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## Calibration of weirs by means of critical flow and specific energy

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CALIBRATION OF WEIRS BY MEANS OF CRITICAL FLOW  
AND  
SPECIFIC ENERGY

BY  
ROBERT A. RAPP

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A  
THESIS

submitted to the faculty of the  
SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI  
in partial fulfillment of the work required for the

Degree of  
MASTER OF SCIENCE, CIVIL ENGINEERING

Rolla, Missouri

1950

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Approved by

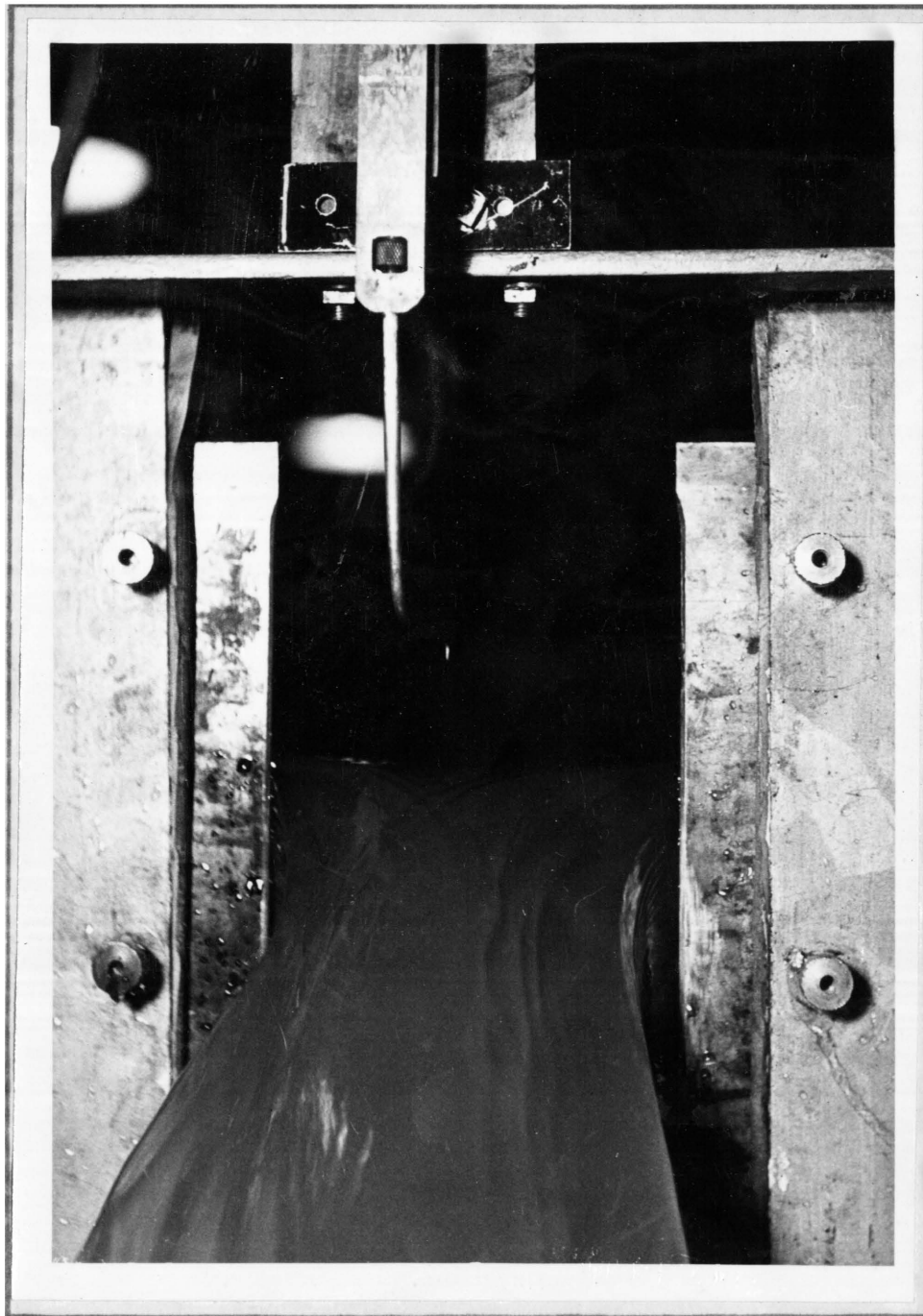


Associate Professor of Civil Engineering

ACKNOWLEDGMENT

The author wishes to express his deepest appreciation to Associate Professor Vernon A. C. Gevecker for his keen interest and very helpful criticism throughout this study. Professor Gevecker instilled within the author a vivid interest for Professor Boris A. Bakhmeteff's textbook "Hydraulics Of Open Channels" which made this study possible.

The author also wishes to express his recognition to Professor Ernest W. Carlton, whose consideration and encouragement in all the writer's research, eased many disgruntled moments.



WEIRS



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## HISTORICAL BACKGROUND

For many years man has been relying more each day upon metering devices to measure the flow of fluids in industry. The orifice, orifice meter, venturi meter, pitot tube, nozzle, flume and the weir have been employed, each having its own particular advantage.

In this parade of metering progress, the contracted weir has long been the forgotten brother of the suppressed weir. In almost every textbook on hydraulics or fluid mechanics, the following words appear, "end contractions are to be avoided where the weir cannot be calibrated".<sup>1</sup> This skepticism and the length or absurdity of some contracted weir formulas led to the development of this study.

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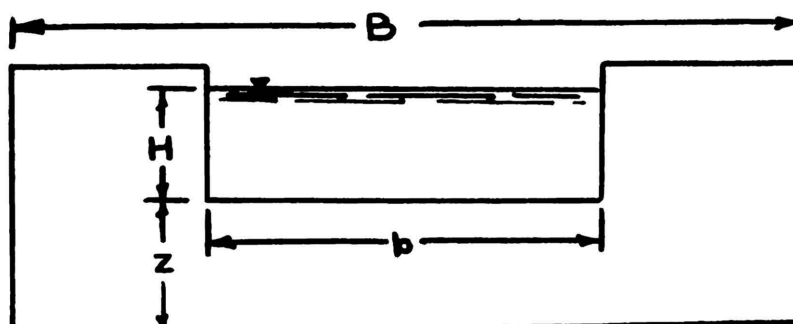
1. Dodge and Thompson, Fluid Mechanics, Mc Graw-Hill, 1937.

"The following equation for flow (cubic feet per second) through a rectangular weir with end contractions appeared in the proceedings of the Swiss Architects' and Engineers' Society in 1924. It has been given in several continental textbooks, so apparently it is considered of value."<sup>2</sup>

$$Q = \frac{2}{3} \sqrt{2g} b H^{3/2} \left[ 0.578 + 0.37 \frac{b^2}{B^2} + \frac{3.615 - 3b^2/B^2}{305H + 1.6} \right] \left[ 1 + 0.5 \frac{b^4 H^2}{B^4 (H+z)^2} \right] \quad (1)$$

---

2. D. S. Ellis, Elements of Hydraulic Engineering,  
D. Van Nostrand, 1947.



From such an equation it becomes apparent that the calibration of a rectangular weir with end contractions has suffered very little thought and energy in the last hundred years. In 1852, James B. Francis published the results of his experiments at Lowell, Massachusetts, in which he proposed the following equation for rectangular weirs with end contractions.<sup>3</sup>

$$Q = C \frac{2}{3} (b - \frac{NH}{10}) \sqrt{2g} H^{3/2} \quad (2)$$

---

3. G. E. Russell, Hydraulics, Henry Holt and Company, 1937.

---

Where  $C$  is the coefficient of contraction,  $b$  is the width of the weir crest,  $H$  is the drawdown head and  $n$  is the number of end contractions. However, the use of the Francis correction leads to an absurdity when the length of the weir becomes small in proportion to the head. For instance, for a weir 0.3 feet long under a head of 1.5 feet,  $L - 0.2H = 0$ , so that  $Q = 0$ , which is evidently not true. The use of the Francis correction is therefore limited to weirs in which  $L$  is at least  $3H$ .

A more precise formula based on tests by Cone<sup>4</sup> for the

- 
4. Cone, V. M., Flow through Weir Notches with thin edges and Full Contraction, Journal of Agriculture Research, U. S. Department of Agriculture, March 1916.
- 

the flow of water through rectangular sharp-crested weirs with complete end and bottom contraction is:

$$Q = 3.247 LH^{1.48} - \left( \frac{0.566 L^{1.8}}{1 + 2L^{1.8}} \right) H^{1.9} \quad (3)$$

Where L is the width of the crest and H is the drawdown head.

Even this formula appears cumbersome when compared to rectangular suppressed weir formulas. Suppressed weir formulas have been thoroughly studied and their results are of a higher precision.

With the publication of "Hydraulics of Open Channels", by Boris A. Bakhmeteff in 1932, a new approach to the solution of all types of open channel flow, of which weir flow is one, was placed at our disposal. Using the basic idea of critical flow and minimum energy, any weir can be calibrated through experimentation to produce a simple formula.

In 1936, Hunter Rouse wrote: "Considerable attention has been given in engineering literature to the measurement of flowing water by means of critical-depth meters of one

form or another..... The shape of the flow profile is directly dependent upon the relationship between a variable discharge and a constant geometrical form of the hydraulic device. Since the surface profile must then vary in shape as the discharge changes, geometrical similarity between any two discharges is impossible and hence complete dynamic similarity is quite out of the question".<sup>5</sup>

---

5. H. Rouse, "Discharge Characteristics of the Free Overfall," Civil Engineering, Vol. 6, No. 4, April 1936.

---

Since it was apparent that water flowing over a sharp-crested weir must pass from the sub-critical range of flow into the super-critical flow range, some relationship must exist between the depth of water above the sharp-crest and the critical depth that would exist in an imaginary open channel having the same dimensions as the weir opening. This study, then, was initiated to determine this relationship.

INTRODUCTION TO THE GENERAL FEATURES OF FLOW

In uniform flow, the work of gravity is entirely consumed in overcoming hydraulic resistance. If the flow is not uniform, then, energy is either stored in or expended from the moving fluid. Therefore, the energy content varies from section to section unless the flow is uniform.

A clear picture of this phenomenon is acquired from the concept of the specific energy flow. In which, the discharge in a channel under varying heads,  $y$ , has an average energy head of:<sup>6</sup>

$$e = Y + \frac{V^2}{2g} = Y + \frac{Q^2}{2ga^2} \quad \text{_____ (4)}$$

---

6. B. A. Bakhmeteff, Hydraulics of Open Channels, McGraw-Hill, 1932.

---

When the flow is referred to a datum line passing through the bottom of the channel section,  $e$  is the specific energy of flow. Where  $Q$  is the discharge in cubic feet per second and  $a$  is the cross-sectional area. The specific energy is a function of the depth of flow,  $y$ , and can be drawn in the form of a curve. This curve is called the specific energy curve.

The potential energy,  $y$ , is represented as a straight line making an angle of 45 degrees with the  $x$ -axis. The kinetic energy of flow is the curve  $k$ , which is asymp-

totic to the x-axis and the y-axis. By the combination of these basic curves, it is possible to produce the specific energy curve. See Figure 1. The specific energy curve is conspicuous for a point of minimum energy which has a corresponding depth known as the critical depth.

CRITICAL DEPTH:

"The particular depth which makes the specific energy a minimum, in other words, the depth under which a certain discharge  $Q$  flows in a given channel with a minimum content of specific energy is called the critical depth."<sup>7</sup> This depth is represented by  $y_{cr}$ .

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7. Bakhmeteff, op. cit., p. 35.

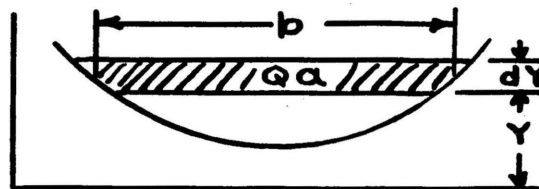
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To determine the value of the critical depth, the specific energy equation can be differentiated and set equal to zero.

$$\frac{de}{dY} = 1 - \frac{Q^2}{ga^3} \cdot \frac{da}{dY} = 0 \quad \text{_____} \quad (5)$$

where:

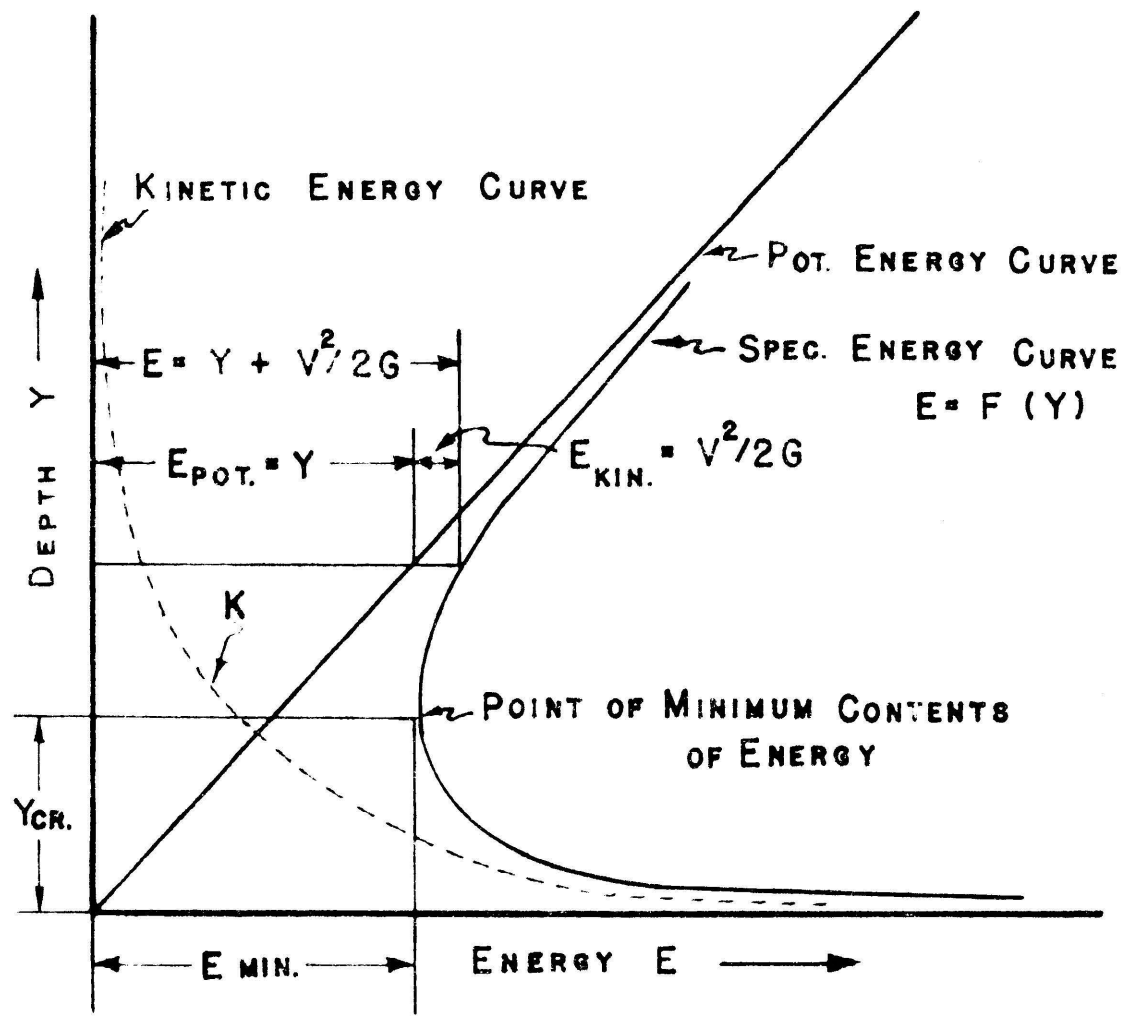
$$\frac{da}{dY} = b \quad \text{_____} \quad (6)$$



then:

$$\frac{de}{dY} = 1 - \frac{Q^2 b}{ga^3} = 0 \quad \text{_____} \quad (7)$$





SPECIFIC ENERGY

FIG. I

Which means the critical depth for a given discharge  $Q$  is the depth  $y_{cr}$  for which the value:<sup>8</sup>

$$\left(a\sqrt{\frac{a}{b}}\right)_{cr} = \frac{Q}{\sqrt{g}} \quad \text{_____} \quad (8)$$

8. Bakhmeteff, op. cit., p. 35.

THE M FUNCTION:

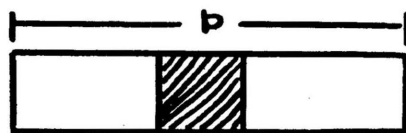
For any given cross-section the value of  $a^3/b$  is a function of the depth only. If we set  $a^3/b$  equal to  $M^2$ , then we can plot a curve that will give us the critical depth for a given discharge  $Q$ .

where:

$$M_{cr} = \frac{Q}{\sqrt{g}} \quad \text{_____} \quad (9)$$

If we consider a rectangular cross-section, the unit discharge or discharge per unit width is:<sup>9</sup>

$$q = \frac{Q}{b} \quad \text{_____} \quad (10)$$



the M function is:

$$M = b\sqrt{y^3} \quad \text{_____} \quad (11)$$

9. Bakhmeteff, op. cit., p. 36.

(See proof on page 11)

and the critical depth is:

$$Y_{CR} = \sqrt[3]{q^2/g} \text{ _____ (12)}$$

### M FUNCTION

A thorn in the side of any calculator of open channel flow has been the long calculations necessary to plot the M Function curve on cross-sectional graph paper. See Fig. 2.

In the lower range of such a curve, it is almost impossible to determine the critical depth with any accuracy. Since the M Function curve is a function of the depth only, the equation of M is always an exponential function and can be plotted on logarithmic graph paper. See Fig. 3.

### M FUNCTION PLOTTING

Taking the basic equation:

$$M = a\sqrt{q/b} \text{ _____ (13)}$$

and substituting for the area, a, and the width, b, the exponential equation can be written in the form of:

$$M = C Y^x \text{ _____ (14)}$$

Where:

C is the M intercept on the line y equal to 1.

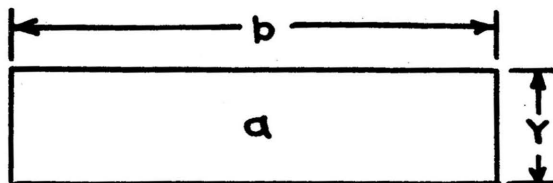
and

x is the slope of the line.

### RECTANGULAR CHANNEL

Where:

$$a = bY \text{ _____ (15)}$$



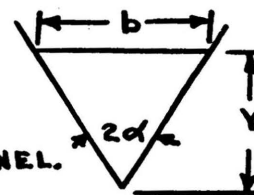
then:

$$M = bY \sqrt{bY/b} = bY^{1.5} \text{ (16)}$$

Therefore, for a rectangular channel the M Function curve can be plotted on logarithmic graph paper with M as the ordinate and y as the abscissa by constructing a line having a slope of 1.5 and passing through M equal to the width b on the line y equal to 1.

#### TRIANGULAR CHANNEL

In a triangular channel having the following dimensions:



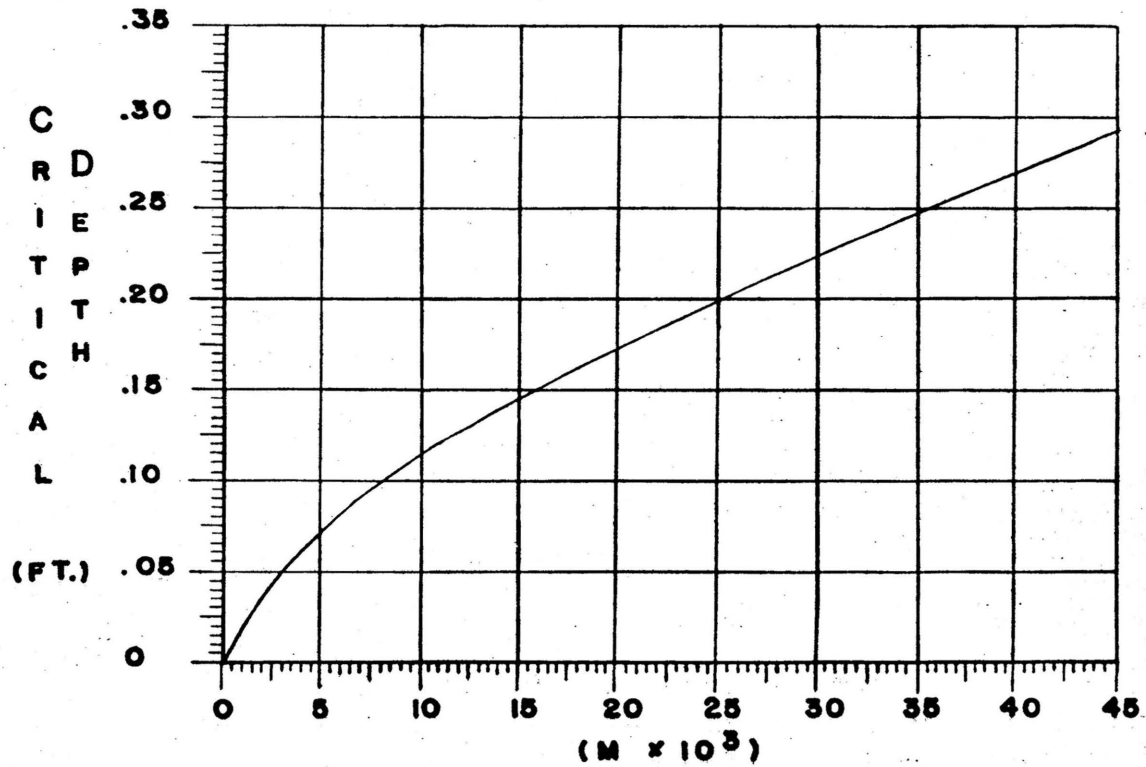
$$\alpha = \frac{1}{2} \text{ THE CENTRAL ANGLE OF THE CHANNEL.}$$

$$b = 2(Y \text{ TAN } \alpha)$$

$$a = \frac{1}{2} bY = \frac{1}{2} Y (2Y \text{ TAN } \alpha)$$

$$a = Y^2 \text{ TAN } \alpha$$

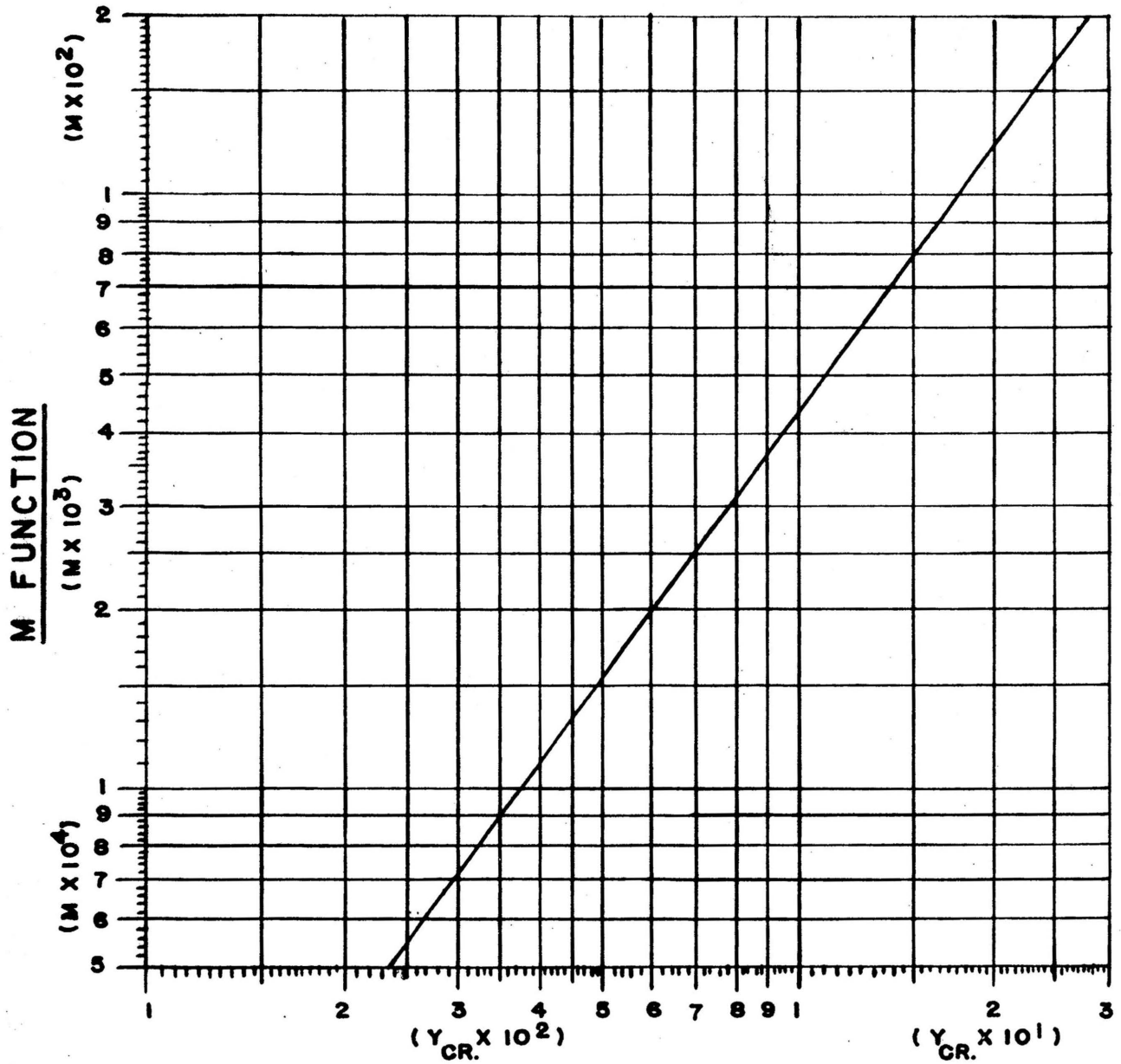
$$M = Y^2 \text{ TAN } \alpha \sqrt{(Y^2 \text{ TAN } \alpha) / 2Y \text{ TAN } \alpha}$$



M

M FUNCTION CURVE

FIG. 2



Y<sub>CR</sub>      CRITICAL DEPTH

M FUNCTION CURVE

FIG. 3

$$M = (\text{TAN } \alpha / \sqrt{2}) Y^{2.5} \text{_____} (17)$$

Similarly, the M Function for a triangular channel can be plotted where 2.5 is the slope and  $\tan \alpha$  divided by the square root of 2 is the intercept.

#### TRAPEZODIAL CHANNEL

The general equation for a trapezodial channel becomes too involved and does not simplify the calculations. However, a basic equation for each particular trapezodial channel is possible so that the logarithmic plot can still be employed with a minimum of calculations.



INTRODUCTION TO THE MAIN STUDY

At the outset of this study, it was doubtful that the critical flow criteria for open channels as set forth by Professor Bakhmeteff<sup>10</sup> would lead to a favorable solution

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10. Bakhmeteff, op. cit.,

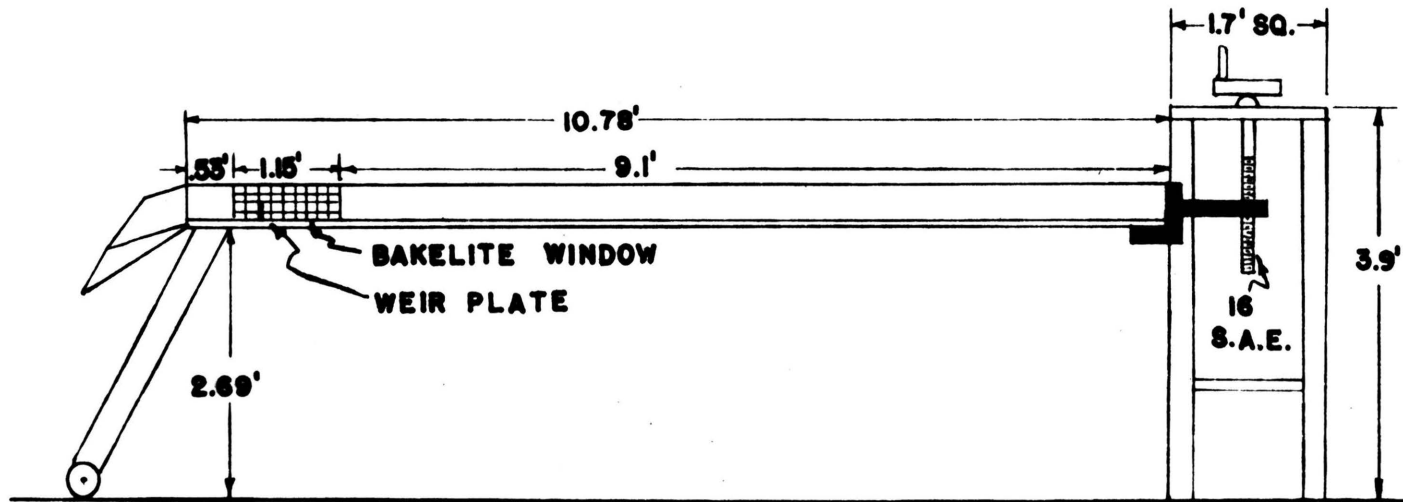
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to flow over a sharp-crested weir because there is no apparent analytical solution to the relationship between the crest depth and the critical depth of an imaginary channel having the same dimensions as the weir opening. Therefore, it was deemed desirable to make a quick preliminary survey and initial tests to determine from those tests the potentialities of the project.

After making a quick survey of the problem, it was decided to use a suppressed weir in a flume having zero slope. See Fig. 4 and 5. In this way, the simplest weir for analysis was used and the zero slope on the flume would reduce the possibilities of increasing the unknowns.

A series of six runs were made as outlined on page 18 and the results were tabulated on page 19. From this series of runs, it was possible to derive the relationship that exists between the crest head,  $y_c$ , and the drawdown head,  $y_d$ , (Fig. 6).

$$Y_c = 0.875 Y_d \text{ ----- (18)}$$



WOOD FLUME WITH WAX COATED BOTTOM

FIG. 4

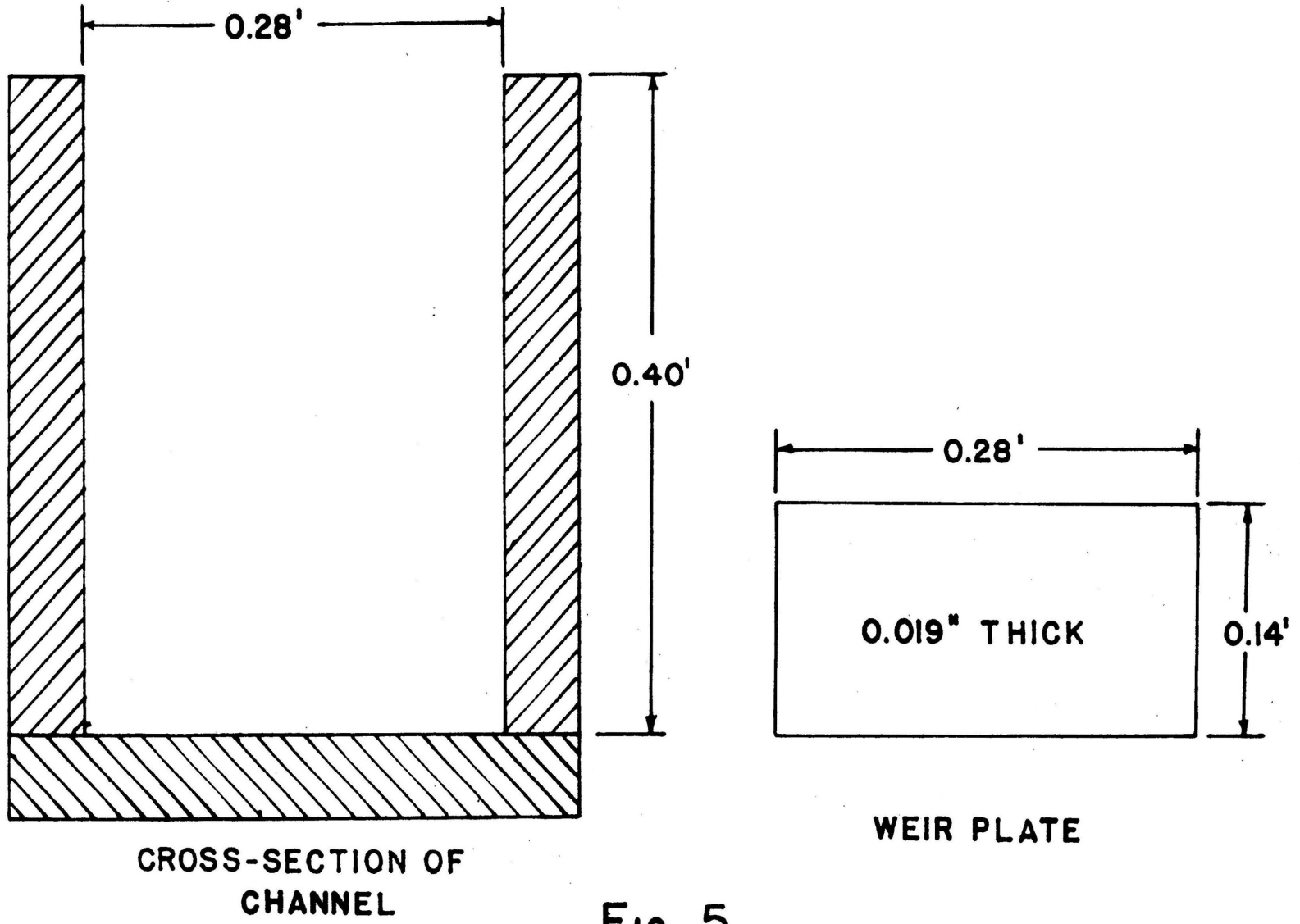


FIG. 5

METHOD OF TESTING

The first runs were made with a zero slope on the flume. The flume was leveled transversely by a small carpenters level. The longitudinal level was checked with a dumpy level. A level rod was placed on the bed of the flume at the discharge end and a reading was made. Then the rod was placed on the bed on the intake end and by means of a screw arrangement the bed was elevated or lowered until the reading at the intake end was the same as the reading at the discharge end. Finally, readings were taken on both ends of the flume to check the level.

Water was allowed to enter the flume. The water was controlled by a valve at the intake end of the flume. It was found that it was impossible to predetermine at what heads the tests would be run because it was found that after opening the valve to the approximate head desired, five (5) minutes would elapse before the head became constant. The crest and drawdown heads were read. Compressed air was bubbled under the nappe because the downstream side of the weir did not provide for the desired ventilation.

EXPERIMENTAL DATADATE: OCTOBER 12, 1949

VENTILATED NAPPE

WATER TEMPERATURE: 24°CSPECIFIC WEIGHT: 62.29FLUME SLOPE: 0

RUN NUMBER	CREST HEAD FT.	QUANTITY POUNDS	TIME SEC	QUANTITY CFS	M FUNCTION	CRITICAL DEPTH FT.	DRAWDOWN HEAD FT.
1	0.084	400	242.2	0.02651	0.004676	0.067	0.097
2	0.083	400	247.6	0.02594	0.0045741	0.066	0.097
3	0.093	400	205.0	0.03132	0.0055245	0.074	0.109
4	0.094	400	213.0	0.030148	0.0053172	0.073	0.109
5	0.117	400	149.4	0.04301	0.0075858	0.092	0.132
6	0.109	400	164.1	0.03913	0.0069016	0.086	0.123

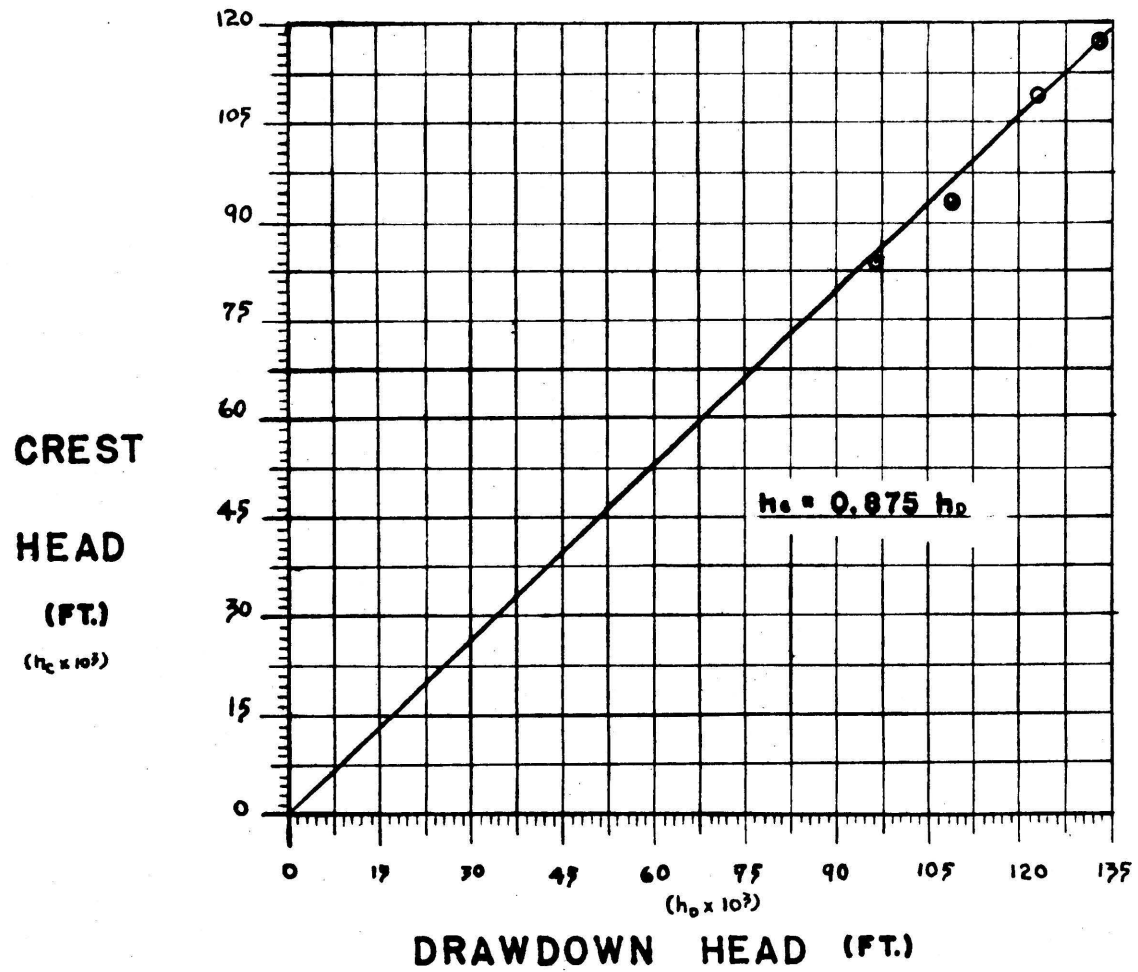


FIG. 6

And by means of an M Function curve, Fig. 2 and 3, the relationship between the critical depth and the crest head was determined. (Fig. 7).

$$Y_c = 1.24 Y_{CR} \text{-----} (19)$$

Combining equation 18 and 19, a third relationship, that of critical depth,  $y_{CR}$ , and drawdown head,  $h_d$ , could be determined.

$$Y_{CR} = 0.705 Y_d \text{-----} (20)$$

Now, assuming that the weir opening is an imaginary open channel and using the equations for unit discharge that Bakhmeteff<sup>11</sup> set forth

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11. Bakhmeteff, op. cit., p. 36.

---

$$q = Q / b \text{-----} (10)$$

$$q = \sqrt{g} Y_{CR}^3 \text{-----} (21)$$

an equation can be derived for total flow by substituting equation 10 into 21.

$$Q = b \sqrt{g} Y_{CR}^{3/2} \text{-----} (22)$$

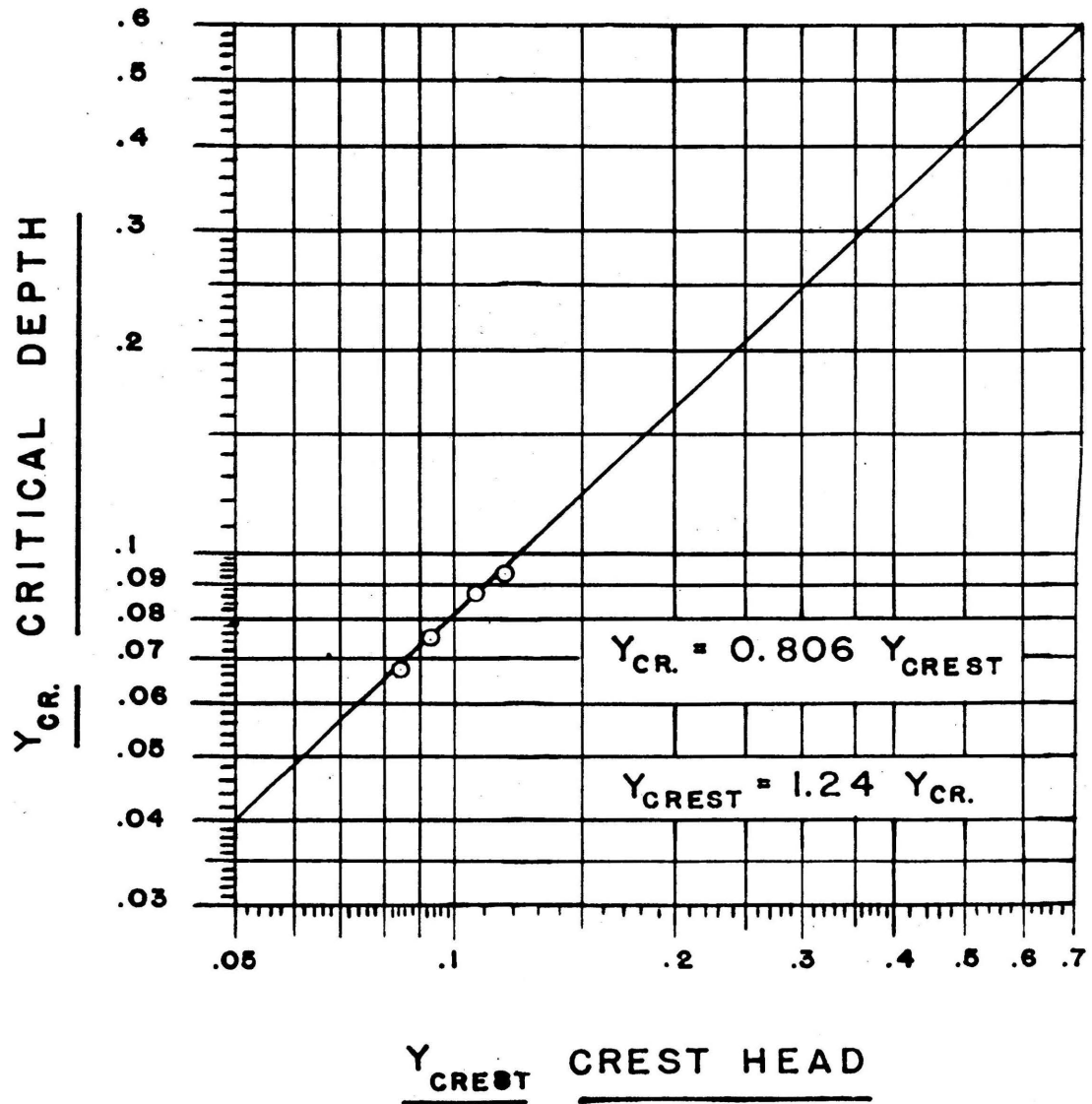


FIG. 7

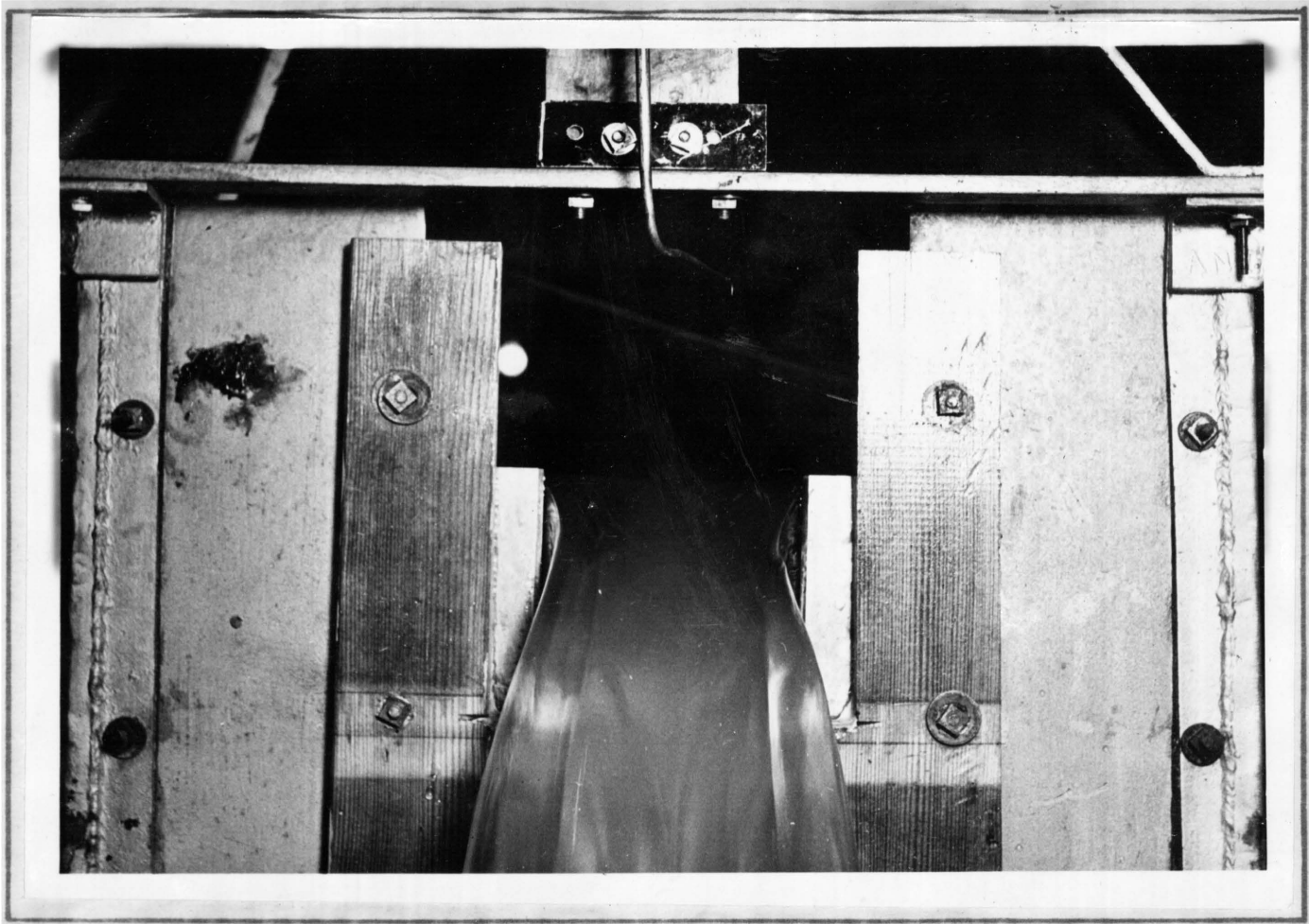


By substituting equation 20 into equation 22 it is possible to derive an equation for flow over a suppressed weir which is familiar to us:

$$Q = 3.36 b Y_d^{3/2} \text{-----} (23)$$

Which you will recognize as being very similiar to the Francis formula.

Having proved that the critical flow criteria would produce a formula that was within 0.1% of the accepted Francis formula the project appeared sound and encouraging.



RECTANGULAR CONTRACTED WEIR

RECTANGULAR WEIR WITH END CONTRACTIONS

After favorable results were obtained from the tests on the suppressed weir, plans were initiated for the work on a series of rectangular contracted weirs of varying widths.

In these tests, four rectangular contracted weirs of varying widths ( 0.2 ft., 0.3 ft., 0.4 ft., and 0.5 ft.) were tested. The weir plates were placed in two different flumes, one of varying slope ( Fig. 8) and one of constant slope,  $s = 0$ . Also, the flumes had different widths. Thus, by the combination of these flumes, the velocity of approach could be introduced or eliminated. See Fig. 9, Fig. 10, Fig. 11, and Fig. 12.

Tests were conducted at slopes of  $s = 0$  and  $s = 0.0346$  and these tests were plotted on Log-Log graph paper. From this graph, a definite relationship was established between the critical depth of flow through an imaginary flume having the dimensions of the weir plate tested and the crest head. However, the crest head is not a constant height across the width of the weir. See Fig. 13 and Fig. 14. On both sides of the nappe, there is a depression. These depressions become more predominate with an increase in discharge. During the tests, all the crest head readings were taken at the center of the crest width. See Fig. 15. The crest depressions fluxuate back and forth across the crest width when the sides of the flume are disturbed.

In a series of seventy tests, the relationship between the critical depth and the crest head was obtained from a Log-Log graph plot. See Fig. 16.

$$Y_{CR} = 0.70 Y_c^{0.94} \text{-----} (24)$$

DERIVATION OF THE FORMULA FOR A RECTANGULAR CONTRACTED WEIR

Using Equation 22:

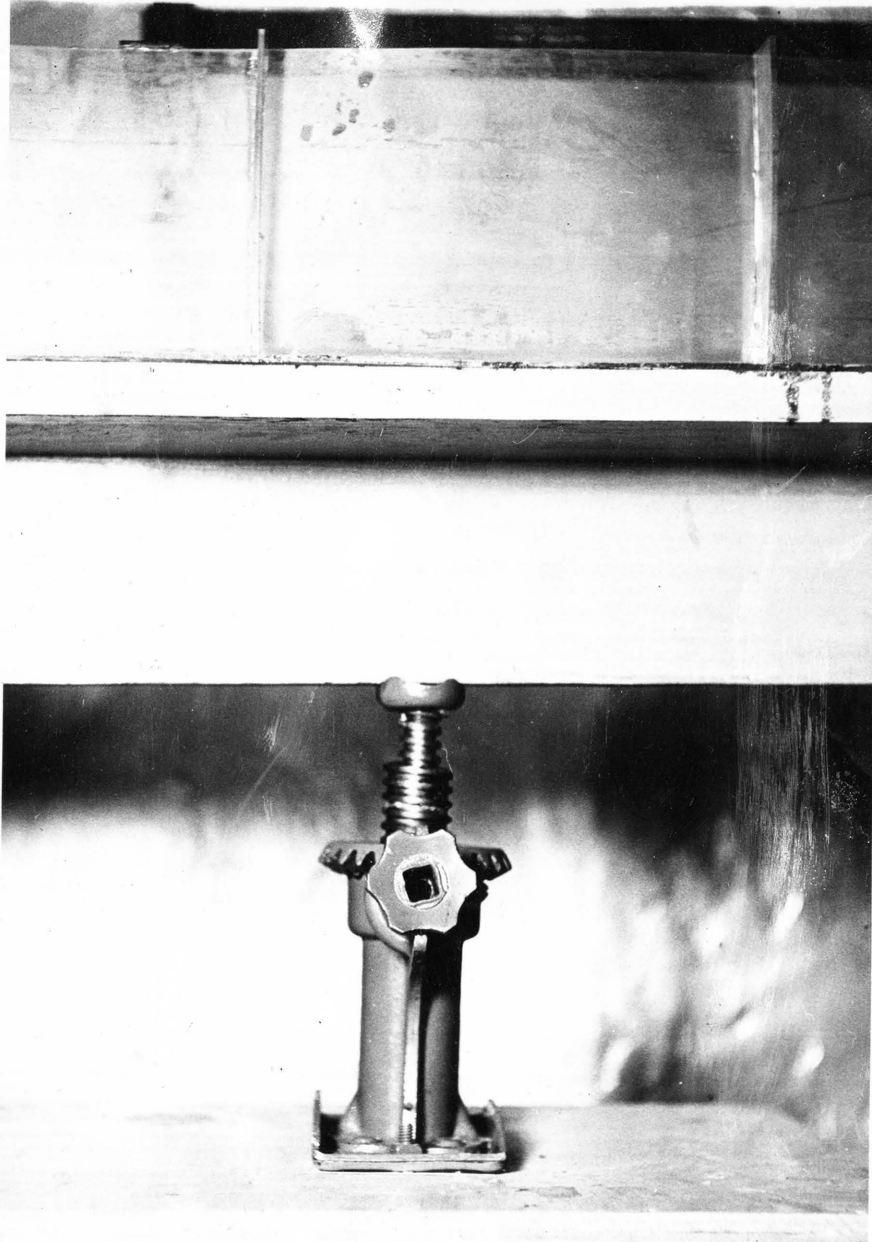
$$Q = b \sqrt{g} Y_{CR}^{1.5} \text{-----} (22)$$

and substituting equation 24 into equation 22 we have:

$$Q = b \sqrt{g} (0.70 Y_c^{0.94})^{1.5}$$

Therefore:

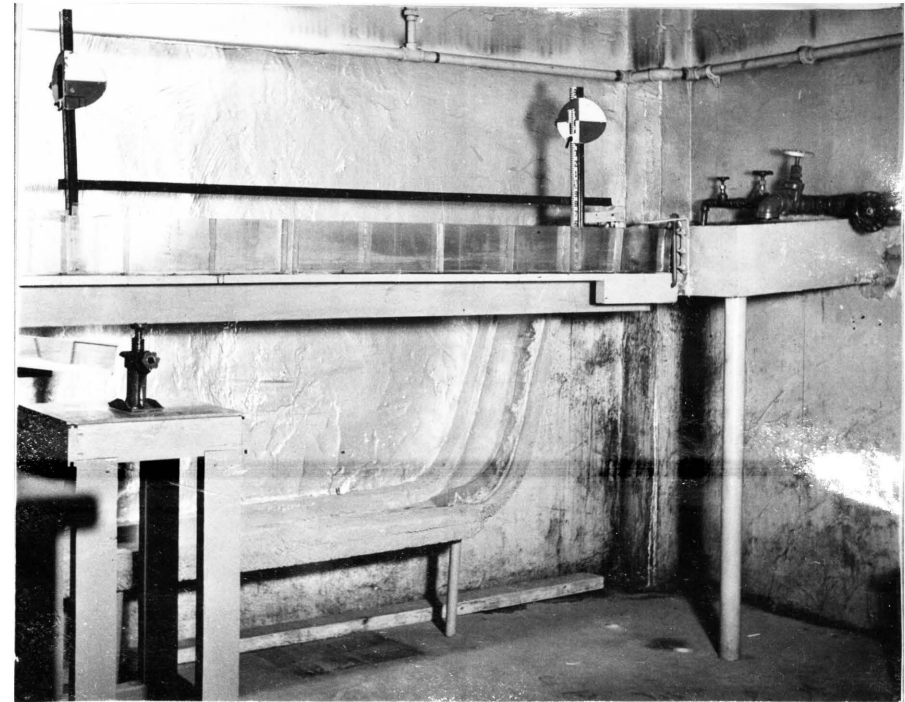
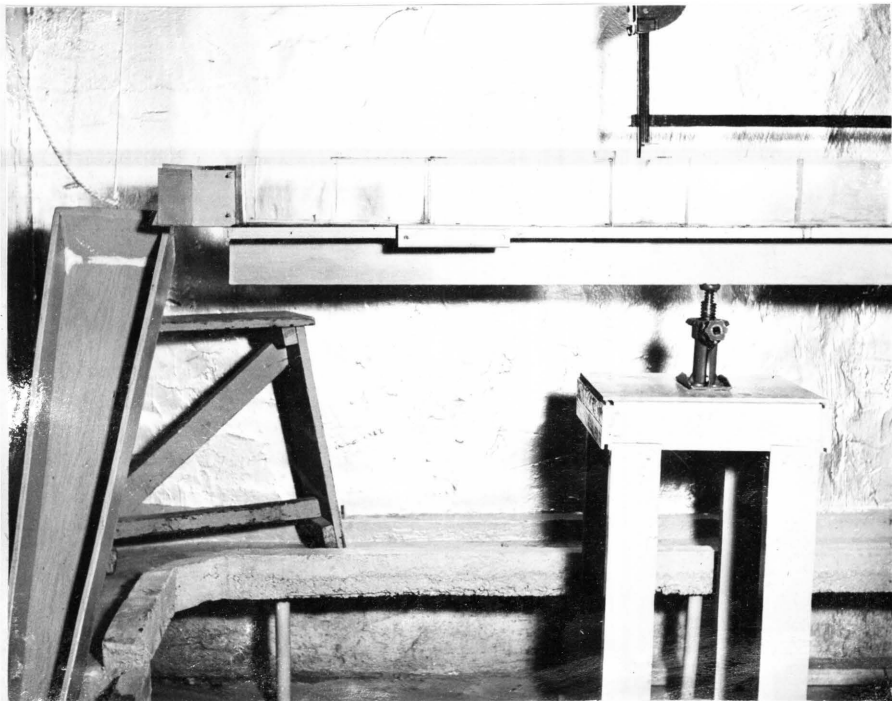
$$Q = 3.32 b Y_c^{1.41} \text{-----} (25)$$



SLOPE CONTROLLING SCREW JACK.

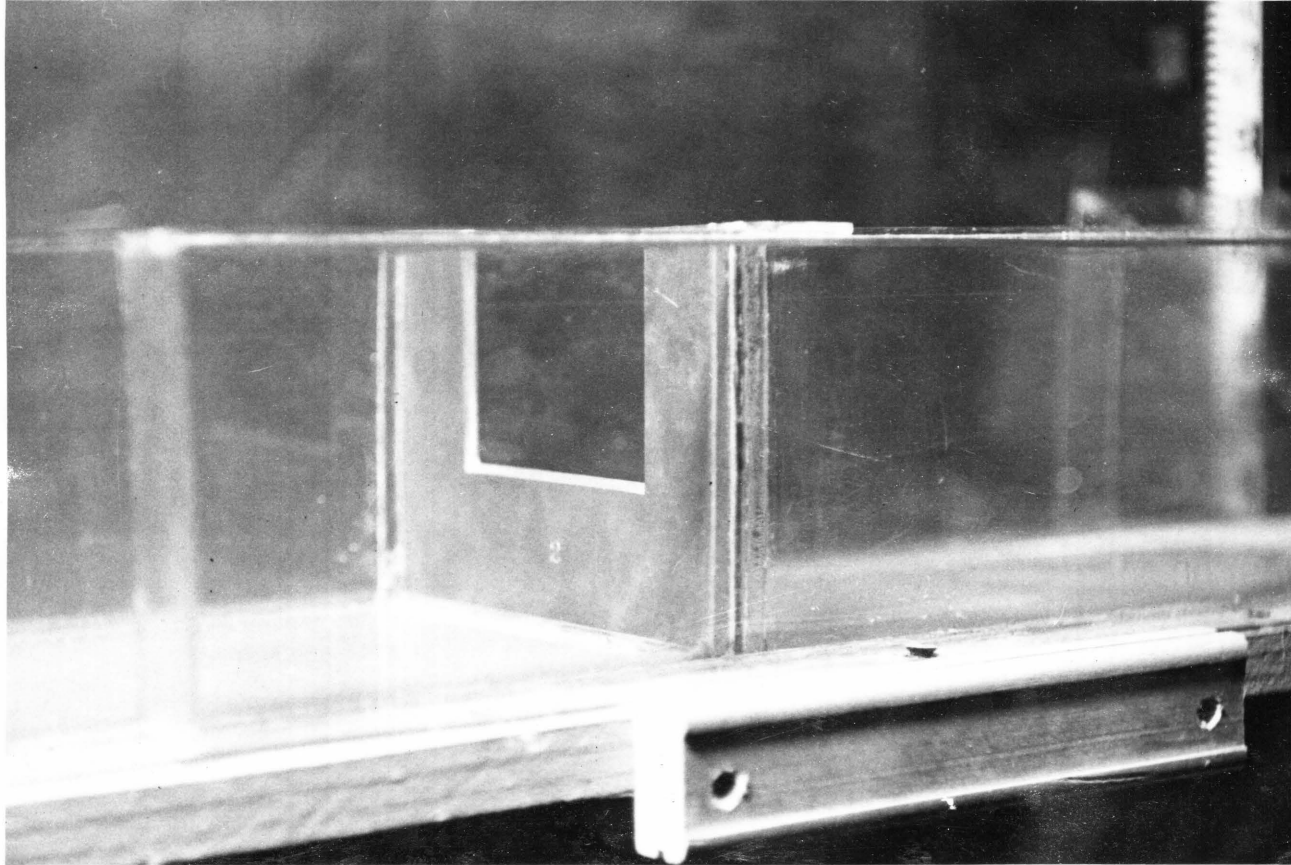
FIG. 8

BELOW. DISCHARGE END OF SMALL  
CELLULOID FLUME. NOTE SLOPE  
CONTROL SCREW JACK AT RIGHT.



ABOVE. INTAKE END OF SMALL  
CELLULOID FLUME. NOTE STILLING  
BASIN AT RIGHT TO WHICH THE  
FLUME IS HINGED.

FIG. 9



WEIR INSTALLATION IN CELLULOID FLUME.

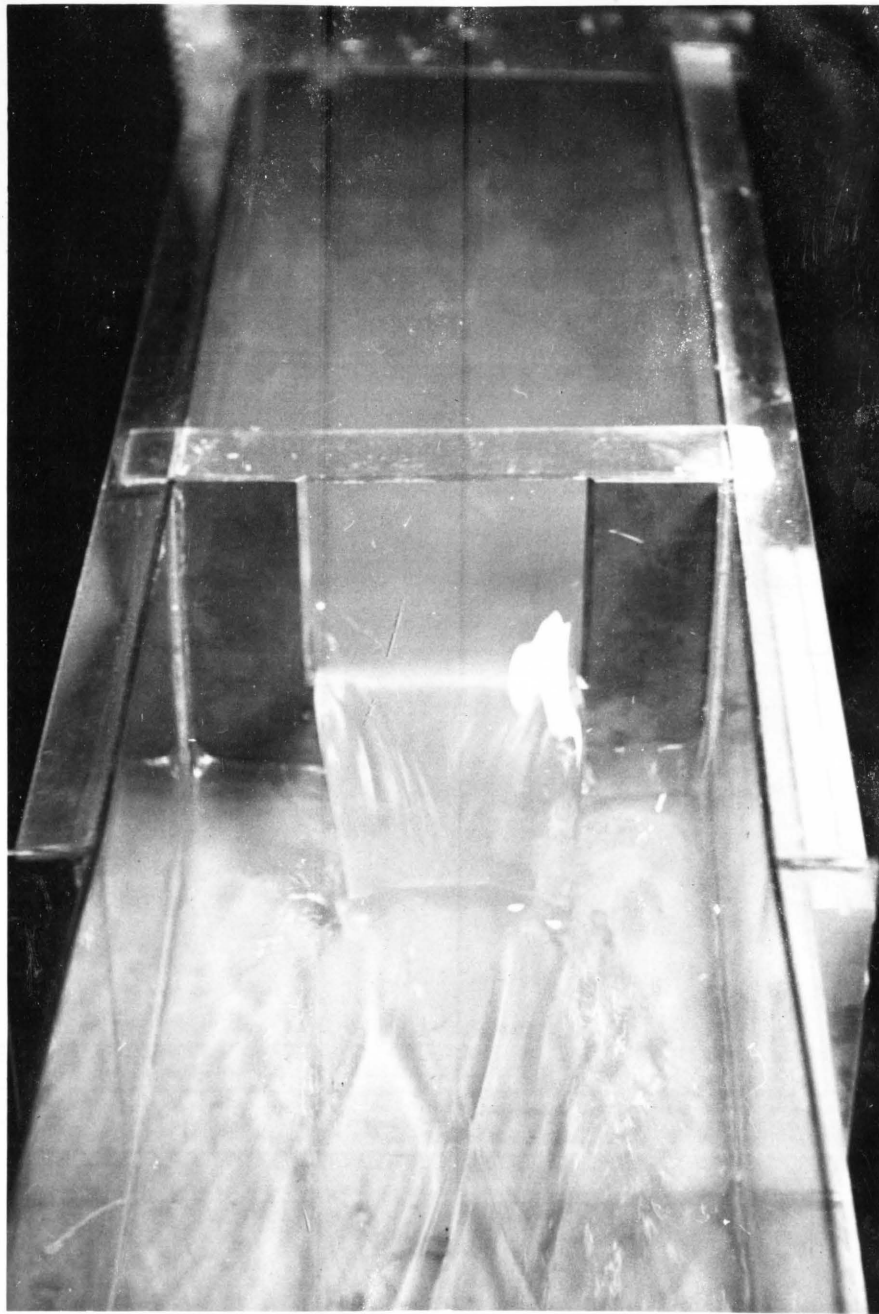
FIG. 10



SIDE VIEW OF WEIR NUMBER 2 DISCHARGING.

FIG. II

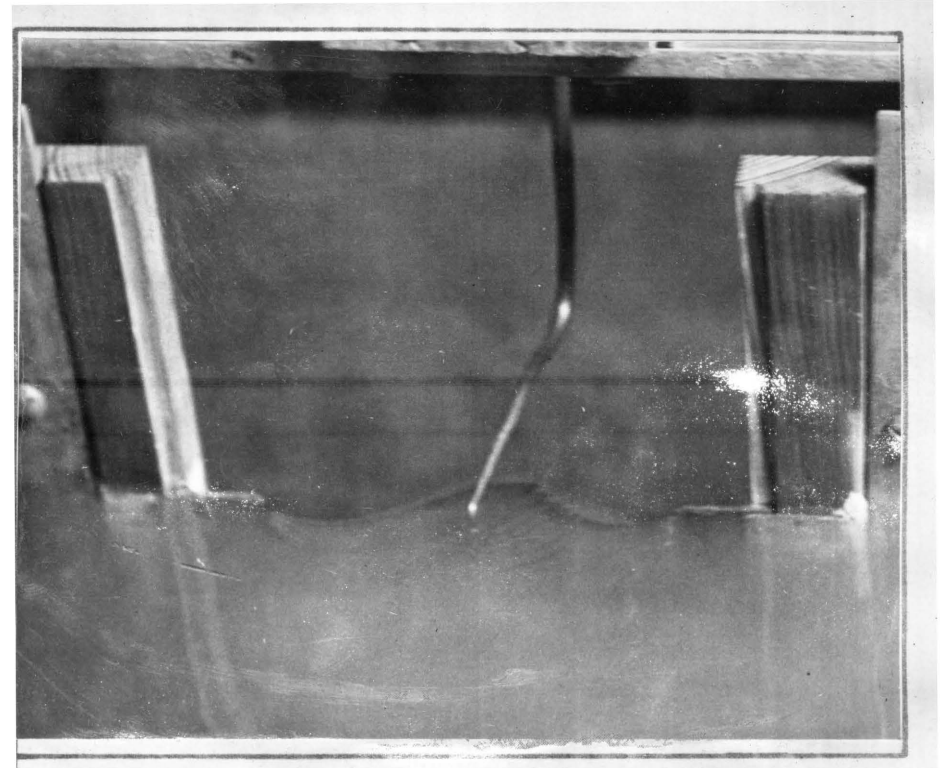
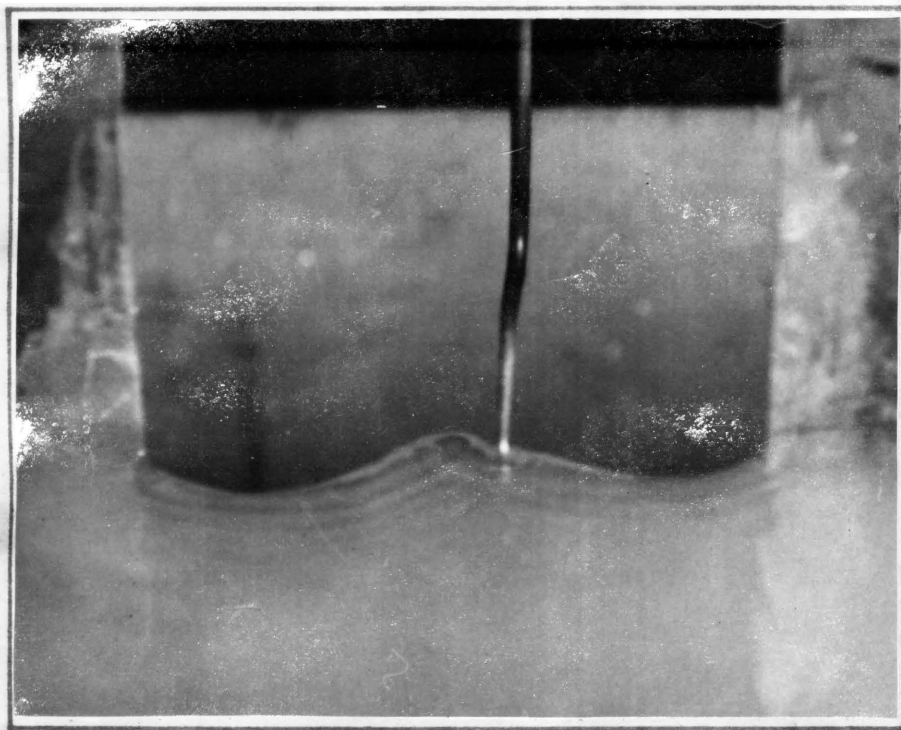




LOOKING DOWN ON WEIR NUMBER 2 IN THE  
CELLULOID FLUME.

FIG. 12.

BELOW. VIEW OF NAPPE DEPRESSIONS.  
DEPRESSIONS ARE MOVING TOWARD  
THE LEFT.



ABOVE. VIEW OF NAPPE SHOWING  
WAVES CAUSED BY MEASURING GAGE.

FIG. 13



SIDE VIEW OF UNCALIBRATED WEIR.

FIG. 14



VIEW OF MEASURING CREST HEAD.

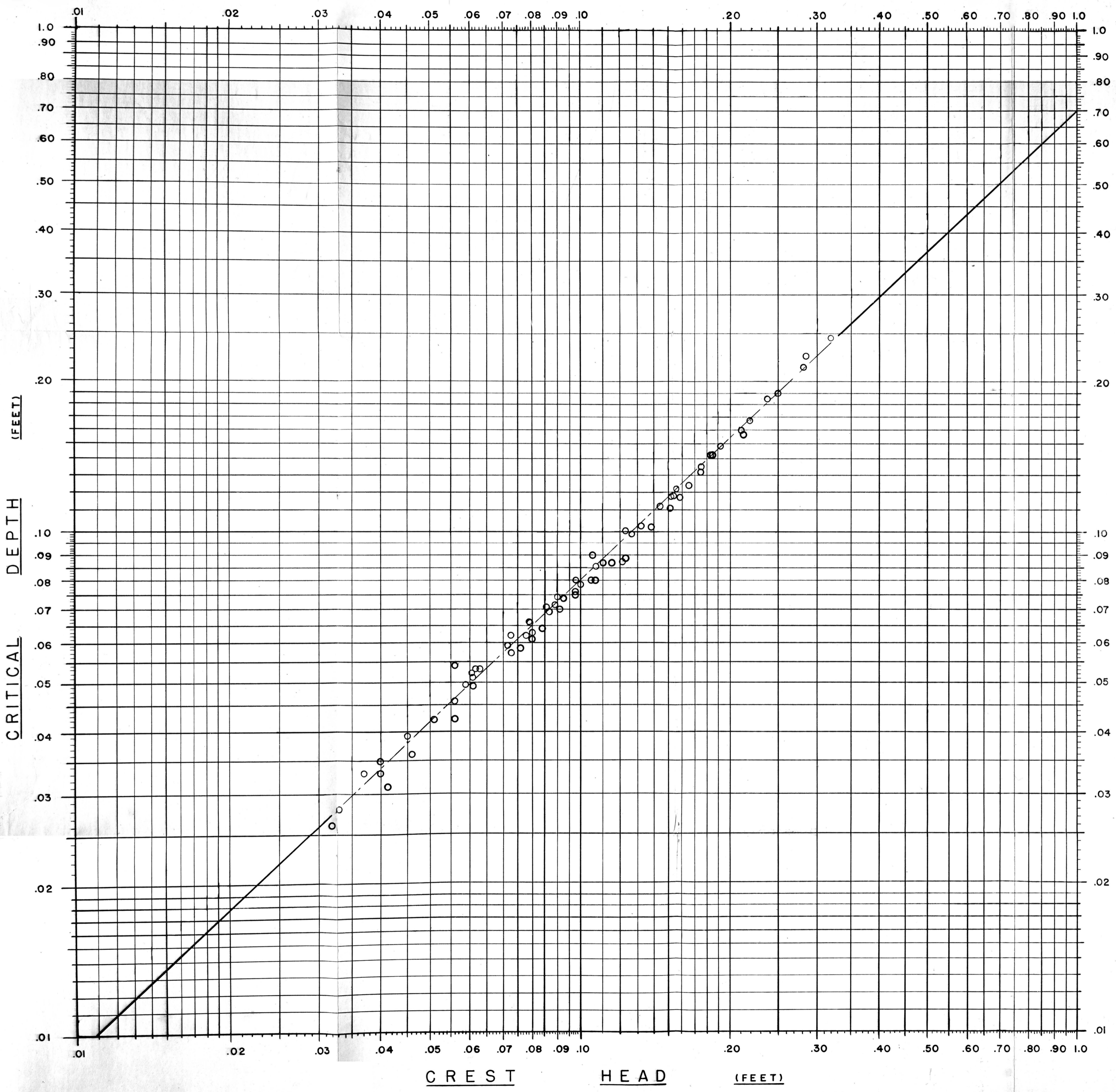
FIG. 15



RELATIONSHIPS

RECTANGULAR CONTRACTED WEIR

$$Y_{\text{CRITICAL}} = 0.70 (Y_{\text{CREST}})^{0.94}$$



CREST HEAD TO CRITICAL DEPTH RELATIONSHIP

FIG. 16

KEY

O ..... RECTANGULAR CONTRACTED WEIRS

### SUMMARY AND CONCLUSIONS

Throughout this study the author tried to keep everything constant and introduce one variable at a time. This attempt was highly successful. In the series of seventy tests, the water temperature varied only one degree Centigrade. Fifty tests were performed at zero slope on the flume and twenty tests were performed at a slope of three hundred forty-six ten thousandths.

To determine what effect the velocity of approach would have upon the relationship between the crest depth and the critical depth of an imaginary open channel having the same dimensions as the weir opening, two flumes of different cross section were used. The small flume (Fig. 9) was constructed of one-eighth of an inch thick celluloid on a wooden base which was hinged at the intake end to a stilling basin and was supported about two-thirds the distance of the flume from the stilling basin by a screw jack. This jack controlled the slope of the flume. The second flume was constructed of steel. It was about four times as wide as the small flume and was constructed at a constant zero slope. In all of the tests, the relationship between the crest depth and the critical depth did not deviate from the straight line (Fig. 16) any greater amount for the tests performed at zero slope than for the tests performed at the slope of three hundred forty-six ten thousandths. The observed data did not deviate any more from the straight

line (Fig. 16) for the wide flume than for the narrow flume. This proves that the velocity of approach has no practical effect upon the crest depth and the critical depth relationship.

The author wishes to warn those who may be interested in performing similar tests, that in the construction of a celluloid flume of variable slope, the flume should be constructed of celluloid with a minimum thickness of three eighths of an inch and the stilling basin should be an integral part of the flume and not a separate unit.

From the results of the tests performed the following conclusions can be enumerated:

1. From the plot of the crest depth and critical depth of an imaginary open channel having the same dimensions as weir opening on logarithmic graph paper with the crest depth as the abscissa and the critical depth as the ordinate (Fig. 16) an equation can be derived for this relationship. For a rectangular weir:

$$Y_{CR} = 0.70 Y_c^{0.94} \quad \text{_____} \quad (24)$$

2. Since a direct relationship can be determined from the log plotting of the critical depth to the crest depth for a rectangular contracted weir, then, a direct relationship can be determined for a sharp crested weir of any geometric shape.

3. After determining the relationship between the crest depth and the critical depth for an open channel

having the same dimensions as the weir opening an equation can be derived for the quantity of water flowing by substituting the equation for the critical depth - crest depth relationship into equation 22 which Professor Bakhmeteff set forth.

$$Q = b\sqrt{g} Y_{CR}^{1.5} \quad (22)$$

For a rectangular contracted weir:

$$Q = 3.32 b Y_c^{1.41} \quad (25)$$

4. It was observed that the crest depth at the center of the crest width and the crest depth at the edges of the weir plate had no appreciable difference, thereby, making it possible to etch the weir plate to measure the quantity of flow directly.

5. This study produced a simple, accurate and quick solution for the plotting of the M function. By means of a log log graph plot, a straight line relationship between the M function and the critical depth is obtained. Laborious tabulation and the errors inherent in fitting a curve are eliminated. This greatly simplifies the determination of the critical flow where the critical depth is known or the critical depth where the flow is known. For a rectangular cross section:

$$M = b Y^{1.5} \quad (16)$$

For a triangular cross section:

$$M = (\text{TAN } \alpha / \sqrt{2}) Y^{2.5} \quad (17)$$



6. A relationship exists between the  $M$  function of channels of the same geometric shape but of different dimensions when plotted on log log graph paper. In the case of rectangular channels the slope of the line is constant while the  $M$  intercept on the line  $Y$  equal to one is the width of the channel  $b$ . Therefore, channels of the same geometric shape produce a family of curves all parallel whose separation depend upon the width of the channel.

7. The velocity of approach does not affect the relationship between the critical depth and the crest depth.

8. The author recommends that this study be continued to include several different geometric weir shapes for which the equation of their curved surface is known.

9. From the continuation of this study the author predicts that a family of curves for the crest depth-critical depth relationship will be produced which will be similar to that obtained for the rectangular contracted weir.

10. From additional tests performed by the author on an uncalibrated weir, equation 25 gave results which were within five per cent of the actual flow. This indicates that any uncalibrated weir can be placed in a stream in which the stream bed has been moderately leveled off and the results obtained will be more accurate than can be obtained from other presently used formulas.

APPENDIX A

GLOSSARY

GLOSSARY

**contracted weir:** Any weir in which the crest width is shorter than the width of the approach channel is called a rectangular notch or contracted weir.

**crest depth or crest head,  $y_c$ :** The vertical height of water above the sharp edged crest of the weir plate.

**crest height:** The distance between the bottom of the weir plate and the sharp-edged crest is the crest height and should be differentiated from the initial crest height.

**drawdown head,  $y_d$ :** The difference in elevation between the crest of the weir and the elevation of the water surface before the drawdown curve begins to form. It is known in textbooks as the "head on the weir".

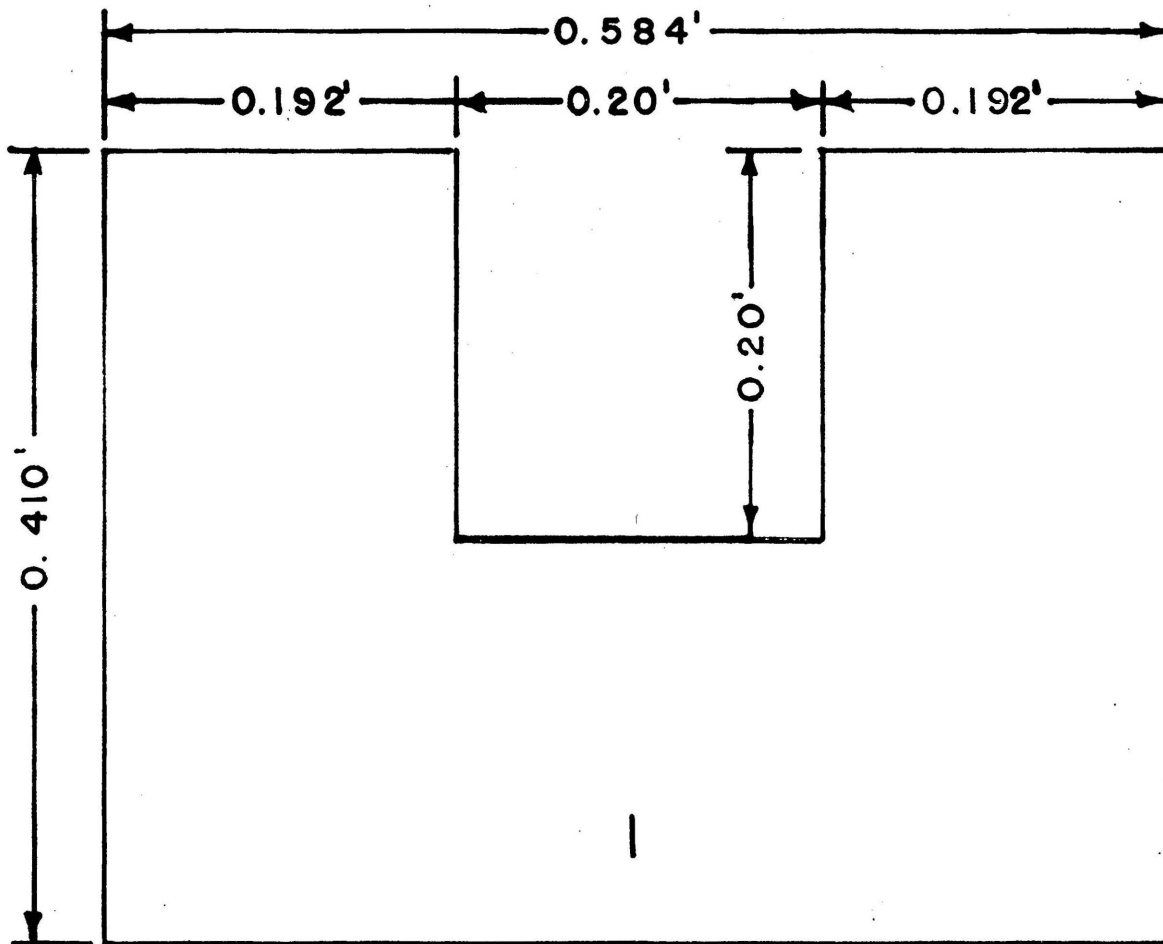
**initial crest height:** Hook gage reading of the water surface when it is just level with the crest of the weir.

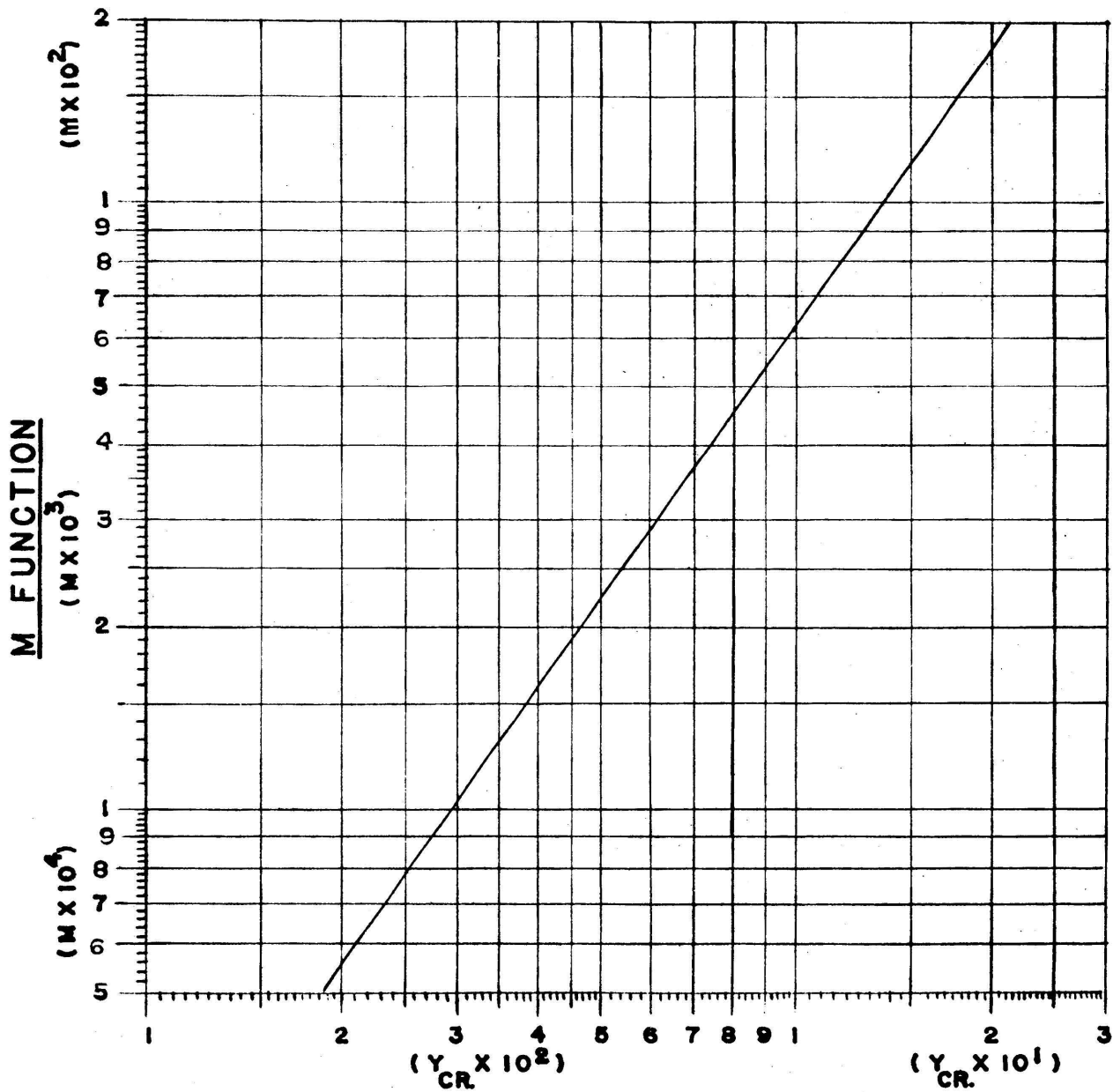
**sub-critical flow:** Flow in which the water is at a greater depth than the critical depth. In other words, the flow is tranquil.

**super-critical flow:** Flow that takes place at a depth less than critical and in the Rapid Flow range is known as super-critical flow.

APPENDIX B

TEST DATA





Y<sub>CR</sub>      CRITICAL DEPTH

WEIR NO.1

M FUNCTION CURVE

EXPERIMENTAL DATADATE: APRIL 30, 1950TEST NUMBER: 1-1WATER TEMPERATURE: 23°CSPECIFIC WEIGHT: 62.27FLUME SLOPE: 0INITIAL CREST HEIGHT: 1.783 FT.

<u>RUN NUMBER</u>	<u>CREST HEIGHT FT.</u>	<u>QUANTITY POUNDS</u>	<u>TIME SEC</u>	<u>QUANTITY CFS</u>	<u>M FUNCTION</u>	<u>CRITICAL DEPTH FT.</u>	<u>CREST HEAD FT.</u>
1	1.8150	200	571.0	.005623	.00098	0.026	0.032
2	1.8245	200	453.5	.007080	.00125	0.031	0.0415
3	1.8390	200	309.5	.010374	.00184	0.042	0.056
4	1.8590	200	201.0	.015974	.00281	0.058	0.076
5	1.8630	200	185.5	.017309	.00305	0.061	0.080
6	1.8670	200	172.0	.018667	.00329	0.064	0.084
7	1.8810	200	137.8	.023300	.00411	0.075	0.098
8	1.8900	200	122.5	.026211	.00462	0.080	0.107
9	1.8985	200	108.0	.029730	.00524	0.087	0.115
10	1.9060	200	104.8	.030637	.00540	0.089	0.123

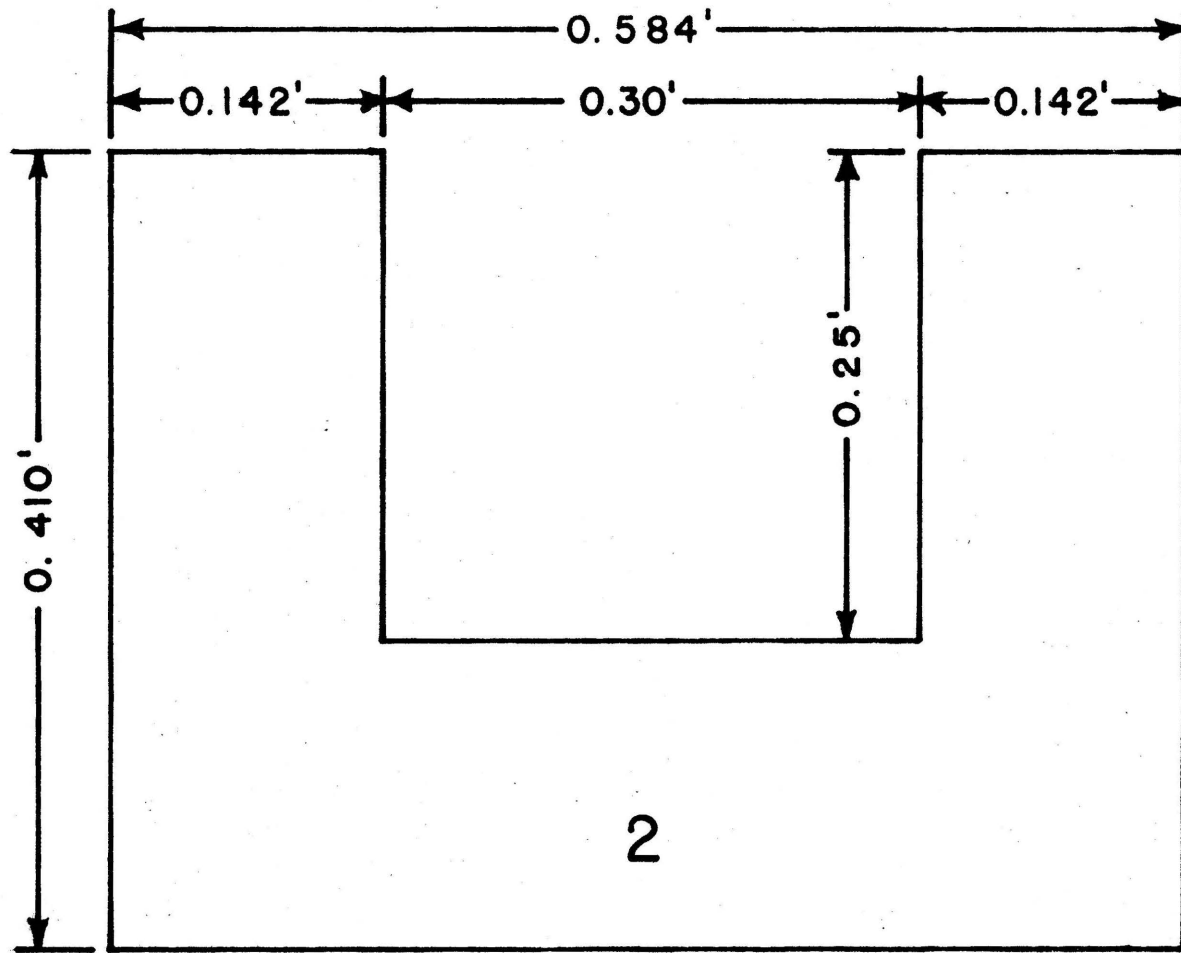
EXPERIMENTAL DATADATE: MAY 18, 1950TEST NUMBER: 1-2WATER TEMPERATURE: 24°CSPECIFIC WEIGHT: 62.29FLUME SLOPE: 0.0346INITIAL CREST HEIGHT: 1.538 FT.

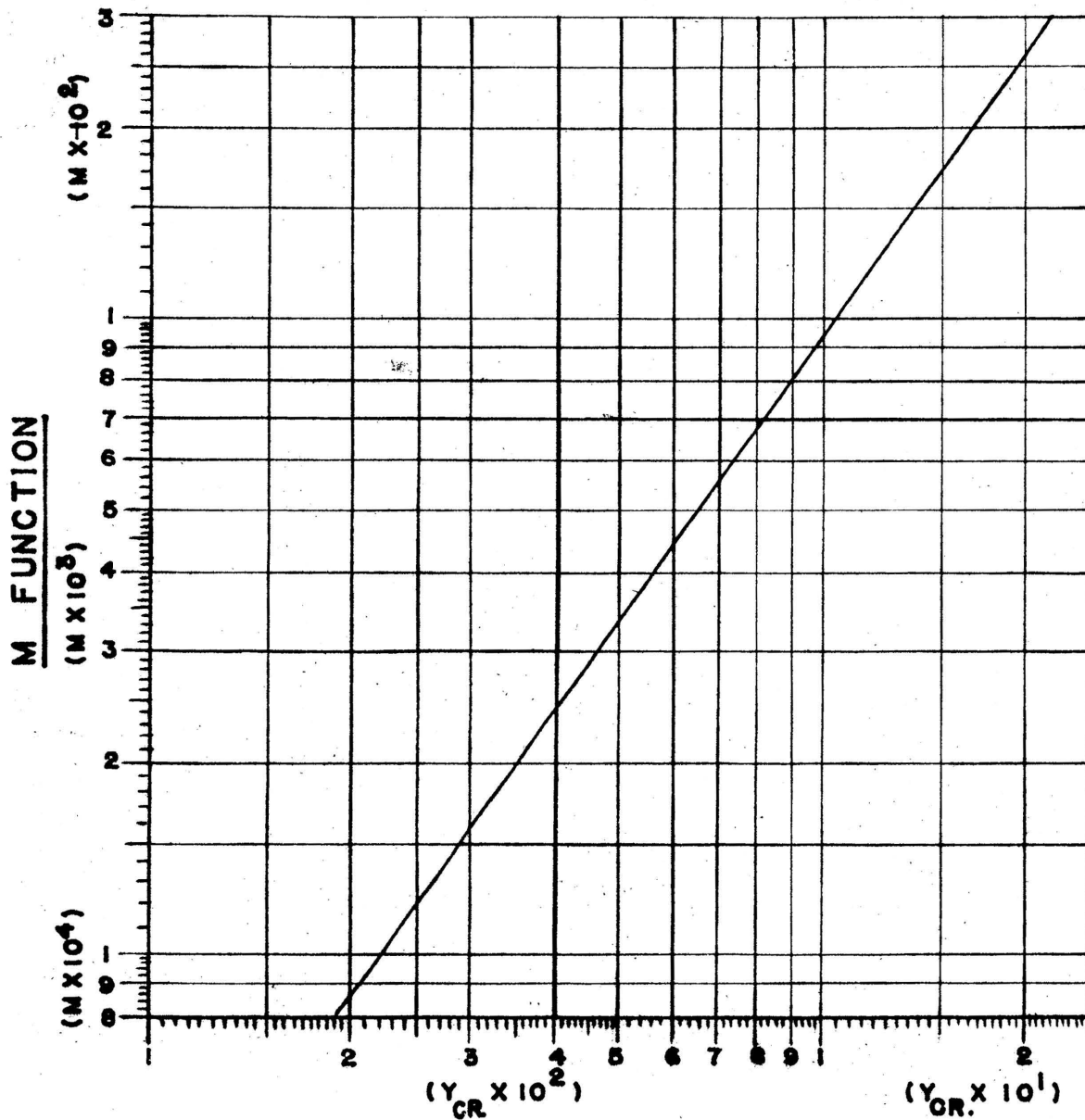
RUN NUMBER	CREST HEIGHT FT.	QUANTITY POUNDS	TIME SEC	QUANTITY CFS	M FUNCTION	CRITICAL DEPTH FT.	CREST HEAD FT.
1	1.653	300	153.6	.03137	.00553	0.090	0.115
2	1.635	200	132.8	.02419	.00427	0.076	0.097
3	1.625	200	155.8	.02062	.00364	0.069	0.087
4	1.618	200	174.7	.01839	.00324	0.063	0.080
5	1.616	200	176.2	.01823	.00322	0.062	0.078
6	1.599	100	123.8	.01297	.00229	0.049	0.061
7	1.594	100	141.2	.01138	.00201	0.046	0.056
8	1.584	100	179.2	.00864	.00152	0.036	0.046
9	1.578	100	220.0	.00730	.00129	0.033	0.040
10	1.571	100	271.2	.00592	.00104	0.028	0.033



EXPERIMENTAL DATADATE: MAY 19, 1950TEST NUMBER: 1-3WATER TEMPERATURE: 23°CSPECIFIC WEIGHT: 62.27FLUME SLOPE: 0INITIAL CREST HEIGHT: 1.326 FT.

<u>RUN NUMBER</u>	<u>CREST HEIGHT FT.</u>	<u>QUANTITY POUNDS</u>	<u>TIME SEC</u>	<u>QUANTITY CFS</u>	<u>M FUNCTION</u>	<u>CRITICAL DEPTH FT.</u>	<u>CREST HEAD FT.</u>
1	1.492	300	97.4	.04946	.00872	0.124	0.166
2	1.484	300	105.2	.04680	.00808	0.117	0.158
3	1.477	200	76.4	.04204	.00741	0.111	0.151
4	1.463	200	87.2	.03683	.00650	0.102	0.137
5	1.447	200	104.4	.03198	.00564	0.0925	0.121
6	1.431	200	124.4	.02582	.00455	0.080	0.105
7	1.417	200	150.0	.02141	.00378	0.070	0.091
8	1.399	100	105.0	.01529	.00270	0.057	0.073
9	1.377	100	170.2	.00944	.00166	0.042	0.051
10	1.366	100	229.8	.00699	.00123	0.035	0.040





Y<sub>CR</sub>      CRITICAL DEPTH

WEIR NO. 2

M FUNCTION CURVE

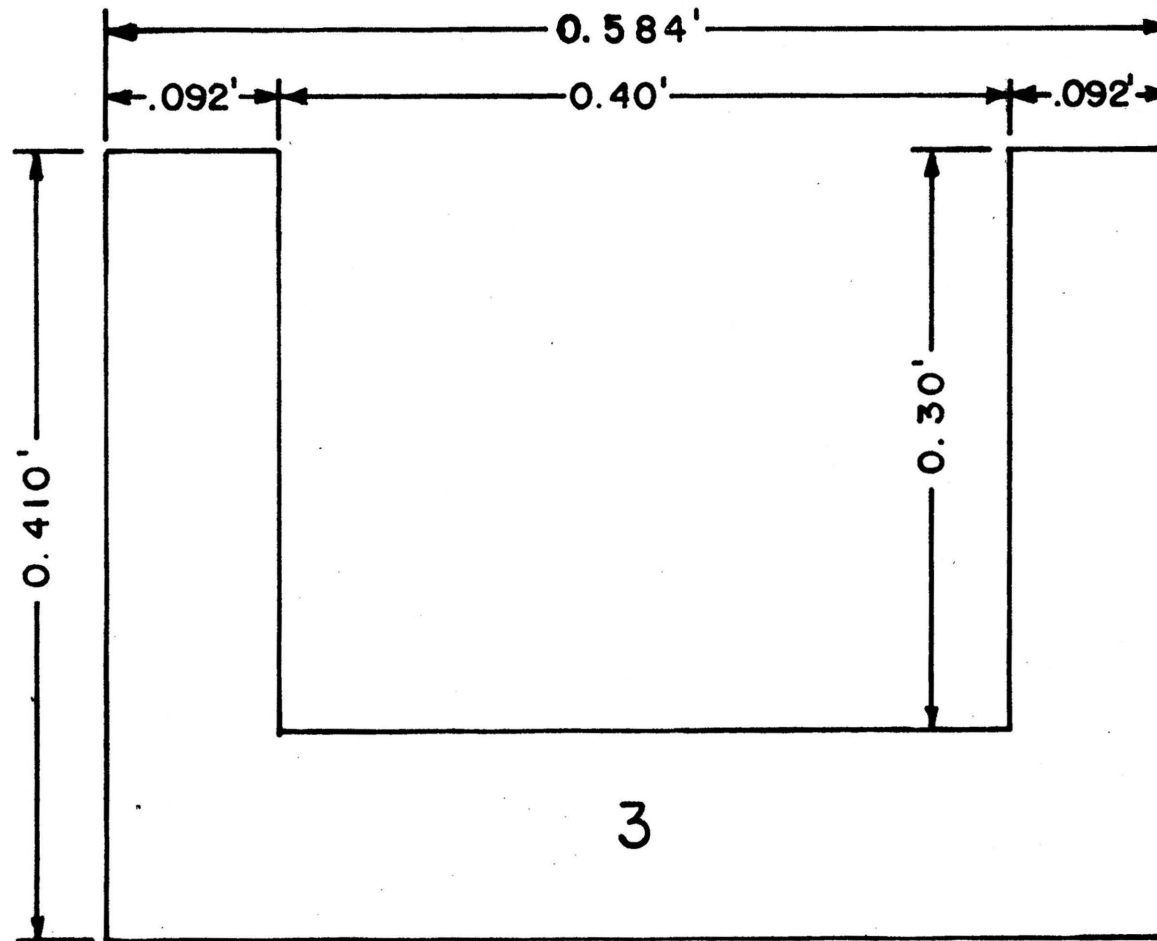
EXPERIMENTAL DATADATE: MAY 16, 1950TEST NUMBER: 2-1WATER TEMPERATURE: 24°CSPECIFIC WEIGHT: 62.29FLUME SLOPE: 0.0346INITIAL CREST HEIGHT: 1.487 FT.

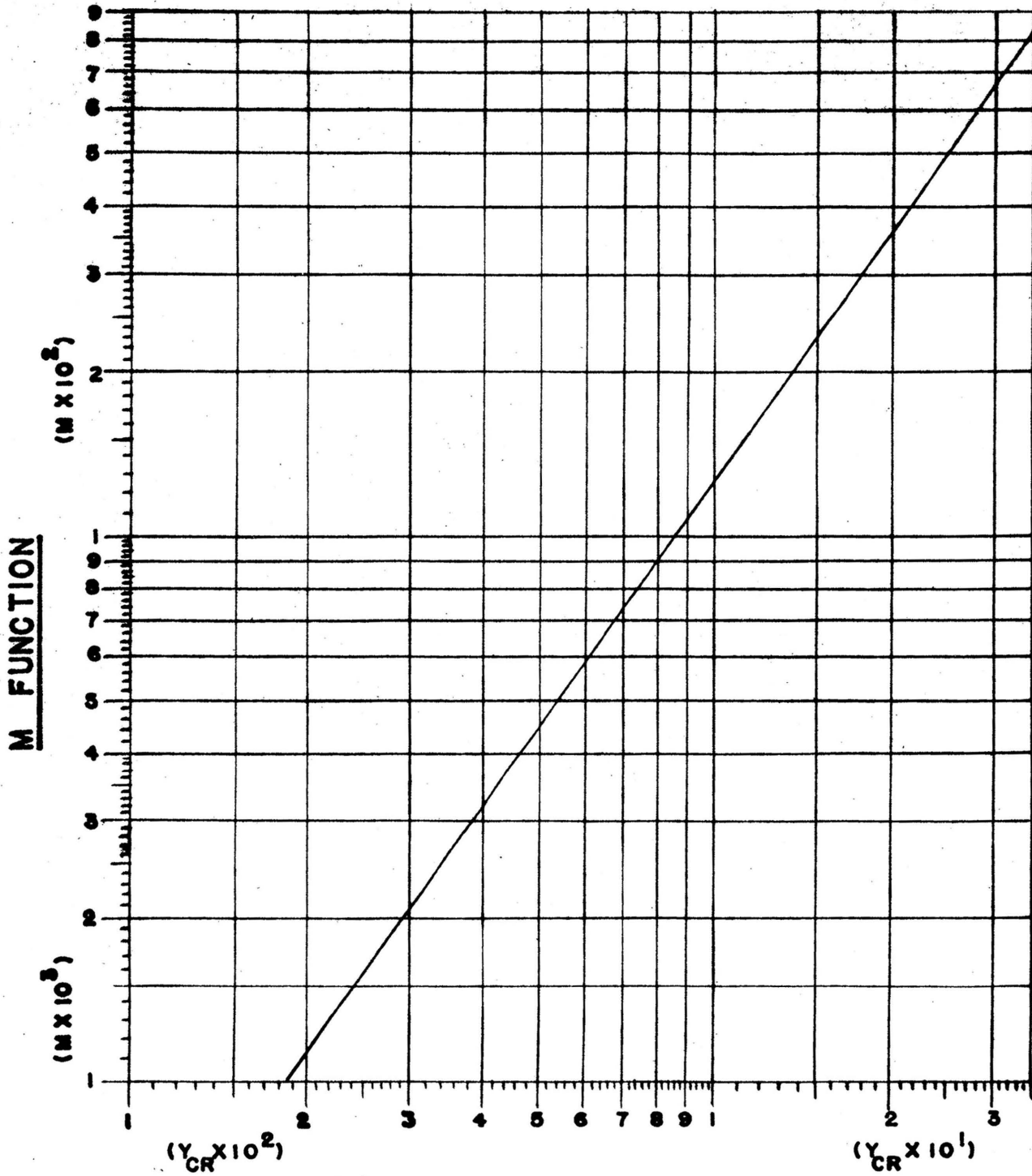
<u>RUN NUMBER</u>	<u>CREST HEIGHT FT.</u>	<u>QUANTITY POUNDS</u>	<u>TIME SEC</u>	<u>QUANTITY CFS</u>	<u>M FUNCTION</u>	<u>CRITICAL DEPTH FT.</u>	<u>CREST HEAD FT.</u>
1	1.630	300	75.0	.06425	.01133	0.112	0.143
2	1.620	200	55.4	.05798	.01023	0.102	0.133
3	1.610	200	60.0	.05354	.00944	0.100	0.123
4	1.593	200	74.8	.04295	.00757	0.086	0.106
5	1.577	200	92.8	.03462	.00611	0.0745	0.090
6	1.566	200	112.0	.02868	.00506	0.0655	0.079
7	1.560	200	122.6	.02620	.00462	0.062	0.073
8	1.549	200	156.2	.02056	.00363	0.053	0.062
9	1.532	100	122.8	.01308	.00231	0.039	0.045
10	1.524	100	160.4	.01001	.00177	0.033	0.037

EXPERIMENTAL DATADATE: MAY 19, 1950TEST NUMBER: 2-2WATER TEMPERATURE: 23°CSPECIFIC WEIGHT: 62.27FLUME SLOPE: 0

INITIAL CREST HEIGHT: 1.273 FT.

RUN NUMBER	CREST HEIGHT FT.	QUANTITY POUNDS	TIME SEC	QUANTITY CFS	W FUNCTION	CRITICAL DEPTH FT.	CREST HEAD FT.
1	1.487	300	45.2	.10659	.018799	0.157	0.214
2	1.480	300	54.0	.08922	.01574	0.142	0.187
3	1.449	300	59.8	.08056	.01421	0.132	0.176
4	1.428	300	70.0	.06882	.01213	0.118	0.155
5	1.383	300	111.4	.04325	.00763	0.087	0.110
6	1.373	300	128.2	.03756	.00663	0.079	0.100
7	1.366	200	93.4	.03439	.00607	0.074	0.093
8	1.345	200	130.8	.02456	.00433	0.059	0.072
9	1.332	100	87.8	.01829	.00323	0.0495	0.059
10	1.334	150	121.6	.01981	.00349	0.051	0.061





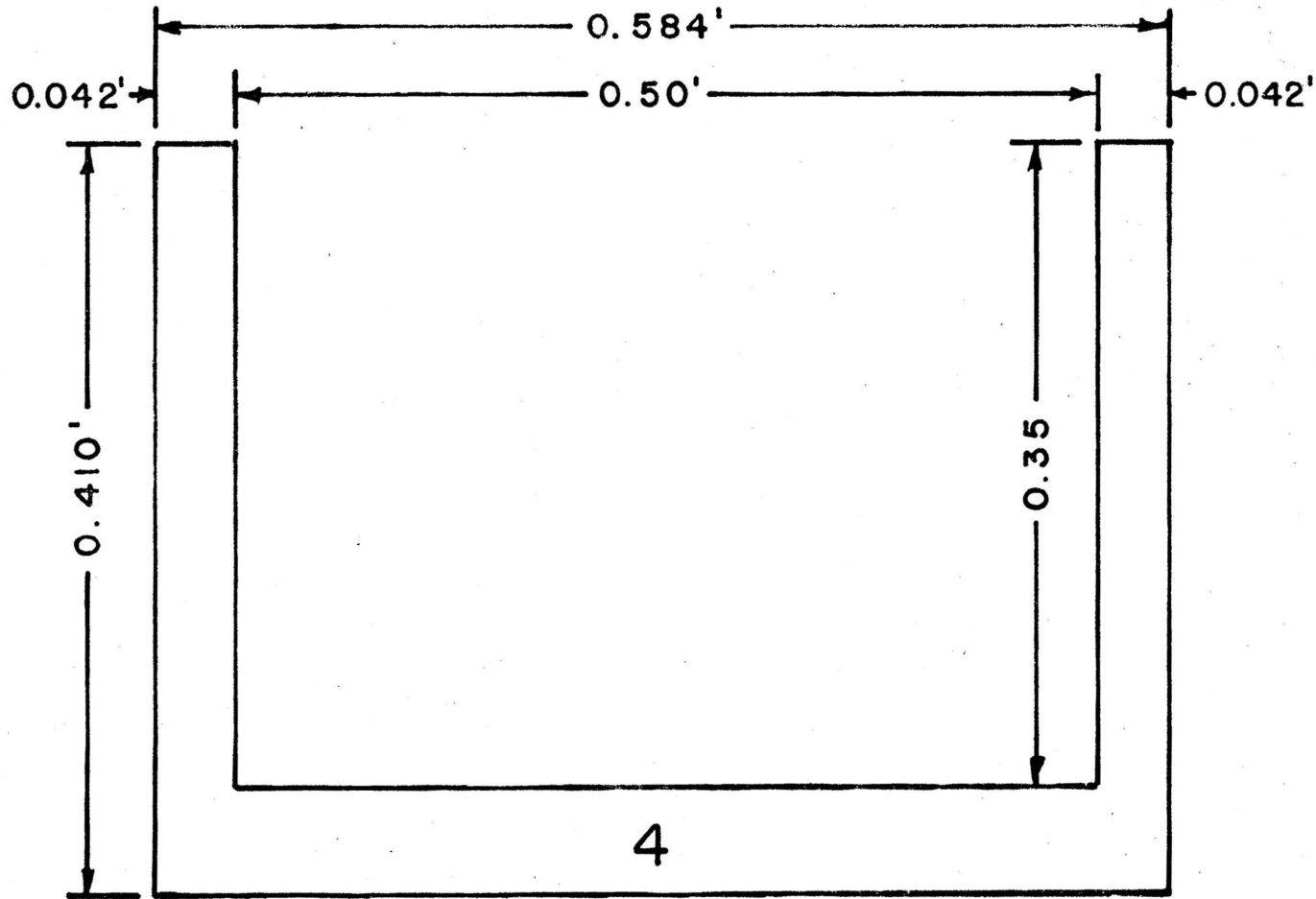
Y<sub>CR</sub>      CRITICAL DEPTH  
WEIR NO. 3

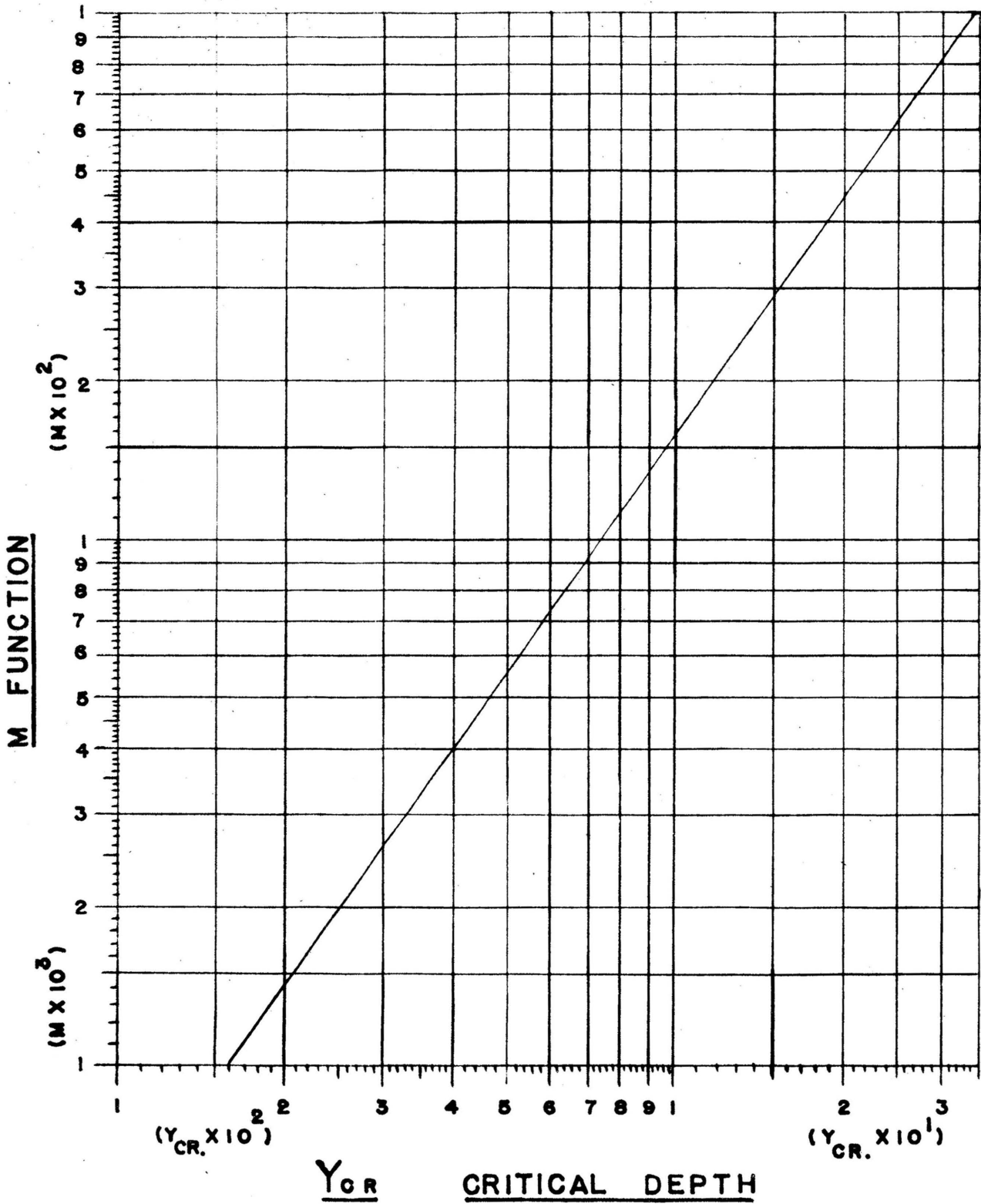
M FUNCTION CURVE

EXPERIMENTAL DATADATE: MAY 22, 1950TEST NUMBER: 3-1WATER TEMPERATURE: 23°CSPECIFIC WEIGHT: 62.27FLUME SLOPE: 0INITIAL CREST HEIGHT: 1.228 FT.

<u>RUN NUMBER</u>	<u>CREST HEIGHT FT.</u>	<u>QUANTITY POUNDS</u>	<u>TIME SEC</u>	<u>QUANTITY CFS</u>	<u>M FUNCTION</u>	<u>CRITICAL DEPTH FT.</u>	<u>CREST HEAD FT.</u>
1	1.507	500	36.8	.218194	.03848	0.212	0.279
2	1.478	500	43.6	.18416	.03248	0.190	0.250
3	1.439	500	54.6	.14706	.02594	0.160	0.211
4	1.410	500	67.2	.11948	.02107	0.142	0.182
5	1.403	500	72.2	.1112	.01961	0.135	0.175
6	1.383	500	85.4	.09402	.01658	0.122	0.155
7	1.326	500	159.0	.0505	.00891	0.080	0.098
8	1.314	500	188.4	.0426	.00752	0.071	0.086
9	1.291	500	287.6	.0279	.00492	0.053	0.063
10	1.284	300	168.0	.0287	.00506	0.054	0.056







WEIR NO. 4

M FUNCTION CURVE

EXPERIMENTAL DATADATE: MAY 22, 1950TEST NUMBER: 4-1WATER TEMPERATURE: 23°CSPECIFIC WEIGHT: 62.27FLUME SLOPE: 0INITIAL CREST HEIGHT: 1.177 FT.

<u>RUN NUMBER</u>	<u>CREST HEIGHT FT.</u>	<u>QUANTITY POUNDS</u>	<u>TIME SEC</u>	<u>QUANTITY CFS</u>	<u>M FUNCTION</u>	<u>CRITICAL DEPTH FT.</u>	<u>CREST HEAD FT.</u>
1	1.497	300	14.0	.34412	.05069	0.244	0.320
2	1.474	300	15.8	.30492	.05378	0.225	0.287
3	1.415	300	21.0	.22942	.04046	0.186	0.238
4	1.395	300	24.4	.19745	.03482	0.168	0.218
5	1.368	300	29.8	.16167	.02851	0.148	0.191
6	1.329	300	41.2	.11694	.02062	0.118	0.152
7	1.303	300	53.8	.08955	.01579	0.099	0.126
8	1.266	300	86.0	.05602	.00988	0.072	0.089
9	1.256	300	100.4	.04799	.00846	0.066	0.079
10	1.238	300	140.6	.03427	.00604	0.052	0.061

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VITA

Robert Albert Rapp was born May 21, 1925, in Johnstown, Pennsylvania, the son of Charles and Veronica (Frick) Rapp.

He received his elementary education in St. Joseph's Parochial School in Johnstown, Pennsylvania. He graduated from Johnstown Catholic High School in June 1943, and entered the Johnstown Center of the University of Pittsburgh.

After one semester, he was called to the Marine Corps, with which he served until 1946. During the time he was in the Marine Corps, he attended Villanova College and the University of Rochester as a member of their V - 12 Detachments.

In December 1945, he married Janet Louise Melvin of Johnstown, Pennsylvania. He has two children, a daughter, Mary Karen, and a son, David Richard.

In June 1946, he resumed his studies at the University of Pittsburgh, Pittsburgh, Pennsylvania, and was graduated with the degree of Bachelor of Science in Civil Engineering, in September 1948.

Immediately following graduation, he came to the Missouri School of Mines and Metallurgy, Rolla, Missouri, as an instructor in Civil Engineering.

His present occupation is as an Instructor in Civil Engineering at the Missouri School of Mines and Metallurgy in Rolla, Missouri.