4}{\pi}z\right)$$

$$= \frac{Ka^4}{\pi}\left(\frac{z^4}{\pi^3} - \frac{2z^3}{\pi^2} + z\right)$$

$$b_m = \frac{2}{\pi}\int_0^{\pi} f(z)\ \sin mz \cdot dz$$

$$= \frac{2Ka^4}{\pi^5}\int_0^{\pi} z^4 \sin mz \cdot dz - \frac{4Ka^4}{\pi^4}\int_0^{\pi} z^3 \cdot \sin mz \cdot dz + \frac{2Ka^4}{\pi^2}\int_0^{\pi} z \cdot \sin mz \cdot dz$$

$$\int_0^{\pi} z^4 \sin mz\, dz = \left[-\frac{1}{m}z^4 \cos mz + \frac{4}{m}\left[\left(\frac{3m^2z^2-6}{m^4}\right)\cos mz + \left(\frac{m^2z^2-6z}{m^3}\right)\sin mz\right]\right]_0^{\pi}$$

$$\frac{2Ka^4}{\pi^5}\int_0^{\pi} z^4 \sin mz\, dz = \frac{2Ka^4}{\pi^5}\left[\left(-\frac{\pi^4}{m} + \frac{12m^2\pi^2 - 24}{m^5}\right)\cos m\pi + \frac{24}{m^5}\right]$$

$$\int_0^{\pi} z^3 \sin mz \cdot dz = \left.\frac{3z^2}{m^2}\sin mz - \frac{6}{m^4}\sin mz - \frac{z^3}{m}\cos mz + \frac{6z}{m^3}\cos mz\right]_0^{\pi}$$

$$-\frac{4Ka^4}{\pi^4}\int_0^\pi z^3 \cdot \sin mz \, dz = -\cos m\pi \left(\frac{24Ka^4}{\pi^3 m^3} - \frac{4Ka^4}{m\pi}\right)$$

$$\int_0^\pi z \cdot \sin mz \, dz = \frac{1}{m2} \sin mz - \frac{1}{m} z \cos mz \Big]_0^\pi$$

$$\frac{2Ka^4}{\pi^2}\int_0^\pi z \cdot \sin mz \cdot dz = -\frac{2Ka^4}{m\pi} \cos m\pi$$

Then, b_m;

$$b_m = \frac{48Ka^4}{\pi^5}\left(\frac{1-\cos m\pi}{m5}\right)$$

call, $\frac{48a^4K}{\pi^5} = K'$

$m = 1$	$b_m = 2K'$
$m = 2$	$b_m = 0$
$m = 3$	$b_m = 2K'$
$m = 4$	$b_m = 0$

odd series.

$$f(z) = 2K' \sum_{m=1}^{\infty} \frac{1}{m5} \sin mz$$

substitute $z = \frac{\pi}{a} x$, $K' = \frac{48Ka^4}{\pi^5}$

and, $K = \frac{q}{24D}$

$$\frac{q}{24D}\left(x^4 - 2ax^3 + a^3x\right) = \frac{4qa^4}{D\pi^5} \sum_{m=1}^{\infty} \frac{1}{m5} \sin \frac{m\pi x}{a}$$

APPENDIX E

Determination of some constants from boundary conditions

$$\left.\begin{array}{l} \omega = 0 \\ \dfrac{\partial^2 \omega}{\partial y^2} = 0 \end{array}\right\} y = \pm b/2$$

$$\omega = \frac{q a^2}{D} \sum_{m=1}^{\infty} \left(\frac{4}{m^5 \pi^5} + C_m \cosh \frac{m\pi y}{a} + B_m \frac{m\pi y}{a} \sinh \frac{m\pi y}{a} \right) \sin \frac{m\pi x}{a}$$

Substitute $y = \pm \frac{b}{2}$, and $\frac{m\pi b}{2a} = \alpha_m$

$$\frac{4}{m^5 \pi^5} + C_m \cosh \alpha_m + \alpha_m B_m \sinh \alpha_m = 0$$

$$\frac{\partial^2 \omega}{\partial y^2} = \frac{q a^4}{D} \left(C_m \frac{m^2\pi^2}{a^2} \cosh \frac{m\pi y}{a} + B_m \frac{m^2\pi^2}{a^2} \cosh \frac{m\pi y}{a} + B_m \frac{m^2\pi^2}{a^2} \cosh \frac{m\pi y}{a} + B_m \frac{m^2\pi^2}{a^2} \frac{m\pi y}{a} \sinh \frac{m\pi y}{a} \right) \sin \frac{m\pi x}{a}$$

substitute; $y = \pm \frac{b}{2}$, and $\frac{m\pi b}{a} = \alpha_m$

$$C_m \cosh \alpha_m + B_m \cosh \alpha_m + B_m \cosh \alpha_m + \alpha_m B_m \sinh \alpha_m = 0$$

$$(C_m + 2B_m) \cosh \alpha_m + \alpha_m B_m \sinh \alpha_m = 0$$

APPENDIX F

Determination of Bending Moments M_x and M_y from Deflection Equation 30

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right)$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right)$$

w = Equation 30, Page 24

$$\frac{\partial^2 w}{\partial x^2} = -\frac{2qa^2}{\pi^3 D} \sum_{m=1}^{\infty} \frac{1}{m^3 \cosh \alpha_m} \Bigg\{ \Bigg[2\cosh \alpha_m - (\alpha_m \tanh \alpha_m + 2) \cosh \frac{m\pi y}{a}$$

$$+ \frac{m\pi y}{a} \sinh \frac{m\pi y}{a} + \left[\frac{\alpha_m - \tanh \alpha_m (1 + \alpha_m \tanh \alpha_m)}{\alpha_m - \tanh \alpha_m (\alpha_m \tanh \alpha_m - 1)}\right]$$

$$\left[\alpha_m \tanh \alpha_m \cosh \frac{m\pi y}{a} - \frac{m\pi y}{a} \sinh \frac{m\pi y}{a}\right]\Bigg]\Bigg\} \sin \frac{m\pi x}{a}$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{2qa^2}{\pi^3 D} \sum_{m=1}^{\infty} \frac{1}{m^3 \cosh \alpha_m} \Bigg\{ \Bigg[-(\alpha_m \tanh \alpha_m + 2) \cosh \frac{m\pi y}{a} + 2\cosh \frac{m\pi y}{a}$$

$$+ \frac{m\pi y}{a} \sinh \frac{m\pi y}{a} + \left[\frac{\alpha_m - \tanh \alpha_m (1 + \alpha_m \tanh \alpha_m)}{\alpha_m - \tanh \alpha_m (\alpha_m \tanh \alpha_m - 1)}\right]$$

$$\left[\alpha_m \tanh \alpha_m \cosh \frac{m\pi y}{a} - 2\cosh \frac{m\pi y}{a} - \frac{m\pi y}{a} \sinh \frac{m\pi y}{a}\right]\Bigg]\Bigg\} \sin \frac{m\pi x}{a}$$

substitute ; $\beta x = \frac{\partial^2 w}{\partial x^2}$, $\beta y = \frac{\partial^2 w}{\partial y^2}$

$$M_x = -D(\beta x + \nu \beta y) \quad ; \quad M_y = -D(\beta y + \nu \beta x)$$

APPENDIX G

Determination of Twisting Moment M_{xy} from Deflection Equation 30

$$M_{xy} = D(1-\nu)\frac{\partial^2 w}{}$$

w = Equation 30, Page 24

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{2qa^2}{\pi^3 D}\sum_{m=1}^{\infty}\frac{1}{m^3 \cosh\alpha_m}\left\{-(\alpha_m \tanh\alpha_m + 2)\sinh\frac{m\pi y}{a} + \sinh\frac{m\pi y}{a}\right.$$

$$+\frac{m\pi y}{a}\cosh\frac{m\pi y}{a} + \left[\frac{\alpha_m - \tanh\alpha_m(1+\alpha_m\tanh\alpha_m)}{\alpha_m - \tanh\alpha_m(\alpha_m\tanh\alpha_m - 1)}\right]$$

$$\left.\left[\alpha_m\tanh\alpha_m \sin\frac{m\pi y}{a} - \sinh\frac{m\pi y}{a} - \frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\right]\right\}\cos\frac{m\pi x}{a}$$

substituting ; $\frac{\partial^2 w}{\partial x \partial y} = \beta_{xy}$

$$M_{xy} = D(1-\nu)\beta_{xy}$$

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