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Development of a set of optimum synchronization codes for a unique decoder mechanization

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DEVELOPMENT OF A SET OF OPTIMUM SYNCHRONIZATION CODES FOR A UNIQUE DECODER MECHANIZATION

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ABSTRACT

Synchronization requirements are specified for and a unique decoder mechanization is associated with a particular communication system. Optimum synchronization codes, defined as codes which are the least susceptible to false synchronization indications, are sought. Existing sets of optimum codes are investigated for applicability. This Thesis shows how these sets were developed from selected criteria and demonstrates why their theoretical nature produces unsatisfactory results in the present application wherein all parameters are known. A computer program was written to examine code pattern performance in the specified decoder under actual operating conditions. From an analysis of the results, a recommended set of optimum synchronization codes was developed.

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CHAPTER I

INTRODUCTION

Synchronization is the process of assuring that two happenings agree in time. Dissimilar and/or remotely located events, actions, or continuing operations may be synchronized. In modern communications systems it is often necessary to synchronize a transmitter and receiver. For instance, television receptors require accurate synchronization to properly reconstruct video information. In radar systems, the reliability and accuracy of data processed by the receiver are dependent upon the relative timing of the transmitter and receiver. Data communication links for a ground station and controlled aircraft or space vehicle require correct and dependable synchronization of both terminals to achieve an informational exchange. Generally, PCM (Pulse Code Modulation) telemetry systems require synchronization information for reconstruction of the channel structure during data collection. Novel synchronization processes exists for each synchronization requirement.

Several types of synchronization are associated with PCM telemetry:

(1) Bit, or digit, synchronization - establishes equal time scales at the two ends of the link.

(2) Group synchronization - pinpoints an origin of time.

(a) Frame synchronization - consists of a short, unique code that precedes every data cycle to identify the new message.

(b) Word synchronization - a one or two bit code, inserted between words, provides sub-frame identification of constituent words.

Bit synchronization is conventionally obtained with a phase lock circuit. Group synchronization is secured with a specific code that is recognized by a matching code detector.

Synchronization customs were not always well defined. Early PCM systems employed relatively crude synchronizing techniques, such as zero crossing bit detection, weighted binary codes, or arbitrary word and frame synchronization code patterns of low error tolerance. Some systems derived bit synchronization from word synchronization with frame synchronization obtained last. Another method used amplitude modulation to obtain a frame or word reference point. These methods required relatively large bandwidths and were susceptible to noise. But, recent advances in reliability and miniaturization together with widespread applications in missiles and spacecraft have created a phenomenal increase in PCM telemetry usage. This proliferation has resulted in more sophisticated synchronization techniques. The most recent recommendations, derived from an Air Force sponsored study[1] at the Naval Ordinance Laboratories (NOL), Corona, California, are:

(1) obtain bit synchronization first, using a phase lock synchronizer;

(2) obtain frame synchronization, using a digital matched filter recognizer;

(3) derive word synchronization from frame synchronization, only; and,

(4) design for low signal-to-noise conditions.

These recommendations appear to oversimplify the issue of group synchronization. Synchronization accuracy, for instance, is influenced by such system design parameters as:

(1) synchronization code length;

(2) synchronization code pattern;

(3) the shape of the transmitted signal; and

(4) receiver response.

Prudent design of these system properties can enhance synchronization reliability. On the other hand, the problem of establishing correct synchronization is adversely affected by:

(1) additive noise, inherent in the RF (radio frequency) link and generated in the transmitting and receiving apparatus;

(2) random transmission times, requiring continuous repeatable synchronization; or

(3) the brevity of time allotted to obtain synchrony, this being one of the severest specifications on an operational PCM system.

Use of a system developing a large signal-to-noise ratio, such as a matched-filter detector, can minimize adverse effects of these factors.

Results of the investigation showed that a universal set of high performance frame synchronization codes cannot be said to exist, per se. Code characteristics are fundamental to synchronization accuracy and are virtually mated to the using system. A designer, implementing a particular detector, providing for an error tolerance, and applying an individual performance yardstick, will evolve a singular criterion of code optimality. Usually, once this criterion has been defined, the binary pattern best fulfilling said standards is subsequently generated. Consequently, there exist sets of "optimum" codes corresponding to the various investigations. Selection of an optimum group synchronization code becomes a matter of matching applications to established criteria, or, for lack of precedence, developing yet another criterion.

One novel application, requiring precise group synchronization, is on a particular military communications link currently under development. The purpose of this link is to reproduce, for near-real time ground observation, aerial reconnaissance data as it is being collected. In this concept, video information from a surveillance radar is suffixed to a synchronizing code and transmitted to a ground terminal for processing. Each video frame contains target reflections associated with a single radar pulse, and represents one radial view from the originating radar. Reassembly, by the ground terminal, of a sequence of radial lines results in the desired reconnaissance picture. Precise realignment of these messages is essential. A timing error, or jitter, of 5 nanoseconds results in a framing misalignment corresponding to approximately 5 feet; an offset of this magnitude is considered sufficient to destroy specification resolution. Thus, the degree of synchronization accuracy is established. Therefore, synchronization codes were investigated to obtain an optimum selection.

Existing optimum codes were tested in the communication link decoder. In comparing resulting decoder outputs, synchronization ambiguities, false synchronization hazards, and low code error tolerances were found. Since these established codes proved ineffective for the proposed system, new codes, predicated upon more applicable criteria, had to be generated.

In this unique system, all parameters affecting decoder output are known. Full advantage of this information was accepted in defining a new criteria of code optimality. The pattern property examined was the crosscorrelation function, which can be accurately written. The criteria applied states that the pattern producing the crosscorrelation function that is the most tolerant of expected code errors is optimum. A computer program was required to produce crosscorrelation functions from which to select a set of optimum synchronization codes.

CHAPTER II

A SUMMARY OF OPTIMUM SYNCHRONIZATION CODES

The objective of synchronization is to designate a precise instant of time as a reference. Theoretically, synchronization requirements could be fulfilled by accurately restoring a brief burst of transmitted energy; unfortunately, channel bandwith limitations, receiver response, and additive noise preclude an unambiguous reproduction of the pulse by the receiver. Ideally, the desired synchronization pulse may be created by transmitting a sample signal and performing a cross-correlation in a matching receiver. In PCM practice, the reference instant is obtained by transmitting a series of pulses and correlating the train in a pulse compression device. The pulse train is known as the synchronization code; one form of a pulse compression device is the matched filter detector.

A. BINARY SYNCHRONIZATION CODES

Synchronization codes, in this Thesis, are constructed of a finite number of binary pulses arranged in a pattern. Binary states may be 0 (ZERO) and 1 (ONE) or +1 (ONE) and -1 (MINUS ONE). Although both alphabets have been used in the references cited, the 0 and 1 symbols will be used henceforth for purposes of uniformity in presentation.

A binary pattern exists in four forms:

(1)	Basic pattern	1110010
(2)	Complement (binary inverse of basic)	0001101
(3)	Mirror (time inverse of basic)	0100111
(4)	Mirror Complement, or Alternate (binary inverse of time inverse of basic)	1011000

In evaluating a pattern's symmetrical autocorrelation function, all four versions of a family produce identical results. When autocorrelation functions are examined, only one representative of a family will be identified.

B. MATCHED FILTER

The digital matched filter is commonly used as the synchronization correlator in PCM systems. A simple, representative correlator, as depicted in Figure 1, consists of:

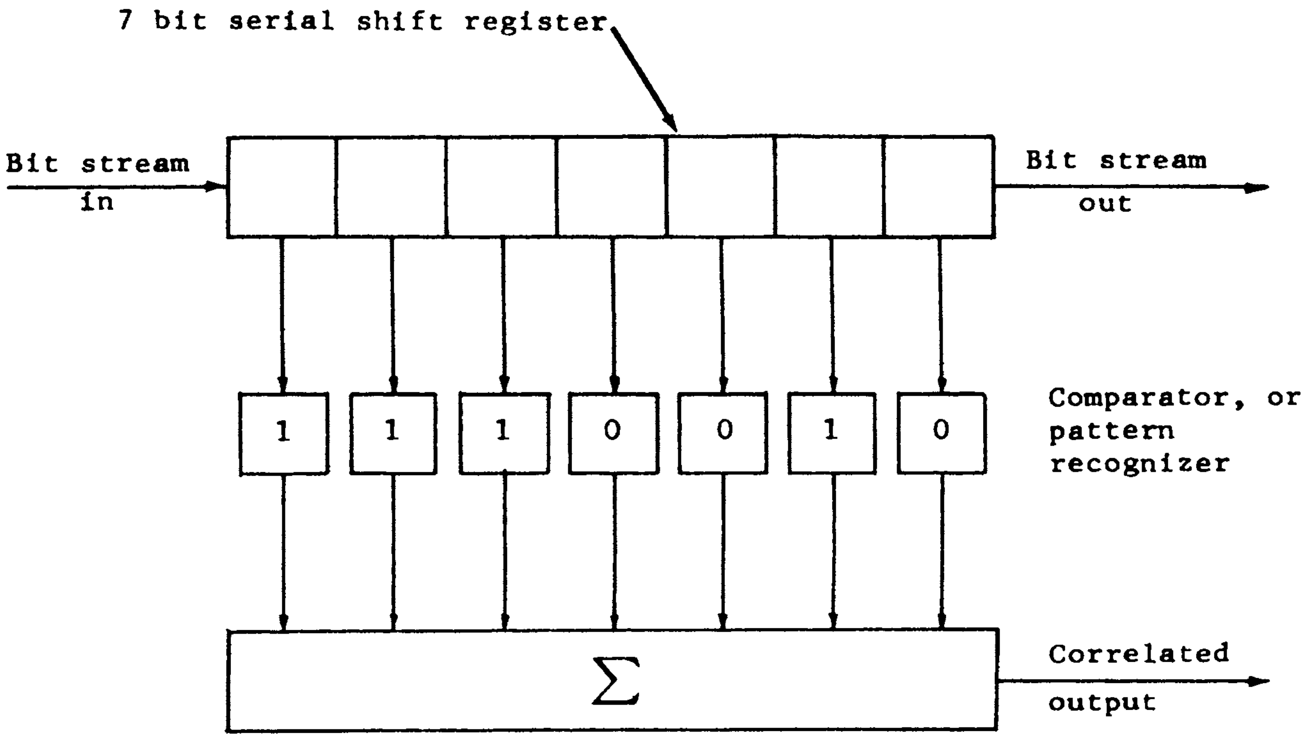
(1) a serial shift register, through which the bit stream is cycled.

(2) a comparator, which stores the synchronization code and matches each comparator stage with a corresponding bit in the shift register. A comparator stage output, for the 0, 1 alphabet, may be governed by these rules:

$$1 \times 1 = 1$$

$$1 \times 0 = -1$$

$$0 \times 0 = 1$$



Synchronization code:
1110010

DIGITAL MATCHED FILTER

FIGURE 1

(3) a summer, whose output is the algebraic sum of all inputs from the comparator stages.

When the shift register contains the synchronization code of 1110010, matching the pattern recognizer stages, the summer output magnitude is 7.

C. AUTOCORRELATION FUNCTION

An n bit code, when inserted into a matched filter, produces a sequence of summations as an output. One consideration of a code's suitability for synchronization purposes is the correlation it has with some aperiodic phase shift with itself. This numerical figure of merit is the autocorrelation function, sometimes referred to as the aperiodic autocorrelation function, represented by:

$$c_k = \sum_{i=1}^{n-|k|} x_i \, x_{i+|k|} \qquad k = 0, \pm 1, \pm 2, \cdots \pm (n-1) \quad x_i = 0,1 \qquad (1)$$

where k = degree of aperiodic phase shift or, the number of code bits not in the shift register.

Maximum value of c_k is $c_o = n$, which occurs at $k = 0$ when the code is exactly in the shift register. This term is the largest c_k value obtainable and is the label used for marking the instant of synchronization. The other terms, C_1 to C_{n-1}, referred to as sidelobes because of their similarity to an antenna radiation pattern, may attain any value within $\pm (n - k)$. Minimum sidelobe amplitudes are desired and the code pattern bits can be manipulated accordingly; but, making some terms more negative assures others will become more positive.

The autocorrelation function of the pattern 1110010 is:

t	1	2	3	4	5	6	7	8	9	10	11	12	13
k	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
c_k	-1,	0,	-1,	0,	-1,	0,	7,	0,	-1,	0,	-1,	0,	-1

Realistically, the sequence of c_k terms represented by the autocorrelation function of Eq. 1 is not the pattern that actually emerges from a matched filter detector. Preceding and succeeding binary bits, cycling through the shift register, affect the sidelobe amplitudes, only. Also, a single detected code error diminishes the magnitude of c_0, the synchronization term, and alters the amplitude of half of the sidelobes.

D. PATTERN CATEGORIES

PCM synchronization codes are normally surrounded by random data containing both ZEROS and ONES: consequently, the autocorrelation function, except for the c_0 term, is distorted. Nonetheless, the detector must unambiguously recognize the true synchronization code, within an allowable error tolerance, among the on-coming bit stream. The matched filter, in the course of continuing inspection, examines three categories of bit patterns:

(1) random region, composed entirely of random data;

(2) overlap region, consisting of both random data bits and synchronization bits; overlapping data bits may number from $k = 1$ to a maximum of $k = n - 1$; and

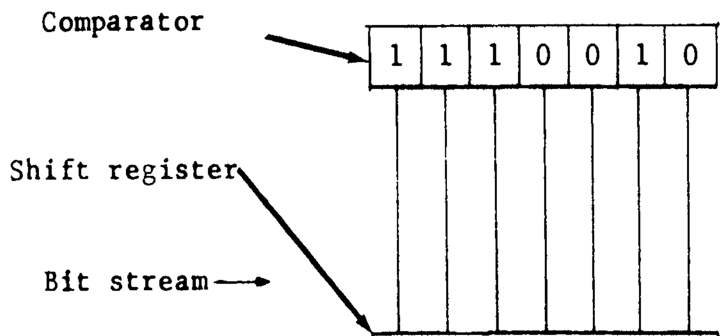
(3) true synchronization, occuring when the true code completely occupies the shift register ($k = 0$).

Figure 2 shows a typical movement of a binary train through the register, illustrating the pattern categories examined in the search for the true 7 - bit code existing amid the digital stream.

In the "random" region, the probability of a false synchronization is $(0.5)^n$ and is completely independent of code pattern. The probability of a false synchronization during the "true synchronization" region is obviously non-existent. Only in the "overlap" region is false synchronization a function of code pattern. Consequently, in evaluating code suitability, its behavior in the "overlap" region is studied.

E. CODE OPTIMALITY

A suitable synchronization code is one that has a minimal probability of causing false synchronization indications, whether caused by detected code errors, or noise or random bits adjacent to the code. The optimum code is the one, for a given length, that is adjudged to have the least probability of producing erroneous synchronization. A commonly used gauge in evaluating a pattern is the autocorrelation function, since this sequence is representative of the developed synchronization term and sidelobes whose amplitudes may be sufficient to cause pre- or post-mature synchronization indication. Several sets of recommended codes have been produced using this direct approach. Other measurable properties of a pattern, not directly related to the autocorrelation statement, have been used for criterion in developing a set of optimum codes.



t	Bit stream →							Shift register							Bit stream →							k Degree of overlap	Region
0	1	1	1	0	0	1	0	X	X	X	X	X	X	X	X	X	X	X	X	X	X	---	Random data
1	X	1	1	1	0	0	1	0	X	X	X	X	X	X	X	X	X	X	X	X	X	-6	Overlap
2	X	X	1	1	1	0	0	1	0	X	X	X	X	X	X	X	X	X	X	X	X	-5	Overlap
3	X	X	X	1	1	1	0	0	1	0	X	X	X	X	X	X	X	X	X	X	X	-4	Overlap
4	X	X	X	X	1	1	1	0	0	1	0	X	X	X	X	X	X	X	X	X	X	-3	Overlap
5	X	X	X	X	X	1	1	1	0	0	1	0	X	X	X	X	X	X	X	X	X	-2	Overlap
6	X	X	X	X	X	X	1	1	1	0	0	1	0	X	X	X	X	X	X	X	X	-1	Overlap
7	X	X	X	X	X	X	X	1	1	1	0	0	1	0	X	X	X	X	X	X	X	0	True synchronization
8	X	X	X	X	X	X	X	X	1	1	1	0	0	1	0	X	X	X	X	X	X	1	Overlap
9	X	X	X	X	X	X	X	X	X	1	1	1	0	0	1	0	X	X	X	X	X	2	Overlap
10	X	X	X	X	X	X	X	X	X	X	1	1	1	0	0	1	0	X	X	X	X	3	Overlap
11	X	X	X	X	X	X	X	X	X	X	X	1	1	1	0	0	1	0	X	X	X	4	Overlap
12	X	X	X	X	X	X	X	X	X	X	X	X	1	1	1	0	0	1	0	X	X	5	Overlap
13	X	X	X	X	X	X	X	X	X	X	X	X	X	1	1	1	0	0	1	0	X	6	Overlap
14	X	X	X	X	X	X	X	X	X	X	X	X	X	X	1	1	1	0	0	1	0	---	Random data

Random data : XXXX
Pattern : 1110010

PATTERN RECOGNITION PROCESS

FIGURE 2

F. BARKER CODES

In a pioneering examination of group synchronization of binary digital systems, Barker[2] reasoned it would be desirable to start with an autocorrelation function having very low sidelobes. The governing code pattern, he insisted, could be unambiguously recognized by the detector. To assure this premise, Barker contended the selected pattern should be sufficiently unlikely to occur, by chance, in a random series of noise generated bits. The patterns examined were correlated in the "simple pattern recognizer" of Figure 3.

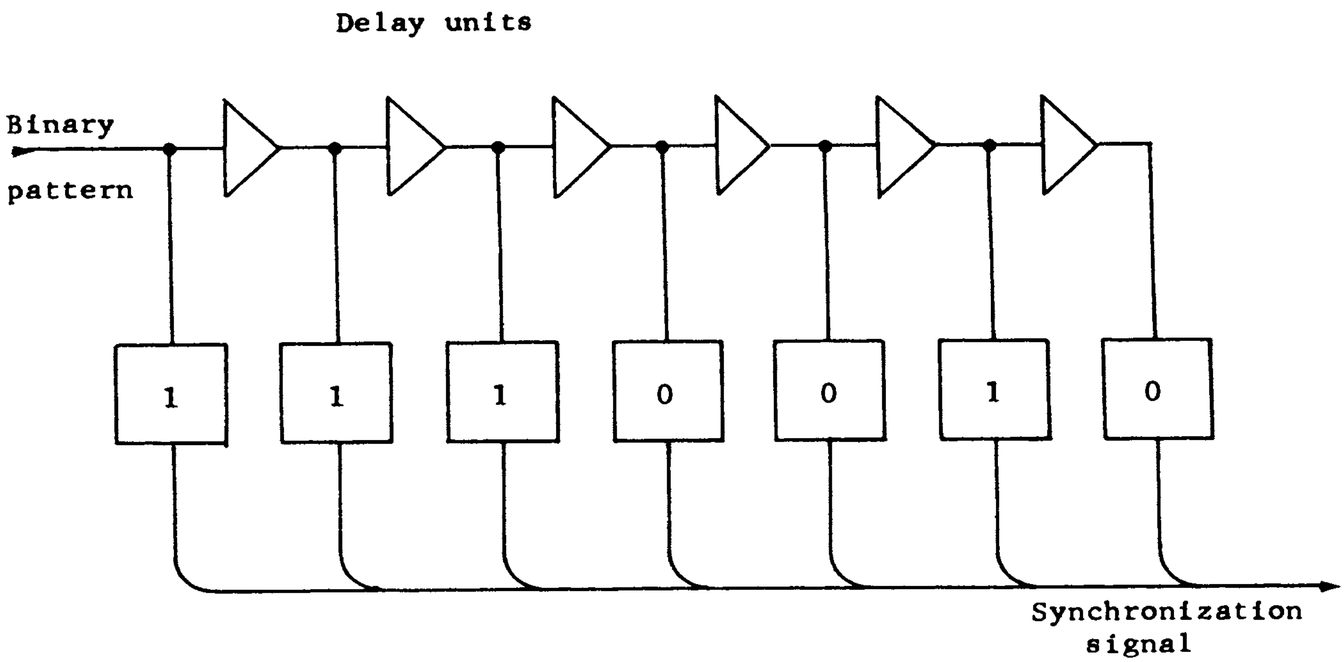
The probability of an n length digital pattern being duplicated by chance is:

$$P(n) = (0.5)^n \tag{2}$$

Longer codes obviously are more immune to duplication but excessive lengths are not necessarily desirable. Among other considerations in determining code length is the accepted error tolerance. If no errors are allowed, only one pattern will be recognized and it will occur with a probability of $(0.5)^n$. If e errors are allowed, a greater number of patterns are qualified for recognition and the probability of pattern recognition becomes:

$$P(r) = (0.5)^n \frac{n!}{e!\,(n-e)!}$$

$$= (0.5)^n C_e^n \tag{3}$$



Synchronization code:
1110010

SIMPLE PATTERN RECOGNIZER

FIGURE 3

A given error tolerance allows for a maximum of y errors among the code bits; summing all possible error combinations for $e = 0$ to $e = y < n$, the probability of randomly duplicating some pattern that will produce a synchronization indication is:

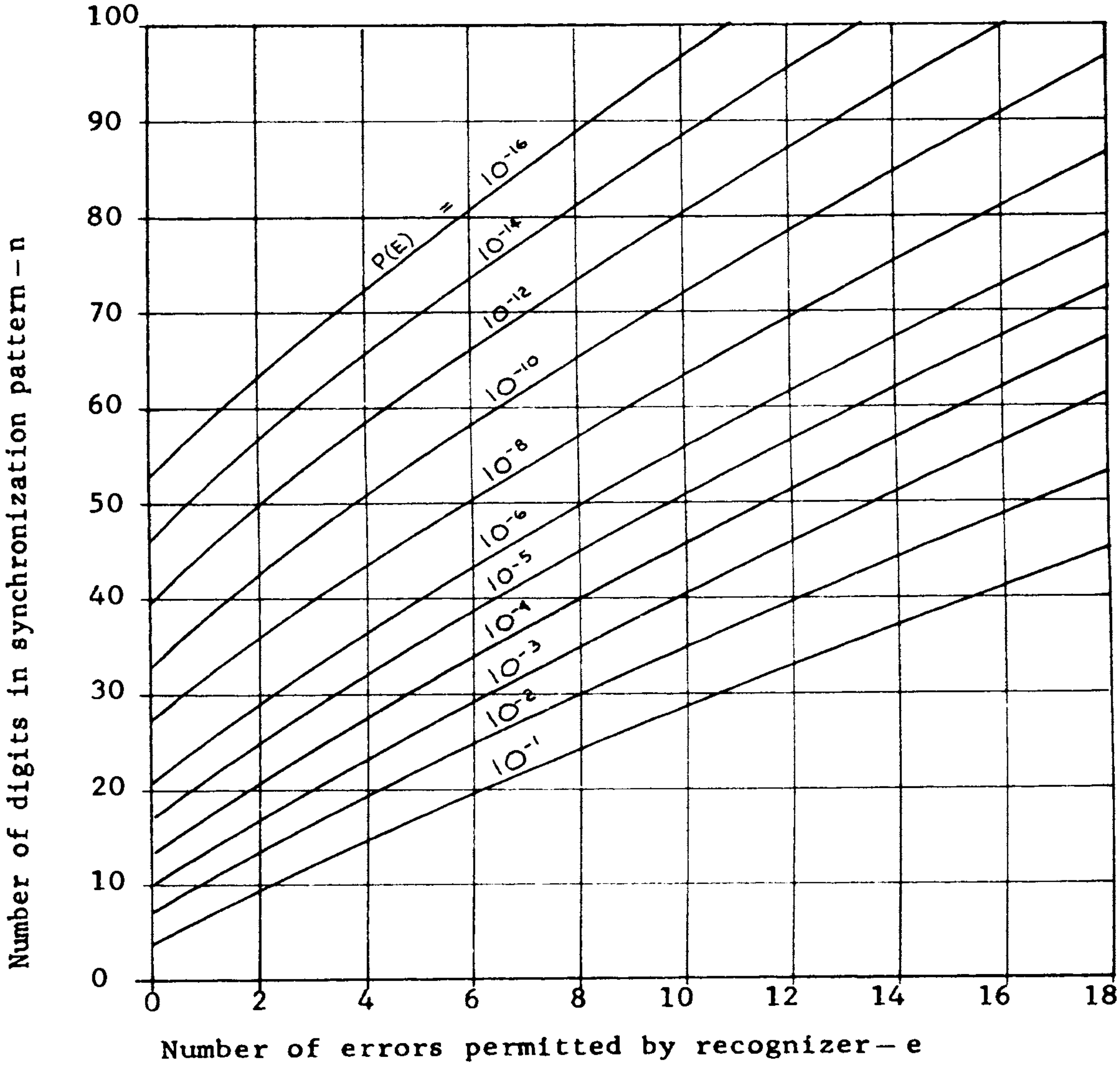
$$P(E) = (0.5)^n \sum_{e=0}^{y} C_e^n \tag{4}$$

This relationship is plotted in Figure 4, which may reasonably be used to calculate a minimum code length once the acceptable false synchronization probability is established.

Having established a minimum code length, a specific code pattern may be determined. From a search of autocorrelation functions, Barker concluded an "ideal" code pattern is one whose autocorrelation function conforms to:

$$C_k = \begin{cases} n, & \text{for } k = 0 \\ 0, & \text{for } k \text{ odd} \\ -1, & \text{for } k \text{ even} \qquad k = 0, 1, 2, \cdots (n-1) \end{cases}$$

The only "ideal" patterns found by Barker are for lengths of 3, 7, and 11 bits; these patterns are noted in Table I. "Ideal" codes were found to possess distinct properties, namely:



PROBABILITY OF FALSE SYNCHRONIZATION

FIGURE 4

TABLE I

BARKER CODES

k	n = 2		n = 3*		n = 4		n = 5		n = 7*		n = 11*		n = 13	
	Code	C_k	Code	C_k	Code	C_k	Code	C_k	Code	C_k	Code	C_k	Code	C_k
0		2		3		4		5		7		11		13
1	1	1	1	0	1	-1	1	0	1	0	1	0	1	0
2	0		1	-1	1	0	1	1	1	-1	1	-1	1	1
3			0		0	1	1	0	1	0	1	0	1	0
4					1		0	1	0	-1	0	-1	1	1
5							1		0	0	0	0	1	0
6									1	-1	0	-1	0	1
7									0		1	0	0	0
8											0	-1	1	1
9											0	0	1	0
10											1	-1	0	1
11											0		1	0
12													0	1
13													1	

* Ideal Barker code

(1) - pattern length n must be 4 q - 1, where q is a positive integer;

(2) - code digits form a symmetry described by:

x_i and x_{n+1-i} are alike if i is even,

x_i and $_{n+1-i}$ are opposite if i is odd.

These properties were not found in any other pattern lengths. However, in offering longer length codes, Barker defined a "very nearly ideal" pattern as one whose autocorrelation function is described by:

$$C_k = \begin{cases} n, & \text{for } k = 0 \\ 0, & \text{for } k \text{ odd} \\ \leq 1, & \text{for } k \text{ even} \end{cases} \qquad k = 0, 1, 2, \cdots (n-1)$$

Conforming patterns may be constructed by combining "ideal" patterns in ideal groups. For instance, the 3 - bit pattern 1 1 0 was used to construct the 9 - bit pattern 1 1 0 1 1 0 0 0 1, which has an autocorrelation sequence of:

1, 0, -3, 0, 1, 0, -3, 0, 9, 0, -3, 0, 1, 0, -3, 0, 1

Similarly, other "very nearly ideal" patterns for lengths of 21, 33, 49, 77, and 121 were found using "ideal" patterns.

In modern literature, "Barker codes" are accepted to be those whose autocorrelation functions correspond to:

$$C_k = \begin{cases} n, & \text{for } k = 0 \\ 0, & \text{for } k \text{ odd} \\ \pm 1, & \text{for } k \text{ even} \end{cases} \qquad k = 0, 1, 2, \cdots (n-1)$$

Included in this expanded category are patterns of length 2, 4, 5, and 13; patterns of length greater than 13 have not been found to exist. The complete set of what are generally referred to as "Barker codes" is presented in Table I.

G. CODRINGTON AND MAGNIN CODES

Barker defined optimum patterns by assigning specific values to each autocorrelation term. In so doing, he limited the number, and length, of conforming codes. If longer length codes are desired, other criteria must prevail. In consonance with Barker's criteria, yet not so restrictive as to dictate specific sidelobe magnitudes, Codrington and Magnin[3] have defined an optimum pattern as one for which the autocorrelation terms have minimum absolute values. A code would be selected by examining autocorrelation functions of virtually all patterns for that code length, a somewhat prohibitive task; a 13- bit code, for instance, has 8192 pattern variations, or over 2000 families of autocorrelation functions to be scrutinized. There exists a need for a systematic method of efficiently generating longer length optimum codes.

"In a search for sequences with flat autocorrelation functions,... [it was]...discovered that the Legendre sequences, arising from quadratic congruences in number theory, possessed the desired property". In fact, sequences of length $n = 4q + 3$ were found with optimum autocorrelation functions. Code lengths of $n = 4q + 1$, although not producing minimum absolute values, proved to be as satisfactory. In all cases, Legendre sequences, as naturally generated, required modification, or optimizing.

1. Calculating Natural Legendre Codes

A form of congruence may be written:

$$w^2 \equiv s \pmod{n} \tag{5}$$

where w is said to be quadratically congruent to s modulo n. If a number w can be found for which Eq. 5 holds, s is said to be a "quadratic residue modulo n"; otherwise, s is a "quadratic non-residue modulo n". A reduced set of residues modulo n may be generated by letting w take on all values from 1 to (n -1). If n is an odd prime, there will be an equal number of quadratic residue modulo n and quadratic non-residue modulo n integers.

In the reduced set of numbers, the Legendre symbol (w/n) is the symbolic weight of w. If n is an odd prime, the following relations hold:

$$(w/_n) = 1 \quad \text{when} \quad w^{\rho} \equiv +1 \pmod{n}$$
$$(w/_n) = 0 \quad \text{when} \quad w^{\rho} \equiv -1 \pmod{n}$$
$$\text{where } \rho = (n-1)/2 \tag{6}$$

To generate a Legendre sequence for n = 5, Eq. 6 is applied as follows:

w	$w^{\rho} \equiv \pm 1 \pmod{n}$	(w/n) = 1, 0
1	$1^2 \equiv +1 \pmod{5}$	(1/5) = 1
2	$2^2 \equiv -1 \pmod{5}$	(2/5) = 0
3	$3^2 \equiv -1 \pmod{5}$	(3/5) = 0
4	$4^2 \equiv +1 \pmod{5}$	(4/5) = 1

The resulting Lengendre sequence is 1 0 0 1. A set of natural Legendre sequences is tabulated in Table II.

2. Optimizing Legendre Codes

Pattern symmetry of the type occurring in natural Legendre codes is to be avoided if the desired minimum values of autocorrelation terms are to be realized. To eliminate symmetry, and optimize the autocorrelation terms, the (0/n) term is added and the code rotated. Several trials may be necessary before a combination of these two arbitrary choices yields an optimum code. The number of trials may be minimized by applying some rules.

Rule 1:	For $n = 4q + 3$, $x_1 = -x_n$
Rule 2:	Long sequences of the same digit, i.e., 1 1 1 1 1, usually should not be split by the rotation.
Rule 3:	Obvious symmetries, e.g., 1 0 1 0 1 0, are to be avoided. The selected (0/n) digit may be governed by this rule.
Rule 4:	The number of digits rotated is generally equal to one fourth of the number of code bits.

A typical optimization, for n = 11, is:

Natural Legendre is:	1 0 1 1 1 0 0 0 1 0
Assume (0/11) is 0:	0 1 0 1 1 1 0 0 0 1 0
Rotating 3 bits :	1 1 1 0 0 0 1 0 0 1 0

TABLE II

THE NATURAL LEGENDRE CODES

n	Pattern	Central* Symmetry
3	1 0	A
5	1 0 0 1	M
7	1 1 0 1 0 0	A
11	1 0 1 1 1 0 0 0 1 0	A
13	1 0 1 1 0 0 0 0 1 1 0 1	M
17	1 1 0 1 0 0 0 1 1 0 0 0 1 0 1 1	M
19	1 0 0 1 1 1 1 0 1 0 1 0 0 0 0 1 1 0	A
23	1 1 1 1 0 1 0 1 1 0 0 1 1 0 0 1 0 1 0 0 0 0	A
31	1 1 0 1 1 0 1 1 1 1 0 0 0 1 0 1 0 1 1 1 0 0 0 0 1 0 0 1 0 0	A

*NOTE: For n = 4q + 1 the 2nd half is the Mirror (M) of the first half and
For n = 4q + 3 the 2nd half is the Alternate (A) of the first half.

"Optimized Legendre" codes are presented in Table III. All the Barker codes are, naturally, included and are optimum codes. The longer length codes shown are considered optimum as their autocorrelation terms best conform to the Codrington and Magnin criteria of "minimum absolute values".

H. GOODE AND PHILLIPS CODES

Use of the autocorrelation function as a guide in determining code optimality is reasonably validated by the agreement of results obtained in using both Barker and Codrington and Magnin criteria. Other pattern properties are also suitable for use as criteria. Goode and Phillips[4] employed two relative measures: cyclic autocorrelation function, c(t), used as a coarse measure, and the least mean squared error, S^2, used as a fine gauge. The resulting selection is a code with the minimum probability of causing false synchronization under all degrees of code overlap and the worst bit error rate allowable. This standard evolved from a requirement to minimize the mean acquisition time of the acquisition mode, generally the most critical problem in PCM systems utilizing frame synchronization.

1. Cyclic Autocorrelation Function

A graphical technique for quickly estimating a code's suitability is to compare its cyclic autocorrelation pattern against the "ideal".

TABLE III
OPTIMUM CODRINGTON - MAGNIN CODES

k	n = 3* Code	C_k	n = 5* Code	C_k	n = 7* Code	C_k	n = 11* Code	C_k	n = 13* Code	C_k	n = 17 Code	C_k	n = 19 Code	C_k	n = 23 Code	C_k	n = 31 Code	C_k
1	1	0	1	0	1	0	1	0	1	0	0	-2	1	0	0	0	0	0
2	1	-1	1	+1	1	-1	1	-1	1	+1	0	-3	1	+1	1	-3	0	+1
3	0		1	0	1	0	1	0	1	0	0	+2	1	0	1	0	0	+2
4			0	+1	0	-1	0	-1	1	+1	1	-3	1	-1	0	+1	1	+1
5			1		0	0	0	0	1	0	1	+2	0	-2	0	-2	0	0
6					1	-1	0	-1	0	+1	0	+3	1	-3	1	-3	1	-1
7					0		1	0	0	0	0	0	0	-2	1	0	0	0
8							0	-1	1	+1	0	-1	1	-1	0	-1	1	+1
9							0	0	1	0	1	-2	0	-2	0	-2	1	-4
10							1	-1	0	+1	0	+1	0	+1	1	-1	1	-1
11							0		1	0	1	-2	0	0	0	+2	0	0
12									0	+1	1	-1	0	+1	1	-3	0	-1
13									1		0	0	1	+2	0	0	0	+2
14											1	-1	1	+1	0	+1	0	-1
15											1	0	0	0	0	0	1	0
16											0	-1	1	-1	0	-1	0	-1
17											1		1	-2	1	+2	0	0
18													0	-1	1	+1	1	-3
19													0		1	-2	0	0
20															1	-1	0	-1
21															1	+2	1	0
22															0	-1	1	+3
23															1		1	-2
24																	0	-1
25																	1	0
26																	1	-1
27																	0	-2
28																	1	-3
29																	1	-2
30																	1	-1
31																	1	

* Barker code

The cyclic autocorrelation function is defined as:

$$c(t) = \sum_{i=1}^{n} (x_i \oplus x_{i+t})' \qquad x_i = 0, 1 \tag{7}$$

where $\oplus$ represents modulo 2 addition, and

$i + t$ is reduced modulo n as required.

The "ideal" cyclic autocorrelation function is described as:

$$c(t) = \begin{cases} n, & \text{at } t = 0 \\ \frac{n}{2}, & \text{for } n \text{ even} \\ \frac{n-1}{2}, & \text{for } n \text{ odd} \end{cases} \quad \Big\} \; t = 1, 2, 3, \cdots$$

This "ideal" pattern is the model against which another n length code's cyclic autocorrelation function is compared. As another judgement, codes producing large sidelobe peaks, particularly near $t = 0$, are likely to cause false synchronization indications in the presence of noise and are to be avoided.

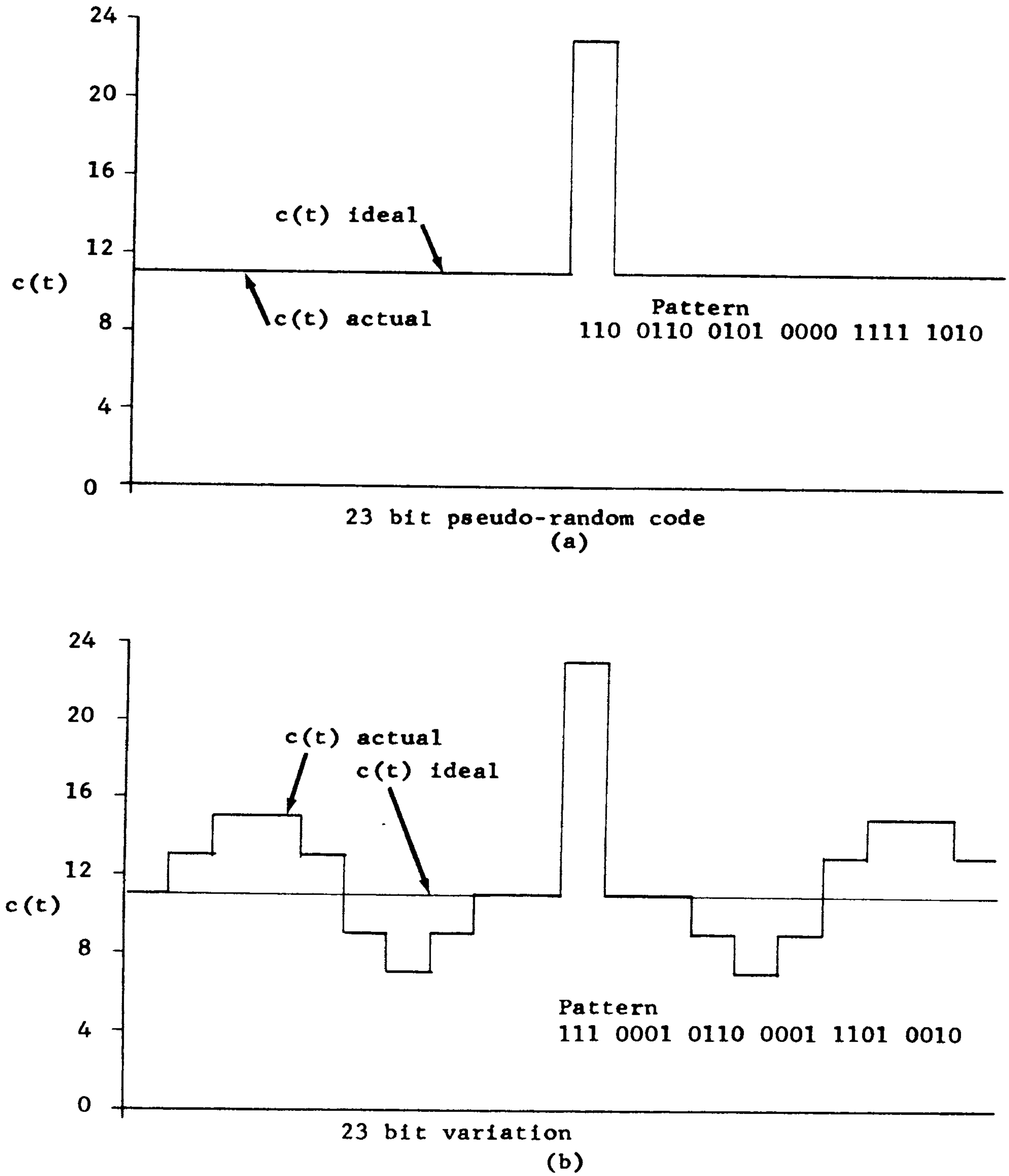
In Figure 5, cyclic autocorrelation functions of two 23 — bit codes are contrasted; Figure 5a represents c(t) for a pseudo-random code:

1 1 0 0 1 1 0 0 1 0 1 0 0 0 0 1 1 1 1 1 0 1 0

Figure 5b represents c(t) for the variation:

1 1 1 0 0 0 1 0 1 1 0 0 0 0 1 1 1 0 1 0 0 1 0

Obviously, the pseudo-random code develops an "ideal" c(t) and is, tentatively, preferred to the other 23-bit code variation.



CYCLIC AUTOCORRELATION FUNCTIONS

FIGURE 5

2. Least Mean Squared Error

For any degree of code entry into the shift register, the digital matched filter output is given by the truncated autocorrelation function figure of merit, defined as:

$$c_m = \sum_{i=1}^{m} (x_i \oplus x_{i+n-m})' \qquad \begin{array}{l} m = 1, 2, \cdots n \\ x_i = 0, 1 \end{array} \tag{8}$$

where m = number of code bits in the shift register, (m = n -k), and $\oplus$ represents modulo 2 addition.

If the synchronizing pulse amplitude of $c_m = n$, at $m = n$, is to be unambiguously prominent under worst error conditions, it is desirable that the correlator output for any other degree of m never exceed $m/_2$.

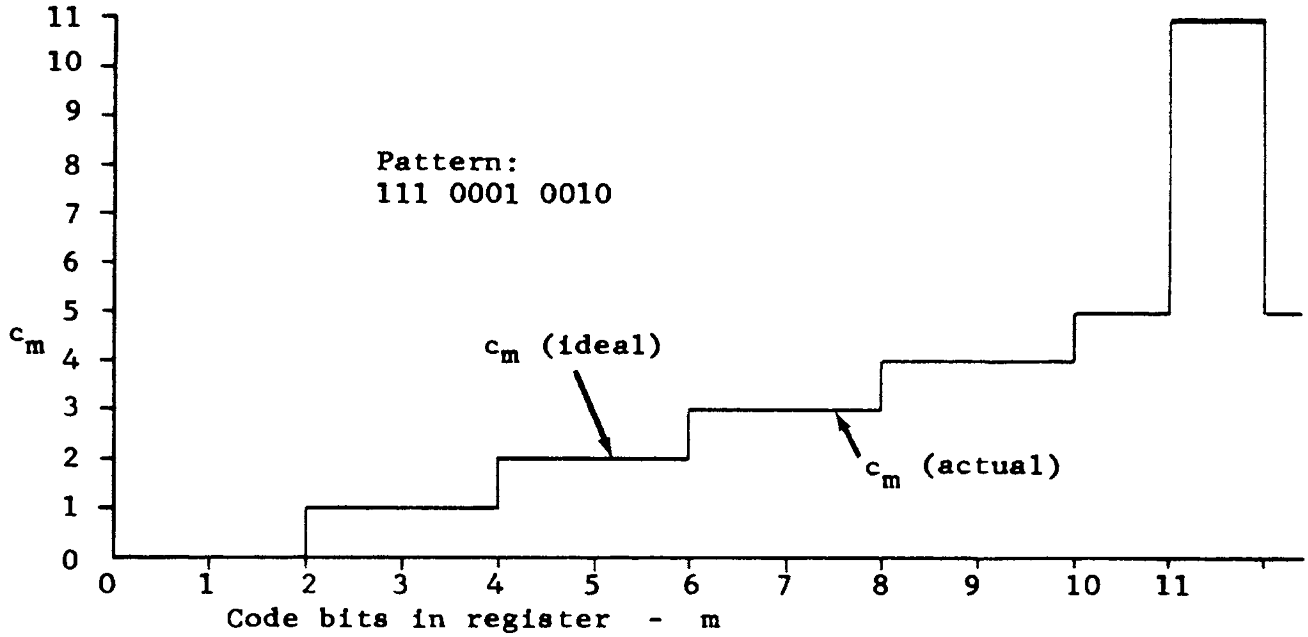
Ideally:

$$c_m = \begin{cases} n, & \text{at } m = n \\ \frac{m}{2}, & \text{for } m \text{ even} \\ \frac{m \pm 1}{2}, & \text{for } m \text{ odd} \end{cases}$$

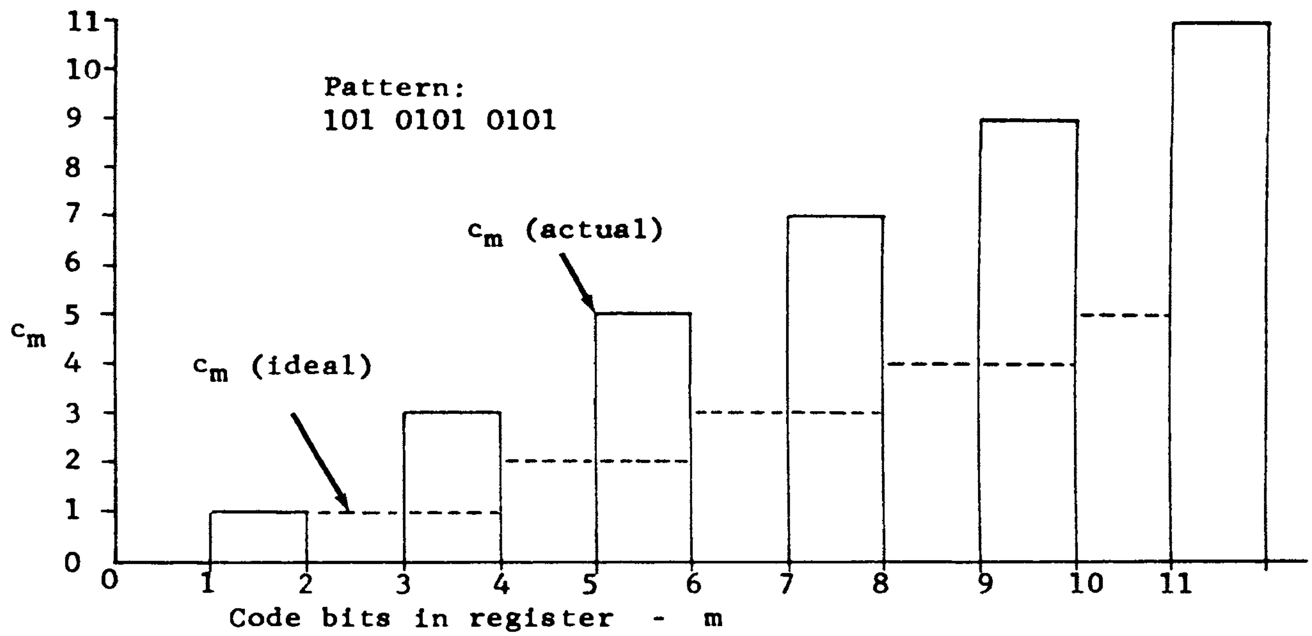
Figures 6a and 6b show comparisons of actual and ideal truncated autocorrelation functions for 11 bit Barker and alternating codes.

The specification for an ideal c_m can also be expressed as:

$$\frac{c_m}{m} \approx \frac{m/2}{m} \approx \frac{1}{2} \tag{9}$$



11 Bit Barker Code
(a)



11 Bit Alternating Code
(b)

TRUNCATED AUTOCORRELATION FUNCTIONS

FIGURE 6

It follows, a code approximating optimality will have a very low value of:

$$\left[\frac{c_m}{m} - \frac{1}{2} \right]^2$$

for any m degree of overlap. Accounting for all degrees of entry from m = 1 to m = n -1:

$$\sum_{m=1}^{n-1} \left[\frac{c_m}{m} - \frac{1}{2} \right]^2$$

and a criteria for determining optimality is available. The least mean squared error (S^2) is now defined:

$$S^2 = \frac{1}{n-1} \sum_{m=1}^{n-1} \left[\frac{c_m}{m} - \frac{1}{2} \right]^2 \tag{10}$$

For any n length code, the pattern yielding the smallest S^2 has the minimum probability of causing a false synchronization indication, and, therefore, is the optimum code.

Referring to Table IV, S^2 values for several codes are compared. Of the two 23 - bit codes, the pseudo-random code is indicated as more desireable, quantitatively corroborating the coarse result previously obtained by graphing c(t). Ideal cyclic autocorrelation functions were found for code lengths of:

$$n = 4q - 1, \qquad q = 1, 2, 3, \cdots 8$$

The set of optimum Goode-Phillips codes is shown in Table V.

TABLE IV

LEAST MEAN SQUARED ERROR (S^2)

Code Length	Pattern	S^2
19	110 0101 1110 0010 0101	0.03349
19*	000 1010 1111 0011 0110	0.02401
23	111 0001 0110 0001 1101 0010	0.03058
23*	110 0110 0101 0000 1111 1010	0.01929
27	110 0010 0100 0011 1011 0100 0101	0.02053
27	000 1100 1001 1111 0001 0101 1010	0.01958
27	101 0101 0101 0101 0101 0101 0101	0.2500

* Pseudo-random code

TABLE V

OPTIMUM GOODE-PHILLIPS CODES

Code Length	Pattern
3	110 *
7	111 0010 *
11	111 0001 0010 *
15	000 1111 0101 1001
19	000 1010 1111 0011 0110
23	110 0110 0101 0000 1111 1010
27	000 1100 1001 1111 0001 0101 1010
31	010 0100 0010 1011 1011 0001 1111 0011

* Barker code

I WILLIARD CODES

In contrast to the methods which used autocorrelation functions as standards and produced optimum patterns of specific length, Williard[5] is able to develop, precisely, an optimum pattern for any code length. Whereas Barker, Codrington and Magnin, and Goode and Phillips compute correlator output to apply their criteria, Williard evaluates a pattern directly. In essence, since the code pattern determines an autocorrelation pattern, Williard asserts the sequence of conflicts, for each degree of overlap, represents the quality of a pattern.

As previously stated, synchronization code length is the only factor affecting the probability of the pattern's random occurrence. For a pattern X bits in length, this probability is $(0.5)^x$. Similarly, an $X + 1$ length pattern has a $(0.5)^{x+1}$ probability-of-occurrence which is, logically, twice as good as the optimum X length code. For an n length series, the optimum code is the one whose pattern is such that sufficient conflicts exist among the overlapping digits, in any degree of the overlap region, to preclude erroneous recognition of a valid code. The instrument employed in developing "sequence-of-conflicts" patterns is the pattern's "relative probability-of-occurrence", P_{mn}. The criteria for selecting an optimum code among patterns so generated is the pattern's "total probability-of-occurrence", P_t.

1. Relative Probability-Of-Occurrence

Among random data or noise bits, the probability-of-occurrence of an n length pattern is $(0.5)^n$. For an n length pattern containing n code bits in the overlap region, the probability-of-occurrence, P(m), of the correct pattern is given by:

$$P(m) = (0.5)^{n-m} (1-H)^l H^p \tag{11}$$

where
- n = number of bits in the code
- m = number of actual code bits in the overlap region
- l = number of overlap code bits which appear correct to the comparator
- p = number of overlap code bits which appear in conflict to the comparator (m = l + p)
- H = random bit error rate on the incoming signal

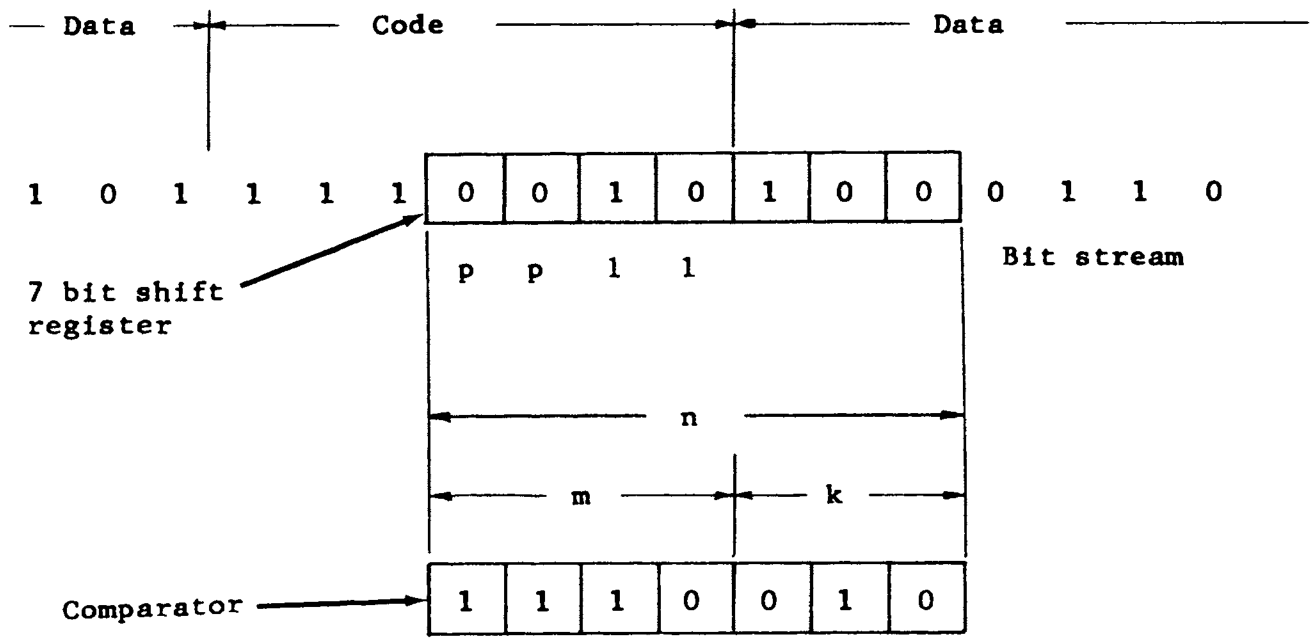
This nomenclature is illustrated in Figure 7.

The relative probability-of-occurrence, P_{mn}, of the correct synchronizing pattern in the overlap region is given by:

$$P_{mn} = \frac{P(m)}{P(n)} = \frac{(0.5)^{n-m} (1-H)^l H^p}{(0.5)^n}$$

$$P_{mn} = 2^m (1-H)^l H^p = 2^m (1-H)^{m-p} H^p \tag{12}$$

P_{mn} is independent of code length; it is a function only of the number of overlapping code bits, the number of these bits in conflict, and the error rate. By definition, a pattern producing sufficient conflicts for every degree of overlap will reduce false synchronization indications, and has a corresponding low P_{mn} value.



OVERLAP REGION

FIGURE 7

Table VI lists P_{mn} values for all combinations of m and p up to m = 7, at an error rate of H = 10%. This table is the tool required for developing optimum codes up to 7 bits in length.

2. Developing a Pattern

To provide any advantage, a synchronizing pattern must produce a P_{mn} less than 1 for all degrees of overlap. From Table VI, p must be at least 1 for m = 1 up to m = 3, and p must be at least 2 for m = 4 thru m = 7.

(a) To meet the requirement that there is one conflict (p = 1) in one degree of overlap (m = 1) it is necessary for the pattern to begin and end with dissimilar bits. Simply:

```
0 · · · · 1
          p
          0 · · · · 1
```

where the dots represent any number of bits inbetween. For this example, n = 6.

(b) For the m = 2 condition:

```
0 · · · · 1
        0 · · · · 1
```

it is seen one conflict is obtained if the second bit is 0, or if the fifth bit is a 1. These two possibilities are represented by:

	0 0 · · X 1	Pattern A
and	0 X · · 1 1	Pattern B

TABLE VI
RELATIVE PROBABILITY-OF-OCCURENCE (P_{mn})

H = 0.1

m Overlapping Code Bits	p Conflicts	P_{mn}
0	0	1.0
1	0	1.8
	1	0.2
2	0	3.24
	1	0.36
	2	0.04
3	0	5.832
	1	0.648
	2	0.072
	3	0.008
4	0	10.4976
	1	1.1664
	2	0.1296
	3	0.0144
	4	0.0016
5	0	18.8955
	1	2.0995
	2	0.2333
	3	0.02592
	4	0.00288
	5	0.00032
6	0	34.0122
	1	3.7791
	2	0.4199
	3	0.04665
	4	0.005184
	5	0.000576
	6	0.000064
7	0	61.2220
	1	6.8024
	2	0.7558
	3	0.08398
	4	0.009331
	5	0.001037
	6	0.0001152
	7	0.0000128

where x denotes the bit state is immaterial. (Since the two derived patterns are mirror complements, only one need be evaluated). Selecting Pattern A, it is seen the required one conflict is assurred:

O O . . X 1
        - p
        O O . . X 1

(c) To determine the third and fourth bits, the $m = 3$ condition is examined.

O O . . X 1
      O O . . X 1

a. Assuming the third bit is O, one conflict is assurred and the fourth digit state is immaterial.
The result is:

O O O X X 1

b. Assuming the third bit is 1, one conflict is assurred if the fourth digit is designated 1. Another pattern is:

O O 1 1 X 1

c. If the fifth digit is O, the fourth digit must be 1 (or the third digit O) and the pattern becomes:

O O . 1 O 1
        p 1 -
        O O . 1 O 1

Table VII contains all acceptable patterns, up to $n = 8$, that were developed in this manner.

3. Total Probability-Of-Occurrence

A figure of merit to evaluate patterns of the same length is Pt, the "total probability-of-occurrence". P_t is the summation of P_{mn} for all degrees of overlap m, viz:

$$P_t = \sum_{m=1}^{n-1} P_{mn} \tag{13}$$

The n bit pattern with the lowest P_t value among all other acceptable n bit patterns shown in Table VII, has the minimum probability of false synchronization and is considered to be optimum. Table VIII is a compilation of codes thusly determined to be optimum by Williard. Exclusive of the simple 3-bit pattern, none of the Barker, Goode and Phillips, or Codrington and Magnin codes are therein contained.

TABLE VII

TOTAL PROBABILITY-OF-OCCURRENCE (P_t)

Code Length	Pattern	P_t
1		0.0
2	01	0.2
3	001*	0.56
4	0011	0.888
	0001	1.208
5	0 0101	1.222
6	00 1011	1.043
	00 1101	1.248
7	000 1011	0.722
	000 1101*	0.832
	001 1101	1.295
8	0001 1011	0.764
	0001 1101	0.895
	0001 0111	0.907
	0000 1101	1.010
	0000 1011	1.064
	0011 1101	1.379
	0011 0101	1.411
	0010 1011	1.464

* Barker Complement

TABLE VIII

OPTIMUM WILLIARD CODES

Code Length	Pattern	P_t
3	001*	0.56
4	0011	0.888
5	0 0101	1.222
6	00 1011	1.043
7	000 1011	0.722
8	0001 1011	0.764
9	0 0010 0111	0.82
10	00 0011 1011	0.70
11	000 1001 0111	0.65
12	0000 0110 1011	0.58
13	0 0000 1101 0111	0.54
14	00 0001 0110 0111	0.55
15	000 0010 1110 0111	0.449
16	0000 0101 1100 1111	0.487
17	0 0001 0101 1011 0111	0.511
18	00 0010 1101 0111 0111	0.405
21	0 0000 1101 1010 1111 0111	0.424
22	00 0001 1011 0101 1110 1111	0.423
23	000 0010 0100 1110 1110 0111	0.381
27	000 0010 0100 1010 1110 1110 0111	0.368
29	0 0000 1001 0010 1110 1110 1110 0111	0.360
31	000 0010 0100 1001 0101 0110 1110 0111	0.361
33	0 0000 1100 1011 0001 0110 1011 0110 1111	0.331

* Barker complement

J. MAURY AND STYLES CODES

After reviewing the literature, which included contributions previously discussed herein, in a search for optimum PCM synchronization codes, Maury and Styles[6] concluded..."that only through the application of an exhaustive technique (i.e., the examination of all binary patterns of a given length against specified criteria) could the truly optimum frame synchronizing codes be established". Like Williard, Maury and Styles also proposed using the pattern itself as the basis for optimum code selection. The comprehensive standard of measure developed was P_{JL}, the probability of a false synchronization occurrence attributed solely to the code pattern.

Maury and Styles reasoned that only in the overlap region is the probability of false synchronization a function of code pattern. In any degree of overlap, each random data bit has a (0.5) probability of agreement with its corresponding pattern recognizer bit; synchronization code bits in agreement with the comparator, for each degree of overlap, are defined by the autocorrelation statement. By combining these values, the probability of a (false) synchronization, P_m, is computed for each degree of overlap. The probability of synchronization over the entire overlap region, P_{JL}, is a summation of all P_m; this probability is a function of a particular pattern arrangement and is the criterion for determining the optimum synchronization code from among all families of a given length n.

In achieving a rigorous analysis, the computation of P_{JL} allows for an error tolerance by the comparator and includes the effects of bit changes due to noise. Maury and Styles' study is considerably more thorough than Williards'. The calculated probability of false synchronization accounts for all combinations of changes in the states of both agreement and disagreement bits in both code and data bits for all degrees of overlap.

P_m has been defined as the probability of a synchronizing indication for an overlap of m degrees. $(1 - P_m)$ is, then, the probability of not having a synchronization for a given m overlap. The probability of not having synchronization during the entire overlap region (from m = 1 to m = n-1) is the product of all $(1 - P_m)$ terms:

$$P_n = \prod_{m=1}^{n-1} (1 - P_m) \tag{14}$$

The probability that synchronization will occur anywhere in the overlap region is $(1 - P_n)$. Thus:

$$P_{JL} = 1 - \prod_{m=1}^{n-1} (1 - P_m) \tag{15}$$

Expanding the terms of Eq. 15:

$$P_{JL} = 1 - \left[(1 - P_1)(1 - P_2)(1 - P_3) \cdots (1 - P_{n-1}) \right]$$

$$= 1 - \left[1 - P_1 - P_2 - P_3 - \cdots P_{n-1} + P_1 P_2 + P_1 P_3 + \cdots P_1 P_{n-1} + \cdots P_{n-2} P_{n-1} - P_1 P_2 P_3 - P_1 P_2 P_4 - \cdots P_1 P_2 P_{n-1} - \cdots P_{n-3} P_{n-2} P_{n-1} + \cdots \right] \tag{16}$$

Since P_m values are quite small, the product terms of Eq. 16 may be deleted and Eq. 15 simplifies to:

$$P_{JL} = 1 - \left[1 - P_1 - P_2 - P_3 - \cdots P_{n-1} \right]$$

$$= \sum_{m=1}^{n-1} P_m \tag{17}$$

1. Nomenclature

Parameters of the overlap region, illustrated in Figure 8, are symbolized:

n = number of bits in the synchronizing code

m = number of code bits in the overlap region

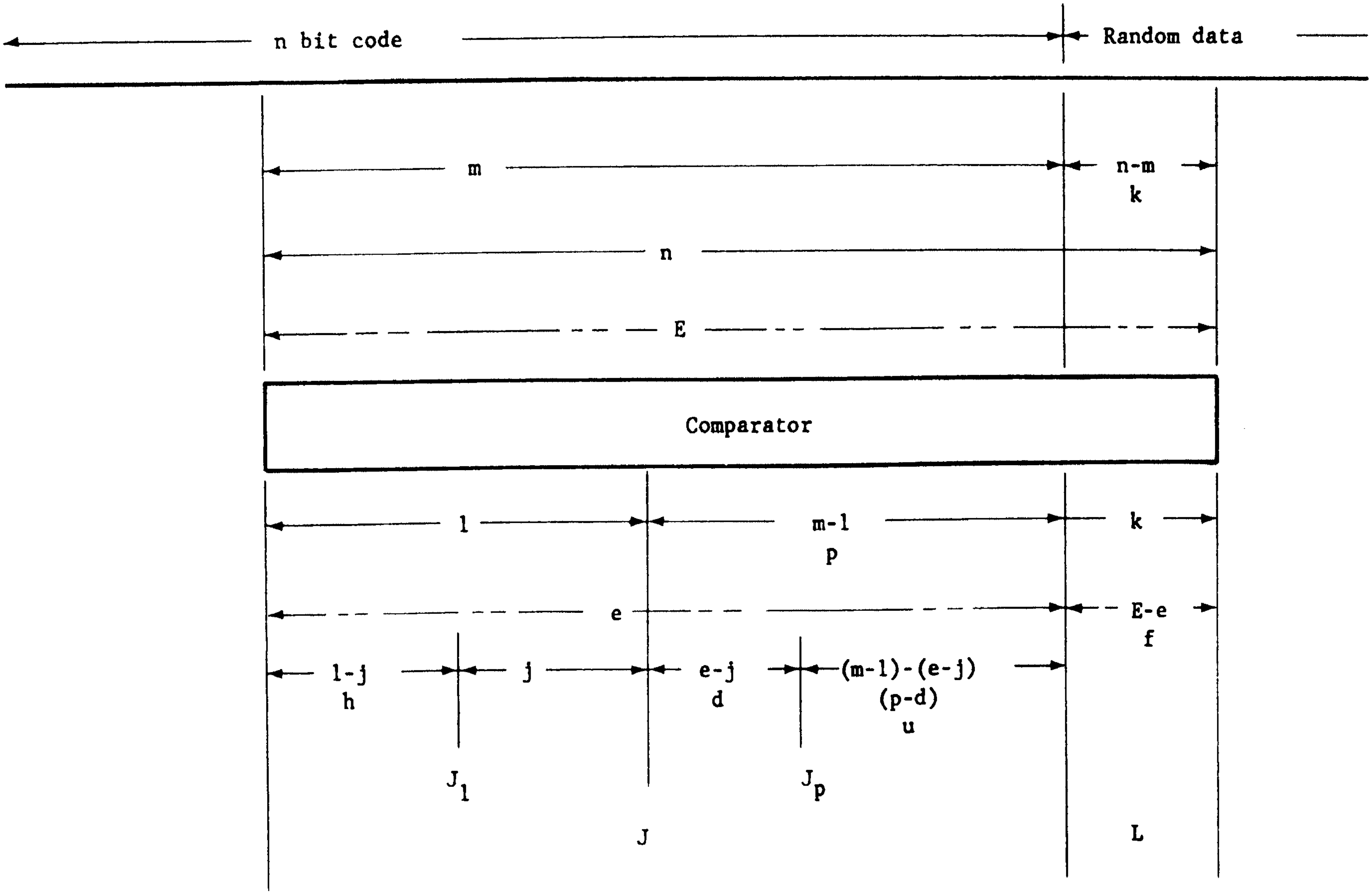
k = number of data bits in the overlap region

= n - m

l = number of overlapping code bits in agreement with corresponding bits of the comparator

p = number of overlapping code bits in disagreement with corresponding bits of the comparator

= m - l



DETAILED OVERLAP REGION PARAMETERS

FIGURE 8

E = number of errors allowed by comparator

e = number of errors allotted to m region ($e \le m$)

f = number of errors allotted to k region ($f \le k$)

= E -e

j = number of agreement bits (1) changed due to noise

d = number of disagreement bits (p) that may remain if j agreement bits are changed

= e -j

h = number of agreement bits (1) that must remain unchanged by noise

= 1 -j

u = number of disagreement bits (p) that must change due to noise

= p -d

J = probability of m overlapping code bits matching corresponding comparator bits

J_1= probability of m agreement bits (1) matching corresponding comparator bits

J_p= probability of m disagreement bits (p) matching corresponding comparator bits

L = probability of k overlapping data bits matching corresponding comparator bits.

2. Evaluation of P_m

The probability of a synchronizing indication, for an overlap of m, can be assumed to be the product of two constituent probabilities: the probability of correctly matching bits in both the code and data overlaps. Then:

$$P_m = J\ L \qquad (18)$$

Synchronization will be indicated if the matching bits have E or less errors. To account for all combinations of error allocation, among k and m bits, the J L products over the range of e = 0 to e = E or e = m (whichever is less) are summed:

$$P_m = \sum_{e=0}^{A} J\ L$$

$$\text{where A represents} \begin{cases} m & \text{if } E > m \\ E & \text{if } E \leq m \end{cases} \qquad (19)$$

3. Evaluation of L

In the data region k, the probability of matching corresponding comparator bits, if zero errors exist, is $(0.5)^k$. The probability of matching comparator bits, if F errors are allowed, must take into account the number of different configurations of F errors distributed among k bits. This probability is expressed as:

$$(0.5)^k \ C_F^k$$

The number of errors may range from $F = 0$ to $F = f$ or $F = k$, whichever is less. The total probability of matching the comparator, allowing for F errors, is the summation of all probabilities within the error range. Therefore:

$$L = (0.5)^k \sum_{F=0}^{B} C_F^k$$

$$\text{where } B \text{ represents } \begin{cases} f & \text{if } f \le k \\ k & \text{if } f > k \end{cases} \qquad (20)$$

4. Evaluation of J

In the code region, consisting of l agreement bits and p disagreement bits, the effects of noise in changing bits in either or both categories, must be considered. If the code bits are to match corresponding comparator bits with j agreement bits changed due to noise, a number of disagreement bits, except for a quantity d, must also be changed. This number is p -d, or u. Expressed as probabilities:

P = probability of a bit being changed due to noise

$1 - P$ = probability of a bit not being changed due to noise

P^j = probability of j agreement bits being changed due to noise

$(1 - P)^h$ = probability of h agreement bits not being changed

$(1 - P)^d$ = probability of d disagreement bits not being changed

P^u = probability of u disagreement bits being changed

The number of ways that j agreement bits can be changed (leaving h bits unchanged) is:

$$C_j^l$$

The probability of j changes in the l agreement bits, while h agreement bits remain unchanged, is:

$$J_l = C_j^l \quad P^j \; (1 - P)^h \tag{21}$$

The number of ways that d disagreement bits can remain unchanged (while the remaining u disagreement bits are changed to agreement) is:

$$C_d^p$$

The probability of u changes among the p disagreement bits, while d disagreement bits remain unchanged, is:

$$Jp = C_d^p \quad P^u \quad (1 - P)^d \tag{22}$$

By definition:

$$J = J_l \; J_p \tag{23}$$

To account for all combinations of bit change apportionment between the l and p bits, the $J_l \; J_p$ products must be summed for every possible value of j. The limits on j are obtained from inspection of Figure 8.

$$0 \le j \le e$$
$$j \le 1$$
$$p \ge d, \qquad \text{or} \quad p \ge e - j$$
$$\therefore \quad j \ge (e - p)$$

Eq. 23 becomes:

$$J = \sum_{j=\min.}^{\max.} J_1 \, J_p$$

$$J = \sum_{N}^{M} J_1 \, J_p$$

$$\text{where } M \text{ represents } \begin{cases} e & \text{if } e \le 1 \\ 1 & \text{if } e > 1 \end{cases}$$

$$\text{and } N \text{ represents } \begin{cases} j = (e - p) & \text{if } (e - p) > 0 \\ j = 0 & \text{if } (e - p) < 0 \end{cases} \tag{24}$$

5. Evaluation of P_{JL}

Substituting Eq. 19 into Eq. 17

$$P_{JL} = \sum_{m=1}^{n-1} P_m = \sum_{m=1}^{n-1} \sum_{e=0}^{A} JL \tag{25}$$

Substituting Eq. 20 and 24 into Eq. 25 and then inserting Eq. 21 along with Eq. 22 into Eq. 25:

$$P_{JL} = \sum_{m=1}^{n-1} \sum_{e=0}^{A} \left[(0.5)^k \sum_{F=0}^{B} C_F^k \right] \sum_{N}^{M} \left[C_j^1 \; P^j \; (1 - P)^h \right] \left[C_d^p \; P^n (1 - P)^d \right] \tag{26}$$

Eq. 26 was programmed for the IBM 7094. Patterns were evaluated with the allowable recognizer error set at $E = 2$ and assuming the probability of a bit change due to noise is $P = 0.10$. Code lengths from $n = 7$ to 30 were evaluated. Even by the most astute programming, computer time for evaluating the 30 bit code was 10.5 hours; longer pattern lengths were not attempted. The optimum Maury - Styles codes, as determined by minimum P_{JL} values, are shown in Table IX. Included are the 7 and 11 bit Barker codes and the 12, 13, and 14 bit Williard codes.

TABLE IX

OPTIMUM MAURY - STYLES CODES

P = 0.10
E = 2

Code Length	Pattern	P_{JL}
7	101 1000*	5.723×10^{-1}
8	1011 1000	4.235×10^{-1}
9	1 0111 0000	2.950×10^{-1}
10	11 0111 0000	1.783×10^{-1}
11	101 1011 1000*	9.065×10^{-2}
12	1101 0110 0000**	5.142×10^{-2}
13	1 1101 0110 0000**	2.821×10^{-2}
14	11 1001 1010 0000**	1.514×10^{-2}
15	111 0110 0101 0000	6.166×10^{-3}
16	1110 1011 1001 0000	3.460×10^{-3}
17	1 1110 0110 1010 0000	1.657×10^{-3}
18	11 1100 1101 0100 0000	8.228×10^{-4}
19	111 1100 1100 1010 0000	3.837×10^{-4}
20	1110 1101 1110 0010 0000	2.175×10^{-4}
21	1 1101 1101 0010 1100 0000	1.051×10^{-4}
22	11 1100 1101 1010 1000 0000	4.906×10^{-5}
23	111 1010 1110 0110 1000 0000	2.533×10^{-5}
24	1111 1010 1111 0011 0010 0000	1.255×10^{-5}
25	1 1111 0010 1101 1100 0100 0000	6.449×10^{-6}
26	11 1110 1001 1010 1100 0100 0000	3.144×10^{-6}
27	111 1101 0110 1001 1001 1000 0000	1.583×10^{-6}
28	1111 0101 1110 0101 1001 1000 0000	8.036×10^{-7}
29	1 1110 1011 1100 1100 1101 0000 0000	4.093×10^{-7}
30	11 1110 1011 1100 1100 1101 0000 0000	2.070×10^{-7}

* Mirror Complement Barker
** Mirror Williard code

CHAPTER III

DEVELOPMENT OF OPTIMUM SYNCHRONIZATION CODES

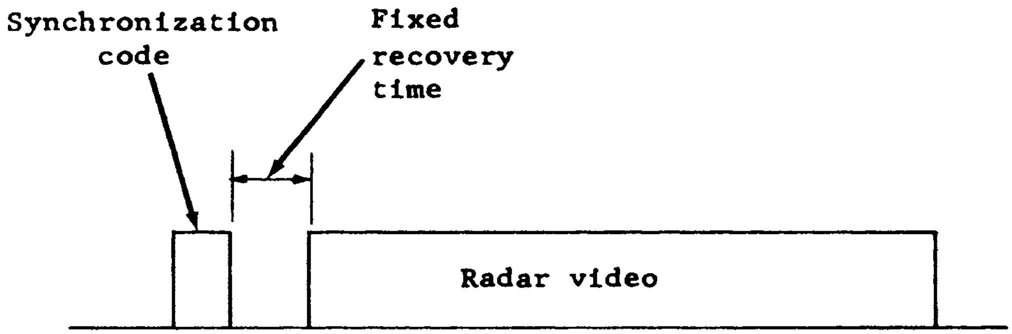
A. SYNCHRONIZATION REQUIREMENTS

The military data communication link employs a binary coded word of ONEs and ZEROs to obtain synchronization. A composite message, transmitted by the aerial reconnaissance radar, is illustrated in Figure 9a. Even though this link is not a PCM telemetry system, the requirements of synchronization are just as severe. Transmission rates are aperiodic, being referenced to aircraft ground speed; every received synchronization code, followed by a single radar message, is independent of any other, and the correlating receiver must establish each reference time on an individual basis; the developed synchronization pulse must be established within a brief interval and with a tolerance of $\pm$ 5 nanoseconds. A timing diagram of the processed transmission is shown in Figure 9b.

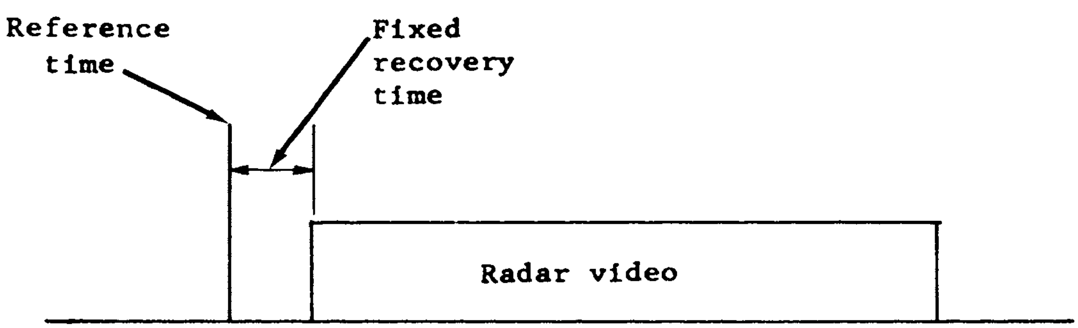
The code presented to the correlator may reflect errors caused by noise in the RF path or introduced during receiving and processing. Because every message is a vital ingredient in reproducing a high resolution composite picture, a maximum error permissivity, within practical limits, is desired. As with PCM telemetry data, an allowable error rate of 10% is acceptable.

B. DECODER MECHANIZATION

The mechanization selected to develop the synchronization pulse is the matched filter shown in Figure 10. Constituent elements are the shift register J-K flip-flops, digital-to-analog converter, slicer,



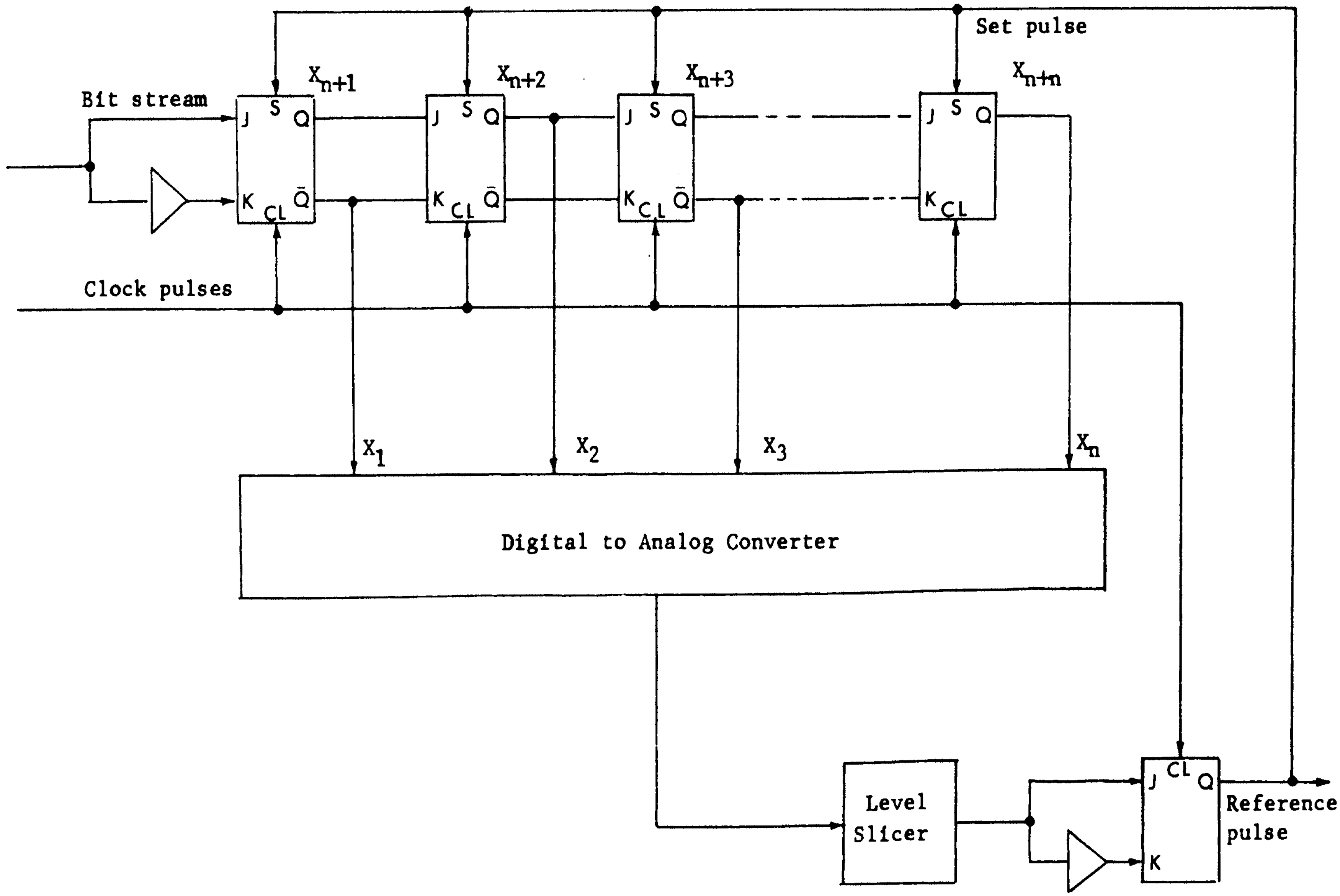
Transmitted Message
(a)



Processed Transmission
(b)

TIMING DIAGRAM

FIGURE 9



DECODER MECHANIZATION

FIGURE 10

and output gate. This design is attractive because it is simple and has a high degree of reliability.

The digital-to-analog converter is an algebraic summer. ONEs are added but ZEROs are subtracted with MINUS ONE weight. Thus, trinary level addition is achieved with binary level signals. Initially, the shift register flip-flops are all "set" (ONE output), or "reset" (ZERO output), to a predetermined state; this initial state should guarantee that the digital-to-analog converter output, under quiescent conditions, is below the slice level. The slicer is adjusted to a level that discriminates against the largest anticipated correlator sidelobe magnitude possible under 10% error conditions.

In operation, the demodulated binary synchronization code is inserted into the shift register J-K flip-flops at the clock rate, displacing the initial shift register pattern in the process. The digital-to-analog converter, in turn, produces a correlated output, which is supplied to the slicer at the clock rate. Under normal conditions, none of these sidelobe terms will be sufficient to pass through the slicer. But, when the correct n length code completely occupies the shift register, a total of n ONES is supplied to the digital-to-analog converter. The summed output passes through the slicer and is applied to an output J-K flip-flop which is enabled by the timing clock. The single output pulse produced is the reference time; this pulse is also used to "set" (or "reset") the shift register flip-flops, eliminating all possibility of a post-synchronization indication.

C. CROSSCORRELATION FUNCTION

In determining an optimum code for the matched filter design of Figure 10, complete information on error effects and overlap regions is available. This knowledge allows a more precise investigation of pattern properties previous to selecting the most unambiguous code. It does not necessarily follow that congruence with any of the existing sets of optimum codes will be found. This expectation is attributed to the criteria, used in evolving said codes, which is not entirely applicable.

The Maury and Styles'criterion is P_{JL}, the "probability of a false synchronization". This factor encompasses detector error tolerance and bit changes due to noise for every degree of overlap. In the overlap regions, the non-code bits are unknown; logically, a random distribution of ONE and ZERO bits was assumed. Williard's "total probability-of-occurrence" figure, P_t, accounts for error rate and a "sequence-of-conflicts" for all degrees of overlap. But, like Maury and Styles, the non-code bits in the overlap regions are taken to be randomly apportioned. Both approaches utilized ambitious computer programs to attach numerical ratings to patterns. In neither study is the correlator's output pattern rendered for evaluation. Yet, it is the autocorrelation expression that contains the desired reference pulse and the accompanying sidelobes that may cause an erroneous synchronization indication.

Goode and Phillips use the truncated autocorrelation function, c_m, in developing the least mean squared error value, S^2. Barker and Codrington and Magnin assay pattern acceptability with the aperiodic autocorrelation function, c_k. Both criterion are only measures of pattern correlation with overlaps of itself. Neither formulation provides for variables such as error rate or detector error tolerance nor is any hint of pattern correlation with known or assumed non-code bits during overlap conditions suggested. The autocorrelation functions for 5 and 7 bit Barker codes are illustrated in Figures 11a and 12a, respectively. Effects of code error on sidelobes and the main term may be included by a modification to Eq. (1):

$$c_k = \sum_{i=1}^{n-|k|} x_{i+Q}\, x_{i+|k|-Q} + 2\, x_a\, x_{a-|k|}$$

$$\text{for } (a-n) < k < (a-1) \qquad (27)$$

where $Q = \dfrac{|k| - k}{k}$

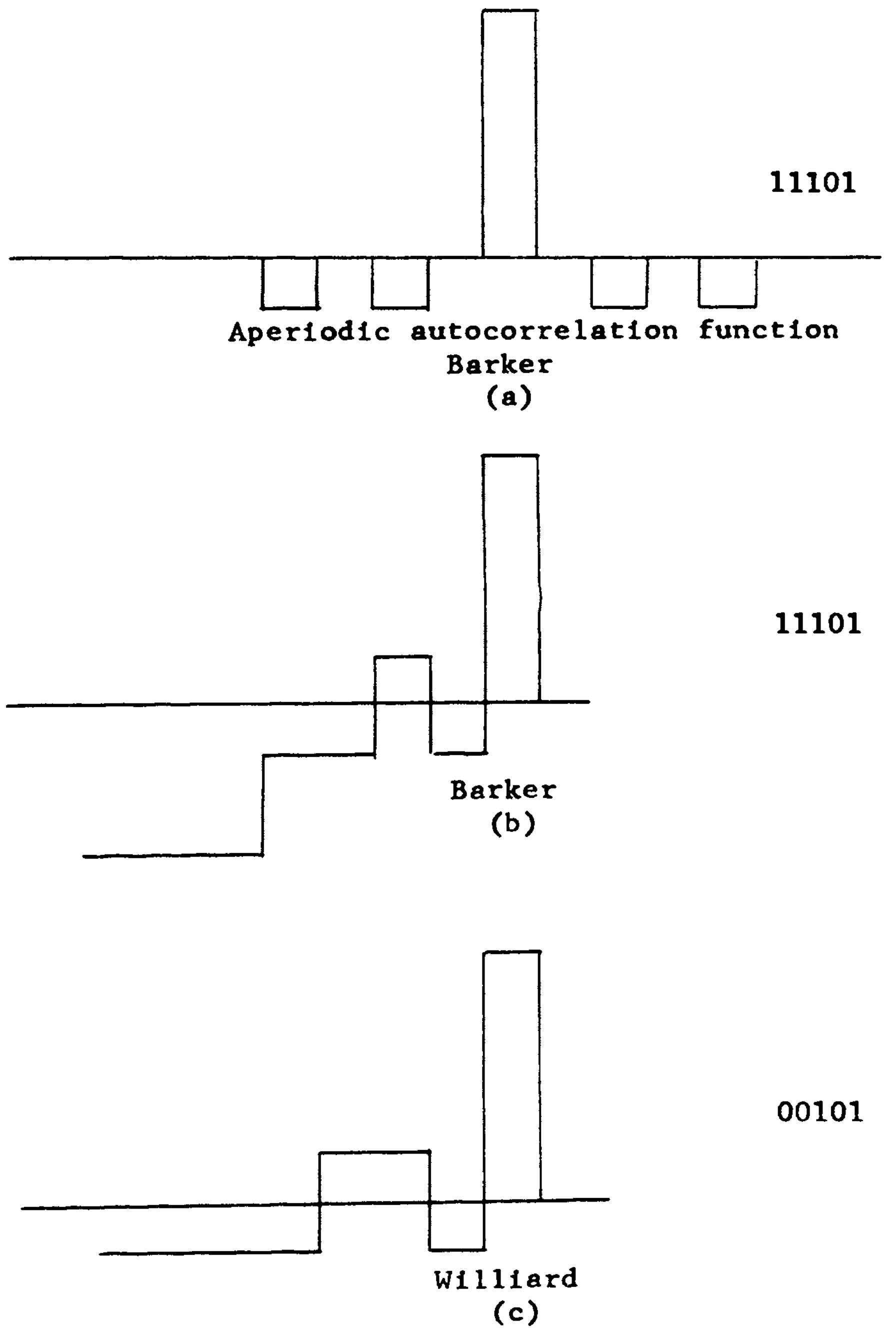
k = degree of aperiodic phase shift

a = number of pattern term in error

X_a = value of term in error.

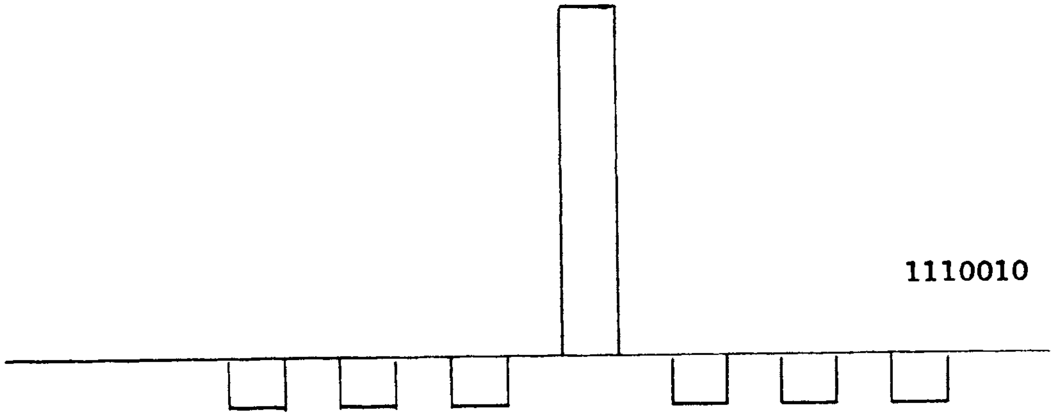
No entry is available for enumerating sidelobe distortions due to actual overlap conditions which is where false synchronizations occur.

In the present study, where precise knowledge of all parameters is accessible, a more quantitative evaluation of synchronization codes than provided by S^2 or c_k yardsticks is possible. Since the shift register is initially "set" (or "reset"), the overlap regions can be exactly defined, unlike the overlap regions postulated by Williard and Maury and Styles. Consequently, an actual crosscorrelation statement of the synchronization pattern may be accurately computed.

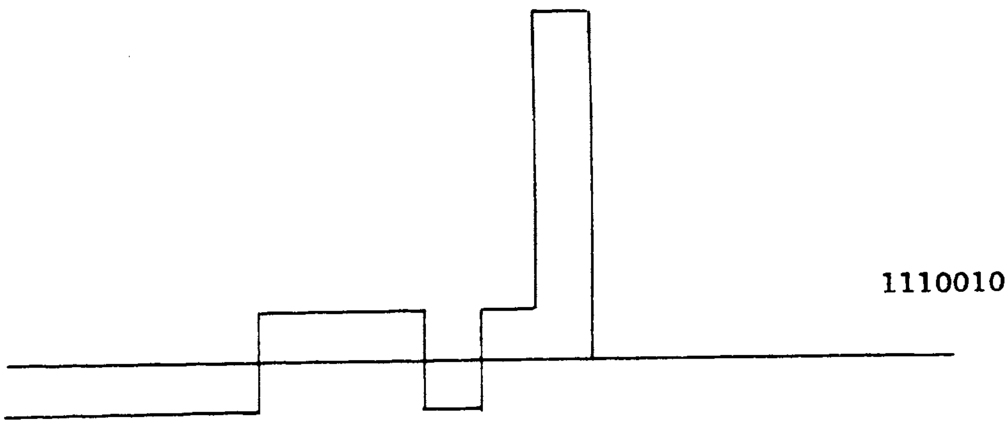


CROSSCORRELATION FUNCTIONS OF 5 BIT CODES

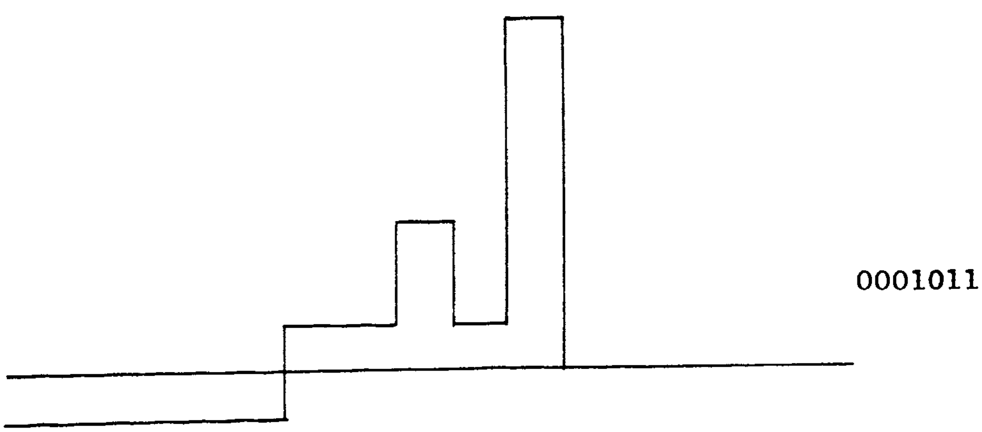
FIGURE 11



Aperiodic autocorrelation function
Barker
(a)



Barker
(b)



Williard
(c)

CROSSCORRELATION FUNCTIONS OF 7 BIT CODES

FIGURE 12

If the predetermined shift register bits are labelled X_{n+1}, X_{n+2}, ... X_{n+n} per Figure 10, the crosscorrelation function is described by:

$$c(k) = \sum_{i=1}^{n} x_i \, x_{i+k}$$

$$k = 0, 1, 2, \cdots (n - 1)$$

$$x_i = 0, 1$$

$$x_{i+k} = \begin{cases} 0, 1 & \text{for } (i + k) \leq n \\ 1 \text{ (or 0)} & \text{for } (i + k) > n \end{cases}$$

Where:

$$1 \times 1 = 1$$
$$0 \times 0 = 1 \qquad (28)$$
$$1 \times 0 = -1$$

An error in the detected code affects the crosscorrelation terms according to:

$$c(k) = \sum_{i=1}^{n} x_i \, x_{i+k} + 2x_a \, x_{a-k}$$

$$\text{for } 0 \leq k < (a - 1) \qquad (29)$$

where a and Xa are as defined in Eq. (27)

The crosscorrelation function, c(k), accurately portrays pattern behavior during the overlap regions and accounts for code errors. It is this test that will be used to grade pattern optimality for the correlator of present interest.

D. CRITERIA OF OPTIMALITY

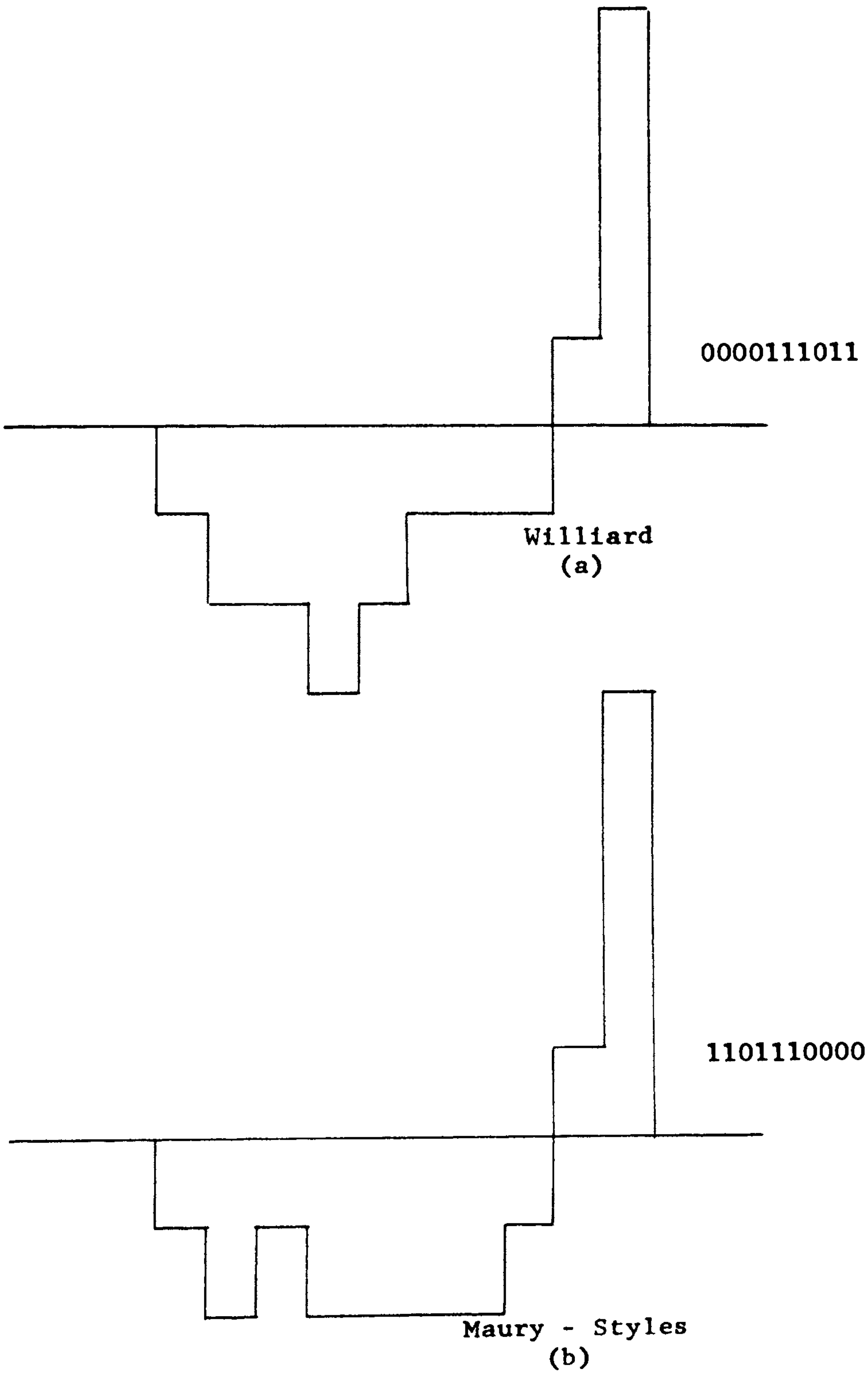
The crosscorrelation function of the 5 bit Barker code is illustrated in Figure 11(b). The positive excursion of sidelobe c(2) represents a false synchronization hazard. A one bit error in the detected code may result in sidelobe c(2)'s having a magnitude as large as the diminished main term c(0); this consequence is precisely what is to be avoided. For the same reason, the 5 bit Williard code, producing the crosscorrelation function shown in Figure 11(c), is also unacceptable.

Crosscorrelation functions of 7 bit Barker and Williard codes are shown in Figures 12(b) and 12(c), respectively. Again, positive sidelobes present a risk. A one bit error in the detected Williard code can result in a premature synchronization; a single Barker code error may create an ambiguity. Where an unambiguous reference time must be established in a noisy environment, these built-in sources of error are to be avoided.

To minimize the possibility of synchronization ambiguities, an acceptable crosscorrelation statement is defined as one whose sidelobe terms are never positive. Specifically:

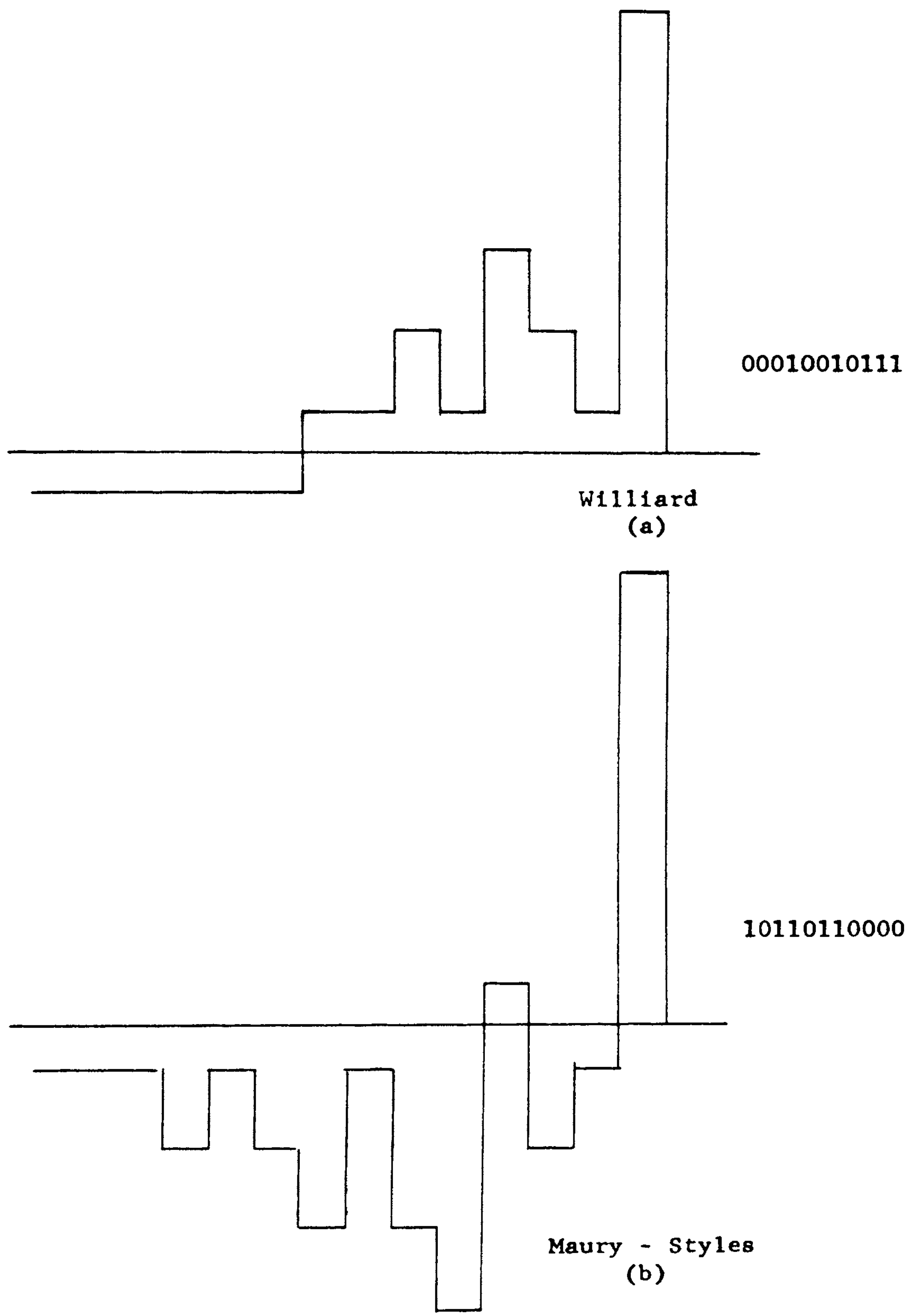
$$c(k) \leq \begin{cases} 0 & \text{for } n \text{ even} \\ -1 & \text{for } n \text{ odd} \end{cases} \qquad (30)$$

This restriction precludes use of the 10 and 11 bit Williard and Maury — Styles codes, whose corresponding crosscorrelation functions are illustrated in Figures 13 and 14.



CROSSCORRELATION FUNCTIONS OF 10 BIT CODES

FIGURE 13



CROSSCORRELATION FUNCTIONS OF 11 BIT CODES

FIGURE 14

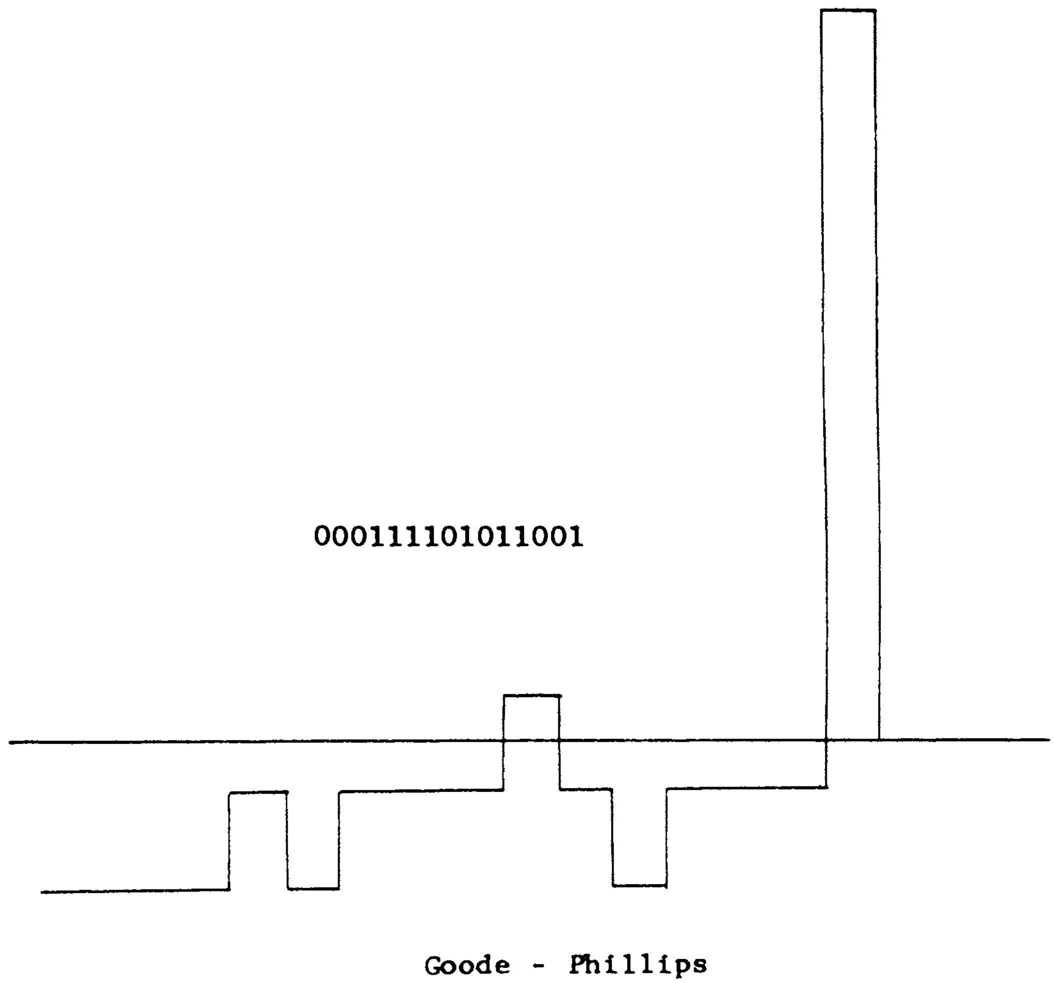
Generally, longer length codes will produce crosscorrelation sidelobes whose amplitudes are of much less prominence than the main term, as illustrated by the 15 bit Goode - Phillips code in Figure 15. Realistically, this code is relatively immune to false synchronization indications in the presence of code errors. Nonetheless, its crosscorrelation statement exceeds the specifications of Eq. 30.

A single error in an otherwise acceptable pattern can cause a sidelobe term to exceed Eq. 30 limits. Since 10% code error is allowed, the maximum adverse effects of errors must be appraised. Accordingly, the crosscorrelation function of the n length code pattern that is the most tolerant of such errors is to be preferred. A simple examination of crosscorrelation functions indicates the optimum code pattern is the one producing the more negative sidelobe values. To arrive at a selection, the crosscorrelation statement of all patterns (except pattern complements) must be computed and compared.

E. COMPUTER PROGRAM

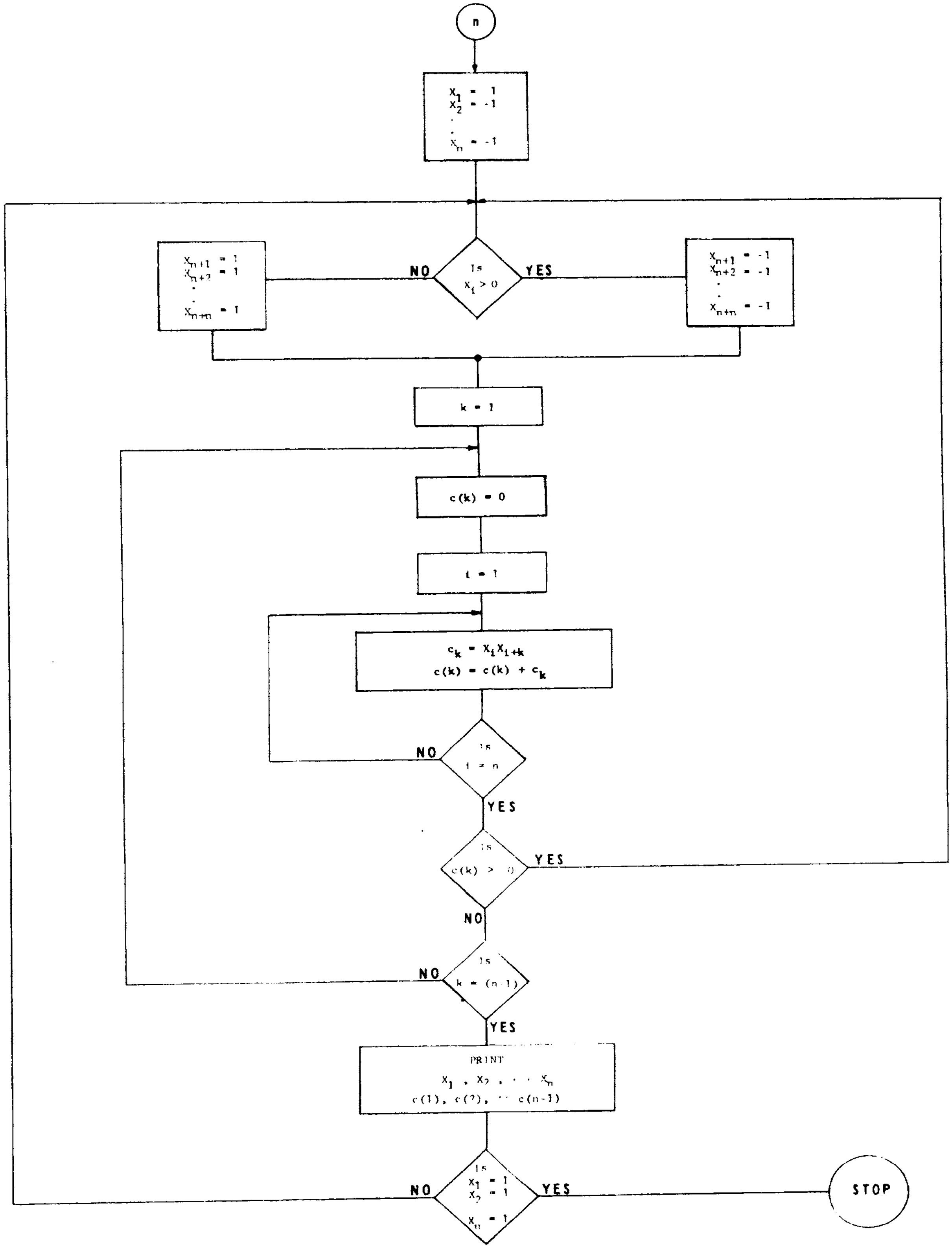
To produce the required crosscorrelation functions from which to select an optimum code, a computer program was written. All permutations of an n length code are examined except the complement. Mirror codes are examined since their crosscorrelation statement is not identical to the basic pattern's statement, as was the case with the symmetrical autocorrelation function.

The program's flow diagram for an even length code is given in Figure 16. For these computations, a binary code ZERO is represented



CROSSCORRELATION FUNCTION OF 15 BIT CODE

FIGURE 15



PROGRAM FLOW DIAGRAM

FIGURE 16

by a -1. The input is n, code length. The code generator starts with $x_1 = 1$ and all other $x_i = -1$. As codes are examined, the generator proceeeds to upcount until all $x_i = 1$; $2^n -1$ permutations are generated in the process. The shift register preset bits, x_{n+1} to x_{n+n}, are computed and set at -1 only if the pattern contains more 1 than -1 terms; otherwise, the shift register starts out with an all 1 sequence.

Crosscorrelation terms c(1) to c(n -1) are generated. If any $c(k) \leq 0$ limit ($c(k) \leq -1$ for n odd) is exceeded, the code is rejected and the next pattern is tested. Acceptable codes and their crosscorrelation functions are printed. From this family of codes, the optimum n length code was selected. The resulting set is compiled in Table X. Crosscorrelation functions for 5, 7, 10, 11, and 15 bit patterns are graphed in Figures 17, 18, 19, and 20.

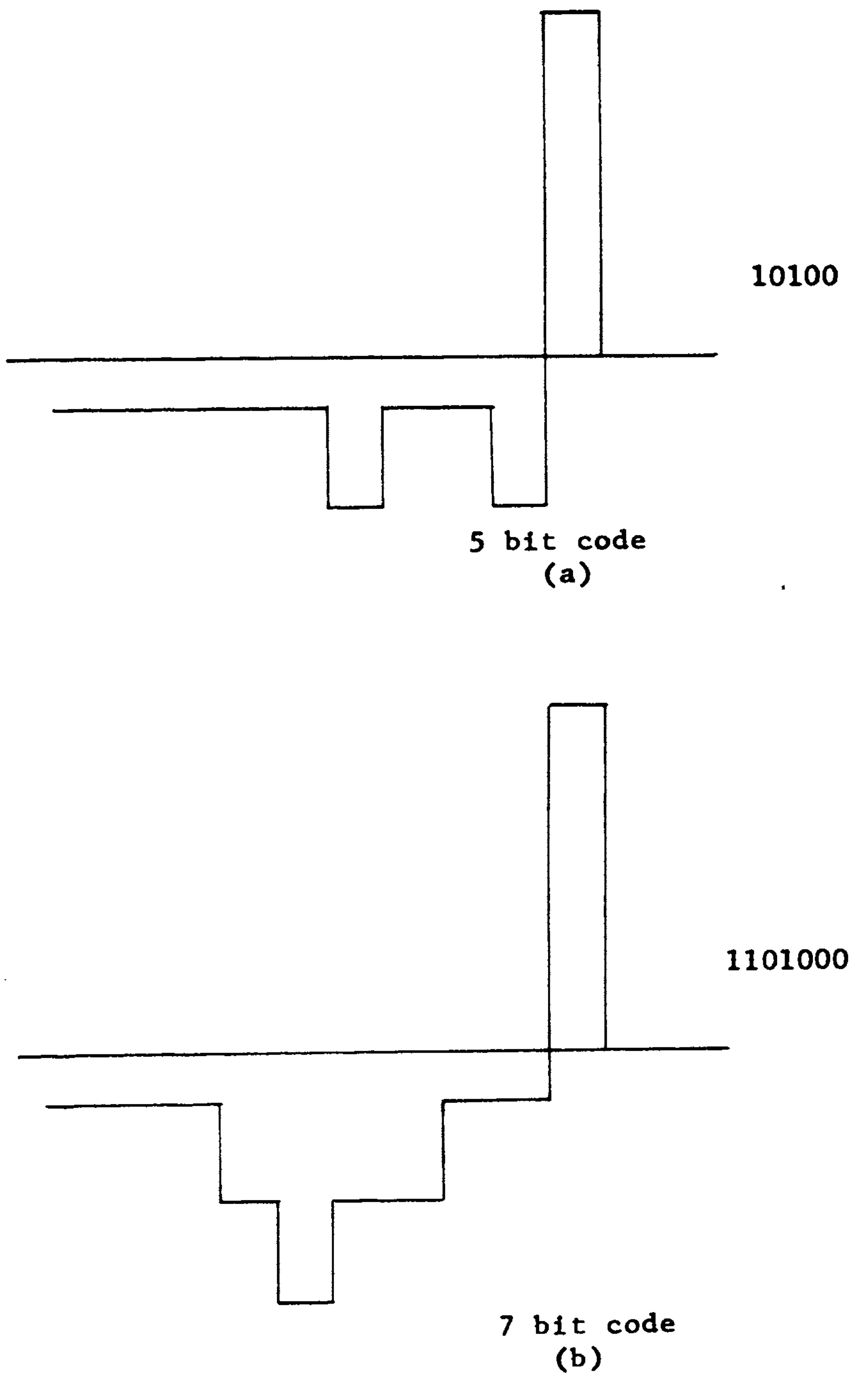
F. CODE LENGTH

For Williard's and Maury and Style's PCM applications, frame synchronization code length is, generally, taken equal to a word length. Barker determined group synchronization code length to be a function of detector error tolerance and a selected value of P(E), the probability of a false synchronization. For applications of the nature herein discussed, the basis for code length is the desired signal-to-noise gain. Having established this value, code length may be determined by reference to Table XI.

TABLE X

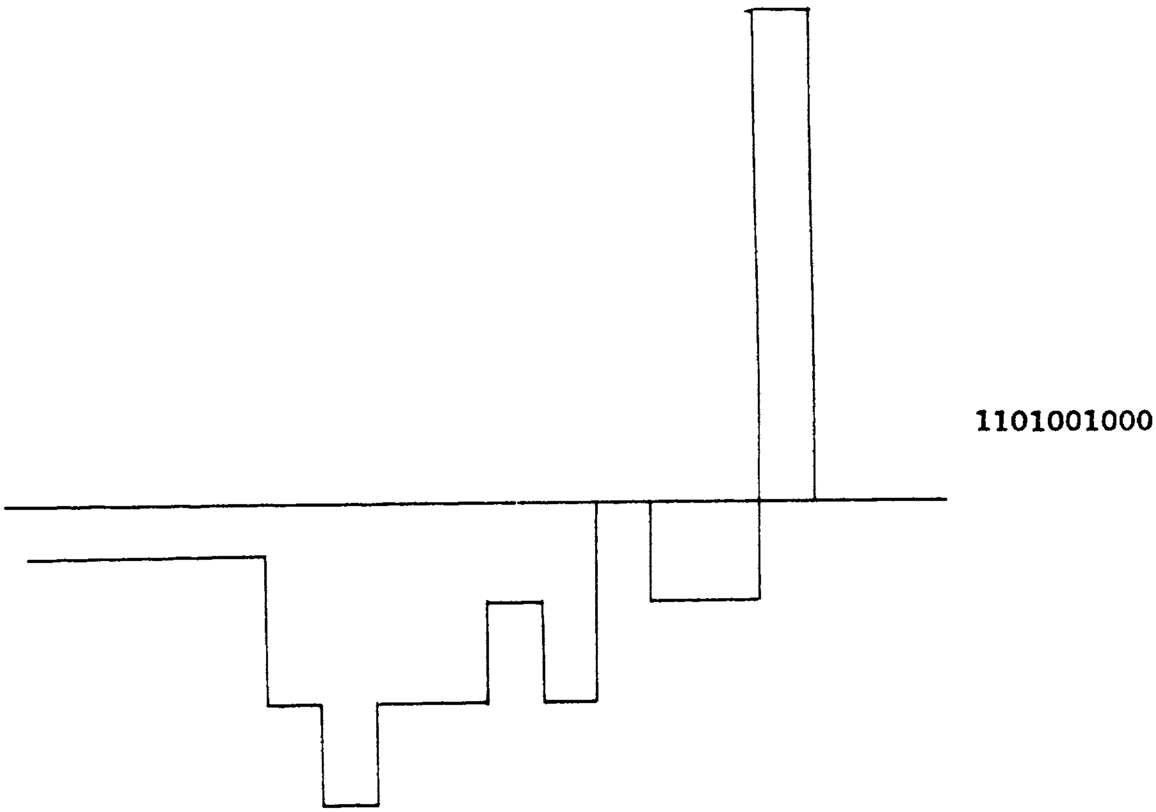
OPTIMUM SYNCHRONIZATION CODES

k	n = 4		n = 5		n = 6		n = 7		n = 8		n = 9		n = 10		n = 11		n = 12		n = 13		n = 14		n = 15	
	Code	c(k)	Code	c(k)	Code	c(k)	Code	c(k)	Code	c(k)	Code	c(k)	Code	c(k)	Code	c(k)	Code	c(k)	Code	c(k)	Code	c(k)	Code	c(k)
0		4		5		6		7		8		9		10		11		12		13		14		15
1	1	0	1	-3	1	-2	1	-1	1	0	1	-3	1	-2	1	-1	1	-2	1	-3	1	-2	1	-1
2	0	-2	0	-1	0	0	1	-1	0	-2	0	-1	1	-2	1	-1	0	-2	0	-1	1	-2	0	-3
3	0	-4	1	-1	1	-4	0	-3	1	-2	1	-1	0	0	0	-3	0	-2	1	-1	0	-4	0	-1
4	0	-2	0	-3	0	-2	1	-3	1	-6	0	-3	1	-4	0	-5	1	-4	0	-3	0	-2	1	-5
5			0	-1	0	-4	0	-5	0	-4	0	-1	0	-2	1	-1	1	-2	1	-3	1	-2	0	-3
6					0	-2	0	-3	0	-2	1	-5	0	-4	0	-3	0	-2	1	-3	1	-6	1	-3
7							0	-1	0	-4	0	-3	1	-4	1	-3	1	-4	0	-9	0	-4	0	-5
8									0	-2	0	-5	0	-6	0	-5	0	-4	0	-3	1	-4	0	-3
9											0	-3	0	-4	0	-7	1	-6	1	-3	0	-2	1	-3
10													0	-2	0	-5	1	-4	0	-5	1	-2	1	-3
11															0	-3	1	-2	0	-3	0	-4	0	-5
12																	1	-4	0	-5	0	-6	0	-3
13																			0	-3	0	-4	0	-5
14																					0	-2	0	-7
15																							0	-5



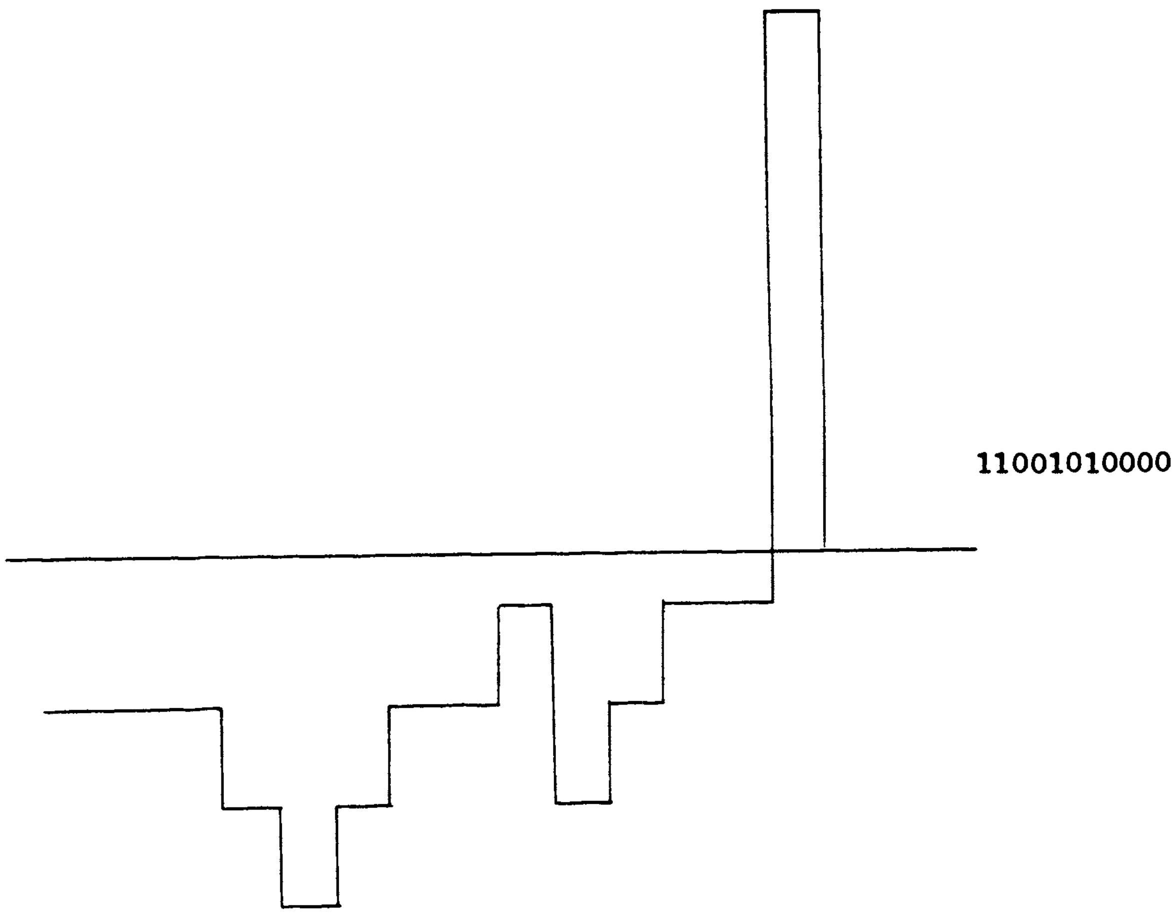
CROSSCORRELATION FUNCTIONS

FIGURE 17



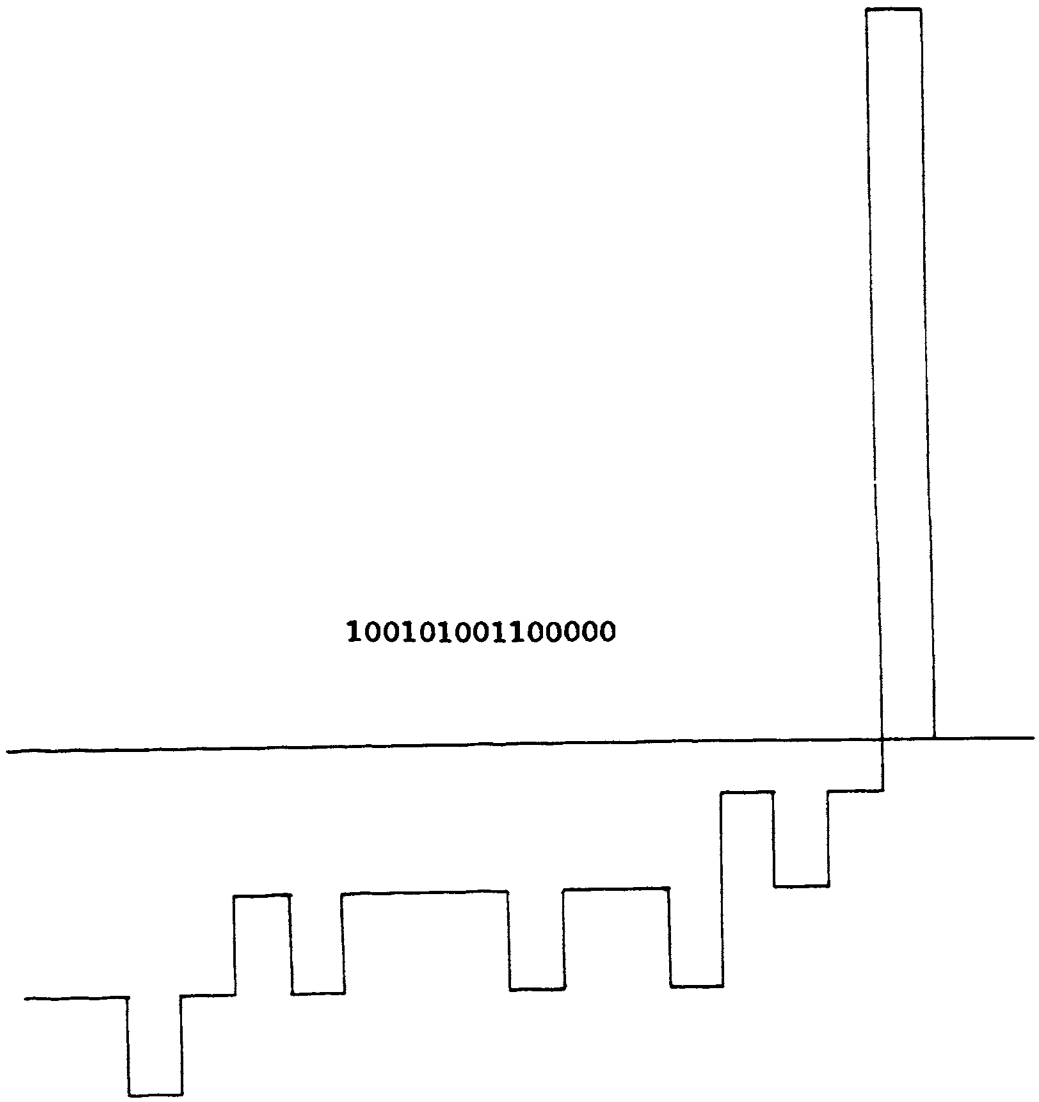
CROSSCORRELATION FUNCTION OF 10 BIT CODE

FIGURE 18



CROSSCORRELATION FUNCTION OF 11 BIT CODE

FIGURE 19



CROSSCORRELATION FUNCTION OF 15 BIT CODE

FIGURE 20

TABLE XI

CODE LENGTH DETERMINATION

n	S/N
4	12
5	14
6	16.6
7	16.9
8	18.1
9	19.1
10	20
11	20.8
12	21.6
13	22.3
14	22.9
16	24.1

CHAPTER IV

CONCLUSION

In this Thesis, the problem of obtaining the optimum synchronization code for a unique application was investigated. The system requiring synchronization and the decoding mechanism were described. Existing sets of optimum codes were surveyed to ascertain their adaptability to the given system. Pattern differences in optimum code sets are due, basically, to the different criteria from which they are derived. None of these criteria were sufficient or satisfactory for the current application. Consequently, some of the established codes proved to be ambiguous to the decoder or intolerant of expected code error. A criterion of code optimality, tailored to system requirements, was stated and a set of conforming codes generated with the use of a computer program.

In the novel system, a binary synchronization code is required to be transformed into a reference pulse with a repeatable accuracy of $\pm$ 5 nanoseconds. The decoding mechanism is a simple, highly reliable matched filter having an error tolerance of 10%. The shift register is pre-set prior to code entry; each synchronizing pulse resets the register to prevent post-synchronization indications. A synchronization pattern that performs dependably with a minimal probability of false indications is desired. Existing sets of optimum codes were tested.

The autocorrelation function pattern, used as criterion by Barker and Codrington and Magnin, measures a pattern's correlation with itself. Effects of code errors and adjacent bits on the decoder output remain unaccounted. This criterion, then, proved too theoretical for application where such influences are known. The synchronization pulse ambiguities and false synchronization hazards, especially in the presence of errors, shown to exist with these codes, supports this contention. Similarly, Goode-Phillips codes, based essentially on the truncated autocorrelation function, were found to be as academic in value for the proposed application.

Williard and Maury and Styles criteria included the effects of non-code bits in the overlap region. In formulating for the general case, these bits were assumed to be of random binary composition. The subsequent codes, when tested in a detector with well defined non-code terms, produced unsatisfactory crosscorrelation functions.

Prior knowledge of the shift register's initial bias permitted a precise definition of the entire overlap spectrum. Correlation output sequences could be stated, for every code pattern examined, with absolute accuracy. As a result, an opportunity was provided for a thorough pattern search and evaluation. The crosscorrelation function was the obvious basis for optimum code selection. The governing criterion designated the pattern whose crosscorrelation function was the most tolerant of allowable errors to be the optimum. This stipulation implies large negative correlator outputs preceding the main pulse are preferred.

A computer program generated patterns and determined shift register initial conditions. Patterns whose crosscorrelation values conformed to the restrictions of Eq. 30 were produced. From among these codes, the most negative crosscorrelation pattern was selected and a set of optimum synchronization codes, for the unique application described, was developed.

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V I T A

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