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DETERMINING THE BEST COMPUTATIONAL
METHOD FOR SIMULATION

BY

NEIL HERTENSTEIN

A

THESIS

submitted to the faculty of

THE UNIVERSITY OF MISSOURI AT ROLLA

in partial fulfillment of the requirements for the

Degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

Rolla, Missouri

1966

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ABSTRACT

Two related problems are treated in the discussion. First, what is the best computational means to be used to simulate a given system? Second, if hybrid is chosen, how do we assign the problem to the different computers to realize the fullest advantage of the hybrid simulation? Most of the available information in the literature is presented in light of the above questions. It is found that, despite the lack of a precise theoretical solution, much insight can be gained into the problem. A proposed procedure for hybrid assignment and a sample problem using this procedure completes the discussion.

PREFACE

The author wishes to thank Dr. R. D. Chenoweth for his help and encouragement throughout the entire project.

The author also wishes to thank Dr. T. L. Noack for his assistance.

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I. INTRODUCTION

In the 1950's, simulation problems in the aircraft and aerospace industries began to exceed the computational limits of the available analog or digital machines. It was then that the first attempts were made to combine the best features of both computers to solve a single problem that was not possible to solve with either one exclusively. Most of the information published was then concerned with specific applications to the simulation of missile and manned spacecraft missions. The first attempts to combine the two computers, such as those described by Wilson¹ and Burns², described systems using general purpose analog equipment and large, general purpose digital computers. The interface problem was solved on a simulation to simulation basis, as no general purpose equipment was available.

Since then, the literature has still been dominated by specific applications, although the applications have become broader as general purpose interface equipment and faster digital machines have become available. Articles written by Mitchel³, Fredrickson⁴, Paquette⁵, Nutting and Roy⁶, Karplus⁷, Korn⁸, Burns⁹, Heartz and Jones¹⁰, and many others demonstrate the breadth of applications to date. While these articles made valuable contributions to the state of the art, they did not provide information of general interest to the problems of hybrid computation. There was still a need to

generalize the approach so that it would apply to more than one kind of problem. About this same time, analog computer manufacturers began developing parallel, patchable logic to augment their analog computers. These provided the hybrid simulation complexes with increased versatility and increased the need for more general information on techniques.

Korn¹¹ and Roth¹² published articles in 1962 and 1963 respectively that represented some of the first attempts to provide general information. However, these were little more than a history of what had been done to date. It was not until 1964, with the widely published work by Truitt¹² entitled "Hybrid Computation - What Is It? Who Needs It?", that a firm foundation was laid for the discussion of general hybrid theory. He provided not only a history and a much needed discussion of the new vocabulary but discussed many of the aspects of selection of a computational means and assignment of the equations to the two computers. Also of value to the inexperienced was a lengthy discussion of the available hardware and its limitations. While he provided a small amount of theoretical basis for his arguments, most of the conclusions drawn were based on experience, a condition that persists to this day. Later, Korn and Korn¹⁴ included an equipment oriented section on hybrid in their textbook on analog computation.

About this same time, information on the development of high-speed digital computers and digital simulation techniques (Brennan

and Sano¹⁵, Palevsky and Howell¹⁶, Linebarger and Brennan¹⁷, Peterson and Sanson¹⁸, Clancy and Fineberg¹⁹) began appearing. These developments challenged the short-lived reign of the hybrid as the ultimate simulation tool. These advancements have added greatly to the computing power of simulation complexes. They have not, however, significantly diminished the importance of the pure analog or the hybrid computer.

All of the above developments lead to the question asked by this paper. Given a simulation problem to be solved, what is the best computational means for simulating the physical system? Little has been published on this question considering the advances on all fronts of simulation, i. e. analog, digital and hybrid. In order to answer the above question we must ask an equally important question. If we choose hybrid, how do we assign the equations of the system to the computers to fully realize the advantages of the hybrid approach?

The main reason that authors have avoided this topic is that there is no firm theoretical method for the prediction of the performance of analog, digital, or hybrid computers. Attempts have been made to evaluate the performance of analog computers for some time e. g. Fifer²⁰ and Korn¹⁹. More recently, significant progress has been made in the area of hybrid systems by Miura and Iwata²¹, Wali²², Applied Dynamics²³ and Karplus²⁴. However, these have only provided the theoretical groundwork. The above questions will

be finally answered, with a high degree of certainty, only when methods of applying this theory are found.

In spite of the absence of a firm theoretical foundation, much information is available concerning the capabilities and limitations of the various computer systems. It is the purpose of this paper to show how this information can be brought to bear on the problems of choosing a computational method and assignment of equations for hybrid computation.

II. SOLUTIONS NEEDED

There are three things that we must know to describe the nature of the solution that will in themselves favor one computational system over another. 1) Can the problem be operated in any time reference or must it be operated in real time? 2) Are we interested in only the final results of the simulation or in the time history? 3) Can the problem be solved piecewise or must the entire device or mission be simulated at once?

How do the above factors influence our decision on computational means and assignment? The digital computer, by its very nature is unrelated to real time. Quite successful attempts with real time computation have been made¹⁶. However, these have only made the digital computer "appear" to be a real time device. The fact remains that for any significant number of closed loop calculations in real time, the analog is vastly superior to the digital. Another aspect of the classification of solution types is the practice of multiple time scaling on an analog computer. If this is used, some form of hybrid or digital automation is helpful.

The analog computer is again at an advantage when a time history of the solution is needed. It is able to provide the operator with an immediate picture of the progress of the solution or with many repeated solutions. When both time-history and final-value type answers are needed, as is often the case in large-scale problems,

the hybrid approach is very attractive. This is especially true since most time-history answers do not require the precision demanded of final-value type answers.

III. EXTERNAL EQUIPMENT

In the simulation of large systems, some of the actual system hardware or special equipment simulating this hardware is associated with the simulation. This external equipment may be analog or digital in nature or both and may be in an open or closed loop configuration with the computers. Usually, if the external equipment is of an analog or digital nature exclusively, then the corresponding computational method should be employed. The hybrid approach is very useful whenever the external equipment is comprised of both analog and digital equipment, as is the case in many large scale problems. In this case, the analog can be used to control the analog devices, and the digital to control the digital devices. This approach eliminates much of the interface problems between analog and digital equipment. In digital to digital communications, however, the complexity of the digital to digital conversion equipment often rivals that of the digital to analog.

An important example of how the external equipment influences not only the choice of computational means but assignment of equations as well is the case where either the digital or analog computer is used to simulate only one subsystem, allowing the actual subsystem to be easily substituted into the system later without significant change to the simulation. The usage of other external equipment, such as recorders, scopes, and printers is determined by the type of solution needed.

IV. COMPUTING EQUIPMENT

The choice of the type of computing equipment to be used in a simulation complex is becoming increasingly difficult. The problems confronting management are: the increasing scope and complexity of simulation problems; the lack of experience in digital, real time simulation techniques; and the reluctance of simulation engineers to depart from all-analog and all-digital methods. These problems can be overcome with proper education of personnel and some careful planning when the purchase of new equipment is proposed. Below are some of the more important factors to be considered in evaluating present or new equipment. The following will also help evaluate the limitations and capabilities of present equipment.

A. Analog

When choosing an analog for either all analog or hybrid application, the most significant factors are flexibility and quality of components. For hybrid applications, the following features are important¹³:

1. Integrators with multiple time scales
2. Track and store capabilities
3. Automatic mode control for individual amplifiers
4. Automatic, remote control of pot setting
5. Fast, accurate multipliers and trigonometric resolvers

6. High speed comparators with logic signal outputs
7. Electronic switches

Equipment requirements for an all-analog simulation becomes prohibitive in some large problems. Also memory requirements may exceed by several orders of magnitude the capabilities of the analog, making a hybrid approach necessary.

B. Digital-Sequential

The sequential type of digital computer is primarily an arithmetic computer and performs its functions sequentially from a stored program. It can be used for logic and control functions, but only at the expense of program time. The most important parameter in selecting this type of digital computer for real time or hybrid applications is speed. Speed is a function of memory access time and multiplier speed and also such equipment as shift registers, etc. Many of the first hybrid problems were performed using large scale, general purpose, digital computers that were primarily used for other applications. As the applications for hybrid work grow, the trend is to smaller, faster, general purpose machines that are used exclusively for hybrid and real time applications.

In this type of computer magnetic core memory is almost a must. Clancy and Fineberg²⁷, in their article "Hybrid Computers", state that a memory of approximately 32K bits with a memory cycle time

of approximately 2 μ sec is a good criterion. Large, off line data storage is not needed and only a minimum of peripheral equipment is useful. With the above memory capacity and with more stringent accuracy requirements, a 24 bit word length is needed. Despite the slight penalty in speed, the ease of handling and programming makes floating point arithmetic essential.

C. Digital-Parallel

The parallel type of logic provides the simulation operator with the ability to control several operations at once and provide several logic decisions simultaneously. While this type of logic is very limited in the amount of arithmetic and memory functions it can perform, it offers flexibility and speed that cannot be achieved with sequential digital equipment. Parallel logic can be provided through either a small amount of logic as an integral part of the analog or a separate parallel logic computer. The important thing here is compatibility with the analog, and it is for this reason that most parallel-type logic computers are designed to be used with a particular analog computer. An alternative to the above would be a small amount of logic on the analog and a small serial-type digital computer to be used only for control and minor logic functions.

D. Conversion and Linkage Equipment

The first hybrid problems were solved using AD and DA conversion equipment designed for a specific problem or class of problems.

The selection of conversion and linkage equipment is still dictated by the intended usage. However, for the large-scale simulation complex, it is desirable to have as much equipment flexibility as possible without requiring the digital program to spend a lot of time controlling the equipment.²⁷

The first consideration in comparing conversion equipment is the number of AD and DA channels available. As the size, variety, and complexity of problems increase so does the need for additional channels. Therefore, the more channels available, the better. Efficient usage of the channels is also desirable. In some conversion equipment, DA channels may be "time-shared". This involves using one channel for setting initial conditions in the re-set mode and for data transfer in the operate mode.

The next consideration is the type of control used.²⁷ Functional control seems to be the best approach if a separate, general purpose, conversion device is employed. With this type of conversion device one or two words from the digital computer initiate all functions necessary for conversion. With the conversion unit controlling most of the details of the conversion process, flexibility is retained without digital program control. An I/O (Input/Output) device, designed for hybrid problems, supplied with the digital computer, retains all of the advantages of functional control. This approach also eliminates the need for programming the conversion equipment separately and

a more direct link to the digital program and main memory is achieved. As in the digital computer, speed and word length are also important considerations.

V. COST

Cost is always a factor when deciding what computational means to use. The method of comparison will vary greatly from installation to installation due to factors such as the reason for solving the problem, the type of work the company performs, the available equipment, the method of paying for the equipment, and the primary mission of the simulation complex. Other factors that will vary from time to time within the company are the work load, the experience of available personnel, how soon the solutions are needed, and the methods of financing the project. Since none of the above factors have a general bearing on the problem, we will only mention them.

There are a few factors that are applicable everywhere; some of them have been mentioned before. These factors fall into three broad categories: 1) programming costs, including set-up time; 2) operating costs; and 3) equipment costs. In all three of the above categories, savings are generally realized if only one form of computer can be successfully used to solve the problem. The savings in programming and operating costs are obvious. If a savings in equipment is not realized, it must be weighed against the savings in the other two.

Hybrid computation usually becomes economically advantageous whenever a significant portion of the problem is not well suited to the type of single computation method being employed. This usually leads

to high costs in both equipment and programming to compensate for the problems arising from attempting to solve an equation for which the computer is not suited. Examples of this would be the need for multiple amplitude scaling when the dynamic range of the analog is exceeded, or the need for exotic compensation when real-time digital techniques are applied to high frequency equations. Another area in which hybrid techniques realize a savings is when a large number of short analog runs can be automated to decrease running and set-up time.

When optimizing costs within a hybrid problem, the most significant factor is efficient usage of the digital computer. Every equation that can successfully be placed on the digital computer without exceeding the computation interval realizes a savings in analog equipment without adding anything to the costs charged against the digital machine. In the analog portion of the simulation, the cost is directly proportional to the number and complexity of the equations solved. In the digital portion, the equipment costs are independent of the size of the digital program. However, as the computational limit of the digital is reached, programming costs begin to mount. Finally, if one computational means has been successfully used, the immediate costs of changing computational methods must be weighed against any long-term gains in equipment.

In the above paragraph, it was assumed that the digital computer was entirely committed to the hybrid problem. The cost criterion

used becomes much more complicated when the digital computer is shared with another problem. Economic factors, too numerous to mention here, will determine for each installation the best cost criterion to use.

VI. TYPES OF EQUATIONS

The equations of the system are the mathematical model of the system being simulated and the simulation is a model of these equations. The extent to which the simulation is an actual model of the system is dependent on how close the mathematical model is to the system, and then how close the simulation is to the mathematical model. Different types of computers will simulate some equations better than others.

Many engineering problems of quite different physical nature will have the same equations and this then becomes one of the more fruitful areas to draw some general conclusions. What are the general conclusions that we can draw? The general classes of equations are algebraic, differential and logic equations. The digital computer, being a discrete device, is more adaptable to the solution of algebraic and logic equations. The analog computer, being a continuous device, is more adaptable to the solution of differential equations.

Another way to classify equations is open and closed loop. Analog systems perform much better on closed loop problems for many reasons and the serial nature of the digital computer makes it ideally suited to open loop equations. Section X lists some of the different types of equations that indicate the need for a hybrid set-up and also lists some of the more common schemes for assignment when only the types of equations are considered.

VII. VARIABLES

Two of the most important things that we must know about the variables of the problem before we can make any meaningful choice of a computational means or separation scheme are the frequencies of the variables and the dynamic range of the variables. We will now describe these two concepts and see how they affect our choice.

A. Dynamic Range and Resolution

The dynamic range of a variable is the ratio of the maximum value that the variable assumes during the problem to the accuracy with which the smallest value must be measured. For example, let the variable be the distance from the target of a missile. If the missile starts 1000 kilometers from the target and we wish to measure the miss distance to the nearest meter, the dynamic range is $1000 \times 10^3 / 1.0$ or 10^6 . This would require the computer to have a static resolution of $1/10^6$ or 10^{-6} . The static resolution of an analog computer is severely limited by drift and noise and is a function of time; the resolving power generally decreases with time. Some of these difficulties can be overcome with multiple amplitude scaling but at high equipment costs. The digital computer, being a discrete device, is capable of a much larger dynamic range. Whenever a large dynamic range is present in a problem, an all digital or hybrid approach is usually advantageous.

B. Frequency Range

The problem of a wide range of frequencies in the simulation is closely related to the problem of resolution. According to Truitt¹³ "The presence of a wide range of signal frequencies in a system to be simulated is the one characteristic that most clearly indicates the need for a hybrid computer". Once the variables are separated according to their frequencies, we can decide how to assign them.

It is fortunate that for most applications the high-speed variables that are very difficult for the digital computer to handle do not require high accuracy and are ideally suited to the analog. On the other hand, low speed variables can be computed on either type, but frequently require an accuracy beyond the range of the analog.

VIII. ACCURACY

Accuracy is always of primary importance when choosing a computational method. In one sense, accuracy is a measure of how successfully the mathematical model has been simulated. We choose the term "mathematical model" because there is no way to improve the accuracy beyond that of the mathematical model. If we think we are, then we are just using a different mathematical model. Without explicitly saying so, most of the criteria that we have mentioned so far have been based on accuracy. However, with the methods presently available, an analysis based entirely on accuracy would be far too unwieldy to be of significant value.

Each computer contributes certain kinds of inaccuracies and these could be calculated for each equation and variable simulated. A much easier approach is to recognize which types of equations and/or variables are most accurately simulated in each type of computational scheme. The purpose of this section is to mention some things that should be considered in determining and improving the accuracy of the entire simulation.

A. Analog

There is not at this time any easily applied theory for computing exactly the inaccuracies in all analog simulation. There is no doubt that these inaccuracies do exist and there has been much written on

improving the accuracy from an equipment standpoint and from a programming standpoint.^{14, 20} It is much easier to recognize the limitations imposed by imperfect integration, drift, noise, band width, component inaccuracies, and set-up procedures.

B. Digital

For a sequential digital program, the truncation and roundoff errors can be monitored and held within certain limits. This information, of limited interest here, can be obtained from any handbook on digital methods. However, when the digital computer is used as a real time simulation device, there are many things to consider when calculating or optimizing the accuracy.

For the digital machine to be used effectively, a high sampling rate and a corresponding short computational time must be used. Errors are caused by the sampling process, the truncation and round-off errors in the program, and the time delay for conversion and computation. The effect of this delay can be reduced by extrapolating the problem variables forward one sample interval.²¹ However, the order of extrapolation provides a trade-off between accuracy of function fit and time for calculation and data handling. When the allowable computation interval is exceeded, instability and prohibitive inaccuracies result and a hybrid approach must be used rather than a real-time digital approach.

C. Hybrid

Many of the errors and limitations in a hybrid approach are similar to those encountered in connection with the analog and the real time digital simulation mentioned above. The most significant sources of error in hybrid computing are the methods that must be used to reconcile the basic differences between the analog and digital computers. These methods include sampling, quantization, and conversion. The analog must appear to the digital as a sequential, discrete, source of data (which it is not) and the digital must appear to the analog as a parallel, continuous source of data (which it certainly is not).

C. R. Wali in his discussion of quantization and sampling errors²² has summarized one of the problems confronting the hybrid programmer.

"...for any specified error bound, how finely should the signal be quantized, how rapidly should it be sampled, and what tradeoffs exist between sampling rate and the number of quantization levels? In short, is there an optimum combination that can be used, or must the maximum possible number of levels be used with the maximum possible sampling rate?"

At this time there are no hard and fast answers to this problem. Some of the "rules of thumb" for approximating the proper sampling rate are provided in the section on programming.

D. Conversion Accuracy

The accuracy of the conversion process is determined to a large extent by the word length used. An increase in the word length causes

an increase in the conversion accuracy, but decreases the speed and increases the cost of the converter. The static accuracy required will place a lower limit on the number of bits that can be used. If only the static accuracy is considered, 10 bits would correspond to 0.1% full scale accuracy ($2^{10} = 1024$, $1/1024 \approx .001$). However, many more bits might be required to obtain this accuracy for smaller signals; this is determined by the static resolution required.

IX. ERROR ANALYSIS

One of the main drawbacks to any meaningful analysis of hybrid systems is the absence of an easily applied theory of error analysis.

Karplus²⁴ in his discussion of error analysis summarizes the present state of the art as follows.

"The basic error-analysis problem can be stated in two ways:

1. Given a hybrid computer system consisting of a specific interconnection of computer components, each with a specified tolerance, what is the probable accuracy of the overall computer solution?
2. If it is desired to obtain a computer solution of a specified accuracy, what must be the accuracy of tolerance of each system component and how should these be interconnected?

That it is unlikely that fully satisfying answers to the above questions will become available in the near future can be inferred from the present status of error analysis in pure analog and digital computation. Although many sophisticated studies have been made of the errors inherent in the linear analog computer components and their effect upon the solution of systems of linear ordinary differential equations, there have been very few extensions of these efforts to complex non-linear differential equations. In most large analog simulations it is virtually impossible to predict an overall solution accuracy. Similarly in digital computations, the effects of accumulated truncation and round-off errors upon the solution of systems of differential equations is never easy and usually impossible. If a system of differential equations is integrated over an appreciable number of time steps, the probable error in the solution is virtually impossible to estimate. A hybrid computer loop contains an analog computer and digital computer as well as linkage equipment. All the errors inherent in analog and in digital computation are therefore present in the hybrid loop, in addition to which the errors introduced by the linkage system must be taken into account."

A complete description of the work done to date in this area would be outside the scope of this paper. However, the following describes briefly some methods that have been proposed for the approximation of the errors inherent in the various parts of the hybrid setup. As stated previously, it is assumed that the digital errors can be monitored so that we consider below only those errors in the analog section and the conversion devices. Finally, we allude to a possible approach to determine the effect the above errors have on the total solution.

A. Analog

One method of estimating the order of magnitude errors in the analog is to compute the difference in the coefficients of the equation being solved and the equation that represents the non-ideal analog circuit. While this will not give the absolute accuracy, at least it will indicate places where a relatively high or low degree of uncertainty exists in the solution. This approach²³ is described briefly below.

First, we will approximate the non-ideal transfer functions of the components: ($p = d/dt$)

$$\text{Summer: } - (1 - T_S p)$$

$$\text{Integrator: } - (1 - T_I p)/p$$

$$\text{Potentiometer: } (1 - T_Q p)a_i$$

To evaluate the transfer functions, we assume the phase shift to be caused by the fundamental equation frequency. We can then

define the constants above as:

$$T_S = 1/2(N_S/W_S) \times (1 + \Sigma R_F/R_I)$$

$$T_Q = N_Q/W_S$$

$$T_I = 1/2(N_I/W_S) \times (1 + \Sigma \frac{1}{C_F R_I})$$

where:

N_S, N_Q, N_I = Phase shift of corresponding component measured in radians at fundamental frequency of the equation.

a_i = Potentiometer setting.

W_S = Fundamental solution frequency in radians.

We now identify the equation we want to solve as the "given equation" and the equation that the machine solves as the "machine equation". To determine the machine equation we write the computer circuit as in Figure 1 so that the highest order derivative is available in the circuit. Imagine the circuit broken at that point and identify the two terminals as y and y' (y' being closest to the integrator). We then evaluate y in terms of y' using the non-ideal transfer functions. To make this step easier, we can re-draw the diagram in block form as in Figure 2. By then letting $y = y'$ in this equation, we have the machine equation. Next, we disregard terms of higher than first order in T , solve for the unknown variable in terms of y or y' from the computer diagram (figure 2), and substitute this into the machine equation. This gives us the form that we seek.

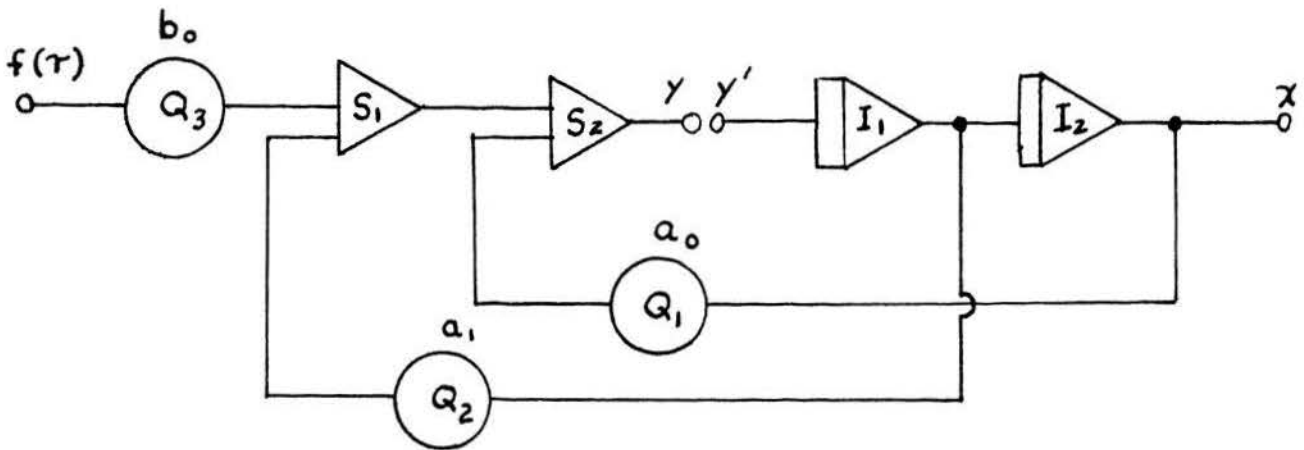


Figure 1 - Example Problem

We can either write the machine equation directly, or redraw the computer circuit as follows:

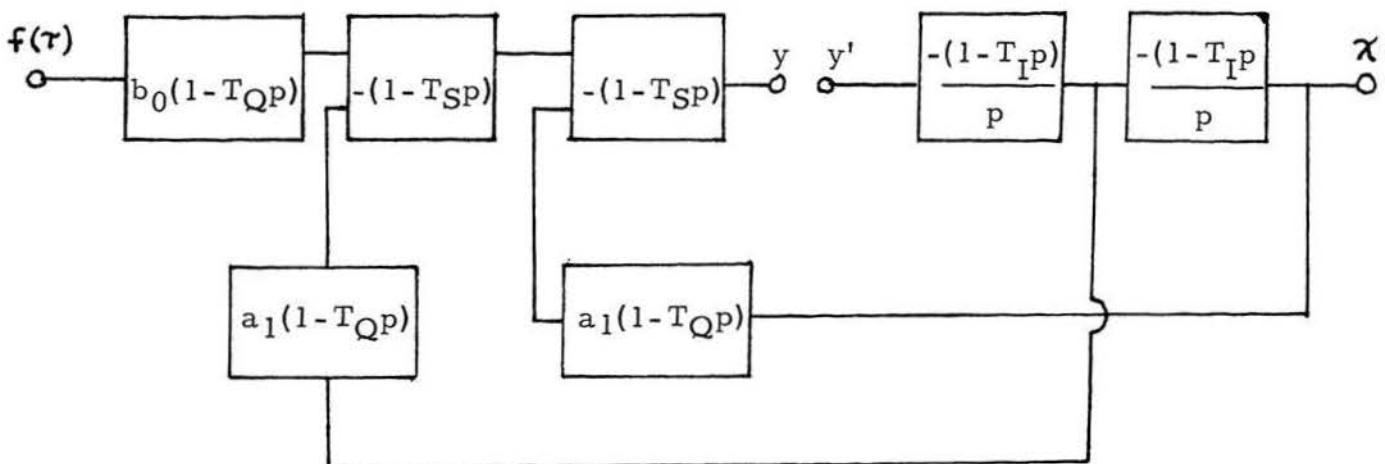


Figure 2 - Block Diagram

If it is necessary to time scale the problem, the T terms should be expressed in time scaled units. The last step is substitution of numerical values for T_S , T_Q , and T_I . This will lead to an equation of the form:

$$(1 - \epsilon_n) \frac{d^n x}{d\tau^n} + (1 - \epsilon_{n-1}) \frac{d^{n-1} x}{d\tau^{n-1}} + \dots x = f(t) - \epsilon_0 \frac{df(\tau)}{d\tau}$$

where ϵ_i is the error in the coefficient of $dx_i/d\tau^i$. This same approach can also be applied to non-linear differential equations.

As an example of the above procedure, consider the typical second order "mass-spring-damper" equation:

$$\ddot{x} + a_1 \dot{x} + a_0 x = b_0 f(t)$$

The computer circuit for this equation is shown in Figure 1.

We can now write the equation for y in terms of y' from Figure 2.

$$y = -y' \left[\left(\frac{1 - T_I p}{p} \right)^2 (1 - T_Q p) (1 - T_S p) + \left(\frac{1 - T_I p}{p} \right) a_1 (1 - T_Q p) (1 - T_S p)^2 \right] + b_0 (1 - T_Q p) (1 - T_S p)^2 f(\tau)$$

Multiplying both sides by p^2 , letting $y = y'$, and dropping terms higher than first order in T we have: $y \left\{ [1 - a_1 (2T_S + T_I + T_Q)] p^2 + a_1 \left[1 - \frac{a_0}{a_1} (T_S + 2T_I + T_Q) \right] p + a_0 \right\} = b_0 [1 - (2T_I + T_Q)] p^2 f(t)$

From Figure 2 we see that: $y \left(\frac{1 - T_I p}{p} \right)^2 = x$

and substituting this into the above equation we have:

$$\left\{ [1 - a_1(2T_S + T_I + T_Q)]p^2 + a_1 \left[1 - \frac{a_0}{a_1} (T_{S2} + 2T_I + T_Q)\right]p + a_0 \right\} x = b_0 [1 - (2T_S + 2T_I + T_Q)p] f(t)$$

This, then, is the form we seek and by substituting in appropriate values for T_I , T_S , and T_Q , we have the coefficient errors that we seek.

The above discussion does not take into account the effects of noise and drift because it assumes that a proper time scale can be chosen to minimize these effects. However, if we must run in real time, we cannot neglect these effects.

If we make a simplifying assumption that drift and noise introduce an uncertainty of K_{dn} volts into any measurable voltage $V(t)$, then the % error in that voltage is:

$$\epsilon_{dn}(t) = \frac{V(t) - K_{dn}(t)}{V(t)} \times 100$$

In general $K_{dn}(t)$ will have a constant white-noise component and a low-frequency drift that increases with time. These will be dependent on the individual computer in question.

B. Conversion Errors

During the conversion process, the continuous analog data is presented to the digital in a discrete form by means of the sampling and quantizing process. This process introduces errors that are

functions of the sample rate and number of quantization levels used. These errors were discussed previously in the sections on accuracy and programming. It should be noted that these errors are also similar to those encountered in real-time, all digital simulation techniques.

After the digital has finished computation, the up-dated values of the variables are presented to the analog. The analog hold circuit is part of the process by which the discrete digital data is presented to the analog in a continuous form. The two methods most commonly employed are the "zero-order hold" and the "first-order hold".

Karplus²⁰ has shown that the instantaneous error introduced by the zero-order hold for a sine wave input ($y = A \sin (wt)$) is

$$\epsilon = A \sin [(n + \tau/T) wt] - A \sin (nwt)$$

and that the maximum occurs when the second term is zero and is approximately $\epsilon_{\max} \approx 2\pi A/p$ where p is the number of times per cycle that the analog is up-dated. Similarly, for a first-order hold $\epsilon_{\max} \approx A(2\pi/p)^2$.

C. Solution Errors For Hybrid Simulation

The final objective of any system of error analysis is the determination of the effects of the individual inaccuracies of the analog, digital, and conversion errors on the final hybrid solution. One of the most promising approaches to this problem is presented by Karplus²⁴. This approach, presented briefly below, involves a sensitivity analysis utilizing state-variable techniques.

First, all sources of error, whether component inaccuracies or inaccuracies due to sampling, quantizing or time delays, which are capable of significantly affecting the state variable of interest (y_n) are identified. Second, the effect of each of these error sources upon the system state equations is expressed by an additive forcing function $h_{nj} \cdot q_j$. Where $H = \{h_{nj}\}$ is a matrix relating each error source j to each state-equation n .

Next, a sensitivity function u_{nj} is associated with each of these forcing functions such that $u_{nj} = \partial y_n / \partial q_j$ and the corresponding sensitivity equations are identified. These equations are then solved on the computer to provide a separate plot of each u_{nj} over the entire time domain of interest. These plots are then employed to determine the maximum q_j which can be permitted without excessive perturbations Δy_n in the state-variable of interest. The bounds on q_j can then be translated into engineering specifications for each of the sub units.

While the above sensitivity approach has yet to be refined to the point where it is useful in hybrid work, another "sensitivity" technique has been employed with success for some time. This technique involves making small changes in each of the parameters in the simulation and noting their effect on the solution. Whenever the simulation exhibits a high sensitivity to small changes in a parameter it is assumed that there is a high degree of uncertainty in that particular portion of the simulation.

X. PROGRAMMING

A. General Considerations

Since much has been written on the subject of both analog and digital programming, we will not spend time here on that subject. However, some insight into some of the things to be considered in hybrid programming will be of value. Programming can be thought of as synthesizing the simulation and, like any synthesis problem, there is not a unique solution and the "best" solution is no easier to define than the "best" computing system.

Most of the methods for improving the accuracy of the digital program result in a longer program. The time necessary to execute the digital program is the limiting factor in most hybrid simulations. Therefore, any of the methods for improving the accuracy should be weighed against their cost in program time. Before starting on any programming scheme it is important to know the speed of the digital computer, the speed of the conversion device and the number of D-A and A-D channels available.

One thing to consider is the optimum word length for the simulation. This is often dictated by the equipment, but when a choice is made it usually represents a trade-off between errors and computing time. As mentioned in the section on accuracy, a capability to use more than one word length is valuable.

If a digital computer is used in the main computing loop some compensation scheme^{21, 25} is needed to improve the accuracy or allow a slower sampling rate which alleviates the stringent speed requirements. Another technique that is valuable for reducing the necessary sampling rate is to integrate high speed variables on the analog before sending them to the digital. Thought must also be given to the control sequence that is used for the conversion equipment.

For special problems such as random events that are not actually equations, special techniques using parallel logic are useful. Another thing that must be considered is the advantages and disadvantages of using more than one sampling rate with the digital computer used on a time shared basis.

Some thought should be given to how much automation of the simulation is necessary. If there is a large number of runs to be made or the simulation is to be used for a long time, the advantages of program automation are obvious. However, if there are only a few runs to be made, the increased complexity of the program and the additional set-up time required may not be justified. Also, consideration must be given to the number of changes that must be made to the program and external simulation equipment. Some changes are more easily made to the analog than to the digital.

B. Determination of Sampling Rate

In order for the use of a digital computer to provide any

improvement in accuracy, the quantizing and sampling process must not introduce prohibitive errors. The accuracy of the quantization and sampling process is improved by reducing the quantization level and by increasing the sampling frequency. As pointed out in the previous section, an increase in the number of quantization levels increases the word length and consequently increases the cost and decreases the speed of A-D conversion. Increasing the sampling frequency decreases the time available for digital calculation which increases cost. The method outlined below is only an approximation but provides a "rule of thumb" method for determining the proper sampling rate.

C. R. Wali²² in his discussion of quantizing and sampling errors treated the process as adding noise to the signal and estimated the relative RMS error in converting a sine wave to be

$$E_r^2 = 1/12 (q^2 + (1.11 \lambda)^2)$$

where q = quantization level

$$\lambda = 4W_o/W_s$$

W_o = frequency of sine wave

W_s = sampling frequency

If we let W_o be the highest frequency that we need to sample and assume that frequencies above this will be filtered out before the sampling is performed then we can say that W_s will be approximately the required sampling frequency.

It should be noted that this equation assumes that some scheme has been used to compensate for the time delay inherent in the process. Also, since it considers only the error in the fundamental component, it can be only a relative measure of the actual error.

As an example, if it is desired to keep this error below .1% and we assume a quantization level of 100_{mV} in a 100 volt analog computer, according to the above equation this would suggest a sampling rate of approximately 100 samples per cycle of the signal of interest.

When the two computers are used in a closed-loop configuration, the phase shift inherent in the conversion process is required to be below a certain level. This can provide an alternate method of determining the proper sampling frequency. Recognizing the phase shift to be $T/2$, where T is the sampling interval, we see that the phase shift in degrees is $\phi = (W_o/2W_s) \times 360^\circ$. If we know the maximum allowable ϕ , and the fundamental frequency of solution, W_o , we can then pick the appropriate sampling frequency, W_s .

XI. GENERAL ASSIGNMENT PHILOSOPHY

The primary consideration in the assignment of the equations to either the analog or digital computers should be the utilization of the best features of each computer. It is important to recognize the mathematics for which each is the best suited. This was outlined in sections V and VI but in summary we will consider these mathematical factors in general terms. The forte of the digital computer is the solution of algebraic equations. Numerical integration comes as a by-product of the digital computer's power in solving these equations. Time is the only penalty. If high precision is not needed, the integration is better performed by the analog computer because the solution of ordinary differential equations is its strong feature.

Efficient computer usage is also an important factor. It is important for computer efficiency that its computation interval be as close as possible to but less than the maximum allowable for real time. Once this is achieved, the requirement for a minimum amount of analog equipment should naturally follow.

The following is a list of those types of equations that are commonly thought to indicate the need for a hybrid approach.^{2, 5, 12, 13}

1. Simultaneous differential equations with widely different parameters which produce both high and low frequencies in their solution.

2. Differential equations to be solved at high speed, their solutions for different initial conditions or parameters being used in a prediction, iteration or optimization process.
3. Combinations of continuous and discrete variables as in the description of a sample data system or a computer control system.
4. Perturbation analysis about slowly changing, precisely established solutions.
5. Partial differential equations to be solved by serial integration procedures.
6. Ordinary differential equations accompanied by transport delays.
7. Boundary value problems.

Next is a listing of the equations and operations that are typically handled by the different computers. This list is based solely on the capabilities of the machines. Many are capable of being solved on either machine and the final selection should be based on the other factors discussed in the paper.

Analog

1. Time integration.
2. Constant by variable products.
3. Implicit calculations.

4. Limits, dead zone, etc.
5. Flight simulator coupling.

Digital

1. Multi-variable arbitrary functions generation.
2. Variable by variable products.
3. Trigonometric calculations.
4. Logic control.

There are probably few, if any, simple hybrid computer applications. In addition there are some problems that are now and probably will always be the exclusive realm of the digital computer. These include data process simulation, information handling systems, business systems, TV coding, character reading, communication coding, etc.

XII. A POSSIBLE APPROACH TO ASSIGNMENT OF EQUATIONS

Once we have decided to use a hybrid approach, we must decide which equations or portions of equations will be solved on each computer. This decision will influence to a large extent the degree of success of the simulation. Many of the characteristics of the equations that indicated the need for hybrid will also indicate where these equations should be solved. As mentioned earlier, this approach has no firm mathematical basis; it is not presumed that the approach can be shown to be "necessary and sufficient".

We will start by assuming that we have available a general purpose analog computer, an associated complement of parallel logic, general purpose interface equipment, and a general purpose digital computer. Second, we will assume that we know three things about the variables of the problem: the approximate dynamic range, the approximate frequencies of the variables and an idea of the accuracy to which each of the variables needs to be calculated.

Next we identify those variables whose values during the simulation are of interest or are needed for external equipment. We then classify the first group as analog outputs (scope displays, recorders, etc.) and digital outputs (tables, final answers, etc.). The second group is classified as to the nature of the equipment used i. e. analog or digital. We then proceed as follows:

1. Those equations that represent a subsystem to be substituted into the simulation at a later time must be grouped on the computer or portion of the computer they are meant to replace.
2. Determine the static resolution that the analog is capable of in terms of its noise and drift figures and the length of time the simulation will be operated. Using this figure determine the maximum dynamic range that the analog can handle. All of those variables that have a higher dynamic range should then be placed on the digital.
3. Using the results of 2, determine the computational interval needed to calculate these variables and add this to the conversion times of the particular interface equipment being used. This time places a lower limit on the sampling interval.
4. Using the method outlined in section XB and the available accuracy bounds on the variables, determine the maximum frequency that can be successfully converted. Any variables whose frequency components exceed this must be placed on the analog.
5. Determine (Section IX A) those variables whose coefficient or multiplier inaccuracies indicate that prohibitive errors will be introduced by analog computation and place them on the digital computer.
6. Examine those variables selected for digital computation in steps 2 and 5 and determine which one has the combination of frequency and accuracy limitations that poses the most stringent limitations on sampling frequency. Use this information, as suggested in section

X B, to determine the proper sampling frequency. This step will also determine the allowable computational interval.

7. At this point careful note should be made of interval that has been used for computation including appropriate compensation schemes (see references 13, 21-26).
8. Next, consideration should be given to those equations of a control nature that will be placed on the parallel logic computer portion of the analog.
9. We will assume that the rest of the equations (if any) can be successfully solved on either computer. The object now is to assign the rest of the equations with a minimum of computing equipment.
10. The primary consideration in minimizing equipment, as pointed out in section V, is to make use of all of the digital computational interval that is left. Assignment should be made according to the functions for which each computer is best suited. Equations of an algebraic and logic nature should be given first priority followed by function generation, etc., as outlined in sections VI, VIII, and IX. This process should be continued until all of the available digital computational interval is exhausted.
11. If it is desired to optimize accuracy rather than equipment cost an alternate approach to step 10 would be to tighten the accuracy requirements on the remaining variables and proceed again with steps 2, 4, 6, and 7 until all of the variables are assigned or the remaining digital computational interval is reached.

A. Example Problem

To illustrate the above procedure, we will apply it to a problem that has been chosen for hybrid computation. This problem is a re-entry guidance problem for the Gemini spacecraft program. The equations of the system are:

1. $\dot{V} = -D - (q - rw^2 \cos^2 i) \sin \gamma$
2. $\dot{\gamma} = \frac{L \cos \phi}{V} + \frac{V}{r} - \frac{q - rw^2 \cos^2 i}{V} \cos \gamma$
3. $\dot{h} = V \sin \gamma + \dot{h}_e$
4. $\dot{\theta} = \frac{V}{r} \frac{(\cos \gamma \sin \psi)}{\cos l}$
5. $\dot{l} = \frac{V}{r} \cos \gamma \cos \psi$
6. $\dot{\psi} = \frac{L \sin \phi}{V \cos \gamma} + \frac{V}{r} \sin \psi \tan l$
7. $\cos i = \cos \phi \sin \psi$
8. $q = 1/2 \rho V^2$
9. $L = (K_1 q C_L) (A/M)$
10. $D = (K_2 q C_D) (A/M)$
11. $A_V = L \cos \gamma \cos \phi + \frac{V^2}{r} - q$
12. $C_D = C_L / (L/D)$
13. $A_L = \frac{L \sin \alpha - D \cos \alpha}{g}$

$$14. \quad A_n = \frac{L \cos \alpha + D \sin \alpha}{g}$$

$$15. \quad \text{Check } \rho \leq \rho_Q$$

$$16. \quad \text{Yes } \dot{Q} = 653 \left(\frac{\rho}{2.38 \times 10^{-3}} \right) \frac{1}{2} \left(\frac{V}{10^4} \right) 3.15$$

$$17. \quad \text{No } \dot{Q} = 4960 \left(\frac{\rho}{2.38 \times 10^{-3}} \right) (.718) \left(\frac{V}{10^4} \right) 3.45$$

$$18. \quad W_x = .02 \int |\ddot{\phi}| \cos \alpha dt$$

$$19. \quad W_z = .055 \int |\ddot{\phi}| \sin \alpha dt$$

$$20. \quad W_{xz} = .00115 \int \dot{\phi}^2 \sin \alpha \cos \alpha dt$$

$$21. \quad W_y = .0714 |\alpha|$$

$$22. \quad W_T = W_x + W_y + W_z + W_{xz}$$

$$23. \quad s\alpha(s) = K_3 (\Delta\alpha) / (1 + \tau_1 s) (1 + \tau_2 s)$$

$$24. \quad s\phi(s) = K_4 (\Delta\alpha) / (1 + \tau_3 s)$$

The constants for the above equations are:

$$r = 2.11 \times 10^7 \quad K_3 = 4$$

$$W = 7.29 \times 10^{-5} \quad K_4 = 10$$

$$A/M = .7 \quad \tau_1 = .1$$

$$g = 32.17 \quad \tau_2 = .2 \quad \tau_3 = .01$$

C_L , K_1 , K_2 , and L/D are furnished in tabular functions of α . \dot{h}_E and ρ are inputs from external equipment.

To make the selection easier, we will list the variables separately along with information on their dynamic range, accuracy, and frequency characteristics.

Variable	Dynamic Range	Frequency (in cps)	Accuracy required
C_L	10^3	--	1 %
C_D	10^3	--	1 %
K_1	10^3	--	1 %
K_2	10^3	--	1 %
A_V	5×10^2	0.4 cps	5° phase
L/D	10^2	--	5 %
ρ	10^4	--	1 %
Q	5×10^5	--	0.5 %
\dot{Q}	5×10^4	--	0.5 %
V	2×10^5	--	0.2 %
\dot{V}	10^2	--	0.2 %
γ	10^2	0.1 cps	3 %
$\dot{\gamma}$	10^2	0.1 cps	3 %
h	5×10^5	--	0.1 %
\dot{h}	10^4	--	0.1 %
\dot{h}_E	5×10^4	--	0.1 %
θ	10^3	0.1 cps	2 %
$\dot{\theta}$	10^2	0.1 cps	2 %
l	10^3	0.1 cps	3° phase
\dot{l}	10^2	0.1 cps	3° phase
ψ	10^3	0.3 cps	3.5° phase
$\dot{\psi}$	10^2	0.3 cps	3.5° phase

Variable	Dynamic Range	Frequency	Accuracy required
W_x	10^3	3.2 cps	3.5^0
W_y	10^3	2.2 cps	3.0^0
W_z	10^3	2.5 cps	3.0^0
W_{xz}	10^3	5.0 cps	3.0^0
W_t	10^3	5.0 cps	3.0^0
ϕ	10^3	0.2 cps	3.5^0
$\dot{\phi}$	10^2	0.2 cps	3.5^0
α	10^3	0.4 cps	5.0^0
$\dot{\alpha}$	10^2	0.4 cps	5.0^0

Next, we will assume that we have a general purpose analog computer, with a dynamic range capability of 10^4 , a general purpose conversion unit whose combined I/O times for the number of variables we want to use is roughly 0.5 ms, and a general purpose digital computer that will be entirely committed to this problem.

With the above information, we will proceed with the steps outlined above and assign the equations to the two computers. To simplify the example, we will assume that the variables can be displayed from either the analog or the digital satisfactorily. The numbered steps below correspond to the numbered steps at the first of this section.

1. For this example, we will assume that all external equipment, such as the guidance computer is available for simulation and that no special grouping is needed.
2. From the list of variables, we see that the dynamic range of V , h , \dot{h} , \dot{h}_E , Q , and \dot{Q} all exceed the dynamic range of the analog. Therefore, we must place equations 2-6, 8, 16, 17, and the second term in equation 11 on the digital computer.
3. When the equations from step 2 were placed on the digital computer, they required 20 ms for a complete cycle which sets a lower limit of $1/20 \times 10^{-3}$ or 50 cps on the sampling frequency.
4. If we assume a maximum allowable phase error of 3.0° , and use the results of section X B, we see that the highest frequency that we can successfully convert is: $W_o = 2W_s (\phi/360) = 2(1/20 \times 10^{-3})(3.5/360) \approx 1$ cps. From the list of variables, we see that W_x , W_y , W_z , and W_{xz} exceed this frequency and must be placed on the analog. This includes equations 18-21.
5. There are none of the equations left that would exhibit prohibitive coefficient inaccuracies, but equation 1 and the first term in equation 11 would exhibit prohibitive multiplier inaccuracies if solved on the analog. We therefore assign equation 1 and the rest of equation 11 to the digital computer.

6. When we examine the equations selected for digital computation, we find that equations 5 and 6 seem to pose the most stringent limitation on conversion. Since $\dot{\psi}$ must be available for external analog equipment, we use its frequency and accuracy limitations to determine the proper sampling interval as explained in section X B.

$W_s = (W_o \times 360)/(2\phi) = (0.4 \times 360)/(2 \times 3.5) \approx 20$ cps. We therefore have 50 ms available for a complete conversion and calculation cycle. Considering the equations already on the digital, we have approximately 25 ms of computation time still available.

7. & 8. We will assume, at this point, that we have already provided for any necessary compensation schemes in the digital program and that their times have been included in the time already used in the digital program.

9. Since K_1 , K_2 , and L/D are available in tabular form, we will use the large memory capabilities and interpolation routines of the digital for their calculations. Since these variables are available on the digital and the equations that use them are of an algebraic nature, we will attempt to assign equations 9, 10, 12, 13, and 14 to the digital computer. Next in our priority comes the logic equation 15 and last in our priority would be equations 22-24.

10. The additional digital program time available for the solution of the remaining equations permits equations 9, 10, and 12-15 to be

solved on the digital computer without exceeding the remaining computation time. However, the addition of equation 22, 23, or 24 would exceed this time. We therefore assign them to the analog.

This completes the assignment of equations for this problem. The digital computer allowed us to handle a wide dynamic range and still operate in real time. By solving the high frequency equations on the analog, we were able to permit a relatively slow sampling rate. This slow sampling rate allowed the digital computer to solve the majority of the equations (1-17) and a relatively small amount of analog equipment was required for the remainder of the equations.

XIII. CONCLUSIONS

The problems of choosing a computational method and of separation for hybrid are complex and the final solutions still elude us. As long as new equipment and techniques continue to develop, the questions may never reach a final conclusion. However, there is much we do know about the nature of the analog and digital computers and much of this can be brought to bear on the problems.

We have shown that proper definition of the problem and proper definition of the solutions needed give us the insight we need to decide on a computation method. Under definition of the problem, we include the types of equations to be solved, the dynamic range and frequency of the variables, the accuracy needed, the economic factors, and the external equipment available for the solution. In the definition of the solution, we take into account whether or not real time operation is required, the types of outputs needed, and how many runs are required. The last technique brought to bear on this problem is error analysis, and we attempt to determine whether the computational method in question will be able to furnish us with meaningful results.

Finally, we see that much of the information above can be used to determine the best way to assign the equations for hybrid computation. From an accuracy point of view, the dynamic range and frequencies of the variables provide the initial separation criterion, while the efficient usage of the computers completes the separation.

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