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Scattering parameter measurements and amplifier design

Thomas J. Herrick

SCATTERING PARAMETER MEASUREMENTS AND AMPLIFIER DESIGN

BY

THOMAS J. HERRICK

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Ralph E. Carson (Advisor) Norman Dillman

Robert Jensen

ABSTRACT

The scattering parameter formulation in describing the small-signal operation of an active two-port device is developed. The inherent advantages of measuring the scattering parameters at UHF frequencies and the clear insight into the design problem are demonstrated. Some of the practical difficulties in a typical design problem are also considered.

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I. INTRODUCTION

The recent development of good transistors capable of significant power gain in the frequency range of 100 MHz to 5 GHz has opened an area of electronic circuit design largely neglected. Generally, it has not been feasible to obtain electronic circuit power gain at frequencies beyond 600 MHz.

Current transistors operating at frequencies greater than 500 MHz have the capability of only small power gains. In order to obtain even this low power gain, it is necessary to optimize the design of each amplifier.

The optimization procedure requires that a set of four parameters for the two-port device be available for computation. The selection of a set of parameters to describe the small signal operation of an active device is not unique. There is actually an infinite set of parameters that could be used. However, a few sets become most popular because their measurement is the simplest.

Historically, there have been only two sets of parameters in common use for two-port active circuit design — the h-parameters and the y-parameters. For example, the h-parameter description of the transistor stage shown in Fig. 1 is:

$$V_1 = I_1 h_{11} + V_2 h_{12} \qquad 1$$

$$I_2 = I_1 h_{21} + V_2 h_{22} \qquad 2$$

where the capital letters indicate phasor notation.

The determination of the h-parameters requires:

$$h_{11} = \left.\frac{V_1}{I_1}\right| V_2 = 0 \qquad h_{21} = \left.\frac{I_2}{I_1}\right| V_2 = 0 \qquad 3$$

$$h_{12} = \left.\frac{V_1}{V_2}\right| I_1 = 0 \qquad h_{22} = \left.\frac{I_2}{V_2}\right| I_1 = 0 \qquad 4$$

Thus, an ac short-circuit test at the output and an open-circuit test at the input is required. The reason for the selection of the h-parameter description is because a bipolar transistor has a low input impedance and high output impedance. In practice, the measurement of the h-parameter is accomplished by meeting the following conditions:

a) h_{11} and h_{21} measurements require $V_2 = 0$.
This can be accomplished by shunting the output terminals by a large capacitor such that $|X_c| << Z_o$, where Z_o is the output impedance of the stage. (Fig. 1)

b) h_{22} and h_{12} measurements require $I_1 = 0$.
This can be accomplished by placing a series inductance in

the input port such that $|X_L| >> Z_i$, where Z_i is the input impedance of the stage. (Fig. 1)

The popularity of the h-parameters comes from the relative ease with which conditions a) and b) above can be met. Since $|Z_o|$ is large, a reasonably small capacitor shunting the output can fulfill the condition $|X_c| << Z_o$, and a small inductor in series with the input port can fulfill the condition $|X_L| >> Z_i$.

At this point a serious difficulty is apparent. The difficulty is with the statement $|X_c| << Z_o$ and $|X_L| >> Z_i$. It is not possible to determine if these inequalities are satisfied until after the measurements are completed such that Z_o and Z_i can be calculated or an entirely separate set of tests must be used to determine Z_i and Z_o.

The major difficulty with the h-parameter set occurs at frequencies greater than 10 MHz. At these frequencies, the open circuit at the input is difficult to obtain because of parasitic capacitance. At frequencies beyond 10 MHz and up to about 500 MHz, the y-parameter description has been used for measurement purposes until very recently.

The y-parameter description.

The y-parameter description of the two-port transistor amplifier in Fig. 1 is:

$$I_1 = y_{11}V_1 + y_{12}V_2 \qquad (5)$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \qquad 6$$

Where:

$$y_{11} = \left.\frac{I_1}{V_1}\right| V_2 = 0 \qquad y_{12} = \left.\frac{I_1}{V_2}\right| V_1 = 0 \qquad 7$$

$$y_{21} = \left.\frac{I_2}{V_1}\right| V_2 = 0 \qquad y_{22} = \left.\frac{I_2}{V_2}\right| V_1 = 0 \qquad 8$$

The y-parameters are dimensionally mhos. The reason for the use of y-parameters at the higher frequencies is the short circuit test at both input and output. This is approximated by shunt capacitors from 1 MHz to 100 MHz. For good accuracy $B_{C1} \gg |Y_i|$ and $B_{C2} \gg |Y_o|$, where B_{C1} and B_{C2} are the susceptances of the shunt capacitors at the input and output port respectively and $Y_i = Z_i^{-1}$, $Y_o = Z_o^{-1}$. Capacitors of reasonable size can easily meet this condition.

At frequencies beyond 500 MHz, it becomes necessary to use quarter-wavelength shorted transmission lines (tuning stubs) since discrete capacitors have sufficient series inductance that they do not approximate a short circuit.

Measurements at frequencies beyond 100 MHz are normally beyond the capability of average laboratory equipment. The General Radio Company Transfer Bridge is the most popular instrument for measuring these parameters at frequencies beyond 100 MHz. This instrument is relatively expensive and requires considerable skill in its use.

Consequently, it is used principally by device manufacturers and not by circuit designers.

The most troublesome disadvantage in measuring y-parameters is the tuning stubs. These require adjustment at each frequency since a quarter-wave shorted stub is a resonant section. Thus, at UHF frequencies, a wide-band short is not available. One of the scattering parameter technique's outstanding advantages is the ability to achieve an extremely wide band test termination without adjustment.

Frequently, it becomes extremely tedious to measure the y-parameters of a device above 100 MHz because the test circuit and the device under test break into oscillation. This happens because stubs and devices which are potentially unstable can satisfy the conditions for oscillation. This oscillation is seldom at the test frequency (where the stub susceptance is very large) but at some other frequency where the conditions for oscillation are satisfied. If the oscillation amplitude is large, such that the device is operating in cutoff and saturation part of the time, nonlinear operation of the device makes measurements meaningless. If the oscillation is soft, then the measurement and the oscillation effects can be separated by filtering. This, however, is not normally done. Usually, the oscillation is suppressed with filter traps. However, it is possible for the circuit to satisfy conditions for oscillations at some new frequency with the filter traps in place, since they are also frequency dependent.

Scattering Parameters.

The scattering parameter technique avoids the mechanical adjustment of terminations and virtually eliminates the possibility of the test circuit from breaking into oscillation.

With the measured scattering parameters, the design of the amplifier proceeds in a clear and simple manner, especially if the device can satisfy the criterion for being approximately unilateral. The design problem is then reduced to the synthesis of matching networks for the input and output ports to achieve the desired transducer gain.

As will be seen later, the features of the last two paragraphs make computer-directed scattering parameter testing and circuit design feasible, since no operating adjustments are necessary.

II. SCATTERING PARAMETER MEASUREMENT THEORY

The selection of a parameter set is usually guided by the ease of measurement. But a second important consideration is the variables to be used with the parameter set. For the h- and y-parameters, the variables are voltages and currents. Normally the specifications that are of interest in circuit design are voltage and current gain and also input and output impedance. These quantities are fairly straightforward to calculate, using voltage and current as variables. In power gain calculations, however, voltage and currents as variables do not lead to simple calculations or interpretations.

The scattering parameter formulation requires a new variable which is somewhat nebulously termed a power wave. The most important property of this variable is that simply squaring it gives power flow directly. Thus for amplifiers where power levels and power gains are of primary interest, calculations are fairly simple, but more importantly, considerable insight into the operation of the device as an amplifier can be obtained from a cursory inspection of the measured scattering parameters.

The earliest mention of scattering parameters applied to circuit theory appears to be the work of Campbell and Foster[1] around 1920. Although the classical study of transmission lines uses the idea of incident and reflected (scattered) voltage and current components, no general development of scattering as a network concept appeared until the early 1940's when physicists familiar with wave-particle scattering relationships applied these principles to microwave wave-guide junction and discontinuity analyses.

The work of Carlin[2] in 1956 formalized much of the scattering method as applied to network theory. And Kurokawa's[3] paper, in 1965, developed many of the ideas which are necessary for network calculations. The definitive work of Bodway[4] specifically applies the scattering parameter method to the design of two-port amplifiers.

The reason that the scattering approach was not generally accepted as a circuit analysis technique earlier was the extreme difficulty in measuring these parameters previous to 1966. The measurements can be made with a slotted line, but the tedious adjustments and the very limited frequency band make the measurements unattractive. Two instruments developed by the Hewlett-Packard Corporation made the measurement not only simple, but in fact much easier than any other existing measurement system. These two instruments are the Hewlett-Packard Network Analyzer Model 8410A which can measure directly the scattering parameters of a two-port network from a frequency of 1 MHz to 12.3 GHz; the other instrument is the Hewlett-Packard Vector Voltmeter Model 8405A which requires some auxilliary test equipment and is much slower, but is much less expensive.

The basic frequency range of the Vector Voltmeter is from 1 MHz to 1.1 GHz. The measurement system discussed in this paper is built around the Model 8405A Vector Voltmeter.

It has previously been stated that there is an infinite set of parameters and variables that can describe the performance of the two-port device. This statement is true because any linear combination of parameter-variable sets is a satisfactory description of the device.

In the scattering parameter formulation, a very special linear combination of voltage and currents is selected so that it produces a unique set of power wave variables (a_i, b_i). The unique property of the a_i, b_i variables is that when they are squared they give the incident and reflected power flow respectively.

The basic circuit variables that will be used in the development of the scattering parameter measurement theory are referred to in Fig. 2. By definition:

$$a_i = \frac{V_i + Z_i I_i}{2\sqrt{\mathrm{Re} Z_i}} \quad \text{and} \quad b_i = \frac{V_i - Z_i^* I_i}{2\sqrt{\mathrm{Re} Z_i}} \qquad 9$$

where:

a_i = incident power wave at $i^{\underline{th}}$ port. $(\text{watts})^{\frac{1}{2}}$

b_i = reflected power wave at $i^{\underline{th}}$ port. $(\text{watts})^{\frac{1}{2}}$

V_i = voltage existing at $i^{\underline{th}}$ port. (volts)

Z_i = reference impedance of $i^{\underline{th}}$ port. (ohms)

I_i = current flow into $i^{\underline{th}}$ port. (ampere)

The reference impedance Z_i can be any complex number, but the simplest and by far the clearest meaning occurs when Z_i is a real positive number (R_i).

The measurement system described in this paper uses sources and terminations (the Z_i's) of 50 ohms. Also, all interconnections are coaxial cables or strip lines of 50 ohms characteristic impedance.

If V_i and I_i are decomposed into incident and reflected (scattered) components:

$$V_1 = V_{i1} + V_{r1} \qquad I_1 = I_{i1} - I_{r1} \tag{10}$$

$$V_2 = V_{i2} + V_{r2} \qquad I_2 = I_{i2} - I_{r2} \tag{11}$$

and then substituted into equation 9, the following results:

$$a_1 = \frac{V_1 + 50\ I_1}{2\sqrt{50}} = \frac{V_{i1} + V_{i1} + 50\ \ I_{i1} - I_{r1}}{2\sqrt{50}} \tag{12}$$

However $V_{i1} = I_{i1}50$ $\qquad$ $V_{r1} = I_{r1}50$, therefore:

$$a_1 = \frac{2V_{i1}}{2\sqrt{50}} = \frac{V_{i1}}{2\sqrt{50}} \tag{13}$$

similarly for the other variables:

$$b_1 = \frac{V_{r1}}{(50)^{\frac{1}{2}}} \tag{14}$$

$$a_2 = \frac{V_{i2}}{(50)^{\frac{1}{2}}} \tag{15}$$

$$b_2 = \frac{V_{r2}}{(50)^{\frac{1}{2}}} \qquad 16$$

The assumption that the circuit to be measured is linear means that the superposition of variables is permissible. Hence one can relate the reflected power waves at both ports to the incident power waves with two linear equations:

$$b_1 = s_{11}a_1 + s_{12}a_2 \qquad 17$$

$$b_2 = s_{21}a_1 + s_{22}a_2 \qquad 18$$

or $$b_i = s_{ij}a_j \qquad 19$$

The values of the parameters s_{ij} can now be determined. The parameters s_{11} and s_{21} are measured by making the reflected component $a_2 = 0$. This is accomplished by matching the 50 ohms transmission system in its characteristic impedance (Z_o) of 50 ohms. Since there is no independent generator in the output and the output line is matched $R_2 = Z_o = 50$, there is no incident power wave ($a_2 = 0$). From equation 17:

$$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2 = 0} \qquad s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2 = 0} \qquad 20$$

From equations 13, 14, and 16:

$$s_{11} = \frac{\frac{V_{r1}}{(50)^{\frac{1}{2}}}}{\frac{V_{i1}}{(50)^{\frac{1}{2}}}} = \frac{V_{r1}}{V_{i1}} \qquad s_{21} = \frac{\frac{V_{r2}}{(50)^{\frac{1}{2}}}}{\frac{V_{i2}}{(50)^{\frac{1}{2}}}} = \frac{V_{r2}}{V_{i1}} \qquad 21$$

Similarly, the measurement of s_{12} and s_{22} requires a test generator at the output port and a matching termination of $R_1 = 50$ at the input port. Therefore, from equation 18:

$$s_{12} = \left.\frac{b_1}{a_2}\right|_{a_1 = 0} \qquad s_{22} = \left.\frac{b_2}{a_2}\right|_{a_1 = 0} \qquad 22$$

from equations 14, 15, and 16

$$s_{12} = \frac{\frac{V_{r1}}{(50)^{\frac{1}{2}}}}{\frac{V_{i2}}{(50)^{\frac{1}{2}}}} = \frac{V_{r1}}{V_{i2}} \qquad s_{22} = \frac{\frac{V_{r2}}{(50)^{\frac{1}{2}}}}{\frac{V_{i2}}{(50)^{\frac{1}{2}}}} = \frac{V_{r2}}{V_{i2}} \qquad 23$$

The test condition that the transmission system be terminated in its characteristic impedance needs special mention. Since the measurement system discussed in this paper uses 50 ohms coaxial cable and the test generator has an internal impedance of 50 ohms, the transistor under test sees an impedance of 50 ohms at both the input and output ports because both lines are matched in their characteristic impedance. A low-loss transmission system terminated in its characteristic impedance is an extremely wide-band non-frequency-dependent termination.

It is certainly not difficult to achieve this 50 ohms termination over a frequency range from DC to 20 GHz.

The consequence of this is that measurements can be made over a wide frequency range without any adjustment of the terminations. This is, of course, far simpler than the adjustment of the mechanical length of tuning stubs for each frequency that measurement of the y-parameters require. Further, since the impedance seen by the input and output ports of the active device is 50 ohms over this wide frequency, it is extremely difficult for the device to break into oscillation since the real part of the input and/or output impedance of the device under test would have to be negative and of a magnitude greater than 50. This is not likely to occur.

These two points of (a), no termination adjustment and (b), extreme stability under measurement, are two of the scattering parameter formulation's most important advantages.

The measurement of the s_{ij} has been shown to reduce simply to the measurement of incident and reflected voltage components on the transmission system. A convenient way to do this is with a dual directional coupler which produces signals proportional to the incident and reflected component. The proportionality is normally a function of frequency. However, assuming that the couplers are identical in construction, the proportionality constant does not enter into the calculations. For example, in the measurement of:

$$s_{11} = \frac{b_1}{a_1}\bigg|_{a_2=0} = \frac{V_{r1}}{V_{i1}} \qquad 24$$

output signals from the coupler V_{ci1} and V_{cr1} which are proportional to V_{r1} and V_{i1} with the proportionality constant frequency dependent would give:

$$V_{ci1} = k_2(f)\ V_{i1} \qquad\qquad V_{cr1} = k_1(f)\ V_{r1} \qquad 25$$

If the couplers are identical, $k_1(f)$ should be very nearly $k_2(f)$ and therefore:

$$s_{11} = \frac{k_1(f)}{k_2(f)} \frac{V_{r1}}{V_{i1}} = \frac{V_{r1}}{V_{i1}} \qquad 26$$

If the coupler arms do not track exactly the calibration of each arm is normally printed on the coupler and can be taken into account in the calculations.

At this point, another advantage of measurement of scattering parameters should be mentioned. The couplers measure the incident and reflected voltage components which do not vary with position along the line; therefore, it does not matter where the couplers are located along the line. They can be removed a considerable distance from the device under test and, provided a low-loss transmission system is used, they give the correct answer. This is not true of measurements of voltage and current required by the y-parameters. The line voltages and line currents vary along the line except for the case of a matched line. It is not usually convenient to measure the voltage and current right at the terminals of the device, so measurements must be made at some other point and then referred back to the terminals of the device under test.

In general, the scattering parameters are complex numbers which means that a phase reference must be established. This is a fairly simple procedure and must be performed only once — when the measurement system is first assembled. Once assembled, there are no further adjustments or calibrations necessary unless the physical length of the system is changed.

The system for measuring the scattering parameters s_{11} and s_{21} is shown in Fig. 3. The transistor is mounted in the center of a special transistor jig constructed of 50-ohm strip line. Dual directional couplers are connected directly to the test jig with Type N connectors. No termination is necessary for port four of the output side since $V_{i2} = 0$, because of the match. Typically, the couplers have a coupling factor of at most -20 db below the main arm voltage component, and drops off about -6 db/octave from some center frequency (600 MHz in this case).

The bias jigs at input and output provide a means of applying the dc bias to the transistor. The shorted section is used only in the initial calibration to establish a reference phase. The shorted section is one-half the length of the transistor jig and has a large plate shorting the strip line at the end of the half section. It is this short which is really replacing the transistor in the transistor jig which calibrates the system. The calibration is accomplished with the measurement of the reflection coefficient of the short circuit. If the voltages at A_1 and B_1 are measured, then:

$$\frac{b_1}{a_1} = \frac{\dfrac{V_1 - R_o I_1}{(R_o)^{\frac{1}{2}}}}{\dfrac{V_1 + R_o I_1}{(R_o)^{\frac{1}{2}}}} = \frac{V_1 - R_o I_1}{V_1 + R_o I_1} = \frac{\dfrac{V_1}{I_1} - R_o}{\dfrac{V_1}{I_1} + R_o} \qquad 27$$

$$\frac{b_1}{a_1} = \frac{Z_1 - R_o}{Z_1 + R_o} \quad \text{where } Z_1 = \frac{V_1}{I_1} = 0 \text{ (the short)} \qquad 28$$

then:

$$\frac{b_1}{a_1} = -\frac{R_o}{R_o} = -1 \qquad 29$$

Therefore, with the short in place, the voltage magnitude read at B_1 should be equal to the voltage magnitude at A_1 and the angle should be 180°. Normally, this angle condition is not met because of the different length paths of Channel A and Channel B. Thus, the requirement of a line stretcher. The length of the Channel A Path is adjusted until the Vector Voltmeter reads 180°. This is a very simple procedure and completes the calibration.

If the transister test jig is then substituted for the half-length shorted section, a reference angle plane is established at the transistor mount. The transistor test jig which is shown in detail in Fig. 4 must have extremely low impedance in the common lead. Any appreciable impedance in the common lead is in a feedback path from output to input.

The center plate of the test jig is designed to be at essentially ac ground but not dc ground since bias to the device's common lead is applied on the center plate. Because of its large dimensions, its series inductance is very low. The connection to ac ground is made through the capacitance of the center plate to the housing of the jig which is at ac and dc ground. To make the capacitance a maximum, only the oxide coating between the aluminum center plate and the housing is used as the dielectric.

The measurement of the scattering is performed by establishing a reference incident signal from the signal generator. This signal is recorded on Channel A. The signal at B_1 is read on Channel B and the phase angle recorded. Channel B test lead is then moved to position B_2 and the voltage and phase angle recorded. The scattering parameters s_{11} and s_{21} are then calculated.

$$s_{11} = \frac{V_{B1} \underline{/ \theta_{A1,B1}}}{V_{A1}} \qquad s_{21} = \frac{V_{B2} \underline{/ \theta_{B2,B1}}}{V_{A1}} \qquad 30$$

The parameters s_{22} and s_{12} are found by reversing the bias jig and the transistor jig positions. The signal is then applied to the output port of the device and s_{22} and s_{12} are calculated exactly the same as s_{11} and s_{21}.

A necessary and sufficient condition for the measurements to be meaningful is that $|s_{11}|$, $|s_{22}| < 1$. This follows from the definition of the reflection coefficient $s_{11} = \frac{V_{r1}}{V_{i1}}$:

$$s_{11} = \frac{Z_1 - 50}{Z_1 + 50} = \frac{ReZ_1 + j\ ImZ_1 - 50}{ReZ_1 + j\ ImZ_1 + 50} \qquad 31$$

$$|s_{11}| = \sqrt{\frac{(ReZ_1 - 50)^2 + (ImZ_1)^2}{(ReZ_1 + 50)^2 + (ImZ_1)2}} \qquad 32$$

and similarly for s_{22}.

If $|s_{11}| > 1$, ReZ_1 must be negative and/or if $|s_{22}| > 1$, ReZ_2 is negative, then special care must be taken in the measurement of the scattering parameters. The measurement system will suppress any tendency for the test circuit to oscillate if ReZ_1 or ReZ_2 are greater than −50 ohms. But if ReZ_1 or ReZ_2, or both, are less than −50 ohms, the measurements are meaningless. However, it is very simple to check on whether Re Z_1 and/or Re $Z_2 < -50$ over the <u>entire</u> frequency range of the measurement system. This is accomplished by setting the signal from the generator to zero, and monitoring the directional couplers for any signals produced by oscillation.

If signals are present due to the oscillation of test circuit and device, then an attempt can be made to stop the oscillations by using filters. To the author's knowledge, no device has ever been able to cause the test circuit to oscillate.

III. AMPLIFIER DESIGN USING SCATTERING PARAMETERS

A convenient method of analysis using scattering parameters is the signal flow graph[4]. Not only are calculations simple, but considerable insight is gained from the representation. The graph for a transistor is shown enclosed in the dotted box of Fig. 5.

When the amplifier is terminated at both ports by the same impedance that was used as a reference (50 ohms), then calculations of power gain are extremely simple.

The parameters s_{11} and s_{22} are reflection coefficients (the ratio of reflected to incident voltage components) that are commonly used in transmission line theory. For the case of matched lines at both ports, see Fig. 5, where:

r_2 = reflection coefficient of the load

$$= \frac{Z_L - 50}{Z_L + 50} = 0 \qquad 33$$

r_1 = reflection coefficient of the generator

$$= \frac{Z_s - 50}{Z_s + 50} = 0 \qquad 34$$

a_s = power wave emanating from the generator

= (the available power from the generator)$^{\frac{1}{2}}$

$$= \frac{E_g}{2(50)^{\frac{1}{2}}} \qquad 35$$

From transmission line theory:

$$E_{r2} = r_2 E_{i2} \text{ or } a_2 = r_2 b_2 \qquad 36$$

since:

$$a_2 = \frac{E_{r2}}{\sqrt{50}} \qquad b_2 = \frac{E_{i2}}{\sqrt{50}} \qquad 37$$

for the amplifier in Fig. 5, then r_1, r_2 = 0 and $a_2 = r_2 b_2 = 0$, because $r_2 = 0$ (output line matched in its characteristic impedance).

$$a_1 = a_s + r_1 b_1 \qquad 38$$

but: $r_1 = 0$, therefore $a_1 = a_s$.

The power wave which travels to the load $R_L = 50$ is $b_2 = E_{r2}/\sqrt{50}$; therefore $(b_2) = s_{21} a_1 = s_{21} a_s$; or, substituting for b_2 and a_s:

$$b_2 = \frac{(E_{r2})}{(50)^{\frac{1}{2}}} \qquad 39$$

Since the output line is matched $E_{i2} = 0$ and $E_2 = E_{r2}$; but $E_2 = E_{i2} + E_{r2}$; therefore:

$$b_2 = \frac{E_2}{(50)^{\frac{1}{2}}} \qquad 40$$

$$a_s = \frac{E_g}{2(50)^{\frac{1}{2}}} \qquad 41$$

$$\frac{b_2}{a_s}^2 = \frac{\left|E_2/(50)^{\frac{1}{2}}\right|^2}{\left|\dfrac{E_g}{(2)(50)^{\frac{1}{2}}}\right|^2} = \frac{\dfrac{E_2}{50}^2}{\dfrac{E_g}{(4)(50)}^2} = \frac{P_{out}}{P_{avail.}} \qquad 42$$

However, $E_2^{\,2}/50$ is the output power delivered to $R_L = 50$ ohms and $E_g^{\,2}/(4)(50)$ is the available power from the generator of $R_s = 50$ ohms.

The ratio of $P_{out}/P_{avail.}$ is defined as transducer gain (G_T). Therefore:

$$\left|\frac{b_2}{a_s}\right|^2 = \left|\frac{b_2}{a_1}\right|^2 = \left|s_{21}\right|^2 = \frac{P_{out}}{P_{avail.}} = \text{Transducer Power gain } (G_T). \quad 43$$

The insight to amplifier circuit design that was previously mentioned is now clear.

The input port reflection coefficient is s_{11}, the output port reflection coefficient is s_{22}, the forward transducer gain for 50 ohms terminations is $\left|s_{21}\right|^2$, $\left|s_{12}\right|^2$ is the reverse transducer gain. The maximum possible forward transducer gain, however, can be considerably greater than $\left|s_{21}\right|^2$.

Fig. 5 shows a signal flow graph for an amplifier that has source and load terminations other than 50 ohms. Therefore, r_1 and r_2 are not zero. The power wave source a_s is now:

$$a_s = \frac{V_s(50)^{\frac{1}{2}}}{Z_s+(50)^{\frac{1}{2}}} \quad 44$$

This represents the power wave launched by the generator.

To calculate transducer power gain the power delivered to the load R_L, and the power available from a source of internal impedance Z_s, are required. The power delivered to the load is:

P_L= (power incident on load) - (power reflected from load)

$$= \left|b_2\right|^2 - \left|a_2\right|^2 \quad 45$$

but:

$$a_2 = b_2 r_2 \text{ or } |a_2|^2 = |b_2|^2 \; |r_2|^2 \qquad 46$$

$$P_L = |b_2|^2 \; (1 - |r_2|^2) \qquad 47$$

Similarly, the available source power is:

$$P_{AVS} = \frac{a_s^{\;2}}{1 - |r_1|^2} \qquad 48$$

then:

$$G_T = \frac{P_L}{P_{AVS}} = \frac{|b_2|^2}{|b_s|^2} \quad (1 - |r_2|^2)(1 - |r_1|^2) \qquad 49$$

The ratio b_2 / b_s is found by application of Mason's Formula[5] to the signal flow graph of Fig. 5.

$$\frac{b_2}{b_s} = \frac{s_{21}}{(1-s_{11}r_1)(1-s_{22}r_2) - s_{12}s_{21}r_2r_1} \qquad 50$$

Substituting for $b_2 / b_s{}^2$ in equation 49:

$$G_T = \frac{|s_{21}|^2 (1-|r_1|^2)(1-|r_2|^2)}{(1-s_{11}r_1)(1-r_2s_{22}) - s_{21}s_{12}r_1r_2} \qquad 51$$

Of special interest is the case where s_{12} is small in the sense that it affects G_T. If $s_{12} = 0$, the unilateral transducer power gain is:

$$G_{Tu} = \frac{|s_{21}|^2 \; (1-|r_1|^2)(1-|r_2|^2)}{(1-s_{11}r_1)\;(1-r_2s_{22})} \qquad 52$$

Bodway[3] developes a relationship for the bounds on the ratio G_T/G_{Tu} in terms of the scattering parameters.

$$\frac{1}{(1+u)^2} < \frac{G_T}{G_{Tu}} < \frac{1}{(1-u)^2} \qquad 53.$$

where:

$$u = \frac{|s_{11}s_{12}s_{21}s_{22}|}{(1-|s_{11}|^2)(1-|s_{22}|^2} \qquad 54$$

If $(G_T/G_{Tu})_{db}$ is bounded by $\pm$ 1 db then a unilateral design is justified.

Since the amplifier is assumed linear, the unilateral transducer gain G_{Tu} will be a maximum when there is a conjugate impedance match at both input and output ports as indicated in Fig. 6. A conjugate impedance match between the input impedance to the device (Z_{in}) and the equivalent source impedance (Z'_s) requires an impedance-transforming network. The same is true of the equivalent load impedance Z'_L and Z_o.

It is not necessary to work with impedance terms, however. The condition that $Z^*_{in} = Z_s$ is the same as $s^*_{11} = r'_1$ That is, for maximum power flow from the generator into the input of the transistor, the reflection coefficient of the generator must be the complex conjugate of the reflection coefficient of the transistor input (s_{11}). The same condition must exist at the output for maximum power flow to Z'_L. Therefore, if $Z'_L = Z^*_o$, then $s^*_{22} = r'_2$. Substituting these conditions into equation 52 results in:

$$G_{Tu} = \frac{|s_{21}|^2 \, (1-|s_{11}|^2)(1-|s_{22}|^2)}{\left(1-|s_{11}|^2\right)^2 \left(1-|s_{22}|^2\right)^2} \qquad 55$$

or:

$$G_{Tu\,(Max.)} = \frac{|s_{21}|^2}{\left(1-|s_{11}|^2\right)\left(1-|s_{22}|^2\right)} \qquad 56$$

Assuming that the amplifier is unconditionally stable, then $|s_{11}|, |s_{22}| < 1$.

$$G_{Tu} = \frac{|s_{21}|^2}{(1-|s_{11}|^2)(1-|s_{22}|^2)} = G_0 G_1 G_2 \qquad 57$$

where:

$$G_0 = |s_{21}|^2 \qquad 58$$

$$G_1 = \frac{1}{1-|s_{11}|^2} \qquad 59$$

$$G_2 = \frac{1}{1-|s_{22}|^2} \qquad 60$$

The forward transmission coefficient s_{21} plays a dominant role very similar to h_{fe} (or h_{21}) in the h-parameter representation. Large values of s_{21} will give large power gains. The terms G_1 and G_2 represent gains in power due to matching the transistor ports to their terminations. If $|s_{11}|$ or $|s_{22}| > 1$, then special care must be taken, since the device is potentially unstable. For this case

$ReZ'_s > |ReZ_{in}|$ and $ReZ'_L > |ReZ_o|$ to insure stability, where Z'_s and Z'_L are the source and load impedance as viewed from the transistor's input and output ports. (Fig. 6).

The design of the matching networks to transform Z_s to Z^*_{in} and Z_L to Z^*_o can be accomplished using a Smith Chart which makes the network design fairly simple. If $|s_{ii}|$ is less than .3, there is little gain in conjugate matching of the i^{th} port. For example, if $s_{11} = .3$:

$$G_1 = \frac{1}{1-|s_{11}|^2} = \frac{}{1-(.3)^2} = 1.1 \ (=.42 \text{ db}) \qquad 61$$

Thus, matching the input results in only a ten percent increase in G_{Tu}. What this indicates is that the input impedance of the device is very nearly 50 ohms and close to maximum available power is being delivered to the input.

The amplifier design theory is sufficiently developed at this point to consider the practical design of a transistor amplifier using the scattering parameter method.

Practical Design Considerations

The scattering parameters of a transistor are a function of the applied dc bias voltages and currents (the quiescent or Q-point). The proper selection of an operating point is an essential step in the design process since the maximum unilateral transducer gain:

$$G_{Tu(Max.)} = \left|s_{21}\right|^2 \left(\frac{1}{1-\left|s_{11}\right|^2}\right)\left(\frac{1}{1-\left|s_{22}\right|^2}\right) = G_0G_1G_2 \qquad 62$$

can be optimized by selection of a Q-point that gives the maximum product of $G_0G_1G_2$. Note that the device is assumed unilateral, which is normally a reasonable assumption. After a Q-point that optimizes $G_{Tu(Max.)}$ has been found, the synthesis of the lossless matching networks completes the design.

It is worth noting that there are frequently other constraints on the operating point and matching networks other than optimizing $G_{Tu(Max.)}$. For example, the amplifier bandwidth, the noise figure, and the bias levels are also of concern. But, to satisfy these often divergent specifications in some optimum sense is a problem best solved by computer testing and design.

A Motorola Type 2N4261 UHF transistor was tested in the measurement system of Fig. 3 at a frequency of 1 GHz. The Q-point which gave the maximum product of $G_0G_1G_2$ was found to be V_{CE} = 11 volts and I_E = 10 milliamps. The s-parameters at this Q-point and frequency were:

$$s_{11} = 0.25\,\angle{-70°} \qquad s_{22} = 0.481\,\angle{-50°} \qquad 63$$

$$s_{21} = 2.12\,\angle{+60°} \qquad s_{12} = 0.208\,\angle{+48°} \qquad 64$$

The optimum Q-point is selected on the basis of a unilateral device. Also, the matching networks are designed from this viewpoint.

Checking the unilateral approximation with equations 53 and 54:

$$\frac{1}{\left(1 + u\right)^2} < \frac{G_T}{G_{Tu}} < \frac{1}{\left(1 - u\right)^2} \qquad 65$$

where:

$$u = \frac{\left| s_{11} s_{12} s_{21} s_{22} \right|}{\left(1 - \left|s_{11}\right|^2\right)\left(1 - \left|s_{22}\right|^2\right)} \qquad 66$$

Substituting for the value of the scattering parameters into equation 66 gives u = 0.0735, and substituting u into equation 65 gives:

$$0.87 < \frac{G_T}{G_{Tu}} < 1.156 \qquad 67$$

Therefore G_T (true) can differ from the assumed unilateral G_{Tu} by approximately $\pm$ fifteen percent; or, in decibels:

$$-\ 0.62 \text{ db} < \left.\frac{G_T}{G_{Tu}}\right|_{db} < 0.61 \text{ db} \qquad 68$$

The maximum error in the transducer gain using the unilateral approximation is 1.234 db. This is certainly small enough to procede on an unilateral basis and to accept the Q-point of $V_{CE} = 11$ volts and $I_E = 10$ milliamps as close to a true optimum.

A device that has $s_{12} = 0$ and $|s_{11}|, |s_{22}| < 1$ is unconditionally stable. For this example, $s_{12} \neq 0$, but the circuit behaves as if this were essentially so. Since $|s_{11}| = 0.25$ and $|s_{22}| = 0.481$

are considerably less than one, no difficulty with instability should occur. If $|s_{11}|$ or $|s_{22}|$ were slightly less than one, assuming the device to be unilateral is a risk, since the value of $|s_{11}|_{true}$ and $|s_{22}|_{true}$ may be greater than one and the circuit could oscillate. The ReZ'_s (Fig. 6) must insure that the net resistance be positive around the input loop if $|s_{11}|>1$. Similarly for ReZ'_L and $|s_{22}|$ in the output loop.

The design of the matching networks to satisfy the condition $s^*_{11} = r'_1$ and $s^*_{22} = r'_2$ is accomplished with the use of a Smith Chart. As was shown in equation 61, if $|s_{11}| < 0.3$, there is little to be gained from attempting a conjugate match at the input. However, the output offers a chance for some additional gain. Substituting s_{22} into equation 62:

$$G_2 = \frac{1}{1 - |s_{22}|^2} = \frac{1}{1 - (0.481)^2} = 1.3 \ (= 1.12 \text{ db}) \qquad 69$$

To achieve this power gain, it is necessary to transform the 50-ohm load resistor ($r_2 = 0$) to $s^*_{22} = 0.481 \underline{/-50°}$.

Regardless of whether a matching network is required or not to achieve a conjugate match, the network shown in Fig. 8 is required to get the dc bias to the transistor and to keep the bias from the 50-ohm terminations. If no conjugate match is desired, then the reactance of the capacitor is made much less than 50 ohms and the reactance of the inductor much greater than 50 ohms. Therefore, the input network consists of simply $X_{C1} \ll 50$ and $X_{L1} \gg 50$. The values of

C_1 and L_1 to achieve this at 1 GHz are calculated from:

$$X_{C1} = \frac{1}{2\pi 10^9 C_1} \ll 50 \qquad X_{L1} = 2\pi 10^9 L_1 \gg 50 \qquad 70$$

solving for L_1 and C_1 gives $L_1 = 8$ nh $C_1 = 318$ pf.

The design of the output network to transform s^*_{22} into 50 ohms is performed on the Smith Chart of Fig. 7. L_2 and C_2 are calculated from the normalized values of x_{c2} and b_{L2} read from the chart.

$$x_{c2} = 1.0$$

$$X_{c2} = x_{c2}50 = 50 = \frac{1}{2\ (10^9)C_2}, \qquad C_2 = 318 \text{ pf} \qquad 71$$

$$b_{L2} = 0.13$$

$$B_{L2} = b_{L2} \times \frac{1}{50} = (0.13)\frac{1}{50} = \frac{}{2\ (10^9)L_2}, \qquad L_2 = 61.3 \text{ nh} \qquad 72$$

The completed amplifier is shown on a 50-ohm strip-line printed-circuit board in Fig. 8.

It should be mentioned that a conjugate match cannot always be achieved with the simple matching network shown in Fig. 8. Nevertheless, the essential feature of the network to apply bias to the device and block the bias from the 50-ohm terminations must be preserved when other network configurations are used.

The measurement of the scattering parameters were referred to a reference impedance of 50 ohms because most UHF sources and loads are also 50 ohms. It is convenient to use a 50-ohm transmission system when constructing an amplifier. The simplest way to do this is with a strip line of 50 ohms characteristic impedance.

It was found that the more common printed circuit materials such as epoxy-fiberglass or phenolic had excessive dielectric losses at a frequency around 1 GHz. The best printed circuit board material that is currently available is 1/16" double-clad copper, teflon-impregnated fiberglass. It has very low loss up to about 10 GHz. A strip line made of this board with a strip width of 0.14 inches has a characteristic impedance of 50 ohms. The characteristic impedance is not a sensitive function of the strip-line width so it is not necessary to hold tolerances low.

The inductors and capacitors used in the amplifier caused considerably more difficulty at 1 GHz than was anticipated. A few turns of #18 copper wire wound on a form proved to be the most practical way to obtain a high quality variable inductor. The actual impedance of the components was measured by the scattering parameter test set of Fig. 3. The one-port component was placed in the test jig and s_{11} recorded. The impedance of the inductor is calculated from the reflection coefficient equation:

$$s_{11} = \frac{Z-50}{Z+50} \quad \text{or} \quad Z = 50\,\frac{1+s_{11}}{1-s_{11}} \qquad 73$$

It was found that a few turns of #18 wire performed essentially as a true inductor (X_L proportional to frequency) over a frequency range of 500 MHz to 1 GHz, the quality factor was about 25. Silver-plated wire could increase this value considerably if necessary.

The only commercial variable capacitors that had very low losses and low series inductance at 1 GHz were the piston air dielectric capacitors. However, it was found that parallel plate capacitors made from the double-clad teflon printed-circuit board were more than adequate. The Q's were high (over 100) and the capacitance could be quickly calculated using the familiar parallel plate formula. Final adjustments were made by trimming the corners.

The value of the components in the matching network is not critical. Maximum power transfer occurs over essentially a two-to-one variation in the impedance levels, so it is not worthwhile to use low tolerance components.

The bypass capacitors, C_E and C_C, around the emitter resistance and collector supply were the most troublesome components. Since these capacitors are in feedback positions, they can increase s_{12} (true) considerably if they do not provide a truly effective ac bypass. The only capacitors that were found effective were silver-plated to reduce losses and had wide ribbon leads to reduce series inductance. A 1000 pf capacitor tested in the scattering parameter measurement system indicated a reflection coefficient of -1 at 1 GHz or a perfect short within measurement accuracy.

IV. CONCLUSIONS AND RECOMMENDATIONS

The scattering parameter representation gives a great deal of insight into amplifier design. An inspection of the parameters reveals the essential transducer gain and stability of the device for the unilateral case. More importantly, scattering parameter measurements are extremely simple. There are no adjustments to be made after the initial calibration. The system, because of the 50-ohm wide-band terminations, is extremely stable. However, many of the problems associated with amplifier design have not been covered in this paper. If the device is not unilateral, then the design process becomes very complicated.

In the general design problem, the device must be treated as bilateral; further, other specifications besides transducer gain must also be satisfied. For example, band width, stability margin, noise figure, component tolerances, and power supplies available must all be considered in many practical problems. A satisfactory solution to these diverse specifications is extremely complicated if not impossible by hand calculations. However, if the device's scattering parameters were measured under the direction of a digital computer, measurements could be completed very rapidly. This is feasible since no adjustments of the measurement system are necessary. Subsequent calculations could then be made to design the circuit to satisfy the design specifications in some optimum sense. A small digital computer should be able to handle this task easily. The author intends to pursue this area of computer testing and optimum design of transistor amplifiers on the Electrical Engineering Department's Digital Computer.

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VITA

The author was born January 25, 1936, in St. Louis, Missouri. He received his primary and secondary education in St. Louis. In June 1958, he received a Bachelor of Science Degree in Electrical Engineering from the University of Missouri School of Mines and Metallurgy.

From 1958 to 1961, he was employed by the United States Navy principally engaged in electronic circuit design and underwater acoustical studies.

From 1961 to the present he has held the position of instructor at the University of Missouri - Rolla, while also taking graduate course work at the same university.

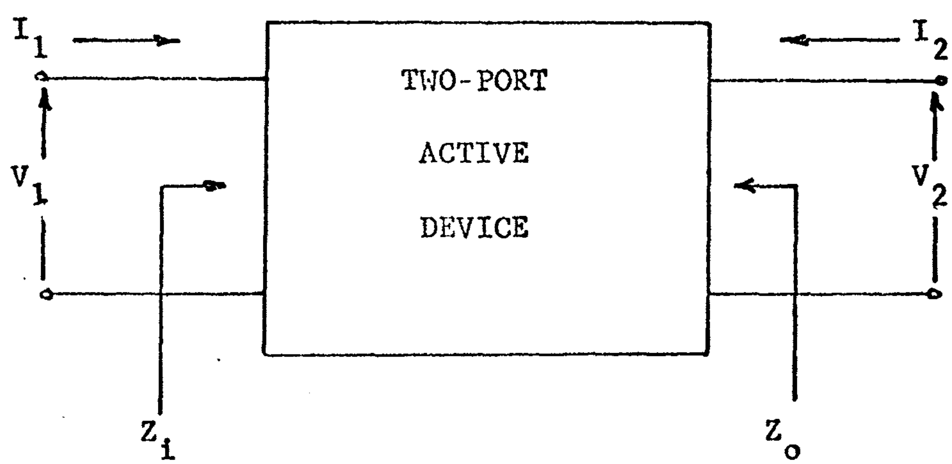


Fig. 1 Voltage, Current and Impedance Definitions for The h- and y-Parameter Representations.

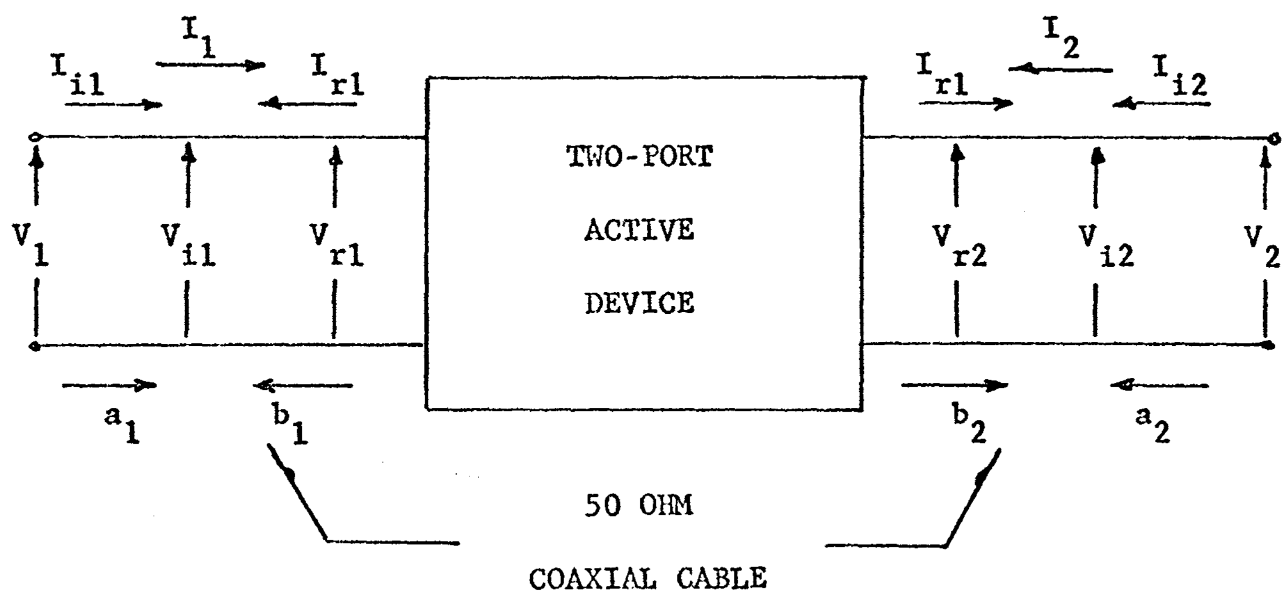


Fig. 2 Voltage, Current and Power Wave Definitions for The Scattering Parameter Formulation.

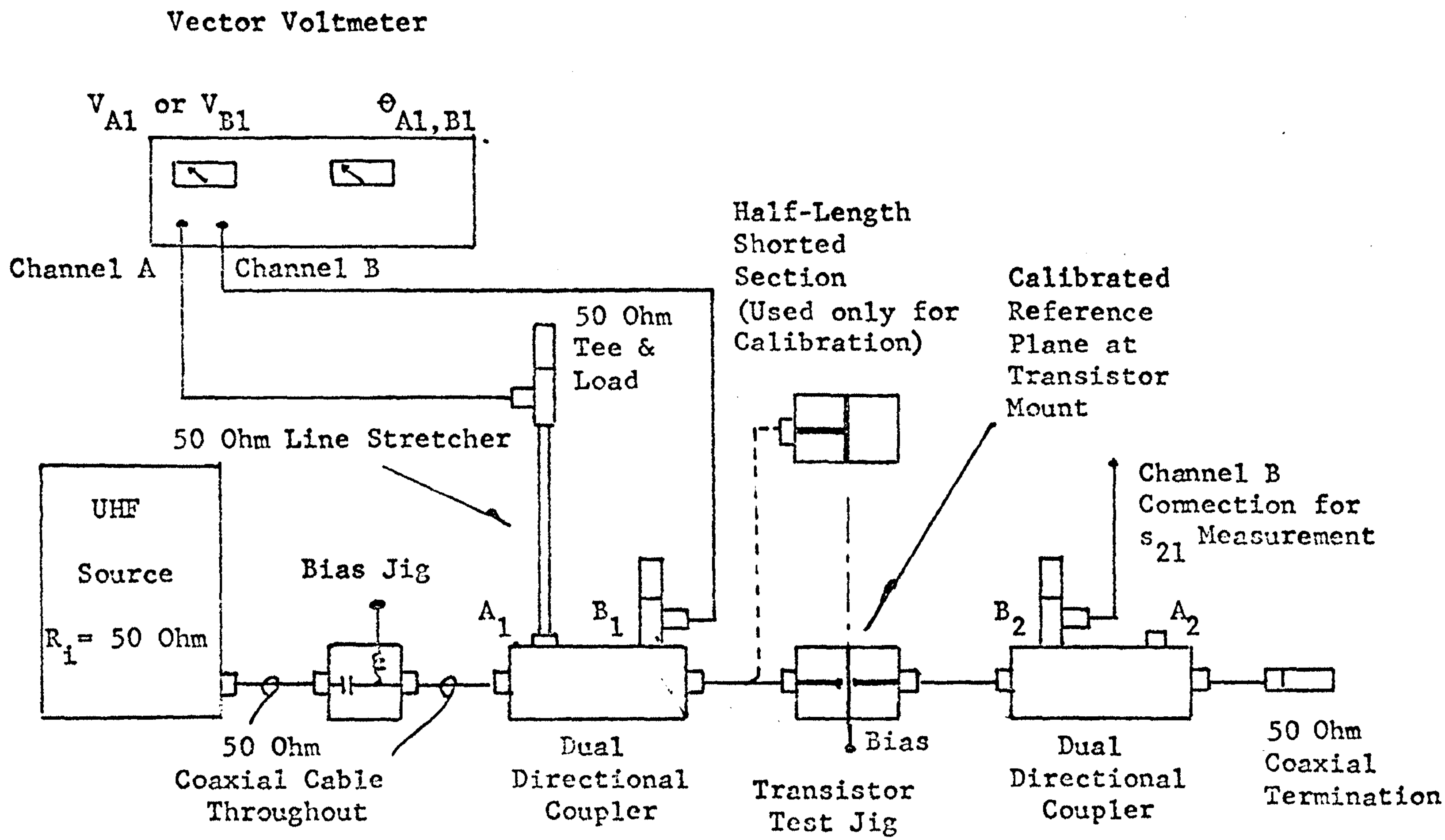


Fig. 3 Scattering Parameter Measurement System

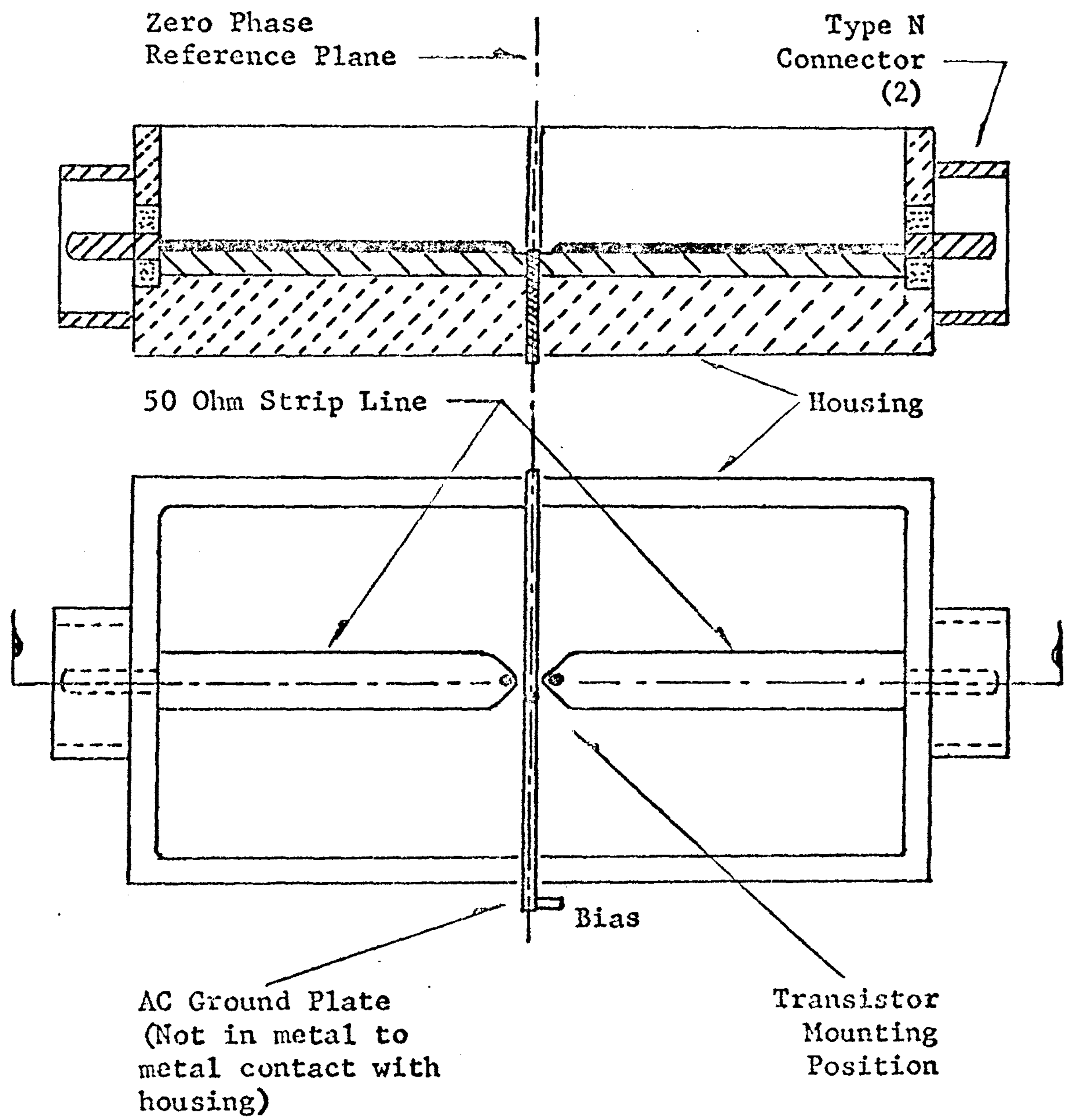


Fig. 4 Transistor Test Jig.

<u>Transistor Equations</u>

$$b_1 = a_1 s_{11} + a_2 s_{12}$$

$$b_2 = a_1 s_{21} + a_2 s_{22}$$

<u>Circuit Equations</u>

$$a_1 = a_s + b_1 r_1$$

$$a_2 = b_2 r_2$$

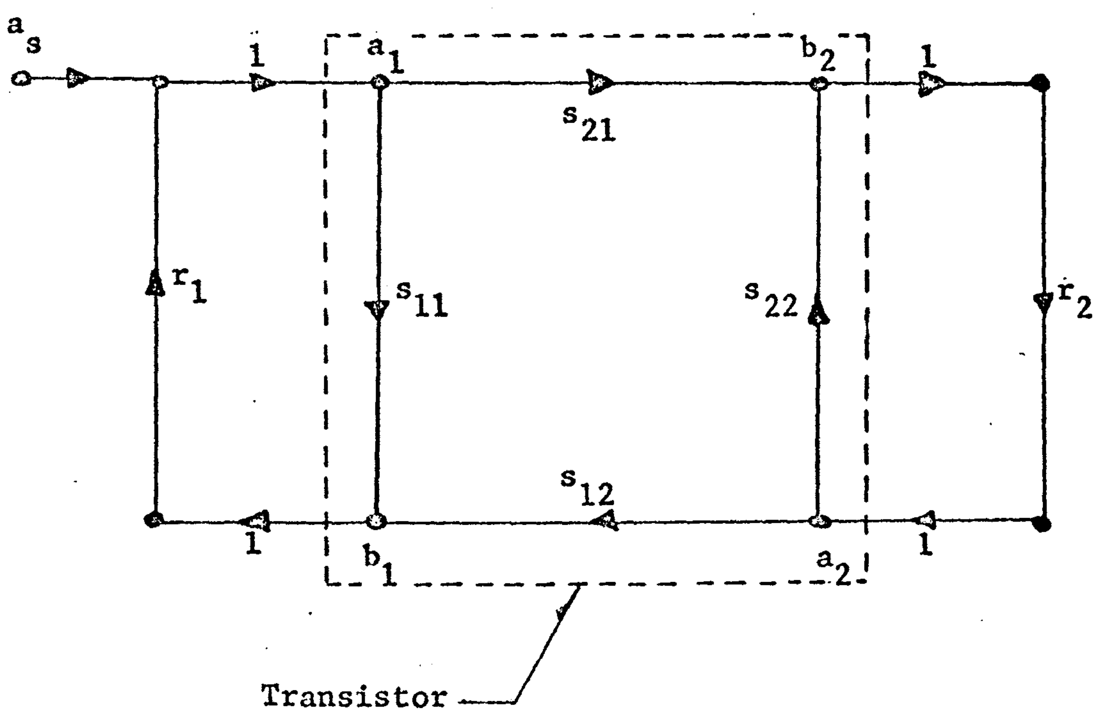
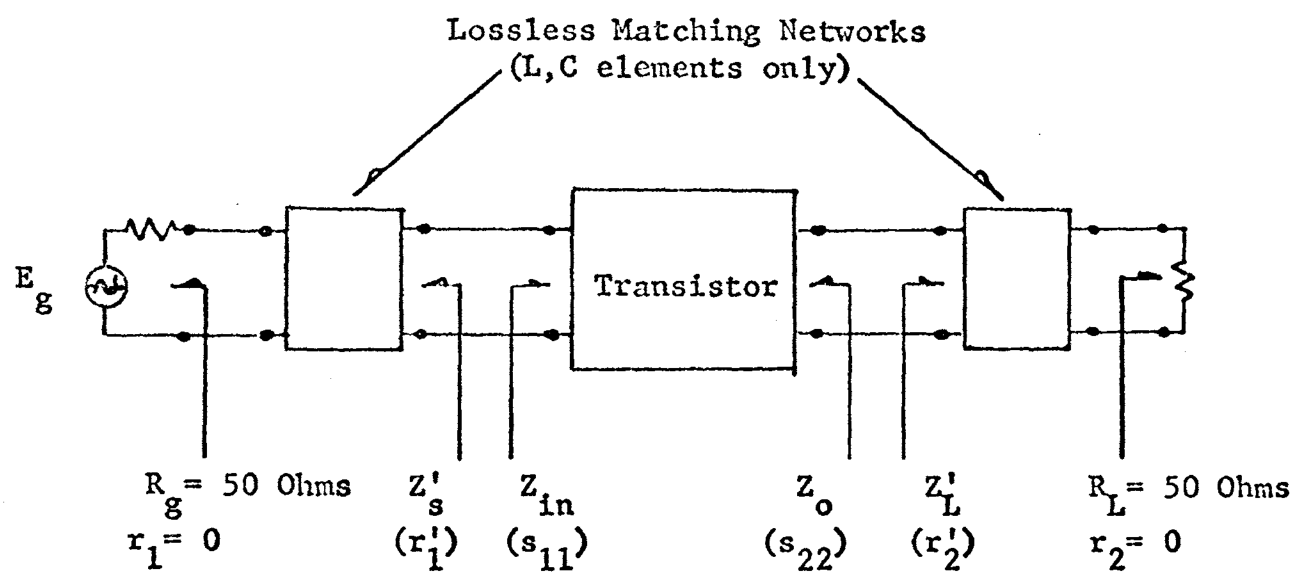


Fig. 5 Signal Flow Graph



Interconnections are 50 Ohm Coaxial Cable.

Fig. 6 Amplifier with Matching Networks

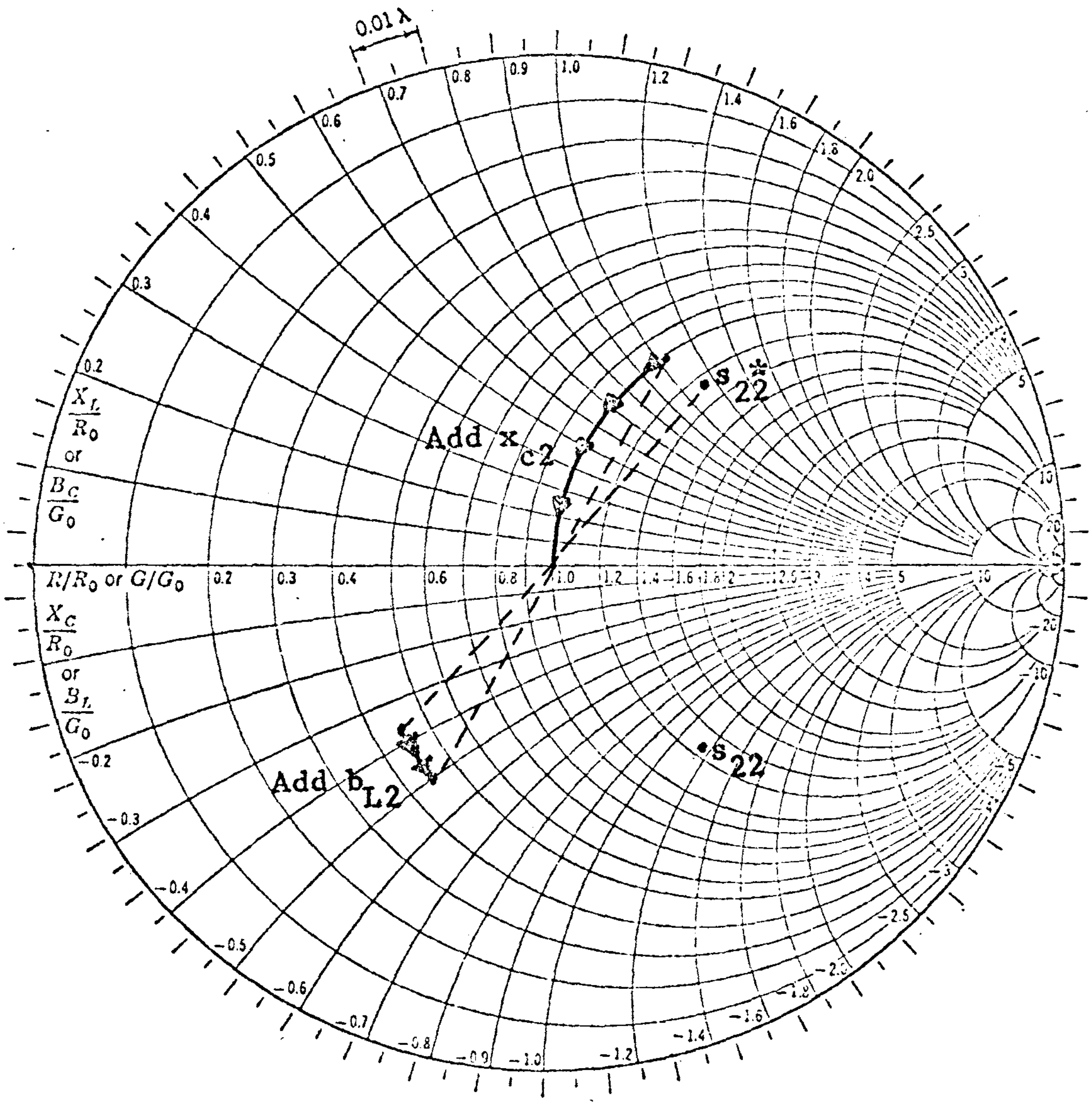


Fig. 7 Smith Chart Design of Output Network

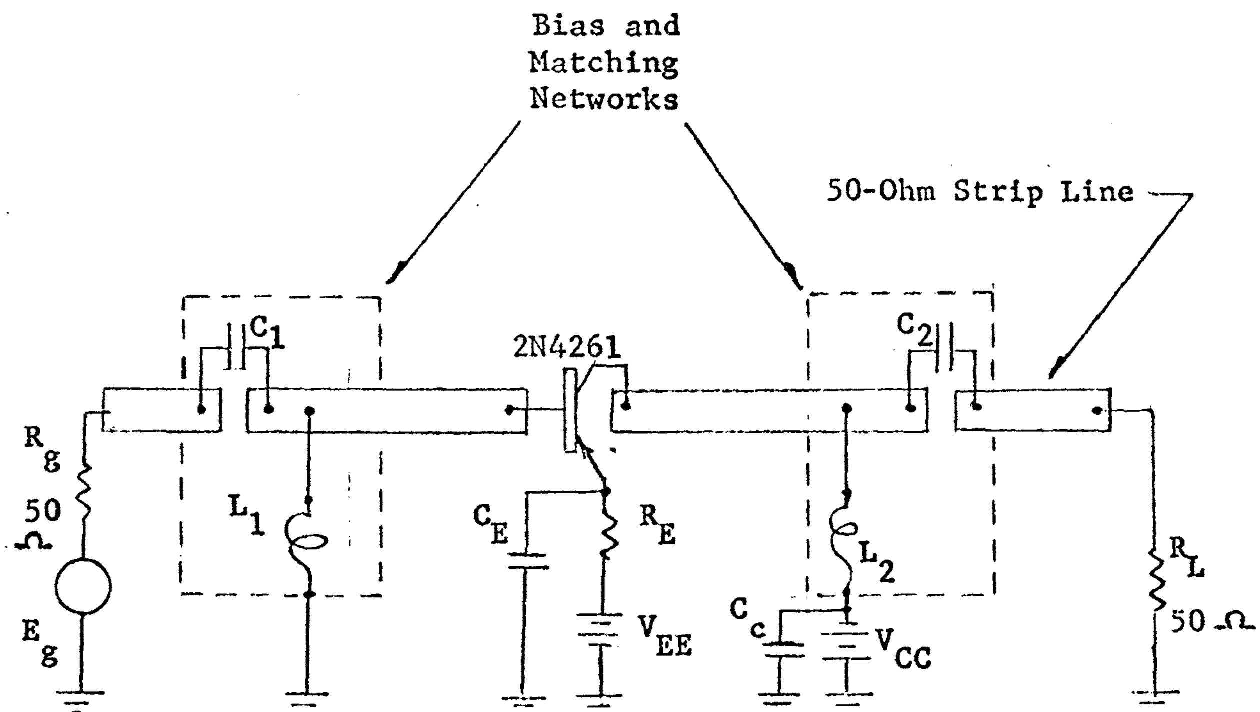


Fig. 8 Transistor Amplifier