EFFECT OF RADIATION ON MAGNETOHYDRODYNAMIC FREE CONVECTION BOUNDARY LAYER FLOW NEAR THE LOWER STAGNATION POINT OF A SOLID SPHERE WITH NEWTONIAN HEATING

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ABSTRACT

In this paper, the effect of radiation on magnetohydrodynamic free convection boundary layer flow near the lower stagnation point of a solid sphere with Newtonian heating, in which the heat transfer from the surface is proportional to the local surface temperature, is considered. The transformed boundary layer equations in the form of partial differential equations are solved numerically using an implicit finite difference scheme known as the Keller-box method. Numerical solutions are obtained for the local wall temperature, the skin friction coefficient, as well as the velocity and temperature profiles. The features of the flow and heat transfer characteristics for various values of the Prandtl number Pr, magnetic parameter M, radiation parameter and the conjugate parameter are analyzed and discussed.

KEYWORDS*: Magnetohydrodynamic (MHD); Newtonian heating; Radiation effects; Solid sphere Stagnation point*

1.0 INTRODUCTION

The effect of radiation on magnetohydrodynamic flow, heat and mass transfer problems has become industrially more important. Many engineering processes occur at high temperatures, the knowledge of radiation heat transfer leads significant role in the design of equipment.

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Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering processes. At high operating temperature, the radiation effect can be quite significant (Sivaiah et al., 2010). Nazar et al., (2002a, 2002b) considered the free convection boundary layer flows on a sphere in a micropolar fluid without effect of radiation and magnetohydrodynamic with constant heat flux (CHF) and constant wall temperature (CWT), respectively. Molla et al., Molla et al., (2011), Akhter & Alim (2008) and Miraj et al., (2010) studied the radiation effect on free convection flow from an isothermal sphere with constant wall temperature, constant heat flux and in presence of heat generation, respectively. The viscous dissipation and magnetohydrodynamic effect on a natural convection flow over a sphere in the presence of heat generation have been presented by Ganesan & Palani (2004), Alam et al., (2007) and Molla et al., (2005).

For the condition Newtonian heating, many of the research were written with this condition It seems that Merkin (1994) was the first to use the term Newtonian heating for the problem of free convection over a vertical flat plate. Lesnic et al., (1999, 2000, 2004) and Pop et al., (2000) to study a free convection boundary layer over vertical and horizontal surfaces as well as over a small inclined flat plate from the horizontal surface embedded in a porous medium with Newtonian heating. Recently Salleh et al., (2010a, 2010b, 2010c, 2012) studied the free and mixed convection boundary layer flows on a sphere with Newtonian heating in a viscous and micropolar fluid, respectively.

The situation with Newtonian heating arises in what are usually termed conjugate convective flows, where the heat is supplied to the convective fluid through a bounding surface with a finite heat capacity Merkin (1994). This configuration occurs in many important engineering devices, for example in heat exchanger where the conduction in solid tube wall is greatly influenced by the convection in the fluid flowing over it. Further, for conjugate heat transfer around fins where the conduction within the fin and the convection in the fluid surrounding it must be simultaneously analyzed in order to obtain the vital design information and also in convection flows set-up when the bounding surfaces absorb heat by solar radiation see Chaudhary & Jain (2006, 2007). This results in the heat transfer rate through the surface being proportional to the local difference in the temperature with the ambient conditions.

Recent demands in heat transfer engineering have requested researchers to develop various new types of heat transfer equipment with superior performance, especially compact and light-weight ones. Increasing the need for small-size units, focuses have been cast on the effects of the interaction between developments of the thermal boundary layers in both fluid streams, and of axial wall conduction, which usually affects the heat exchangers performance. Therefore, we conclude that the conventional assumption of the absence of interrelation between coupled conduction convection effects is not always realistic, and this interrelation must be considered when evaluating the conjugate heat transfer processes in many practical engineering applications Chaudhary & Jain (2007). Excellent reviews of the topic of conjugate heat transfer problems can be found in the book by Martynenko & Khramtsov (2005).

Therefore, the aim of the present paper is to study the effect of radiation on magnetohydrodynamic free convection boundary layer flow on a solid sphere with convective boundary conditions. The governing boundary layer equations are first transformed into a system of non-dimensional equations via the non-dimensional variables, and then into non-similar equations before they are solved numerically by the Keller-box method, as described in the book Cebeci & Bradshaw (1984).

2.0 MATHEMATICAL ANALYSES

Consider a heated sphere of radius a, which is immersed in a viscous and incompressible fluid of ambient temperature T_∞ . The surface of the sphere is subjected to a Newtonian heating (NH). We assume that the equations are subjected to a Newtonian heating (NH). Under the Boussinesq and boundary layer approximations, the basic equations are

$$\frac{\partial}{\partial \bar{x}}(\bar{r}\,\bar{u}) + \frac{\partial}{\partial \bar{y}}(\bar{r}\,\bar{v}) = 0 \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta(T - T_\infty)\sin\left(\frac{\bar{x}}{a}\right) - \frac{\sigma\beta^2}{\rho}\bar{u} \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\frac{\partial^2 T}{\partial \bar{y}^2} - \frac{1}{\rho c_\rho}\frac{\partial q_r}{\partial \bar{y}} \tag{3}$$

Subject the boundary conditions of (Salleh et al., 2010c)

$$\bar{u} = \bar{v} = 0 \quad \frac{\partial T}{\partial \bar{y}} = -h_s T \quad \text{at} \quad \bar{y} = 0$$
$$\bar{u} \to 0 \quad T \to T_\infty \quad \text{as} \quad \bar{y} \to \infty, \tag{4}$$

where $\bar{r}(\bar{x}) = a\sin(\bar{x}/a)$, $\bar{u}$ and $\bar{v}$ are the velocity components along the $\bar{x}$ and $\bar{y}$ directions, respectively, T is the local temperature, g is the gravity acceleration, β is the thermal expansion coefficient, v is the kinematic viscosity, ρ is the fluid density, σ is the electric conductivity, c_ρ is the specific heat, α is the thermal diffusivity, k is the thermal conductivity, and h_s is the heat transfer coefficient for Newtonian heating condition.

We introduce now the following non-dimensional variables (Salleh et al., 2010c):

$$x = \frac{\bar{x}}{a},\; y = Gr^{1/4}\left(\frac{\bar{y}}{a}\right),\; r = \frac{\bar{r}}{a},$$
$$u = \left(\frac{a}{\nu}\right)Gr^{-1/2}\bar{u},\; v = \left(\frac{a}{\nu}\right)Gr^{-1/4}\bar{v},$$
$$\theta = \frac{T - T_\infty}{T_\infty} \tag{5}$$

where is the $Gr = g\beta T_\infty a^3/\nu^2$ Grashof number for Newtonian heating (NH).

Using the Rosseland approximation for radiation (Bataller Bataller (2008)), the radiative heat flux is simplified as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial \bar{y}} \tag{6}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. We assume that the temperature differences within the flow through the porous medium such as that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, we get

$$T^4 \cong 4T_\infty^3 - 3T_\infty \tag{7}$$

Substituting variables (5) to (7) into (1) to (3) then become

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial y}(rv)=0, \tag{8}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+\theta\sin x-Mu, \tag{9}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\left(1+\frac{4}{3N_R}\right)\frac{\partial^2 \theta}{\partial y^2}, \tag{10}$$

where $\Pr=\frac{\nu}{\alpha}$ is the Prandtl number $M=\frac{\sigma\beta^2 a^2}{\nu\rho Gr^{1\backslash 2}}$ is the magnetic parameter and $N_R=\frac{\alpha k^{*}\rho c_\rho}{4\sigma^{*}T_\infty^3}$ is the radiation parameter. The boundary conditions (4) become

$$u=v=0,\ \frac{\partial\theta}{\partial y}=-\gamma(1+\theta)\ \text{ on }\ y=0$$
$$u\to 0,\ \ \theta\to 0\ \text{ as }\ y\to\infty \tag{11}$$

where $\gamma=ah_s Gr^{-1/4}$ is the conjugate parameter for the Newtonian heating. It is noticed that $\gamma=0$ is for the insulated plate and $\gamma\to\infty$ is when the surface temperature is prescribed. To solve (8) to (10), subjected to the boundary conditions (11), we assume the following variables:

$$\psi=xr(x)f(x,y),\ \ \theta=\theta(x,y), \tag{12}$$

where ψ is the stream function defined as

$$u=\frac{1}{r}\frac{\partial\psi}{\partial y}\ \text{ and }\ v=-\frac{1}{r}\frac{\partial\psi}{\partial x}, \tag{13}$$

which satisfies the continuity equation (8). Thus, (9) and (10) become

$$\frac{\partial^3 f}{\partial y^3}+\left(1+x\cot x\right)f\frac{\partial^2 f}{\partial y^2}-\left(\frac{\partial f}{\partial y}\right)^2+\frac{\sin x}{x}\theta-M\frac{\partial f}{\partial y}=x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}-\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right) \tag{14}$$

$$\frac{1}{\Pr}\left(1+\frac{4}{3N_R}\right)\frac{\partial^2\theta}{\partial y^2}+\left(1+x\cot x\right)f\frac{\partial\theta}{\partial y}=x\left(\frac{\partial f}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial\theta}{\partial y}\right), \tag{15}$$

subject to the boundary conditions

$$f = \frac{\partial f}{\partial y} = 0, \ \frac{\partial \theta}{\partial y} = -\gamma(1+\theta) \ \text{at } y = 0$$
$$\frac{\partial f}{\partial y} \to 0, \ \theta \to 0 \ \text{as } \ y \to \infty \qquad (16)$$

It can be seen that at the lower stagnation point of the sphere, $(x \approx 0)$ equations (14) and (15) reduce to the following ordinary differential equations:

$$f''' + 2ff'' - f'^2 + \theta - Mf' = 0 \qquad (17)$$
$$\frac{1}{\Pr}\left(1 + \frac{4}{3N_R}\right)\theta'' + 2f\theta' = 0 \qquad (18)$$

and the boundary conditions (16) become

$$f(0) = f'(0) = 0, \ \theta'(0) = -\gamma(1+\theta)$$
$$f' \to 0, \theta \to 0 \ \text{as } \ y \to \infty \qquad (19)$$

where primes denote differentiation with respect to y.

3.0 RESULTS AND DISCUSSION

Equations (17) and (18) subject to the boundary conditions (19) were solved numerically using an efficient, implicit finite-difference method known as the Keller-box scheme for Newtonian heating (NH) with several parameters considered, namely, magnetic parameter M, radiation parameter N_R, the Prandtl number Pr, the conjugate parameter γ and the coordinate running along the surface of the sphere, x.

In this paper, the results focused only on the case at the lower stagnation point, $x \approx 0$. Values of Pr considered are Pr = 0.7, 1 and 7. It is worth mentioning that small values of Pr («1) physically correspond to liquid metals, which have high thermal conductivity but low viscosity, while large values of Pr (»1) correspond to high-viscosity oils. It is worth pointing out that specifically, Prandtl number Pr = 0.7, 1.0 and 7.0 correspond to air, electrolyte solution and water, respectively.

Table 1 present the values of the wall temperature $\theta(0)$ and the skin friction coefficient $f''(0)$ at the lower stagnation point of the sphere,

$x \approx 0$, when Pr = 0.7, 1, 7, M = 0, $N_R = \infty$ and $\gamma = 1$. In order to verify the accuracy of the present method, the present results are compared with those reported by Salleh et al., (2010c). It is found that the agreement between the previously published results with the present ones is very good. We can conclude that this method works efficiently for the present problem and we are also confident that the results presented here are accurate.

Table 2 shown the values of the wall temperature $\theta(0)$ and the skin friction coefficient $f''(0)$ at the lower stagnation point of the sphere, $x \approx 0$, for various values of N_R when Pr = 0.7, $\gamma = 1$ and M = 0, 5, 10. It is observed that, when the magnetic parameter M is fixed an increasing of the radiation parameter N_R show that both values of $\theta(0)$ and $f''(0)$ decreases, and also when N_R is fixed, an increasing of M the values of and increases.

Figures 1 and 2 shows the temperature and velocity profiles near the lower stagnation point of the sphere, $x \approx 0$, when Pr = 0.7, M = 5, $N_R = 1, 5, 10, \infty$ and $\gamma = 1$, respectively. It is found that as N_R increases, the temperature and velocity profiles decreases.

The temperature and velocity profiles presented in Figure 3 and 4, respectively, near the lower stagnation point of the sphere, $x \approx 0$, when Pr = 0.7, $N_R = 0$, M = 0,5,10 and $\gamma = 1$ shows that when the value of M increases, it is found that the temperature profiles also increases, but the velocity profiles decreases.

Table 1. Values of the wall temperature $\theta(0)$ and the skin friction coefficient $f''(0)$ at the lower stagnation point of the sphere, $x \approx 0$, when Pr = 0.7, 1, 7, M = 0, $N_R = \infty$ and $\gamma = 1$

	$\theta(0)$		$f''(0)$	
Pr	**Salleh et al. (2010c)**	**present**	**Salleh et al. (2010c)**	**present**
0.7	26.4584	26.4595	8.9609	8.9626
1	17.2861	17.2884	6.1409	6.1413
7	3.3651	3.3669	1.2489	1.2490

Table 2. Values of the wall temperature $\theta(0)$ and the skin friction coefficient $f''(0)$ at the lower stagnation point of the sphere, $x \approx 0$, for various values of N_R when Pr = 0.7, M = 0, 5, 10 and $\gamma = 1$

	M = 0 Present		M = 5 Present		M = 10 Present	
N_R	$\theta(0)$	$f''(0)$	$\theta(0)$	$f''(0)$	$\theta(0)$	$f''(0)$
1	84.6126	24.2288	112.7021	26.5229	140.1570	28.63586
3	42.6999	13.5465	61.28889	15.2316	79.03272	16.71782
5	35.8107	11.6383	52.39761	13.1705	68.11037	14.50669
7	33.0155	10.8461	48.73030	12.3084	63.56162	13.57723
10	30.9817	10.2623	46.03631	11.6703	60.20188	12.88732
100	26.8867	9.0660	40.53631	10.3544	53.29070	11.45869
1000	26.4904	8.9487	39.99804	10.2246	52.61029	11.31734
∞	26.4595	8.9626	39.93837	10.2102	52.53481	11.30165

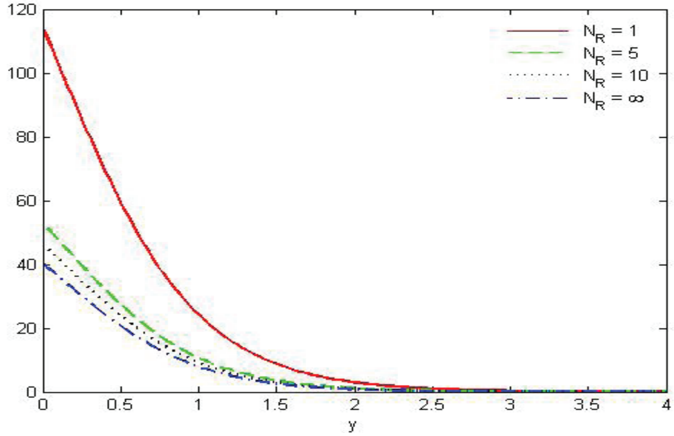

Figure 1. Temperature profiles θ near the lower stagnation point of the sphere, $x \approx 0$, when Pr = 0.7, M = 5, $N_R = 1, 5, 10, \infty$ and $\gamma = 1$

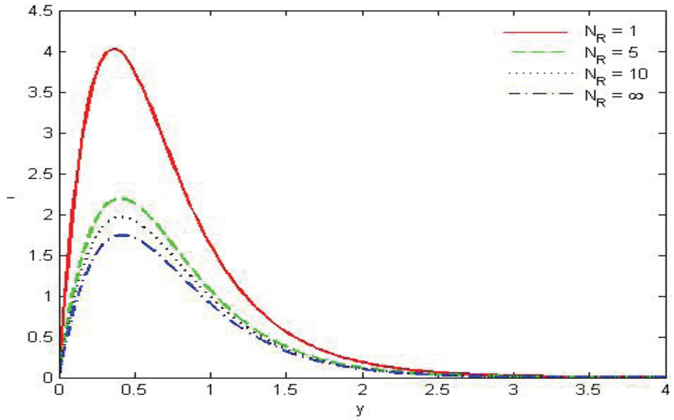

Figure 2. Velocity profiles f' near the lower stagnation point of the sphere, $x \approx 0$, when Pr = 0.7, M = 5, $N_R = 1, 5, 10, \infty$ and $\gamma = 1$

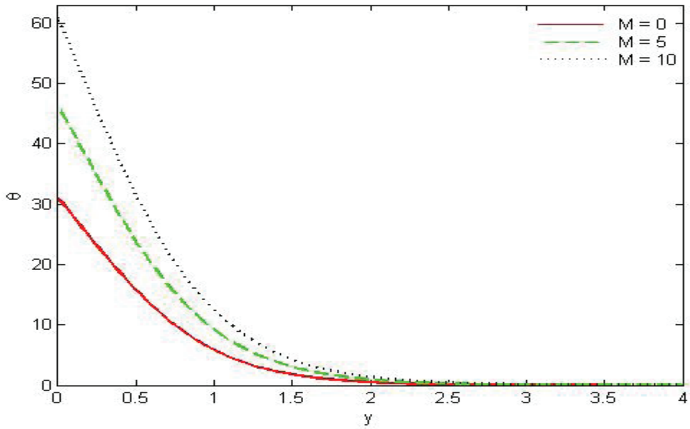

Figure 3. Temperature profiles θ near the lower stagnation point of the sphere, $x \approx 0$, when Pr = 0.7, $N_R = 10$, M = 0,5,10 and $\gamma = 1$

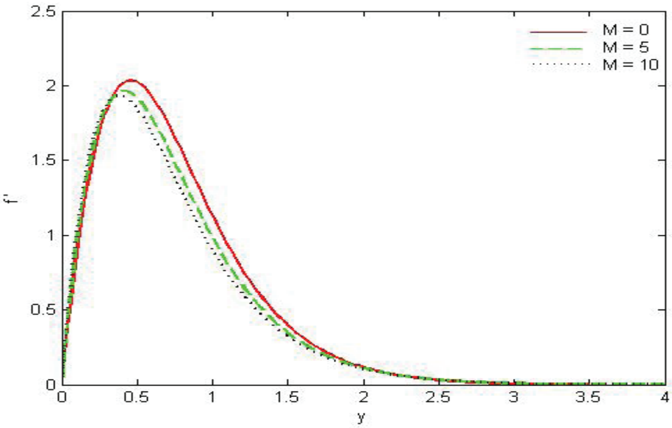

Figure 4. Velocity profiles f' near the lower stagnation point of the sphere, $x \approx 0$, when Pr = 0.7, $N_R = 10$, M = 0,5,10 and $\gamma = 1$

4.0 CONCLUSIONS

In this paper, we have numerically studied the problem of the effect of radiation on magnetohydrodynamic free convection boundary layer flow near the lower stagnation point of a solid sphere with Newtonian heating (NH). It is shown how the Prandtl number Pr, magnetic parameter *M*, thermal radiation parameter N_R and conjugate parameter γ affects the values of the temperature profiles $\theta(0)$, the skin friction coefficient $f''(0)$. We can conclude that

i. when Pr and M are fixed, as increases, the values of skin friction coefficient, temperature and velocity profiles decreases,

ii. when Pr and are fixed, as M increases, the temperature profiles increases, but skin friction coefficient and velocity profiles decreases.

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