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# Performance characteristics of a tubular regenerative heat exchanger

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# PERFORMANCE CHARACTERISTICS OF A TUBULAR REGENERATIVE HEAT EXCHANGER

BY

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THESIS

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Approved by ______________________
Professor of Mechanical Engineering

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# TABLE OF CONTENTS

# LIST OF ILLUSTRATIONS

# LIST OF PLATES

# INTRODUCTION

The term regenerator is widely used to define any type of heat exchanger in which the heat is alternately stored and removed. In the ordinary heat exchanger the two working fluids exchange heat through a solid wall, whereas the distinguishing feature of the regenerator is that the same space is alternately occupied by two gases. During the flow period of the hot gas, heat is transferred to the confining walls or to solid material within the heater and the colder gas picks up this heat while flowing through this space at another time interval. In most cases the method is to pass the two fluids alternately through the same passages in opposite directions.

Hereafter, throughout this paper the term regenerator will refer to the storage type of heat exchanger; one in which matrix material is first heated by the flow of hot gases and subsequently cooled by air flow. The term "matrix" refers to that part of the heater involved in the exchange of heat. A further limitation on the term regenerator is that it will refer to the "single-blow" type, which employs a single heating period followed by a single cooling period.

The storage heater has three major advantages: (1) A much more compact heat transfer surface can be employed. (2) The heat transfer surface in general is much less expensive. (3) Instead of sealing all passages to prevent leakage between the two fluids, only overall sealing is needed. These features widen the possible range of materials for regenerators and thus the variety of materials considered has ranged from rocks to bundles of steel tubes and corrugated metallic ribbons.

In the storage type heater energy can be stored at a relatively low rate and then withdrawn at a high rate as may be required for operation. Thus the storage heater is very adaptable for use in connection with intermittent or blowdown wind tunels since large amounts of heat are needed during the short run period and can be replaced when the tunnel is not in operation. This heat, required for establishment of air flow at desired values of stagnation temperature, is often supplied from bundles of tubes or sheets which are preheated to approximately the desired stagnation temperature of the air. Little information is available on the design of this type of heater.

The present investigation is concerned with the calculation of the characteristics of the "single-blow", tubular regenerative heater, its dimensions and performance. In the analysis in this thesis, the heat is considered to be stored in bundles of tubes initially preheated by hot gases flowing through the heater and subsequently cooled by the flow of air. This paper makes no attempt to deal with the effects of radiation from the outer row of tubes to the heater shell.

This subject was chosen because previous solutions for the "single-blow" regenerator are limited in scope. For this type of heater, the problem of first obtaining the temperature distribution along the tube after the gas flow period and then determining the heater performance during the air flow period has not been previously considered.

## REVIEW OF LITERATURE

A survey of the literature on regenerators indicates that two distinct procedures have been employed for determining the performance of storage heaters. The first procedure consists of simple approximate equations determined from empirical data. However, the regenerator process is so complicated that only in special cases will this method provide reasonable results. The other procedure is the analytical approach.

Some of the major publications on the analytical side are due to Schumann,[1] Hansen,[2] Schack,[3] and Ackermann.[4]

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(1) Schumann, T.E.W., Heat Transfer: A Liquid Flowing Through a Porous Prism, Journal of the Franklin Institute, Vol. 208, 1929, pp. 405-416.

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(2) Hansen, Von H., Naherungsverfahren zur Berechnung des Warmeaustausches in Regeneratoren, Zeitschrift fur Angewandte Mathematik und Mechanik, Vol. 11, 1931, pp. 105-114.

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(3) Schack, A., Industrial Heat Transfer, Translated by H. Goldschmidt and E.P. Partridge. New York, John Wiley and Sons, Inc., 1933, pp. 237-264.

(4) Ackermann, Von G., Die Theorie der Warmeaustauscher mit Warmespeicherung, Zeitschrift fur Angewandte Mathematik und Mechanik, Vol. 11, 1931, pp. 192-205.

Several analytical solutions plus detailed approximate numerical and graphical methods on a theoretical basis may be found in Jakob.[5]

(5) Jakob, Max. Heat Transfer, Vol. II, New York, John Wiley and Sons, Inc., 1957, pp. 261-341.

An excellent historical review has been presented by Iliffe.[6]

(6) Iliffe, C.E., Thermal Analysis of the Contra-flow Regenerative Heat Exchanger, The Institution of Mechanical Engineers, London, 1948. Advanced Copies.

The majority of the forementioned literature pertains to regenerators used for air preheating for furnances and boilers or for air liquefaction. Most of the available information dealing with regenerators designed for use in connection with wind tunnels is found in the form of papers or reports.

Judd[7] presents an analytical method for the com-

(7) Judd, J.H., Transient Temperatures in Heat Exchangers for Supersonic Blowdown Tunnels, National Advisory Committee for Aeronautics, Technical Note 3078, Washington, 1954.

putation of tube and fluid temperatures in a heat exchanger consisting of bundles of tubes preheated to a constant axial temperature.

One design procedure, essentially a compromise of allowable lengths based on available pressure drop and required lengths based on an idealized heating process corrected for time lag, is found in reference (8).

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(8) AEDC TM-56-4, Design Studies of Tunnel D, Gas Dynamics Facility, 1956.

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For storage heaters filled with spherical particles, one method of calculating the size of the heater and the characteristics of its performance is presented by Krahn.[9]

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(9) Krahn, E., The Storage Heater for Tunnel 8 at NOL, Navord Report 4449, 1957, White Oak, Maryland.

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Excellent summaries of the extensive investigations on convective heat transfer to and from gases flowing in tubes may be found in McAdams[10] and Jakob.[11]

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(10) McAdams, W.H., Heat Transmission, Third Edition, 1954, New York, McGraw Hill, pp. 202-250.

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(11) Jakob, Max. Heat Transfer, 1st. Edition, Vol. 1, New York, John Wiley & Sons, Inc., 1949, pp. 443-480.

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Kays and London[12] present, in graphical form a

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(12) Kays, W.M. and London, A.L., Compact Heat Exchangers, Palo Alto, California, The National Press, 1955, pp. 47-53.

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number of useful analytical solutions for heat transfer and fluid friction for flow in both circular and rectangular tubes.

Throughout all of the available literature, no general analytical solution was established for the tubular, single blow regenerative heater.

# NOMENCLATURE

| SYMBOL | UNITS | SIGNIFICANCE |
|---|---|---|
| $\theta$ | second | time |
| x | feet | axial distance along heater |
| q | Btu | quantity of heat |
| h | Btu/sec.ft.$^{o}$F | heat transfer coefficient |
| P | feet | flow perimeter |
| $A_c$ | sq. ft. | total frontal area |
| $A_f$ | sq. ft. | flow area |
| t | $^{o}$F | temperature |
| T | $^{o}$F | temperature difference |
| $\rho$ | lb./cu. ft. | density |
| c | Btu/lb.$^{o}$F | specific heat |
| $c_p$ | Btu/lb.$^{o}$F | specific heat at constant pressure |
| V | ft./sec. | fluid velocity |
| D | feet | tube diameter |
| L | feet | total flow length |
| $J_o$ | | Bessel function of zero order |
| w | lbs./sec. | mass rate of flow |

SUBSCRIPTS

a - air; air period

g - gas; gas period

w - matrix

i - initial; inside

o - outside

# DISCUSSION

This investigation is concerned with the analysis of heat exchangers of the storage type in which the matrix consists of a bundle of tubes through which the working fluid flows. Except for the outer row of tubes where a small amount of heat may be transferred to the surrounding shell, the fluid provides the only means of heat transfer during the operation.

One of the basic characteristics of a storage type heater is that as heat is transferred to or from it by the flowing fluid, the temperature of both the heat storage material and the fluid changes with time. The nature of this variation depends primarily upon the initial temperature distribution within the storage heater. Figure No. 1, page 10, is a graphical representation of the axial temperature distribution during the heating period.

In order to study the transient performance of any heat exchanger, a sufficiently general analytical model must be established which is both an adequate idealization of the physical system and capable of reasonable mathematical description. Such a model is pictured schematically in Figure 2, page 11, for a counterflow storage heater of the type considered.

The analytical treatment employs the following

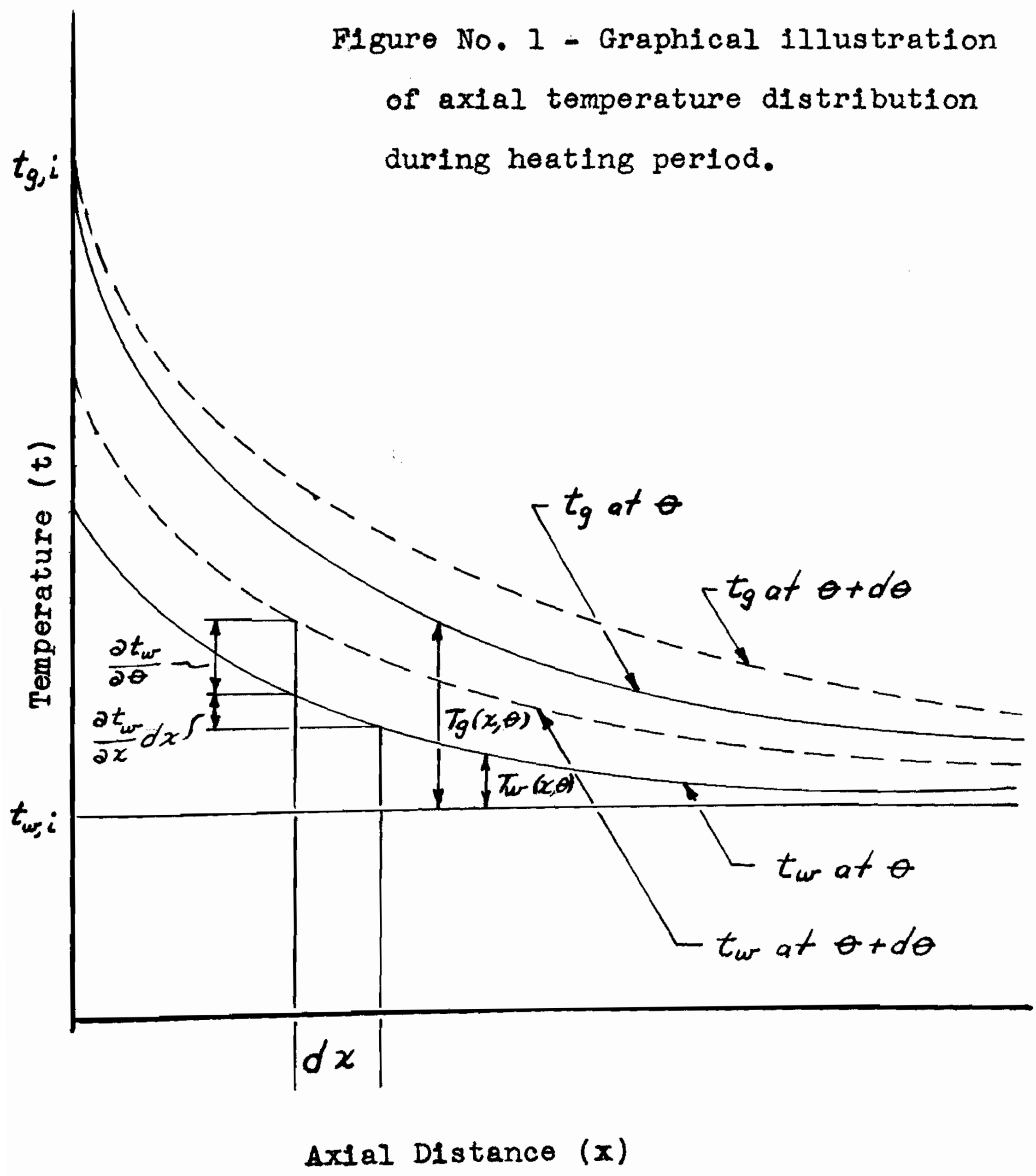


Figure No. 1 - Graphical illustration of axial temperature distribution during heating period.

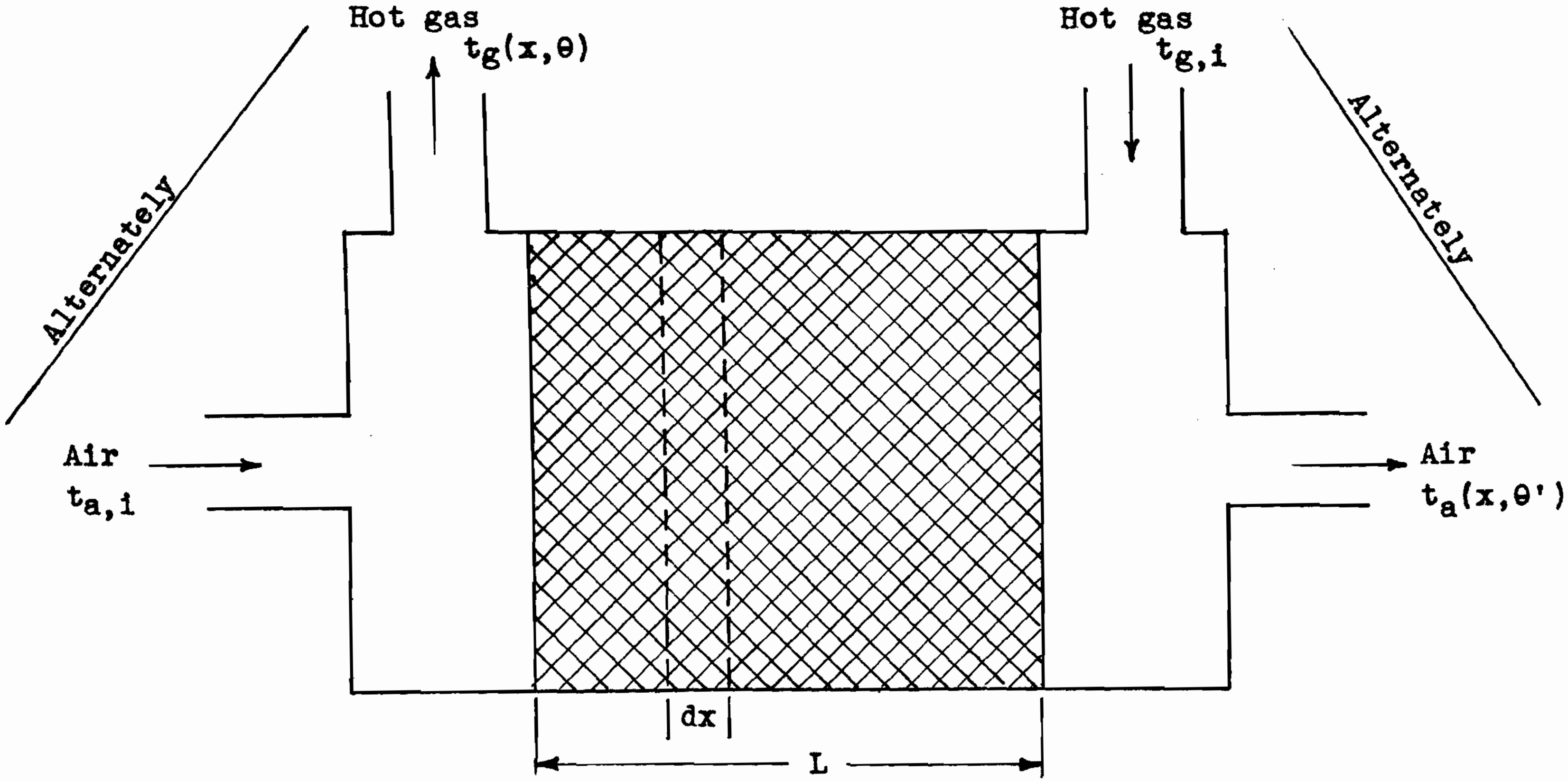


Figure No. 2 - Heat Exchanger Model

idealizations useful for this transient study:

1. The thermal conductivity of the matrix is zero in the gas and air flow directions, and infinite in the normal direction to the flow.

2. Physical properties of the fluids and the wall and also the heat transfer coefficient are independent of time and position and are to be evaluated at the average temperature within the exchanger.

3. The temperature of each of the fluids and the wall are functions of time $\theta$ and distance $x$; $t = t(x,\theta)$. This idealization one-dimensionalizes the problem.

4. The system is overall adiabatic; that is, perfect insulation surrounds the exchanger and thus the effect of radiation from the outer row of tubes to the shell is neglected.

5. The mass flow rates, $w_a$ and $w_g$, are constant.

6. Entering fluid temperatures are constant with time.

The first of these idealizations was investigated by Hansen[13] and shown to be satisfied for most cases

(13) Hansen, op.cit., pp. 105-114.

of practical importance. Thermal conduction in the direction of flow may be prevented almost entirely, but even matrices of reasonable wall thickness have negligible longitudinal heat transfer. Large thickness of the matrix material leads to considerable waste heat being trapped within the material. The use of very thin tubular elements will minimize wasted heat, and tend to validate the first idealization.

Saunders and Smoleniec(14) found that the variation

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(14) Saunders, O.A., and Smoleniec, S., Heat Transfer in Regenerators, IME-ASME General Discussion on Heat Transfer, London, England, 1951, p. 443.

---

in fluid and matrix specific heat resulted in little error.

The fifth idealization corresponds to the usual assumption of steady-state flow conditions.

The other idealizations parallel those usually made in conventional heat exchanger design theory.

Consideration of the performance of the storage type heater can be logically divided into two phases. First there is the heating-up or charging period during which hot gases flow through the heater and heat the matrix material to the desired temperature. The second phase consists of the blowdown period during which the storage heater performs its primary function of heating the air passing through it.

# ANALYSIS OF MATRIX TEMPERATURE DISTRIBUTION DURING GAS PERIOD

Based upon the forementioned idealizations, the differential equations relating the system temperatures may be derived from energy balance and heat-transfer-rate considerations.

Considering first an element of the heat exchanger of differential length dx, flow perimeter P, total frontal area $A_c$ and flow area $A_f$; the heat energy received from the hot gases in the time element $d\theta$ is

$$d^2q = h_g P\,dx\,(t_g - t_w)\,d\theta$$

where $t_g$ denotes the gas temperature and $t_w$ the matrix temperature. To subsequently reduce the mathematical expressions, let

$$T_g = t_g - t_{w,i}$$

and

$$T_w = t_w - t_{w,i}$$

where $t_{w,i}$ represents the uniform initial temperature throughout the heater. Thus

$$d^2q = h_g P\,dx\,(T_g - T_w)\,d\theta .$$

The heat stored in the matrix is

$$d^2q' = \rho_w c_w (A_c - A_f)\,dx\,\frac{\partial T_w}{\partial \theta}\,d\theta$$

where $\rho_w$ is the density of the matrix material and $c_w$ is the specific heat of the material.

Since the heat received equals the heat stored, it follows that

$$h_g P dx (T_g - T_w) d\theta = \rho_w c_w (A_c - A_f) dx \frac{\partial T_w}{\partial \theta} d\theta.$$

Thus

$$\frac{\partial T_w}{\partial \theta} = \frac{h_g P (T_g - T_w)}{\rho_w c_w (A_c - A_f)}$$

where for simplification

$$A = \frac{h_g P}{\rho_w c_w (A_c - A_f)} \qquad \text{(a)}$$

is introduced since these terms are independent of x and θ, at least by the initial idealizations.

Therefore,

$$\frac{\partial T_w}{\partial \theta} = A (T_g - T_w) \qquad \text{(b)}$$

Now consider the transfer of heat from an element of the gas; the heat imparted to this element by the matrix will be

$$d^2 q_1 = -h_g P dx (T_g - T_w) d\theta.$$

The heat carried in by the flowing gas is

$$d^2 q_2 = -\rho_g C_{p_g} A_f \, dx \frac{dx}{d\theta} \frac{\partial T_g}{\partial x} d\theta .$$

If the gas velocity $V_g$ is substituted for $\frac{dx}{d\theta}$, then

$$d^2 q_2 = -\rho_g C_{p_g} A_f \, dx \, V_g \frac{\partial T_g}{\partial x} d\theta .$$

The heat energy stored in the element of the hot gas is

$$d^2 q_3 = \rho_g C_{p_g} A_f \, dx \frac{\partial T_g}{\partial \theta} d\theta .$$

For the necessary heat balance

$$d^2 q_1 + d^2 q_2 = d^2 q_3$$

or

$$-h_g P \, dx (T_g - T_w) d\theta - \rho_g C_{p_g} A_f V_g \, dx \frac{\partial T_g}{\partial x} d\theta = \rho_g C_{p_g} A_f \, dx \frac{\partial T_g}{\partial \theta} d\theta$$

or

$$\frac{\partial T_g}{\partial \theta} + V_g \frac{\partial T_g}{\partial x} = -\beta (T_g - T_w) \qquad \text{(c)}$$

where a constant was introduced; defined as

$$\beta = \frac{h_g P}{A_f c_{p_g} \rho_g} \quad . \tag{d}$$

Equations (b) and (c) determine the transfer of heat. For a complete solution to the problem under consideration, the following boundary and initial temperature conditions must be introduced and satisfied:

$$T_g(0,\theta) = t_{g,i} - t_{w,i} = T_1 \text{ (constant).} \tag{e}$$

$$T_w(x,0) = 0 . \tag{f}$$

Hence, from equations (e) and (b)

$$\frac{dT_w}{d\theta} = A(T_1 - T_w) \quad \text{at} \quad x=0$$

or

$$\frac{dT_w}{T_w - T_1} = -A\,d\theta .$$

By integration, with lower limits of zero

$$\ln \frac{(T_w - T_1)}{-T_1} = -A\theta$$

or

$$\frac{T_w - T_1}{-T_1} = e^{-A\theta}$$

or

$$T_w(0,\theta) = T_1\left[1 - e^{-A\theta}\right]. \tag{g}$$

At $x = V_g\theta$, the foremost element of the gas is always in contact with the part of the solid which has an initial temperature excess of $T_w = 0$.

At $x = V_g\theta$:

$$\frac{\partial T_g}{\partial x} = 0$$

and from equation (c)

$$\frac{\partial T_g}{\partial \theta} = -\beta T_g$$

or

$$\frac{dT_g}{T_g} = -\beta d\theta.$$

By integration,

$$T_g(V_g\theta, \theta) = Ne^{-\beta\theta}$$

where N is a constant of integration. From $x = V_g\theta$ it follows that $x = 0$ for $\theta = 0$. Therefore, from eq. (e)

$$T_g(0,0) = T_1$$

and

$$T_g(0,0) = N$$

and

$$T_g(V_g\theta, \theta) = T_1 e^{-\beta\theta}. \qquad \text{(h)}$$

The differential equations relating the system temperatures and the associated boundary conditions are restated here for convenience:

$$\frac{\partial T_w}{\partial \theta} = A(T_g - T_w). \qquad \text{(b)}$$

$$\frac{\partial T_g}{\partial \theta} + V_g \frac{\partial T_g}{\partial x} = -\beta(T_g - T_w). \qquad \text{(c)}$$

$$T_g(0, \theta) = T_1. \qquad \text{(e)}$$

$$T_w(x, 0) = 0. \qquad \text{(f)}$$

$$T_w(0, \theta) = T_1[1 - e^{-A\theta}]. \qquad \text{(g)}$$

$$T_g(V_g\theta, \theta) = T_1 e^{-\beta\theta}. \qquad \text{(h)}$$

With the exception of the constants $A$ and $\beta$, the differential equations are the same as those at which Schumann(15) arrived for flow through a porous prism.

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(15) Schumann, T.E.W., op. cit. pp. 407-412.

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The classical method of Schumann is given in Appendix A and only the solution is presented here. Both are reworked due to the difference in constants and nomenclature.

The resulting equation, expressing the axial temperature distribution throughout the heater in terms of time, axial distance and initial conditions, is

$$t_w = (t_{g,i} - t_{w,i})\left[e^{-\xi-\eta}\sum_{n=1}^{\infty}\eta^n M_n(\xi\eta)\right] + t_{w,i}; \qquad \text{(j)}$$

where:

$$\xi = \beta \frac{x}{V_g} ;$$

$$\eta = k\left(\theta - \frac{x}{V_g}\right) ;$$

$$M_0(\xi, \eta) = J_0(2i\sqrt{\xi\eta}) ;$$

$$M_n = \frac{d^n J_0(2i\sqrt{\xi\eta})}{d(\xi\eta)^n} .$$

By substituting the heating time (length of the gas flow period) into this equation, along with the values for the physical properties, the axial temperature distribution throughout the heater after the heating period is determined. Assuming instantaneous switching from gas flow to air flow, this will be the temperature distribution at the beginning of the air flow period.

It is obvious that Equation (j) is not practical for use by a design engineer. To alleviate this situation, a graph has been plotted of $T_w/T_1$ for values of $\xi$ and $\eta$ ranging from zero to 10. This graph is shown in Plate 1. The points necessary for plotting this graph were obtained from Schumann's curves.

## PERFORMANCE ANALYSIS DURING AIR FLOW PERIOD

When the length of the heating period is substituted into equation (j), the heater has a specified initial temperature distribution along its length for the air flow period. A stream of air is introduced, at a specified mass flow rate, with the entering temperature constant in respect to time. The temperature history of the air and the matrix is to be determined. For a specified heating period, the axial temperature distribution of the matrix at the start of the air flow reduces to a function of x; $t'_{w,i} = t'_w(x',0) = f(x)$, where $x' = x$ for parallel flow and for counterflow $= L - x$.

Again considering an element of the heat exchanger of differential length dx, the heat energy transferred to the air in the time element $d\theta$ is

$$d^2q_a = h_a P\,dx\,(t_w - t_a)\,d\theta,$$

where $t_a$ denotes the air temperature. To subsequently reduce the mathematical expressions, let

$$T_a = t_a - t'_{w,i}$$

and

$$T'_w = t_w - t'_{w,i}$$

where $t'_{w,i}$ = initial matrix temperature at start of air flow.

Thus

$$d^2 q_a = h_a P dz (T_w' - T_a) d\theta .$$

The heat lost by the matrix is

$$d^2 q_a' = \rho_w c_w (A_c - A_f) dz \frac{\partial T_w'}{\partial \theta} d\theta .$$

Since the heat transferred is equal to the heat lost, it follows that

$$h_a P dz (T_w' - T_a) d\theta = \rho_w c_w (A_c - A_f) dz \frac{\partial T_w'}{\partial \theta} d\theta .$$

Thus

$$\frac{\partial T_w'}{\partial \theta} = \frac{h_a P (T_w' - T_a)}{\rho_w c_w (A_c - A_f)}$$

where for simplification

$$A' = \frac{h_a P}{\rho_w c_w (A_c - A_f)} \qquad (k)$$

is introduced. Therefore,

$$\frac{\partial T_w'}{\partial \theta} = A' (T_w' - T_a) . \qquad (m)$$

Now consider an element of the air; the heat imparted to this element by the matrix will be

$$d^2 q_{a,1} = h_a P dz (T_w' - T_a) d\theta .$$

The excess of outflow over inflow thermal energy in the air stream is

$$d^2 q_{a,2} = \rho_a C_{pa} A_f \, dx \, V_a \frac{\partial T_a}{\partial x} d\theta.$$

The heat energy stored in the element of the air is

$$d^2 q_{a,3} = - \rho_a c_{pa} A_f \, dx \frac{\partial T_a}{\partial \theta} d\theta.$$

The energy balance yields

$$\frac{\partial T_a}{\partial \theta} + V_a \frac{\partial T_a}{\partial x} = -\beta' (T_w' - T_a) \qquad \text{(n)}$$

where

$$\beta' = \frac{h_a P}{A_f C_{pa} \rho_a} . \qquad \text{(p)}$$

The boundary conditions for the air flow period based upon an initial matrix temperature represented by f(x) are:

$$T_a(0, \theta) = t_{a,i} - t_{w,i} = \phi(x).$$

$$T_w(x, 0) = 0.$$

A strict mathematical solution based upon these boundary conditions would be quite complex and it is doubtful as to its usefulness, especially for design engineering. It would hardly be possible to precalculate curves for all the situations that might arise.

For this reason, an approximate method has been devised. In applying this approximate method, the continuous process is replaced by a stepwise one, which might be termed an increment method.

After first determining the axial temperature distribution in the matrix after the heating period, the length of the heater is divided up into a number of finite increments each having a relatively constant axial temperature.

Figure 3, page 25, illustrates graphically a possible temperature distribution in the matrix at the beginning of the air flow period and the stepwise division into relatively isothermal lengths. The temperature $t_1$ approximates the average temperature of the crosshatched zone. The temperature along this crosshatched zone is then assumed to be constant at $t_1$.

Thus if $t'_{w,i} = f(x)$ is replaced by a constant temperature over the increment, the solution follows the method employed for the heating period. The boundary conditions for a constant initial matrix temperature are:

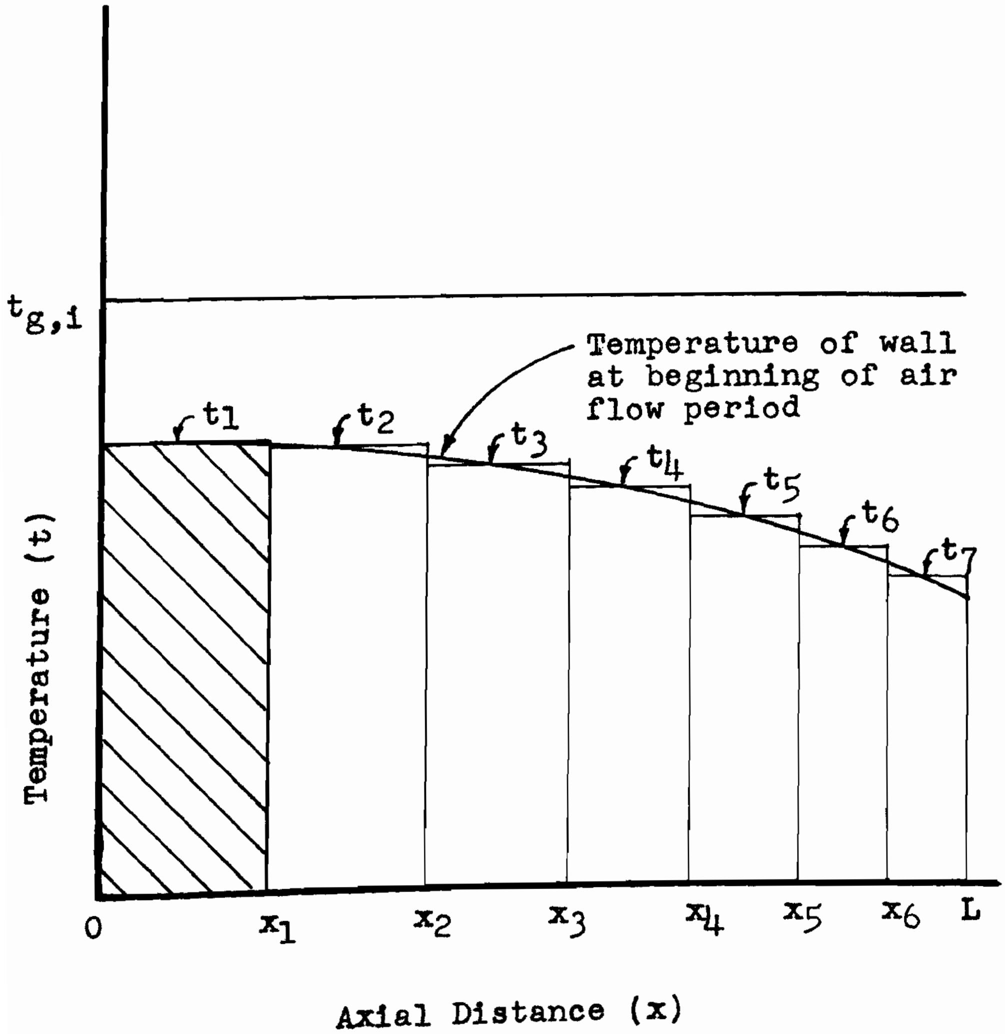
$$T_a(0,\theta) = t_{a,i} - t'_{w,i} = T_o \quad (\text{e.g.} - T_o = t_{a,i} - t_1 \text{ from } 0 \text{ to } x_1). \tag{r}$$

$$T'_w(x,0) = 0. \tag{s}$$

$$T'_w(0,\theta) = T_o[1 - e^{-A'\theta}]. \tag{t}$$

$$T_a(V_a\theta,\theta) = T_o e^{-B'\theta}. \tag{u}$$

Figure No. 3 - Graphical illustration of initial temperature distribution for air flow period and stepwise division into relatively isothermal increments.



It is seen that with the exception of the constants, the differential equations and boundary conditions are the same as those derived for the heating period. Thus, the resulting solution, expressing the axial temperature of the matrix and the air in terms of time, axial distance and initial conditions, is

$$t_a = t'_{w,i} - (t'_{w,i} - t_{a,i})\left[e^{-\xi-\eta}\sum_{n=0}^{\infty}\eta^n M_n(\xi\eta)\right] \qquad (v)$$

$$t_w = t'_{w,i} - (t'_{w,i} - t_{a,i})\left[e^{-\xi-\eta}\sum_{n=1}^{\infty}\eta^n M_n(\xi\eta)\right], \qquad (w)$$

where $\xi, \eta, M_n$ are as defined on page 20, replacing $A$ and $B$ with $A'$ and $B'$ and $V_g$ with $V_a$.

While the air temperature entering the first isothermal length increment is constant, the temperature of the air entering succeeding sections of the heater will vary with time. Thus, in order to apply equations (v) and (w) to sections other than the entrance, it will be necessary to use an average value for the air temperature entering any prescribed section evaluated over a relatively isothermal time interval.

Therefore, the air temperature leaving the first section will have to be evaluated for various values of time and the temperature distribution with time broken up into relatively isothermal increments in the same manner as for the initial axial temperature distribution. The temperature of the air leaving that section is the entering air temperature for the second section. The initial value for the temperature of the air entering the second section during the first time interval is substituted along with the initial matrix temperature into equations (v) and (w) and the temperature of the air leaving the section and the new axial temperature distribution over the section at the end of the interval are evaluated. The axial temperature distribution is again assumed constant over the section at the new average value and the process repeated for the next interval of time. The temperature of the air leaving the second section must be averaged over the time interval or broken down into smaller increments to obtain a constant value for the air temperature entering the next section during a definite time interval. The calculations are then repeated for this interval. By combining these two processes and reapplying equations (v) and (w) over the length of the heater and the desired time range, the temperature history throughout the heater is determined.

The number of subdivisions of length and time will determine the accuracy; the more increments, the greater the accuracy. As the number of steps increases, the approximate method approaches the analytical solution.

The author believes that for this type of problem, it is better to use the simplest working equations and reach the desired degree of precision by close subdivision rather than to adopt complex equations.

Plate No. 2, $T_a/T_o$ versus $\xi$ for different values of $\eta$ as parameter, is provided for expediency in applying the solution for the air flow period. The points are again obtained from Schumann's curves.[16]

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(16) Schumann, T.E.W., op.cit. p. 413.

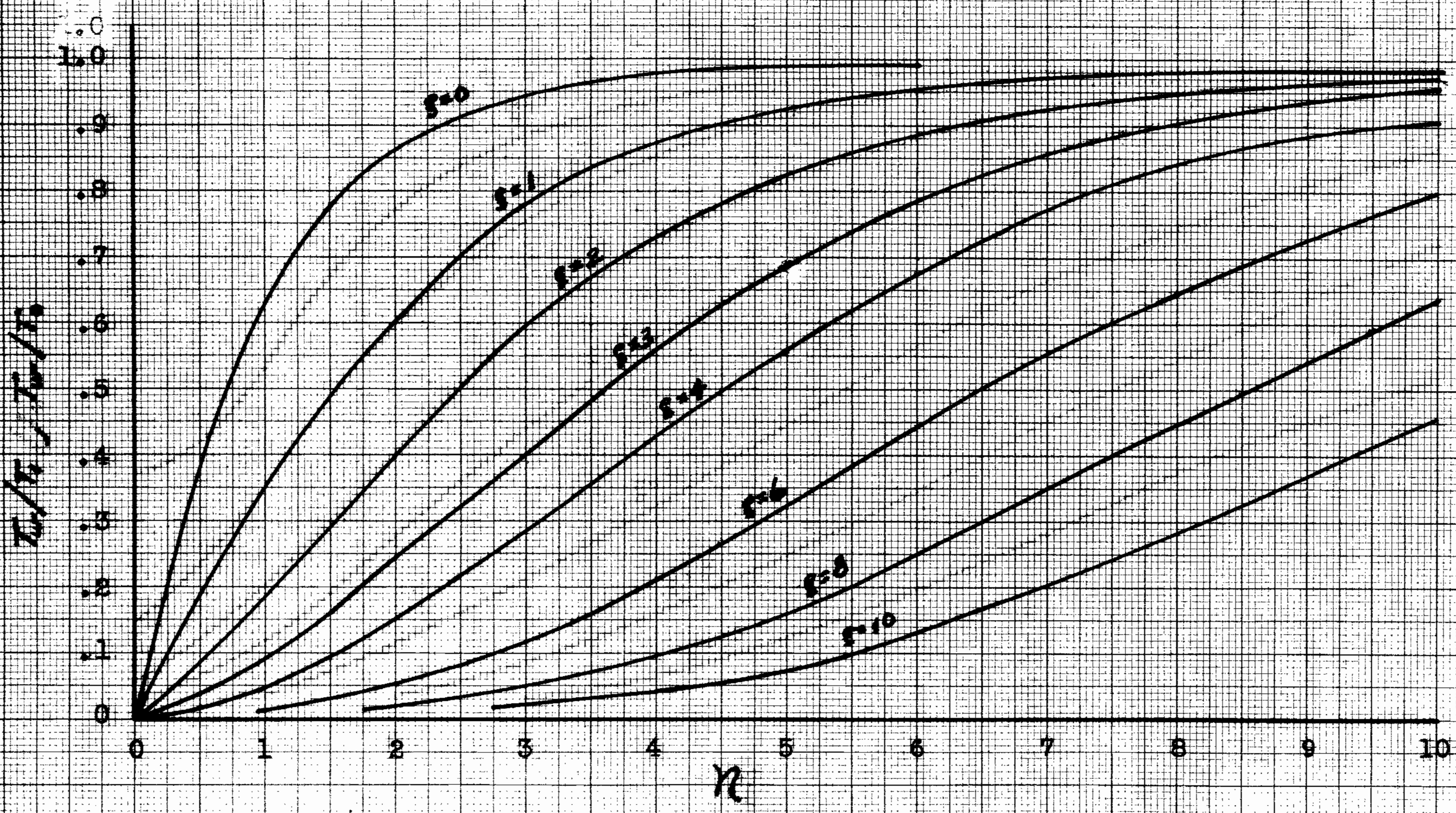


Plate No. 1 - Wall temperature variation.

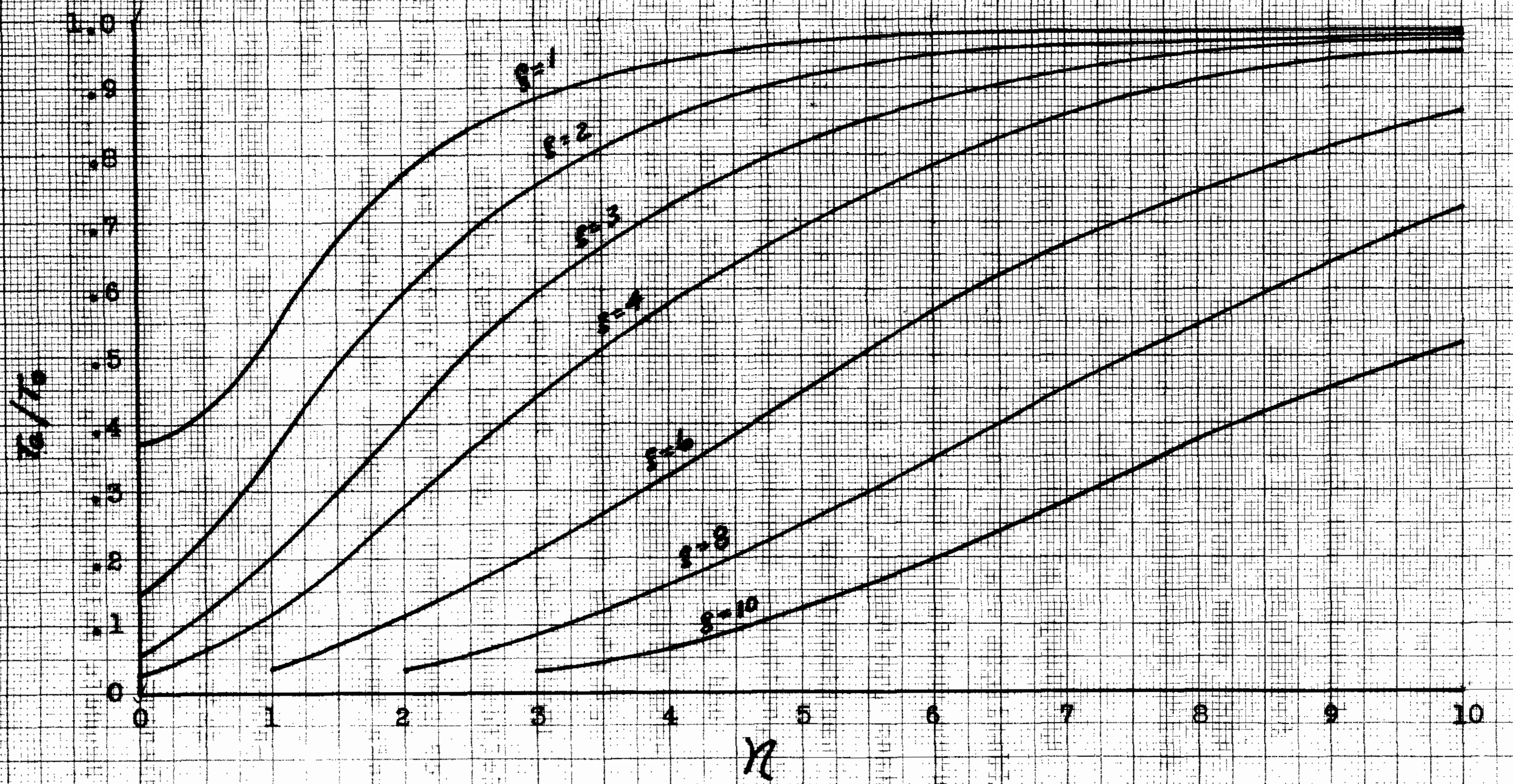


Plate No. 2 - Air temperature variation.

## CONCLUSION

It should be noted that although the internal configuration of the heater has not been considered in the general analysis, the idealizations make this solution particularly adaptable to regenerators which use thin tubular elements as the matrix.

Although the results, as presented in Equations (j), (v), and (w) and Plates (1) and (2) are applicable to many types of regenerators since the equations were derived in general terms, such as flow perimeter, heat transfer area, and total frontal area; the idealizations limit the accuracy. In the general solution, it is seen that only the constants $\alpha$ and $\beta$ and the velocity are directly dependent upon the matrix configuration. Thus, with proper evaluation of $\alpha$, $\beta$, and V, the results may be applied to the following: heating elements which form a heating surface parallel to the flow, such as plates; staggered rods normal to the flow; porous material; and other similar heat storage material.

The equations for the matrix and fluid temperatures were obtained by integrating the differential equations derived from energy balance and heat-transfer rate considerations applied to a differential element dx of the heat exchanger. The solution also met the boundary conditions which were characteristic to this particular problem.

In the analysis in this thesis, no attempt was made to deal with frictional heating effects and pressure drops.

## SUMMARY

The important features of this paper may be summarized as follows:

1. An analytical method has been presented for determining the axial temperature distribution in the matrix during the heating period.

2. An approximate analytical solution has been derived for determining the performance characteristics during the air flow period.

3. Graphs are included of $T_w/T_1$, $T_w/T_o$, and $T_a/T_o$ versus $\xi$ for different values of $\eta$ as parameter for the heating and cooling periods of the regenerator. Since the mathematical solutions are applicable only after the execution of rather complicated calculations, these graphs were deemed necessary to alleviate this situation.

Several problems are suggested from the idealizations and the method of solution presented in this thesis. The major ones are: (1) Consideration of finite conductivity normal to the flow. For solid matrix material considerable error may be introduced if the thickness is great and this conductivity is neglected. (2) Consideration of the variation in the values of the fluid properties both in the flow direction and normal to it.

# APPENDIX A

## Schumann's Analytical Method

The differential equations and boundary conditions are:

$$\frac{\partial T_g}{\partial \theta} + V_g \frac{\partial T_g}{\partial x} = -\beta (T_g - T_w). \tag{1}$$

$$\frac{\partial T_w}{\partial \theta} = A(T_g - T_w). \tag{2}$$

$T_w(x,0) = 0.$ (5) $T_g(0,\theta) = T_1.$ (7)

$T_w(0,\theta) = T_1[1 - e^{-A\theta}].$ (6) $T_g(V_g\theta,\theta) = T_1 e^{-\beta\theta}.$ (8)

By introducing two new independent variables

$$\xi = \beta x / V_g, \tag{9}$$

$$\eta = A(\theta - x/V_g), \tag{10}$$

equations (1) and (2) are reduced to the simplier forms,

$$\frac{\partial T_w}{\partial \eta} = T_g - T_w, \tag{11}$$

$$\frac{\partial T_g}{\partial \xi} = T_w - T_g. \tag{12}$$

These can be further simplified by introducing the two new dependent variables U and V, where

$$T_w = T_1 (U - V) e^{-\xi-\eta}, \tag{13}$$

$$T_g = T_1 (U + V) e^{-\xi-\eta}. \tag{14}$$

Substituting these values in (11) and (12), we find that

$$\frac{\partial U}{\partial \eta} - \frac{\partial V}{\partial \eta} = U + V, \tag{15}$$

$$\frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial \xi} = U - V, \tag{16}$$

and by further differentiation

$$\frac{\partial^2 V}{\partial \xi \partial \eta} = V. \tag{17}$$

The boundary conditions for U and V will obviously be:

(a) when $\xi = 0$,

$$U = e^{\eta} - \frac{1}{2}, \quad (18) \qquad V = \frac{1}{2}; \quad (19)$$

(b) when $\eta = 0$,

$$U = \frac{1}{2}, \quad (20) \qquad V = \frac{1}{2}. \quad (21)$$

The first step is to solve equation (17) subject to the boundary conditions (19) and (21).

Put $$\phi^2 = -4\xi\eta \tag{22}$$

then equation (17) reduces to

$$\frac{d^2V}{d\phi^2} + \frac{1}{\phi}\frac{dV}{d\phi} + V = 0 \tag{23}$$

which is a form of Bessel's equation, of which a well-known solution is

$$V = A\, J_0(\phi) \tag{24}$$

where A is a constant and $J_0(\phi)$ is a Bessel function of the first kind and zero order.

The boundary conditions (19) and (21) are both satisfied if $A = \frac{1}{2}$; hence the final solution is

$$V = \frac{1}{2} J_0(\phi) = \frac{1}{2} J_0(2i\sqrt{\xi\eta})$$

$$= \frac{1}{2} M_0(\xi\eta), \tag{25}$$

where we introduce the function $M_0$ which is defined thus: $M_0(\xi\eta) = J_0(2i\sqrt{\xi\eta})$

$$= 1 + \xi\eta + \frac{(\xi\eta)^2}{(2!)^2} + \frac{(\xi\eta)^3}{(3!)^2} + \cdots. \qquad (26)$$

Having obtained the value of V, the next step is to find a solution for U which will satisfy equations (15) and (16) subject to the given boundary conditions.

Integrating equation (15) as an ordinary linear differential equation, we find

$$U = e^{\eta}\int e^{-\eta}\left(V + \frac{\partial V}{\partial\eta}\right)d\eta + e^{\eta}f(\xi)$$

$$= V + 2e^{\eta}\int e^{-\eta}V\,d\eta + f(\xi)e^{\eta}, \qquad (27)$$

where $f(\xi)$ is a function of $\xi$. But by means of successive partial integration we find that

$$2e^{\eta}\int e^{-\eta}V\,d\eta = -2\left(V + \frac{\partial V}{\partial\eta} + \frac{\partial^2 V}{\partial\eta^2} + \cdots\right)$$

$$= -\sum_{n=0}^{\infty}\xi^n M_n(\xi\eta), \qquad (28)$$

where the M functions are thus defined:

$$M_0(\xi\eta) = J_0(2i\sqrt{\xi\eta}),$$

$$M_n(\xi\eta) = \frac{d^n M_0(\xi\eta)}{d(\xi\eta)^n}. \qquad (29)$$

The series obtained above can be shown to be converging so that the expansion is valid. It follows that

$$U = \frac{1}{2}M_0(\xi\eta) + f(\xi)e^{\eta} - \sum_{n=0}^{\infty}\xi^n M_n(\xi\eta). \qquad (30)$$

Now, when $\eta = 0$,

$V = 1/2$,

$U = 1/2$,

and

$$\sum_0^\infty \xi^n M_n(\xi\eta) = e^\xi.$$

Therefore

$$\frac{1}{2} = \frac{1}{2} + f(\xi) - e^\xi,$$

or

$$f(\xi) = e^\xi$$

and equation (30) becomes

$$U = \frac{1}{2} M_0(\xi\eta) + e^{\xi+\eta} - \sum_{n=0}^\infty \xi^n M_n(\xi\eta). \tag{31}$$

This solution satisfies both the boundary conditions, but it must still be demonstrated that it also satisfies equation (16), which states that

$$\frac{\partial U}{\partial \xi} - U = -\frac{\partial V}{\partial \xi} - V.$$

From equations (27) and (28) it is evident that the expression for U can be written in the form

$$U = V + e^{\xi+\eta} - 2\left(V + \frac{\partial V}{\partial \eta} + \frac{\partial^2 V}{\partial \eta^2} + \cdots\right).$$

Hence

$$\frac{\partial U}{\partial \xi} = \frac{\partial V}{\partial \xi} + e^{\xi+\eta} - 2\left(\frac{\partial V}{\partial \xi} + \frac{\partial^2 V}{\partial \xi \partial \eta} + \frac{\partial^3 V}{\partial \xi \partial \eta^2} + \cdots\right),$$

$$= e^{\xi+\eta} - \frac{\partial V}{\partial \xi} - 2\left(V + \frac{\partial V}{\partial \eta} + \frac{\partial^2 V}{\partial \eta^2} + \cdots\right),$$

since

$$\frac{\partial^2 V}{\partial \xi \partial \eta} = V.$$

Substituting these values for U and $\frac{\partial U}{\partial \xi}$ in (16), it is evident that (16) is satisfied. Equation (31) therefore gives the value of U which satisfies the conditions of the problem completely.

Schumann also shows that, between the last term of equation (31) and the corresponding expression $\sum_{0}^{\infty} \eta^n M_n(\xi\eta)$, the following simple relation exists:

$$\sum_{0}^{\infty} (\xi^n + \eta^n) M_n(\xi\eta) = e^{\xi+\eta} + M_0(\xi\eta),$$

which can be verified by expansion of the expressions.

By substituting the values found for U and V in equations (13) and (14), the final solution is obtained

$$\frac{T_w}{T_1} = e^{-\xi-\eta} \sum_{n=1}^{\infty} \eta^n M_n(\xi\eta) \ ,$$

$$\frac{T_g}{T_1} = e^{-\xi-\eta} \sum_{n=0}^{\infty} \eta^n M_n(\xi\eta) .$$

# APPENDIX B

## Method for Determining the Approximate Size of a Tubular Storage Heater

First, assume that at the end of the air flow period, the temperature of the matrix at the air inlet is the temperature of the incoming air and at the outlet is still the desired stagnation temperature and that there is a linear temperature distribution in the heater. Then the heater has given to the air, half of the amount of stored heat. The heat transferred to the air is $w_a c_{p_a}$ (Btu/°F) and thus the necessary amount of heat to be stored initially in the heater is $2w_a c_{p_a}$ (Btu/°F).

Now, suppose that the heater matrix consists of a material with a specific heat of $c_w$ Btu/lb.°F, then the weight of matrix material required is $2w_a c_{p_a}/c_w$ (lbs.). The necessary volume of the matrix is then $2w_a c_{p_a}/c_w \rho_w$. $\rho_w$ is the density of the matrix material in lbs./cu.ft.

Upon choosing a tube size and internal geometry, the volume of the heater can be approximated. Then the heater design problem reduces to determining a length to cross-section ratio which will permit an adequate matrix length for the required heat transfer without introducing an excessive pressure drop.

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