Spring 1989

System modeling and vibration reduction of a flexible beam under rotary motion

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SYSTEM MODELING AND VIBRATION REDUCTION OF

A FLEXIBLE BEAM UNDER ROTARY MOTION

by

JUI-HUNG CHIEN, 1961-

A THESIS

Presented to the Faculty of the Graduate School of the

UNIVERSITY OF MISSOURI-ROLLA

In Partial Fulfillment of the Requirements for the Degree

MASTER OF SCIENCE

in

MECHANICAL ENGINEERING

1989

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ABSTRACT

The objective of this thesis work is to reduce the end-point vibration of a flexible beam using the feedback control of the partial state variables. The dynamic model is derived from the assumed-modes method. The new feature of this model is that it is applicable to control system analysis and synthesis. A practical example is presented to illustrate the use of control law to improve the transient response.

ACKNOWLEDGEMENTS

I wish to acknowledge my mother for her endless patience throughout these years.

I also wish to express my sincere thanks to Dr. F. W. Liou for his guidance and support. He led me right from the selection of the topic and patiently guided me in all aspects of problems.

Also, I wish to express my appreciation for my most respected teachers Dr. Clark R. Barker and Dr. W-Z Ben Wu for refining my knowledge of the subject and serving on the committee.

NOMENCLATURE

{ }	column vector
[ ]	matrix
EI(x)	bending stiffness
ρ	linear density of the beam
M_t	tip mass
J	mass moment of inertia with respect to the neutral axis
L	beam length
$q_i(t)$	the i-th generalized coordinate
$Q_i(s)$	the Laplace transformation of $q_i(t)$
$\tau(t)$	the joint torque
T(s)	the Laplace transformation of $\tau(t)$
T	kinetic energy
V	potential energy
$\phi_i(x)$	mode shape of the i-th mode
$(\dot{\ })$	first derivative with respect to time
$(\ddot{\ })$	second derivative with respect to time
$(\ )'$	first derivative with respect to spatial variable
$(\ )''$	second derivative with respect to spactial variable
$(\hat{\ })$	a unit vector
B	the integration domain over the beam
$\delta\bar{W}$	the virtual work
δr_i	the virtual displacement

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I. INTRODUCTION

A. THESIS GOALS:

The aim of the dynamic control of an industrial robot is to force its end effector to follow a specified path that is required to complete its task satisfactorily. Sometimes, the task could be complicated by constraints imposed in the working space.

In practice, structures which tend to increase the overall weight are preferably used. However, bulky designs introduce limitations in terms of speed, energy consumption and mobility. An intuitive approach to overcome these disadvantages is to design light weight manipulators whose chaotic response is not affected by structural flexibility.

Recently, a lot of work has been done successfully using numerical methods to study the dynamics of a flexible link. However, not much work has been done to improve the end effector behavior.

In this thesis a single flexible beam with tip mass is considered as a simplified manipulator which contains a certain amount of distributed elasticity. A general procedure to derive distributed characteristics and to approximate this flexible dynamic system is formulated. In addition, a feedback control strategy to reduce the end-point vibration is also presented in terms of realistic consideration.

B. SUMMARY:

The computer simulation of the dynamic model and control results of the single flexible beam are presented in this thesis. Chapter III and Chapter IV describe the mathematical model. Chapter III formulates the kinetic energy and potential energy. Based on these results, the equations of motion are derived in Chapter IV by means of Lagrange's equations as well as the principle of virtual work. The beam is considered to be flexible and able to rotate horizontally such that the gravity effects can be neglected. For the system analysis, the model of the dynamic system is developed as transfer functions.

Chapter V mentions the theoretical approach to the mathematical model and the results are compared with those from Chapter IV.

Chapter VI and Chapter VII describe the control scheme and simulation. Chapter VI presents the way to modify the characteristics of the system. Chapter VII presents the simulation of the end-point trajectory before/after control.

Chapter VII comments on the practical features of the formulation presented here and suggests future work for extending this approach for more complex systems.

II. REVIEW OF LITERATURE

The literature pertinent to this thesis is classified into two catagories. First, researchers in applied mechanics have analyzed problems of vibration for elastic components. Second, in the application area of control systems there are several typical methods which have been used in the vibrating manipulators. Upon reviewing these publications, it is evident that most investigators have focused upon numerical approaches and only a small number of papers have been dedicated to studying the system transfer function of this flexible manipulator.

Research dealing with the dynamic response of mechanisms containing elastic links has been reviewed by Cetinkurt and Book [1], Thompson and Sung [2], and Erdman and Sandor [3]. Some early attempts to include elastic effects in the analysis of manipulators in references [4 to 14] have focused on the four-bar or slider crank mechanisms.

The second group of researchers [15 to 22] have been working on the area of vibration reduction of elastic component. The feed-forward control strategy [23,24] has been a useful technique. In addition, optimal control techniques [25], which use the state space model, have also been used to design beams.

III. SYSTEM DESCRIPTION

A. PHYSICAL MODEL:

The system, as shown in Figure 1 is composed of a tip mass and a flexible beam rotating with respect to Z-axis. It is assumed that this is a direct drive system and the flexible beam is built in on the motor shaft. Two coordinate frames are assigned to the system:

[X,Y,Z] - a Newtonian reference frame with origin at O.

[η, ζ, ξ] - a rotating frame with origin at O and the η-axis tangent to NN' (the neutral axis of the beam) at O.

B. BEAM CONFIGURATION:

The arm is modeled as a continuous clamped-free beam of length L and linear density ρ.

The following assumptions are made.

a. The flexible link can bend and rotate freely in the XY plane but does not deform out of the XY plane [26,27].

b. An ideal planar motion does not result in torsion [28].

c. Elastic deformation $u(\eta,t)$, shown in Figure 1, is always so small (<0.1L) that any extension is negligible [29].

d. For transverse vibrations due to small rotary and shearing effects, approximately 1.7% [30], bending effect is negligible.

e. Tip mass is considered as a point mass.

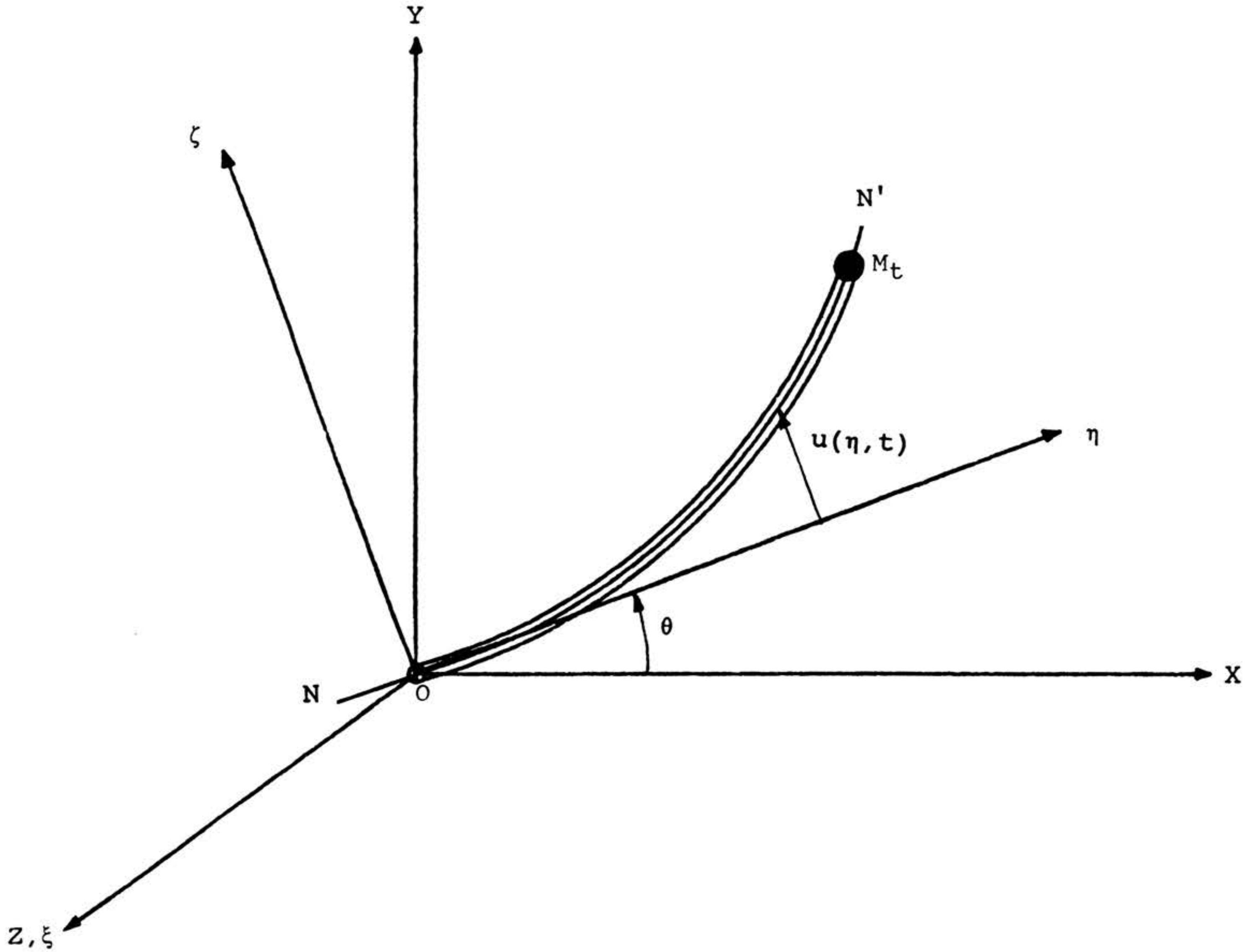


Figure 1: The Flexible Beam

f. $\phi_1, \phi_2, \ldots, \phi_{n-1}$ are shape functions of the first n-1 modes in clamped-free beam, respectively.

Following the assumptions d and f, the local deflection $u(\eta, t)$ of the points along the deformable beam can be expressed as below.

$$u(\eta, t) = \sum_{i=1}^{n-1} \phi_i(\eta) q_i(t) \tag{3.1}$$

where $q_i(t)$ = the time dependent portion of i-th mode.

Consequently, the first derivative and second derivative with respect to time and spatial variables, respectively, can be derived as follows.

$$\dot{u}(\eta, t) = \sum_{i=1}^{n-1} \phi_i(\eta) \dot{q}_i(t) \tag{3.2}$$

$$u''(\eta, t) = \sum_{i=1}^{n-1} \phi''_i(\eta) q_i(t). \tag{3.3}$$

Applying assumption c, it is easily understood that

$$\dot{R}_d = (\eta\dot{\theta} + \dot{u})\hat{e}_\theta \tag{3.4}$$

where R_d = vector of the point interested

e_θ = unit vector in θ direction.

C. KINETIC ENERGY:

The kinetic energy of the beam and the tip mass can be expressed as

$$T = \frac{1}{2}\int_B (\dot{R}_d \cdot \dot{R}_d)\rho d\eta + \frac{1}{2}M_t(L\dot{\theta} + \dot{u}(L,t))^2. \tag{3.5}$$

Since
$$\begin{aligned} \dot{R}_d \cdot \dot{R}_d &= (x\dot{\theta} + \dot{u})\hat{e}_\theta \cdot (x\dot{\theta} + \dot{u})\hat{e}_\theta \\ &= (x\dot{\theta} + \dot{u})^2 \\ &= x^2\dot{\theta}^2 + 2x\dot{\theta}\dot{u} + \dot{u}^2. \end{aligned} \tag{3.6}$$

Therefore,

$$\begin{aligned} T &= \frac{1}{2}\sum_i^n\sum_j^n m_{ij}\dot{q}_i\dot{q}_j \\ &= \frac{1}{2}\{\dot{q}\}^T[M]\{\dot{q}\} \end{aligned} \tag{3.7}$$

where

$$[M] = [m_{ij}]$$

$$m_{ij} = \int_B \phi_i\phi_j\rho d\eta + \delta_{ij}M_t\phi_i(L)\phi_j(L) \qquad i,j = 1,2,..,n-1$$

$$\begin{aligned} m_{in} &= m_{ni} \\ &= \int_B \eta\phi_i\rho d\eta + M_t L\phi_i(L) \qquad i = 1,2,..,n-1 \end{aligned}$$

$$m_{nn} = \int_B \eta^2\rho d\eta + M_t L^2. \tag{3.8}$$

D. POTENTIAL ENERGY:

As the motion in ideal planar gravitational energy is negligible and the potential energy of the system will be assumed to be the elastic strain energy of the link only.

$$V = \frac{1}{2}\int_B EIu''^2 dx. \tag{3.9}$$

Since

$$\begin{aligned} u''^2 &= \left[\sum_{i=1}^{n-1} \phi''_i(\eta) q_i(t)\right] \cdot \left[\sum_{j=1}^{n-1} \phi''_j(\eta) q_j(t)\right] \\ &= \sum_i^{n-1}\sum_j^{n-1} \phi''_i \phi''_j q_i(t) q_j(t). \end{aligned} \tag{3.10}$$

Consequently
$$\begin{aligned} V &= \frac{1}{2}\sum_i^n\sum_j^n K_{ij} q_i q_j \\ &= \frac{1}{2}\{q\}^T [K]\{q\} \end{aligned} \tag{3.11}$$

where $[K] = [K_{ij}]$

$$K_{ij} = \int_B EI\phi''_i \phi''_j d\eta \qquad i,j = 1,2,..,n-1$$

$$K_{in} = K_{ni} = 0 \qquad i = 1,2,..,n.$$

IV. EQUATIONS OF MOTION AND A SIMPLIFIED MODEL

A. PRINCIPLE OF VIRTUAL WORK:

To apply Lagrange's equations, one needs to formulate the generalized forces $\tau_i(t)$ associated with generalized coordinates q_i by means of the principle of virtual work.

The principle of virtual work relates the work done by external forces F_i during a virtual displacement of generalized forces. As a sequence, we can easily show that

$$\delta\overline{W} = \sum_{j=1}^{n} \tau_j \delta q_j \tag{4.1}$$

where n = No. of generalized coordinates

$$\delta\overline{W} = \sum_{i=1}^{p} F_i \delta r_i \tag{4.2}$$

where p = No. of external forces, and deduce the form

$$\tau_n = \sum_{i=1}^{n} F_i \frac{\partial r_i}{\partial q_n} \qquad n = 1,2,..,n. \tag{4.3}$$

As in this case, it is noted that the generalized forces need not necessarily be forces and sometimes could be torques.

$$\delta\bar{W} = \sum_{j=1}^{n} \tau_j \delta q_j \tag{4.4}$$

$$\delta\bar{W} = \sum_{i=1}^{p} F_i \delta r_i = \tau\delta\theta . \tag{4.5}$$

By comparison of both equations (4.4) and (4.5), it is easily concluded that $\tau_i=0$, $i=1,..,n-1$ and $\tau_n=\tau(t)$. Therefore,

$$\{\tau\} = \begin{Bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \tau(t) \end{Bmatrix} . \tag{4.6}$$

B. LAGRANGE'S EQUATIONS OF MOTION:

Equations (3.7), (3.9) and (4.6) can be rewritten as following.

$$T = \frac{1}{2} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \cdot \\ \cdot \\ \dot{q}_n \end{Bmatrix} \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & \cdots & m_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ m_{n1} & \cdots & \cdots & m_{nn} \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \cdot \\ \cdot \\ \dot{q}_n \end{Bmatrix} \tag{4.7}$$

$$V = \frac{1}{2} \begin{Bmatrix} q_1 \\ q_2 \\ \cdot \\ \cdot \\ q_n \end{Bmatrix} \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ k_{n1} & \cdots & \cdots & k_{nn} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ \cdot \\ \cdot \\ q_n \end{Bmatrix} \tag{4.8}$$

$$\{\tau\} = \begin{Bmatrix} 0 \\ 0 \\ . \\ . \\ \tau(t) \end{Bmatrix}. \tag{4.9}$$

Here, both the kinetic energy and potential energy are in a form generally known as quadratic. Thus,

$$\begin{aligned}
\frac{\partial L}{\partial \dot{q}_j} &= \frac{\partial}{\partial \dot{q}_j}(T - v) \\
&= \frac{1}{2}\sum_{r=1}^{n}\sum_{s=1}^{n} m_{rs}\left(\frac{\partial \dot{q}_r}{\partial \dot{q}_j}\dot{q}_s + \dot{q}r\frac{\partial \dot{q}_s}{\partial \dot{q}_j}\right) \\
&= \frac{1}{2}\sum_{r=1}^{n}\sum_{s=1}^{n} m_{rs}(\dot{q}_s\delta_{rj} + \dot{q}_r\delta_{sj}) \qquad (4.10) \\
&= \frac{1}{2}\sum_{s=1}^{n} m_{js}\dot{q}_s + \frac{1}{2}\sum_{r=1}^{n} m_{rj}\dot{q}_r \\
&= \sum_{s=1}^{n} m_{js}\dot{q}_s \qquad j = 1, 2, .., n.
\end{aligned}$$

Where δ_{rj} is the Kronecker delta, which is equal to zero for $r \neq s$ and equal to unity for $r = s$.

Moreover, by analogy the following equations can be derived.

$$
\begin{aligned}
\frac{\partial L}{\partial q_j} &= \frac{\partial (T - v)}{\partial q_j} \\
&= -\frac{\partial v}{\partial q_j} \\
&= \frac{1}{2} \sum_{r=1}^{n} \sum_{s=1}^{n} k_{rs}(q_s \delta_{rj} + q_r \delta_{sj}) \qquad\qquad (4.11) \\
&= \frac{1}{2} \sum_{s=1}^{n} k_{js} q_s + \frac{1}{2} \sum_{r=1}^{n} k_{rj} q_r \\
&= \sum_{s=1}^{n} k_{js} q_s \qquad\qquad j = 1,2,..,n.
\end{aligned}
$$

Introducing equations (4.9), (4.10) and (4.11) into Lagrange's equations:

$$
\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = \tau_j \qquad\qquad j = 1,2,...,n. \qquad (4.12)
$$

One obtains Lagrange's equations of motion for a general linear system.

$$
\sum_{s=1}^{n} [m_{js} \ddot{q}_s(t) + k_{js} q_s(t)] = \tau_j(t) \qquad\qquad j = 1,2,...,n. \qquad (4.13)
$$

Equation (4.13) constitutes a set of n simultaneous second-order differential equations in the generalized coordinates $q_s(t)$ $(s=1,2,...,n)$.

To obtain a more compact form, the equations can be written in matrix form.

$$[M]\{\ddot{q}\} + [K]\{q\} = \{\tau\}. \qquad (4.14)$$

The above matrix equations form the basis of an explicit relationship between generalized q_i $(i=1,2,..,n)$ and joint torque $\tau(t)$. Such relationship is particularly attractive from a control point of view. Taking Laplace transformation and imposing zero initial conditions, the following equations can be obtained.

$$\begin{bmatrix} m_{11}s^2 + k_{11} & m_{12}s^2 + k_{12} & \cdots & m_{1n}s^2 + k_{1n} \\ m_{21}s^2 + k_{21} & m_{22}s^2 + k_{22} & \cdots & m_{2n}s^2 + k_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ m_{n1}s^2 + k_{n1} & m_{n2}s^2 + k_{n2} & \cdots & m_{nn}s^2 + k_{nn} \end{bmatrix} \times$$

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ T(s) \end{Bmatrix}. \qquad (4.15)$$

C. <u>THE SIMPLIFIED SYSTEM</u>:

For computational simplicity, equation (4.15) can be truncated down to the first two modes (i.e. n-1=2) and this should not lose its generality and the final result will not be effected [21].

Thus, equation (4.15) reduces to the compact form

$$\begin{bmatrix} m_{11}s^2 + k_{11} & m_{12}s^2 + k_{12} & m_{13}s^2 + k_{13} \\ m_{21}s^2 + k_{21} & m_{22}s^2 + k_{22} & m_{23}s^2 + k_{23} \\ m_{31}s^2 + k_{31} & m_{32}s^2 + k_{32} & m_{33}s^2 + k_{33} \end{bmatrix} \begin{Bmatrix} Q_1(S) \\ Q_2(S) \\ Q_3(S) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ T(s) \end{Bmatrix}.$$

D. SOLUTIONS OF GENERALIZED COORDINATES Q_i'S :

It is easy to relate each Q_i's to T(s) by means of matrix operations. Therefore,

$$Q_1(S) = \left\{\frac{Amp_{11}}{S^2+\omega_1^2} + \frac{Amp_{12}}{S^2+\omega_2^2}\right\}T(S) \tag{4.16}$$

$$Q_2(S) = \left\{\frac{Amp_{21}}{S^2+\omega_1^2} + \frac{Amp_{22}}{S^2+\omega_2^2}\right\}T(S) \tag{4.17}$$

$$Q_3(S) = \left\{\frac{Amp_{30}}{S^2} + \frac{Amp_{31}}{S^2+\omega_1^2} + \frac{Amp_{32}}{S^2+\omega_2^2}\right\}T(S) \tag{4.18}$$

where Amp_{ij} is the effective amplitude of the system and its value depends on the system matrices [M] and [K]. For all practical purposes, it is customary to add to each mode a damping term proportional to the natural frequency and damping coefficient ξ_i.

Rewrite equations (4.16), (4.17) and (4.18).

$$Q_1(S) = \left\{\frac{Amp_{11}}{S^2+2\xi_1 S+\omega_1^2} + \frac{Amp_{12}}{S^2+2\xi_2 S+\omega_2^2}\right\}T(S) \tag{4.19}$$

$$Q_2(S) = \left\{\frac{Amp_{21}}{S^2+2\xi_1 S+\omega_1^2} + \frac{Amp_{22}}{S^2+2\xi_2 S+\omega_2^2}\right\}T(S) \tag{4.20}$$

$$Q_3(S) = \left\{\frac{Amp_{30}}{S^2} + \frac{Amp_{31}}{S^2+2\xi_1 S+\omega_1^2} + \frac{Amp_{32}}{S^2+2\xi_2 S+\omega_2^2}\right\}T(S). \tag{4.21}$$

Based on equations (4.19), (4.20) and (4.21), the end point trajectory can be written as follows:

$$U(L,S) = \mathcal{L}\{u(L,t)\} = Q_1(S)\phi_1(L) + Q_2(S)\phi_2(L) + Q_3(S)L. \qquad (4.22)$$

The block diagram representation of equation (4.22) is given in Figure 2 for the case of zero damping.

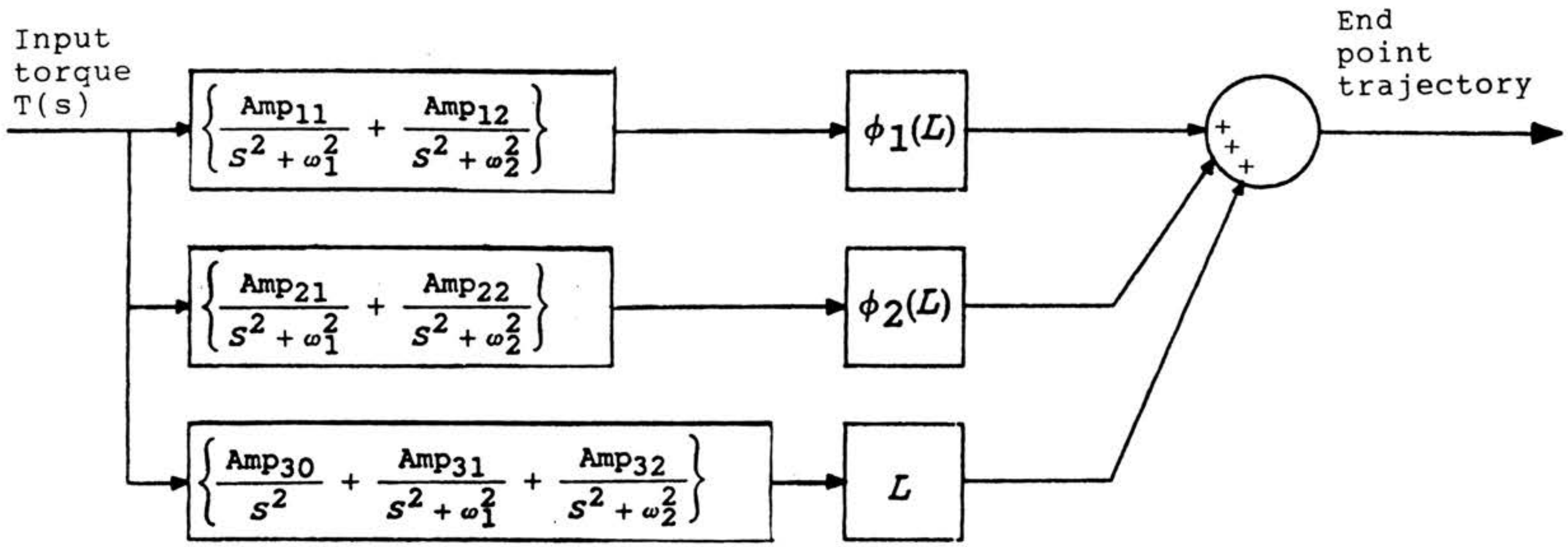


Figure 2: System Block Diagram Representation

V. VERIFICATION OF BEAM MODEL

A. EFFECT OF STIFFNESS:

Results for the flexible arm model were compared with those of a rigid arm, which has the same corresponding characteristics except the flexibility. Clearly as the stiffness, EI(x), of the link increases, the system response of the flexible model converge to the rigid model response. This convergence is graphically shown in Figures 3 and 4 which are plots of tip position vs. time for beams with stiffness $14.7MP_a/MP_a$ and $100MP_a$ respectively.

B. ANALYTICAL SOLUTION - EULER EQUATION FOR BEAMS:

For simplification consider the beam shown in Figure 5(a) with constant EI(x), m(x) but without tip mass.

1. Boundary-Value Problem Formulation: At any point x the bar has a mass per unit length m(x), a cross-sectional area A(x), and an area moment of inertia I(x) about the neutral axis as shown in Figure 5(b). This is a one-dimensional problem because only one space variable, x, is involved.

The total deflection y(x,t) of the bar at a point x consists of two parts, one caused by bending and one by shear. Therefore, the slope of the deflection curve at the point x can be written

$$\frac{\partial y(x,t)}{\partial x} = \psi(x,t) + \beta(x,t) \tag{5.1}$$

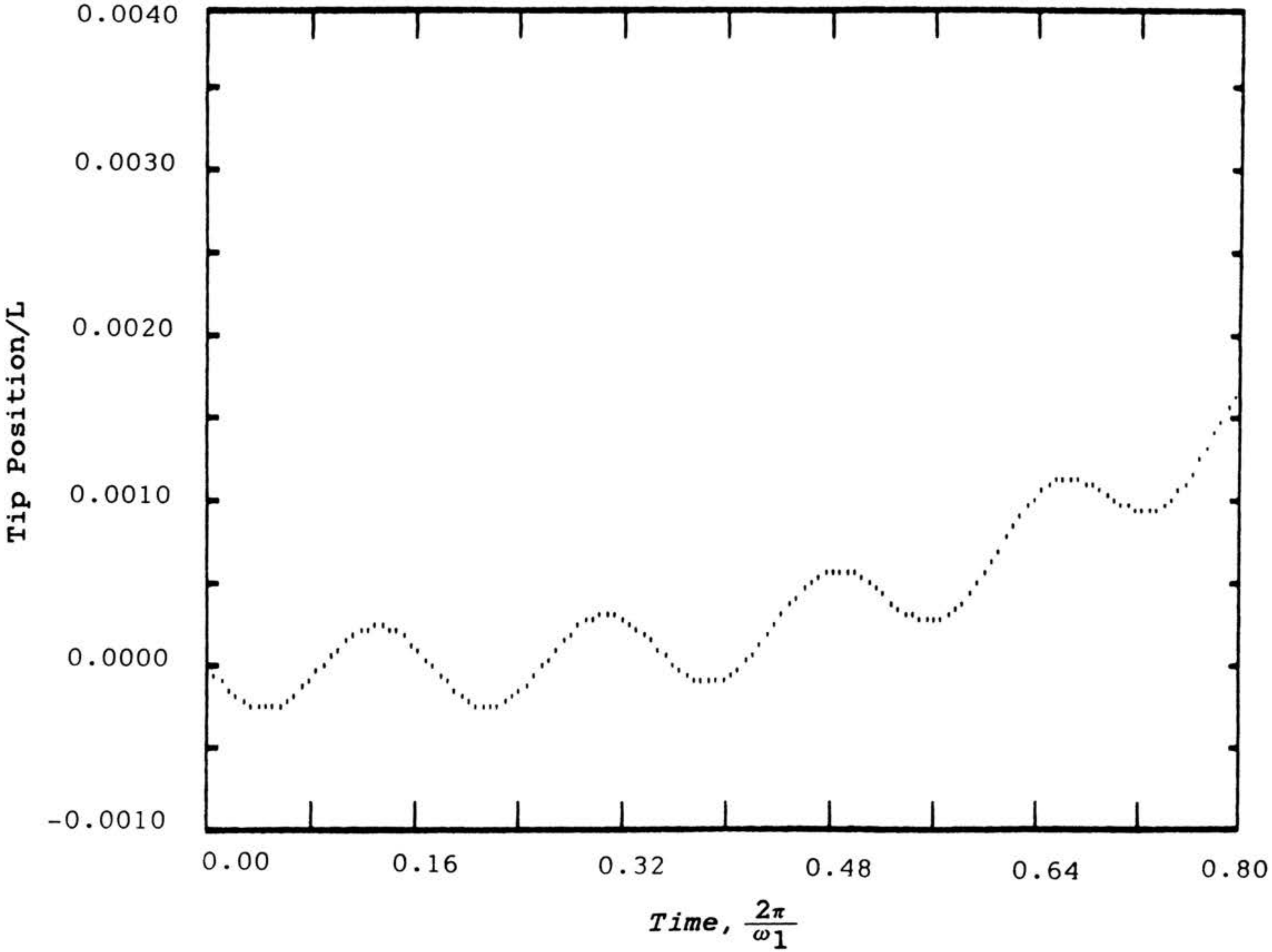


Figure 3: Response of a Flexible Beam

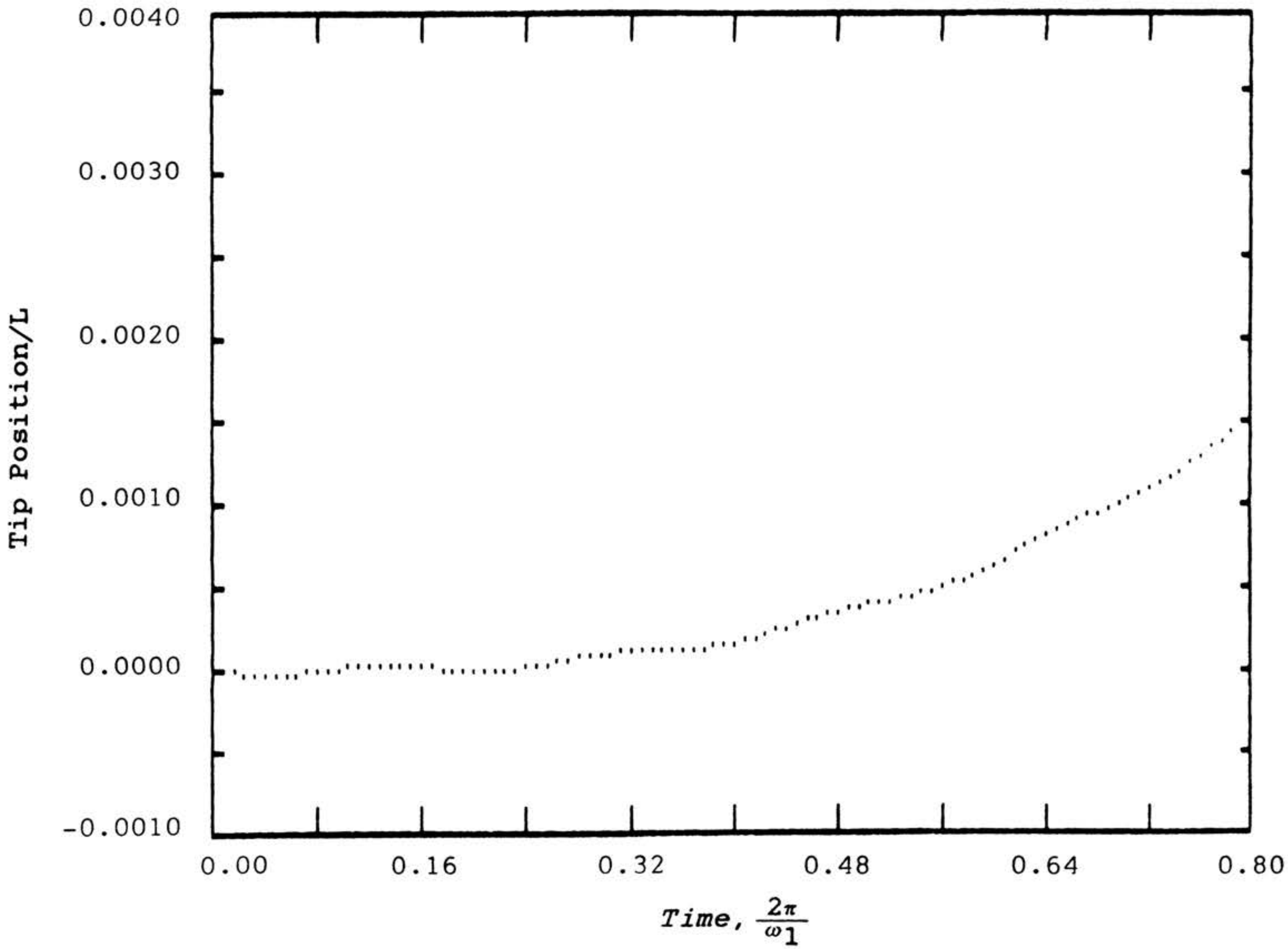
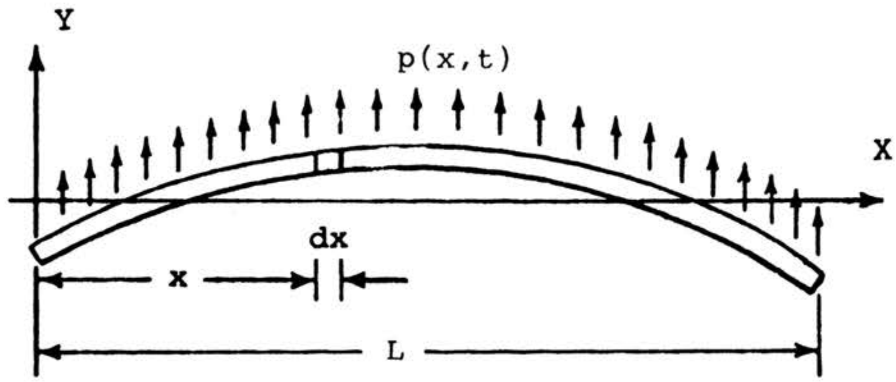
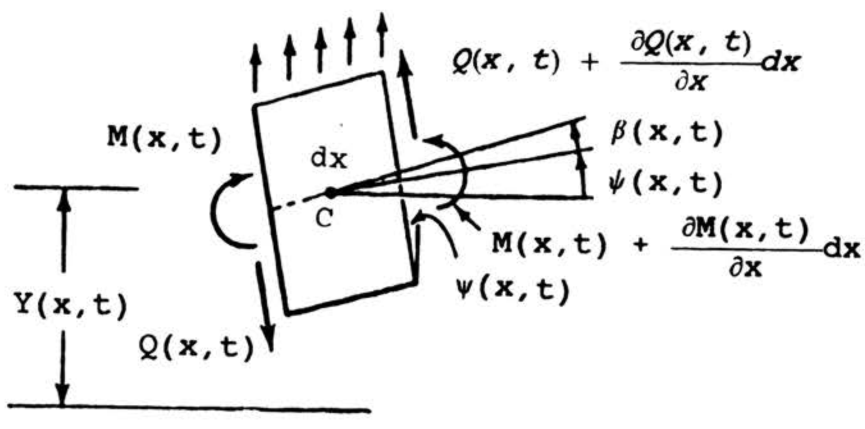


Figure 4: Response of a Beam With High Stiffness



(a) The Nonuniform Bar



(b) Free-body Diagram of a Beam Element

Figure 5: A Beam Under General Consideration

where $\psi(x,t)$ is the angle of rotation due to bending and $\beta(x,t)$ is the angle of distortion due to shear. As usual, the linear deflection and angular deflection are assumed small.

The relation between the bending moment $M(x,t)$ and the bending deformation is

$$M(x,t) = EI(x)\frac{\partial\psi(x,t)}{\partial x} \tag{5.2}$$

and the relation between the shearing force $Q(x,t)$ and shearing deformation is given by

$$Q(x,t) = k'GA(x)\beta(x,t) \tag{5.3}$$

where G is the shear modulus and k' is a numerical factor that depends on the shape of the cross section. Shear alone will cause distortion without rotation.

To formulate the boundary-value problem we shall make use of the extended Hamilton principle.

Here, kinetic energy is due to translation and rotation and is expressed as

$$T(t) = \frac{1}{2}\int_0^L\left[\frac{\partial y(x,t)}{\partial t}\right] m(x)dx + \frac{1}{2}\int_0^L\left[\frac{\partial\psi(x,t)}{\partial t}\right] J(x)dx \tag{5.4}$$

where L is the length of the bar and J(x) is the mass moment of inertia per unit length about the neutral axis which passes through the center C as shown in Figure 5(b). But J(x) is related to I(x) by

$$J(x) = \sigma I(x) = \frac{m(x)}{A(x)} I(x) = k^2(x) m(x) \tag{5.5}$$

where σ is the mass density and k(x) is the radius of gyration about the neutral axis. The variation of T can be readily written as

$$\delta T = \int_0^L m \frac{\partial y}{\partial t} \delta\left(\frac{\partial y}{\partial t}\right) dx + \int_0^L k^2 m \frac{\partial \psi}{\partial t} \delta\left(\frac{\partial \psi}{\partial t}\right) dx. \tag{5.6}$$

The virtual work consists of conservative and nonconservative work. Since the external load is in the direction of the displacement, the virtual work for the whole bar is

$$\delta W(t) = \delta W_c(t) + \delta W_{nc}(t) = -\delta V(t) + \int_0^L p(x,t) \delta y(x,t) dx \tag{5.7}$$

where V(t) is the potential energy given by

$$\begin{aligned} V(t) &= \frac{1}{2}\int_0^L M(x,t) \frac{\partial \psi(x,t)}{\partial x} dx + \frac{1}{2}\int_0^L Q(x,t) \beta(x,t) dx \\ &= \frac{1}{2}\int_0^L EI(x) \left[\frac{\partial \psi(x,t)}{\partial x}\right]^2 dx + \frac{1}{2}\int_0^L k'GA(x) \beta^2(x,t) dx. \end{aligned} \tag{5.8}$$

Hence the variation of potential energy has the form

$$\begin{aligned} \partial V &= \int_0^L EI \frac{\partial \psi}{\partial x} \delta\left(\frac{\partial \psi}{\partial x}\right) dx + \int_0^L k'GA\beta \; \delta\beta \; dx \\ &= \int_0^L EI \frac{\partial \psi}{\partial x} \delta\left(\frac{\partial \psi}{\partial x}\right) dx + \int_0^L k'GA \left(\frac{\partial y}{\partial x} - \psi\right) \delta\left(\frac{\partial y}{\partial x} - \psi\right) dx. \end{aligned} \tag{5.9}$$

Introducing equations (5.6), (5.7) and (5.9) in the variational principle leads to

$$
\begin{aligned}
&\int_{t_1}^{t_2}(\delta T + \delta W)dt \\
&= \int_{t_1}^{t_2}\left[\int_0^L m\frac{\partial y}{\partial t}\delta\left(\frac{\partial y}{\partial t}\right)dx + \int_0^L k^2 m\frac{\partial \psi}{\partial t}\delta\left(\frac{\partial \psi}{\partial t}\right)dx\right. \\
&- \int_0^L EI\frac{\partial \psi}{\partial x}\delta\left(\frac{\partial \psi}{\partial x}\right)dx - \int_0^L k'GA\left(\frac{\partial y}{\partial x} - \psi\right)\delta\left(\frac{\partial y}{\partial x} - \psi\right)dx \\
&\left. + \int_0^L p\delta y dx\right]dt = 0.
\end{aligned}
\tag{5.10}
$$

Since the order of integrations with respect to x and t is interchangeable and the variation and differentiation operators are commutative, we can perform the following integrations by parts:

$$
\begin{aligned}
\int_{t_1}^{t_2} m\frac{\partial y}{\partial t}\delta\left(\frac{\partial y}{\partial t}\right)dt &= \int_{t_1}^{t_2} m\frac{\partial y}{\partial t}\frac{\partial}{\partial t}(\delta y)dt = m\frac{\partial y}{\partial t}\delta y\Big|_{t_1}^{t_2} \\
&- \int_{t_1}^{t_2}\frac{\partial}{\partial t}\left(m\frac{\partial y}{\partial t}\right)\delta y\, dt = -\int_{t_1}^{t_2} m\frac{\partial^2 y}{\partial t^2}\delta y\, dt
\end{aligned}
$$

because δy vanishes at $t=t_1$ and $t=t_2$. In a similar fashion we obtain

$$
\int_{t_1}^{t_2} k^2 m\frac{\partial \psi}{\partial t}\delta\left(\frac{\partial \psi}{\partial t}\right)dt = -\int_{t_1}^{t_2} k^2 m\frac{\partial^2 \psi}{\partial t^2}\delta\psi\, dt.
$$

On the other hand, integration over the spatial variable yields

$$\int_0^L EI\frac{\partial\psi}{\partial x}\delta\left(\frac{\partial\psi}{\partial x}\right)dx$$

$$= \int_0^L EI\frac{\partial\psi}{\partial x}\frac{\partial}{\partial x}(\delta\psi)dx = \left(EI\frac{\partial\psi}{\partial x}\right)\delta\psi\Bigg|_0^L - \int_0^L\frac{\partial}{\partial x}\left(EI\frac{\partial\psi}{\partial x}\right)\delta\psi\, dx$$

$$\int_0^L k'GA\left(\frac{\partial y}{\partial x} - \psi\right)\delta\left(\frac{\partial y}{\partial x} - \psi\right)dx$$

$$= \int_0^L k'GA\left(\frac{\partial y}{\partial x} - \psi\right)\frac{\partial}{\partial x}(\delta y)dx - \int_0^L k'GA\left(\frac{\partial y}{\partial x} - \psi\right)\delta\psi\, dx$$

$$= \left[k'GA\left(\frac{\partial y}{\partial x} - \psi\right)\right]\delta y\Bigg|_0^L - \int_0^L\frac{\partial}{\partial x}\left[k'GA\left(\frac{\partial y}{\partial x} - \psi\right)\right]\delta y\, dx$$

$$- \int_0^L k'GA\left(\frac{\partial y}{\partial x} - \psi\right)\delta\psi dx.$$

Using the equation (5.10) produces

$$\int_{t_1}^{t_2}\Bigg\{ - \int_0^L m\frac{\partial^2 y}{\partial t^2}\delta y dx - \int_0^L k^2 m\frac{\partial^2\psi}{\partial t^2}\delta\psi dx - \left(EI\frac{\partial\psi}{\partial x}\right)\delta\psi\Bigg|_0^L$$

$$+ \int_0^L\frac{\partial}{\partial x}\left(EI\frac{\partial\psi}{\partial x}\right)\delta\psi dx - \left[k'GA\left(\frac{\partial y}{\partial x} - \psi\right)\right]\delta y\Bigg|_0^L$$

$$+ \int_0^L\frac{\partial}{\partial x}\left[k'GA\left(\frac{\partial y}{\partial x} - \psi\right)\right]\delta y dx$$

$$+ \int_0^L k'GA\left(\frac{\partial y}{\partial x} - \psi\right)\delta\psi dx + \int_0^L p\delta y dx\Bigg\}dt$$

$$= \int_{t_1}^{t_2} \left[\int_0^L \left\{ \frac{\partial}{\partial x}\left[k'GA\left(\frac{\partial y}{\partial x} - \psi \right) \right] - m\frac{\partial^2 y}{\partial t^2} + p \right\} \delta y dx \right.$$

$$\int_0^L \left\{ \left[\frac{\partial}{\partial x}\left(EI\frac{\partial \psi}{\partial x} \right) + k'GA\left(\frac{\partial y}{\partial x} - \psi \right) \right] - k^2 m\frac{\partial^2 \psi}{\partial t^2} \right\} \delta \psi dx \tag{5.11}$$

$$\left. - \left(EI\frac{\partial \psi}{\partial x} \right)\delta \psi \Big|_0^L - \left[k'GA\left(\frac{\partial y}{\partial x} - \psi \right) \right]\delta y \Big|_0^L \right] dt = 0.$$

The virtual displacements $\delta\psi$ and δy are arbitrary and independent, so they can be made equal to zero at x=0 and x=L and arbitrary for 0<x<L. Therefore, we must have

$$\frac{\partial}{\partial x}\left[k'GA\left(\frac{\partial y}{\partial x} - \psi \right) \right] - m\frac{\partial^2 y}{\partial t^2} + p = 0 \tag{5.12}$$

$$\frac{\partial}{\partial x}\left(EI\frac{\partial \psi}{\partial x} \right) + k'GA\left(\frac{\partial y}{\partial x} - \psi \right) - k^2 m\frac{\partial^2 \psi}{\partial t^2} = 0 \tag{5.13}$$

throughout the domain. In addition, if we write

$$\left(EI\frac{\partial \psi}{\partial x} \right)\delta \psi \Big|_0^L = 0 \tag{5.14}$$

$$\left[k'GA\left(\frac{\partial y}{\partial x} - \psi \right) \right]\delta y \Big|_0^L = 0 \tag{5.15}$$

there is a possibility that either $EI(\partial\psi/\partial x)$ or $\delta\psi$, on one hand, and either $k'GA[(\partial y/\partial x) - \psi]$ or δy, on the other, vanishes at any of the ends x=0 and x=L. Equations (5.12) and (5.13) are the differential equations of motion that must be satisfied over the length of the bar, and (5.14) and (5.15) represent the boundary conditions. The four equations together constitute the boundary-value problem. Equation (5.14) requires that either the bending moment or the bar rotation variation vanish at each end and (5.15) requires that either the shearing force or the deflection variation be zero at each end. It is the satisfaction of these boundary conditions that renders the solution of the differential equations unique.

2. <u>The Eigenvalue Problem</u>: The formulation given by equation (5.12) and (5.13), includes the shear deformation effect and the rotary inertia effect, the latter caused by the angular acceleration of a bar element. The shear deformation effect is reflected in the second integral in the potential energy expression, equation (5.8), and the rotatory inertia effect is represented by the second integral in the kinetic energy expression, equation (5.4). When the cross-sectional dimensions are small compared with the length of the bar, both shear and rotatory inertia effects can be neglected. If this is the case and if the external load is zero, i.e. p(x,t)=0, equations (5.12) and (5.13) can be combined into a single equation,

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 y(x,t)}{\partial x^2}\right] = -m(x)\frac{\partial^2 y(x,t)}{\partial t^2}. \qquad (5.16)$$

The boundary conditions used to solve the above equations are

$$y = 0 \quad , \quad EI\frac{\partial^2 y}{\partial x^2} = \text{Torque} = \tau(t) \quad \text{at} \quad x = 0 \tag{5.17}$$

$$\frac{\partial^2 y}{\partial x^2} = 0 \quad , \quad \frac{\partial^3 y}{\partial x^3} = 0 \qquad \text{at} \quad x = L. \tag{5.18}$$

3. Solution of Eigenvalue Problem: Let us consider a one-dimensional system described over the domain $0<x<L$ by the differntial equation of motion

$$EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} = 0 \qquad 0<x<L \tag{5.19}$$

and by the time-dependent boundary conditions

$$y(0,t) = 0 \tag{5.20}$$

$$\left.\frac{\partial^2 y}{\partial x^2}\right|_{x=0} = \tau(t) \tag{5.21}$$

$$\left.\frac{\partial^2 y}{\partial x^2}\right|_{x=L} = 0 \tag{5.22}$$

$$\left.\frac{\partial^3 y}{\partial x^3}\right|_{x=L} = 0. \tag{5.23}$$

To transform this boundary-value problem with the time-dependent boundary conditions into a problem consisting of a nonhomogeneous differential equation with homogeneous boundary conditions, one assumes a solution of the boundary-value problem described by equations (5.19) through (5.23) in the form

$$y(x,t) = v(x,t) + g(x)\,\tau(t). \tag{5.24}$$

Introducing equation (5.24) in equations (5.20) through (5.23) the transformed boundary condition is obtained

$$y(0,t) = v(0,t) + g(0)\tau(t) = 0 + 0 \tag{5.25}$$

$$\frac{\partial^2 y}{\partial x^2}(0,t) = \frac{\partial^2 v}{\partial x^2}(0,t) + g''(0)\tau(t) = 0 + \tau(t) \tag{5.26}$$

$$\frac{\partial^2 y}{\partial x^2}(L,t) = \frac{\partial^2 v}{\partial x^2}(L,t) + g''(L)\tau(t) = 0 + 0 \tag{5.27}$$

$$\frac{\partial^3 y}{\partial x^3}(L,t) = \frac{\partial^3 v}{\partial x^3}(L,t) + g''(L)\tau(t) = 0 + 0. \tag{5.28}$$

Imposing the conditions g(0)=0, g"(0)=1, g"(L)=0, and g"(L)=0, in equations (5.25) through (5.28), one obtains the nonhomogeneous differential equation

$$EI\frac{\partial^4 v}{\partial x^4} + m\frac{\partial^2 v}{\partial t^2} = -EIg^{iv}\tau - mg(x)\ddot{\tau}(t) \tag{5.29}$$

with time-independent boundary conditions

$$v(0,t) = 0 \tag{5.30}$$

$$\frac{\partial^2 v}{\partial x^2}(0,t) = 0 \tag{5.31}$$

$$\frac{\partial^2 v}{\partial x^2}(L,t) = 0 \tag{5.32}$$

$$\frac{\partial^3 v}{\partial x^3}(L,t) = 0. \tag{5.33}$$

Since the time dependence for all the conservative systems discussed is harmonic the boundary-value problem reduces to the eigen value problem

$$EI\frac{d^4}{dx^4}[X(x)] = \omega^2 m(x)X(x) \qquad 0<x<L \tag{5.34}$$

with

$$v(x,t) = X(x)F(t)$$

(5.34) can be rewritten as

$$\frac{d^4}{dx^4}[X(x)] = \beta^4 X \qquad \text{with} \qquad \beta^4 = \frac{m}{EI}\omega^2 \tag{5.35}$$

The function X(x) must satisfy appropriate boundary conditions.

$$X(0) = 0 \tag{5.36}$$

$$X''(0) = 0 \tag{5.37}$$

$$X''(L) = 0 \tag{5.38}$$

$$X''\ (L) = 0 \tag{5.39}$$

For these boundary conditions general solution of equation (5.34) can be shown to be

$$X(x) = \sin \beta L \sinh \beta x + D \sinh \beta L \sin \beta x.$$

The natural frequencies of vibration found from equation (5.34) are

$$\omega_n = \frac{\alpha_n}{L^2}\sqrt{\frac{EI}{m}} \qquad (5.40)$$

where α_n is given in Table I.

TABLE I - CHARACTERISTIC VALUES FOR EQUATION (5.34)

n	0	1	2	3	4
α_n	0	15.4	50.0	104.0	178.3

C. <u>SYSTEM PARAMETERS</u>:

The data for the flexible beam in our discussion is shown in Table II.

TABLE II - PARAMETERS OF THE BEAM

Length (m)	Thickness (m)	Width (m)	Linear Density(kg/m)	Young's Modulus(GPa)
5	0.005	0.02	0.27126	14.7917

Table III presents the system model parameters derived by equations (4.16), (4.17) and (4.18).

TABLE III - THEORETICAL NATURAL FREQUENCY OF THE FLEXIBLE BEAM

Mode No.	Natural Frequency ω_n (rad/sec)
1	0
2	4.55
3	14.97

A basic computer code called FANCY, which computes the parameters used in equations (4.16) through (4.22), has been generated for determining system parameters. The program also computes the natural frequency shown in Table IV.

TABLE IV - NATURAL FREQUENCY DERIVED BY PROGRAM FANCY

Modal No.	Natural Frequency Ω_n (rad/sec)
1	0
2	4.54
3	21.69

As the tip trajectory of a flexible link converges to that of a rigid link (Figure 3 and Figure 4) and as the first two natural frequencies are almost equal, the analytical and numerical results obtained for the case without tip mass can be assumed accurate.

VI. CONTROL STRATEGY

The lack of tip trajectory accuracy of the flexible link limits the work capability in industrial applications. It also clearly limits loading ability to levels far below their structural limitation.

Most manufacturing and many assembly tasks require position tolerances in the order of 0.025mm, yet sufficiently robust industrial manipulators are always characterized by repeatabilities close to 0.25mm. To overcome such an inaccuracy, prior results of flexible systems are defined by a single set of partial differential equations. However, this study using expressions of system transfer function is easier than those prior efforts, specially in control application.

A. THE FORMULATION OF CONTROL LAW :

The main goal in designing a controller is to relocate the position of poles of the system such that a better performance can be derived.

Rearranging equation (4.22), the system transfer function H(S) can be written in the form suitable to control system design:

$$H(S) = \frac{U(L,S)}{T(S)} = \frac{b_1S^4 + b_2S^3 + b_3S^2 + b_4S + b_5}{S^6 + a_1S^5 + a_2S^4 + a_3S^3 + a_4S^2} = \frac{b(S)}{a(S)} \tag{6.1}$$

where

$$b_1 = (Amp_{11} + Amp_{21} + Amp_{31}) + (Amp_{12} + Amp_{22} + Amp_{32}) + Amp_{30}$$

$$b_2 = 2(Amp_{11} + Amp_{21} + Amp_{31})\xi_2\omega_2 + 2(Amp_{12} + Amp_{22} + Amp_{32})\xi_1\omega_1 + 2Amp_{30}(\xi_1\omega_1 + \xi_2\omega_2)$$

$$b_3 = (Amp_{11} + Amp_{21} + Amp_{31})\omega_2^2 + (Amp_{12} + Amp_{22} + Amp_{32})\omega_1^2 + Amp_{30}(\omega_1^2 + \omega_2^2 + 4\xi_1\xi_2\omega_1\omega_2)$$

$$b_4 = 2Amp_{30}\omega_1\omega_2(\omega_1\xi_2 + \omega_2\xi_1)$$

$$b_5 = Amp_{30}\,\omega_1^2\omega_2^2$$

$$a_1 = 2(\xi_1\omega_1 + \xi_2\omega_2)$$

$$a_2 = \omega_1^2 + \omega_2^2 + 4\xi_1\xi_2\omega_1\omega_2$$

$$a_3 = 2\omega_1\omega_2(\omega_1\xi_2 + \omega_2\xi_1)$$

$$a_4 = \omega_1^2\omega_2^2$$

An auxiliary variable $\psi(t)$ referred to as the partial state variables, can be defined as the transfer function from $\Psi(S)$ to T(S):

$$\frac{\Psi(S)}{T(S)} = \frac{1}{a(S)} \tag{6.2}$$

or

$$\psi^{vi}(t) + a_1\psi^{v}(t) + a_2\psi^{iv}(t) + a_3\psi''(t) + a_4\psi''(t) = \tau(t). \tag{6.3}$$

Similarly, the transfer function from U(L,S) to $\Psi(S)$ is

$$\frac{U(L,S)}{\Psi(s)} = b(S) \tag{6.4}$$

or

$$u(L,t) = b_1\psi^{iv}(t) + b_2\psi''(t) + b_3\psi''(t) + b_4\psi'(t) + b_5\psi(t). \tag{6.5}$$

Combining equations (6.2) and (6.4), the block diagram shown in Figure 6 is obtained.

In order to formulate the control law, one defines the state variable of the system as below.

$$\dot{x}_1 = \psi^{vi} = -a_1x_1 - a_2x_2 - a_3x_3 - a_4x_4 + \tau \tag{6.6}$$

$$\dot{x}_2 = \psi^{v} = x_1 \tag{6.7}$$

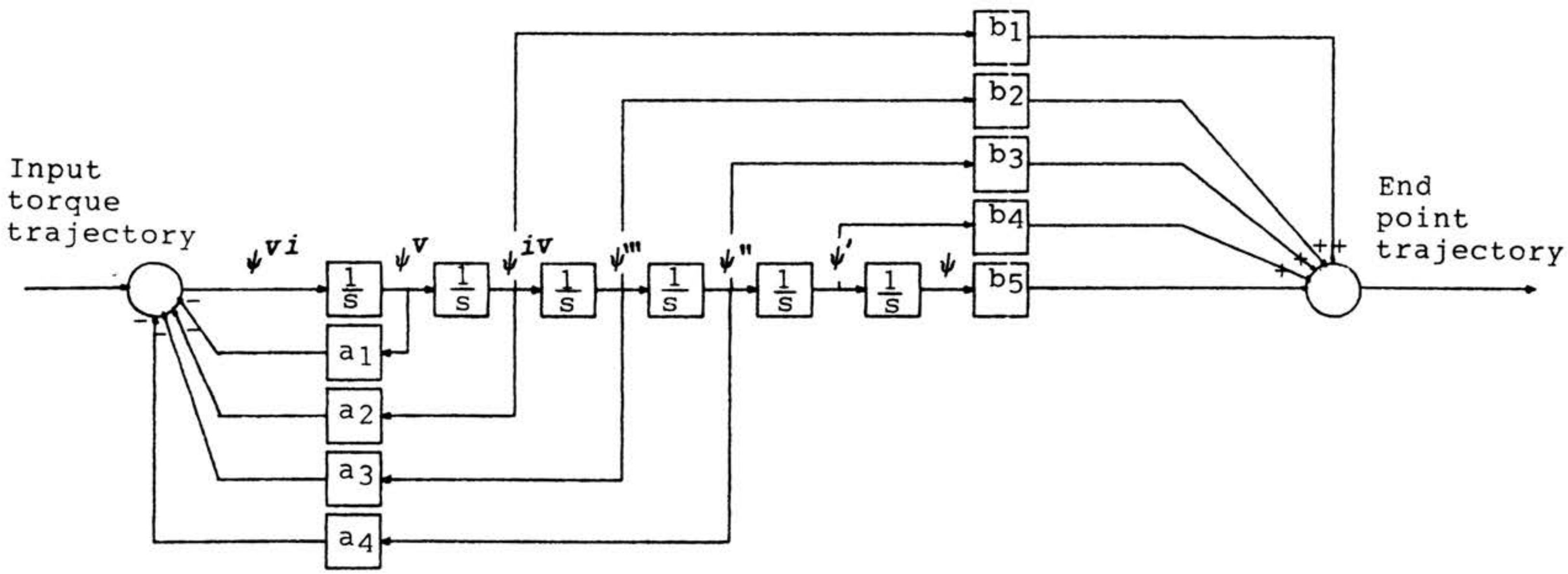


Figure 6: System Block Diagram in Terms of Partial State Variables

$$\dot{x}_3 = \psi^{iv} = x_2 \tag{6.8}$$

$$\dot{x}_4 = \psi'' = x_3 \tag{6.9}$$

$$\dot{x}_5 = \psi'' = x_4 \tag{6.10}$$

$$\dot{x}_6 = \psi' = x_5. \tag{6.11}$$

These equations can be arranged into a matrix form.

$$\{\dot{X}\} = [F]\{X\} + \{G\}\tau \tag{6.12}$$

where
$$\{X\} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{Bmatrix} \tag{6.13}$$

$$\{\dot{X}\} = \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{Bmatrix} \tag{6.14}$$

$$[F] = \begin{bmatrix} -a_1 & -a_2 & -a_3 & -a_4 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{6.15}$$

$$\{G\} = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{6.16}$$

In this case, the form (Figure 6) in which all the feedback loops return to the point of application of the input is referred to as the control canonical form.

B. SELECTION OF GAINS:

Applying matrix operations, the appropriate feedback gain values $K_1, \ldots, K_6$ (as shown in Figure 7) can be determined by selection of the desired pole locations which correspond to the roots of new characteristic equation.

$$a'(S) = S^6 + a'_1 S^5 + a'_2 S^4 + a'_3 S^3 + a'_4 S^2 + a'_5 S + a'_6 = 0. \tag{6.17}$$

Further more, the characteristic equation of the feedback system is:

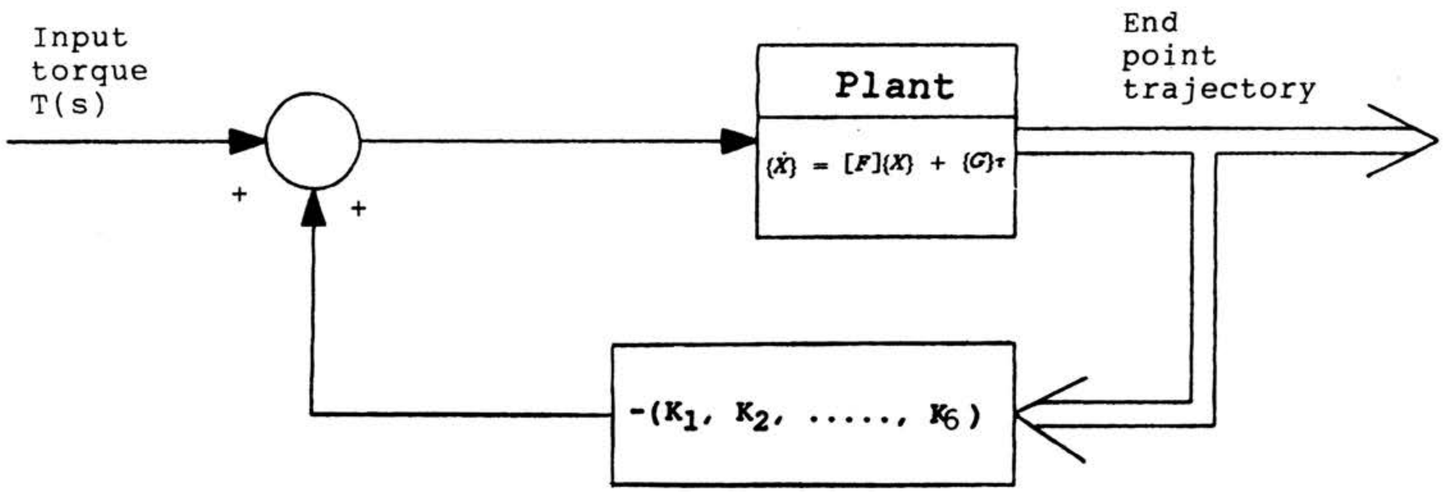


Figure 7: Control System Block Diagram Representation

$$\begin{aligned}\alpha_C(S) &= \det[SI - (F - GK)] \\ &= S^6 + (a_1 + K_1)S^5 + (a_2 + K_2)S^4 + (a_3 + K_3)S^3 + (a_4 + K_4)S^2 \\ &\quad + K_5 S + K_6 \\ &= 0\end{aligned} \tag{6.18}$$

where

I = Unity matrix

$$F\text{-}GK = \begin{bmatrix} (-a_1 - K_1) & (-a_2 - K_2) & (-a_3 - K_3) & (-a_4 - K_4) & -K_5 & -K_6 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

The feedback gains can then be found by equating equations (6.17) and (6.18). The results are

$$\begin{aligned}K_1 &= -a_1 + a'_1 \\ &= -a_1 + 2(\xi_1\omega_1 + \xi_2\omega_2 + \xi_3\omega_3)\end{aligned} \tag{6.19}$$

$$\begin{aligned}K_2 &= -a_2 + a'_2 \\ &= -a_2 + \omega_1^2 + \omega_2^2 + \omega_3^2 + 4(\xi_1\xi_2\omega_1\omega_2 + \xi_2\xi_3\omega_2\omega_3 + \xi_1\xi_3\omega_1\omega_3)\end{aligned} \tag{6.20}$$

$$\begin{aligned}K_3 &= -a_3 + a'_3 \\ &= -a_3 + 2\omega_1^2\omega_2\xi_2 + 2\omega_2^2\omega_1\xi_1 + 2\omega_1^2\omega_3\xi_3 + 2\omega_3^2\omega_1\xi_1 \\ &\qquad + 2\omega_2^2\omega_3\xi_3 + 2\omega_3^2\omega_2\xi_2 + 8\omega_1\omega_2\omega_3\xi_1\xi_2\xi_3\end{aligned} \tag{6.21}$$

$$K_4 = -a_4 + a'_4$$

$$= -a_4 + 4\omega_1^2\omega_2\omega_3\xi_2\xi_3 + 4\omega_2^2\omega_1\omega_3\xi_1\xi_3 + 4\omega_3^2\omega_1\omega_2\xi_1\xi_2 + \omega_1^2\omega_2^2 + \omega_1^2\omega_3^2 + \omega_2^2\omega_3^2 \tag{6.22}$$

$$K_5 = a'_5$$

$$= 2\omega_1^2\omega_2^2\omega_3\xi_3 + 2\omega_2^2\omega_3^2\omega_1\xi_1 + 2\omega_1^2\omega_3^2\omega_2\xi_2 \tag{6.23}$$

$$K_6 = a'_6$$

$$= \omega_1^2\omega_2^2\omega_3^2 \tag{6.24}$$

VII. SIMULATION OF DYNAMIC SYSTEM

The response is simulated for a total of two periods of the fundamental frequency. The sampling duration in computation is fixed at 0.003 seconds, which is approximately 110 times higher than the highest frequency of the truncated system. To evaluate the system performance, a step torque (0.1Nm) is utilized as the input torque.

Figures 8 and 9 are the simulation results. Notice that the deflections and time elapsed are all normalized quantities and these will provide a good picture about the system performance no matter how fast or slow the system response is.

Figure 8 illustrates the tip position and its commanded trajectory. It is noted that the flexible link is deflected backwards and forwards with respect to its commanded trajectory all the time. And the actual response is always lower than its trajectory. This feature results from the fact that the moment of inertia for the link is increased due to the link flexibility.

Figure 9 shows the feedback control simulation. For this case the poles are moved to left (Figure 10) such that damping ratios of the system are increased to 0.5. It is noted from the figures that the feedback control is successful in suppressing the structural vibration of the

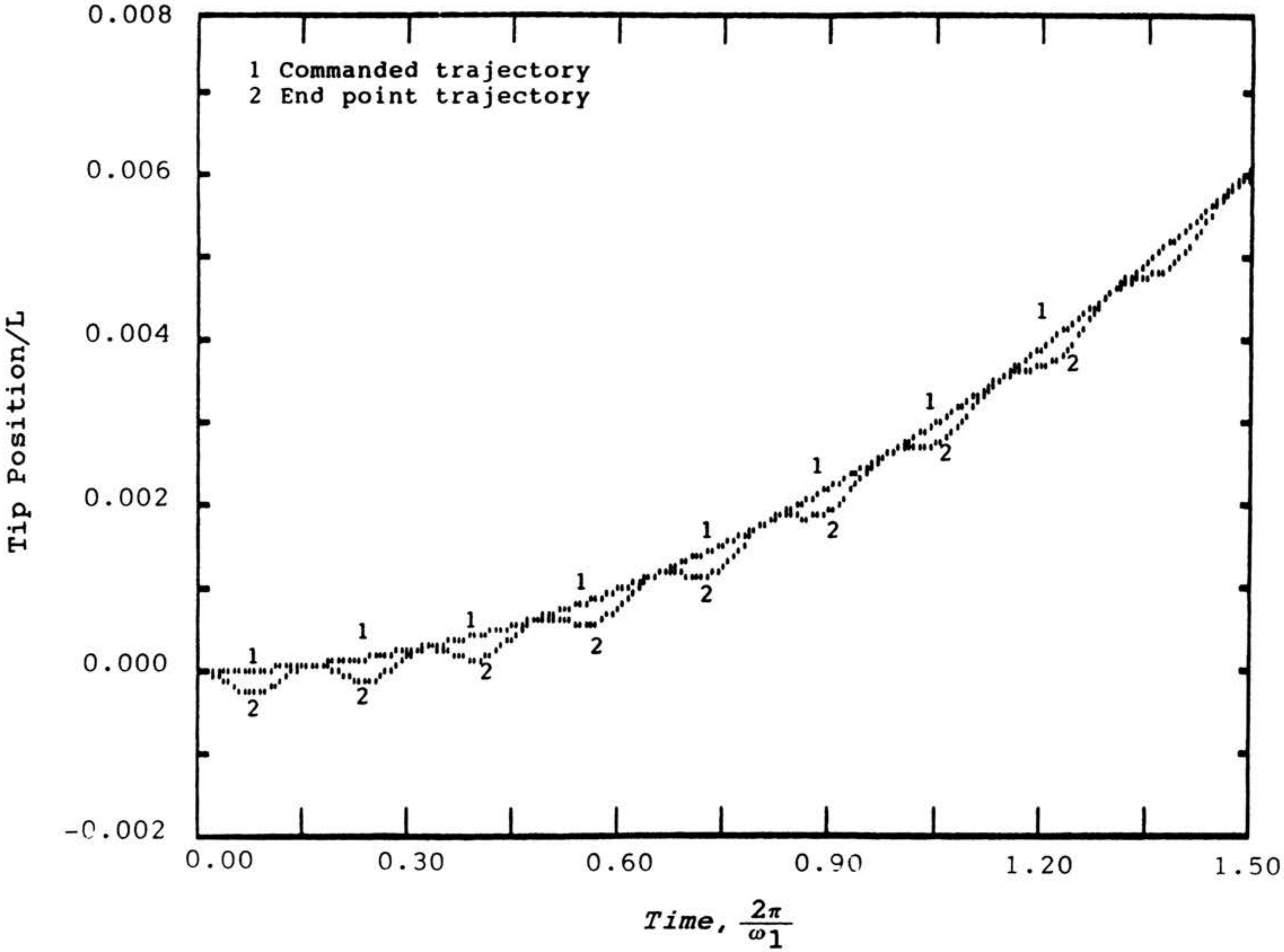


Figure 8: Commanded Trajectory and End Point Trajectory

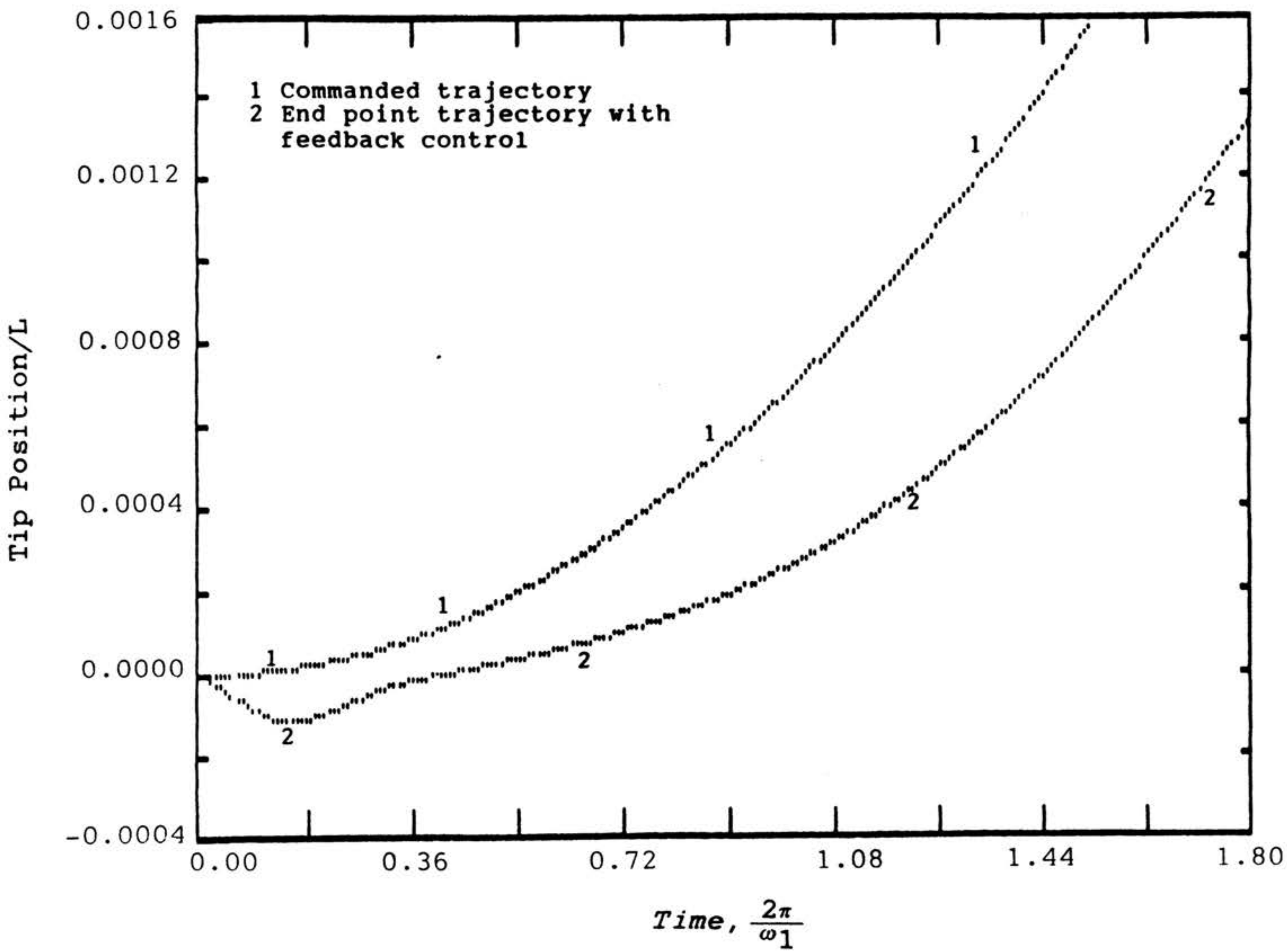


Figure 9: Feedback Control Simulation

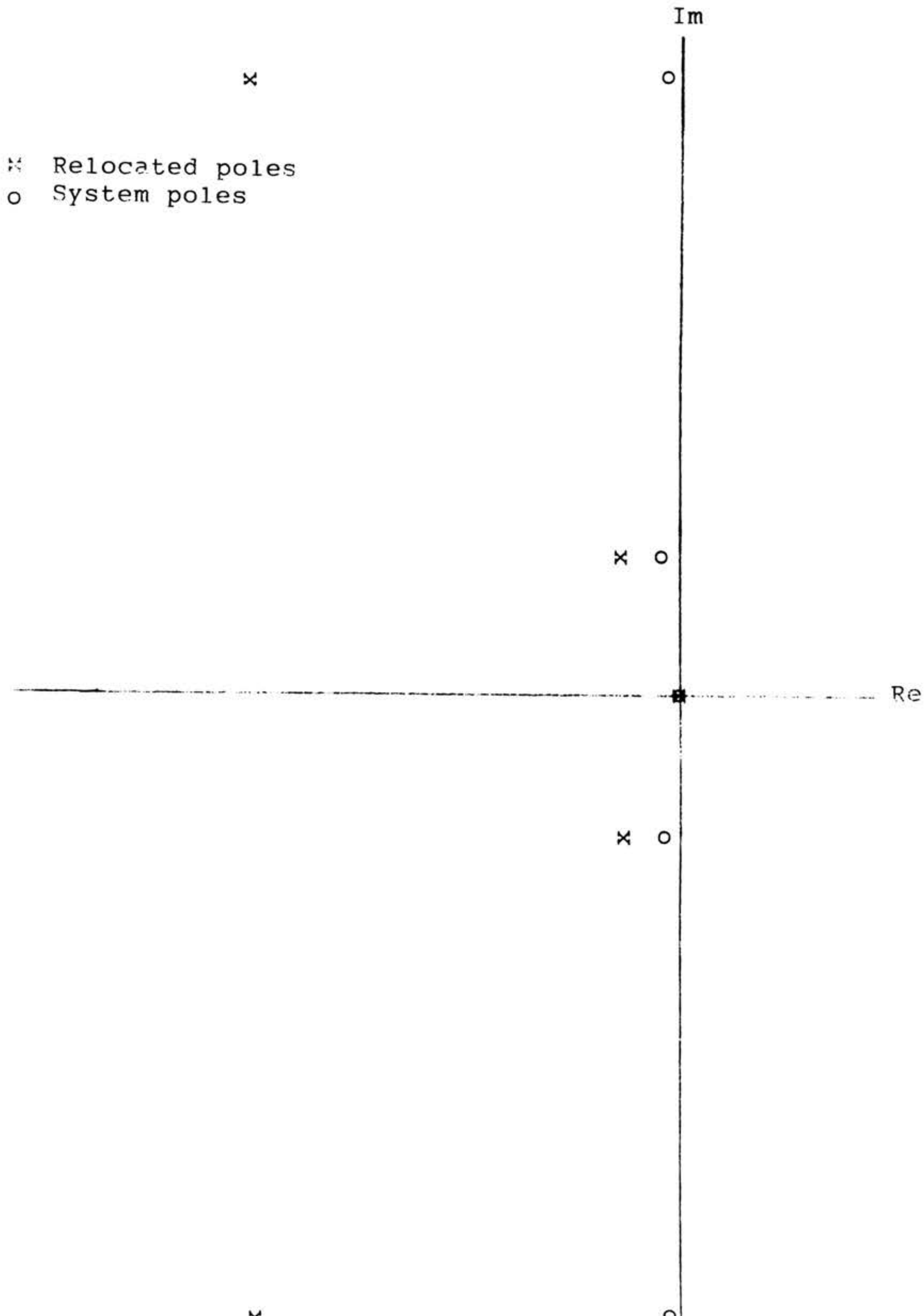


Figure 10: Positions of Poles and Relocated Poles

flexible beam and also results in a slow transient response (Figure 9).

Finally, Figure 11 indicates the control efforts for this case. It is not accidental that the settling time is very satisfactory.

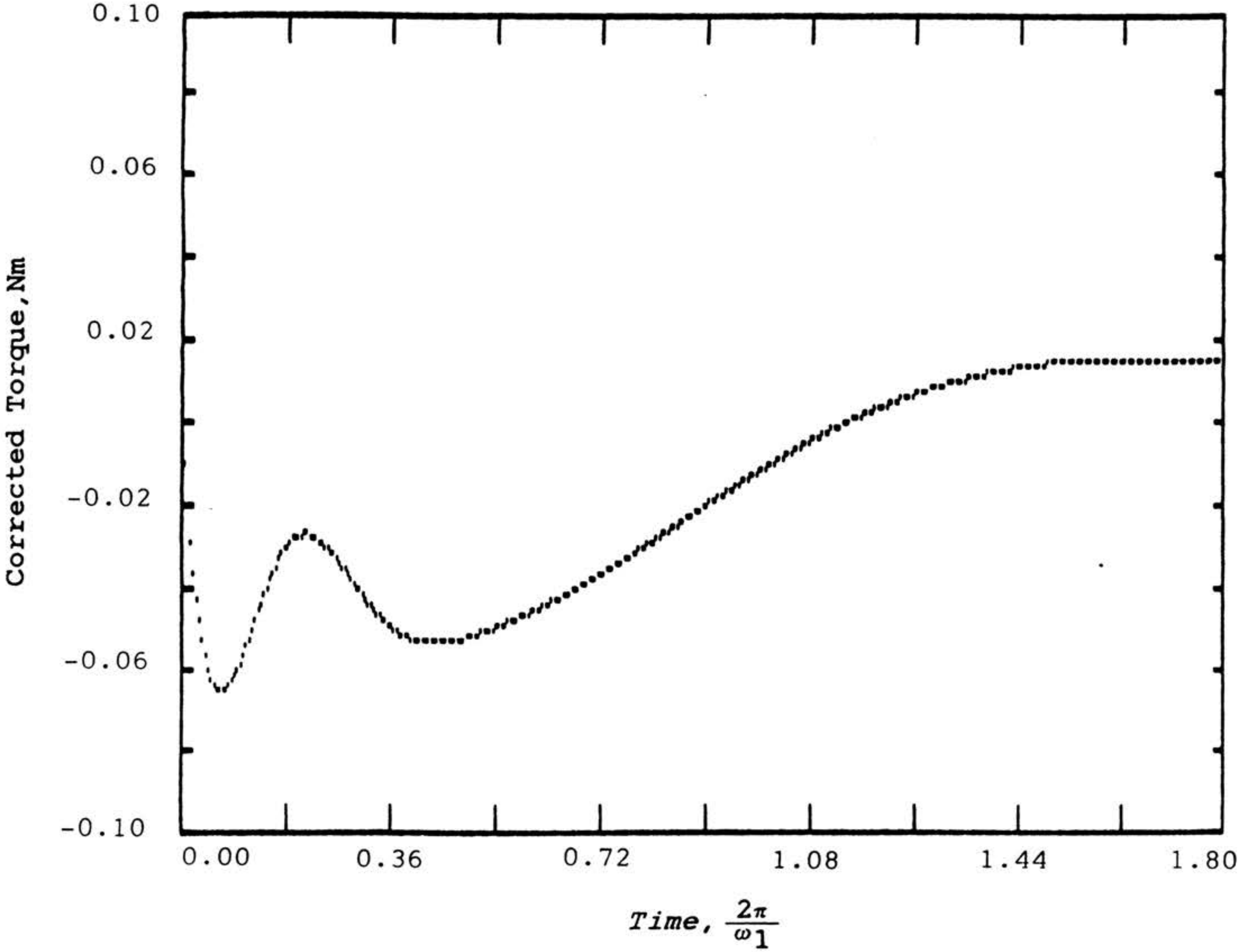


Figure 11: Control Efforts

VIII. CONCLUSIONS AND RECOMMENDATIONS

A. SUMMARY OF CONCLUSION:

The results of this thesis support the conclusions summarized in this section.

1. The transfer function approach is a valid, useful, and convenient way to model the dynamic system of flexible manipulators for the purpose of controller analysis and synthesis.

   It has the adaptability for use in the structure design. It also has the efficiency for extensive analysis of particular configurations of deformable components.

2. For simple controller, flexibility limits the system performance achieved by feedback control.

3. For manipulators with the common control scheme assumed, flexibility is the most critical factor in the design of the arm structure for general practical requirements. More sophisticated control schemes are of interest in these cases if the disadvantanges due to inherent flexibility cannot be overcome.

B. RECOMMENDATIONS FOR FUTURE WORK:

The recommendations for future work can be classified into three categories.

1. Systematic methods for evaluating system parameters are really needed for the application of control law

proposed. This will be especially important if the additional number of arms is present.

2. Torsional effects should be considered for those cases with heavy payload and in high speed. This will result in a more complex driving mechanism than the ideally planar motion.

3. Additional control schemes should be explored. Due to the inaccessibility of partial state variables used in feedback control, a reduced observer will be greatly required to implement the control law.

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