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A correlation of formulas for the flow of fluids in pipes

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A

CORRELATION OF FORMULAS

FOR THE

FLOW OF FLUIDS IN PIPES

A

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

A BRIEF HISTORICAL REVIEW

Although the origin of the science of hydraulics dates back to Biblical times, its growth was slow and spasmodic until about the seventeenth century. Such men as Galileo, Huygens, Pascal, Guglielmini, Torricelli, and Newton did much to solidify the fragments of knowledge on the subject, and a fundamental understanding of basic hydraulic principles was evident for the first time in this period.

Closely following was the work of Poleni, Pitot, Bernoulli, and Lecchi. It might be noted that the bulk of the work in hydraulics up to this time was of a theoretical nature. Little effort had been made to correlate the theoretical with the experimental.

In 1774 a new era in hydraulics was in evidence for Turin and Bossut established as a fundamental principle that formulae must be deduced from experiment. Bossut's experiments were among the first on the flow of water through pipes. Perhaps the most famous engineer of that day, at least to the present day student, was Antoine de Chezy who in 1775 developed the basic formula,

$$V = C\sqrt{RS} \qquad (1)$$

an expression which carries his name, for the flow in pipe and open channels.

The growth of hydraulics was phenomenal from this day on as the science grew in scope and content. In the following century countless contributions to science were made by men of almost all European nations.

In about the middle of the nineteenth century a much used pipe formula came into use.

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \tag{2}$$

Credit for its origin is given to Darcy, Weisbach, Fanning, or Eytelwein by various authors of the present day. It is widely known as the Darcy-Weisbach equation and will be so called in this paper.

At about the same time the law of laminar flow was first brought to light by Hagen. This work was almost immediately confirmed by Poiseuille, who expressed his findings in equation form. In terms of head loss, the equation is,

$$h_f = \frac{32 \mu L V}{D^2 g \rho} \tag{3}$$

Comparing this equation to the Darcy-Weisbach formula, it is evident that the friction factor is,

$$f \doteq \frac{64 \mu}{D V \rho} = \frac{64}{R} \tag{4}$$

This relationship has since been substantiated and is in general use to-day.

Since the latter part of the nineteenth century when such men as Stokes and Reynolds were in the forefront, a noticeable split in the manner of treatment of hydraulic pipe problems has taken place.

Hiram Mills, Hamilton Smith, Jr., and John R. Freeman, on the one hand, were leaders in the determination of friction factors and coefficients from experimental data. Their work was widely accepted and much used by practicing engineers

In the twentieth century others continued this work including Scobey and Schoder in this country. These men, however, did not continue experiments solely to determine the friction factor or Chezy coefficient. Instead they used their experimental findings as a basis for the development of the so called "exact" or exponential type formula which will be discussed later.

On the other hand, leaders such as Blasius, Schiller, Prandtl, von Karman, Bakhmeteff, and Rouse appear to have favored a theoretical treatment of hydraulics.

Since 1883 when Osborne Reynolds performed his classic experiments, the parameter which carries his name,

$$\underline{R} = \frac{DV\rho}{\mu} \qquad (5)$$

has proved a boon to the further development of pipe flow theory and practice.

It remained for Stanton and Pannell of the National Physical Laboratory in London, England, to utilize the Reynolds number and put it in a usable form. In 1914 they evolved the much used curve found by plotting experimental data and correlating Reynolds number with the friction factor.

Lees, Lander, and others quickly verified the work of Stanton and Pannell. Scores of engineers have since studied and written of this relationship. Foremost among them in the United States were Wilson, McAdams, and Seltzer in 1922 and W. G. Heltzel in 1926 and 1930.

The chemical and petroleum engineer interested in pipeline flow eagerly accepted this new found criterion and have used it advantageously for a quarter of a century. Advancements have been made continuously in practice and theory largely through this use.

Engineers soon noticed that pipe roughness also affected the friction factor determination and plotted new curves from experimental data, most of which approximately paralled the Stanton and Pannell curve in the turbulent flow region.

Since 1930 many laboratory experiments on the roughness effect have been made. Nikuradse was the first to publish his findings in 1933. He noted that the Reynolds number-friction factor relationship in the laminar flow region remained unchanged, but that an increase in the relative roughness of a pipe caused a corresponding increase in the friction factor in the turbulent flow region. V. L. Streeter conducted similar experiments on artificially roughened pipe and published his findings in 1935.

Since then much has been written on the subject by Rouse, Bakhmeteff, Colebrook, Kalinske, Bardsley, Aude, Moody, and others. As well, several textbooks, which treat rather comprehensively even the more recent material, have been published.

Meanwhile engineers interested primarily in the flow of water have continued to use long standing formulas such as the Chezy, Kutter, Darcy-Weisbach, and Hazen-Williams with experimentally determined factors and coefficients.

Hence, at present there exist two quite distinct fields in fluid flow in pipes, the one in water supply and the other in the petroleum industry. It is the purpose of this paper to discuss the various pipeline flow formulas as used in both fields and to correlate the exponential type formula which is much used in practice with the Nikuradse curves obtained in the laboratory.

THE FLOW OF WATER IN PIPES

Of the flow formulas in use to-day, that devised by Chezy is the oldest. It has enjoyed wide use and is still favored by some engineers. Based on experiment, Chezy published it in the following form in 1775.

$$V = C\sqrt{RS} \qquad (1)$$

If we substitute the value of $D/4$ for the hydraulic radius and h_f/L for the slope we have,

$$V = C\sqrt{D/4 \times h_f/L}$$

Squaring,

$$V^2 = C^2 \frac{D}{4} \frac{h_f}{L}$$

And solving for h_f and multiplying through by $\frac{2g}{2g}$, we obtain,

$$h_f = \frac{4 \times 2g}{C^2} \frac{L}{D} \frac{V^2}{2g}$$

It is evident that this is the Chezy equation in the Darcy-Weisbach form where,

$$f = 8g/C^2 \qquad (6)$$

and

$$C = 2\sqrt{2g/f} \qquad (6a)$$

Although Chezy presumed that his coefficient C was both dimensionless and constant, this has since been disproven.

C has been shown to have the dimension of $\sqrt{g}$, and being a function of the friction factor it must also vary with the roughness of the pipe.[1] The Darcy-Weisbach formula is,

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(1) H. Rouse, "Elementary Mechanics of Fluids," 1st ed., p. 217, John Wiley and Sons, New York, 1946.

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then, the result of the Chezy formula. We thus obtain,

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \qquad (2)$$

Present day analysts prefer this latter form because f is dimensionless. This fact is of little consequence in a fixed gravitational field however.

Through the years tables of values of C and f have been laboriously compiled for pipes of various composition, condition, and size and for varying velocities. Tables 1 and 2 are typical of the innumerable tables in existence. It should be noted that any table is valueless without a complete description of the pipe and its condition. Nevertheless, they have seen widespread use and have served their purpose.

The magnitude of the coefficient C in the Chezy formula has been the subject of much investigation. Various relationships have been suggested for finding this value, the more common being the empirical forms of Ganguillet and Kutter, Manning, and Bazin. These are, respectively, as follows:

Table 1[2]

Values of C for Clean, Smooth, Cast Iron, Steel and Concrete Pipes

Diameter in inches	Velocity in feet per second				
	1	2	3	5	10
4	95	101	104	107	114
6	99	104	107	111	116
8	101	106	110	114	120
10	103	108	112	116	121
12	105	110	114	118	123
15	106	112	115	120	125
18	108	114	116	121	127
24	111	116	120	125	131
30	114	118	121	127	134
36	115	120	123	129	136
42	116	121	125	131	138
48	118	123	127	131	138
60	120	125	129	134	141

(2) G. E. Russell, "Hydraulics," 5th ed., p. 222, Henry Holt and Co., New York, 1942

Table 2[3]

Values of f in the Darcy-Weisbach Formula

For water flowing in straight smooth pipe

Diameter in inches	Mean velocity in feet per second				
	1	2	3	5	10
1	.035	.032	.030	.027	.024
2	.033	.030	.028	.026	.024
4	.031	.028	.026	.025	.023
6	.029	.026	.025	.024	.022
8	.028	.025	.024	.023	.021
10	.026	.024	.023	.022	.021
12	.025	.023	.022	.021	.020
18	.022	.021	.020	.020	.019
24	.020	.019	.019	.018	.018
30	.019	.018	.018	.017	.017
36	.017	.017	.017	.016	.016
42	.016	.016	.015	.015	.015
48	.015	.015	.015	.014	.014

(3) H. W. King, "Handbook of Hydraulics," McGraw-Hill Book Co., 1939, P. 205

$$C = \frac{41.65 + \frac{0.00281}{S} + \frac{1.811}{n}}{1 + \left(41.65 + \frac{0.00281}{S}\right)\frac{n}{\sqrt{R}}} \qquad (7)$$

$$C = \frac{1.486}{n} R^{1/6} \qquad (8)$$

$$C = \frac{157.6}{1 + \frac{m}{\sqrt{R}}} \qquad (9)$$

Of the three, the Manning formula is the only one that has seen wide use for both the flow in pipes and open channels.(4)

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(4) H. W. King, "Manning Formula Tables," Vol. 1 Flow in Pipes, Vol. 2 Flow in Open Channels, McGraw-Hill, New York, 1937

The Chezy formula with the Manning evaluation of C (called the Manning formula in this form) would be,

$$V = \frac{1.486}{n} R^{2/3} S^{1/2} \qquad (8a)$$

For pipes, we might obtain a more convenient form by solving for h_f after introducing $D/4$ for R and h_f/L for S. The Manning formula in this form is

$$h_f = 2.87\, n^2 \frac{LV^2}{D^{4/3}} \qquad (8b)$$

By assigning a value to n, problems in pipe flow are readily solvable. Table 3 contains typical values of n recommended for water flowing in pipes.(5)

(5) King, Wisler, and Woodburn, "Hydraulics," 4th ed., p. 184, John Wiley and Sons, New York, 1941.

Another type of formula that has seen extensive use in the United States is the exponential type. The Chezy formula would fall in this category if it were expressed as follows:

$$V = C\, R^{1/2}\, S^{1/2} \qquad (1a)$$

However, perhaps the most famous formula of this type is the Hazen and Williams formula. They published in 1905 a formula based on all available experimental data on pipe flow.(6) It is

$$V = 1.318\, C_1\, R^{0.63}\, S^{0.54} \qquad (10)$$

(6) Williams and Hazen, "Hydraulic Tables," 3rd ed., John Wiley and Sons, New York, 1933.

Table 3

Values of n To Be Used in the Manning Formula

Kind of Pipe	Variation.	
	From	To
Clean cast iron pipe	0.010	0.015
Tuberculated cast iron pipe	.015	.035
Riveted steel pipe	.013	.017
Welded steel pipe	.010	.014
Corrugated iron pipe	.013	.017
Brass and glass pipe	.009	.013
Wood-stave pipe	.010	.014
Concrete pipe	.010	.017
Vitrified sewer pipe	.010	.017
Common clay drainage tile	.011	.017
Asbestos-cement pipe	.010	.012

In their book the authors of the formula have recommended that the following values of C_1 be used for the flow of water in pipes.

Description of Pipe	Value of C_1
Extremely smooth and straight	140
Very smooth	130
Smooth wooden or wood-stave	120
New riveted steel	110
Vitrified	110

F. C. Scobey of the U. S. Department of Agriculture has also done much with this type of formula. From 1910 to 1930 Scobey published several formulas based on a large number of field tests. Three of his formulas which have been widely used in the irrigation field are noted below.

Wood Stave Pipe

$$V = 1.62\, D^{0.65}\, H_f^{0.55} \qquad (11)$$

Concrete Pipe

$$V = C_s\, d^{0.625}\, H_f^{0.5} \qquad (12)$$

Riveted Steel and Other Pipe (13)

$$H_f = \nu^{0.1}\, M_s \frac{V^{1.9}}{D^{1.1}}$$

These formulas were published in the order listed. It should be noted that equation 13 is the only one so far presented that contains a viscosity term. Scobey introduced this term, ν , to allow for temperature changes.

Another approach to the solution of pipe problems was made by E. W. Schoder who arbitrarily divided all pipe into four categories of roughness and devised a formula of the exponential type for each category.(7)

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(7) Schoder and Dawson, "Hydraulics," 2nd ed., p. 198, McGraw-Hill, New York, 1934.

Schoder's formulas for extremely smooth pipes, fairly smooth pipes, rough pipes, and extremely rough pipes are respectively as follows:

$$H_f = 0.30 \frac{V^{1.75}}{D^{1.25}} \qquad (14)$$

$$H_f = 0.38 \frac{V^{1.86}}{D^{1.25}} \qquad (15)$$

$$H_f = 0.50 \frac{V^{1.95}}{D^{1.25}} \qquad (16)$$

$$H_f = 0.69 \frac{V^{2}}{D^{1.25}} \qquad (17)$$

In using Schoder's formulas, it is left to the engineer to decide in which category the pipe under consideration would fall. Following is a description of the "categories of roughness" as given by Schoder.

<u>Notes on the Several Categories of Roughness</u>

"EXTREMELY SMOOTH PIPES: New seamless-drawn brass, block-tin and lead, glass, porcelain-like glazed pipes; all with interior surfaces both appearing very even to the eye and feeling very firm and smooth to the touch.

Intermediate between the above category and the one below are all sorts of newly laid so-called "smooth" common commercial pipes such as coated cast iron, wrought iron, and wood stave. High grade rubber-lined fire hose causes about one-third less loss of head than the following category.

FAIRLY SMOOTH PIPES: All ordinary pipes after a few (say about five, more or less) years in ordinary service, such as asphalt-coated cast-iron and spiral-riveted steel pipes (latter of thin metal and with very flat rivet heads), wrought iron, both "black" and galvanized (but the latter in the small sizes may be "rough pipes" even when new), wood stave, reinforced concrete, galvanized, spiral-riveted steel. This category is rough enough to be called fairly conservative for general water supply designing purposes.

Intermediate between the above category and the one below are the above-mentioned pipes after being fairly long (say about ten years or so) in service and subjected to average deterioration. Unlined linen "mill fire hose" causes about one-third more loss of head than the previous category of "fairly smooth pipes."

ROUGH PIPES: Originally "fairly smooth pipes" that have deteriorated fairly rapidly for some ten or fifteen years after being laid; also ordinary lap-riveted steel pipes some years in service; also large well-laid brick storm-water sewers flowing full. This category represents a roughness such that its use in design is quite conservative in cases where full capacity will not be demanded for some dozen years after laying.

Intermediate between the above category and the one below are pipes having more local roughness or more frequent joints than ordinary water pipes, e.g., ordinary glazed clay sewer pipes in average good-service condition, also small brick-lined sewers, also small riveted-steel pipes made of sections only some two or three diameters long.

EXTREMELY ROUGH PIPES: This category represents a degree of roughness or deterioration beyond anything that would ordinarily be allowed for in design of water pipes, say the condition of small street mains after some thirty or forty years of service. In this category come small sewer pipes considerably fouled by slime and deposits or laid with poor alignment."

Through the years engineers have used scores of formulas in solving pipe flow problems. Many have seen only a limited use before being discarded for one reason or another. The formulas heretofore mentioned are among those that have seen constant use and are still considered as giving reliable results. For mention and discussion of other formulas the reader is referred to texts and handbooks on hydraulics published thirty or so years ago.

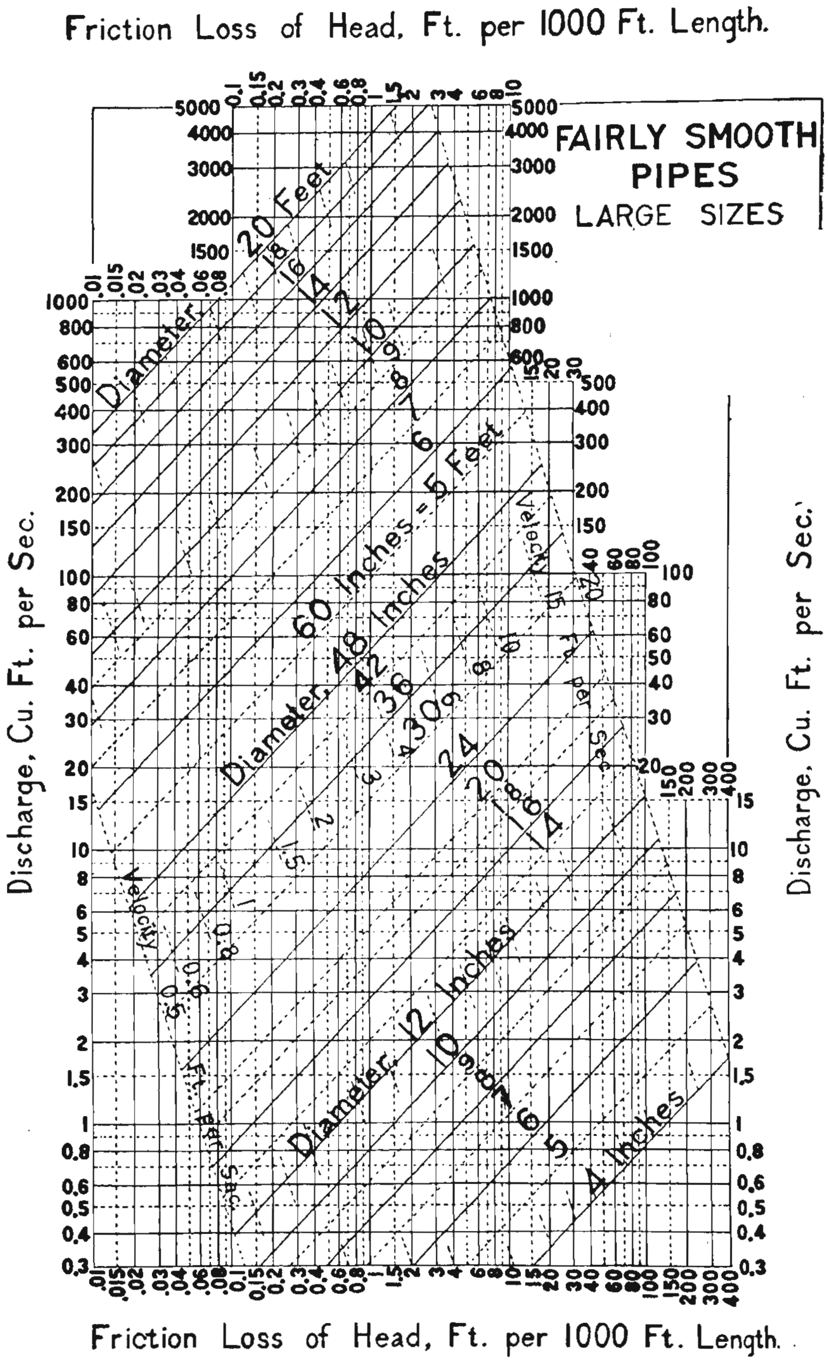
Engineers long ago noted that computations of pipe flow problems often proved to be very time consuming. An effort was made to simplify their work by devising any number of time saving devices such as hydraulic charts, tables, and special slide rules.

Typical of the charts that have been used is the one on the following page. It is based on Schoder's formula for fairly smooth pipes. Since most problems in pipe flow require a solution for either H_f , Q, or D, the diagram has been set up so that given any two values, the remaining one can be obtained directly.

Some engineers prefer the use of tables to charts or diagrams. As a result numerous tables have been compiled based on the various formulas. King's "Manning Formula Tables" in two volumes is an extreme example of the extent to which engineers will go in compiling tables for solution of the hydraulic flow formulas.

The Hazen-Williams slide rule based on their formula was invented specifically for the solution of problems by that one formula. Other slide rules have been used based on the Kutter and Manning formulas. However, the use of the special slide rules has been limited.

Present day engineers interested solely in the flow of water almost universally use one or more of the formulas mentioned. Little effort has been made to utilize the advances made by those interested in the flow of viscous fluids, and the friction factor-Reynolds' number relationship


Friction Loss of Head, Ft. per 1000 Ft. Length.
FAIRLY SMOOTH PIPES
LARGE SIZES
Diameter 20 Feet
18
16
14
12
10
9
8
7
6
60 Inches = 5 Feet
Diameter, 48 Inches
42
36
30
24
20
18
16
14
Diameter, 12 Inches
10
9
8
7
6
5
4 Inches
Velocity, Ft. per Sec.
Discharge, Cu. Ft. per Sec.
Friction Loss of Head, Ft. per 1000 Ft. Length.

is almost ignored. This simply implies a backwardness in those concerned for it will be shown that comparable results may be obtained as quickly as by other formulas, and the method of solution also can be used with equal accuracy for any fluid at any temperature.

In the following section several typical problems will be solved by each of the formulas that has been discussed hereto. A solution of a problem by using the friction factor-Reynolds' number relationship will also be shown to illustrate that the procedure used for the solution of viscous flow problems will also yield acceptable results for the flow of water.

SOLUTION OF PROBLEMS ON THE FLOW OF WATER IN PIPES

Problem No. 1.) It is required to find the discharge of a concrete pipe 48 inches in diameter and 4500 feet long in which the loss of head is 18 feet.

A.) Solution by the Manning Formula

$$V = \frac{1.486}{n} R^{2/3} S^{1/2} \qquad (8a)$$

From Table 3, 0.012 is taken as an average value of n for a concrete pipe of this size.

$$R = D/4 = \frac{4}{4} = 1 \qquad\qquad S = \frac{h_F}{L} = \frac{18}{4500} = 0.004$$

$$V = \frac{1.486}{0.012}(1)^{2/3}(0.004)^{1/2} = 7.8\ {}'/\text{sec.}$$

$$Q = AV = 12.57\,(7.8) = \underline{98 \text{ cfs}}$$

B.) Solution by the Kutter Formula

$$C = \frac{41.65 + \frac{0.00281}{S} + \frac{1.811}{n}}{1 + \left(41.65 + \frac{0.00281}{S}\right)\frac{n}{\sqrt{R}}} \qquad (7)$$

$$\text{in} \quad V = \sqrt{RS} \qquad (1)$$

Using $n = 0.012$ as above.

$$C = \frac{41.65 + \frac{0.00281}{0.004} + \frac{1.811}{0.012}}{1 + \left(41.65 + \frac{0.00281}{0.004}\right) \frac{0.012}{1}} = 129$$

$$V = 129 \sqrt{1(0.004)} = 8.1 \text{ '/sec.}$$

$$Q = 12.57 (8.1) = \underline{102 \text{ cfs}}$$

C.) Solution by the Scobey Formula

$$V = C_s \, d^{0.625} \, H_f^{0.5} \qquad (12)$$

From Scobey's publication, The Flow of Water in Concrete Pipe, from which the formula was taken, C_s was chosen as 0.345.

$$H_f = \frac{18}{4.5} = 4$$

$$V = 0.345 \, (48)^{0.625} (4)^{0.5} = 7.73 \text{ '/sec.}$$

$$Q = 12.57 (7.73) = \underline{97 \text{ cfs}}$$

D.) Solution by the Hazen and Williams Formula

$$V = 1.318 \, C_1 R^{0.63} S^{0.54} \qquad (10)$$

From Hydraulic Tables by Hazen and Williams, take C_1 as 120

$$V = 1.318 (120) (1)^{0.63} (0.004)^{0.54} = 7.9 \ '/sec$$

$$Q = 12.57 (7.9) = \underline{99.5 \text{ cfs}}$$

E.) Solution by the Schoder Formula

Assuming the pipe to fall in the "fairly smooth" category, Formula 15 may be used in the following form:

$$V = 1.68 \, D^{0.67} \, H_f^{0.54} \qquad (15a)$$

$$V = 1.68 \, (4)^{0.67} (4)^{0.54} = 9.0 \ '/sec$$

$$Q = 12.57 (9.0) = \underline{113 \text{ cfs}}$$

F.) Solution by Schoder's Chart

Enter the chart at the bottom at a value of H_f = 4 feet.

Trace the line vertically to the inclined line for d = 48 inches.

Proceed horizontally to the edge of the chart.

Read Q = <u>110 cfs</u>.

G.) Solution by Darcy-Weisbach Equation

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \qquad (2)$$

Or

$$V = 8.02 \sqrt{h_f/L \; D/f} \qquad (2a)$$

$$V = 8.02 \sqrt{\frac{18}{4500} \frac{4}{0.014}} = 8.55 \ '/sec$$

Where f is assumed to be 0.014 (See Table 2)

$$Q = 12.57 (8.55) = \underline{107.5 \text{ cfs}}$$

Problem No. 2.) Determine the diameter of a welded steel pipe required to convey 8 cfs a distance of 5100 feet with a head loss of 10 feet.

A.) Solution by Manning Formula

$$D = \left(\frac{2.159\ Q\ n}{S^{1/2}}\right)^{3/8} \qquad (8c)$$

Assuming $n = 0.010$ (See Table #3)

$$D = \left(\frac{2.159\ (8)\ (0.010)}{(10/5100)^{1/2}}\right)^{3/8} = \underline{1.67'}$$

B.) Solution by Scobey Formula

Combining $v^{0.1}$ and M_s into a factor K_s

$$H_f = K_s \frac{V^{1.9}}{D^{1.1}} \qquad (13a)$$

Or $$D = \sqrt[4.9]{\frac{K_s}{H_f}\left(\frac{4Q}{\pi}\right)^{1.9}} \qquad (13b)$$

Using a value of $K_s = 0.32$ (From Handbook of Welded Steel Pipe)

$$D = \sqrt[4.9]{\frac{0.32}{1.96}\left(\frac{4(8)}{\pi}\right)^{1.9}} = \underline{1.69'}$$

C.) Solution by Hazen and Williams Formula

$$D = \sqrt[2.63]{\frac{2.31}{C_1} \frac{Q}{S^{0.54}}} \qquad (10a)$$

Use $C_1 = 130$ (From authors' text)

$$D = \sqrt[2.63]{\frac{2.31}{130} \frac{8}{(0.00196)^{0.54}}} = \underline{1.725'}$$

D.) Solution by Schoder's Formula

Assume pipe to be "fairly smooth" (See description)

$$D = 0.9 \frac{Q^{0.374}}{H_f^{0.201}} \qquad (15b)$$

$$D = 0.9 \frac{8^{0.374}}{1.96^{0.201}} = \underline{1.715'}$$

E.) Solution by the Schoder Chart

Enter the chart at the left where $Q = 8$ cfs. Trace a line horizontally to the intersection of the line projected vertically from the point at the bottom of the chart where $H_f = 1.96$ feet. Read $d = 20.5$ inches or 1.7 feet.

F.) Solution by the Darcy-Weisbach Equation

$$D = \sqrt[5]{\frac{8f}{g} \frac{L}{\pi^2} \frac{Q^2}{h_f}} \qquad (2b)$$

Assume f = 0.020 (Then check to confirm assumption)

$$D = \sqrt[5]{\frac{8(0.020)(5100)(64)}{32.2 \ \pi^2 \ (10)}} = \underline{1.75'}$$

Problem No. 3.) What is the head required to convey 1 cfs a distance of 4000 feet in a cast iron pipe 8 inches in diameter?

A.) Solution by the Manning Formula

$$V = 1/0.349 = 2.87 \ '/sec.$$

$$h_f = 2.87 \ n^2 \ \frac{LV^2}{D^{4/3}} \qquad (8b)$$

Set n = 0.011 (See Table 3)

$$h_f = 2.87(0.011)^2 \frac{4000 \ (2.87)^2}{(0.667)^{4/3}} = \underline{19.2'}$$

B.) Solution by the Schoder Formula

From Schoder's description of the categories of roughness the "fairly smooth" pipe formula is chosen.

$$H_f = 0.38 \frac{V^{1.86}}{D^{1.25}} \qquad (15)$$

$$H_f = 0.38 \frac{(2.87)^{1.86}}{(0.667)^{1.25}} = 4.48'$$

$$\therefore h_f = 4(4.48) = \underline{17.9'}$$

C.) Solution by Schoder's Chart

Enter the chart at the left or right for a value of Q of 1 cfs.

Trace horizontally to the intersection with the inclined line for a diameter of 8 inches.

Drop vertically to the bottom of the chart and read H_f = 4.5 feet.

$$h_f = 4(4.5) = \underline{18'}$$

D.) Solution by the Darcy-Weisbach Formula

Use f = 0.024 (See Table 2)

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \qquad (2)$$

$$h_f = 0.024 \frac{4000}{8/12} \frac{(2.87)^2}{64.4} = \underline{18.4'}$$

E.) Solution by the Scobey Formula

$$H_f = K_s \frac{V^{1.9}}{D^{1.1}} \qquad (13a)$$

From the Handbook of Welded Steel Pipe (p. 87), use $K_s = 0.38$

$$H_f = 0.38 \frac{(2.87)^{1.9}}{(0.667)^{1.1}} = 4.39'$$

$$h_f = 4(4.39) = \underline{17.6'}$$

F.) Solution by the Hazen and Williams Formula

$$h_f = \left(1.825 \frac{V L^{0.54}}{C_1 D^{0.63}}\right)^{1.85} \qquad (10b)$$

Using $C_1 = 120$ as recommended by the authors

$$h_f = \left(1.825 \frac{2.87 (4000)^{0.54}}{120 (0.667)^{0.63}}\right) = \underline{19.7'}$$

G.) Solution by the Darcy-Weisbach Formula

(Using the Reynolds' Number- Friction Factor Diagram)

$V = 2.87\ '/sec$ $\qquad D = 0.667'$

$\nu = 1 \times 10^{-5}\ ft^2/sec$

$$\underline{R} = \frac{VD}{\nu} = \frac{2.87\,(0.667)}{10^{-5}} = 191{,}000$$

Using Figure 4, Curve 7, f = 0.023

$$h_f = 0.023\ \frac{4000}{8/12}\ \frac{(2.87)^2}{64.4} = \underline{17.6'}$$

THE FLOW OF OILS IN PIPES

As has been previously mentioned, the petroleum engineer early accepted the Reynolds' number-friction factor relationship as an invaluable tool for solving pipe flow problems with the Darcy-Weisbach formula. The formula has been "simplified" or "improved" by many engineers. Hence, at present scores of adaptations are in use. For the most part, however, changes are minor, amounting to use of different symbols or units only.

Before embarking on a discussion of the problems involved the writer believes it prudent to summarize and define where necessary the principles and terms that will be encountered.

Density, ρ , may be defined as the mass of fluid contained in a unit of volume. It has the dimensions of pound seconds^2 per foot^4 or slugs per cubic foot. In the metric system, ρ is measured in grams per cubic centimeter and is numerically equal to the specific gravity.

The specific weight, γ , is defined as the weight of fluid contained in a unit volume. (Hence $\gamma = \rho g$) The specific weight is expressed as pounds/cubic foot or grams/cubic centimeter.

The specific gravity is the ratio of the density or specific weight of a substance to the density or specific weight of pure water at a specified temperature. The specific gravity of oils is influenced by both chemical composition and physical properties. In practical operation in the petroleum industry the specific gravity is generally expressed in A.P.I. (American Petroleum Institute) degrees. The conversion of the A.P.I. scale into specific gravity, and vice versa, may be effected by using the relationships shown below.

$$\text{Degrees A.P.I.} = \frac{141.5}{\text{Sp. Gr. } 60^\circ/60^\circ\text{F}} - 131.5 \qquad (18)$$

$$\text{Sp. Gr. } 60^\circ/60^\circ\text{ F} = \frac{141.5}{131.5 + \text{degrees A.P.I.}} \qquad (18a)$$

It should be noted that as the specific gravity increases, the A.P.I. gravity decreases.

The <u>absolute viscosity</u>, μ, is defined as the force required to move a flat surface of unit area at unit relative velocity parallel to another surface at unit distance away, the space between the surfaces being filled with the fluid. In foot-pound-second units, μ is expressed as pound seconds/square foot or slugs/foot second. In the metric system the unit of viscosity is called the poise which is equal to one dyne second/square centimeter. The term centipoise (0.01 poises) is often used. It has been noted that water at a temperature of 68° F has an absolute viscosity of one centipoise.

<u>Kinematic viscosity</u>, ν , is a term used for the recurring ratio of the absolute viscosity of a fluid to its

density. Or symbolically,

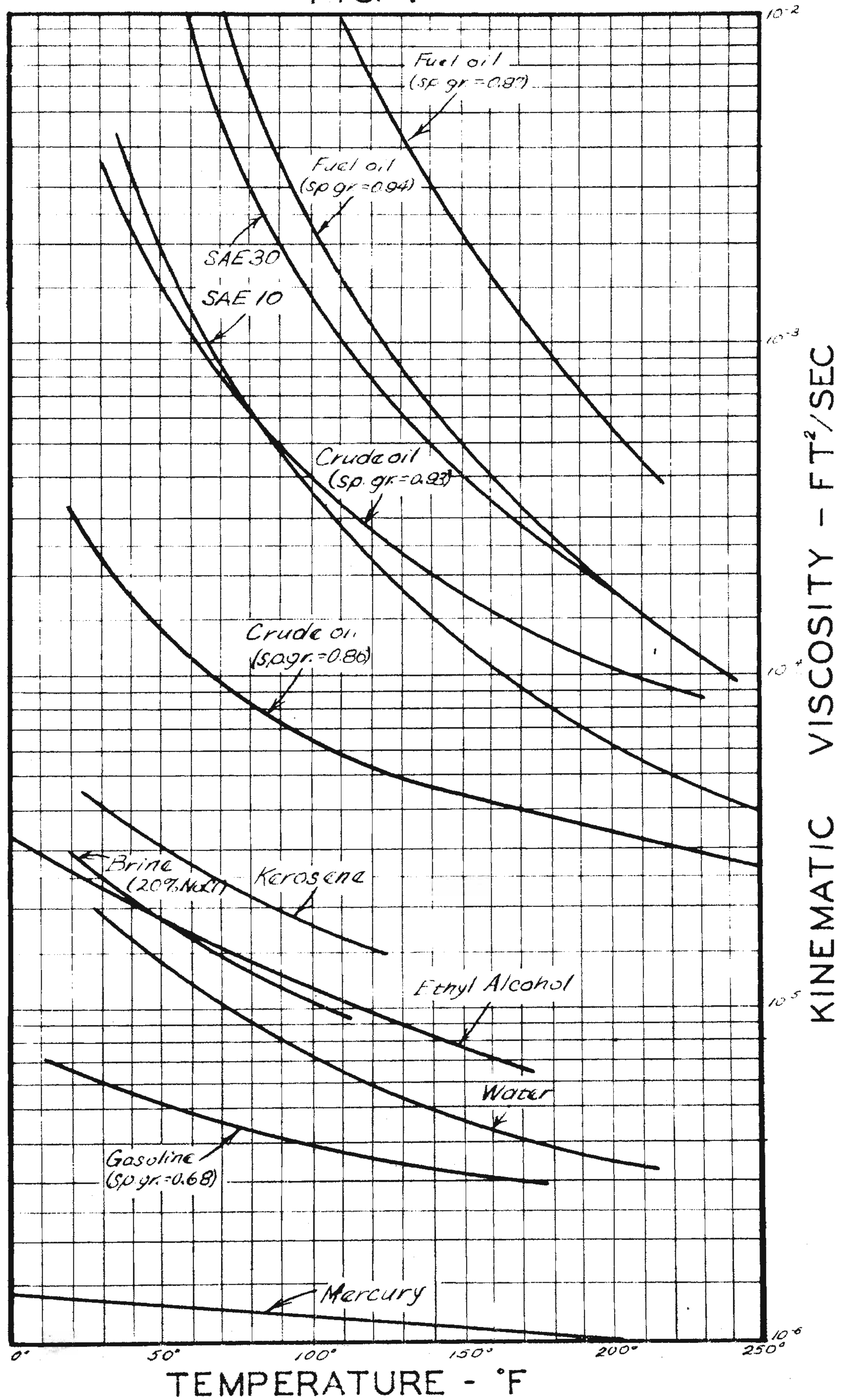
$$\nu = \mu/\rho \qquad (19)$$

The units used are square feet/second or square centimeters/second. The latter term is called a stoke. Figure 1 shows the change in kinematic viscosity with temperature for some common liquids.

The viscosity of oils is measured by a viscosimeter. The viscosity is usually stated in terms of the time necessary for a definite volume of oil at a specified temperature to flow through a small opening. The oil is first heated in a metallic cup surrounded by an oil bath. When the oil has been heated to the desired temperature a small orifice in the bottom of the cup is opened. The time necessary for a given quantity of oil to pass through the orifice is taken as a measure of the viscosity.

The more common viscosimeters are the Saybolt Universal, the Redwood, and the Engler. For the first mentioned, results are expressed in seconds Saybolt Universal or S.S.U. The Redwood results are also expressed in seconds, but the Engler results are expressed in Engler degrees. If extremely heavy oils are to be measured, results are usually expressed either in seconds Saybolt Furol or seconds Redwood Admiralty. It might be noted that Saybolt Universal readings are about ten times as great as Saybolt Furol readings. Engler degrees are a measure of the viscosity as compared to water and are

FIG. I


Fuel oil (sp. gr. = 0.87)
Fuel oil (sp gr = 0.94)
SAE 30
SAE 10
Crude oil (sp. gr. = 0.93)
Crude oil (sp. gr. = 0.86)
Brine (20% NaCl)
Kerosene
Ethyl Alcohol
Water
Gasoline (sp. gr. = 0.68)
Mercury
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
0°
50°
100°
150°
200°
250°
TEMPERATURE - °F
KINEMATIC VISCOSITY - FT²/SEC

an indication of the absolute viscosity. Figure 2 indicates how the Saybolt Universal, Redwood, and Engler results may be converted to kinematic viscosity expressed in stokes.

Crude petroleums differ greatly in viscosity. Some are very mobile while others are quite viscid. The viscosity increases with the density. However, the viscosities of oils of the same specific gravity may not be the same. This is due to a difference in the chemical composition of some oils.

Reynolds' number, R, is a hydraulic parameter that is used to distinguish between laminar and turbulent flow. In its true form it is a dimensionless number as is shown below.

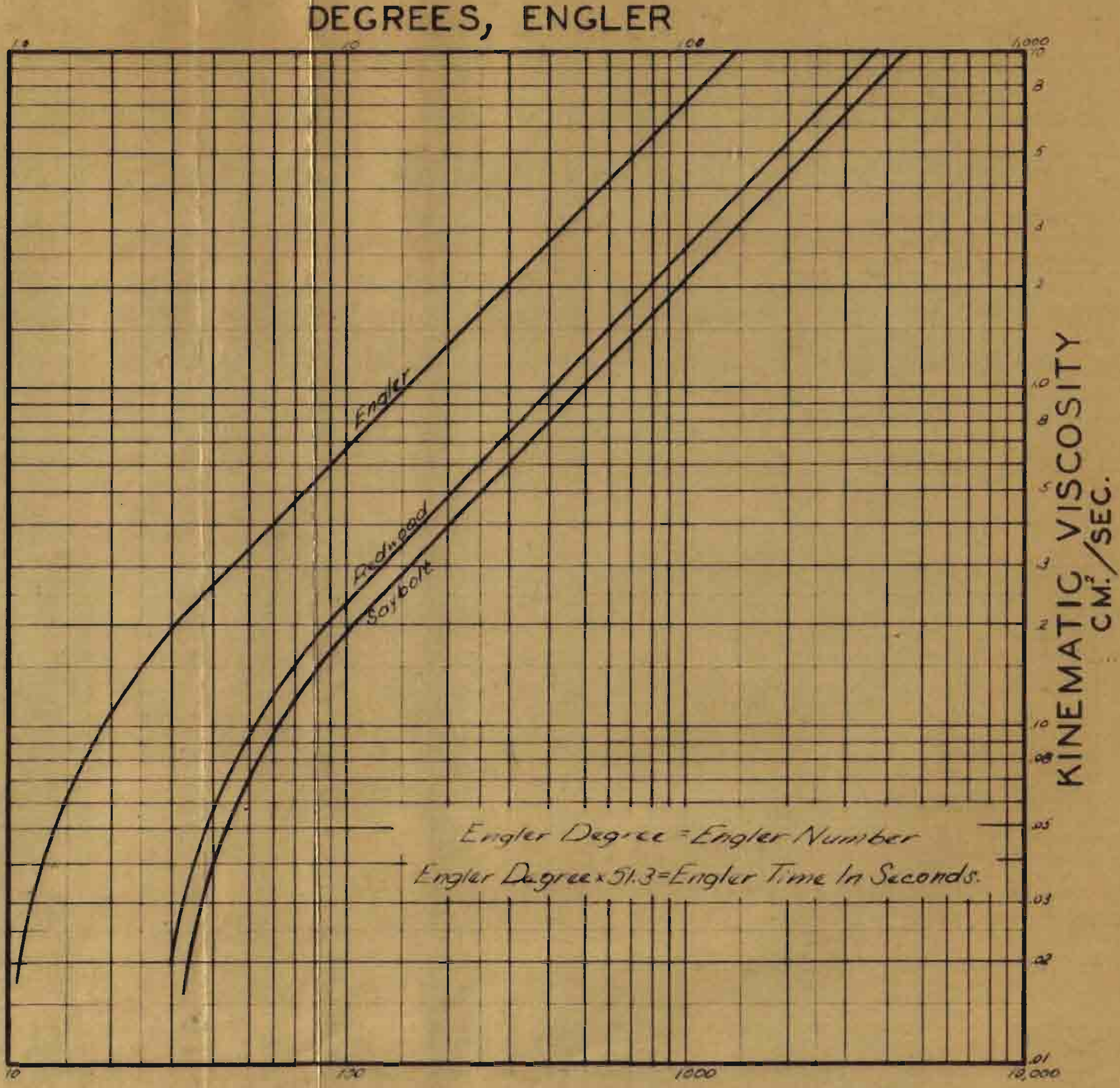
$$\underline{R} = \frac{DV\rho}{\mu} = \frac{DV}{\nu} \qquad (5)$$

$$\frac{ft \left(\frac{ft}{sec}\right)\left(\#\ sec^2/ft^4\right)}{\#\ sec/ft^2} = \text{a dimensionless number}$$

If fundamental units in either the c.g.s. or f.p.s. systems are used, the relationship between Reynolds' number and the Darcy-Weisbach friction factor for viscous flow is:

$$f = \frac{64}{\underline{R}} \qquad (4)$$

FIG. 2


DEGREES, ENGLER
Engler
Redwood
Saybolt
Engler Degree = Engler Number
Engler Degree x 51.3 = Engler Time In Seconds.
KINEMATIC VISCOSITY
CM.²/SEC.
TIME IN SECONDS; REDWOOD, SAYBOLT

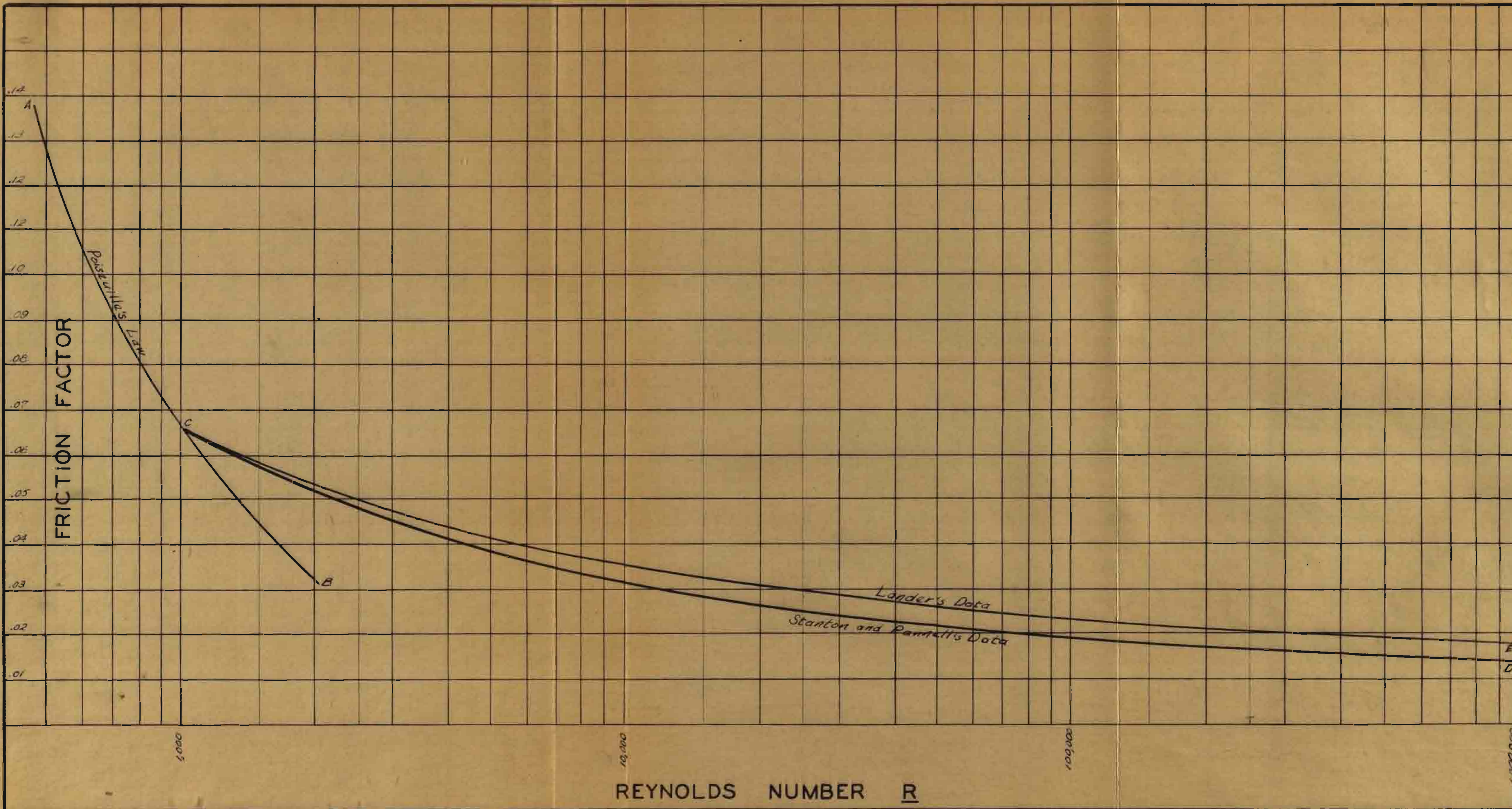
Equation 4 holds for Reynolds' numbers of about 2000 and below. These numbers indicate viscous flow. For values of R between 2000 and 2400 there is a transition zone about which little is known. For all higher Reynolds' numbers, in the turbulent flow region, Stanton and Pannell found a different curve. Figure 3 shows the Stanton and Pannell curve as published by W. G. Heltzel.(8) The line A-B is

(8) W.G. Heltzel, Fluid Flow and Friction in Pipe Lines, Oil and Gas Journal, Volume 29, Number 3, p. 203, June 5, 1930.

a plot of f versus R in the viscous flow region and clearly follows Poiseuille's law. The curve C-D indicates the relationship found by Stanton and Pannell for the turbulent flow region. Their data was based on experiments on the flow of air and water flowing at different velocities through smooth drawn brass pipe of diameters eight inches and larger. It is not surprising then to note that using new commercial steel pipe with diameters of one to six inches Lander found a similar curve in the turbulent flow region, but one, C-E, which fell above the original and thus gives larger values of the friction factor for the same Reynolds' number.

Through the years thousands of experiments have been conducted and new curves have been evolved which more or less parallel those of Stanton and Pannell and Lander until in the present day it is not uncommon to find ten or more

FIG. 3


FRICTION FACTOR
REYNOLDS NUMBER R
.14
.13
.12
.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
1,000
10,000
100,000
1,000,000
A
B
C
D
E
Poiseuille's Law
Lander's Data
Stanton and Pannell's Data

curves (in the turbulent flow range) plotted for pipes of different roughness. One of the most used sets of curves was published by P. J. S. Pigott.(9) Figure 4 and Table 4

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(9) R. J. S. Pigott, The Flow of Fluids in Closed Conduits, Mechanical Engineering, Volume 55, Number 8, p. 497, August 1933.

---

are reproductions from the above mentioned work. These curves are approximately equally spaced between the Stanton and Pannell curve and one for which the friction factor was 0.054 throughout the turbulent range of flow. It should be evident that in choosing the proper curve, the engineer must exercise a certain degree of judgement.

The writer, in corresponding with some fifty oil pipe line companies, noted that only a relatively few used the f versus R diagram in the conventional form. Those that indicated that they used the diagram were far from being in agreement as to what curve to use. Some used curves obtained largely from data obtained from field tests. One used the original Stanton and Pannell curve. Two companies recommended the use of the Danforth curve.(10) It is inter-

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(10) R. S. Danforth, Oil Flow in Pipe Lines, 525 Market Street, San Francisco, California.

---

esting to note that this curve lies almost directly on the Stanton and Pannell curve.

Mr. L. E. Davis of the Sinclair Refining Company, Pipe Line Department, sent the results of some field tests conducted in the early 1930's. This data has not been published.

FIG. 4

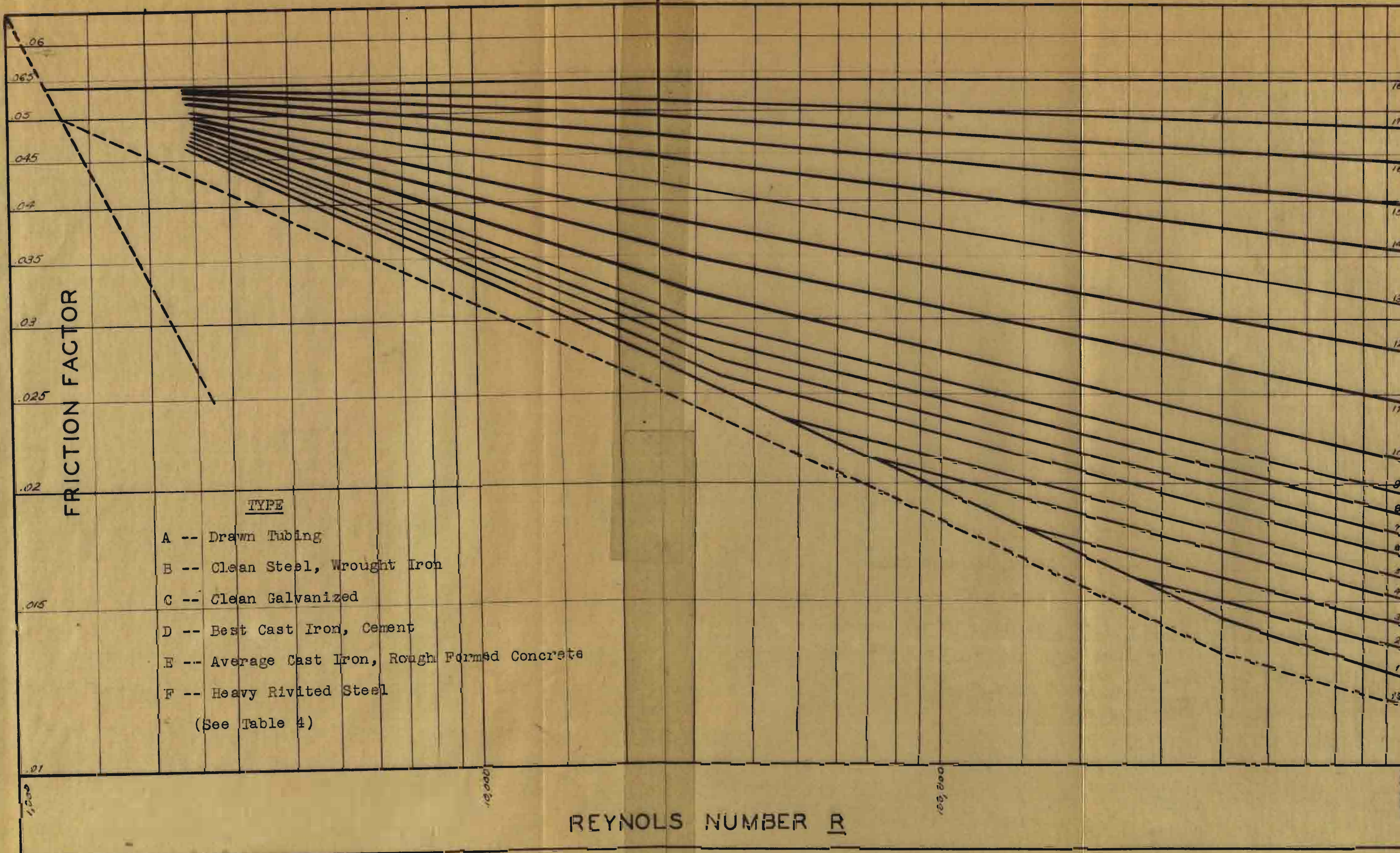

FRICTION FACTOR
.06
.05
.045
.04
.035
.03
.025
.02
.015
.01
TYPE
A -- Drawn Tubing
B -- Clean Steel, Wrought Iron
C -- Clean Galvanized
D -- Best Cast Iron, Cement
E -- Average Cast Iron, Rough Formed Concrete
F -- Heavy Rivited Steel
(See Table 4)
10,000
100,000
REYNOLS NUMBER R

Table 4

Selected Location of "f" by Roughness Relation

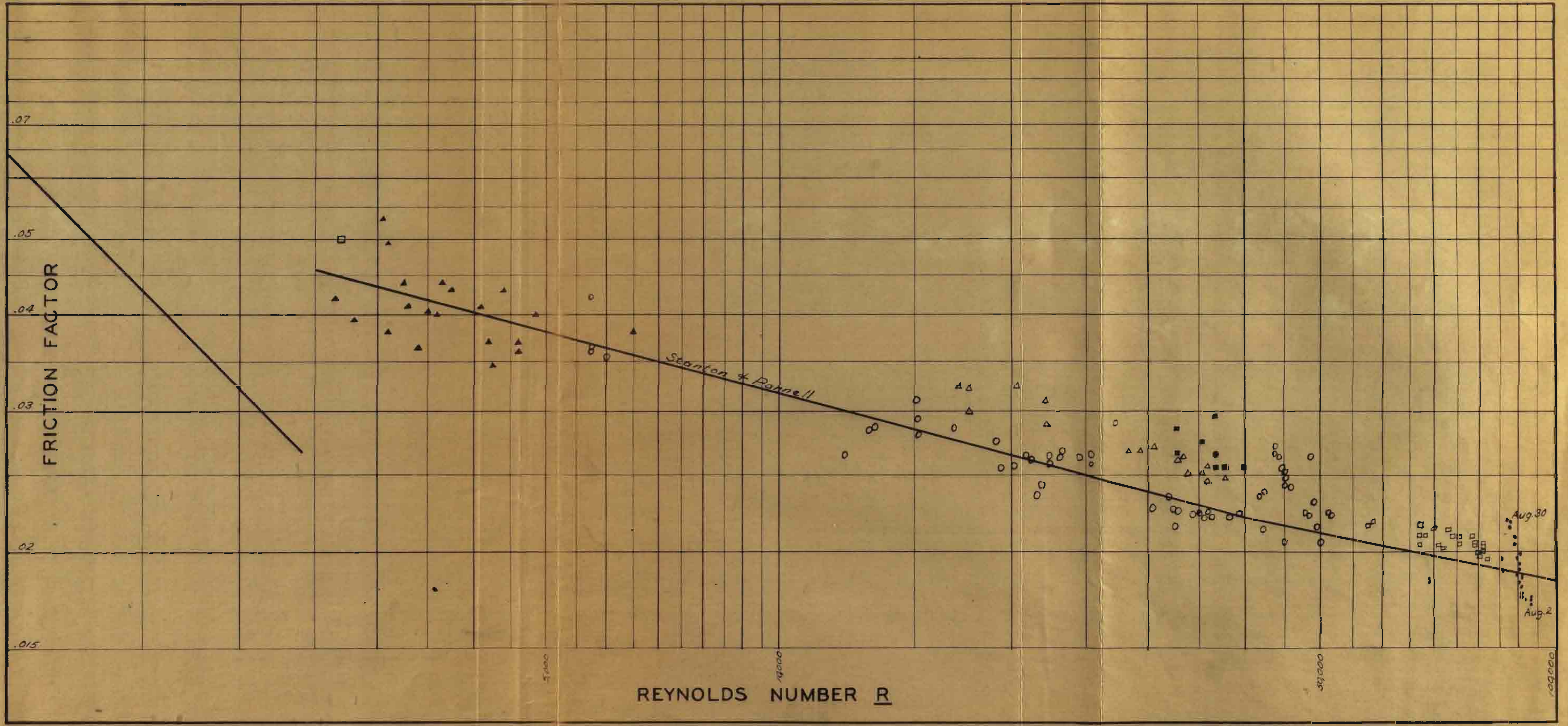
Curve No.	Roughness %	Diameter of Pipe, in Inches Type A	B	C	D	E	F
1	0.2	0.35 up	72	...	...	...	...
2	0.45	. . .	48-66	...	...	...	...
3	0.81	. . .	14-42	30	48-96	96	220
4	1.35	. . .	6-12	10-24	20-48	42-96	84-204
5	2.1	. . .	4-5	6-8	12-16	24-36	48-72
6	3.0	. . .	2-3	3-5	5-10	10-20	20-42
7	3.8	. . .	$1\frac{1}{2}$	$2\frac{1}{2}$	3-4	6-8	16-18
8	4.8	. . .	$1-1\frac{1}{4}$	$1\frac{1}{2}-2$	$2-2\frac{1}{2}$	4-5	10-14
9	6.0	. . .	3/4	$1\frac{1}{4}$	$1\frac{1}{2}$	3	8
10	7.2	. . .	1/2	1	$1\frac{1}{4}$	...	5
11	10.5	. . .	3/8	3/4	1	...	4
12	14.5	. . .	1/4	1/2	...	...	3
13	19.0	. . .	1/8	...	...	...	...
14	24.0	0.125	...	3/8	...	...	...
15	28.0	. . .	...	...	...	...	...
16	31.5	. . .	...	1/4	...	...	...
17	34.0	. . .	...	...	...	...	...
18	37.5	0.0625	...	1/8	...	...	...

Pipes checked ranged from 8 to 12 inches in diameter. The viscosity generally was between 46 and 65 S.S.U. at 70° F. and the discharge varied from 450 to 2600 barrels per hour. In some cases Mr. Davis explained that high or seemingly erratic results may have been caused by faulty pressure gauges or deposits inside the pipe.

A most interesting observation was made on tests conducted in 1931 on a 12 inch line carrying 2600 barrels per hour of East Texas Crude. The values of "f" increased steadily from August 2 to August 30. Whether this was due entirely to the formation of deposits within the pipe is entirely a matter of conjecture. The data mentioned is at the extreme right of Figure 5 at the point where R is equal to 90,000. In studying the data contained on Figure 5 the writer has come to the conclusion that using the Stanton and Pannell curve, or an adaptation thereof, that the actual capacity of a pipeline would always be less than calculations would indicate. It is for this reason that some companies use curves falling higher on the f versus R diagram. One should never fail to remember that smaller sizes of commercial pipe would yield correspondingly higher values for the friction factor.

A majority of the oil pipe line companies indicated that they used charts set up for a direct solution of discharge or pressure loss per mile. Figure 6 is typical of this type of chart. In this case discharge is expressed in barrels per hour, pressure drop in pounds per square

FIG. 5


.07
.05
.04
.03
.02
.015
FRICTION FACTOR
Stanton + Pannell
Aug.30
Aug.2
5000
10000
50000
100000
REYNOLDS NUMBER R

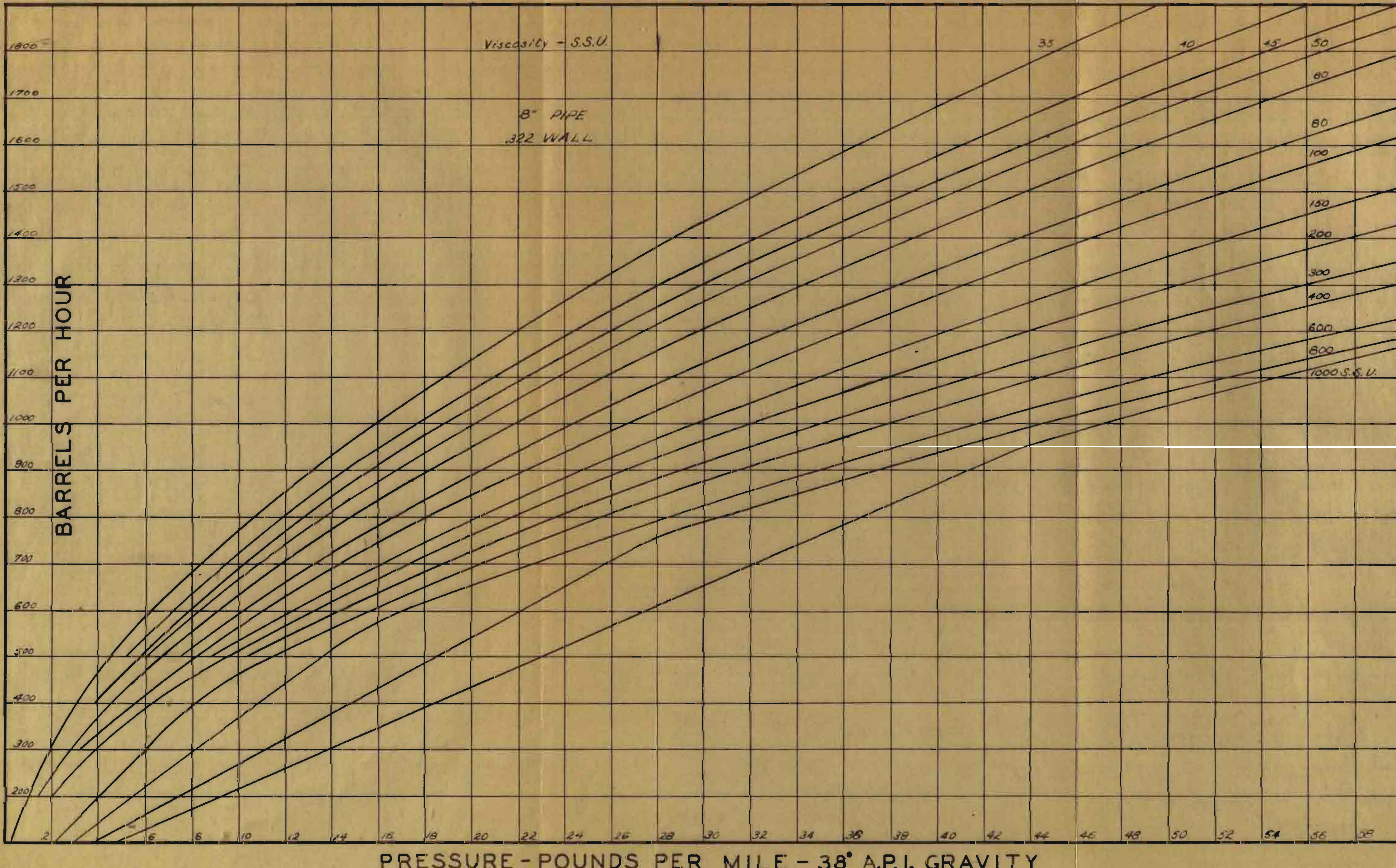
inch per mile, and viscosity in S.S.U. The chart is set up for an 8 inch pipe through which a fluid of 38° A.P.I. is flowing. A separate chart would be required for every pipe diameter in use, and if the gravity of the flowing liquid was other than 38° A.P.I., a correction would be in order. Many pipeline companies have constructed an entire set of such charts and find that they yield entirely satisfactory results.

A few pipe line engineers indicated that they favor a hydraulic slide rule instead of charts, diagrams, or tables for solution of pipe flow problems. T. R. Aude has devised a much used rule which is patented and sold by the Stanolind Pipe Line Company. The Hazen-Williams formula has been adapted for slide rule and is used for the flow of gasoline. By adding a viscosity factor it could be equally as useful for solving problems on the flow of viscous fluids.

In no case has the writer noted that pipe line companies use other than the Darcy-Weisbach formula for the flow of viscous oils. Many correspondents indicated that the formula had been "improved" to suit themselves, but changes were always either in the form of the formula or the units used. Mention was made of the Hazen-Williams formula for the flow of gasoline and the Weymouth formula for the flow of natural gas, fluids which will not now be considered.

Typical flow problems will be treated in the next section by each of the several methods mentioned. An attempt

FIG. 6


Viscosity - S.S.U.
8" PIPE
.322 WALL
35
40
45
50
60
80
100
150
200
300
400
600
800
1000 S.S.U.
1800
1700
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
BARRELS PER HOUR
2
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
42
44
46
48
50
52
54
56
58
PRESSURE - POUNDS PER MILE - 38° A.P.I. GRAVITY

will be made to show that it is a matter of personal likes or dislikes which approach is used, as the same net result is obtained in every case.

SOLUTION OF PROBLEMS ON THE FLOW OF OIL IN PIPES

Problem No. 1.) It is required to find the pressure loss in pounds per square inch in 8000 feet of 8 inch (8.071") commercial steel pipe of a crude oil having a gravity of 38° A.P.I. and viscosity of 100 S.S.U. when 500 barrels per hour are flowing.

Solution A.) In foot- pound- second system:

Using the Saybolt Universal Viscosimeter formula for f.p.s. system,

$$\nu = 0.00000237t - \frac{0.00194}{t}$$

$$\nu = 0.00000237(100) - \frac{0.00194}{100} = 0.0002176 \ ft^2/sec$$

$$Q = 500(0.00156) = 0.78 \ cfs$$

$$V = Q/A = 0.78/0.355 = 2.2 \ '/sec.$$

$$\underline{R} = \frac{VD}{\nu} \doteq \frac{2.2(0.667)}{0.0002176} = 6740$$

Using the curve of Lander (See Fig. 3) f = 0.038

$$Sp. Gr. = \frac{141.5}{131.5 + 38} = 0.835$$

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = 0.038 \frac{8000(2.2)^2}{0.667(64.4)} = 34.2' \text{ of oil}$$

$$p_f = wh = 0.433(34.2)(0.835) = \underline{12.4 \ psi}$$

Using Pigott's Chart, Figure 4, f = 0.037

Hence, $p_f = 12.4 \frac{0.037}{0.038} = \underline{12.1 \text{ psi}}$

Solution B.) The Darcy-Weisbach formula may be expressed as shown below by introducing the discharge in barrels per hour.

$$h_f = \frac{f}{65.6} \frac{L Q_B^2}{d^5} \qquad (2c)$$

In similar units,

$$\underline{R} = 22.13 \frac{Q_B}{\nu d}$$

where ν is expressed in stokes

From Figure 2 $\nu = 0.20$

$$\underline{R} = 22.13 \frac{500}{0.20 (8.071)} = 6860$$

As before, f = 0.038

$$\therefore h_f = \frac{0.038}{65.6} \frac{(8000)(500)^2}{(8.071)^5} = 33.8' \text{ of oil}$$

$$p_f = 0.433 (33.8)(0.835) = \underline{12.25 \text{ psi}}$$

Solution C.) Using the Pressure Drop- Discharge Chart, Figure 6

Read Q = 500 barrels per hour at the left margin of the chart.

Trace this line horizontally to the intersection with the curved line for a value of 100 S.S.U.

Read

$$P_f/\text{mile} = 8.2\ \text{psi}$$

$$\therefore P_f = \frac{8000}{5280}(8.2) = \underline{12.4\ \text{psi}}$$

Solution D.) Using the Moody Curves

As before,

$$V = 2.2\ '/\text{sec}$$

$$s.g. = 0.835$$

$$\underline{R} = 6740$$

From Table 5, k = 0.00015

$$\therefore r/k = \frac{4.036}{0.00015} = 269$$

From Figure 9, f = 0.036

$$P_f = 12.4\ \frac{0.036}{0.038} = \underline{11.75\ \text{psi}}$$

Problem No. 2.) What would be the pressure drop if the gravity of the oil in the preceding problem had been 30° A.P.I., all other conditions remaining the same?

Solution A.) Using Darcy-Weisbach Formula

$$s.g. = \frac{141.5}{131.5 + 30} = 0.876$$

From the previous problem,

$$h_f = 34.2'$$

$$P_f = 0.433(34.2)(0.876) = \underline{12.95 \text{ psi}}$$

Solution B.) Using Figure 6

$$P_f = 12.4 \text{ psi}$$

For a gravity of 30° A.P.I.,

$$P_f = 12.4 \frac{0.876}{0.835} = \underline{13.0 \text{ psi}}$$

A COMPARISON OF THE WORKS OF SCHODER AND NIKURADSE

Numerous studies have been made in the past to determine experimentally the effect of pipe surface roughness on the flow of fluids. Early investigators artificially roughened pipes by cutting screw threads of varying depths on the interior of the pipes. These early investigators noted that the friction factor increased with the surface roughness for all Reynolds' numbers. For high degrees of surface roughness it was noted that the friction factor was independent of the Reynolds' number and the friction loss varied as the velocity squared.

Dr. J. Nikuradse conducted a series of painstaking experiments during the period from 1928 to 1931 and in 1933 published his now famous findings.(11) He coated the

(11) J. Nikuradse, Laws of Fluid Flow in Rough Pipes, Petroleum Engineer, Volume 11, March, May, June, July, August 1940. (A translation of the 1933 article)

interior surfaces of pipes with grains of sand so that the surfaces resembled those of sandpaper. In each case the sand was sifted and carefully graded so that all sands in any one group were of uniform size. Hence, in each case a definite relationship between the mean diameter of the sand grains and the radius of the pipe was obtained. Nikuradse called the ratio of the pipe radius to the mean elevation of roughness the "relative roughness" of the pipe surface.

In his experiments the relative roughness varied from 15 to 507, the pipe diameter from 0.61" to 2.51", and the water temperature from 54° F. to 61° F. Figure 7 shows a plot of Nikuradse's experimental findings.

As might be expected, the single curve in the viscous flow region follows Poiseuille's law and has the equation,

$$f = \frac{64}{\underline{R}} \tag{4}$$

The base curve through the turbulent flow range follows the conventional curve of Stanton and Pannell up to a Reynolds' number of about 100,000. Blasius found by analyzing an extensive series of measurements made by Saph and Schoder that the turbulent flow data for smooth pipes lay along this line. In a logarithmic plot the data formed a straight line, the slope and position indicating the following exponential form:[12]

---

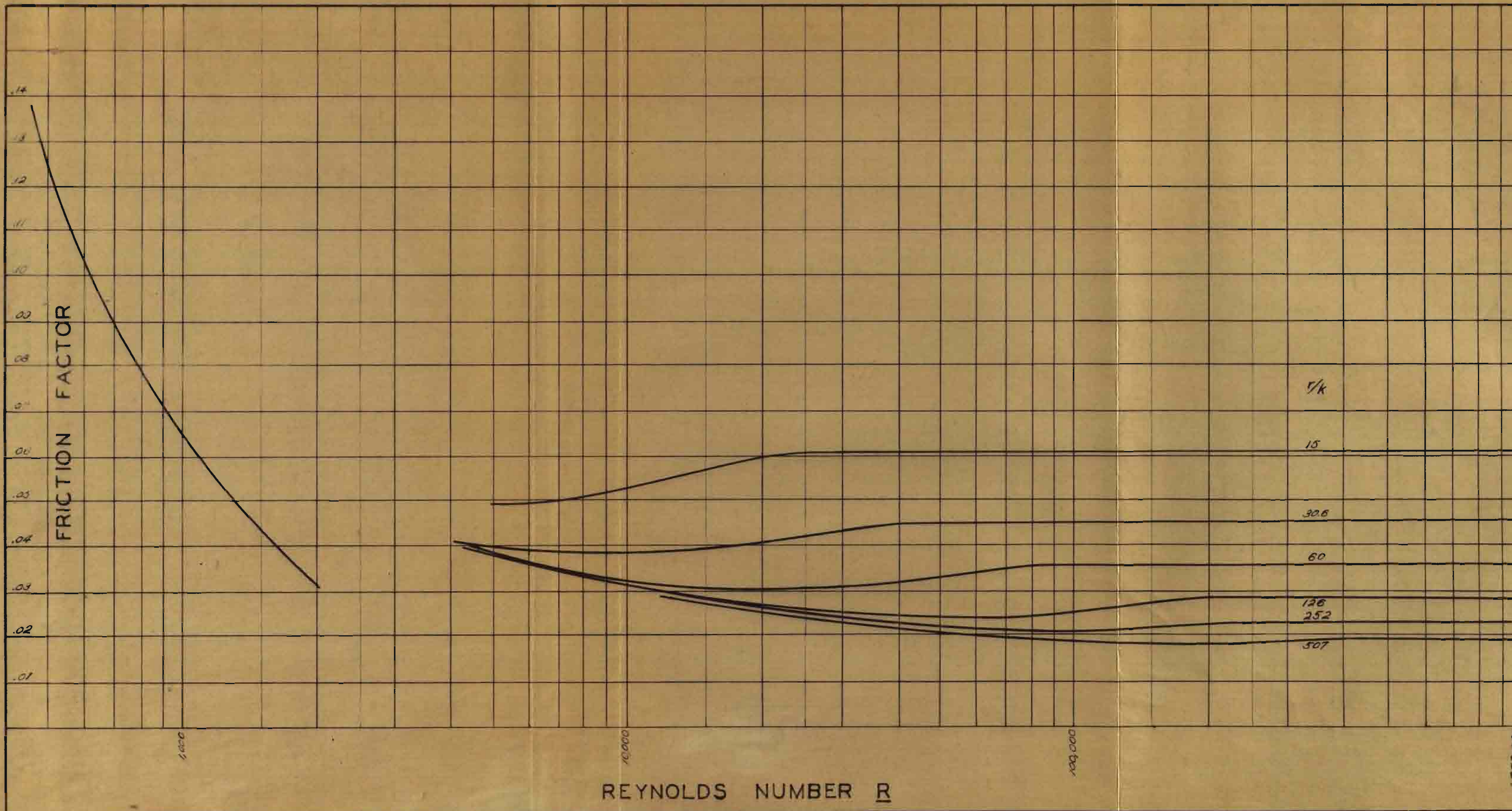
(12) H. Rouse, Fluid Mechanics for Hydraulic Engineers, McGraw-Hill Book Company, 1938, p. 246.

$$f = \frac{0.3164}{\underline{R}^{1/4}} \tag{20}$$

Beyond the Blasius range ($\underline{R} \approx 100{,}000$) Nikuradse found that the exponential form no longer holds. He proposed the following empirical relationship based on his experimental findings for the extended Stanton and Pannell curve.

$$f = 0.0032 + \frac{0.221}{\underline{R}^{0.237}} \tag{21}$$

FIG. 7


FRICTION FACTOR
.14
.13
.12
.11
.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
r/k
15
30.6
60
126
252
507
1000
10,000
100,000
1,000,000
REYNOLDS NUMBER R

In discussing his findings Nikuradse suggests dividing the chart into three zones. In the first, a smooth pipe zone, the laminar film makes pipe surface roughness of no consequence and the relation of f to R is the same for both smooth and rough pipes.

In the second or transition zone the thickness of the laminar film has been reduced to the point where a portion of the pipe surface projections penetrate into the turbulent flow area and thereby cause an increased friction loss.

In the rough pipe zone all projections penetrate the laminar film. The turbulence produced by the pipe roughness becomes a maximum and the friction factor is observed to be independent of the Reynolds' number. In this zone the friction loss is seen to vary as the square of the velocity.

In order to compare the Schoder formulas with Nikuradse's work the writer has taken each of the four formulas (14), (15), (16), and (17) and put it in the Darcy-Weisbach form. Each was then equated to the Darcy-Weisbach formula and a relation for the friction factor in terms of the diameter and velocity was obtained as shown below:

$$H_f = 0.30 \frac{V^{1.75}}{D^{1.25}} \qquad (14)$$

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \qquad (2)$$

For "extremely smooth" pipes

$$f = \frac{0.30\,(2g)}{1000\ V^{0.25}\ D^{0.25}} = \frac{0.0193}{V^{0.25}\ D^{0.25}} \qquad (22)$$

Similarly, for "fairly smooth" pipes,

$$f = \frac{0.0245}{D^{0.25}\ V^{0.14}} \qquad (23)$$

For "rough" pipes,

$$f = \frac{0.0322}{D^{0.25}\ V^{0.05}} \qquad (24)$$

For "extremely rough" pipes,

$$f = \frac{0.0445}{D^{0.25}} \qquad (25)$$

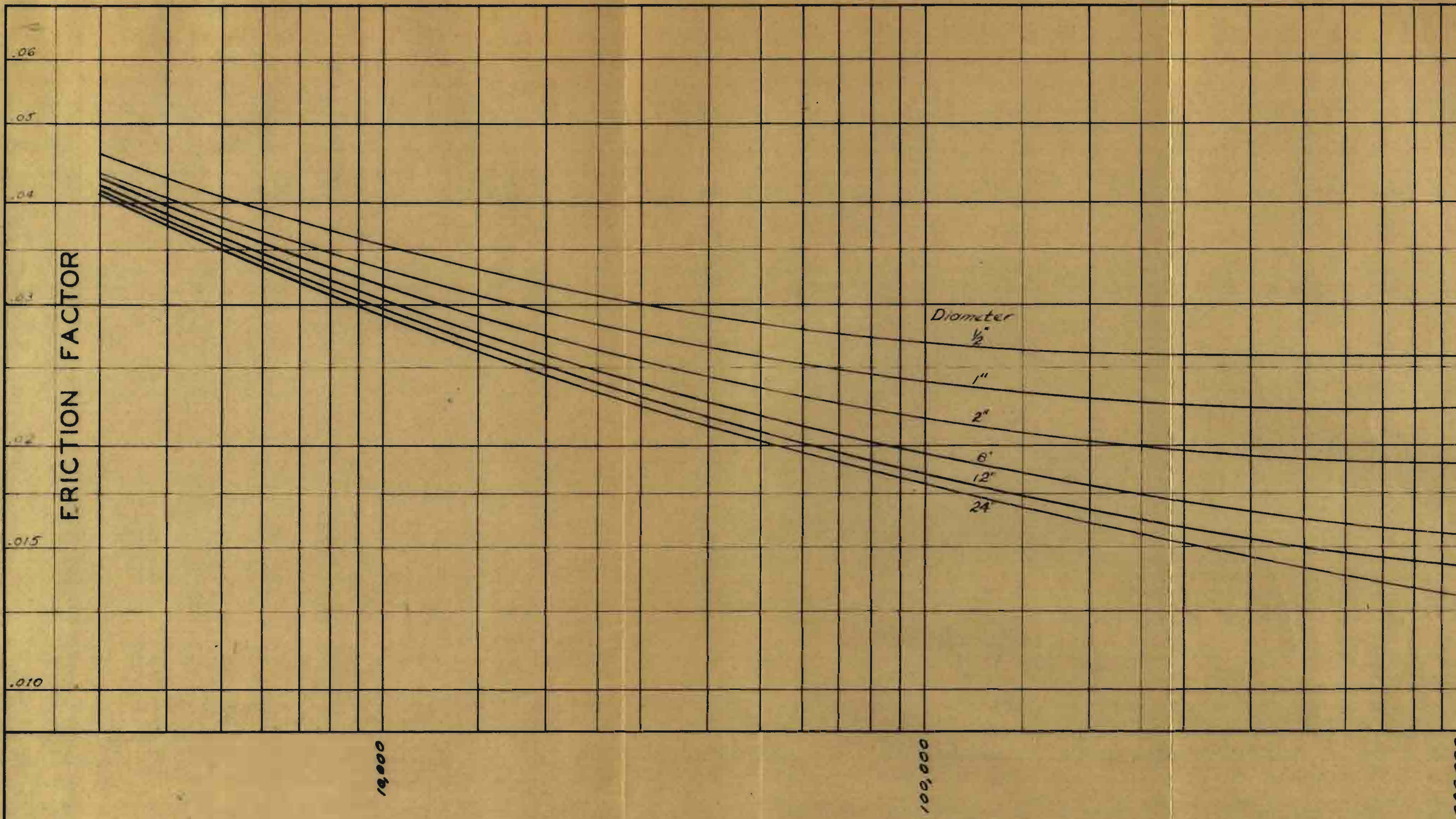
If values of the friction factor are plotted against Reynolds' number for each of Schoder's formulas, the result would be three charts similar to Figure 8[13] and one for

(13) Pipe Friction - Tentative Standards of Hydraulic Institute, 1948, p. 17.

"extremely rough" pipes which would consist of a series of horizontal lines. Such charts are limited in use as each may be used for only the one type of pipe.

On Figure 9 the f versus R relationship for Schoder's formulas (while holding the diameter constant) has been

FIG. 8



REYNOLDS NUMBER R
FRICTION FACTORS FOR STEEL OR WROUGHT-IRON PIPE

plotted over the work of Nikuradse. In this way it is observed that the curve for "extremely smooth" pipe falls just above the lowest Nikuradse curve. The writer has found that Schoder's curve follows Lander's data closely while Nikuradse's agrees with the Stanton and Pannell curve up to a value of R of approximately 100,000. Beyond this point the Nikuradse curve follows a path that has been substantiated only for pipes of uniform roughness.

The curve for "fairly smooth" pipes, as expected, falls somewhat above the "extremely smooth" curve and crosses it at a point where R is 8000. Similarly, the "rough" pipe curve falls yet higher and crosses the original curve at a point where R is between 6000 and 7000. The corresponding curve for "extremely rough" pipes would be a straight horizontal line as shown. It is interesting to note that it is possible to make the latter curve fall on each of the Nikuradse curves simply by varying the diameter. Thus, for Nikuradse's curves having a relative roughness of 15, 30.6, 60, 126, 252, and 507, the diameters required would be 3.6", 9.6", 2.5', 6.5', 14', and 30' respectively. This conformity takes place only above and to the right of the line A - B, however.

Also plotted on Figure 9 are several curves plotted after data presented in similar form by Moody.[14] The

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(14) L. F. Moody, Friction Factors for Pipe Flow, A.S.M.E. Transactions, pp. 671-690, November 1944.

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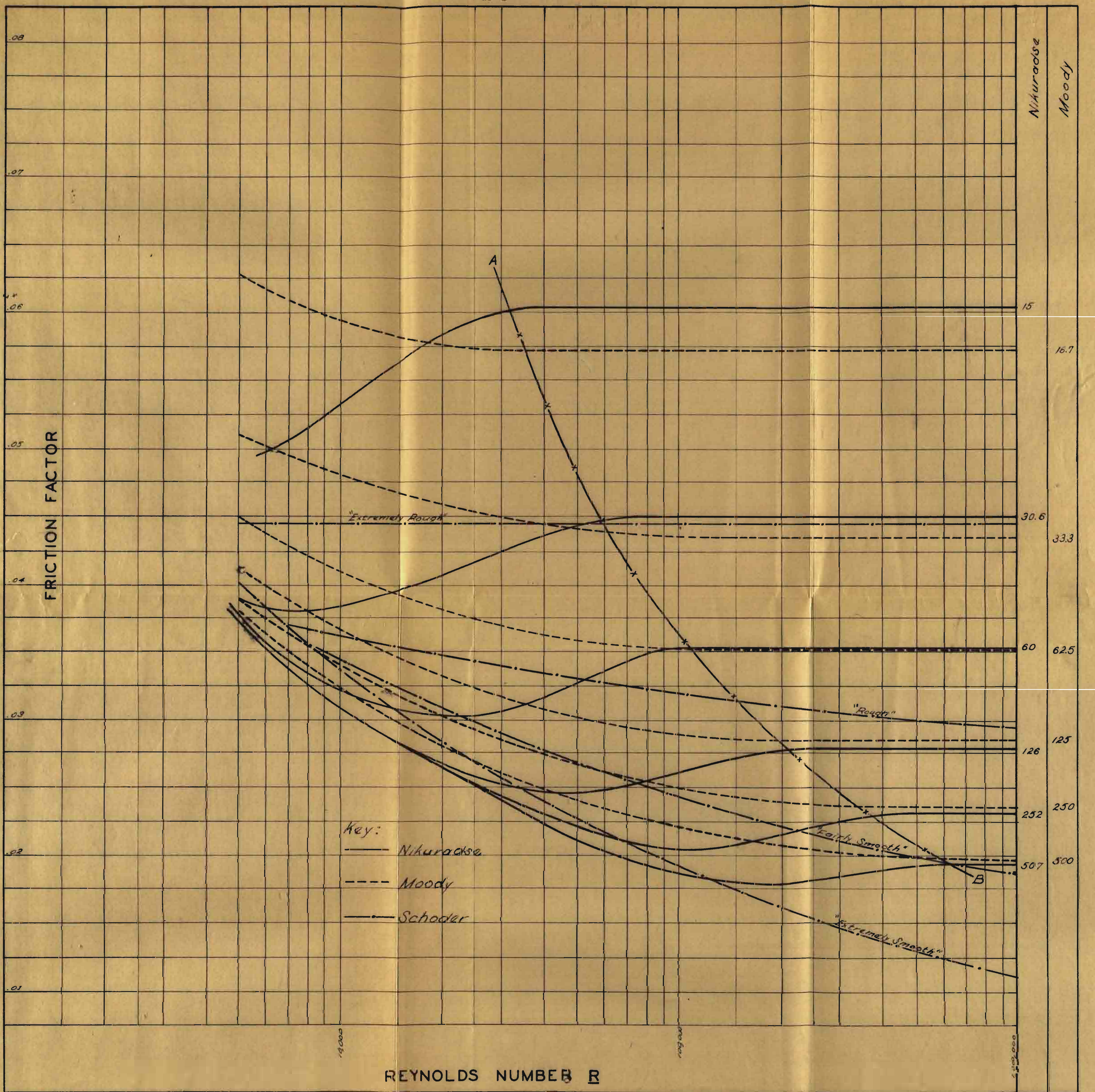
numbers in the columns to the right of the figure indicate

FIG. 9



.08
.07
.06
.05
.04
.03
.02
.01
FRICTION FACTOR
Nikuradse
Moody
A
B
15
16.7
30.6
33.3
60
62.5
126
125
252
250
507
500
"Extremely Rough"
"Rough"
"Fairly Smooth"
"Extremely Smooth"
Key:
Nikuradse
Moody
Schoder
10000
100000
1,000,000
REYNOLDS NUMBER R

what relative roughness value a curve represents. From Moody's work six curves were chosen which corresponded closely to those of Nikuradse. It is immediately evident that Moody and Nikuradse agree only in the range of complete turbulence, i.e., to the right of the line A-B. To the left of this line Moody's curves follow the pattern set by Schoder's curves. In fact, Moody's curve for a relative roughness of 250 agrees closely with the Schoder "fairly smooth" pipe curve up to a value of R of about 100,000. Beyond this point the Schoder curve continues to drop while the Moody curve flattens out.

In comparing the Pigott chart, Figure 4, with the Schoder curves on Figure 9 a striking similarity in the manner in which the curves denoting increasing values of roughness "sprout" from a base curve is to be noted. If curves for intermediate categories of roughness were added to Figure 9, the resemblance would be even more evident.

Since the work of Nikuradse, Moody, Pigott, and Schoder can be compared, at least in certain regions of flow, it should be possible to estimate the relative value of each. The Nikuradse curves, as has been noted, have a transition zone peculiar to themselves. Nikuradse's work has filled a great gap in our knowledge of fluid flow but his curves are applicable only for pipes having a uniform roughness unheard of in present day commercial pipe. For this reason it is impossible to use his work to advantage in practice.

Moody's curves are also based on the relative roughness concept but have been developed for commercial pipe. Although new, his work promises to be of great value after it has been tested. He has assigned the values of mean elevation of roughness in the table below to the various kinds of pipe.

Table 5

Mean Elevation of Roughness for Pipes of Various Materials

Material	k in Feet
Brass, Lead, Glass, Transite, etc.	0.000005
Commercial Steel or Wrought Iron	0.00015
Cast Iron-Asphalt Dipped	0.0004
Galvanized Iron	0.0005
Wood Stave	0.0006-0.003
Cast Iron - Uncoated	0.0085
Concrete	0.001-0.01
Riveted Steel	0.003-0.03

Moody's curves check the work of Pigott and Schoder except at high Reynolds' numbers. They check Nikuradse's work in this region. It would appear that these curves, if substantiated by use, would prove invaluable to the engineer for herein a fundamental description of pipe surface roughness has been established.

Pigott's curves probably see more widespread use than any other set. They have been included in numerous text-books and handbooks. The curves check pretty well those of Moody and Schoder. Their popularity is doubtlessly due in part to the accompanying description (see Table 4) which enables one to choose a suitable curve for determination of the friction factor.

Schoder's curves check those of Moody and Pigott through a fair range of flow. However, at low and high Reynolds' numbers they are at variance. In fact, at low values of R they cross each other, indicating that in this range an "extremely smooth" pipe could be assigned a higher value of f than a pipe which falls in one of the other categories of roughness. As well, these curves are based on formulas for the flow of water only. The formulas are not readily adaptable for the flow of viscous fluids.

From the preceding the writer concludes that the determination of the head loss in fluid flow problems is at present dependent largely on the experience and judgement of the engineer. The definition, specification, and estimation of pipe roughness should be reduced to a universal form. Nikuradse has suggested that the "relative roughness" of the pipe is the needed description. His work, however important, was carried out on relatively small pipes of a roughness too uniform to be of value commercially. Hence, until further experiments on larger pipes for other roughness types is carried out, his curves are of no real value.

Pigott's organization of data in Table 4 and Figure 4 has been well received, and his chart has been much used in the past decade for the solution of practical problems. His description of the type of pipe with a range of diameters listed for each curve has limited somewhat the possibility of error in determining the friction factor. His work is not perfect though, for all types of pipes are not included and the range of pipe diameters listed is incomplete.

The curves representing Schoder's formulas clearly indicate that the exponential type formula should be restricted in use. At high values of $\underline{R}$ the friction factor is too low while at low values of $\underline{R}$ the reverse is true. Clearly the determination of the pipe roughness is a matter of judgement, although Schoder does give a general description of his "categories of roughness." One should also remember that Schoder's formulas, and others of this type, do not as a rule include a viscosity term. This, of course, restricts their use to a fluid over not too great a range of temperature.

The writer favors Moody's set of curves which has been incorporated in the Tentative Standards of the Hydraulic Institute. Moody also uses the relative roughness concept but in a different form. He uses the ratio of the pipe surface roughness to the diameter of the pipe as the relative roughness. Although Moody lists values for the mean elevation of roughness (see Table 5) the final determination of the relative roughness is still subject to some discretion on the part of the engineer in that some allowance for the effect of age must be made. This allowance, of course, will be dependent upon local conditions and the requirements of each particular installation. It is important that we note that Moody's curves are a relatively new innovation and their use must be found advantageous and reliable before they will be acceptable to the engineering profession.

CONCLUSIONS

In comparing the many pipe flow formulas, the writer has been confronted with the fact that each has certain limitations. It is to be feared that the average engineer uses one or more formulas blindly, without regard to the experimental data on which the formula is based. Even the determination of the Darcy-Weisbach friction factor is subject to this criticism for the manner of description of pipe roughness remains unfixed.

The solutions of problems on the flow of water presented earlier in this paper indicate that throughout an appreciable range all formulas used will yield reliable results. Despite this fact many engineers have the conception that a particular formula is superior to all others. In many cases the belief that some one formula is to be favored can be traced to the classroom. The student often accepts the teacher's opinion as conclusive and continues to use whatever formula was used in school.

It is suggested that the engineering student be shown the limitations of the various formulas used in the field. The writer believes it would be wise to promote the use of the Darcy-Weisbach formula for the solution of problems involving the flow of water. This formula is not subject to the limitations of many of the empirical formulas. The Reynolds' number- friction factor relationship used to determine f is applicable for all fluids. The writer has learned by correspondence that a great majority of the petroleum pipeline companies have used the Darcy-Weisbach formula and the Reynolds' number- friction factor relationship successfully for over twenty-five years. The hydraulic engineers' desire to use familiar formulas has probably been the predominant reason why the Reynolds' number- friction factor relationship has not been used for problems involving the flow of water.

An inspection of the numerous texts, handbooks, and articles that have been published on the subject would indicate that most of the formulas discussed herein have been handed down through the years. The use of the Kutter formula for pipe flow problems has been suggested by a few authors. Only a relatively few engineers use the Kutter formula for similar results may be obtained by using the less complicated Manning formula. The Manning, Scobey, and Hazen-Williams formulas are all expressed in the same form. The exponents for each of the formulas are only slightly different. The main difference is in the factor denoting roughness. In the Manning formula, n is placed in the denominator. For the Hazen-Williams formula the coefficient is the product of 1.318 and a constant, C_1. Scobey has determined a roughness factor for each kind of pipe used. Any of the above formulas may be used with confidence once the applicable roughness factor has been determined.

Schoder's formulas differ from those previously mentioned only in that Schoder has introduced an exponential formula for each of four arbitrary "categories of roughness." Each of Schoder's formulas yields reliable results, although at times it might be desirable to consider a pipe as falling in an intermediate category. The writer feels that the description of the "categories of roughness" leaves much to be desired in that only a hint of the effect of pipe diameter on roughness is given.

In the past, the description of pipe roughness has been made largely from the appearance of the surface to the eye or the smoothness to the hand. Descriptions such as "a smooth glassy" or "a slimy" surface were common. Pigott's classification of the type of material coupled with a range of diameters was marked improvement. The presentation of the relative roughness concept by Moody appears to be the logical method of describing surface roughness. The Hydraulic Institute has tentatively adopted Moody's work, and it appears that the ratio of the pipe radius or diameter to the mean elevation of surface roughness will soon be accepted as the best method for describing pipe roughness.

Notation

C ---- Chezy coefficient

C_1 ---- Hazen-Williams constant

C_s ---- a Scobey coefficient

D ---- Diameter in feet

d ---- Diameter in inches

f ---- Darcy-Weisbach friction factor

g ---- Acceleration due to gravity

h_f ---- Head loss due to friction

H_f ---- Head loss per thousand feet of length

k ---- Mean elevation of roughness

K_s ---- A Scobey coefficient

L ---- Length

m ---- Bazin's roughness term

M_s ---- a Scobey coefficient

n ---- Manning's coefficient of roughness

p_f ---- Pressure drop in psi

Q ---- Discharge in cfs

Q_B ---- Discharge in barrels per hour

r ---- Radius of pipe

R ---- Hydraulic radius

$\underline{R}$ ---- Reynolds' number

S ---- Slope

t ---- Time in seconds

V ---- Mean velocity

w ---- Specific weight

w.p. - Wetted perimeter

r/k -- Relative roughness

γ ---- Specific weight

μ ---- Absolute viscosity

ν ---- Kinematic viscosity

ρ ---- Density

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VITA

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He received his grade school and high school education in the public schools at Newport, Rhode Island. In June 1939 he graduated from Rogers High School, then worked for a year before entering Rhode Island State College in September, 1940.

Except for summer employment as a toolkeeper with the Platt Construction Company and as a material inspector with Merritt-Chapman and Scott Corporation, he remained with his studies until April, 1943.

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In February, 1946 he enrolled at Washington University and graduated from there with the degree of Bachelor of Science in Civil Engineering in June, 1947.

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