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Calibration of an instrument to calibrate accelerometers

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CALIBRATION OF AN INSTRUMENT
TO
CALIBRATE ACCELEROMETERS

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ABSTRACT

The purpose of this research was to calibrate an instrument with which an accelerometer could be calibrated accurately. The instruments used were a piezoelectric shaker and a modified Michelson interferometer together with necessary electronic equipment. This equipment was used to calibrate a Kistler Model 808A accelerometer.

Weight was added to the shaker in the second part of this experiment in order to determine its effect on the amplitude and frequency of oscillation of the shaker.

Results show that this instrument may be used to calibrate an accelerometer quite accurately. The weight added to the shaker will effect only the amplitude of vibration and not the frequency of oscillation.

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LIST OF SYMBOLS

Symbol	Name	Units
d	Thickness of air film	cm
r	Radius of Fringe	cm
R	Radius of curvature of convex lens	cm
PC_b	Charge	micro-microcoulombs
mv	Voltage	milli-volt
I	Brightness	Lumens/cm^2
K	Constant	Lumens/cm^2
h	Fringe width	cm
X	Distance	cm
T	Period	sec
B_o	Bessel Function	
λ	Wavelength	Angstrom
A	Amplitude	in
g	Acceleration of gravity	in/sec^2
S	Displacement	in
V	Velocity	in/sec
a	Acceleration	in/sec^2
f	Frequency	cycles/sec
G	"g-level"	

I. INTRODUCTION

The dynamic loading on structures and components in various environments has put great emphasis upon the development of new measuring devices. The need for accurate and reliable data has increased primarily because of the complexity of the applications. Due to widespread use of accelerometers in modern engineering research, a need has developed for a more accurate means of calibrating this instrument. The object of this research was to produce a motion to check the calibration of an already calibrated accelerometer, and to find the effects of added weights to the response of the instrument.

An acceleration was supplied from the shaker to the accelerometer. Its output voltage was recorded and the acceleration was calculated, establishing a calibration point. The amplitude and frequency of these points must be determined in order to calculate the acceleration and to construct the calibration curve. A piezoelectric shaker produced the oscillation. The shaker was driven by a power amplifier, which amplified the signal from an audio oscillator. A modified Michelson interferometer was used in determining the amplitude of oscillation. This amplitude could be determined by the disappearance of the Newton rings.

The effect of additional weights on the resonant points was investigated in order to see whether the resonant points of the system differ under various weights.

The system produced a suitable motion over a frequency range of approximately 1,000 to 5,000 cycles per second. The output of the shaker became closer to sinusoidal motion as the frequency was increased. All accelerations were calculated in terms of "g", which is the standard gravitational acceleration constant.

II. REVIEW OF LITERATURE

Man's earliest production of an electric effect came through the agency of mechanical forces. As more was learned about electricity, its various manifestations were distinguished by special prefixes, as galvanic, frictional, thermo-, photo-, hallo-, pyro-, and piezo-, some of which are now obsolete or abandoned (1).*

It had long been observed that tourmaline crystal when placed in hot ashes first attracted and then repelled the hot ashes. This fact first became known in Europe about 1703, when tourmalines were brought from Ceylon, which was sometimes called "Ceylon Magnet," and its electrical character was established in 1656 by Aepinus, who noted the opposite polarities at the two ends of heated tourmaline (1).

Piezoelectricity was discovered in 1880 by two French brothers, Pierre Curie and Jacques Curie, for which they were awarded the Plante Prize in 1895 (2).

Piezoelectricity is defined by Walter G. Cady (1) as "electric polarization produced by mechanical strain in crystals belonging to certain classes, the polarization being proportional to the strain and changing sign with it." This definition is known as direct effect. Closely related to it is the converse effect, whereby, a piezoelectric crystal becomes strained when electrically polarized, by an amount proportional to the polarizing field.

*All references may be found in Bibliography.

The "equivalent network" of any electromechanical system is generally understood to mean an assemblage of resistors (R), inductors (L), and capacitors (C), each independent of frequency, so interconnected that the assemblage when substituted for the actual system will be affected the same as that of the electromechanical system itself, at least over a certain range of frequency. The equivalent network universally adopted for piezoelectric systems was derived by Butterworth (3). The equivalent electrical impedance consists of a certain resistance, inductance, and capacitance in series; the whole being shunted by a second capacitance.

If the convex surface of a lens is placed in contact with a plane glass plate, a thin film of air is formed between the two surfaces. The thickness of the film is very small at the point of contact, gradually increasing as one proceeds outward. The loci of points of equal thickness are circles concentric with the points of contact. Since this was studied by Newton, they are called Newton's rings. The thickness of the air film is inversely proportional to R, which is the radius of curvature of the convex lens (4), and is given by:

$$d = \frac{r^2}{2R} \qquad (1)$$

where: d is the thickness of air film and r is the radius of any ring for which d is calculated.

The earliest shakers were mechanically driven and consisted of a crankshaft, connecting rods, and driving head (5). But these shakers had a very limited operational frequency range with great distortion due to the friction in bearings and other losses.

The piezoelectric shaker was selected in this project because of its wide frequency response. There are several papers presented by Edelman, Jones, Smith, Thomas and Schmidt on various types of piezoelectric shakers (6, 7, 8, 9, 10, 11, and 12).

Interferometer methods of determining the amplitudes of small mechanical vibrations have been investigated by S. H. Cartez (13), Fujimoto (14), H. Osterbert (15), and W. Jay Kennedy (16), with various degrees of success. Fujimoto (14) was able to estimate the amplitude of oscillation by varying the voltage impressed across the electrodes of a quartz resonator, and observing variation in the clearance of the monochromatic interference fringes formed by using the crystal as the mirror of the interferometer.

III. APPARATUS

A. Electrical System: (Figures 1 and 3)

1. A Hewlett-Packard audio oscillator, model 201C, supplied the input signal. The input frequency was monitored by using a Beckman Counter, model 5230B. An adjustment of the attenuation caused the input signal to produce a sine wave signal, which was observed on an Analab Dual Trace, model 1120, oscilloscope. The oscilloscope was also used to measure the output of the accelerometer.

2. A Stromberg-Carlson, model 70807, power amplifier was used to provide sufficient voltage gain, and to produce a signal without distortion to the shaker.

The amplifier produced a linear output at approximately 100 watts over a frequency range of 500 to 12,000 cycles per second. To avoid damage to the amplifier and retain a continuous load, a resistor of 2000 ohms was placed across the output of the amplifier, which resulted in higher output voltages. A variable inductor was placed parallel with the shaker, and resistor on the amplifier output, in order to tune the circuit for different frequencies.

A Helipot, precision potentiometer with a resistance of 1,000 ohms, was connected to the input of the amplifier to provide a precise amplitude control of the input signal (Figure 2).

3. A Kistler quartz accelerometer, model 808A, weight 0.7 ounces, that was already calibrated by the manufacturer at 1.210 P_{cb}/g, was used

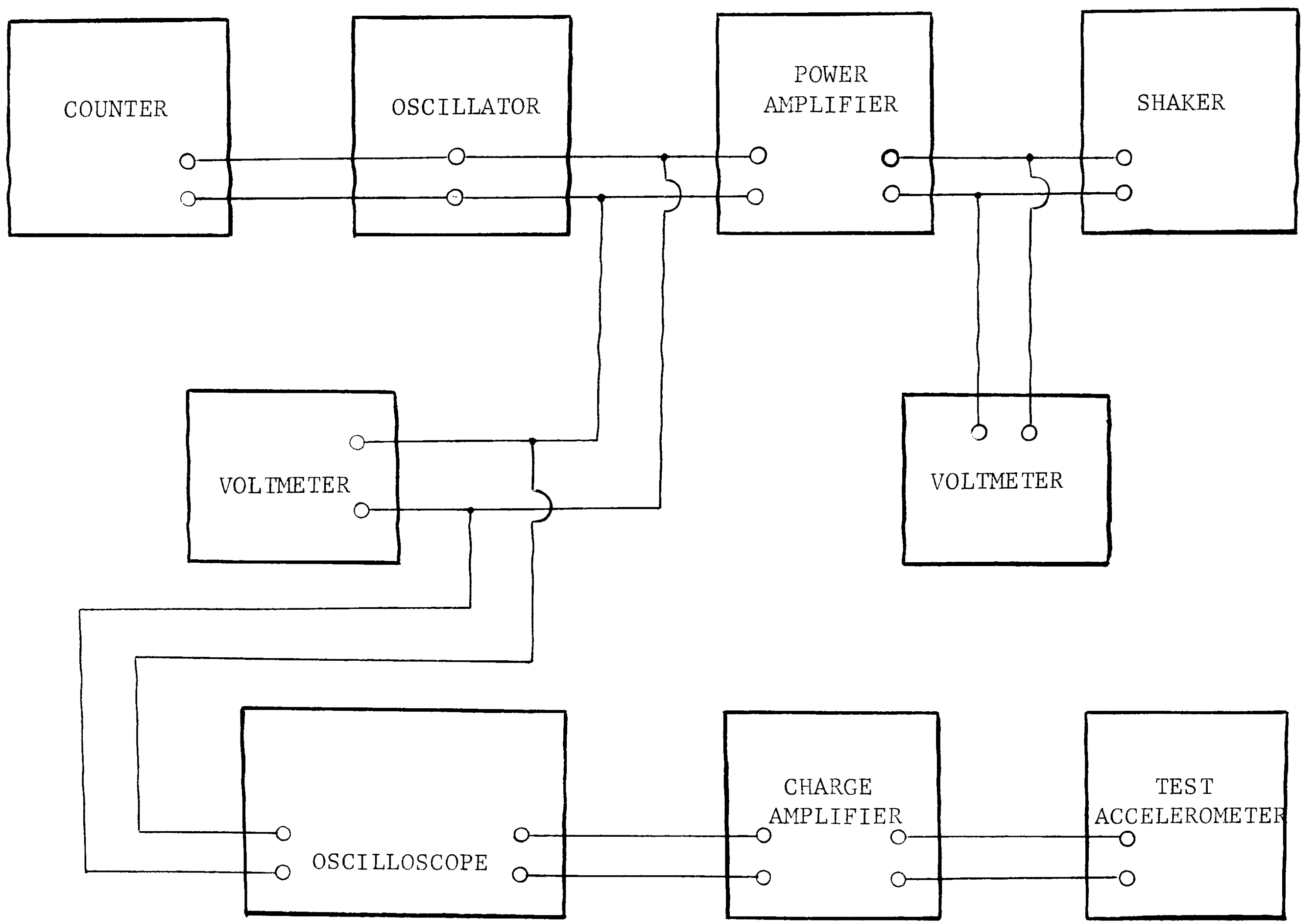

Figure 1. Electrical Schematic

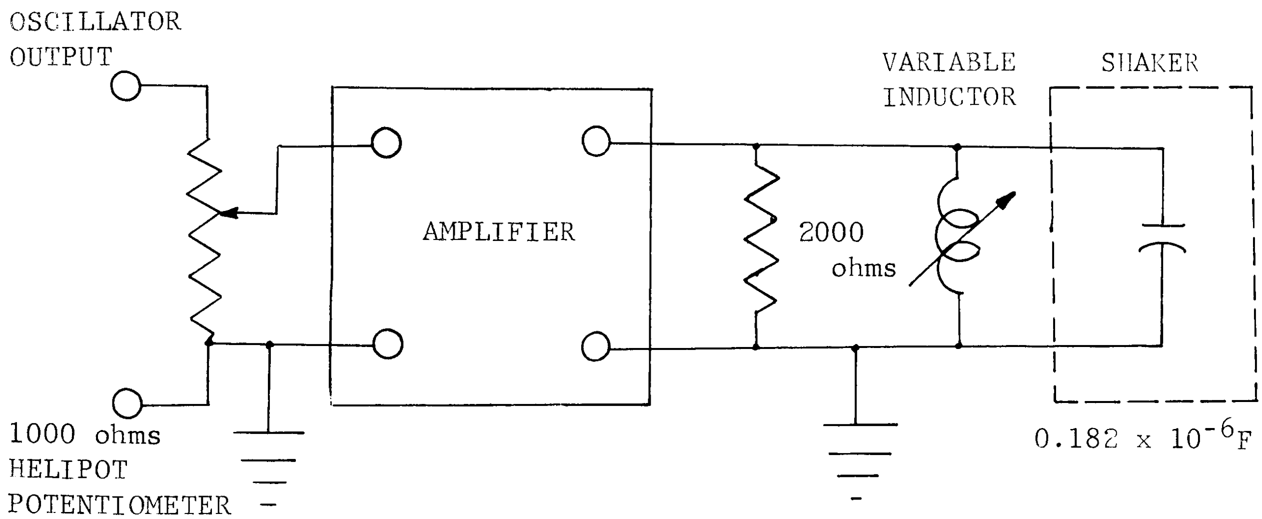

Figure 2. Electrical Control System.

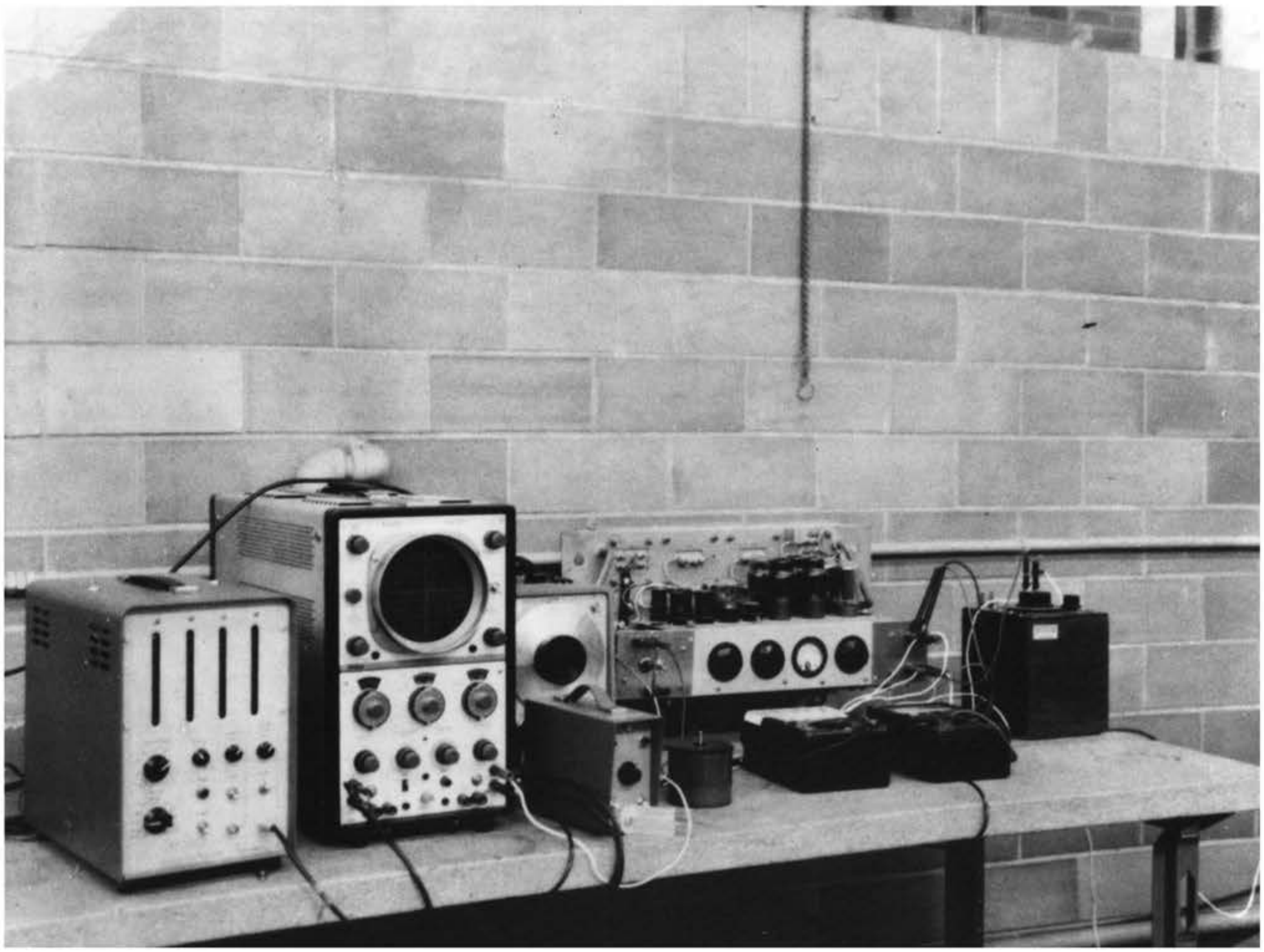

Figure 3. Electrical System.

as a standard. The results obtained from the interferometer were compared to those of the accelerometer. An amplifier was required to connect the high-impedance charge of the accelerometer to a low-impedance voltage which could be displayed on the oscilloscope. A Kistler Electrostatic charge amplifier, model 566 multi-range was used, and the output voltage was displayed on the oscilloscope.

B. Shaker and Cage:

1. Shaker: (Figure 4)

The shaker consisted of barium titanate crystals. The dimensions of the crystals are as shown in Figure 5. This type of pizoelectric crystal was chosen, because of its availability and its wide range of operating frequency. The primary disadvantage was that it has only discrete points corresponding to mechanical resonance. The points were distinguished by varying the amplitude of oscillation, and observing points of resonance of the accelerometer on the oscilloscope.

The crystals were polarized radially and both inside and outside surfaces had been polished and silvered. Due to this polarization, when an alternating electric signal was applied to the crystals, radial expansion and contraction resulted. This radial expansion and contraction was accompanied by longitudinal expansion and contraction as predicted by Poisson's Law.

Because of the elastic properties of the crystals, the expansion and contraction of one crystal is very small. In order to obtain measurable oscillation, twelve crystals were used, which also allowed a

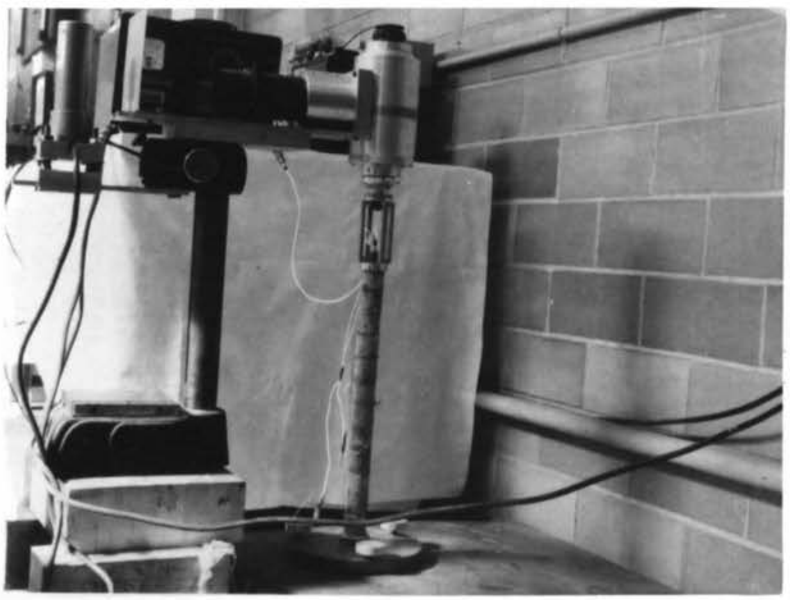

Figure 4. Shaker and Interferometer.

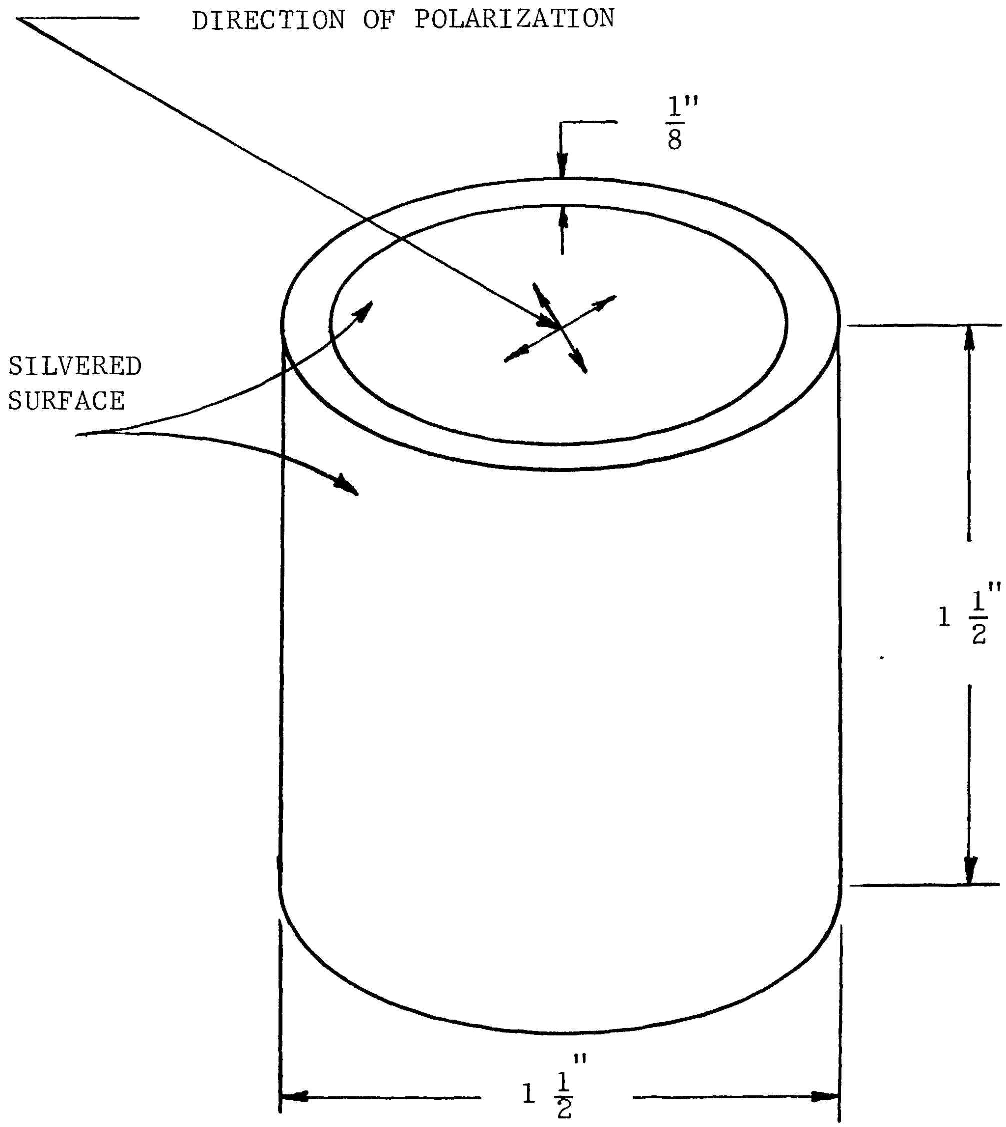

Figure 5. Typical Barium Titanate Crystal Used.

wider range of operating frequency. These crystals were connected electrically in parallel by soldering the leads to the crystal surfaces.

In order to have a system of one degree of freedom, the crystals had to be fastened together in such a way as to act as a solid stack. However, due to the high frequency vibration under which the system was operating, high shear forces resulted at the joints between crystals. Difficulty was experienced in the selection of an epoxy which would withstand these dynamically produced forces. The following list includes the epoxies that were tested and states their advantages and disadvantages:

(i) Sauereisen Adhesive Cement No. 1 paste proved to be a very strong adhesive, under static loads. However, after a matter of weeks the epoxy became brittle and failed under operating conditions.

(ii) Eastman 910: The instructions for this adhesive stated that the surfaces to be joined should be smooth and matched. It is also necessary to apply a relatively high pressure to the crystals while curing to insure a good bond. These requirements were determined to be impractical for this application since some of the surfaces were chipped. The requirement for a high pressure between the joints also proved impractical, due to the brittleness of the crystals.

(iii) Dow Corning A-4000 Adhesive was the epoxy finally selected. To improve adhesion, Dow Corning, A-4014

Primer was used to pre-condition the surfaces. The junctions bonded by this epoxy did not fail and results were satisfactory.

In the process of using the various adhesives, if failure occurred at the bond, a different adhesive was substituted. The end result was that, three bonds were of Sauereisen Cement No. 1, three bonds were of Eastman 910, and six bonds were of Dow Corning A-4000. After all the experimental work was completed, the stack was oscillated at a resonant point until failure occurred. It was concluded that Dow Corning Adhesive A-4000, was the best as it did not fail.

The bottom end of the stack of crystals was attached to a brass disc which weighed approximately fifteen pounds. It was then placed on a heavy steel pad which was isolated by a cushion from the floor of the laboratory. The other end, of the shaker was attached to the collet so it could be fastened to the shaker.

2. Construction of Cage: (Figure 6)

The cage was designed in order to mount the accelerometer and additional weights on the shaker. The parts of the cage were made of T-6 aluminum in order for it to possess a high natural frequency and also to be light and rigid.

Figure 7 shows how the parts of the cage were assembled. It consisted of a lower disc which was fastened to the collet of the crystals by eight screws. Four of these screws were used to hold the legs which supported the top discs and the mirror. The accelerometer was mounted

Figure 6. Cage and Accelerometer.

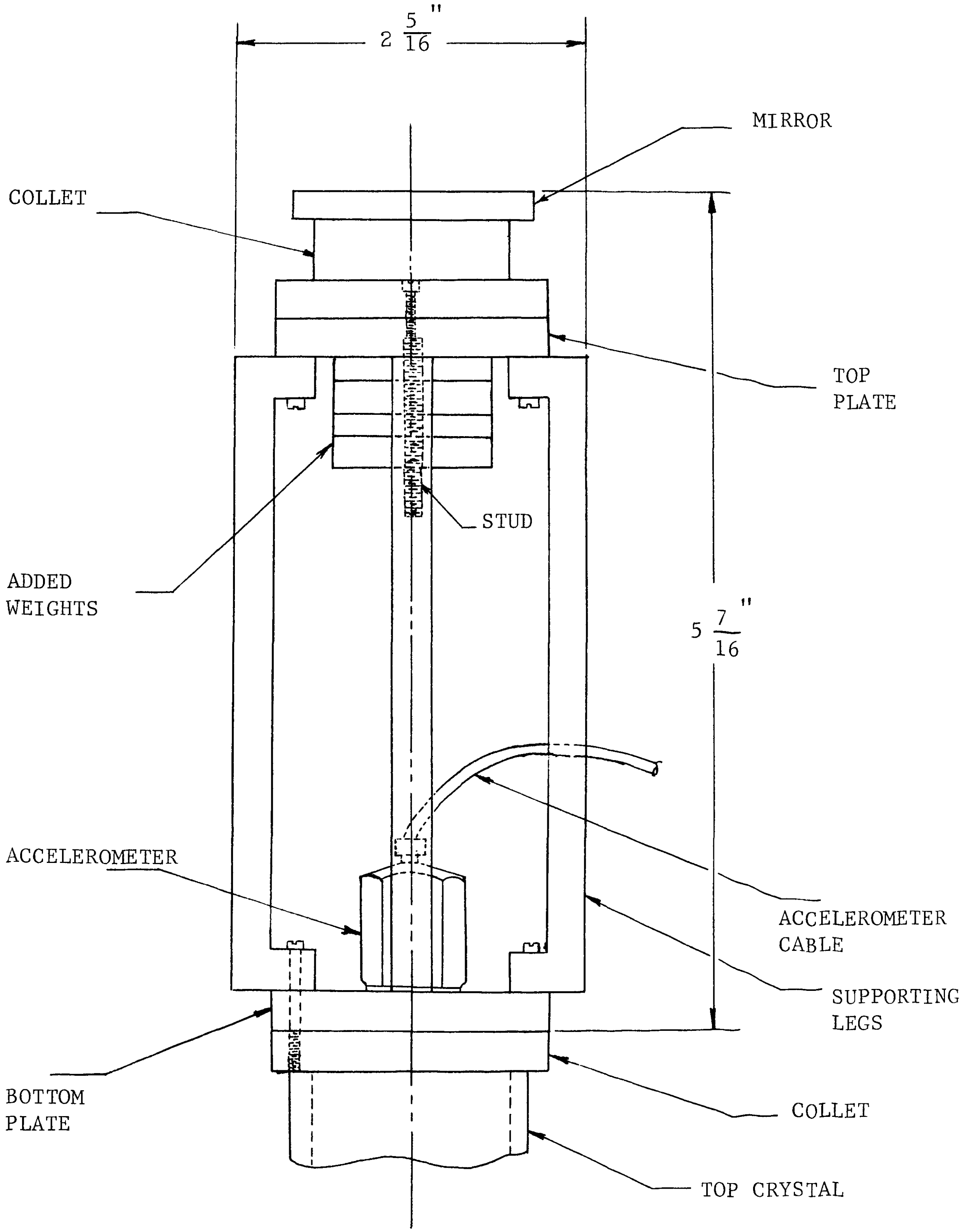

Figure 7. Cage Assembly.

with a stud to the lower disc, concentric to the centerline of the shaker. The upper part of the cage consisted of two discs which were identical to the lower disc. A stud was placed in the lower disc, where the additional weights were added, while the upper disc was cemented to a front surface mirror. A front surface mirror was used in order to minimize defraction of light.

C. Interferometer:

A modified Michelson interferometer built by Richard Rabenau (5), was used to produce and observe the Newton rings. Whereby appearance and disappearance of these Newton rings at a certain frequency forms the basis for determining the amplitude of oscillation, Figure 8. The figure shows a point source of monochromatic sodium light to illuminate the field. The light after passing through a condenser lens, was reflected by the partially silvered mirror, which was placed between the eyepiece and the objective lens. The light passes the objective lens as a collimated beam, which indicates that the beam reflected from the partial mirror must pass through the focal point of the objective lens. The collimated beam passes through the plano-convex lens, and is reflected back from the mirror of the shaker. Upon return it produces Newton rings, which was accomplished by adjusting the distance between the shaker mirror and plano-convex lens.

The plano-convex lens was selected, with a focal length of 453 mm, in order to produce a good image of the rings.

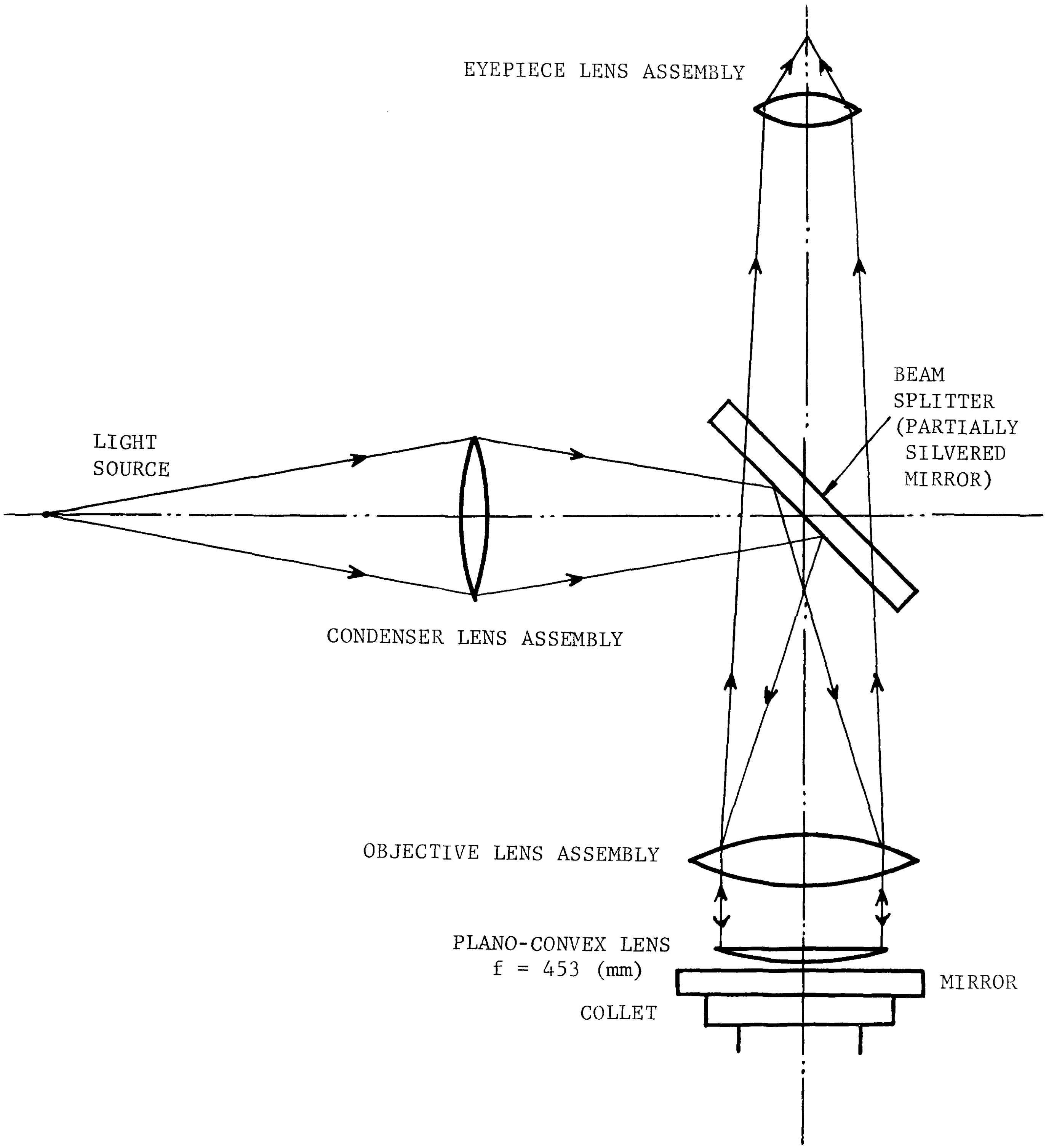

Figure 8. Optical Components of Interferometer.

1. Theory of Operation:

When operating a Michelson interferometer, using a monochromatic light source, Newton rings will shift laterally when the returning mirror of the shaker is moving in a direction perpendicular to its plane. Simple harmonic motion of the mirror and consequently the fringe system can be achieved by keeping the output circuit tuned to a given frequency. The resultant brightness at any point in the oscillating fringe system is, in general, a function of both the time and the amplitude of oscillation. The frequency of oscillation must be a minimum of 25 cycles per second so that visual brightness at any point will be the time average of the instantaneous brightness (17).

The following derivation for finding the relationship of the amplitude of vibration to the Newton rings was given by Harold Osterberg in the Journal of the Optical Society of America, Vol. 22 in 1932 (18).

If both reflecting surfaces of the mirror are at rest and uniformly illuminated, and two interfering beams are at equal intensity, the brightness of the fringes is given by:

$$I_o = K\left[1 + \cos\frac{2\pi X}{h}\right] \qquad (2)$$

where: K = constant (Lumens/cm^2)

h = fringe width (cm)

x = distance measured perpendicularly across the fringes.

With sinusoidal vibration of the mirror, a simple harmonic motion of amplitude A_o and frequency 1/T is imparted to the fringe system. At any point the instantaneous brightness I_i is given by:

$$I_i = K\left[1 + \cos \frac{2\pi}{h}\left(X - A_o \cos \frac{2\pi t}{T}\right)\right] \tag{3}$$

As was mentioned before the visual brightness at any point is the time average of the instantaneous brightness provided that the frequency is greater than 25 cycles per second, then I can be written as:

$$I = \frac{2K}{T} \int_o^{T/2} \left[1 + \cos \frac{2\pi}{h}\left(X - A_o \cos \frac{2\pi t}{T}\right)\right] dt \tag{4}$$

Now let

$$\frac{2\pi A_o}{h} = Z \text{ and } \frac{2\pi t}{T} = \phi$$

By substituting and rearranging, I becomes:

$$I = K\left[1 + \frac{1}{\pi}\cos\frac{2\pi X}{h}\int_o^{\pi} \cos(Z \cos\ ) d\phi + \frac{1}{\pi}\sin\frac{2\pi X}{h}\int_o^{\pi} \sin(Z \cos\phi) d\phi\right]. \tag{5}$$

By evaluating the integrals it can be found that:

$$\int_o^{\pi} \sin(Z \cos\phi) d\phi = 0 \tag{6}$$

and

$$\frac{1}{\pi}\int_o^{\pi} \cos(Z \cos\phi) d\phi = B_o(Z) \tag{7}$$

where: $B_o(Z)$ is a Bessel function of order zero (19).

In order to calculate the amplitude of oscillation, the relationship between the amplitude and the wavelength, λ, of the light used is necessary. Let A denote the amplitude of oscillation of the mirror surface, then:

$$\frac{A_o}{h} = \frac{2A^*}{\lambda} \qquad (8)$$

By substituting (6), (7) and (8) into (4), then:

$$I = K\left[1 + B_o\left(\frac{4\pi A}{\lambda}\right)\cos\frac{2\pi X}{h}\right] \qquad (9)$$

An examination of this expression shows that for $B_o\left(\frac{4\pi A}{\lambda}\right) = 0$, $I = K$ and the fringes disappear and the field is uniformly illuminated. That is, for all values of A which give $B_o\left(\frac{4\pi A}{\lambda}\right) = 0$, the fringes disappear; this disappearance is quite sharp. Since the maximum value of $B_o\left(\frac{4\pi A}{\lambda}\right) = B_o(0) = 1$, the fringes display the greatest contrast only when the interferometer mirror is stationary.

There are an infinite number of values of A which satisfy $B_o\left(\frac{4\pi A}{\lambda}\right) = 0$ for a particular light source.

In Table I the number of zeros is given with the corresponding roots of $B_o\left(\frac{4\pi A}{\lambda}\right)$ in columns 1 and 2, while column 3 contains the appropriate values of A/λ. The corresponding values of A are in inches and centimeters in columns 4 and 5. A sodium vapor lamp was used with a wavelength of 5,892 A°.

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*See Bibliography (5)

Table I

Number of Zeros	Root of $B_o\left(\frac{4\pi A}{\lambda}\right)$	A/λ	A (in. x 10^5)	A (cm x 10^5)
1	2.40	.191	.444	1.12
2	5.52	.439	1.020	2.59
3	8.65	.688	1.590	4.06
4	11.80	.938	2.180	5.53
5	14.90	1.190	2.750	7.00

The values given in Table I are in close agreement with those that were published by S. H. Cortz (13) and W. J. Kennedy (16).

In order to develop an expression for calculating the acceleration, simple harmonic motion was assumed. The equation of motion of a body in simple harmonic motion can be written as:

$$S = \frac{A}{2} \sin \omega t \qquad (10)$$

where: $A = 2A_o$

S = displacement (inches)

A = amplitude of motion (inches peak to peak)

ω = angular velocity $\left(\frac{\text{rad}}{\text{sec}}\right)$

t = time (sec).

By obtaining the first and second derivatives, the expressions for velocity and acceleration may be obtained as follows:

$$\dot{S} = \frac{ds}{dt} = V = \frac{A}{2} \omega \cos \omega t \qquad (11)$$

and

$$S = \frac{d^2s}{dt^2} = a = -\frac{A}{2}\omega^2 \sin\omega t \tag{12}$$

where: V = velocity (in/sec)

a = acceleration (in/sec^2).

Now, by substituting $\omega = 2\pi f$ and dividing both sides of the equation by the standard gravitional acceleration g, the "g level" can be obtained as follows:

$$G = \frac{a}{g} = \frac{-A4\pi^2 f^2 \sin 2\pi ft}{2 \times 386} \tag{13}$$

$$= .0511\ Af^2 \sin 2\pi ft$$

and

$$G_{max.} = .0511\ Af^2 \tag{14}$$

where: f = frequency (cycle/sec).

By obtaining the value of A from Table 1 which corresponds to a certain number of the zero of $B_o\left(\frac{4\pi A}{\lambda}\right)$, and the frequency, the maximum acceleration can be calculated from equation (14).

IV. DISCUSSION

A. Operating Procedure:

1. Calibration:

The accelerometer was attached to the cage by means of a stud. The stud was tightened so as to assure a minimum of relative motion between the accelerometer and the cage (Figure 7).

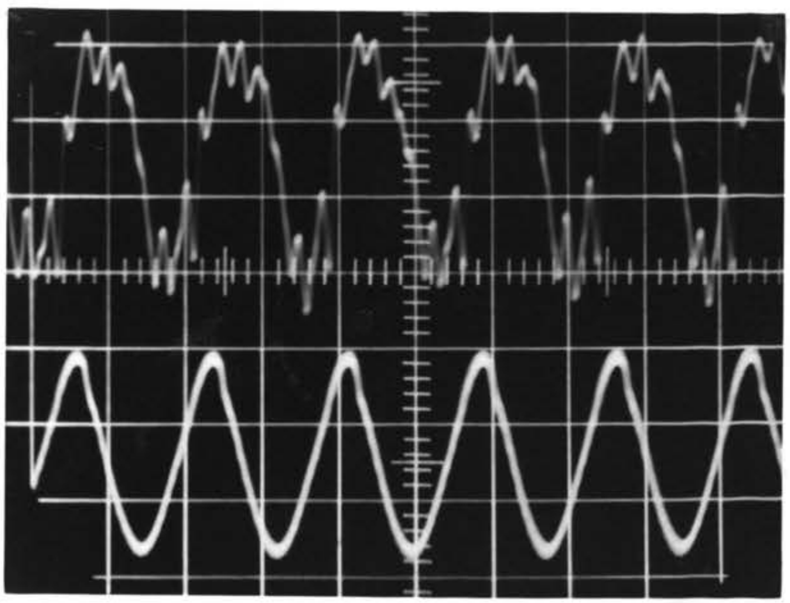
The interferometer was properly aligned over the shaker in accordance with Appendix B. The interferometer was then lowered over the mirror on the shaker head until a fringe pattern could be observed. Extreme caution was exercised in lowering the interferometer so as to prevent damage to the mirror. A gap of a few thousandths of an inch was maintained between the mirror and the plano-convex lens in order to allow the mirror to oscillate freely.

The electrical equipment was connected as shown in Figure 1. This equipment was allowed to warm up for a period of at least one hour prior to taking any data, in order to assure steady state operation. The oscillator frequency was then increased until the fringes disappeared. Then, without readjusting the frequency of the oscillator, the variable inductance was set so as to tune the circuit to have a harmonic signal to the shaker. The fringes did reappear occasionally due to the increase of voltage across the crystals. In this case the amplitude of the oscillation was increased slowly using the Helipot potentiometer until the fringes finally disappeared. When the fringes disappeared the output voltage of the accelerometer was measured by the vernier of the oscilloscope.

The frequency, voltage across the crystal, and the oscillator output voltage were also recorded (Appendix A). Throughout the runs the output voltage of the oscillator was monitored on the oscilloscope to check for distortion (see Figures 9, 10, and 11). The amplitude of oscillation was then increased until the fringes again disappeared and the data was taken again as explained above. At the same frequency the amplitude was then increased and the above procedure was repeated until five points were determined where the fringes disappeared. Then frequency was adjusted to the next calibration point and the same procedure was followed. The instrument was calibrated for three resonant points; this was the limit of this equipment.

2. Effect of Added Weight:

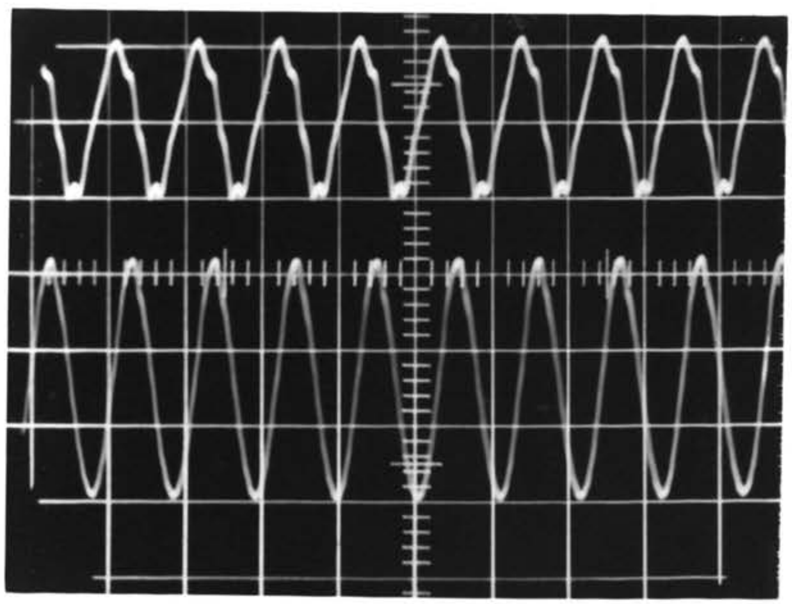
The interferometer was set as described in the calibration run, and a 0.7 ounce weight was then added to the cage as shown in Figure 8. As before the frequency was increased up to the first calibration point at which the fringes disappeared. The amplitude of oscillation was then increased by the Helipot potentiometer, until the last fringe disappearance was obtained; this was the point with maximum amplitude for that particular frequency. The output voltage of the accelerometer, frequency, and voltage across the crystal were recorded and output voltage of the oscillator was monitored on the oscilloscope to check the distortion. The frequency was then increased to the next calibration point and the variable inductance was set so as to tune the circuit. If the fringes reappeared the frequency was increased or decreased slightly until the fringes again disappeared. A trial and error method was used in order to find the right combination of inductance and frequency



ACCELEROMETER
OUTPUT

OSCILLATOR
OUTPUT

Figure 9. Output Waveforms at f = 1037 cps.



ACCELEROMETER
OUTPUT

OSCILLATOR
OUTPUT

Figure 10. Output Waveforms at f = 2348 cps.

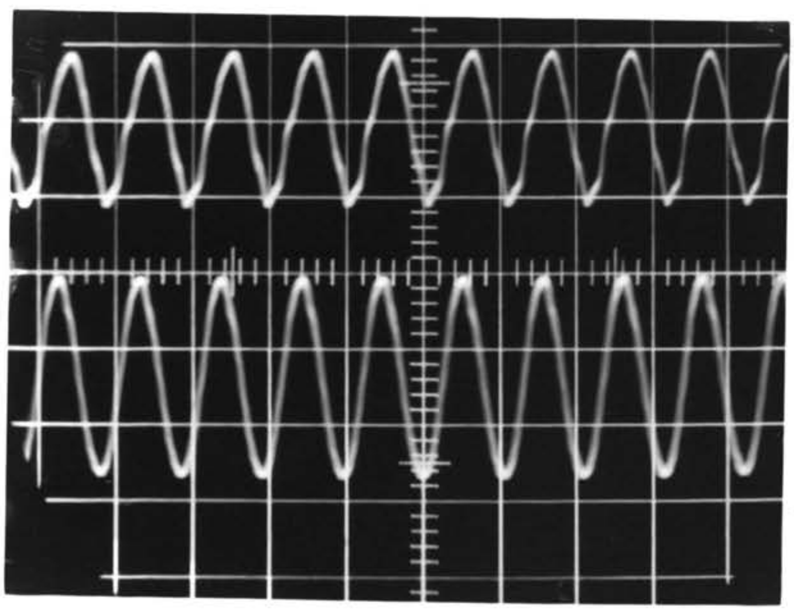


Figure 11. Output Waveforms at f = 3667 cps.

such that the circuit was simultaneously tuned and the fringes disappeared. This had to be done since the value of the resistance of the Helipot potentiometer was not readjusted after the first time, in order to study the effect of weight on the oscillation. After the last calibration point was reached and data was taken, additional weight was added and the same procedure was repeated for 1.0, 1.7, and 2.7 ounces added weight.

B. Discussion of Results:

The primary purpose of this research was to determine whether accelerometers can be calibrated accurately with this instrument; that is whether the system is capable of producing a stable known acceleration which can be used for calibration.

The calculations to find acceleration were based upon the assumption that simple harmonic motion was obtained on the shaker. For this reason, an attempt was made throughout the runs to keep the output circuit of the amplifier tuned while selecting the proper calibration frequencies for the resonant points of the shaker.

These objectives were achieved quite closely in the frequency range of 1,000 to 4,000 cycles per second, but for higher frequencies a different variable inductor was necessary in order to tune the circuit.

The accelerometer was calibrated by calculating the acceleration using equation 14, and the amplitudes in column four of Table I. These results were then compared with the output of the accelerometer. Difficulties were encountered in finding all five zeros for $B_o(\frac{4\pi A}{\lambda})$ at a particular frequency during runs due to the weakness of the light source.

As a result, it was necessary to make each run a number of times to determine all the data points. The calculated values of acceleration and their percent of deviation are listed in Table II.

Percent of deviation is defined as:

$$\% \text{ Deviation} = \frac{a_c - a_a}{a_c} \times 100$$

where: a_c = Acceleration calculated by the interferometer method (Equation 14).

a_a = Acceleration found from the Kistler 808A Accelerometer output.

Table II shows that percent deviation ranges from approximately 0.4% to 8%.

Due to the fact that the plano-convex lens was loosely mounted on the interferometer, other relative motion, in addition to the shaker motion, existed. This made it more difficult to find the disappearance points.

The mechanical structure of the shaker provided some damping. This also resulted in motion other than simple harmonic.

Another source of error was in assuming that the shaker was a pure capacitive load when making calculations to tune the circuit. As mentioned in the first section of this thesis, the shaker actually has both resistance and inductance. This was partially overcome by monitoring the amplifier input signal on the oscilloscope.

TABLE II

Number of Zeros	Frequency (cps)	Voltage Across the Shaker (volts)	Accele-rometer Output Voltage (mv)	Acceleration Calculated by Equation 14 (g)	Accele-ration from the Accele-rometer (g)	Percent Devia-tion
1	1037	3.00	2.90	0.244	0.240	1.600
2	"	3.60	6.50	0.560	0.538	3.930
3	"	6.00	10.75	0.874	0.884	1.150
4	"	6.72	14.40	1.200	1.190	0.834
5	"	7.00	18.50	1.510	1.540	2.650
1	2348	3.10	15.70	1.305	1.293	0.922
2	"	6.20	37.00	3.000	3.060	2.000
3	"	7.10	57.00	4.670	4.710	0.869
4	"	8.90	82.50	6.400	6.800	0.625
5	"	11.00	105.00	8.909	8.250	1.980
1	3667	8.00	41.50	3.050	3.300	8.200
2	"	11.00	84.50	7.010	6.980	0.427
3	"	13.30	130.00	10.920	10.710	1.920
4	"	14.50	180.00	15.000	15.300	2.000
5	"	14.90	226.00	18.900	18.700	1.060

The sodium spectrum lines of the light source vary between 5890 A° and 5896 A° (5), for calculation a value of 5893 A° was used which could have produced an error in acceleration of up to $\pm$ 0.05 percent.

Deviation of $\pm$ 5.0 cycles per second were quite common during the runs. This was a source of error because the acceleration is a function of frequency squared.

The acceleration produced by the shaker was converted to an electrical signal by the accelerometer, and transmitted to the oscilloscope. From the output voltage indicated on the oscilloscope, the acceleration could be calculated. The acceleration was also determined by using the interferometer method. The values of acceleration obtained by both methods were plotted against the frequency of oscillation as shown in Figures 12 through 16. It may be seen from the graphs that the values for acceleration obtained by the interoferometer method correspond quite closely to those obtained by the accelerometer. The largest deviation as shown in Table II was found to be 8.2%.

The procedure of calibrating an unknown accelerometer will now be given: In this procedure the standard accelerometer was used as the unknown.

The g level was calculated from the observed frequency at which disappearance of the Newton rings occur as discussed in the operating procedure. The amplitude of oscillation of the accelerometer was measured on the oscilloscope in milli-volts. The setting on the range switch of the charge amplifier was recorded (mv/P_{cb}). The output charge of the accelerometer was then determined in P_{cb}. After a

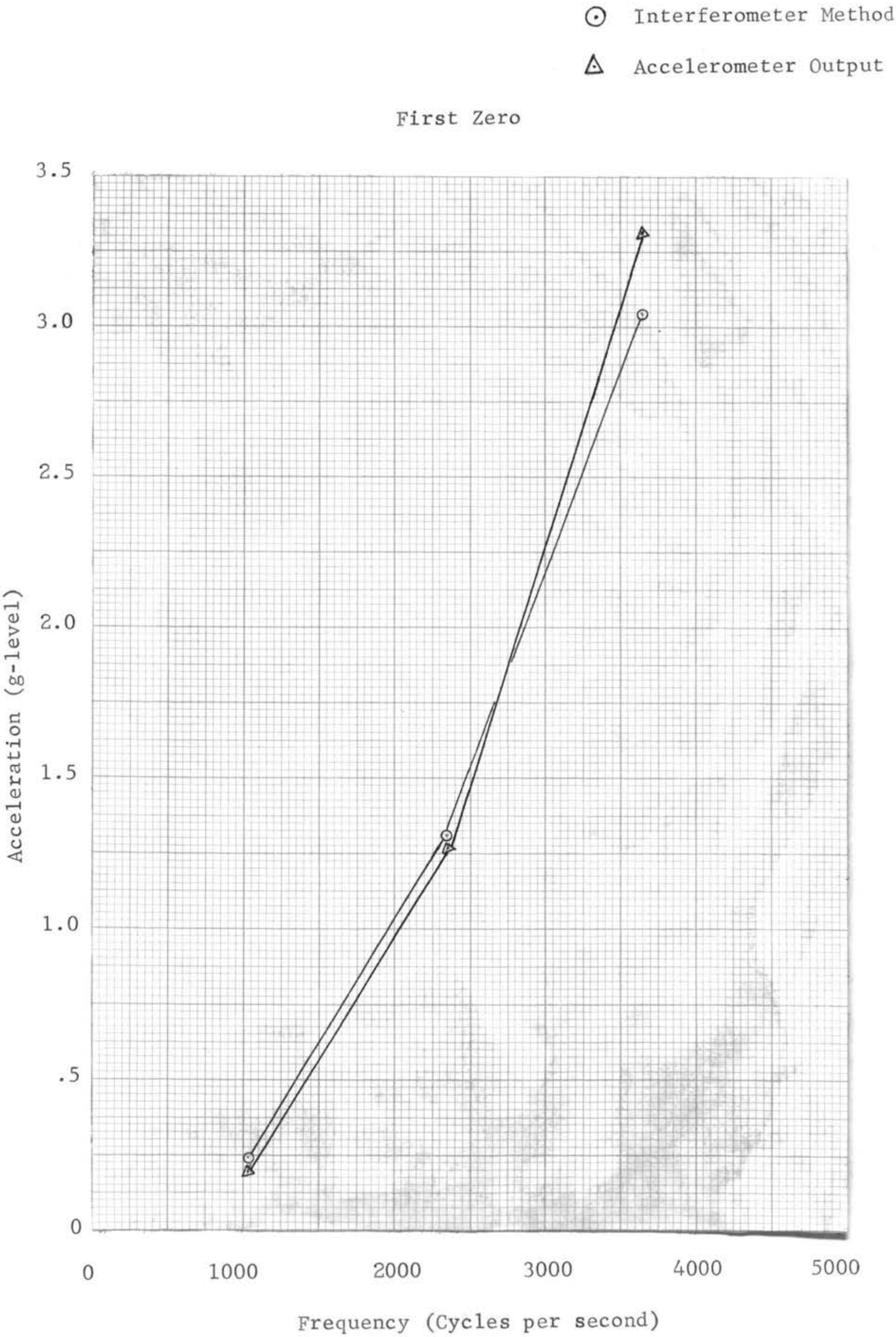


Figure 12. Acceleration Versus Frequency (First Zeros).

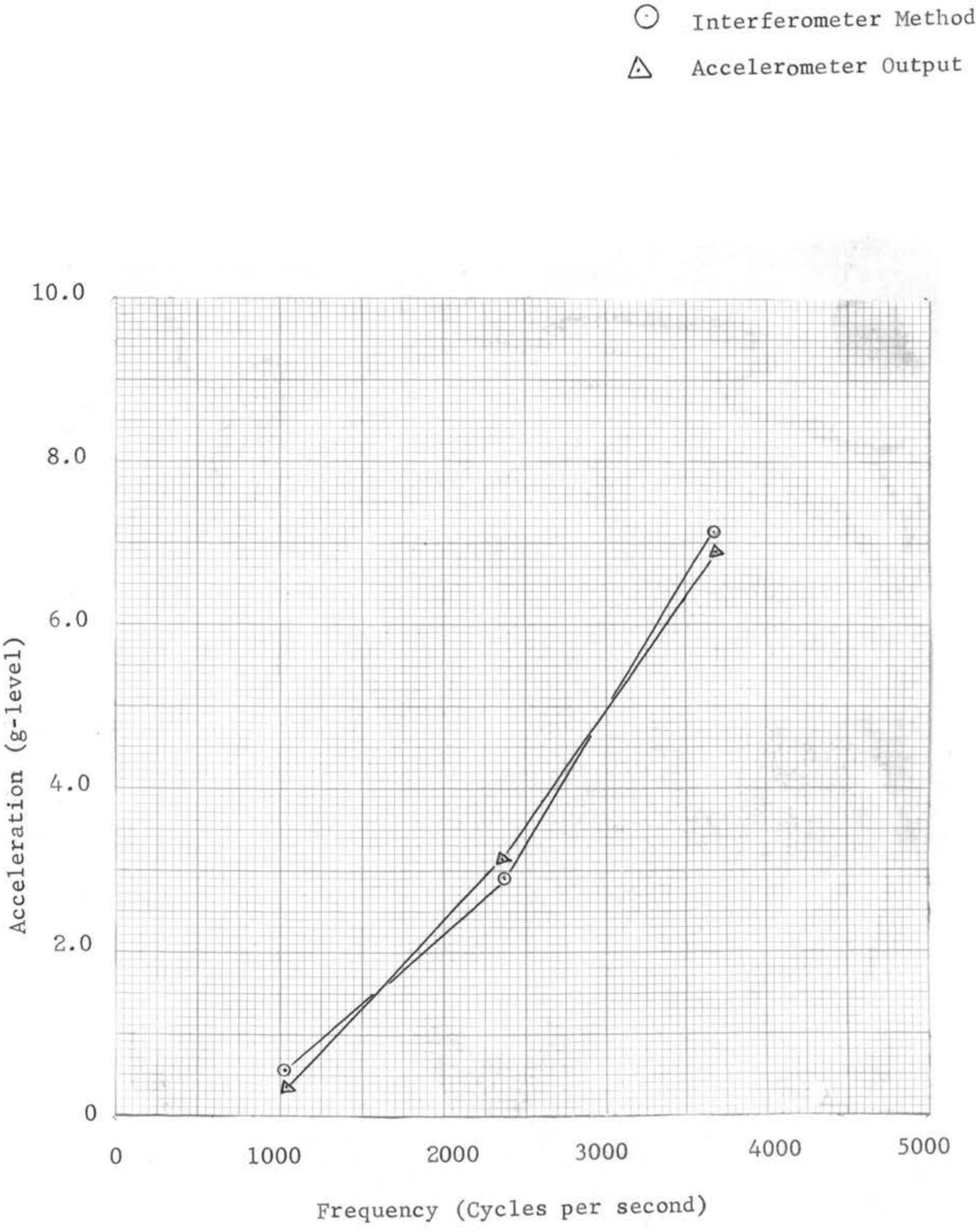


Figure 13. Acceleration Versus Frequency (Second Zeros).

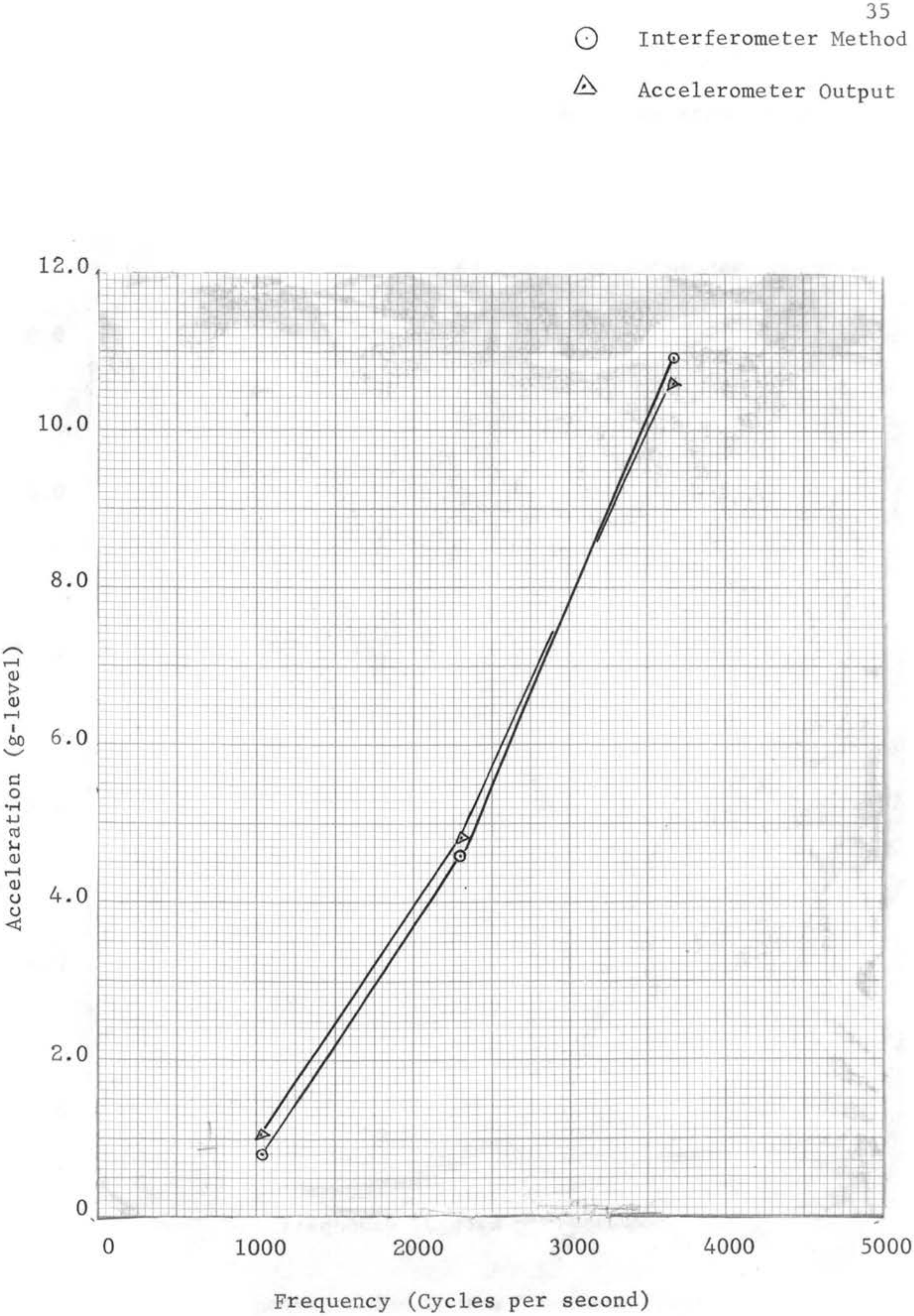


Figure 14. Acceleration Versus Frequency (Third Zeros).

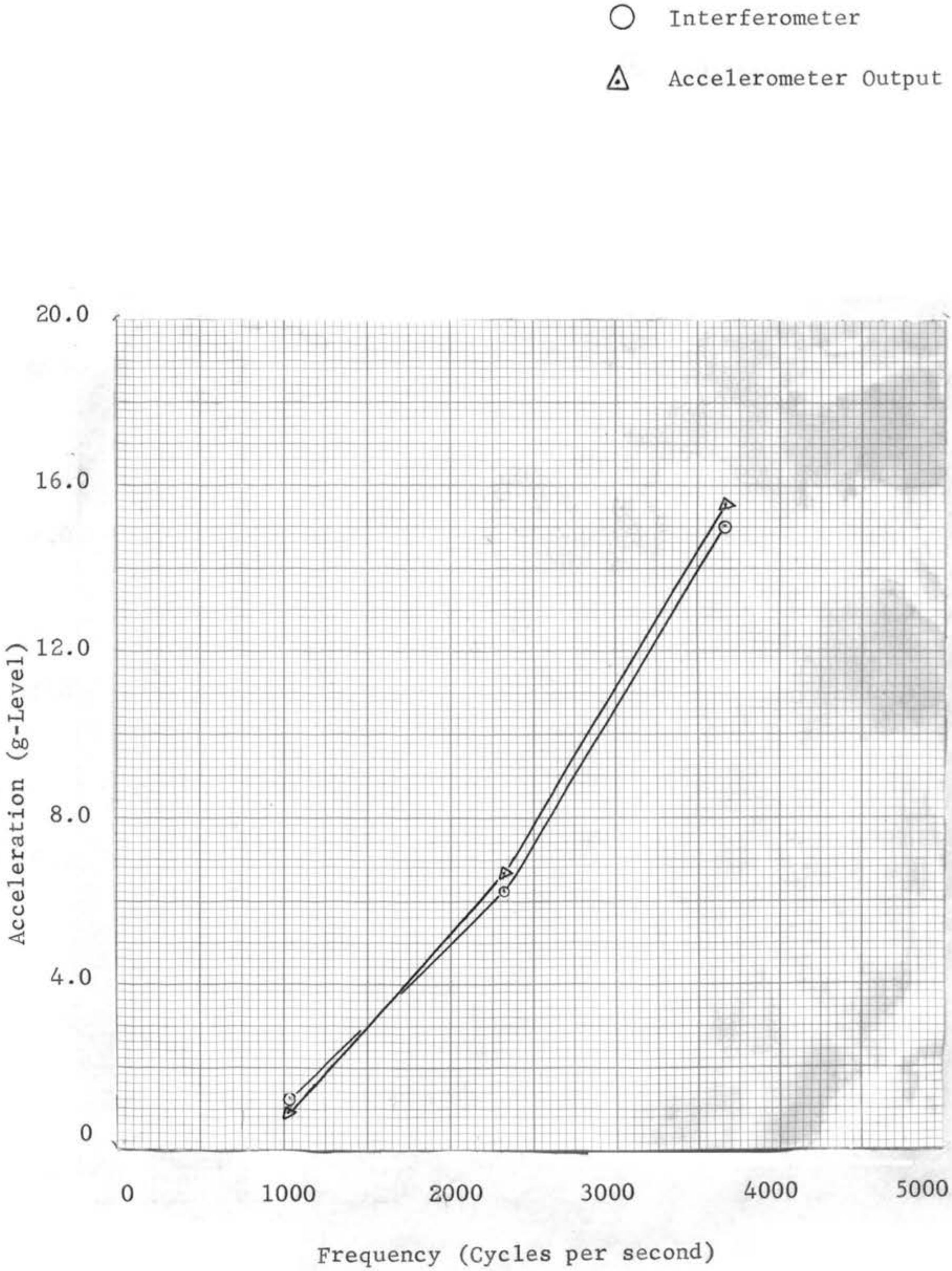


Figure 15. Acceleration Versus Frequency (Fourth Zeros)

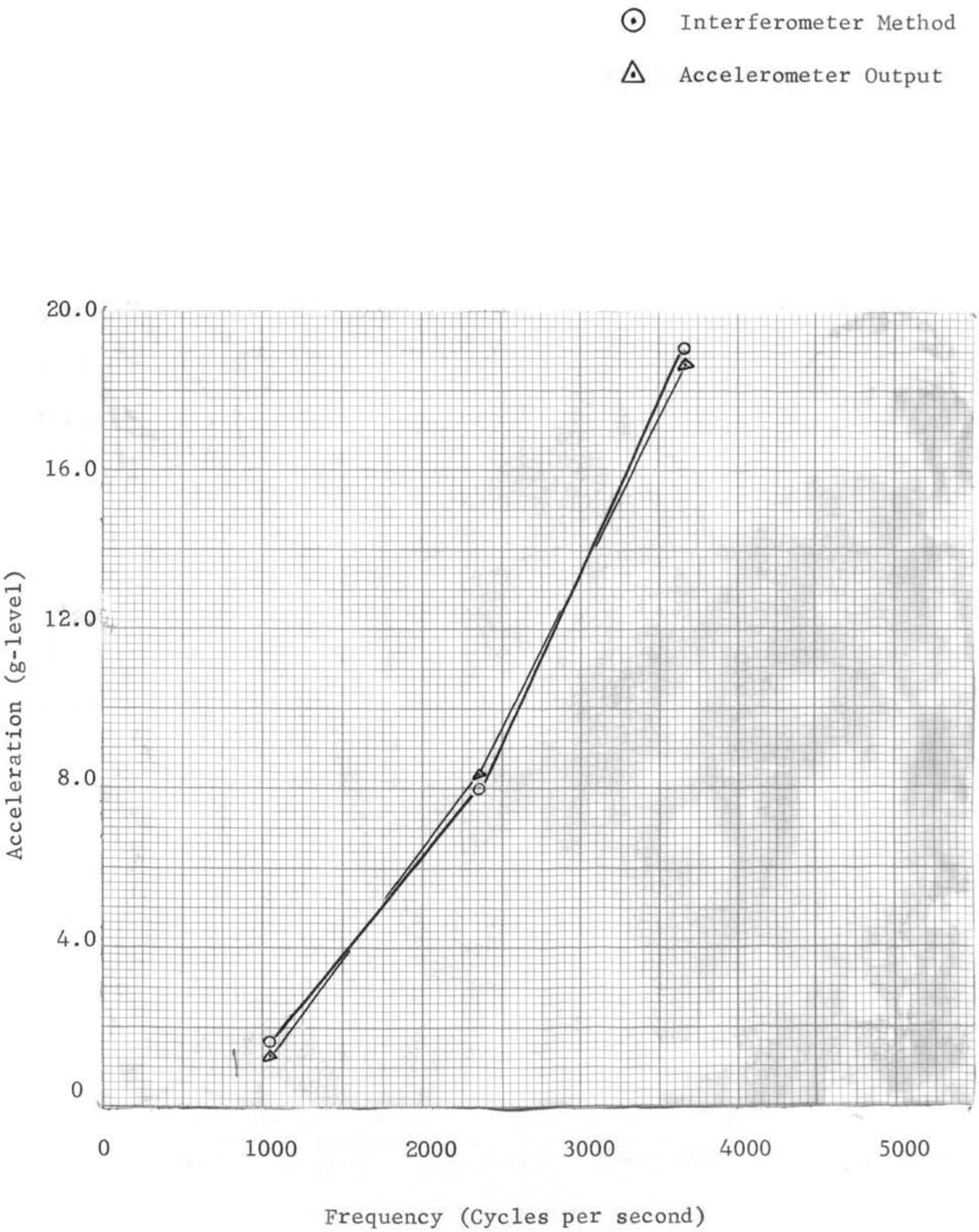


Figure 16. Acceleration Versus Frequency (Fifth Zeros).

number of observations were made, the calibration curve was plotted (Figure 17). The slope of the calibration curve was found to be 1.23 P_{cb}/g, which is the sensitivity of the accelerometer.

The actual sensitivity of the accelerometer as determined by the manufacturer is 1.21 P_{cb}/g. This shows that the results of the interferometer method of calibration is quite close to that of the manufacturers.

Weights were added to the shaker. It may be seen from Figure 18 that these added weights did not noticably alter the mechanical resonance frequencies, but the amplitude of oscillation was noticeably reduced. This was as predicted considering that the Helipot Potentiometer was maintained at its highest value and the added weights served as an energy absorber. The final value of additional weight used was 2.7 oz. With this weight the fringes did not disappear due to the amplitude of the oscillation dropping below $.445 \times 10^{-5}$ inches. This could be overcome by using a more powerful oscillator.

Figures 9, 10 and 11 show the output waveform of the accelerometer and the oscillator at the first zero of resonant points. It can be seen that as the frequencies increase the output of the accelerometer will approach a sine wave. This could be justified, due to the fact that as the frequencies increase the oscillation of the shaker approaches a simple harmonic motion.

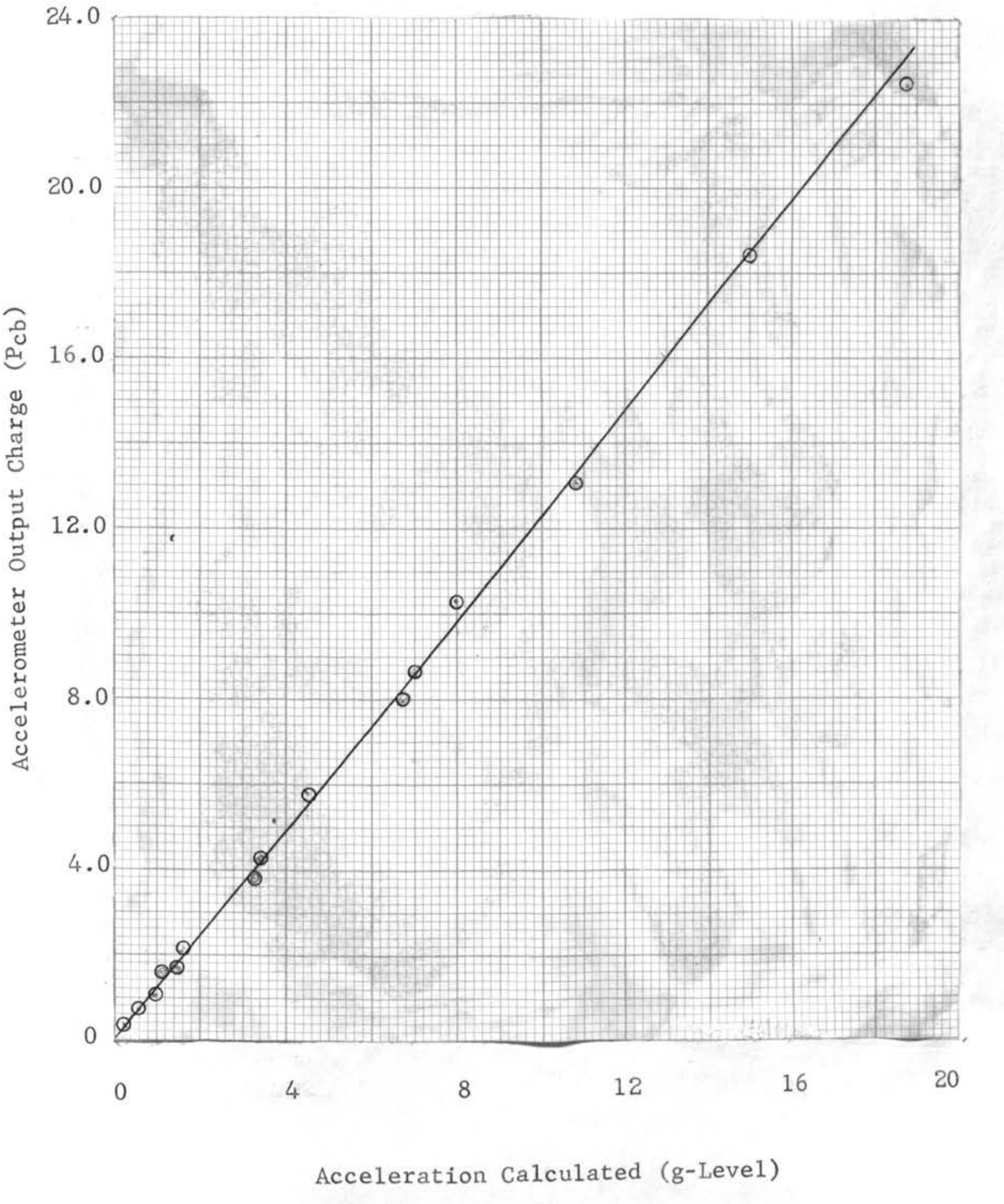


Figure 17. Calibration Curve.

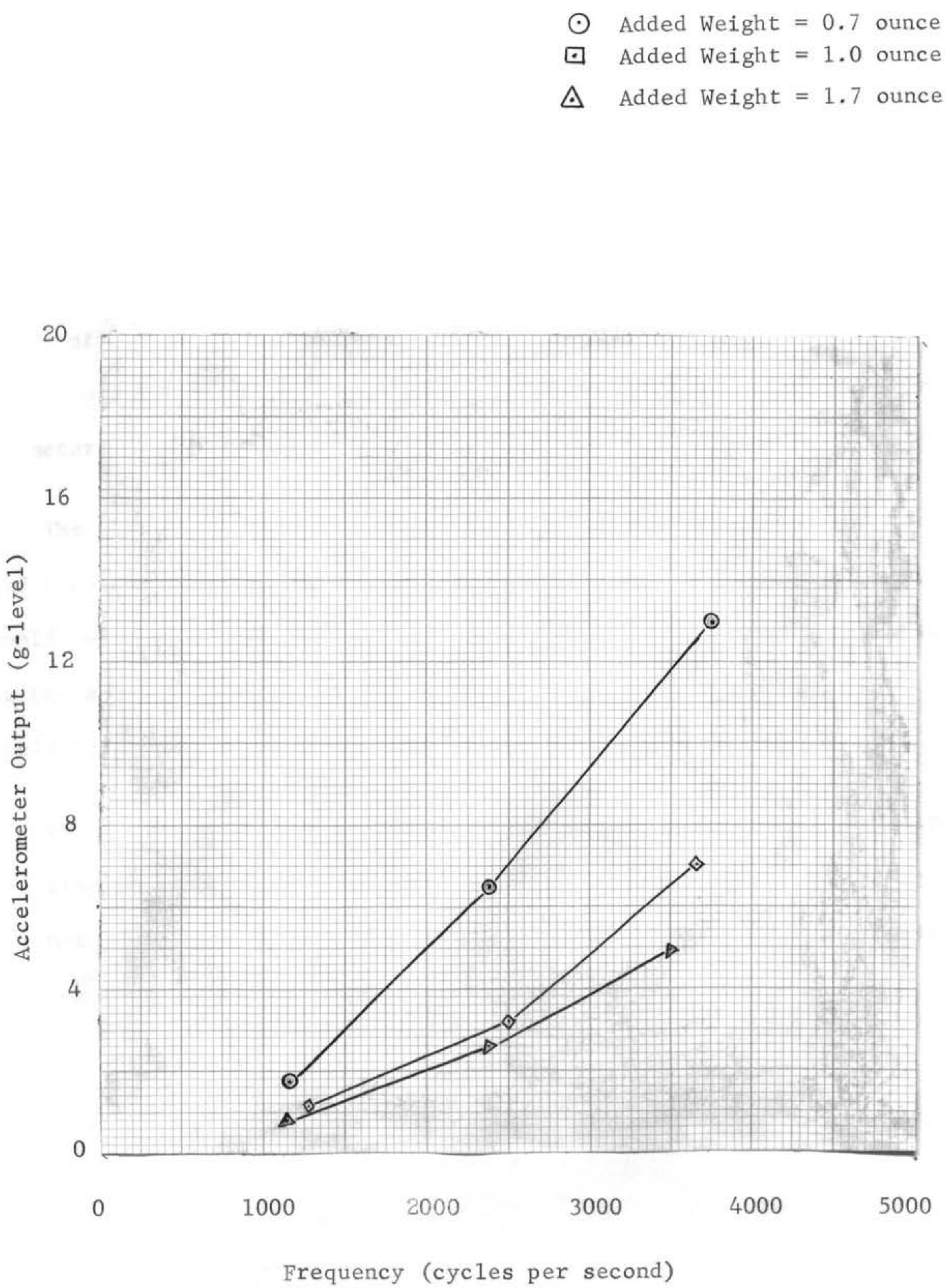


Figure 18. Effect of Added Weight.

V. RECOMMENDATIONS

Certain changes can be made in order to improve the accuracy and ease of operation of the entire system.

In order to increase the range of operation of the interferometer a very strong light source is needed, so any slight change in fringe pattern can be noticed. Also, the power of the microscope of the interferometer should be increased by selecting more powerful lenses.

The plano-convex lens at the end of the interferometer was assembled loosely so as to prevent damage to the mirror of the shaker. This should be changed to a system more rigid and fine adjustment for adjusting the spacing between the mirror and the lens. Also, the fixtures should be redesigned so that a higher focal length lens can be used.

The stack of crystals that were used for the shaker may be replaced by a single crystal with the same dimension, in order to prevent relative motion between the different segments of the crystal and to increase the range of operating frequency of the shaker.

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VITA

The author, Nader Khorzad, was born on June 14, 1939, in Tehran, Iran. He received his primary and secondary education in Shemiran, Iran, his college education at Queens College, Queens, New York and at the University of Missouri at Rolla, Rolla, Missouri. In August, 1964, he received a Bachelor of Science in Mechanical Engineering from the University of Missouri at Rolla, and was employed as a design engineer at Lindberg Steel Treating Equipment Company, Melrose Park, Illinois, from September, 1964 to September, 1965. Since September, 1965, he has been a graduate student at the University of Missouri at Rolla.

APPENDICES

APPENDIX A

The results of the calibration runs are listed below:

Frequency (cps)	Oscillator Output Voltage (volts)	Voltage Across the Shaker (volts)	Accelerometer Output Voltage (mv)	Charge Amplifier Switch (mv/P_{cb})	Variable Inductance (mh)
1037	3.20	3.00	2.90	10	52
1036	4.00	3.60	6.50	10	52
1038	5.90	6.00	10.75	10	52
1037	6.20	6.72	14.40	10	52
1035	7.10	7.00	18.50	10	52
2398	3.35	3.10	15.70	10	23
2396	6.60	6.20	37.00	10	23
2395	7.20	7.10	57.00	10	23
2398	9.00	8.90	82.50	10	23
2398	11.00	11.00	104.00	10	23
3667	8.40	8.00	41.50	10	18
3668	11.20	11.00	84.50	10	18
3663	13.80	13.30	130.00	10	18
3667	14.80	14.50	185.00	10	18
3668	15.10	14.90	226.00	10	18

APPENDIX B

Instructions for setup of the Michelson Interferometer used in this research:

1. Allow the sodium vapor lamp approximately one hour to stabilize and reach its maximum intensity.

2. Level the shaker base with adjusting screws.

3. Align the axis of the shaker with the optical axis of the interferometer.

4. Lower the interferometer until the plano-convex lens barely touches the mirror on the shaker head.

5. Adjust the condenser lens, in or out, until the entire field of view is uniformly illuminated.

6. If the circle of light is not in the center of the optical lens, move the center of the circle to the center of the sight picture by adjusting the beam splitter mirror.

7. The focus is now adjusted by turning the knurled knob on the eyepiece until a sharp image is obtained.

8. Raise the interferometer a few thousandths of an inch while still retaining the fringes.

9. The interferometer is now properly adjusted for calibration.