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The static nonequilibrium characteristics of planar germanium probes in a slightly ionized hydrogen plasma for low injections

Alfred Gene Williams

THE STATIC NONEQUILIBRIUM CHARACTERISTICS OF PLANAR GERMANIUM PROBES IN A SLIGHTLY IONIZED HYDROGEN PLASMA FOR LOW INJECTIONS

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ABSTRACT

The theory for a planar semiconductor probe in a slightly ionized gas with small dc current densities is developed for a germanium probe immersed in a hydrogen plasma. First, the equilibrium characteristics due to the probe in the plasma are developed from Poisson's equation and current density equations. Then, the static nonequilibrium characteristics due to the probe are found by perturbing the equilibrium characteristics and substituting the perturbation terms into Poisson's equation and current density equations. The total current, I, is found to vary linearly with the applied voltages, V, and the ratios I/V are essentially the same for both intrinsic and n-type germanium probes if the width of the probes is much smaller than the dimensions of the plasma.

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I. INTRODUCTION

The problem in this thesis is to investigate a planar semiconductor probe in a plasma for dc currents by utilizing the theory of the metal probe and the theory of the surface of semiconductors.

The theory for a metal planar probe in a plasma has been given by Langumiur(1) in the 1920's. Then in 1955, Kingston and Neustadter(2) extended Shockley's theory of a p-n junction in semiconductors(3) to develope the theory for the surface of semiconductors.

The problem will be considered in a one dimensional model where the x axis will be perpendicular to the plasma and semiconductor interface and the origin of x will be located at the semiconductor surface, as is shown in Figure 1. Only the low current injection case will be investigated because high injections may produce degeneracy in the semiconductor and other complications(4).

First, the expressions for the charge number densities and electric fields due to the plasma semiconductor interface will be determined. Then, perturbation techniques similar to those used by Vol'kenstein and Karpenko(5) will be used to determine the static nonequilibrium characteristics.

The plasma investigated will be hydrogen, H_2, with a neutral density on the order of $10^{15}/cm^3$ (6). It will

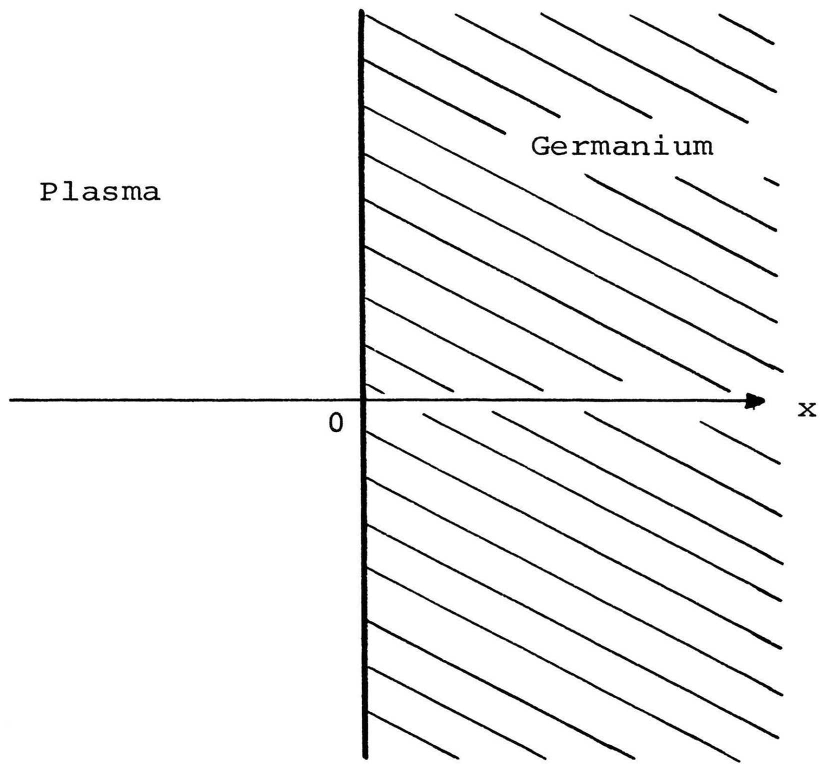

Figure 1. Diagram of the Plasma Germanium Interface Geometry.

be assumed that the plasma is slightly ionized and is in thermal equilibrium. The ions are assumed to be H_2^+ and to have a density on the order of $10^{10}/cm^3$ (7) far from the interface. Electrical neutrality is assumed in the bulk of the plasma(8), therefore, the electron number density in the bulk is $10^{10}/cm^3$ since the ions are singly ionized. The kinetic temperatures of the electrons, ions, and neutrals are the same because the plasma is in thermal equilibrium(8) and is assumed to be on the order of 10^4 K.

The semiconductor investigated will be germanium at room temperature, 300 K, with no oxidation layers on its surface. Germanium is chosen because its properties have been studied extensively. The germanium will be studied for two extreme cases, intrinsic and heavily doped n-type. The p-type germanium will not be discussed because the static nonequilibrium solutions are obtainable only by numerical techniques.

In the subsequent development, it will be assumed that the density gradients and electric fields are small enough so that the current density equations may be expressed as the sum of the drift current density and the diffusion current density(9). It is also assumed there are no temperature gradients in either the plasma or semiconductors or there would also be current flow due to the temperature gradients(10).

II. EQUILIBRIUM CHARACTERISTICS

The equilibrium characteristics of the planar semiconductor probe in a plasma gas will be solved by examining the plasma and semiconductors separately. Then, by using the boundary conditions that the electric displacement and electrostatic potentials are continuous at the interface(11), the equilibrium characteristics of the plasma and semiconductors will be altered to form the equilibrium characteristics of a planar semiconductor probe in a plasma. The equilibrium characteristics will be found by using the current density equations and Poisson's equations.

The expressions for the equilibrium characteristics are found subject to the following assumptions:

(1) Einstein's relation is valid;

(2) the materials are isotropic and homogeneous far from the interface;

(3) the drift current density of each charge species opposes and cancels its diffusion current density;

(4) the electric fields generated by the plasma semiconductor interface, $E(x)$, is related to the electrostatic potential, $\phi(x)$, by

$$E(x) = -\frac{d\phi(x)}{dx} \qquad 2.1$$

and vanishes for $|x|$ large, therefore $\phi(x)$ for $|x|$ large becomes a constant;

(5) the electrostatic potential will be referenced to the plasma bulk potential;

(6) charge neutrality exists in the bulk of the plasma.

A. HYDROGEN PLASMA EQUILIBRIUM CHARACTERISTICS

To analyze the equilibrium characteristics of the hydrogen plasma, consider equations A.1 and A.2 with the above assumptions:

$$J_i(x) = -e[\mu_i n_i(x) \frac{d\phi(x)}{dx} + D_i \frac{dn_i(x)}{dx}] = 0, \; x \leq 0 \qquad 2.2$$

$$J_e(x) = -e[\mu_e n_e(x) \frac{d\phi(x)}{dx} - D_e \frac{dn_e(x)}{dx}] = 0, \; x \leq 0 \qquad 2.3$$

where the subscripts i and e represent the ions and electrons, respectively. The ion and electron mobilities were calculated in Appendix B to be 4.56×10^4 and 1.56×10^7 cm^2/(volt sec), respectively. If charge neutrality exists far from the interface, then

$$n_i(-\infty) = n_e(-\infty) \qquad 2.4$$

for a singly ionized plasma.

Integrating equations 2.2 and 2.3 from x to $x = -\infty$ and using Einstein's relation results in

$$n_i(x) = n_e(-\infty)\ \exp[-\frac{e}{kT_e}\ \phi(x)],\ x \leq 0 \qquad 2.5$$

$$n_e(x) = n_e(-\infty)\ \exp[\frac{e}{kT_e}\ \phi(x)],\ x \leq 0 \qquad 2.6$$

where k is Boltzmann's constant and T_e is the kinetic temperature of the electrons and ions.

By relating the electric field to its potential and assuming that $\varepsilon_r = 1$ for this plasma, Poisson's equation for the plasma, equation A.13, becomes

$$\frac{d^2\phi(x)}{dx^2} = -\frac{e}{\varepsilon_o}\ [n_i(x) - n_e(x)]. \qquad 2.7$$

Now, define a new variable $y_1(x)$ as follows:

$$y_1(x) \triangleq \frac{e}{kT_e}\ \phi(x). \qquad 2.8$$

Substituting equations 2.5, 2.6, and 2.8 into equation 2.7 yields

$$\frac{d^2y_1(x)}{dx^2} = \frac{e^2 n_e(-\infty)}{\varepsilon_o kT_e}\ [e^{y_1(x)} - e^{-y_1(x)}],\ x \leq 0$$

or

$$\frac{d^2y_1(x)}{dx^2} = \frac{2e^2 n_e(-\infty)}{\varepsilon_o kT_e}\ \sinh y_1(x), \qquad x \leq 0. \qquad 2.9$$

Multiply equation 2.9 by 2 $\frac{dy_1(x)}{dx}$ and integrate.

$$\left(\frac{dy_1(x)}{dx}\right)^2 = \frac{4e^2 n_e(-\infty)}{\varepsilon kT_e} [\cosh y_1(x) + C], \quad x \leq 0 \qquad 2.10$$

where C is the constant of integration. At $x = -\infty$, $\phi(-\infty)$ and $E(-\infty)$ are zero therefore $y_1(-\infty)$ and $\left.\frac{dy_1(x)}{dx}\right|_{x=-\infty}$ are zero. Substitution of these boundary conditions into equation 2.10 yields

$$C = -\frac{4e^2 n_e(-\infty)}{\varepsilon_o kT_e} . \qquad 2.11$$

Therefore, equation 2.10 simplifies to

$$\frac{dy_1(x)}{dx} = \pm\sqrt{\frac{4e^2 n_e(-\infty)}{\varepsilon_o kT_e}} [\cosh y_1(x) - 1]^{\frac{1}{2}}, \quad x \leq 0 \qquad 2.12$$

or

$$\frac{dy_1(x)}{dx} = \pm\sqrt{\frac{8e^2 n_e(-\infty)}{\varepsilon_o kT_e}} \sinh\left(\frac{y_1(x)}{2}\right), \quad x \leq 0. \qquad 2.13$$

Integrating equation 2.13 from x = 0 to x yields

$$\tanh\left(\frac{y_1(x)}{4}\right) = \tanh\left(\frac{y_1(0)}{4}\right) e^{\pm\sqrt{\frac{8e^2 n_e(-\infty)}{\varepsilon_o kT_e}}\, x}, \quad x \leq 0 \qquad 2.14$$

or

$$\tanh\left(\frac{e\phi(x)}{4kT_e}\right) = \tanh\left(\frac{e\phi(0)}{4kT_e}\right) e^{\pm\sqrt{\frac{8e^2 n_e(-\infty)}{\varepsilon_o kT_e}}\, x}, \quad x \leq 0. \qquad 2.15$$

To satisfy the boundary condition that $\phi(-\infty) = 0$, the sign in the exponent of equation 2.15 must be positive, therefore

$$\tanh\left(\frac{y_1(x)}{4}\right) = \tanh\left(\frac{y_1(0)}{4}\right) e^{+\sqrt{\frac{8e^2 n_e(-\infty)}{\varepsilon_o kT_e}}\, x}, \quad x \leq 0. \qquad 2.16$$

Since equation 2.13 is the derivative of equation 2.16, equation 2.13 becomes

$$\frac{dy_1(x)}{dx} = \sqrt{\frac{8e^2 n_e(-\infty)}{\varepsilon_o kT_e}} \sinh\left(\frac{y_1(x)}{2}\right), \quad x \leq 0 \qquad 2.17$$

or

$$\frac{d\phi(x)}{dx} = \sqrt{\frac{8kT_e n_e(-\infty)}{\varepsilon_o}} \sinh\left(\frac{e\phi(x)}{2kT_e}\right), \quad x \leq 0. \qquad 2.18$$

Therefore, the electric field in the plasma, $E_1(x)$, is

$$E_1(x) = -\sqrt{\frac{8kT_e n_e(-\infty)}{\varepsilon_o}} \sinh\left(\frac{e\phi(x)}{2kT_e}\right), \quad x \leq 0. \qquad 2.19$$

In summary, the equilibrium characteristics of the plasma are given by the following equations:

$$E_1(x) = -\sqrt{\frac{8kT_e n_e(-\infty)}{\varepsilon_o}}) \sinh\left(\frac{e\phi(x)}{2kT_e}\right), \quad x \leq 0 \qquad 2.20$$

$$\tanh\left(\frac{e\phi(x)}{4kT_e}\right) = \tanh\left(\frac{e\phi(0)}{4kT_e}\right) e^{+\sqrt{\frac{8e^2 n_e(-\infty)}{\varepsilon_o kT_e}}\, x}, \quad x \leq 0 \qquad 2.21$$

$$n_i(x) = n_e(-\infty)\, e^{-\frac{e}{kT_e}\phi(x)}, \quad x \lesssim 0 \qquad 2.22$$

$$n_e(x) = n_e(-\infty)\, e^{+\frac{e}{kT_e}\phi(x)}, \quad x \lesssim 0. \qquad 2.23$$

B. INTRINSIC GERMANIUM EQUILIBRIUM CHARACTERISTICS

By the same analysis as was used in the plasma, the equilibrium current density equations for the semiconductor are

$$J_p(x) = -\, e[\mu_p p(x) \frac{d\phi(x)}{dx} + D_p \frac{dp(x)}{dx}] = 0, \; x \gtrsim 0 \qquad 2.24$$

$$J_n(x) = -\, e[\mu_n n(x) \frac{d\phi(x)}{dx} - D_n \frac{dn(x)}{dx}] = 0, \; x \gtrsim 0 \qquad 2.25$$

where the subscripts p and n represent the holes and electrons, respectively, and the terms p(x) and n(x) are the number densities of the holes and electrons, respectively. The values of the mobilities for the holes and electrons are 1900 and 3900 cm^2/(volt sec), respectively[4].

Since the germanium is intrinsic in this case[12],

$$n(+\infty) = p(+\infty) = n_I \simeq 2.5x10^{13}/cm^3 \qquad 2.26$$

where n_I is the intrinsic number density. Integrating equations 2.24 and 2.25 from x to x = +∞ and using Einstein's relation yields

$$p(x) = n_I e^{-\frac{e}{kT_s}(\phi(x)-\phi(+\infty))}, \quad x \geq 0 \qquad 2.27$$

$$n(x) = n_I e^{\frac{e}{kT_s}(\phi(x)-\phi(+\infty))}, \quad x \geq 0 \qquad 2.28$$

where T_s is the temperature of the semiconductor, T_s = 300 K.

By relating the electric field to its potential, Poisson's equation for intrinsic germanium, equation A.12 becomes

$$\frac{d^2\phi(x)}{dx^2} = -\frac{e}{\varepsilon_r \varepsilon_o}[p(x) - n(x)], \quad x \geq 0 \qquad 2.29$$

where ε_r is 16 for germanium.

Now define a new variable $y_2(x)$ as follows:

$$y_2(x) \triangleq \frac{e}{kT_s}[\phi(x)-\phi(+\infty)], \quad x \geq 0. \qquad 2.30$$

By following the same techniques as were used in the plasma, the expressions for the equilibrium electric field, $E_2(x)$, and potential in the intrinsic germanium are, respectively,

$$E_2(x) = \sqrt{\frac{8kT_s n_I}{\varepsilon_r \varepsilon_o}} \sinh\left(\frac{y_2(x)}{2}\right), \quad x \geq 0 \qquad 2.31$$

and

$$\tanh\left(\frac{y_2(x)}{4}\right) = \tanh\left(\frac{y_2(0)}{4}\right) e^{-\sqrt{\frac{8e^2 n_I}{\varepsilon_r \varepsilon_o kT_s}}\,x}, \quad x \geq 0 \qquad 2.32$$

where $y_2(x)$ is defined by equation 2.30.

In summary, the equilibrium characteristics for intrinsic germanium are as follows:

$$n(x) = n_I e^{y_2(x)} \qquad 2.33$$

$$p(x) = n_I e^{-y_2(x)} \qquad 2.34$$

$$E_2(x) = \sqrt{\frac{8kT_s n_I}{\varepsilon_r \varepsilon_o}} \sinh\left(\frac{y_2(x)}{2}\right) \qquad 2.35$$

$$\tanh\left(\frac{y_2(x)}{4}\right) = \tanh\left(\frac{y_2(0)}{4}\right) e^{-\sqrt{\frac{8e^2 n_I}{\varepsilon_r \varepsilon_o kT_s}}\,x}, \quad x \geq 0 \qquad 2.36$$

where

$$y_2(x) = \frac{e}{kT_s}\left[\phi(x) - \phi(\infty)\right], \quad x \geq 0. \qquad 2.37$$

C. N-TYPE GERMANIUM EQUILIBRIUM CHARACTERISTICS

By using the same techniques as were used for the intrinsic germanium, the electron and hole number densities are

$$n(x) = n(\infty)\, e^{y_3(x)} \qquad 2.38$$

and

$$p(x) = p(\infty)\, e^{-y_3(x)}, \qquad 2.39$$

respectively, where

$$y_3(x) \triangleq \frac{e}{kT_s} (\phi(x) - \phi(\infty)), x \geq 0. \qquad 2.40$$

Since the germanium is heavily doped n-type, $n(\infty)$ and $p(\infty)$ are[4]

$$n(\infty) = N_d \qquad 2.41$$

$$p(\infty) = \frac{n_I^2}{N_d} \qquad 2.42$$

where N_d is the number density of the donors, $N_d \simeq 10^{18}/cm^3$ in this problem. All the donors are assumed to be fully ionized.

Using equation A.11, Poisson's equation for the n-type germanium becomes

$$\frac{d^2\phi(x)}{dx^2} = -\frac{e}{\varepsilon_r \varepsilon_o} [p(x) - n(x) + N_d], \; x \geq 0. \qquad 2.43$$

Now, using similar techniques as were employed in the plasma to determine the equilibrium electric field, yields

$$\frac{dy_3(x)}{dx} = \pm\sqrt{\frac{2e^2}{\varepsilon_r\varepsilon_o kT_s}}\,[N_d(e^{y_3(x)} - y_3(x) - \frac{n_I^2}{N_d}\,e^{-y_3(x)} + C]^{\frac{1}{2}}, x \geq 0$$

2.44

or

$$E_3(x) = \mp\sqrt{\frac{2kT_s}{\varepsilon_r\varepsilon_o}}\,[N_d(e^{y_3(x)} - y_3(x)) - \frac{n_I^2}{N_d}\,e^{-y_3(x)} + C]^{\frac{1}{2}},\ x \geq 0$$

2.45

where $E_3(x)$ is the equilibrium electric field in the n-type germanium and C is the constant of integration. Applying the boundary conditions that $E_3(\infty)$ and $y_3(\infty)$ are zero, yields

$$E_3(x) = \mp\sqrt{\frac{2kT_sN_d}{\varepsilon_r\varepsilon_o}}\,[e^{y_3(x)} - y_3(x)) - 1 - (\frac{n_I}{N_d})^2\,(e^{-y_3(x)} - 1)]^{\frac{1}{2}}, x \geq 0.$$

2.46

Since $\frac{n_I}{N_d} << 1$, the terms multiplied by $\frac{n_I}{N_d}$ may be neglected if $y_3(x)$ is a small variable. If $y_3 << 1$, the exponentials in equation 2.46 may be expanded in a Taylor series expansion about $y_3(x) = 0$. Neglecting third and higher order terms, equation 2.46 becomes

$$E_3(x) = \mp\sqrt{\frac{kT_sN_d}{\varepsilon_r\varepsilon_o}}\,y_3(x),\ x \geq 0 \qquad 2.47$$

or

$$\frac{dy_3(x)}{dx} = \pm\sqrt{\frac{e^2N_d}{kT_s\varepsilon_r\varepsilon_o}}\; y_3(x), \quad x \geq 0. \qquad 2.48$$

Integration of equation 2.48 from x = 0 to x yields

$$y_3(x) = y_3(0)\; e^{\pm\sqrt{\frac{e^2N_d}{\varepsilon_r\varepsilon_o kT_s}}\,x}, \quad x \geq 0 \qquad 2.49$$

or

$$\phi(x) = \phi(\infty) + (\phi(0)-\phi(\infty))e^{\pm\sqrt{\frac{e^2N_d}{\varepsilon_r\varepsilon_o kT_s}}\,x}, \quad x \geq 0. \qquad 2.50$$

Since $\phi(\infty)$ has to be finite, the sign in the exponent of equation 2.50 is negative. Therefore

$$E_3(x) = +\sqrt{\frac{kT_sN_d}{\varepsilon_r\varepsilon_o}}\; y_3(x), \quad x \geq 0 \qquad 2.51$$

and

$$y_3(x) = y_3(0)\; e^{-\sqrt{\frac{e^2N_d}{kT_s\varepsilon_r\varepsilon_o}}\,x}, \quad x \geq 0. \qquad 2.52$$

To check the assumption that $y_3(x) << 1$, it is necessary only to show that the maximum value of $y_3(x) << 1$. Equation 2.53 indicates that $y_3(x)$ has a maximum value at $x = 0$. To evaluate $y_3(0)$, the continuity of the electric displacement at $x = 0$ is used.

$$\varepsilon_o E_1(0) = 16\ \varepsilon_o E_3(0). \qquad 2.53$$

Using the value of the equilibrium surface potential found in Appendix C and evaluating equation 2.53, it is found that

$$y_3(0) \simeq 4.96\text{x}10^{-4}. \qquad 2.54$$

Therefore, the assumption that $y_3 << 1$ is valid.

To summarize, the equilibrium characteristics for the heavily doped n-type germanium, one has

$$n(x) = N_d e^{y_3(x)}, \quad x \geq 0 \qquad 2.55$$

$$p(x) = \frac{n_I^2}{N_d} e^{-y_3(x)}, \quad x \geq 0 \qquad 2.56$$

$$E_3(x) = \sqrt{\frac{kT_s N_d}{\varepsilon_r \varepsilon_o}}\ y_3(x), \quad x \geq 0 \qquad 2.57$$

and

$$y_3(x) = y_3(0)\ e^{-\sqrt{\frac{e^2 N_d}{kT_s \varepsilon_r \varepsilon_o}}\ x}, \quad x \geq 0 \qquad 2.58$$

where

$$y_3(x) = \frac{e}{kT_s}[\phi(x) - \phi(\infty)], \quad x \geq 0 \qquad 2.59$$

D. EQUILIBRIUM PLANAR GERMANIUM PROBE CHARACTERISTICS

To evaluate the equilibrium probe characteristics, the boundary conditions that the electric displacement and potential are continuous at $x = 0$ are used.

Referring to Appendix C, the surface potential of the germanium is dependent only on the plasma and is equal to -1.768 volts approximately. Therefore, equation 2.19 states that the electric field in the plasma is independent of the type of germanium probe. Evaluation of equation 2.19 at $x = 0$ yields

$$E_1(0) \simeq 429.4 \text{ volts/cm.} \qquad 2.60$$

The application of the continuity of the electric displacement at $x = 0$

$$\varepsilon_o E_1(0) = 16\ \varepsilon_o E_2(0) \qquad 2.61$$

and

$$\varepsilon_o E_1(0) = 16\ \varepsilon_o E_3(0) \qquad 2.62$$

yields

$$E_2(0) = E_3(0) \simeq 26.8 \text{ volts/cm.} \qquad 2.63$$

With the use of the above boundary conditions, the theory for the planar germanium probes in equilibrium with a hydrogen plasma is completed. The resulting probe equilibrium characteristics are discussed in Chapter 4. In order to show the change in $\phi(x)$ and the change in charge number density due to plasma and germanium interface, the potentials and charge number densities are discussed in Chapter 4 relative to the surface values and bulk values, respectively, and are designated by

$$\Delta\phi(x) = \phi(0) - \phi(x) \qquad 2.64$$

$$\Delta n_i(x) = n_i(x) - n_e(-\infty) \qquad 2.65$$

and

$$\Delta n_e(x) = n_e(x) - n_e(-\infty) \qquad 2.66$$

in the plasma and

$$\Delta\phi(x) = \phi(0) - \phi(x) \qquad 2.67$$

$$\Delta n(x) = n(x) - n(\infty) \qquad 2.68$$

$$\Delta p(x) = p(x) - p(\infty) \qquad 2.69$$

in the germanium probes.

III. STATIC NONEQUILIBRIUM PROBE CHARACTERISTICS

A perturbation technique applied to the equilibrium characteristics will be used to determine the static nonequilibrium characteristic equations of the probe. It will be assumed that the result of applying small static electric fields across the surface of the probe can be expressed in terms of the sum of the equilibrium characteristics and static nonequilibrium characteristics. The perturbation terms of the charge number densities and electric fields are the excess charge carrier number densities and applied electric fields, respectively. Only the low injection problem will be examined, therefore the perturbation terms will be considered to be much smaller than the equilibrium terms. The perturbation terms will be designated by a superscript $'$.

Since the total current density, J_t, given by (4)

$$J_t = J_+(x) + J_-(x) \qquad 3.1$$

is continuous and a constant for static nonequilibrium[13], therefore

$$\frac{dJ_t}{dx} = 0. \qquad 3.2$$

A. GENERAL PERTURBATION TECHNIQUES

The static nonequilibrium terms are assumed to be as follows:

$$E_t(x) = E(x) + E'(x) \qquad 3.3$$

$$\phi_t(x) = \phi(x) + \phi'(x) \qquad 3.4$$

$$n_{t-}(x) = n_-(x) + n_-'(x) \qquad 3.5$$

$$n_{t+}(x) = n_+(x) + n_+'(x) \qquad 3.6$$

where the subscript t represents the total terms and the primed terms represent the first order perturbation terms.

Substitution of equations 3.3, 3.4, 3.5, and 3.6 into equations A.1 and A.2 yields the following current density equations:

$$J_+(x) = e[\mu_+(n_+(x)E'(x) + n_+'(x)E(x)) - D_+ \frac{dn_+'(x)}{dx}] \qquad 3.7$$

$$J_-(x) = e[\mu_-(n_-(x)E'(x) + n_-'(x)E(x)) + D_- \frac{dn_-'(x)}{dx}] \qquad 3.8$$

where second and higher order terms have been neglected.

Referring to Van der Ziel[(4)], if the length of the sample is much greater than the diffusion length of the semiconductor, then the excess charge carrier number density will be approximately zero for large x. Since the plasma behaves as if it were an intrinsic semiconductor, the excess charge carrier number density will also be approximately zero for x large compared to its diffusion length. Therefore, at $x = \pm\infty$, the boundary conditions are

$$n_+^{'}(\pm\infty) = 0 \qquad 3.9$$

$$n_-^{'}(\pm\infty) = 0. \qquad 3.10$$

If it is assumed that the applied electric fields are constants in the bulk of the materials and that the deviation from these constant values are small, then

$$\frac{dE^{'}(x)}{dx} \approx 0. \qquad 3.11$$

Therefore,

$$E^{'}(x) \simeq E^{'} = \text{constant}. \qquad 3.12$$

Now using Poisson's equation for the applied electric fields and equation 3.11 yields

$$n_+^{'}(x) \simeq n_-^{'}(x). \qquad 3.13$$

Since the total current density is a constant, it may be evaluated at any value of x. To obtain a simple expression for J_t, evaluate the sum of equations 3.7 and 3.8 at $x = \pm\infty$.

$$J_t = e[\mu_+ n_+(\pm\infty) + \mu_- n_-(\pm\infty)]E' . \qquad 3.14$$

Substitute equation 3.14 into the sum of equations 3.7 and 3.8 for finite x.

$$[\mu_+ n_+(\pm\infty) + \mu_- n_-(\pm\infty)]E' = [\mu_- n_-(x) + \mu_+ n_+(x)]E' +$$

$$+[\mu_+ + \mu_-]E(x)n_-'(x) + [D_- - D_+] \frac{dn_-'(x)}{dx}. \qquad 3.15$$

Defining a new variable, y(x), as

$$y(x) \triangleq \frac{e}{kT} (\phi(x) - \phi(\pm\infty)) \qquad 3.16$$

and changing the derivative in equation 3.16 from the derivative with respect to x to the derivative with respect to y(x) by using the chain rule, yields

$$\frac{d}{dx} = \frac{d\phi(x)}{dx} \frac{dy(x)}{d\phi(x)} \frac{d}{dy(x)} \qquad 3.17$$

or

$$\frac{d}{dx} = - E(x) \frac{e}{kT} \frac{d}{dy(x)} . \qquad 3.18$$

Substitution of equation 3.18 into equation 3.14 and normalizing the coefficient of the derivative to one yields

$$\frac{dn_-'(x)}{dy} + [\frac{\mu_+ + \mu_-}{\mu_+ - \mu_-}]n_-'(x) = \frac{E'[\mu_+\{n_+(\pm\infty) - n_+(x)\} + \mu_-\{n_-(\pm\infty) - n_-(x)\}]}{-E(x)(\mu_- - \mu_+)}$$

3.19

where Einstein's relation was used.

Equation 3.19 has the form of

$$\frac{dy}{dx} + P(x)y = Q(x). \qquad 3.20$$

The solution of equation 3.20[14] is

$$y = e^{-\int P(x)\,dx}\left[\int e^{\int P(x)\,dx}\, Q(x)\,dx + C\right] \qquad 3.21$$

where C is the constant of integration. Therefore, the solution of equation 3.19 is

$$n_-^{'}(x) = e^{-\left(\frac{\mu_+ + \mu_-}{\mu_+ - \mu_-}\right)y(x)}\left[\int e^{\left(\frac{\mu_+ + \mu_-}{\mu_+ - \mu_-}\right)y(x)} E^{'}\{\mu_+(n_+(\pm\infty) - n_+(x))\right.$$

$$\left. + \mu_-(n_-(\pm\infty) - n_-(x))\} + C\right]. \qquad 3.22$$

B. EXPRESSIONS FOR THE CHARGE CARRIERS

To evaluate the charge carrier density for intrinsic germanium, substitute equations 2.27, 2.28, and 2.31 into 3.22. Integrating the resulting equation with the boundary condition that $n^{'}(\infty) = 0$ yields

$$n'(x) = E_2' \sqrt{\frac{2\varepsilon_r\varepsilon_o n_I}{kT_s}} \left[\frac{\mu_p}{3\mu_n+\mu_p} \left(e^{-\frac{y_2(x)}{2}} - e^{-\left(\frac{\mu_p+\mu_n}{\mu_p-\mu_n}\right) y_2(x)}\right)\right.$$

$$\left. - \frac{\mu_n}{3\mu_p+\mu_n} \left(e^{+\frac{y_2(x)}{2}} - e^{-\left(\frac{\mu_p+\mu_n}{\mu_p-\mu_n}\right) y_2(x)}\right)\right], \quad x \geq 0$$

3.23

where E_2' is the applied electric field in the intrinsic germanium.

To determine the charge carrier number density in the plasma, substitute the equations 2.20, 2.22, and 2.23 into equation 3.22. Integrating this equation with the boundary condition $n_e'(-\infty) = 0$ yields

$$n_e'(x) = E_1' \sqrt{\frac{2\varepsilon_o n_e(-\infty)}{kT_e}} \left[\frac{\mu_e}{3\mu_i+\mu_e} \left(e^{\frac{y_1(x)}{2}} - e^{-\left(\frac{\mu_i+\mu_e}{\mu_i-\mu_e}\right) y_1(x)}\right)\right.$$

$$\left. - \frac{\mu_i}{3\mu_e+\mu_i} \left(e^{-\frac{y_1(x)}{2}} - e^{-\left(\frac{\mu_i+\mu_e}{\mu_i-\mu_e}\right) y_1(x)}\right)\right], \quad x \geq 0 \qquad 3.24$$

where E_1' is the applied electric field in the plasma.

To obtain the charge carrier number density for the heavily doped n-type germanium, substitute equations 2.56 and 2.57 into equation 3.22. The resulting equation is

$$n'(x) = e^{-\left(\frac{\mu_p+\mu_n}{\mu_p-\mu_n}\right) y_3(x)} \left[\int e^{\left(\frac{\mu_p+\mu_n}{\mu_p-\mu_n}\right) y_3(x)} E_3' \frac{\mu_n N_d (e^{y_3(x)} - 1)\, dy(x)}{E_3(x)} + C\right],\ x \geq 0, \qquad 3.25$$

where E_3' is the applied electric field in the n-type germanium. It is observed that

$$\frac{N_d (e^{y_3(x)} - 1)}{E_3(x)} = \sqrt{\frac{N_d \varepsilon_r \varepsilon_o}{kT_s}} . \qquad 3.26$$

Therefore, equation 3.25 becomes

$$n'(x) = \mu_n E_3' \sqrt{\frac{\varepsilon_r \varepsilon_o N_d}{kT_s}}\, e^{-\left(\frac{\mu_p+\mu_n}{\mu_p-\mu_n}\right) y_3(x)} \left[\int e^{\frac{\mu_p+\mu_n}{\mu_p-\mu_n} y(x)}\, dy(x) + C\right],\ x \geq 0. \qquad 3.27$$

Integrating equation 3.27 with the boundary condition that $n'(\infty) = 0$ yields

$$n'(x) = E_3' \frac{\mu_n}{\mu_p+\mu_n} \sqrt{\frac{\varepsilon_r \varepsilon_o N_d}{kT_s}} \left(e^{-\left(\frac{\mu_p+\mu_n}{\mu_p-\mu_n}\right) y_3(x)} - 1\right), x \geq 0. \qquad 3.28$$

C. STATIC CURRENT DENSITIES

The static current densities in the plasma obtained from this investigation are

$$J_i(x) = e[\mu_i(n_i(x)E_1' + n_e'(x)E_1(x)) - D_i \frac{dn_e'(x)}{dx}], \quad 3.29$$

$$J_e(x) = e[\mu_e(n_e(x)E_1' + n_e'(x)E_1(x)) + D_e \frac{dn_e'(x)}{dx}], \quad 3.30$$

and

$$J_t = J_i(x) + J_e(x) \quad 3.31$$

where $x \leq 0$.

Similarly, the current densities in the intrinsic and n-type germanium probes are

$$J_p(x) = e[\mu_p(p(x)E_{2,3}'+n'(x)E_{2,3}(x)) - D_p \frac{dn'(x)}{dx}] \quad 3.32$$

$$J_n(x) = e[\mu_n(n(x)E_{2,3}'+n'(x)E_{2,3}(x)) + D_n \frac{dn'(x)}{dx}] \quad 3.33$$

and

$$J_t = J_p(x) + J_n(x) \quad 3.34$$

where $x \geq 0$.

IV. RESULTS

All the resulting graphs of the previous derivations are obtained by using single precision WATFIVE language on the IBM 360/50 computer at the University of Missouri - Rolla. All the variables are plotted versus the distance from the plasma semiconductor interface.

A. EQUILIBRIUM PROPERTIES IN THE GERMANIUM PROBES

To emphasize the variation of the electrostatic potentials in the germanium probes, $\Delta\phi$, the change in the electrostatic potentials relative to the surface potential are plotted in Figures 2 and 3 for the n-type and intrinsic germanium probes, respectively. Figures 2 and 3 show that $\Delta\phi$ decays from zero at x = 0 to approximately -1.3×10^{-5} and -1.5×10^{-3} volts at $x \simeq 2.8 \times 10^{-6}$ cm in the n-type probe and $x \simeq 1.3 \times 10^{-4}$ cm in the intrinsic probe, respectively. The $\Delta\phi$ in the bulk of the n-type case is less than $\Delta\phi$ in the bulk of the intrinsic case and obtains its bulk value nearer to the surface of the n-type probe due to the higher electron concentration in the bulk of the n-type germanium.

The deviation of the equilibrium electron density from its bulk value in the n-type germanium is plotted in Figure 4 and is designated by Δn. Similarly, the deviation from the bulk values of the number densities of the holes and electrons in the intrinsic germanium are plotted in

Figure 5. The Δn in the n-type germanium varies from approximately zero at $x \simeq 2x10^{-6}$cm to approximately $4.9x10^{14}/cm^3$ at x = 0. In the intrinsic case Δn and Δp are approximately $2.5x10^{11}/cm^3$ and $-2.5x10^{11}/cm^3$ at $x \simeq 1.3x10^{-4}$cm, respectively, and are approximately $1.8x10^{12}/cm^3$ and $-1.8x10^{12}/cm^3$ at x = 0, respectively.

The equilibrium electric fields for the intrinsic and n-type probes are plotted in Figures 6 and 7, respectively. It is observed that the electric fields in both types of germanium are identical at x = 0, but the electric field in the intrinsic germanium decays to zero slower than in the n-type case because the electrostatic potential in the intrinsic probe reaches its bulk value at farther distance from the surface of the semiconductor than in the n-type probe.

B. EQUILIBRIUM PROPERTIES IN THE HYDROGEN PLASMA

Since the potentials in the germanium were plotted relative to the surface potential of the probe, the same is done for the hydrogen plasma in Figure 8. Figure 8 shows that the variation of $\Delta\phi$ from the probe surface to its bulk value is approximately 1.768 volts. This is many orders of magnitude greater than the total variations of $\Delta\phi$ in the germanium probes.

The deviation of n_i and n_e from their bulk values are plotted in Figure 9. At x = 0, Δn_i and Δn_e are approximately 6.75×10^{10} and $-9 \times 10^{9}/cm^{3}$, respectively, and decay to zero as becomes more negative.

The equilibrium electric field in the H_2 plasma is plotted in Figure 10. $E_1(x)$ decays from approximately 429 volts/cm at x = 0 to zero as x becomes more negative.

It is observed in all the equilibrium characteristics of the hydrogen plasma that the influence of the probe extends approximately -2.2×10^{-2} cm into the plasma, which is greater than the debye length (6.9×10^{-3}cm) but less than the mean free path between collisions of electrons with molecules (5.5×10^{-1}cm) and ions with molecules (9.8×10^{-2}cm). The effect of the plasma and semiconductor interface penetrates deeper into the plasma than into the germanium probes because the charge number densities in the bulk of the plasma are much smaller than in either semiconductor probe.

C. STATIC NONEQUILIBRIUM PROPERTIES OF THE HYDROGEN PLASMA AND GERMANIUM PROBES

The charge carrier number densities of the plasma intrinsic germanium, and n-type germanium are normalized to their respective applied electric fields and are plotted, respectively, in Figures 11, 12, and 13. It is indicated in Figure 11 that the normalized charge carrier number density in the plasma has a maximum at approximately $x = -1.75 \times 10^{-3}$cm. This indicates that the diffusion term in the current density is zero at that distance from the interface.

The total current density, ion current density, and electron current density are normalized by the applied electric field in the plasma and are plotted in Figure 14. Similarity for the semiconductor, the total current density, hole current density, and electron current density are normalized by the applied electric fields in the semiconductor and are plotted in Figures 15 and 16 for the intrinsic and n-type cases, respectively. From Figures 14, 15, and 16, it is observed that the current densities are related to the applied electric fields by

$$J(x) = \sigma(x)E' \qquad 4.1$$

where $\sigma(x)$ is an effective conductivity.

Since the total static current density is a constant and continuous, J_t in the plasma is equal to J_t in the intrinsic and n-type germanium. Setting the J_t in the plasma equal to J_t of the intrinsic and n-type germanium yields

$$\sigma_2 E_2' = \sigma_1 E_1' \qquad 4.2$$

$$\sigma_3 E_3' = \sigma_1 E_1' \ . \qquad 4.3$$

Using the data from Figures 14, 15, and 16, equations 4.2 and 4.3 reduce to

$$E_2' \simeq 1.08 x E_1' \qquad 4.4$$

$$E_3' \simeq 4.01 x 10^{-5} x E_1'. \qquad 4.5$$

Now using the condition that the difference of the electric displacements of two dissimilar materials is equal to the surface charge on the interface between the materials[13] yields

$$\varepsilon_o(16E_2'-E_1') = \rho_2 \qquad 4.6$$

$$\varepsilon_o(16E_3'-E_1') = \rho_3 \qquad 4.7$$

where ρ_2 and ρ_3 are the charges on the surface of the intrinsic and n-type germanium probes, respectively. Substitution of equations 4.4 and 4.5 into equations 4.6 and 4.7 produces

$$\rho_2 \simeq (1.44 x 10^{-12} \text{ coul/volt-cm}) E_1' \qquad 4.8$$

$$\rho_3 \simeq -(8.85 x 10^{-14} \text{ coul/volt-cm}) E_1'. \qquad 4.9$$

Therefore, the surface charge on the interface is directly proportional to the applied electric fields.

Examining Figures 11, 12, and 13 with the use of equations 4.4 and 4.5, it is observed that the charge carrier number densities are discontinuous at the interface. The discontinuity of carrier density is due to the buildup of the surface charge.

If the length of the plasma is a and the length of the semiconductor probes is b and equations 3.9 and 3.10 are valid, then the applied voltage, V, is related to the electric fields by[13]

$$V = E_1'a + E_2'b \qquad 4.10$$

for the intrinsic probe and

$$V = E_1'a + E_3'b \qquad 4.11$$

for the n-type probe. Using equations 4.4 and 4.5, equation 4.1 now becomes

$$J_t = \frac{\sigma_1 V}{a+1.08b} \qquad 4.12$$

for the intrinsic case and

$$J_t = \frac{\sigma_1 V}{a+(4.01\times10^{-5})b} \qquad 4.13$$

for the extrinsic case. To obtain the total current, I, multiply equations 4.12 and 4.13 by the cross-sectional area of the probe, A.

$$\frac{I}{V} = \frac{\sigma_1 A}{a+1.08b} \qquad 4.14$$

for the intrinsic probe and

$$\frac{I}{V} = \frac{\sigma_1 A}{a+4.01\times10^{-5}b} \qquad 4.15$$

for the n-type probe. Equations 4.14 and 4.15 state that the V-I curves for both germanium probes are linear, but the slope of the n-type probe's V-I curve is greater. It should be noted that equations 4.14 and 4.15 are valid only near V = 0.

To check the assumption that $E_1^{'}$ can be approximated as a constant, consider the total static current density at x = 0 with $E_1^{'}(0)$ different from $E_1^{'}(-\infty)$. If the total current density of the plasma at x = 0 is approximately electron current density, then

$$\frac{J_T}{e\mu_e} \simeq n_e^{'}(x)E_1(0) + n_e(0)\, E_1^{'}(0) + \frac{kT_e}{e}\frac{dn_e^{'}(x)}{dx}\Bigg|_{x=0}. \qquad 4.16$$

Now assume that

$$\frac{n_e^{'}(0)}{n_e(0)} >> \frac{E_1^{'}(0)}{E_1(0)} \qquad 4.17$$

and

$$\left.\frac{dn_e'(x)}{dx}\right|_{x=0} << n_e'(0)E_1(0). \qquad 4.18$$

Equation 4.16 becomes

$$\frac{J_t}{e\mu_e} \simeq n_e'(0)E_1(0). \qquad 4.19$$

It is known that the current density is a constant with respect to x, therefore J_t at $x = -\infty$ is given by

$$\frac{J_t}{e\mu_e} \simeq n_e(-\infty)E_1'(-\infty). \qquad 4.20$$

Equating equation 4.19 and 4.20 yields

$$\frac{n_e'(0)}{E_1'(-\infty)} \simeq 2.32\text{x}10^7/\text{Vcm}^2. \qquad 4.21$$

The value of $n_e'(0)/E_1'$ obtained by assuming that E_1' is a constant is

$$\frac{n_e'(0)}{E_1'} \simeq 2.55\text{x}10^7/\text{Vcm}^2 \qquad 4.22$$

where its value is obtained from Figure 11. Comparing equations 4.21 and 4.22, it is observed that the assumption E_1' is a constant is a reasonable assumption.

Equations 4.19, 4.20, and 4.22 imply that the total current density in the plasma is dependent upon the ratio

$$\frac{n_e'(0)}{E_1'} \simeq \frac{n_e(-\infty)}{E_1(0)} . \qquad 4.23$$

Therefore, the total current density is dependent upon the bulk ion and electron density, type of ion, kinetic temperature of the ions and electrons, and the surface potential because equations

$$\frac{n_e(-\infty)}{E_1(0)} = \frac{n_e(-\infty)}{-\sqrt{\frac{8kT_e n_e(-\infty)}{\varepsilon_o}} \sinh \left(\frac{e\phi(0)}{kT_e}\right)} \qquad 4.24$$

and

$$\phi(0) = -\frac{kT_e}{4e} \ln \frac{m_i}{m_e} \qquad 4.25$$

contain the characteristics of the plasma. Therefore, J_t contains the characteristics of the plasma and may be expressed as

$$J_t \simeq -e\mu_e 4n_e(-\infty)\left[\frac{\mu_e}{3\mu_i+\mu_e}\left(e^{\frac{e\phi(0)}{2kT_e}} - e^{\left(\frac{\mu_i+\mu_e}{\mu_e-\mu_i}\right)\left(\frac{e\phi(0)}{kT_e}\right)}\right.\right.$$

$$\left.\left. - \frac{\mu_i}{3\mu_e+\mu_i}\left(e^{-\frac{e\phi(0)}{2kT_e}} - e^{\left(\frac{\mu_i+\mu_e}{\mu_e-\mu_i}\right)\frac{e\phi(0)}{kT_e}}\right)\right] \sinh\frac{e\phi(0)}{2kT_e}$$

4.26

where equations 4.19 and 3.24 were used.

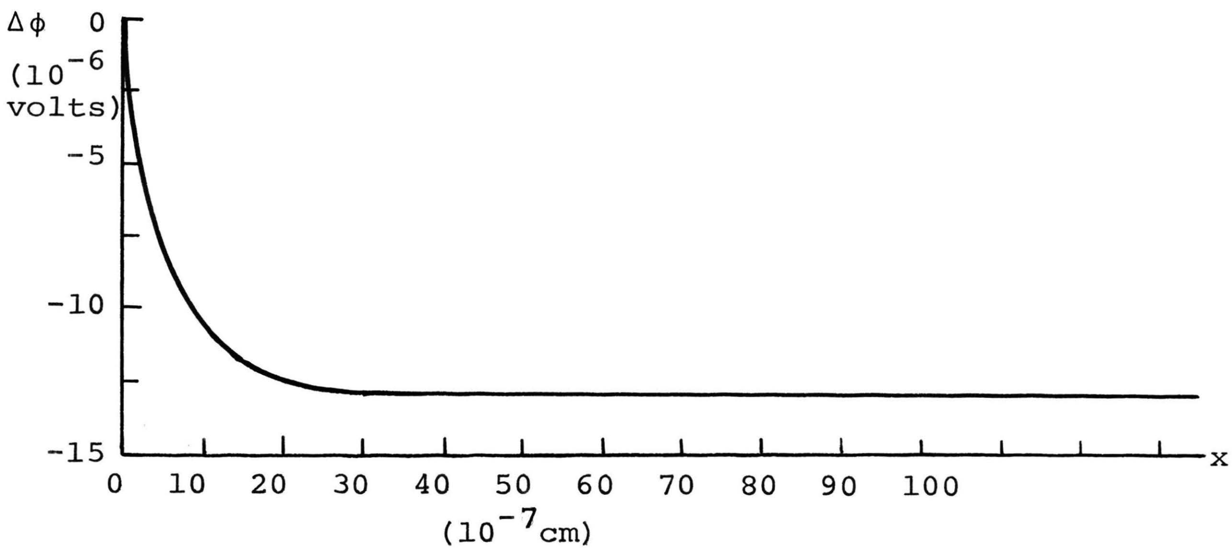

Figure 2. Δφ vs x in the N-type Germanium Probe

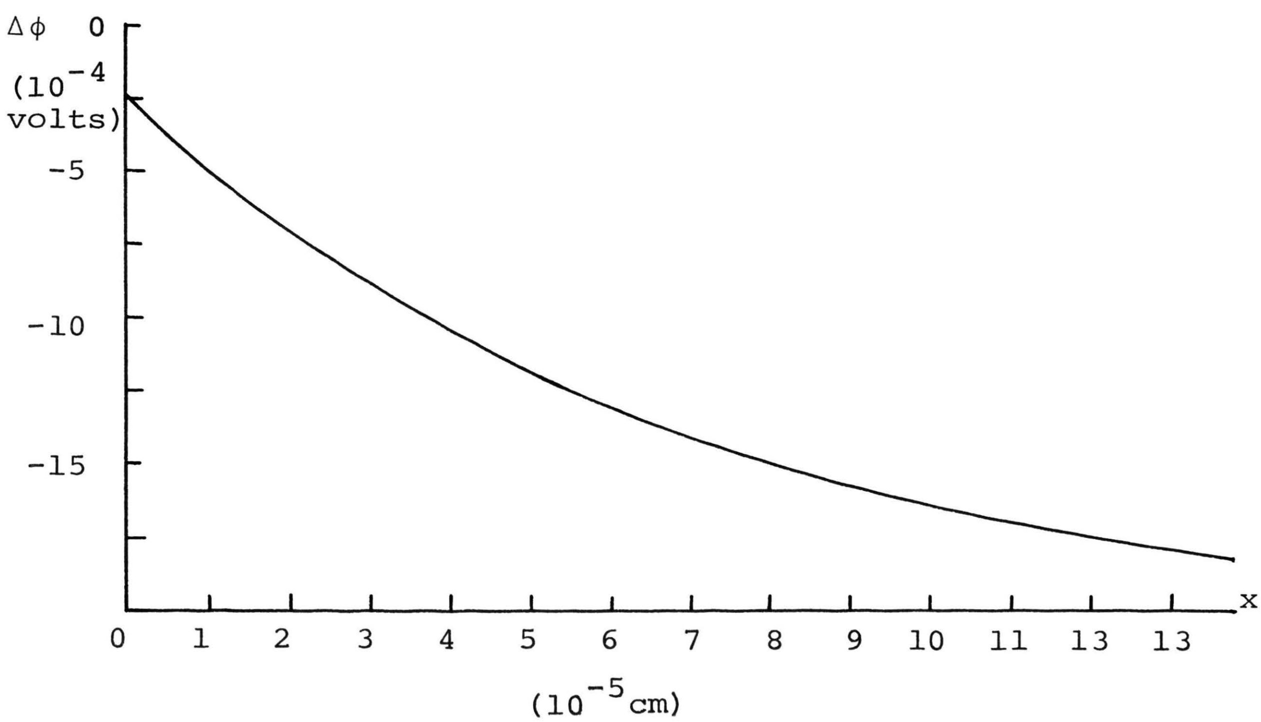

Figure 3. Δφ vs x in the Intrinsic Germanium Probe

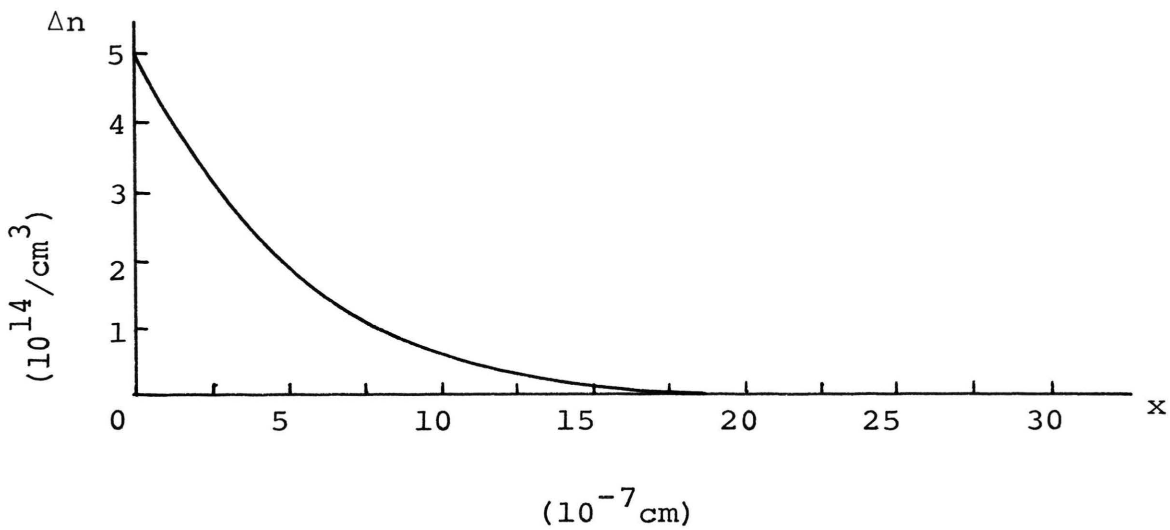

Figure 4. Δn vs x in the N-type Germanium Probe

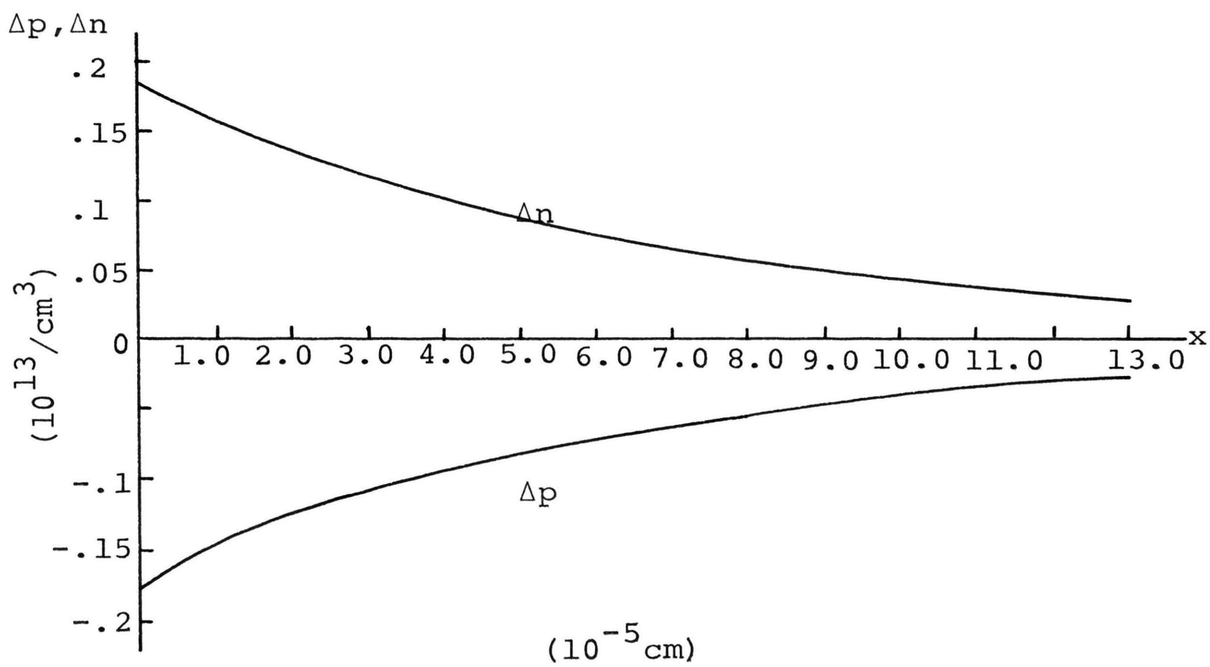

Figure 5. Δn and Δp vs x in the Intrinsic Germanium Probe

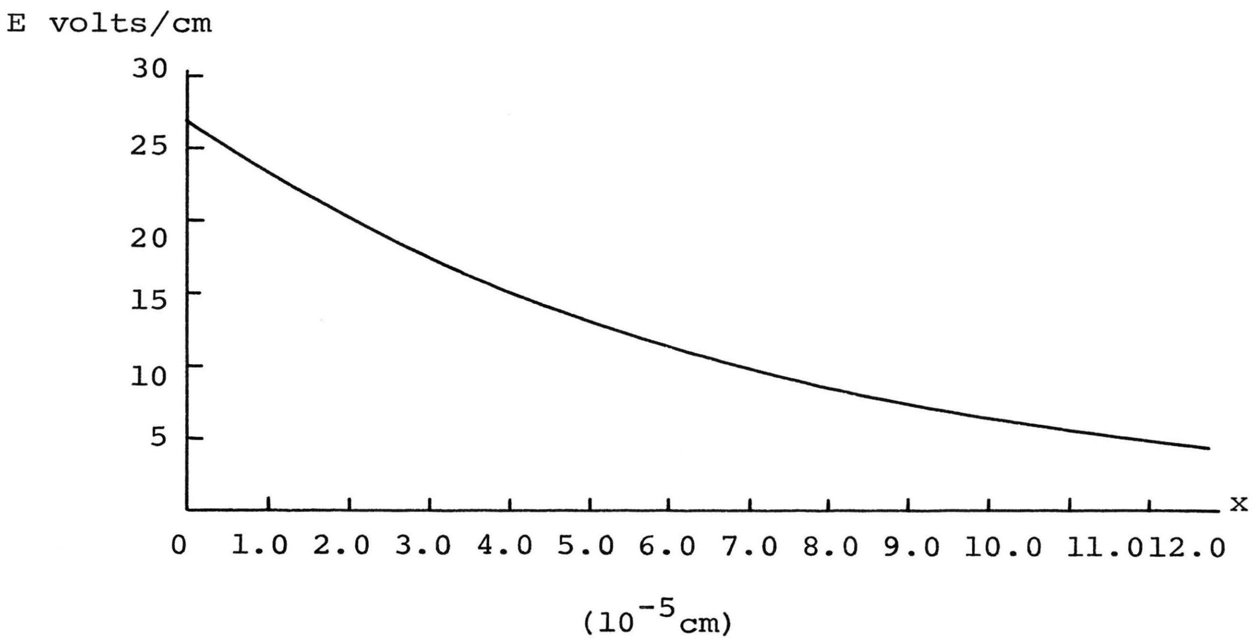

Figure 6. E_2 vs x in the Intrinsic Germanium Probe

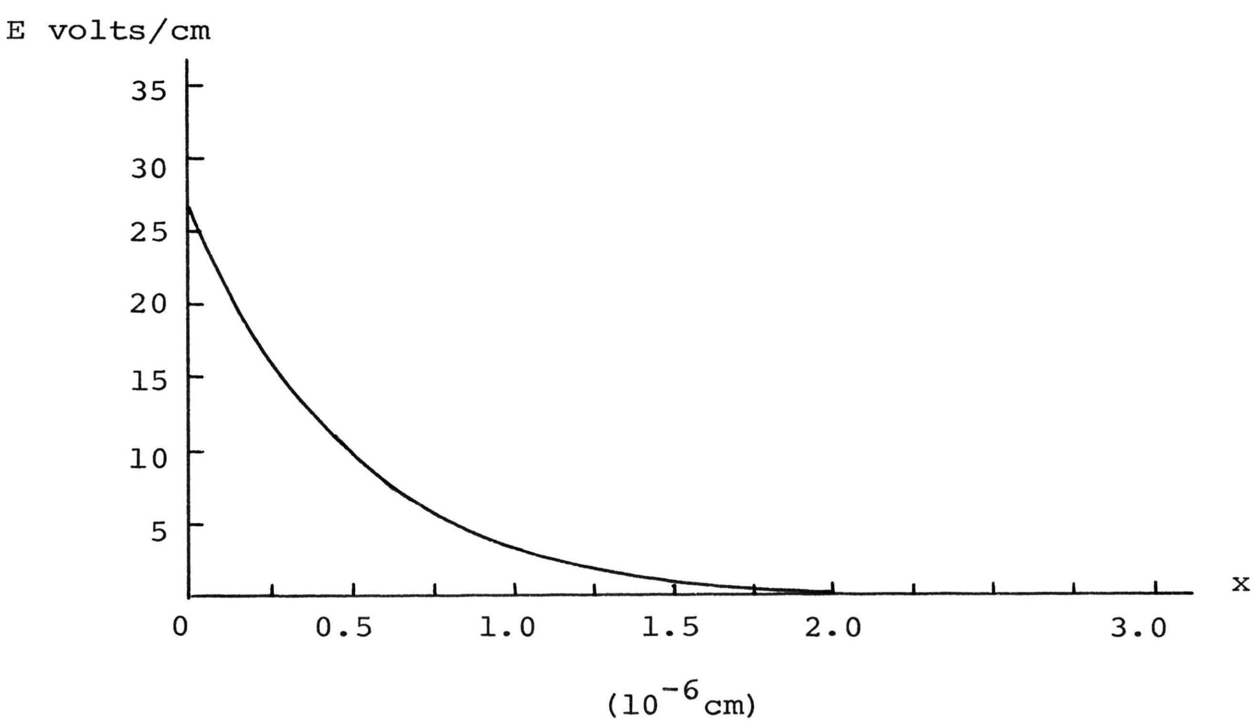

Figure 7. E_3 vs x in the N-type Germanium Probe

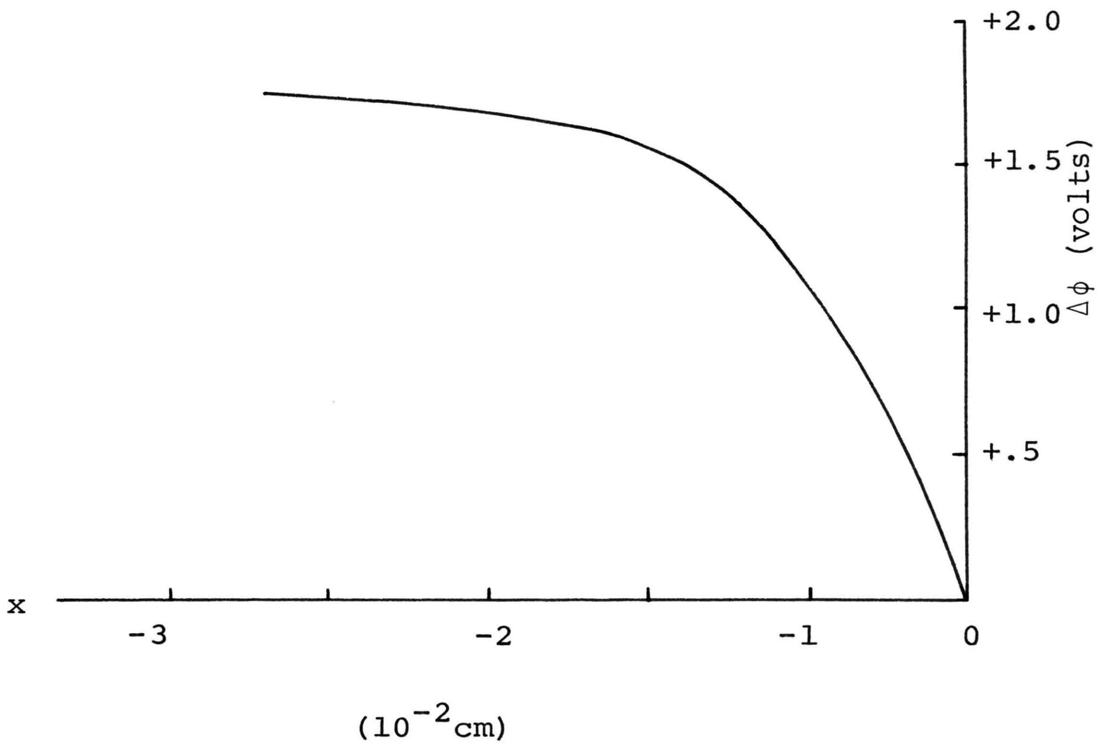

Figure 8. $\Delta\phi$ vs x in the Hydrogen Plasma

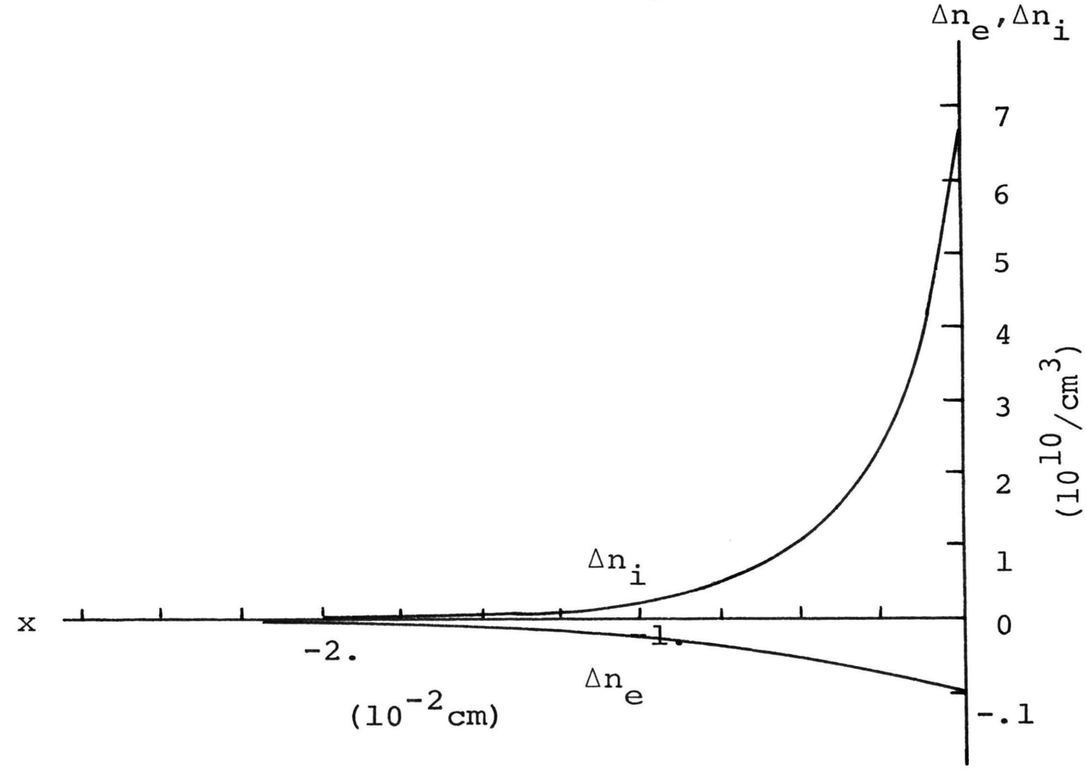

Figure 9. Δn_e and Δn_i vs x in the Hydrogen Plasma

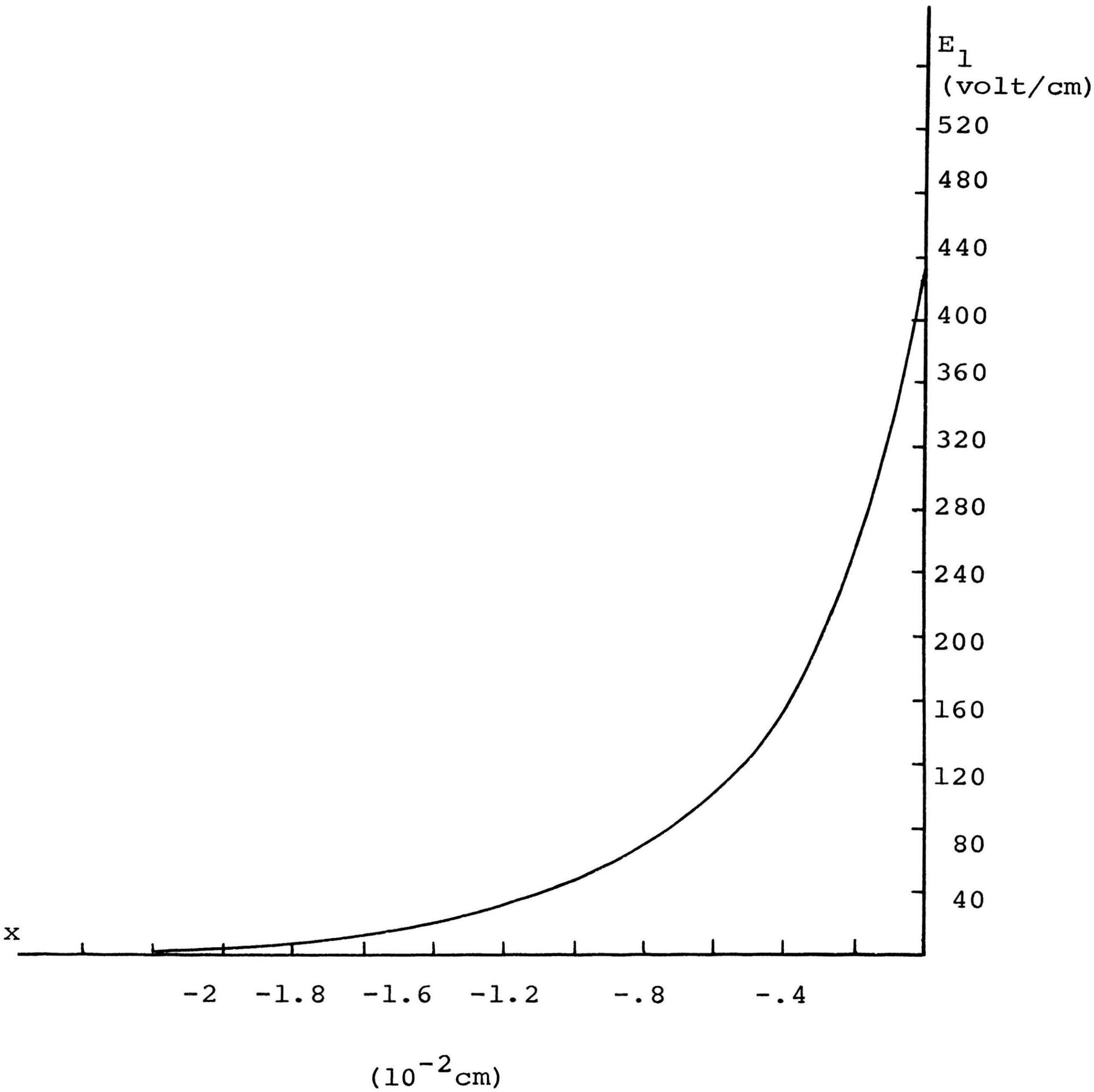

Figure 10. E_1 vs x in the Hydrogen Plasma

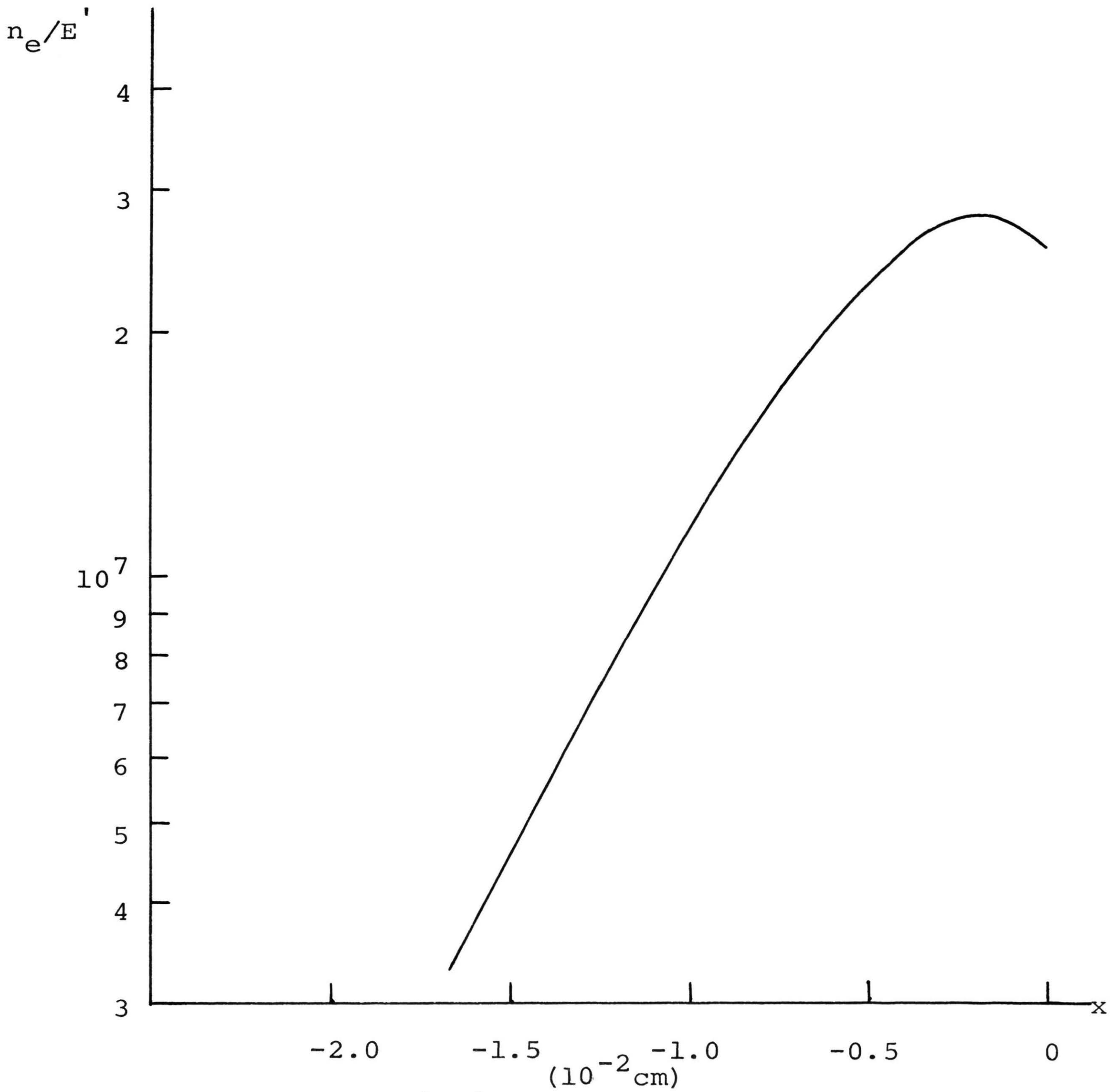

Figure 11. n_e'/E' vs x in the Hydrogen Plasma

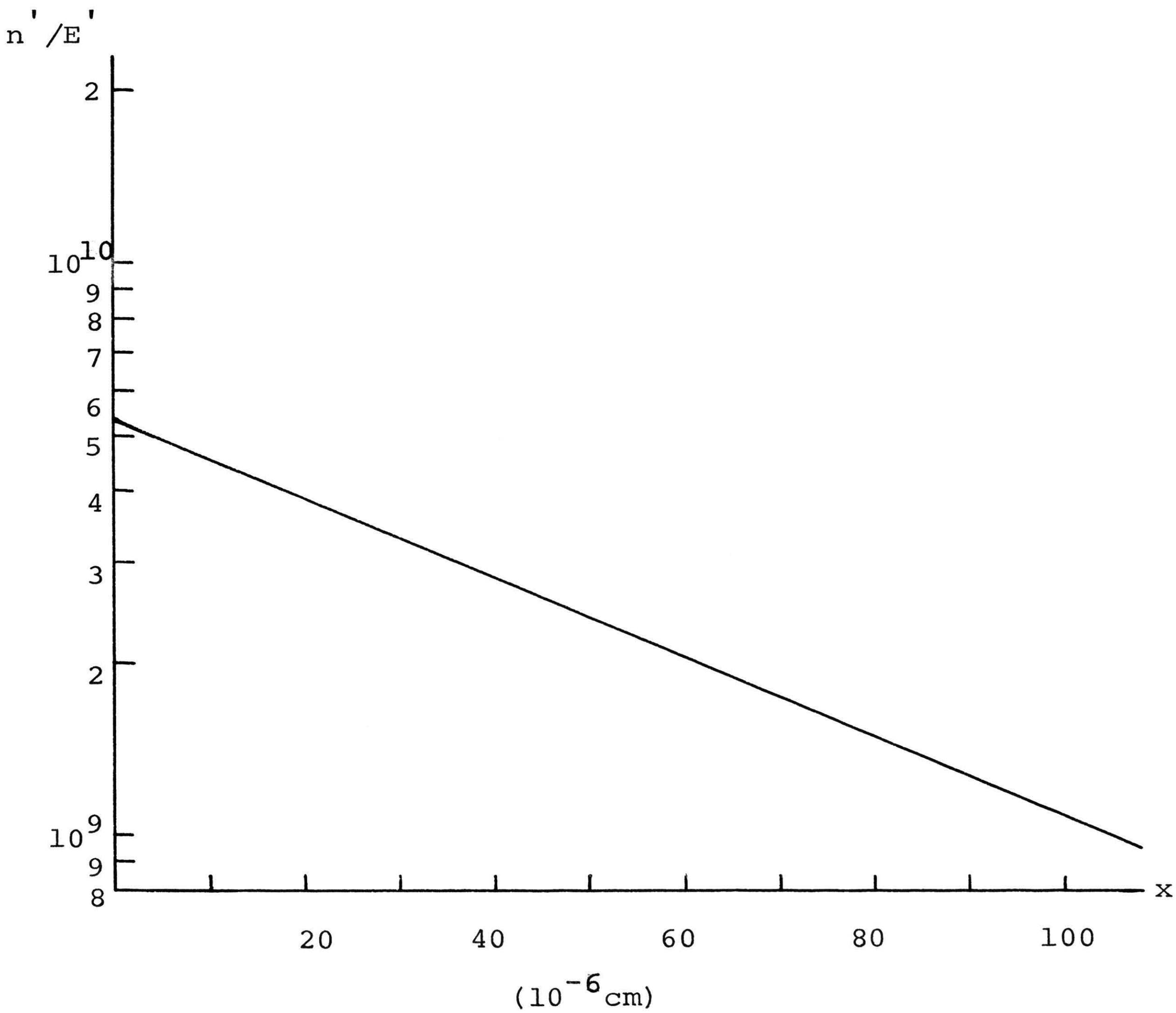

Figure 12. n'/E' vs x in the Intrinsic Germanium Probe

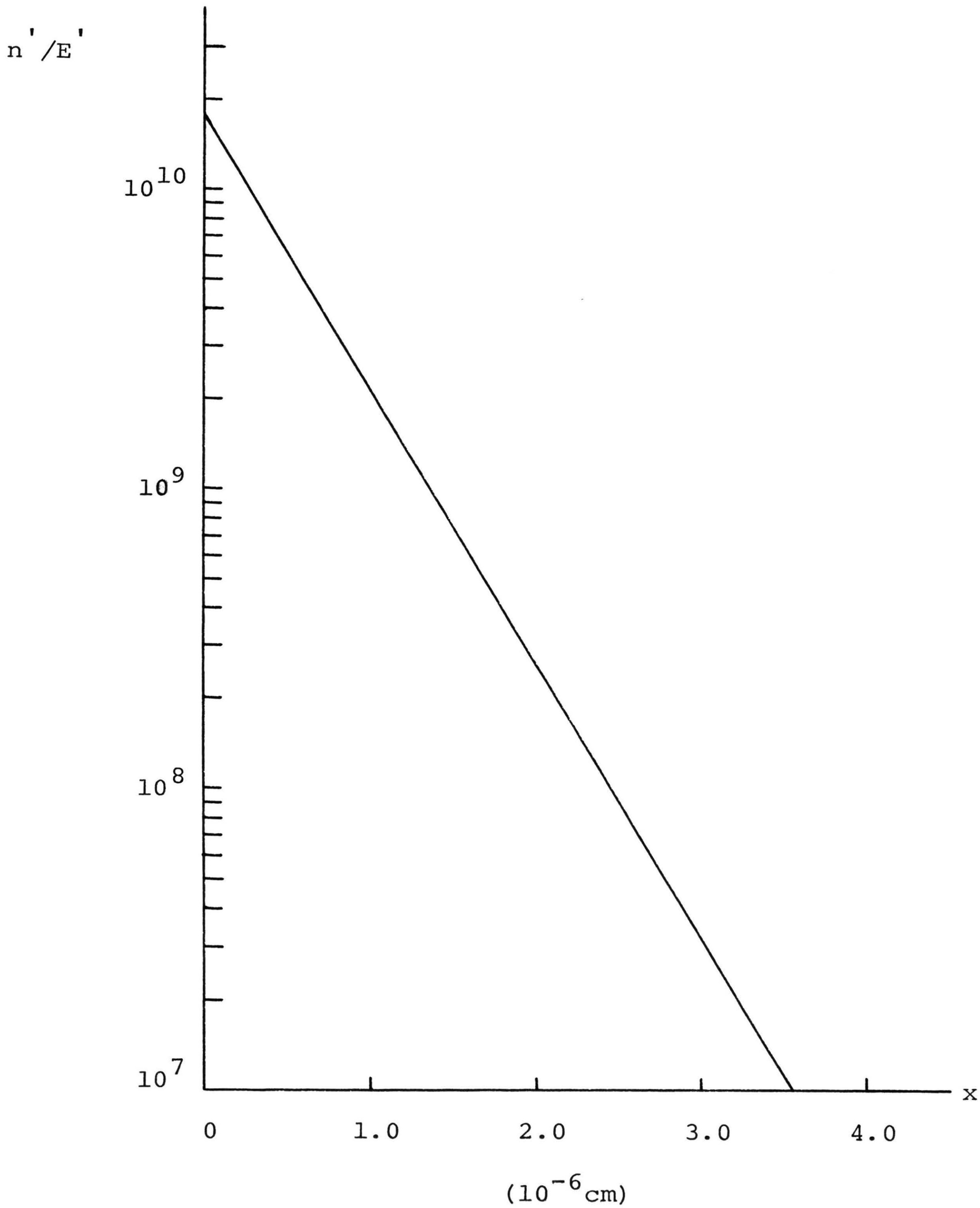

Figure 13. n'/E' vs x in the N-type Germanium Probe

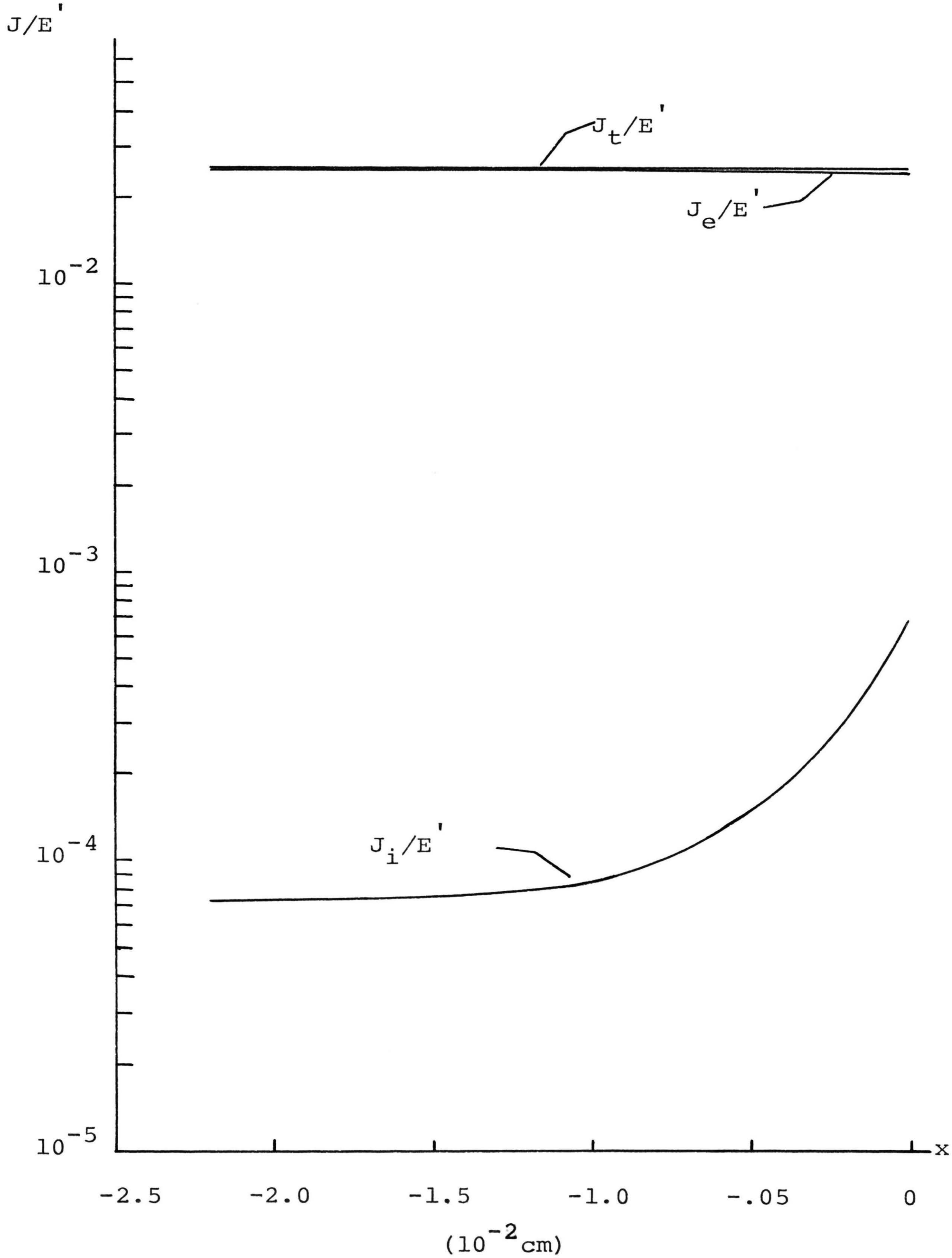

Figure 14. J_t/E', J_e/E', and J_i/E' vs x in the Hydrogen Plasma

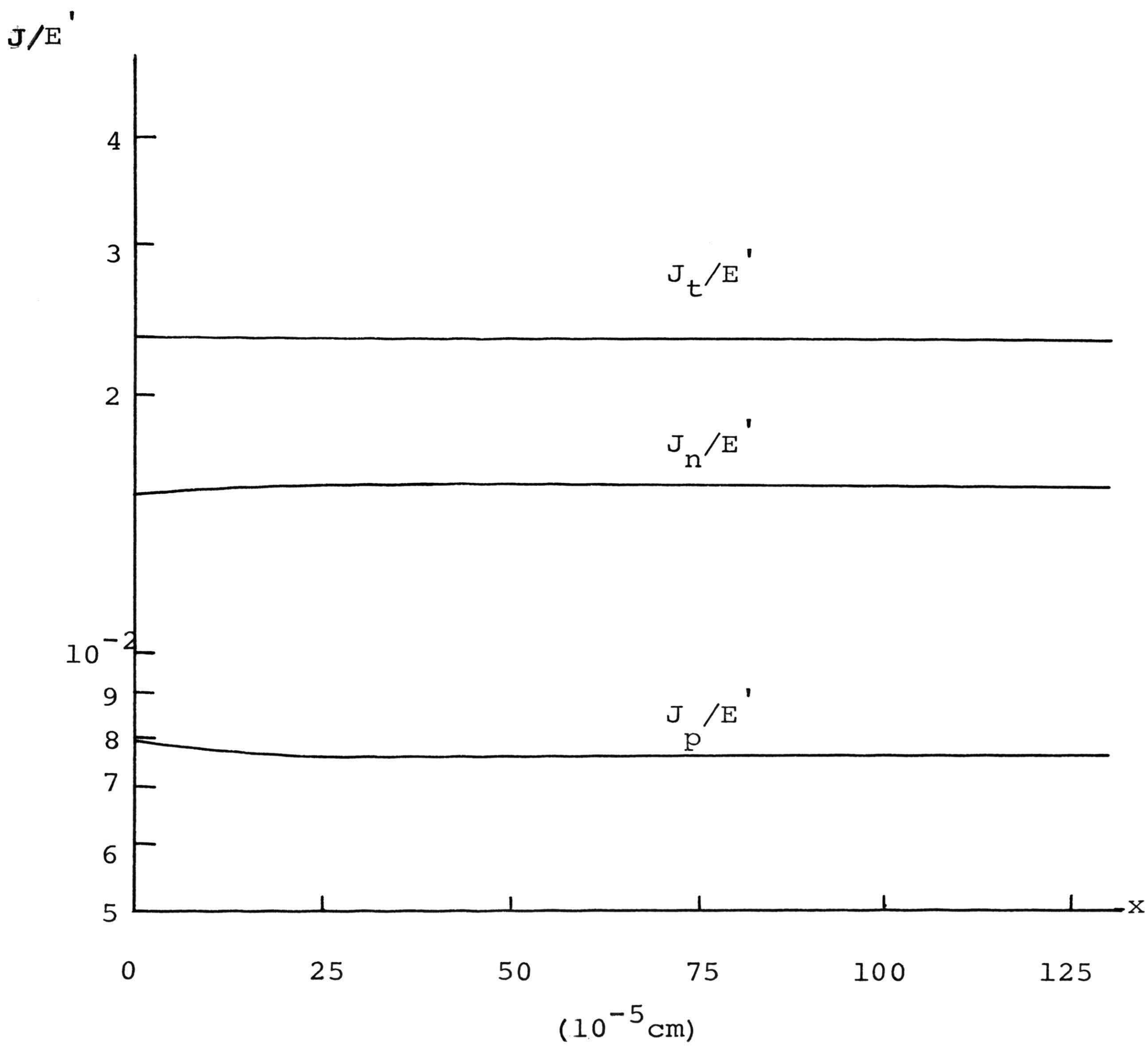

Figure 15. J_t/E', J_n/E', and J_p/E' vs x in the Intrinsic Germanium Probe

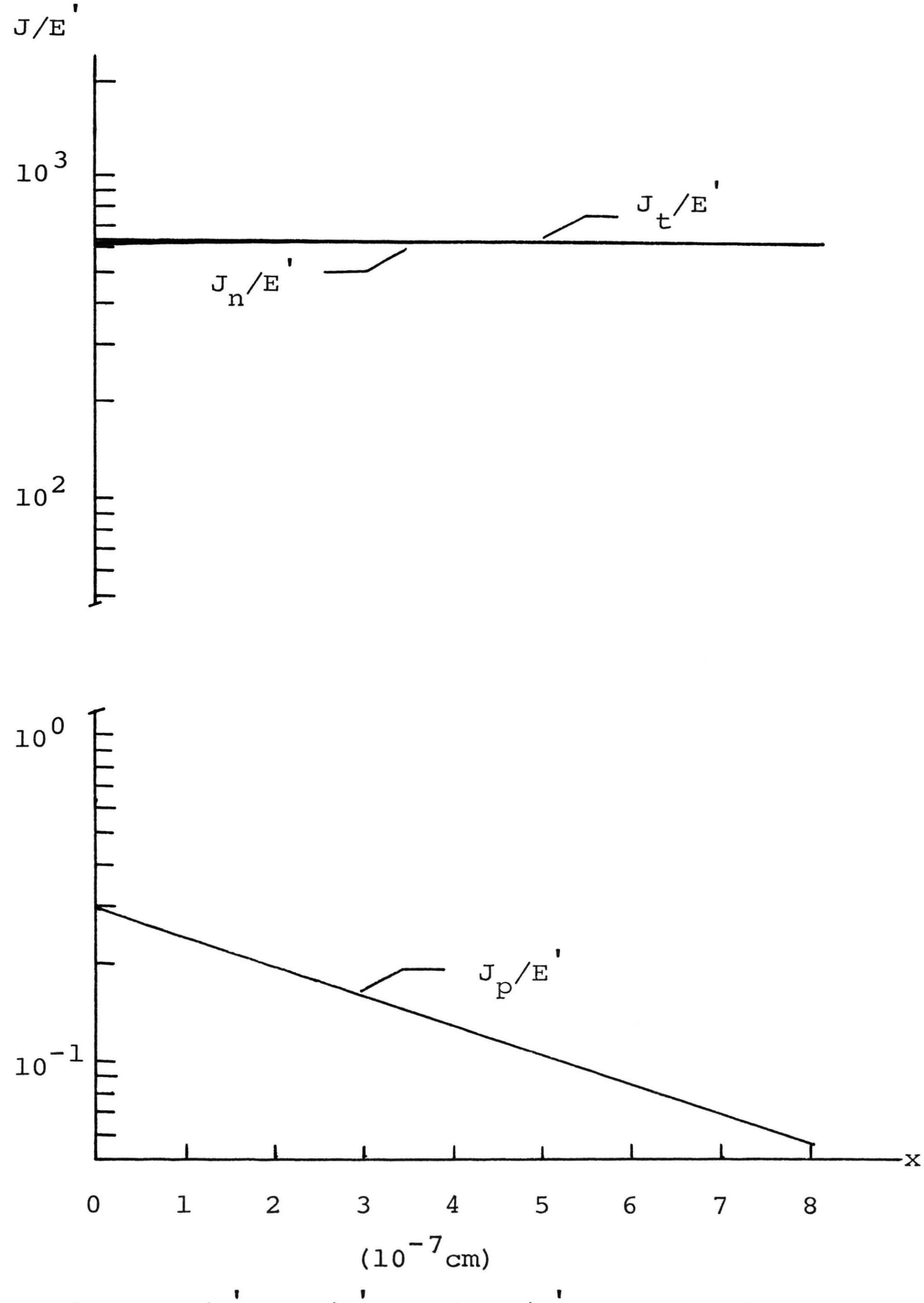

Figure 16. J_t/E', J_n/E', and J_p/E' vs x in the N-type Germanium Probe

V. CONCLUSIONS

The important result of this investigation is the V-I probe characteristics because they are the measurable characteristics. The V-I curves for both intrinsic and n-type germanium planar probes vary linearly near V = 0. The slope the V-I curve is greater for the n-type probe than for the intrinsic probe, but the magnitude of the difference between the slopes is dependent upon the dimensions of the probes relative to the plasma. If the probe length, b, is much smaller than the length of the plasma, a, then the V-I curves are essentially independent of the doping level in the germanium probes and appears to be a metal probe[7] near V = 0. This result is caused by the condition that the plasma is slightly ionized, therefore the number of charged particles available for conduction is limited by the ionization of the plasma and not by the doping level of the germanium probes.

Since a theoretical investigation for germanium probes immersed in a hydrogen plasma has been solved for V near zero, it would be of interest to check the results of this theory in the laboratory. This investigation studied only small probe bias characteristics, therefore the usefulness of this theory is confined to that of determining the effective resistance of the probe plasma interface.

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VITA

Alfred Gene Williams was born on July 8, 1948 in Birch Tree, Missouri. He was graduated from Birch Tree's High School in May 1966. After that he attended the University of Missouri - Columbia from September 1966 until August 1967. He then entered the University of Missouri - Rolla and was graduated with a Bachelor of Science Degree in Physics in May 1970. He has been enrolled in the Graduate School of the University of Missouri - Rolla since January 1971. The author held a position as a Graduate Teaching Assistant from January 1971 until May 1972.

He is married to the former Brenda Sue Taylor of Rolla, Missouri, and they have one son, Alfred Gene, Jr.

APPENDIX A

BASIC EQUATIONS

The three basic equations used in solving the planar semiconductor probe characteristics in a plasma are the current density equations, the current continuity equations, and Poisson's equations.

1. CURRENT DENSITY EQUATIONS

The current density equations in the x direction for a semiconductor[4] and a plasma[15] are given by

$$J_+(x) = e(\mu_+ n_+(x)E(x) - D_+ \frac{dn_+(x)}{dx}) \qquad \text{A.1}$$

and

$$J_-(x) = e(\mu_- n_-(x)E(x) + D_- \frac{dn_-(x)}{dx}) \qquad \text{A.2}$$

where the subscripts + and - represent the holes and electrons, respectively, in the semiconductor and represent the ions and electrons, respectively, in the plasma. The terms J, e, μ, n, E, and D are, respectively, the current density, electronic charge, mobility constant, number density of the charge, electric field, and diffusion constant.

2. CURRENT CONTINUITY EQUATIONS

Referring to Van der Ziel[4], the divergence of the static current density in the x direction for the negative charge current density and positive charge current density are given, respectively, by

$$-\frac{1}{e}\frac{dJ_-(x)}{dx} = g - rn_+n_- \qquad \text{A.3}$$

and

$$\frac{1}{e}\frac{dJ_+(x)}{dx} = g - rn_+n_- \qquad \text{A.4}$$

where g is the generation rate of the charge pairs and r is the recombination coefficient. Equations A.3 and A.4 show that the spacial derivative of J_+ is equal to the negative of the spacial derivative of J_-, therefore, it is necessary to examine only equation A.3. If excess charge pairs, n_-', are injected into either the semiconductor or plasma, equation A.3 becomes

$$-\frac{1}{e}\frac{dJ_-(x)}{dx} = g - r(n_+ + n_-')(n_- + n_-') \ . \qquad \text{A.5}$$

Using $g = rn_+n_-$ [4] and the assumption of low injections,

$$n_-' << n_+, n_- \qquad \text{A.6}$$

yields

$$-\frac{1}{e}\ \frac{dJ_-(x)}{dx} = -\ r[n_+(x) + n_-(x)]\ n_-'(x). \qquad A.7$$

Now define τ, the recombination time, as follows:

$$\frac{1}{\tau} = r(n_+(x) + n_-(x)). \qquad A.8$$

Equations A.3 and A. 4 become

$$\frac{1}{e}\ \frac{dJ_-(x)}{dx} = \frac{n_-'(x)}{\tau} \qquad A.9$$

$$\frac{1}{e}\ \frac{dJ_+(x)}{dx} = -\ \frac{n_-'(x)}{\tau}\ . \qquad A.10$$

Equations A.9 and A.10 are valid only for low injections of charge pairs and time invarient conditions.

3. POISSON'S EQUATIONS

The one dimensional Poisson's equation in a semi-conductor with both donors, N_d, and accepters, N_a,[4] where the impurity ions are assumed to be fully ionized is given by

$$\frac{dE(x)}{dx} = \frac{e}{\varepsilon_r \varepsilon_o}\ [n_+(x) - n_-(x) + N_d(x) - N_a(x)] \qquad A.11$$

where ε_o is the dielectric constant of a vacuum and ε_r is the relative dielectric constant of the material. If $N_d = N_a = 0$, then the semiconductor is intrinsic and equation A.11 becomes

$$\frac{dE(x)}{dx} = \frac{e}{\varepsilon_r \varepsilon_o} [n_+(x) - n_-(x)]. \qquad \text{A.12}$$

Assuming charge neutrality in the bulk of the plasma, the following one dimensional Poisson's equation is obtained[16]

$$\frac{dE(x)}{dx} = \frac{e}{\varepsilon_r \varepsilon_o} [n_+(x) - n_-(x)]. \qquad \text{A.13}$$

A comparison of equations A.12 and A.13 shows that Poisson's equations for the plasma and the intrinsic semiconductor are of the same form.

APPENDIX B

THE EFFECTS OF COLLISIONS IN THE H_2 PLASMA

Since the number densities of the electrons and ions are much smaller than the number density of the molecules, the dominate collisional processes in this hydrogen plasma are the electron-molecule and ion-molecule collisions. It is also assumed that no energy exchange occurs during the collisions because the plasma is in thermal equilibrium, therefore only momentum exchange occurs.

Referring to Sutton and Sherman[8], the collision frequency of species r colliding with species s is given by

$$<\nu_{rs}> = \frac{2Q_{rs}n_s(2k)^{\frac{1}{2}}}{\sqrt{\pi}} \left(\frac{T_r}{m_r} + \frac{T_s}{m_s}\right)^{\frac{1}{2}} \qquad \text{B.1}$$

where Q_{rs} is the total collisional cross-section for momentum transfer, n_s is the number density of the scatters, T is the kinetic temperature of the species, and m is the mass of the species. Q_{rs} for momentum transfer collisions is

$$Q_{rs} = \pi(R_s + R_r)^2 \qquad \text{B.2}$$

where R is the radii of the species.

If the incident particle is an electron (regarded as a point mass) and the scatters are the hydrogen molecules, then Q_{rs} becomes

$$Q_{rs} = \pi R_s^2 \qquad \text{B.3}$$

where R_s is equal to 1.2 A[17]. If $T_s = T_r$, then

$$\frac{T_r}{m_r} >> \frac{T_s}{m_s} \qquad \text{B.4}$$

because the mass of the hydrogen molecule is much greater than the mass of the electron. The collision frequency of an electron and the hydrogen molecules is found to have the value

$$\langle \nu_{rs} \rangle \simeq 112\text{x}10^6/\text{sec}. \qquad \text{B.5}$$

The collision frequency is equal to the reciprocal of the mean free time between collisions, τ_2, of an electron with the hydrogen molecules. Therefore,

$$\frac{1}{\tau_e} \simeq 112\text{x}10^6/\text{sec}. \qquad \text{B.6}$$

If the incident particle is an H_2^+ ion and the scatters are hydrogen molecules, it is assumed that $m_r = m_s = 3.346\text{x}10^{-27}$kg, $T_r = T_s$, and the radius of the ion is equal to the radius of the molecule. Therefore, equation B.1 has the value

$$\frac{1}{\tau_i} \simeq 105\text{x}10^5/\text{sec} \qquad \text{B.7}$$

where τ_i is the mean time between collisions for ions.

If the mean free time between collisions is assumed to be a constant in velocity space, the mobilities for the electrons and ions, respectively, are approximately[4]

$$\mu_e = \frac{e}{m_e} \tau_e \simeq 1.56\text{x}10^7 \text{cm}^2/\text{volt-sec} \qquad \text{B.8}$$

$$\mu_i = \frac{e}{m_i} \tau_i \simeq 4.56\text{x}10^4 \text{cm}^2/\text{volt-sec}. \qquad \text{B.9}$$

APPENDIX C

THE SURFACE POTENTIAL OF MATERIALS IN A HYDROGEN PLASMA

Referring to Uman[18], the total current density of a singly ionized plasma to any material in equilibrium with the plasma is

$$\frac{en_i\langle v_i\rangle}{4} - \frac{en_e\langle v_e\rangle}{4} = 0 \qquad \text{C.1}$$

where $\langle v_i\rangle$ and $\langle v_e\rangle$ are the thermal velocities of the ions and electrons, respectively. If the plasma has a homogeneous Maxwell-Boltzmann distribution, then $\langle v_i\rangle$ and $\langle v_e\rangle$ are given by

$$\langle v_i\rangle = \sqrt{\frac{8kT_i}{\pi m_i}} \qquad \text{C.2}$$

$$\langle v_e\rangle = \sqrt{\frac{8kT_e}{\pi m_e}} \qquad \text{C.3}$$

where T_i and T_e are the kinetic temperatures of the ions and electrons, respectively. The terms m_i and m_e are the masses of an ion and an electron, respectively. For a plasma in thermodynamic equilibrium $T_i = T_e$.

If the plasma is hydrogen with H_2^+ ions and is in thermal equilibrium, then equation C.1 becomes

$$\phi(0) = -\frac{kT_e}{4e} \ln \frac{m_i}{m_e} \qquad \text{C.4}$$

where equations 2.5 and 2.6 were employed. For the plasma under consideration

$$T_e = 10^4 k \qquad C.5$$

and

$$m_i \simeq 3.346 \times 10^{-27} kg. \qquad C.6$$

Therefore, equation C.4 reduces to

$$\phi(0) \simeq -1.768 \text{ volts}. \qquad C.7$$

According to equation C.7, the materials in equilibrium with the plasma will always have a surface potential negative relative to the plasma bulk potential.

APPENDIX D

LIST OF SYMBOLS

A cross-sectional area of probe

a length of plasma

b length of probe

D_i diffusion constant of ions

D_e diffusion constant of electrons in plasma

D_n diffusion constant of electrons in probe

D_p diffusion constant of holes in probe

D_+ diffusion constant of positive charges

D_- diffusion constant of negative charges

E equilibrium electric field

E_t total electric field

E_1 equilibrium electric field in plasma

E_2 equilibrium electric field in intrinsic probe

E_3 equilibrium electric field in n-type probe

E' applied electric field

E_1' applied electric field in plasma

E_2' applied electric field in intrinsic probe

E_3' applied electric field in n-type probe

e electronic charge

g generation rate of charges

I static current

J_e current density of electrons in plasma

J_i current density of ions in plasma

J_n current density of electrons in probe

J_p current density of holes in probe

J_t total current density

J_+ current density of positive charge

J_- current density of negative charge

k Boltzmann's constant

m_e mass of electron

m_i mass of ion

m_r mass of particle r

m_s mass of particle s

N_a acceptor number density

N_d donor number density

n equilibrium number density of electrons in probe

n_I intrinsic number density of germanium

n_e plasma's electron equilibrium number density

n_s number density of scatters

n_i plasma's ion equilibrium number density

n_{t+} total number density of positive charge

n_{t-} total number density of negative charge

n_+ equilibrium number density of positive charge

n_- equilibrium number density of negative charge

n' charge carrier number density in probe

n_e' charge carrier number density in plasma

n_+' charge carrier number density of positive charge

n_-' charge carrier number density of negative charge

Δn equilibrium electron number density in the germanium probes relative to the bulk value

Δn_e equilibrium electron number density in the plasma relative to its bulk value

Δn_i equilibrium number density of ions relative to its bulk value

p equilibrium number density of holes

Δp equilibrium number density of holes relative to their bulk value

Q_{rs} total collisional cross-section

R_r radius of particle r

R_s radius of particle s

r recombination rate coefficient

T_e kinetic temperature of plasma particles

T_i kinetic temperature of ions

T_r kinetic temperature of particles r

T_s temperature of probe

V applied dc voltage

$\langle v_e \rangle$ thermal velocity of electrons in plasma

$\langle v_i \rangle$	thermal velocity of ions in plasma
x	distance from plasma and semiconductor interface
ε_o	dielectric constant of a vacuum
ε_r	relative dielectric constant
μ_e	mobility of electrons in plasma
μ_i	mobility of ions in plasma
μ_n	mobility of electrons in probe
μ_p	mobility of holes in probe
μ_+	mobility of positive charge
μ_-	mobility of negative charge
σ_1	effective conductivity in plasma
σ_2	effective conductivity in intrinsic germanium
σ_3	effective conductivity in n-type germanium
$\langle \nu_{rs} \rangle$	collisional frequency between a particle r and particles s
ρ_1	surface charge on the surface of the intrinsic germanium probe
ρ_2	surface charge on the surface of the N-type germanium probe
ϕ	equilibrium potential relative to plasma bulk potential
ϕ_t	total potential

ϕ' perturbed potential

$\Delta\phi$ equilibrium potential relative to the surface of the germanium probes

τ recombination time

τ_e mean free time between collisions for electrons in plasma

τ_i mean free time between collisions for ions in plasma