1970

Digital simulation of a Costas loop demodulator in Gaussian noise and CW interference

Ajay Mugatlal Mehta

Department:

DIGITAL SIMULATION OF A COSTAS LOOP DEMODULATOR IN GAUSSIAN NOISE AND CW INTERFERENCE

By

AJAY MUGATLAL MEHTA, 1947 -

A

THESIS

submitted to the faculty of

THE UNIVERSITY OF MISSOURI - ROLLA

in partial fulfillment of the requirements for the

Degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

Rolla, Missouri

1970

Approved by

R. E. Zuemer (Advisor) William H. Tranter

Thomas Baird

ABSTRACT

A digital computer model of a Costas loop has been employed to study the effects of Gaussian noise and continuous-wave interference on the detection of biphase modulated signals. The dependence of bit-error probability, mean-square error between input and output, phase-error variance and threshold on signal-to-Gaussian-noise power ratio, interference-to-Gaussian-noise power ratio and frequency offset of the interference have been investigated and compared with previously obtained experimental results. The model has been verified for its accuracy in the case of no noise and when Gaussian noise is present at its input.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ACKNOWLEDGEMENTS

The author wishes to express his sincere gratitude to Dr. R.E. Ziemer for his very valuable guidance and help during the course of this study. Thanks are also due to Dr. William H. Tranter for his very useful comments in the writing of this thesis.

The author is also thankful to all others who contributed towards the completion of this thesis.

I. INTRODUCTION

A numerical technique for conducting statistical sampling experiments on a model of a system to obtain a probabilistic approximation to the behavior of the system is called Monte Carlo simulation. The model of the system must be describable in a logical and/or mathematical manner. In general, the simulation is carried on a digital computer, not because of any basic relationship, but because of the very large amount of calculations required.

This thesis is concerned with the development of a digital computer model of a Costas loop demodulator and simulating the effects of Gaussian noise and continuous-wave (cw) interference on the demodulation of phase shift keyed (PSK) signals. Interference is commonly encountered in many applications of Costas loops, e.g., in telemetry systems where the interference is due to extraneous sources. The simulation technique consists of integrating the loop equations by using the Runge-Kutta method of order IV [1].

Costas loops are similar to phase-lock loops (PLLs) in that both can be used to demodulate signals which are digitally phase modulated. The difference, however, is that a PLL requires carrier component to lock onto, but a

Costas loop does not. A more complete description of the operation of a Costas loop will be given later. Because of the intractable mathematical nature of the problem, computer modeling appears to be one of few ways available for obtaining useful results.

Digital computer simulation of PLL's in Gaussian noise has been carried out before [2,3]. Simulation of PLL operation in multipath backgrounds has also been done previously [4], and theoretical analyses of Costas loops operating in Gaussian noise are also obtainable in the literature [5]. However, few results are available on the effects of interference on such devices.

The performance of the Costas loop considered in this thesis is characterized in terms of (1) bit-error probability, (2) normalized mean-square error between input and output, (3) threshold, and (4) phase-error variance. Results are obtained for different values of (i) signal-to-Gaussian-noise power ratio, (ii) interference-to-Gaussian-noise power ratio, and (iii) frequency offset of interference. Previously obtained experimental results [6] will be used to compare with and verify the accuracy of the simulation.

II. DERIVATION OF THE COMPUTER MODEL

A. Equations for Costas Loop Operation in Noise and Interference

In this chapter the equations describing the operation of the Costas loop are derived. These equations will then be used to develop the computer model. Figure 1 shows a block diagram representation of a Costas loop demodulator. The equations which describe the operation of the loop with signal, Gaussian noise, and cw interference at its input will now be obtained. The digital computer model consists of a numerical solution of these equations, as will be described later.

Let $y(t)$ be the input to the Costas loop. It consists of the suppressed carrier signal, $s(t)$, plus Gaussian noise, $n_g(t)$, plus cw interference, $n_i(t)$. Thus, $y(t)$ can be written as

$$y(t) = s(t) + n_g(t) + n_i(t) \qquad (2\text{-}1)$$

The signal $s(t)$ is represented as

$$s(t) = \sqrt{2}\, A\, m(t)\, \sin(\omega_o t+\theta), \qquad (2\text{-}2)$$

where

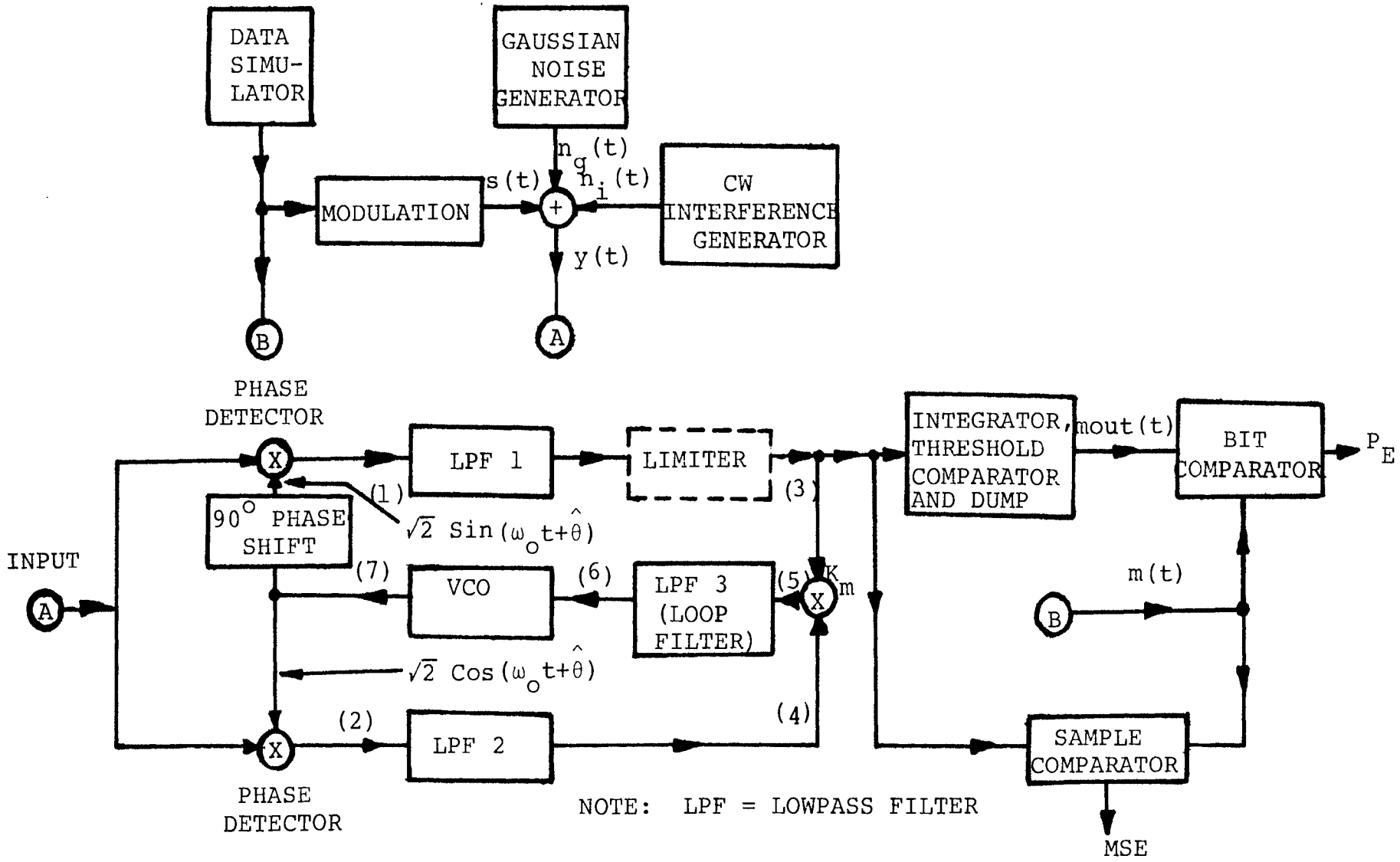


Figure 1. Block Diagram of a Costas Loop Demodulator

A = constant amplitude factor,

$m(t)$ = modulation or signal envelope,

ω_o = frequency of the carrier in radians/second,

and

θ = random phase angle of the carrier.

Assuming that the noise $n_g(t)$ is narrow-band, zero-mean, and Gaussian, it can be represented as [7]

$$n_g(t) = \sqrt{2}\, n_1(t)\mathrm{Cos}(\omega_o t+\theta) + \sqrt{2}\, n_2(t)\mathrm{Sin}(\omega_o t+\theta) \quad (2\text{-}3)$$

where $n_1(t)$ and $n_2(t)$ are statistically independent sample functions of a jointly stationary Gaussian process. The power spectral densities of $n_1(t)$ and $n_2(t)$ are N_o watts/Hz, single-sided, over the passband 0 to W/2 Hz and zero elsewhere, if that of $n_g(t)$ is N_o watts/H_z, single-sided, over the passband $f_o - W/2$ to $f_o + W/2$Hz ($f_o = \omega_o/2\pi$) and zero elsewhere. Thus,

$$\overline{n_g^2(t)} = 2\overline{n_1^2(t)} = 2\overline{n_2^2(t)} = N_o W \quad (2\text{-}4)$$

The cw interference can be represented as

$$\begin{aligned} n_i(t) &= \sqrt{2}\, b_o\, \mathrm{Sin}[(\omega_o+\Delta\omega)t+\theta+\delta] \\ &= \sqrt{2}\, b_o\, \mathrm{Sin}(\Delta\omega t+\delta)\mathrm{Cos}(\omega_o t+\theta) \\ &\quad + \sqrt{2}\, b_o\, \mathrm{Cos}(\Delta\omega t+\delta)\mathrm{Sin}(\omega_o t+\theta), \quad (2\text{-}5) \end{aligned}$$

where the randomly varying phase difference between the signal and the interference is represented as δ.

Equation (2-5) can be written in the same form as Eq. (2-3), so that

$$n_i(t) = \sqrt{2}\, n_{ic}\mathrm{Cos}(\omega_o t+\theta) + \sqrt{2}\, n_{is}\mathrm{Sin}(\omega_o t+\theta) \quad (2\text{-}6)$$

where

$$n_{ic} = b_o\, \mathrm{Sin}(\Delta\omega t + \delta) \quad (2\text{-}7a)$$

and

$$n_{is} = b_o\, \mathrm{Cos}(\Delta\omega t + \delta) \quad (2\text{-}7b)$$

The lowpass filters LPF 1 and LPF 2 are assumed to have the following effects on the signal, noise, and interference components at points (1) and (2) in Figure 1:

(1) The low frequency components of signal are passed without distortion;

(2) The low frequency components of the noise are passed without distortion;

(3) All double frequency components are completely rejected;

(4) The only effect on the cw interference is attenuation and phase shift of its low frequency components in accordance with the filter transfer function, which is represented as

$$H(\omega) = B(\omega)\ \exp[j\theta(\omega)], \qquad (2\text{-}8)$$

where

$B(\omega)$ = amplitude response function of the lowpass filters

and

$\theta(\omega)$ = phase shift function of the lowpass filters.

Let the output of the VCO (point (7) in Figure 1) be

$$V_7(t) = \sqrt{2}\ \mathrm{Cos}\ (\omega_o t + \hat{\theta}) \qquad (2\text{-}9)$$

where $\omega_o/2\pi$ is the free-running frequency of the VCO in Hz, and $\hat{\theta}$ is the Costas loop's estimate of the unknown phase θ. The output $V_3(t)$ of LPF 1 is the same as the low frequency components of the output $V_1(t)$ of the upper phase detector except that the low frequency components of interference will be attenuated and phase shifted according to the transfer function $H(\omega)$ as defined by Eq. (2-8). Thus,

$$\begin{aligned} V_3(t) = {} & Am(t)\mathrm{Cos}\phi - n_1(t)\mathrm{Sin}\phi + n_2(t)\mathrm{Cos}\phi \\ & - b_o B(\Delta\omega)\mathrm{Sin}[\Delta\omega t + \delta + \theta(\Delta\omega)]\mathrm{Sin}\phi \\ & + b_o B(\Delta\omega)\mathrm{Cos}[\Delta\omega t + \delta + \theta(\Delta\omega)]\mathrm{Cos}\phi \end{aligned} \qquad (2\text{-}10)$$

where

$$\phi = \theta - \hat{\theta} \qquad (2\text{-}11)$$

is the phase-error.

Similarly, the output of LPF 2 can be written as

$$V_4(t) = A\, m(t)\mathrm{Sin}\phi + n_1(t)\mathrm{Cos}\phi + n_2(t)\mathrm{Sin}\phi$$
$$+ b_o B(\Delta\omega)\ \mathrm{Sin}[\Delta\omega t+\delta+\theta(\Delta\omega)]\mathrm{Cos}\phi$$
$$+ b_o B(\Delta\omega)\ \mathrm{Cos}[\Delta\omega t+\delta+\theta(\Delta\omega)]\mathrm{Sin}\phi \qquad (2\text{-}12)$$

The differential equation describing the loop is

$$\frac{d\hat{\theta}(t)}{dt} = K_v\, F(p)\, V_5(t) \qquad (2\text{-}13)$$

where

$p = d/dt$ is the differential operator,

K_v = multiplying constant for the VCO,

$F(p)$ = transfer function of the loop filter LPF 3,

and

$V_5(t)$= voltage at point (5) in Figure 1.

From Figure 1,

$$V_5(t) = K_m\, V_3(t)\, V_4(t), \qquad (2\text{-}14)$$

where K_m is the multiplier constant.

In practice, the characteristics of a second order loop are conveniently specified in terms of its damping factor ζ and the natural frequency of oscillation ω_n, or the equivalent noise bandwidth, W_L, of the linearized loop.

These parameters can be defined in terms of various loop and signal constants. From Eqs. (2-10) and (2-14), the differential equation for the linearized loop with only signal present at the input is

$$\frac{d\hat{\theta}}{dt} = K_v K_m A^2 F(p) (\theta - \hat{\theta}) \qquad (2\text{-}15)$$

where the approximation $\sin 2\phi \simeq 2\phi$ has been used. The closed loop transfer function $H_o(s)$ can be defined as the ratio of the Laplace transform of $\theta(t)$ to that of $\hat{\theta}(t)$, and from Eq. (2-15), is

$$H_o(s) = \frac{Y(s)}{1+Y(s)}, \qquad (2\text{-}16)$$

where

$$Y(s) = K_v K_m A^2 F(s)/s \qquad (2\text{-}17)$$

is referred to as the open loop transfer function. Equation (2-17) is written under the assumption that $m(t) = \pm 1$, i.e., $m(t)$ is a binary digital sequence.

Assuming $F(s)$ to be the filter transfer function of a perfect second-order active loop, i.e.,

$$F(s) = 1 + \frac{a}{s}, \qquad (2\text{-}18)$$

it can be easily shown that

$$H_o(s) = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad (2\text{-}19)$$

where

$$a = \omega_n/2\zeta, \tag{2-20}$$

and

$$2\zeta\omega_n = A^2K_vK_m. \tag{2-21}$$

A commonly used value for the damping factor in PLL work is $\zeta = 0.707$ [8]. This value for ζ will be used here. The natural frequency ω_n is conveniently specified in terms of ζ and the equivalent noise bandwidth W_L of the loop, by

$$\omega_n = \frac{4\zeta W_L}{1+4\zeta 2} \tag{2-22}$$

The parameter W_L is defined by

$$W_L = 2B_L = \int_{-\infty}^{\infty} \left| \frac{Y(f)}{1+Y(f)} \right|^2 df \tag{2-23}$$

where $Y(f)$ is given by Eq. (2-17) with $s = j2\pi f$.

Even for the nonlinear case, it is usually most convenient to describe the second-order loop in terms of ζ and ω_n (or W_L) which are parameters that have been defined for the linearized loop. When Eq. (2-13) is expressed in terms of ζ and ω_n, for a loop filter with transfer function given by Eq. (2-18), it becomes

$$\frac{d^2\hat{\theta}}{dt^2} = 2\zeta\omega_n \frac{d}{dt}\left(\frac{V_5}{K_m}\right) + \omega_n^2\left(\frac{V_5}{K_m}\right) \tag{2-24}$$

This last equation along with Equations (2-10), (2-12), (2-14) and (2-22) completely describes the loop action in terms of the desired parameters W_L and ζ in response to the signal, noise and interference.

For the purpose of numerical integration, Eq. (2-24) can be represented as two simultaneous differential equations of first order [1]. If $\hat{\theta} = y$, and z is introduced as a dummy variable, Eq. (2-24) is equivalent to

$$\frac{dy}{dt} = z + 2\zeta\ \omega_n \left(\frac{V_5}{K_m}\right) \tag{2-25}$$

and

$$\frac{dz}{dt} = \omega_n^{\ 2}\ \left(\frac{V_5}{K_m}\right) \tag{2-26}$$

These are the equations which are numerically integrated in the digital computer model of the loop. This model will be described in the next section.

B. Implementation of the Loop Equations as a Computer Model

The equations derived in the previous section that describe the Costas loop operation are first represented as a flow diagram to facilitate the development of the computer program for the model. Two such flow diagrams are shown in Appendices A and C. The first one is for the noiseless case and is a much simplified version of

the second one, which includes Gaussian noise and cw interference. The flow diagrams are presented in the logical form and show the major steps implemented in the computer program (Appendices B and D) written for the model.

From the flow diagram for the model with interference it will be noted that a limiter is included in the in-phase channel of the loop. This was done mainly to simulate the actual loop used to obtain the experimental results which included a limiter. It stabilizes the loop response at high noise and interference levels.

The results presented in the next section were obtained only for a few specific cases, because (i) only a limited number of experimental results [6] were available for comparison and verification of the model and (ii) availability of computer time was limited. However, the flow diagram presented in Appendix C is quite general and is flexible enough to allow a selection of loop characteristics such as amplitude response function $B(\omega)$ of the in-phase and quadrature-phase channel filters (LPF1 and LPF2) to the low frequency components of interference, the loop bandwidth W_L, and the damping factor ζ. Two types of modulating signals can be selected, namely, a random binary sequence or periodic binary sequences such as a pseudo-random maximal-length

sequence [9]. In practice, purely random sequences with known properties are difficult to generate and use. Therefore, periodic pseudo-random sequences of finite length are often employed to simulate digital data.

A normal random number generator (available in the Scientific Subroutine Package (SSP) memory of the computer) was employed to generate noise samples for the two Gaussian processes $n_1(t)$ and $n_2(t)$ (refer to Figure 7). The standard deviation for these Gaussian samples was calculated as follows.

Assume a sample is taken every

$$T_I = \frac{T_b}{SPB} \text{ seconds,} \qquad (2\text{-}27)$$

where

T_b = period of a data bit,

and

SPB = number of samples per bit

Also, let the variance of each noise sample be σ^2. We want σ^2 in terms of the signal-to-Gaussian-noise ratio (SNR) and SPB. Let the noise bandwidth be B Hz and the noise power spectral density be N_o watts/Hz, as shown in Figure 2.

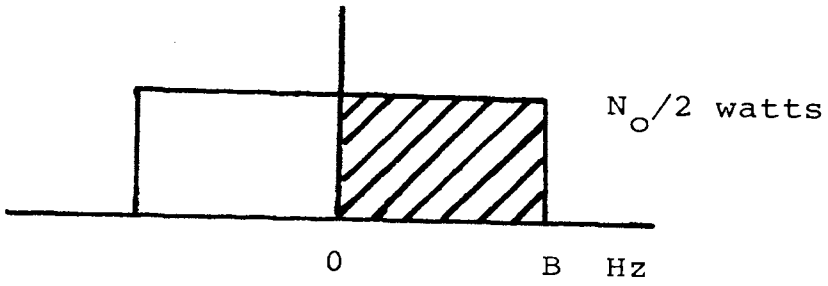


Figure 2: Power Spectral Density of the Gaussian Sampling Processes

Thus,

$$\sigma^2 = N_o B \tag{2-28}$$

and, for the samples to be independent, we must sample at the Nyquist rate [10], or

$$T_I = \frac{1}{2B} \tag{2-29}$$

Substituting for B in Equation (2-28) from Equation (2-29) we obtain

$$\sigma^2 = \frac{N_o}{2T_I} \tag{2-30}$$

The signal-to-Gaussian noise ratio is defined as

$$SNR = \frac{A^2 T_b}{N_o} \quad , \tag{2-31}$$

where the noise power is measured in a bit-rate bandwidth $f_b = 1/T_b$. From Equations (2-27), (2-30) and (2-31) we obtain

$$\sigma^2 = \frac{SPB}{2 \times SNR} \tag{2-32a}$$

where A = 1 for convenience (i.e., all power levels are normalized to the power in the signal). Thus, the standard deviation σ of each noise sample is

$$\sigma = \left(\frac{SPB}{2 \times SNR}\right)^{1/2} . \qquad (2\text{-}32b)$$

The filters LPF 1 and LPF 2 were assumed to be a cascade of two one-pole lowpass stages, having an amplitude response function

$$B(f) = \frac{1}{1 + (f/\alpha)^2} \qquad (2\text{-}33)$$

where α is the cut-off frequency of the filter.

From Equation (2-5), the interference power is b_o^2. However, the signal power has been assumed to be unity (for convenience), so the amplitude b_o of the cw interference can be calculated for the model as

$$b_o^2 = \text{Interference Power}$$

$$= \frac{\text{Interference Power}}{\text{Signal Power}} = \frac{INR}{SNR} \qquad (2\text{-}34a)$$

or

$$b_0 = \left(\frac{INR}{SNR}\right)^{1/2} . \qquad (2\text{-}34b)$$

The relations derived in this section were employed to calculate different parameters for the computer model from the given values of the standard parameters ζ, W_L, INR, SNR, Δf, f_b, etc. for the Costas loop.

III. VERIFICATION OF THE MODEL AND RESULTS

Once the computer model was obtained, its validity and accuracy were tested by comparing simulation results with known theoretical results for loop operation under various conditions but without cw interference. The effects of cw interference on Costas loop operation were then studied for a few specific cases by using the model.

A. Verification of Model

The major steps taken in verifying the accuracy of the digital computer model of a Costas loop operating in interference were the following:

1. Selection of a proper method for solving the differential equation of the Costas loop

Before adopting the Runge-Kutta method of order IV for the differential equation of the loop, the RKGS and HPCG subroutines available in the SSP memory of the computer were first used. Neither of these methods were suitable,as in trying to satisfy a specified error, they have the inherent tendency to reduce the step size. This in turn, made it impossible to calculate the standard deviation of the Gaussian-noise samples used to simulate

$n_1(t)$ and $n_2(t)$ from the known signal-to-Gaussian-noise ratio, because of the resulting unknown number of samples used per bit. The Runge-Kutta method of order IV for solving two simultaneous differential equations of first order [1] with a fixed size integration (i.e. sampling) interval, h, was then used and found acceptable.

2. Testing the model for transient response under noiseless conditions

Testing of the model for transient response was accomplished for two inputs, namely, signal with initial frequency offset from the VCO frequency, and an input signal with increasing frequency offset.

Case 1: Constant Frequency Offset

For the constant frequency offset case, the input signal is

$$s(t) = \sqrt{2}\, Am(t) \cos(\omega_o t + \theta) \qquad (3\text{-}1)$$

where

$$\theta = \Omega t\, u(t) \qquad (3\text{-}2)$$

That is, the initial frequency offset of the signal is Ω radians/second from the initial frequency of the VCO.

With small phase error and for no noise and interference, Eqs.(2-10), (2-12) and (2-14) simplify to

$$\frac{V_5(t)}{K_m} = \frac{1}{2} \sin 2\phi \simeq \phi \ , \qquad (3\text{-}3)$$

when A = 1 (assumed for convenience).

Substituting into Eq. (2-24) and solving for the phase error ϕ we get, for $\zeta = 0.707$,

$$\phi(t) = \sqrt{2}\,\frac{\Omega}{\omega_n}\,\exp\left(-\frac{\omega_n}{\sqrt{2}}t\right)\mathrm{Sin}\left(\frac{\omega_n}{\sqrt{2}}t\right)u(t). \quad (3\text{-}4)$$

Linear operation of the loop imposes the condition that $|\phi_{max}| << 1$ radian. It can be shown that in terms of Ω this requires

$$\Omega << 3.2W_L \quad (3\text{-}5)$$

for the damping factor used.

A plot of phase error, ϕ, versus time, as obtained from Eq. (3-4), is shown in Figure 3. The phase error obtained from the computer model is also shown. For Ω=10 and W_L=100, the condition of linearity is satisfied and the two plots coincide exactly. This indicates that the basic computer model is accurate for noiseless inputs. Also shown in Figure 3 is the response of the model for Ω=200, which shows the deviation from linearity when a value of Ω is used that does not satisfy the inequality in Eq. (3-5).

Case 2. Linearly increasing frequency

The phase of the carrier for a linearly increasing frequency is

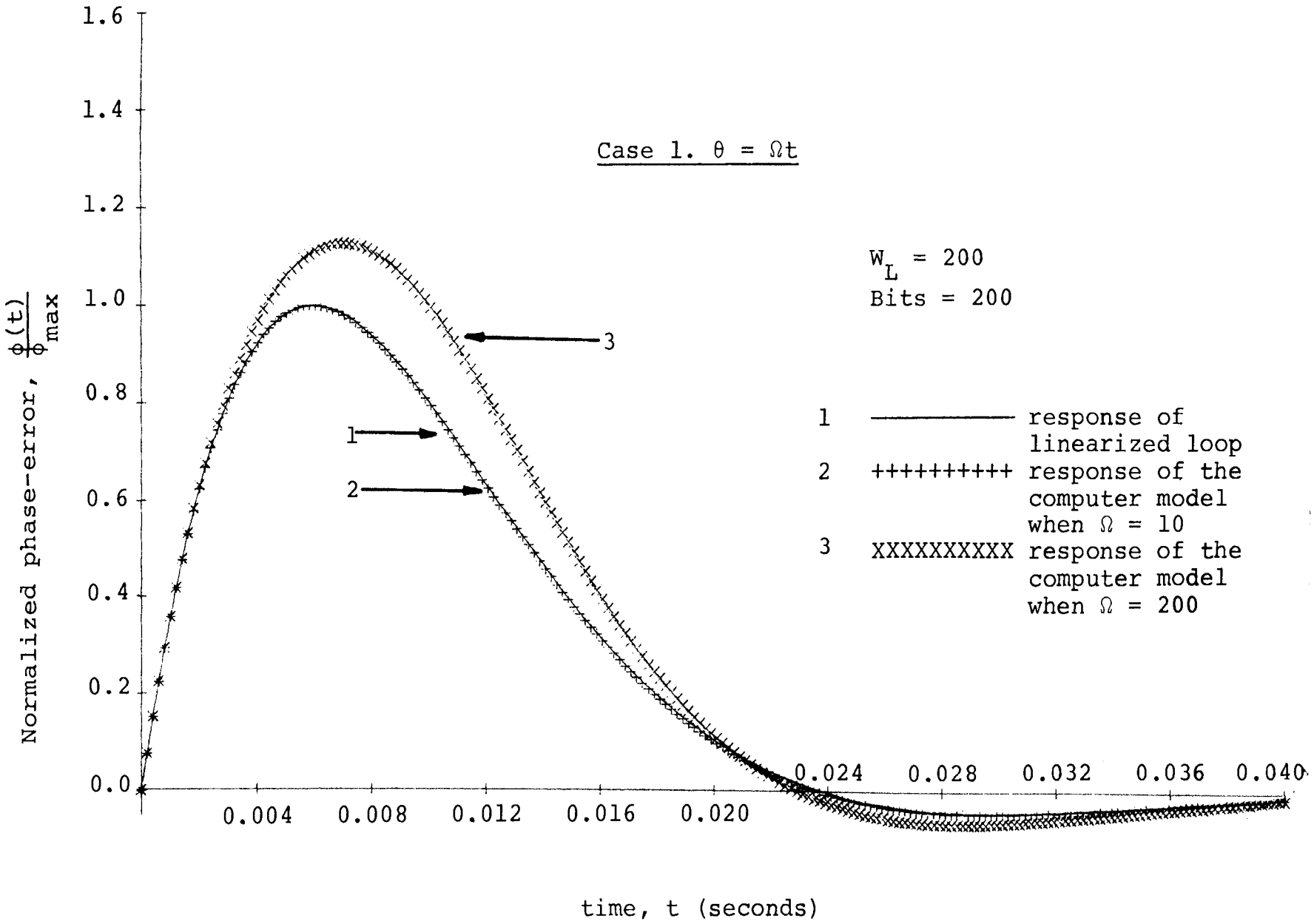


Figure 3. Transient Response of the Model - Noiseless Case 1.

$$\theta = \frac{1}{2}Dt^2 \; u(t). \qquad (3\text{-}6)$$

The solution of the linearized differential equation of the loop for this input phase is

$$\phi(t) = \frac{D}{\omega_n^2}\left[1 - \sqrt{2}\exp\left(-\frac{\omega_n t}{\sqrt{2}}\right)\cos\left(\frac{\omega_n}{\sqrt{2}}\, t - \frac{\pi}{4}\right)\right] \qquad (3\text{-}7)$$

and the condition of linearity, $|\phi_{max}| << 1$, leads to

$$D << 8/9 \; W_L^2 \qquad (3\text{-}8)$$

for $\zeta = 0.707$.

A comparison of $\phi(t)$ calculated from the linearized differential equation with $\phi(t)$ obtained from the computer model is given in Fig. 4 for two values of D. For D=10, the condition for linearity, Eq. (3-8), is satisfied, while for D=10000, the model is operating in the non-linear region. Again, as in the previous case, the plot obtained from the linearized solution coincides exactly with the response of the model as long as condition for linearity is satisfied. This further verifies the accuracy of the model under noiseless conditions.

3. Selection of proper seeds for the random Gaussian number generators

Seed for a random number generator is a number to be specified and used as a starting point in generating the random number sequence. Proper selection of seeds

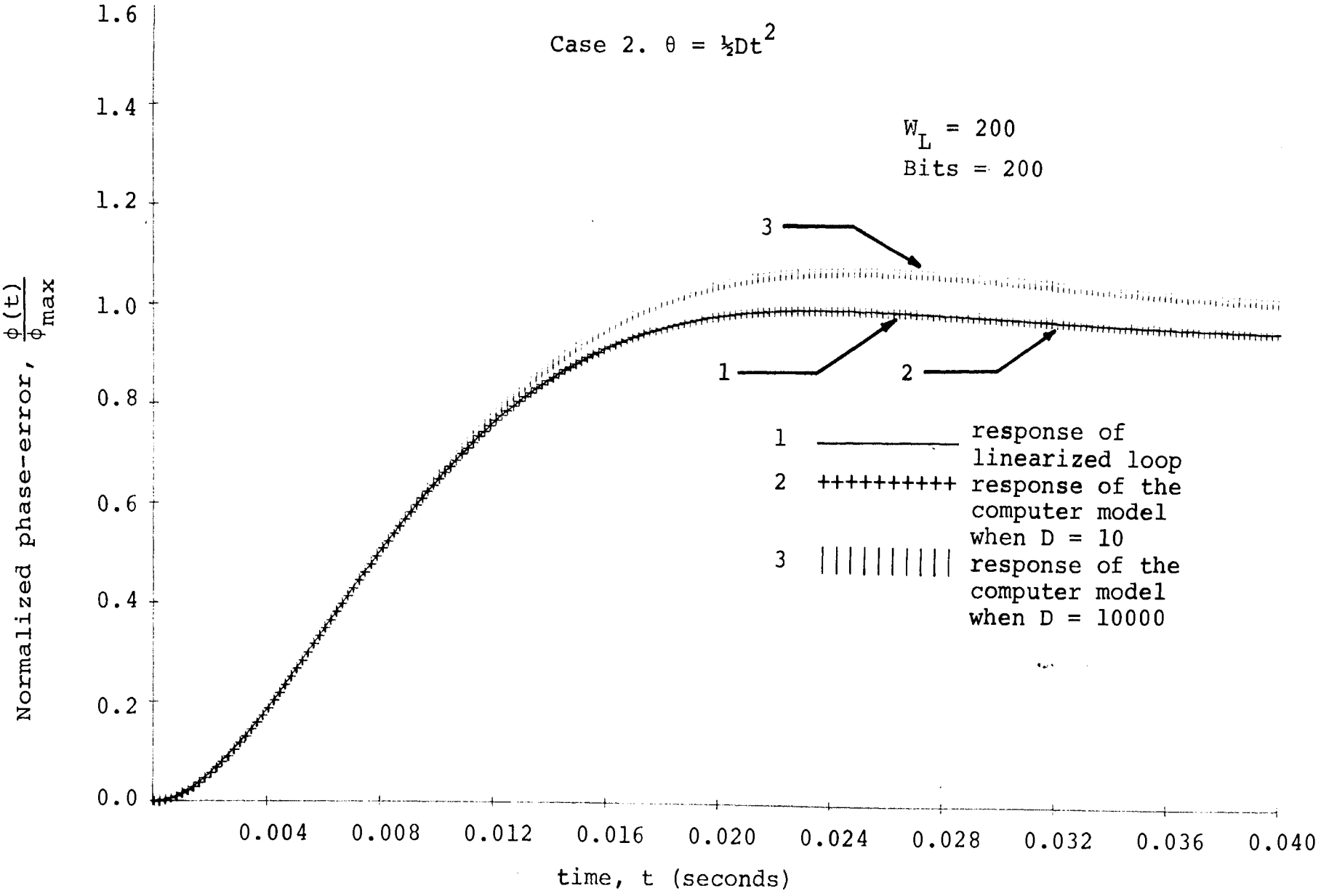


Figure 4. Transient Response of the Model-Noiseless Case 2.

for the random number generators had a significant influence on the number of replications of the simulation model required to reach steady state conditions. Good seeds were selected by trial and error. The sample mean and standard deviation of the output of the Gaussian random number generator used to generate the noise samples were plotted against the number of iterations. The two seeds selected for generating $n_1(t)$ and $n_2(t)$ were the ones that generated processes for which the mean and the standard deviation settled down to their nominal values in minimum number of iterations. Example of the effect of seed selection is given in Fig. 5.

4. Selecting the number of initial bits which could be considered as the transient response of the model

The initial bits that constitute the transient response of the model are omitted from the calculation of bit-error probability, mean-square error, cycle-skips, etc. to allow the transient response of the model to die out. The number of bits omitted was selected in conjunction with step 3 above, and for the parameters and bit-rates chosen was about 200 bits, as can be seen from Fig. 5.

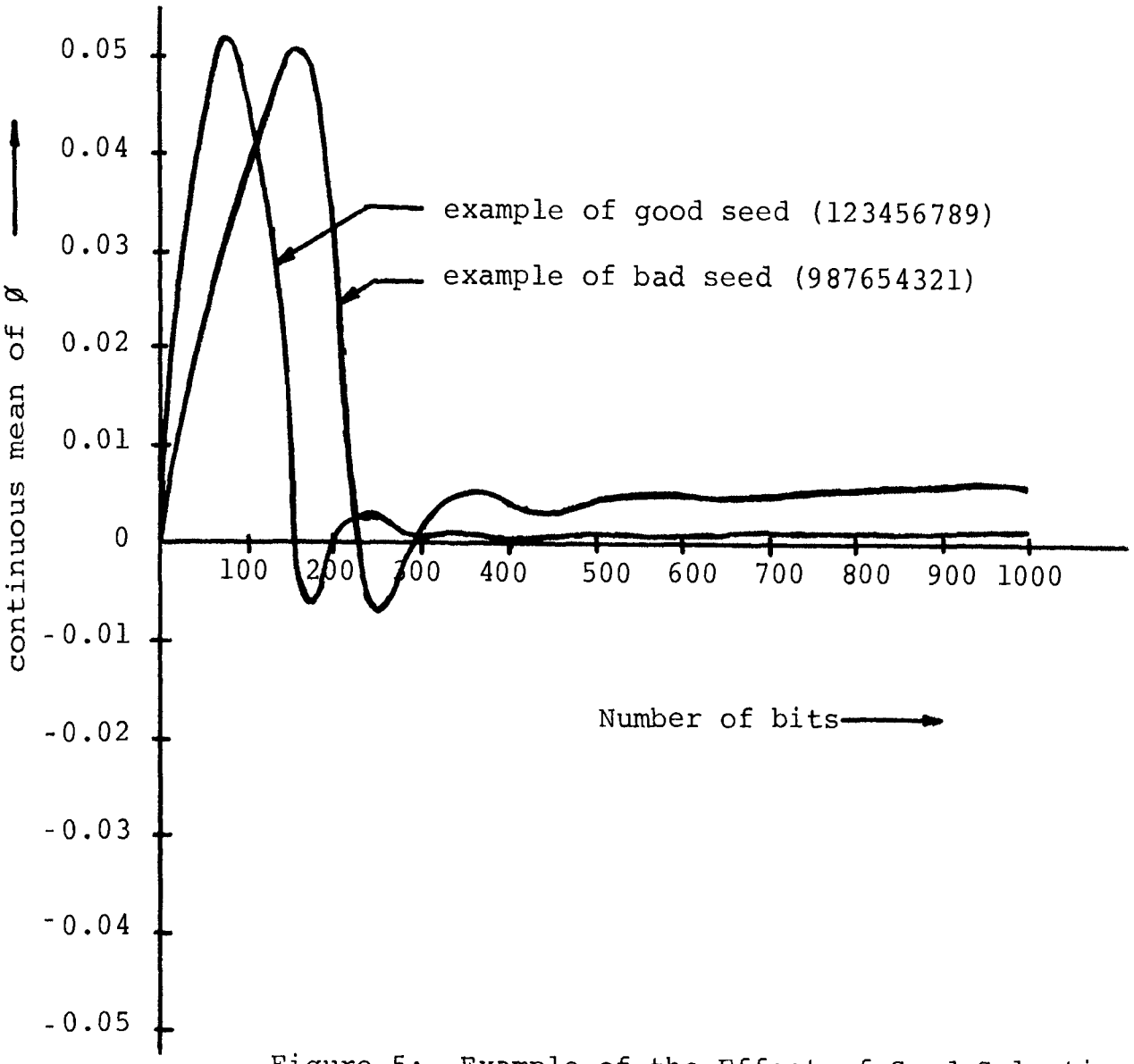


Figure 5: Example of the Effect of Seed Selection

5. Testing the model when Gaussian noise is present

The model was run for signal-to-noise power ratios (noise referred to a bit-rate bandwidth) of 0,1,2,3 and 4 decibels (SNRDB=0,1,2,3 and 4). The bit-error probability P_E obtained from the model was compared with the theoretical values obtained by Didday and Lindsey [5] for a first-order Costas loop (i.e., where LPF3, shown in Figure 1 is absent). This comparison is shown in Table 1.

The phase-error variance is one more yardstick to test the model. The variance σ_ϕ^2 of the phase-error was obtained from the model and compared with the theoretical results calculated from a linearized analysis of the loop. It has been shown [5] that for high signal-to-noise power ratios the following approximate relation holds:

$$\sigma_\phi^2 = \frac{W_L}{W}\left[\frac{1}{SNRBR} + \frac{1}{2(SNRBR)^2}\right] \qquad (3\text{-}9)$$

where SNRBR is the signal-to-Gaussian-noise power ratio in a bit-rate bandwidth. A comparison between σ_ϕ^2 obtained from the computer model and the corresponding values for a linearized model given by Eq. (3-9) is presented in Table 2. This was done for SNRDB=7,8, and 9dB.

Table 1: Bit-Error Probability vs SNRDB*

SNRDB	P_E	
	Theoretical	Model + (with limiter)
0	0.078	0.099
1	0.055	0.076
2	0.035	0.041
3	0.022	0.024
4	0.010	0.012

* Signal-to-Gaussian-noise power ratio measured in a bit-rate bandwidth and expressed in decibels.

+ A step-limiter was introduced in the in-phase channel of the Costas loop

Table 2: Phase-Error Variance vs SNRDB

SNRDB	σ_ϕ^2		
	Theoretical	Model	
		with limiter	without limiter
7	0.00219	0.00295	0.00236
8	0.00171	0.00161	0.00167
9	0.00134	0.00118	0.00120

The mean of the phase-error is expected to be zero because of a zero mean assumed for the Gaussian noise. The values obtained from the computer model were very small, but not quite equal to zero. The reason for this is discussed in the next chapter.

Threshold can be defined as the signal-to-noise ratio at which a loop looses lock which is more or less arbitrarily defined.

An arbitrary method of determining the occurrence of loss of lock in the computer model is used. Each time the absolute value of the phase-error ϕ exceeds the threshold value, $\pi/4$ radian, a cycle-skip results and is noted. A count is also kept of the number of cycle-skips in every ten consecutive input bits, and a loss of lock is said to occur when this count exceeds the value five. The value of the SNR for which this occurs is defined as the threshold value for the model. Also note that after a cycle-skip has been registered, the conditions in the loop (model) are initialized to those at time t=0 second.

It has been suggested [5] that threshold occurs in a Costas loop at $\sigma_{\phi}^{2} \simeq 1/8$. With $W/W_L = 100$ in Eq. (3-9), the value used in the model, a threshold of -9.2 dB results (measured in a bit-rate bandwidth).

SPB*	Model (without limiter)	Theoretical	Experimental
10	-10dB+	-9.2dB	-7dB
20	-9dB		

* SPB - Noise samples taken per bit

+ Signal-to-Gaussian-noise power ratio (measured in a bit-rate bandwidth) expressed in decibels.

Table 3: Threshold Results for Costas Loop

Threshold obtained from the model is compared with this value and the previously obtained experimental value, in Table 3.

Results were also obtained for phase-error variance for the case when a limiter was inserted in the in-phase channel of the loop. These results are included in Table 2 and conclusions drawn in the next chapter. The limiting process has an interesting effect on the threshold. This is discussed in the next chapter.

The effect on the threshold of varying the number of samples per input bit was also investigated. The results are included in Table 3 and the comments follow in the next chapter.

B. Results for interference backgrounds

The final step was to obtain results for cw interference once the model was tested and verified (steps 1 through 5).

Bit-error probability and normalized mean-square error between input and output were computed for different signal-to-Gaussian-noise power ratios, interference-to-Gaussian noise power ratios and frequency offsets of the interference. These are compared with previously obtained experimental results in Tables 4 and 5.

SNRDB	INRDB	Δf=10KHz		Δf=5KHz		Δf=0	
		Experimental MSE	Model MSE	Experimental MSE	Model MSE	Experimental MSE	Model MSE
7	0	0.140	0.166	0.160	0.168	0.220	0.187
	3	0.140	0.167	0.190	0.171	0.340	0.187
	6	0.155	0.170	0.230	0.175	Lock Lost	0.215
8	0	0.120	0.136	0.130	0.137	0.160	0.150
	3	0.120	0.136	0.150	0.141	0.220	0.162
	6	0.125	0.137	0.170	0.170	Lock Lost	0.179
9	0	0.100	0.108	0.105	0.110	0.140	0.116
	3	0.100	0.108	0.130	0.111	0.175	0.124
	6	0.105	0.109	0.150	0.123	0.330	0.153

Notes: INRDB = cw interference-to-Gaussian-noise power ratio (in decibels)
Δf = frequency-offset of interference from the carrier frequency f_o Hz
MSE = normalized mean-square error between input and output

Table 4: Summary of Results for MSE in Presence of cw Interference

SNRDB	INRDB	$\Delta f=10KH_z$		$\Delta f=5KH_z$		$\Delta f=0$	
		Experimental P_E	Model P_E	Experimental P_E	Model P_E	Experimental P_E	Model P_E
7	0	0.002	0.001	0.002	0.001	0.010	0.021
	3	0.002	0.001	0.0025	0.001	0.037	0.032
	6	0.002	0.001	0.003	0.002	Lock Lost	0.070
8	0	0.0005	0.001	0.0006	0.001	0.005	0.004
	3	0.0005	0.0	0.0009	0.001	0.015	0.009
	6	0.0005	0.0	0.001	0.002	Lock Lost	0.042
9	0	0.001	0.0	0.0002	0.0	0.002	0.0
	3	0.001	0.0	0.0003	0.0	0.007	0.005
	6	0.0001	0.0	0.0004	0.0	0.030	0.032

Table 5: Summary of Results for P_E in Presence of cw Interference

IV. DISCUSSION OF RESULTS AND CONCLUSIONS

From the results presented in the previous chapter, the following comments can be made and conclusions drawn:

A. For the ideal noiseless case, the transient response (Figs. 3 and 4) of the model coincided closely with the analytically calculated response of the linearized ($\sin 2\phi \simeq 2\phi$) loop, when the conditions of linearity were satisfied. This verified the accuracy of the computer model for noiseless conditions.

B. Selection of seeds for the random number generators (subroutine GAUSS in the computer program shown in Appendix D) had a significant influence on the length of the initial transient response of the model and on the values for P_E and σ_ϕ^2 calculated from the model. A bad seed had the effect of increasing the P_E and σ_ϕ^2 beyond the values expected theoretically, while a good seed tended to produce more acceptable values for these quantities.

C. For the Gaussian noise case, the P_E calculated from the model was always higher than the theoretically calculated values (Table 1). This can be attributed

to (i) the theoretical values used for comparison are for a first-order Costas loop (i.e. when LPF 3 shown in Figure 1 is absent) while the model is for the second-order Costas loop, and (ii) for the number of noise samples used in the model, the noise may not have been perfectly Gaussian in character. Also, the two noise processes $n_1(t)$ and $n_2(t)$ were not truly independent statistically, as they should be.

D. For high signal-to-Gaussian-noise ratios (SNRDB= 7,8, and 9), the introduction of a limiter in the in-phase channel of the loop (model) did not have any noticeable effect on the values of σ_ϕ^2 (Table 2). With or without a limiter, the values of σ_ϕ^2 calculated from the model, compared favorably with the theoretical values calculated from Eq. (3-9) which holds for high signal-to-noise ratios.

E. Without a limiter in the in-phase channel, the value obtained for the threshold was close to the theoretically predicted value in the presence of Gaussian noise alone. Increasing the number of noise samples per bit had the interesting effect of raising the threshold. However, the most

interesting result found was that the limiter in the in-phase channel of the loop had the effect of stabilizing the loop near threshold and as a result, the loop did not go out of lock even at -11dB (SNRDB) and had only 4 cycle-skips in 800 bits.

F. The results obtained for the cw interference were compared only with previously obtained experimental results. No theoretical results are available in the literature for the performance of Costas loops operating in cw interference. From the majority of the results obtained for the MSE (Table 4) it can be concluded that the effects of interference as obtained with the model were much less severe than shown by the corresponding experimental results. Possible explanations for this are: (i) measurement error in obtaining the experimental results; (ii) inadequate representation of the cw interference in the model. However, no such conclusion could be made from the results for the P_E. For the very low values of P_E corresponding to high SNRDB's, the model did not employ a sufficient number of bits for good accuracy. Only 1000 bits were used due to limited availability of computer time.

REFERENCES

1. Conte, S.D., Elementary Numerical Analysis (1965), McGraw-Hill.

2. Sanneman, R.W., and Rowbotham, J.R., "Unlock Characteristics of the Optimum Type II Phase-Locked Loop", IRE Trans. on Aerospace and Navigational Electronics, Vol. ANE-11 (March 1964), 15-24.

3. Rowbotham, J.R., and Sanneman, R.W., "Random Characteristics of the Type II Phase-Locked Loop", IEEE Trans. on Aerospace and Electronic Systems, Vol. AES-3 (July 1967), 604-612.

4. Smith, A.E., and Johnson, R.S., "A Digital Simulation of a Carrier Demodulation/Tracking Phase-Locked Loop in a Noisy, Multipath Environment", EASCON '68 Record (1968), 206-216.

5. Didday, Richard L. and Lindsey, William C., "Subcarrier Tracking Methods and Communication System Design", IEEE Trans. on Communication Technology, Vol. COM-16 (August 1968), 541-550.

6. Ziemer, R.E., "Experimental Comparison of Costas and PLL Demodulator in RFI Environments", NASA Report (Listed in STAR) No. X-520-69-355 (September 1969).

7. Downing, John J., Modulation Systems and Noise (1964), Englewood Cliffs: Prentice-Hall.

8. Jaffe, R.M., and Rechtin, E., "Design and Performance of Phase-Lock Circuits Capable of Near-Optimum Performance Over a Wide Range of Input Signals and Noise Levels", IRE Trans. on Information Theory, Vol. IT-1 (March 1955), 66-76.

9. Golomb, S.W., Digital Communications with Space Applications (1964), Englewood Cliffs: Prentice-Hall.

10. Carlson, Bruce A., Communication Systems (1968), McGraw-Hill.

11. Nichols, Myron H. and Rauch, Lawrence L., "Telemetry", Prepared for USAF Systems Command Under Contract No. AF 19(628)-4048, ESD-TR-66-464 (July 1966), Chapter 6 (pp. 6.42 to 6.51).

VITA

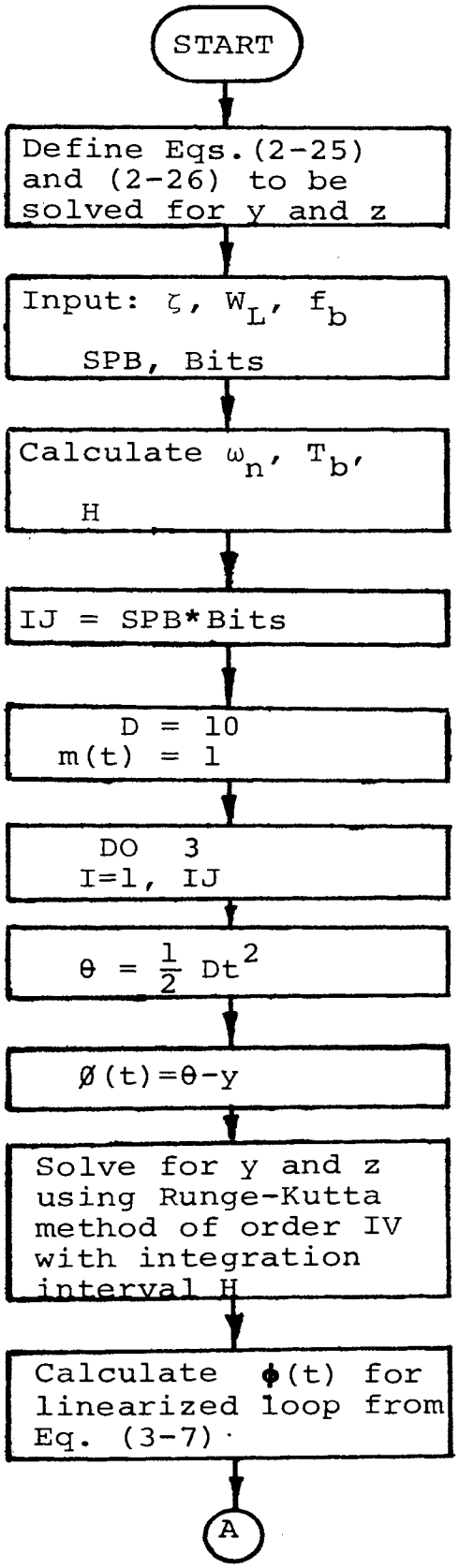
The author, Ajay Mugatlal Mehta, was born on January 16, 1947 in India. He got his Bachelor's degree in Electrical Engineering with honors at the Indian Institute of Technology, Bombay, in 1968.

He has been enrolled as a Master's candidate in the Electrical Engineering department of the University of Missouri - Rolla since September, 1968.

Mr. Mehta is a member of IEEE and Eta Kappa Nu.

APPENDIX A

FLOW DIAGRAM - NOISELESS CASE



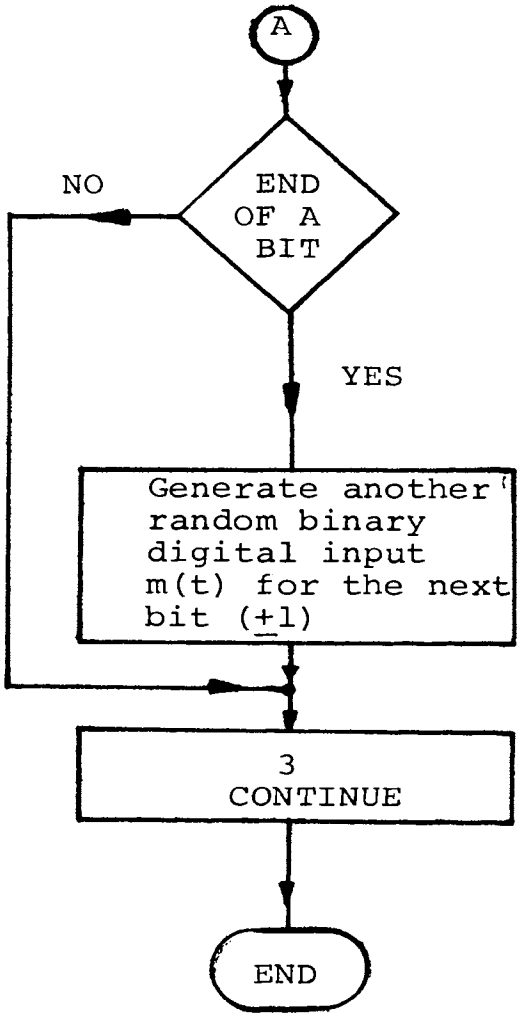


Figure 6: Flow Diagram for Computer Model - Noiseless Case 2

APPENDIX B

COMPUTER PROGRAM - NOISELESS CASE

```
C
C     COMPUTER PROGRAM FOR DIGITAL SIMULATION OF COSTAS LOOP OPERATION
C     ----------------------------------------------------------------
C
C     CASE OF LINEARLY INCREASING FREQUENCY AT THE INPUT
C
C     P1 STORES THE VALUE OF PHI AT THE END OF EACH BIT, TPHI STORES THE
C     THEORATICALLY CALCULATED VALUE OF PHI AT THE END OF EACH BIT
0001  DIMENSION P1(200),TPHI(200)
C
C     DEFINE THE TWO SIMULTANEOUS DIFFERENTIAL EQUATIONS OF THE COSTAS LOOP
C          T - TIME IN SECONDS, Y =θ̂ , Z - DUMMY VARIABLE INTRODUCED
0002  F1(T,Y,Z)=Z+2.*ZETA*WN*V5
0003  G1(T,Y,Z)=WN*WN*V5
C
C          ZETA - DAMPING FACTOR, WL - EQUIVALENT NOISE BANDWIDTH
C          WN - NATURAL FREQUENCY, WI - INPUT SIGNAL BANDWIDTH
C          FB - BIT-RATE BANDWIDTH, TB - PERIOD OF AN INPUT BIT
C          SPB - GAUSSIAN-NOISE SAMPLES TAKEN PER BIT, BITS -
C          TOTAL NUMBER OF BITS CONSIDERED, H - INTEGRATION AND
C          SAMPLING INTERVAL, SAMPLE - TOTAL NUMBER OF SAMPLES
C          E - MODULATING DIGITAL SIGNAL
C
0004  ZETA=0.707
0005  PI=3.1416
0006  WL=200.
0007  WN=4.*ZETA*WL/(1.+4.*ZETA**2)
0008  WI=10000.
0009  FB=WI/2.
0010  TB=1./FB
0011  SPB=20.
0012  NSPB=SPB
0013  BITS=200.
0014  NMAX=BITS
0015  H=TB/SPB
0016  SAMPLE=SPB*BITS
0017  IJ=SAMPLE
0018  CALL PENPOS(12HMEHTA AJAY M,12,1)
```

```
C
C       D - RATE OF INCREASE IN THE INPUT FREQUENCY (HZ/SEC)
0019            D=10.
C
C       DO LOOP FOR TWO VALUES OF D (D = 10, CONDITION OF LINEARITY
C       SATISFIED ; AND D = 10000, NONLINEAR CASE)
0020            DO 55 K=1,2
C
0021            T=0.
0022            Y=0.
0023            Z=0.
0024            F=1.
0025            TPHI(1)=0.
0026            P1(1)=0.
C
C       MAIN DO LOOP PERFORMING ITERATIONS
0027            DO 3 I=1,IJ
C
0028            THETA=D*T*T/2.
0029            PHI=THETA-Y
0030            V3=F*COS(PHI)
0031            V4=F*SIN(PHI)
0032            V5=V3*V4
C
C       EMPLOYING RUNGE-KUTTA METHOD OF ORDER IV TO SOLVE THE LOOP EQUATIONS
C
0033            A1=H*F1(T,Y,Z)
0034            B1=H*G1(T,Y,Z)
0035            A2=H*F1(T+H/2.,Y+A1/2.,Z+B1/2.)
0036            B2=H*G1(T+H/2.,Y+A1/2.,Z+B1/2.)
0037            A3=H*F1(T+H/2.,Y+A2/2.,Z+B2/2.)
0038            B3=H*G1(T+H/2.,Y+A2/2.,Z+B2/2.)
0039            A4=H*F1(T+H,Y+A3,Z+B3)
0040            B4=H*G1(T+H,Y+A3,Z+B3)
0041            Y=Y+(A1+2.*A2+2.*A3+A4)/6.
0042            Z=Z+(B1+2.*B2+2.*B3+B4)/6.
0043            T=T+H
C
```

```
C       CHECK FOR THE END OF A BIT
0044    IF(MOD(I,NSPB))3,4,3
C
0045  4 CONTINUE
0046    J=I/SPB+1.
0047    PI(J)=PHI
0048    TPHI(J)=D*(1.-1.414*EXP(-WN*T*0.707)*COS(WN*T*0.707-PI/4.))/WN/WN
C
C       GENERATING A RANDOM DIGITAL MODULATING SIGNAL
C
0049    ZZ=RAND(0)
0050    IF(ZZ.LT.0.5)F=-1.
0051    IF(ZZ.GE.0.5)F=1.
C
0052  3 CONTINUE
0053    PHIMIN=0.
0054    PHIMAX=D*(1.+EXP(-PI))/WN/WN
0055    WRITE(3,100)PHIMIN,PHIMAX
0056  100 FORMAT(1H,'PHIMIN =',F7.4,10X,'PHIMAX =',F7.4)
0057    YMIN=1.6*PHIMIN
0058    YMAX=1.6*PHIMAX
0059    CALL NEWPLT(2.0,2.0,10.0)
0060    CALL ORIGIN(0.0,0.0)
0061    CALL TSCALE(0.0,BITS*TB,8.0)
0062    CALL YSCALE(YMIN,YMAX,5.0)
0063    CALL TPLT(PI,NMAX,2,10*(K-1)+3)
0064    D=10000.
0065  55 CONTINUE
0066    CALL NEWPLT(2.0,2.0,10.0)
0067    CALL ORIGIN(0.0,0.0)
0068    CALL TSCALE(0.0,BITS*TB,8.0)
0069    CALL YSCALE(YMIN,YMAX,5.0)
0070    CALL TPLT(TPHI,NMAX,1,-1)
0071    CALL TAXIS(SPB*TB)
0072    CALL YAXIS(PHIMAX/5.)
0073    CALL ENDPLT
0074    CALL LSTPLT
0075    CALL EXIT
0076    STOP
0077    END
```

APPENDIX C

FLOW DIAGRAM - GAUSSIAN NOISE AND CW INTERFERENCE PRESENT

Nomenclature for notations used in Appendix C (but not defined earlier)

Bits - Total number of bits considered

OMIT - Number of initial bits omitted as transient response of the computer model

IJ - Total number of noise samples considered

NI - Number of initial samples omitted as transient response

NUSKIP-Count of the number of cycle-skips per every 10 consecutive bits

NSKIP- Number of cycle-skips

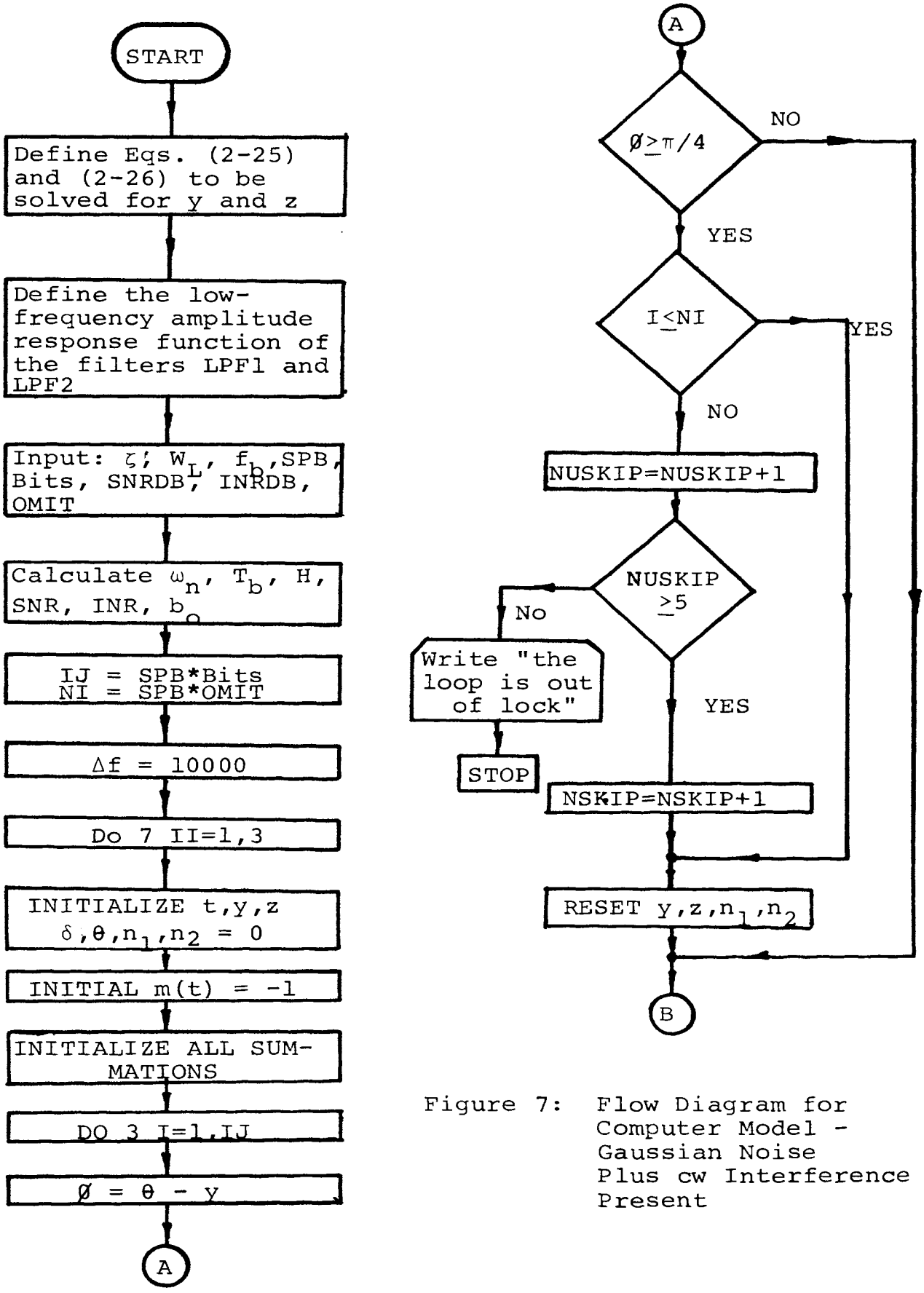


Figure 7: Flow Diagram for Computer Model - Gaussian Noise Plus cw Interference Present

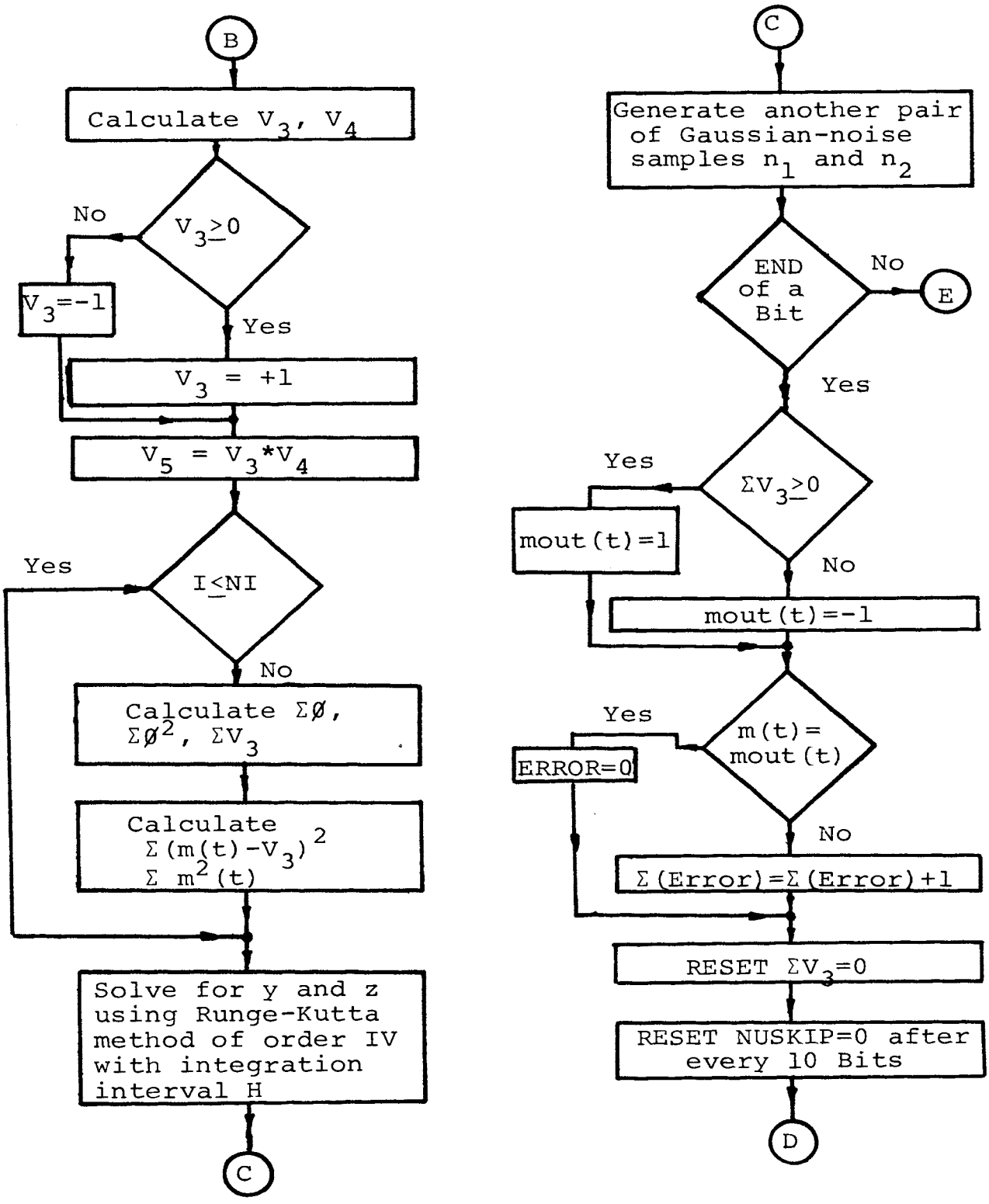


Figure 7 (contd.): Flow Diagram for Computer Model - Gaussian Noise Plus cw Interference Present

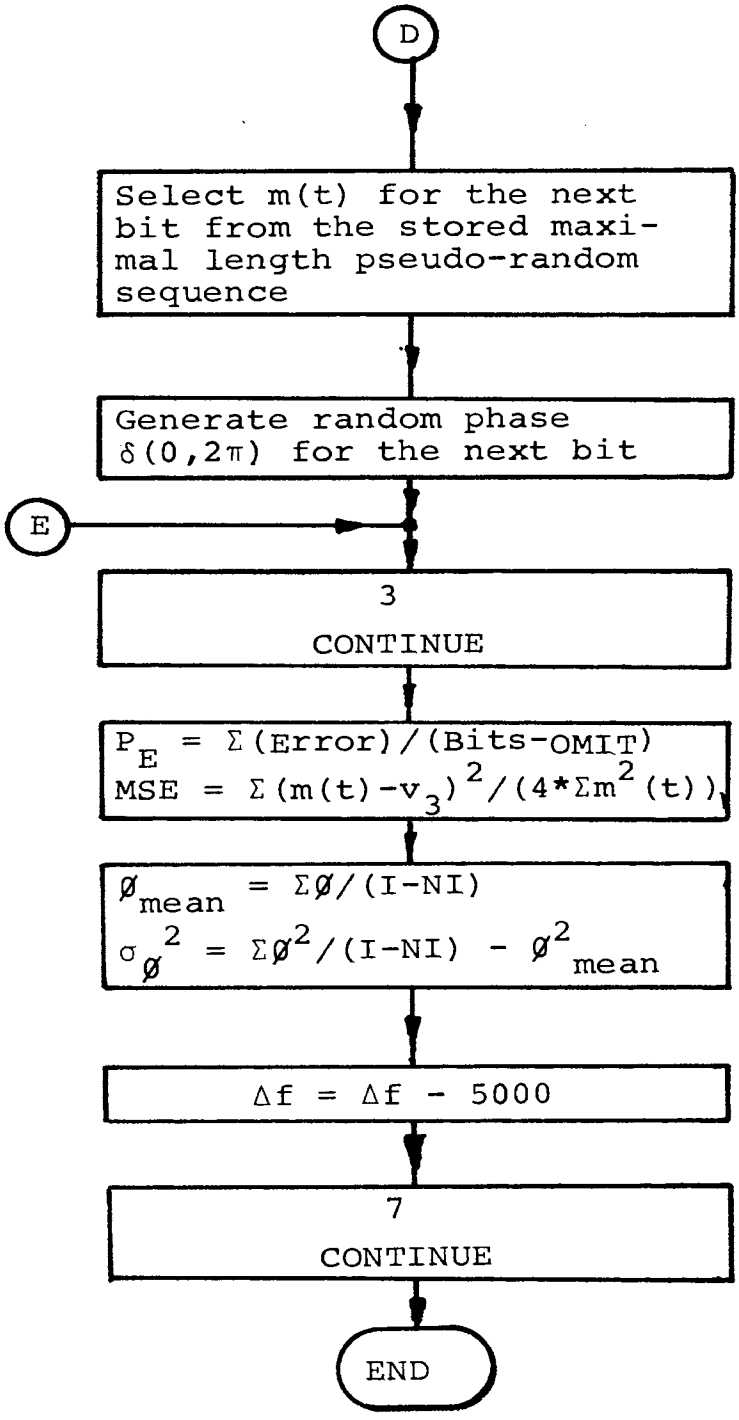


Figure 7 (contd.): Flow Diagram for Computer Model - Gaussian Noise Plus cw Interference

APPENDIX D

COMPUTER PROGRAM - GAUSSIAN NOISE AND CW INTERFERENCE PRESENT

```
C
C       COMPUTER PROGRAM FOR DIGITAL SIMULATION OF COSTAS LOOP OPERATION
C       ----------------------------------------------------------------
C
C       CASE OF SIGNAL CORRUPTED BY GAUSSIAN NOISE AND CW INTERFERENCE
C
0001            REAL*4 INR,INRDB,N1,N2
0002            DIMENSION F2(31)
0003            DIMENSION IJKLM(3),JKLMN(3)
C
C       DEFINE THE TWO SIMULTANEOUS DIFFERENTIAL EQUATIONS OF FIRST-ORDER
C       DESCRIBING THE COSTAS LOOP
C       T - TIME IN SECONDS, Y - θ̂, Z - DUMMY VARIABLE INTRODUCED
0004            F1(T,Y,Z)=Z+2.*ZETA*WN*V5
0005            G1(T,Y,Z)=WN*WN*V5
C
C       DEFINE AMPLITUDE RESPONSE FUNCTION OF THE FILTERS LPF 1 AND LPF 2
C       FOR THE LOW-FREQUENCY COMPONENTS OF THE CW INTERFERENCE
0006            BB(DELTAF,N,ALPHA)=1./(1.+(DELTAF/ALPHA)**N)
0007            N=2
0008            ALPHA=5.3E3
C
C       CONSTANTS AND PARAMETERS DEFINED
0009            PI=3.1416
0010            THRESH=PI/4.
C
C               ZETA - DAMPING FACTOR, WL - EQUIVALENT NOISE BANDWIDTH
C               WN - NATURAL FREQUENCY, WI - INPUT SIGNAL BANDWIDTH
C               FB - BIT-RATE BANDWIDTH, TB - PERIOD OF AN INPUT BIT
C               SPB - GAUSSIAN-NOISE SAMPLES TAKEN PER BIT, BITS -
C               TOTAL NUMBER OF BITS CONSIDERED, H - INTEGRATION AND
C               SAMPLING INTERVAL, SAMPLE - TOTAL NUMBER OF SAMPLES
C               F - MODULATING DIGITAL SIGNAL, CMIT - NUMBER OF INITIAL
C               BITS OMITTED AS TRANSIENT RESPONSE OF THE MODEL
C
0011            ZETA=0.707
0012            WL=100.
0013            WN=4.*ZETA*WL/(1.+4.*ZETA**2)
0014            WI=10000.
0015            FB=WI/2.
0016            TB=1./FB
0017            SPB=20.
```

```
0018                NSPB=SPB
0019                OMIT=200.
0020                NI=OMIT*SPB
0021                H=TB/SPB
0022                BITS=700.
0023                SAMPLE=SPB*BITS
0024                IJ=SAMPLE
                C
                C   DEFINE THE MAXIMAL-LENGTH PSEUDO-RANDOM DIGITAL MODULATING
                C   SEQUENCE OF +1'S AND -1'S
0025                READ(1,201)(F2(L), L=1,31)
                C
                C   INPUT : SIGNAL-TO-GAUSSIAN-NOISE POWER RATIO IN DECIBELS
0026                READ(1,104)SNRDB
0027                SNR=10.**(SNRDB/10.)
                C
                C   CALCULATE THE STANDARD DEVIATION OF RANDOM GAUSSIAN SAMPLING
                C   PROCESSES N1 AND N2
0028                STDDEV=SQRT(SPB/SNR/2.)
                C
                C   CALCULATE ESTIMATED VALUE OF VARIANCE(PHI), HOLDS GOOD AT HIGH SNR'S
0029                VARPHI=WL/WI*(1./SNR+0.5/SNR**2)
                C
                C   INPUT : INTERFERENCE-TO-GAUSSIAN-NOISE POWER RATIO IN DECIBELS
0030                READ(1,104)INRDB
0031                INR=10.**(INRDB/10.)
                C
                C   CALCULATE THE AMPLITUDE OF CW INTERFERENCE
0032                B=SQRT(INR/SNR)
                C
                C   DO LOOP FOR DIFFERENT VALUES OF FREQUENCY-OFFSET OF INTERFERENCE
                C
0033                DELTAF=10000.
0034                DO 7 II=1,3
0035                WRITE(3,101)SNRDB,STDDEV,INRDB,N,ALPHA,DELTAF
0036                JKLMN(II)=123456789
0037                IJKLM(II)=753214587
0038                C=BB(DELTAF,N,ALPHA)
0039                DELTAW=2.*PI*DELTAF
                C
                C   INITIALIZE THE CONDITION OF THE LOOP
                C
```

```
0040                    I=0.
0041                    Y=0.
0042                    Z=0.
0043                    F=-1.
0044                    N1=0.
0045                    N2=0.
0046                    BIGPHI=0.
0047                    BIGPH2=0
0048                    BIGERR=0.
0049                    BIGO=0.
0050                    BIGF2=0.
0051                    BIGFO2=0.
0052                    DELTA=0.
0053                    THETA=0.
0054                    NUSKIP=0
0055                    NSKIP=0
                C
                C       MAIN LOOP PERFORMING ITERATIONS
0056                    DO 3 I=1,IJ
                C
                C       PHI = PHASE ERROR
0057                    PHI=THETA-Y
                C
                C       CHECK FOR A CYCLE-SKIP
0058                    IF(ABS(PHI).GE.THRESH)GO TO 11
                C
0059                    GO TO 12
0060                11  CONTINUE
0061                    IF(I.LE.NT)GO TO 20
0062                    NUSKIP=NUSKIP+1
                C
                C       IF NUMBER OF CYCLE-SKIPS PER EVERY 10 CONSECUTIVE BITS IS GREATER
                C       THAN 5, DECLARE THE LOOP AS BEING OUT OF LOCK
0063                    IF(NUSKIP.GE.5)GO TO 66
0064                    NSKIP=NSKIP+1
0065                20  CONTINUE
                C
                C       IN CASE OF CYCLE-SKIP, RESET THE CONDITION OF THE LOOP AS AT TIME
                C       T=0, THIS IS EQUIVALENT TO MANUALLY RESETTING IN AN EXPERIMENT
0066                    PHI=0.
0067                    Y=0.
0068                    Z=0.
0069                    N1=0.
0070                    N2=0.
                C
0071                    GO TO 12
```

```
0072          66  WRITE(3,103)
0073              STOP
0074          12  CONTINUE
0075              X=DELTAW*T+DELTA
0076              V3=(F+N2+B*C*COS(X))*COS(PHI)-(N1+B*C*SIN(X))*SIN(PHI)
0077              V4=(F+N2+B*C*COS(X))*SIN(PHI)+(N1+B*C*SIN(X))*COS(PHI)
                C
                C     CASE OF OUTPUT THROUGH A LIMITER IN THE IN-PHASE CHANNEL OF THE LOOP
0078              IF(V3.GE.0.)V3=1.
0079              IF(V3.LT.0.)V3=-1.
                C
0080              V5=V3*V4
0081              IF(I.LE.NI)GO TO 21
0082              BIGPHI=BIGPHI+PHI
0083              BIGPH2=BIGPH2+PHI*PHI
0084              Q=V3
0085              FQ=F-Q
0086              BIGQ=BIGQ+Q
0087              BIGFQ2=BIGFQ2+FQ*FQ
0088              BIGF2=BIGF2+(F-(-F))**2
0089          21  CONTINUE
                C
                C     RUNGE-KUTTA METHOD OF ORDER IV USED FOR NUMERICAL INTEGRATION
                C
0090              A1=H*F1(T,Y,Z)
0091              B1=H*G1(T,Y,Z)
0092              A2=H*F1(T+H/2.,Y+A1/2.,Z+B1/2.)
0093              B2=H*G1(T+H/2.,Y+A1/2.,Z+B1/2.)
0094              A3=H*F1(T+H/2.,Y+A2/2.,Z+B2/2.)
0095              B3=H*G1(T+H/2.,Y+A2/2.,Z+B2/2.)
0096              A4=H*F1(T+H,Y+A3,Z+B3)
0097              B4=H*G1(T+H,Y+A3,Z+B3)
0098              Y=Y+(A1+2.*A2+2.*A3+A4)/6.
0099              Z=Z+(B1+2.*B2+2.*B3+B4)/6.
0100              T=T+H
                C
                C     GENERATE SAMPLES FOR THE TWO RANDOM GAUSSIAN PROCESSES N1 AND N2
0101              CALL GAUSS(IJKLM(II),STDDEV,0.,N1)
0102              CALL GAUSS(JKLMN(II),STDDEV,0.,N2)
                C
                C     CHECK FOR THE END OF A BIT
0103              IF(MOD(I,NSPB))3,4,3
                C
0104          4   CONTINUE
```

```
0105                J=FLOAT(I)/SPB
0106                IF(J.LE.200)GO TO 22
0107                IF(MOD(J,10).EQ.0)NUSKIP=0
0108                IF(BIGQ.LT.0.)FOUT=-1.
0109                IF(BIGQ.GE.0.)FOUT=1.
                C
                C       COMPUTING THE NUMBER OF ERRORS
0110                IF(F.NE.FOUT)BIGERR=BIGERR+1.
                C
0111                BIGQ=0.
0112             22 CONTINUE
0113                JJ=MOD(J+1,31)
0114                IF(JJ.EQ.0)JJ=31
                C
                C       GENERATING A PSEUDO-RANDOM FINITE-LENGTH DIGITAL MODULATING SIGNAL
0115                F=F2(JJ)
                C
                C       GENERATING A RANDOM PHASE, DELTA, FOR THE CW INTERFERENCE
0116                ZZ=RAND(0)
0117                DELTA=2.*PI*ZZ
                C
0118              3 CONTINUE
                C
                C            PE - BIT-ERROR PROBABILITY, SQMEAN - MEAN-SQUARE
                C            ERROR BETWEEN INPUT AND OUTPUT, PHIM - MEAN OF PHI
                C            PHIVAR - VARIANCE OF PHI
                C
0119                PE=BIGERR/(BITS-OMIT)
0120                SQMEAN=BIGQ2/BIGF2
0121                PHIM=BIGPHI/(BITS-OMIT)
0122                PHIVAR=BIGPH2/(BITS-OMIT)-PHIM**2
0123                WRITE(3,100)PE,SQMEAN,PHIM,PHIVAR,NSKIP
0124                WRITE(3,100)PE,SQMEAN,PHIM,VARPHI,NSKIP
0125                DELTAF=DELTAF-5000.
0126              7 CONTINUE
0127            100 FORMAT(/1X,'PE='F5.3,10X,'MEAN SQ. ERR.=',F9.4,10X,'MEAN PHI=',F8.
                   /5,10X,'VAR.PHI=',F12.5,10X,'CYCLES SKIPPED=',I3)
0128            101 FORMAT('1','SNRDB = ',F5.1,10X,'STD. DEVIATION =',F7.4,10X,'INRDB
                   /=',F5.1/' N   = ',I1,10X,'ALPHA = ',E16.7,10X,' DELTAF =',E16.7)
0129            103 FORMAT(//10X,'THE LOOP IS OUT OF LOCK'//)
0130            104 FORMAT(F7.2)
0131            201 FORMAT(F4.1)
0132                STOP
0133                END
```