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# A method for studying sequential faults on a three phase distribution transformer

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# A METHOD FOR STUDYING SEQUENTIAL FAULTS ON A THREE PHASE DISTRIBUTION TRANSFORMER

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# ABSTRACT

A method has been presented for studying sequential faults on opposite sides of a three-phase distribution transformer using the theory of symmetrical components and Alpha, Beta, and Zero components of three-phase systems. The effect of the faults on the distribution system has also been analyzed and improvements in the protection scheme suggested.

The sequential faults occurred, one after the other, on the distribution system of the Central Electric Power Cooperative of Jefferson City in the state of Missouri resulting in a double unbalanced fault condition. The data used in the example problem has been supplied through the courtesy of the above Cooperative. The problem has been solved in three stages, namely, for a two line to ground fault, an open conductor and the resulting double fault conditions with grounded transformer as well as with the ungrounded transformer when the grounding wire of the transformer burns out.

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# I. INTRODUCTION

The configuration of the three-phase distribution transformer, with its primary winding delta connected and the secondary winding wye connected with a solid neutral grounding, is shown in Figure 1. The system was protected by fuses in each phase of the primary side of the transformer.

At time $t = 0$, a two line to ground fault on phases B and C occurred on the secondary side of the system transformer. As a result of this fault, the fuse of phase a was blown on the primary side of the transformer at time $t = t_1$ and thus the system developed a double unsymmetrical fault condition on the opposite sides of the distribution transformer.

With the above two faults existing, the grounding wire (neutral wire) of the secondary wye connected winding burned through at time $t = t_2$ because of excessive current which further unbalanced the system, resulting in the destruction of customers' appliances on phase A.

The theory of symmetrical components has been applied to solve the two line to ground fault by using a Thevenin equivalent circuit assuming the circuit to be linear and bilateral and neglecting the saturation effects as well as the magnetizing current of the transformer. The system currents under fault conditions were calculated in the primary winding, taking the appropriate phase-shift into consideration, and the time ($t_1$) for melting the phase a fuse was then obtained from the available current against elapsed time characteristics of the fuse.

Now, with an open conductor in the primary circuit and a two line to ground fault in the secondary circuit, the system was reduced to a

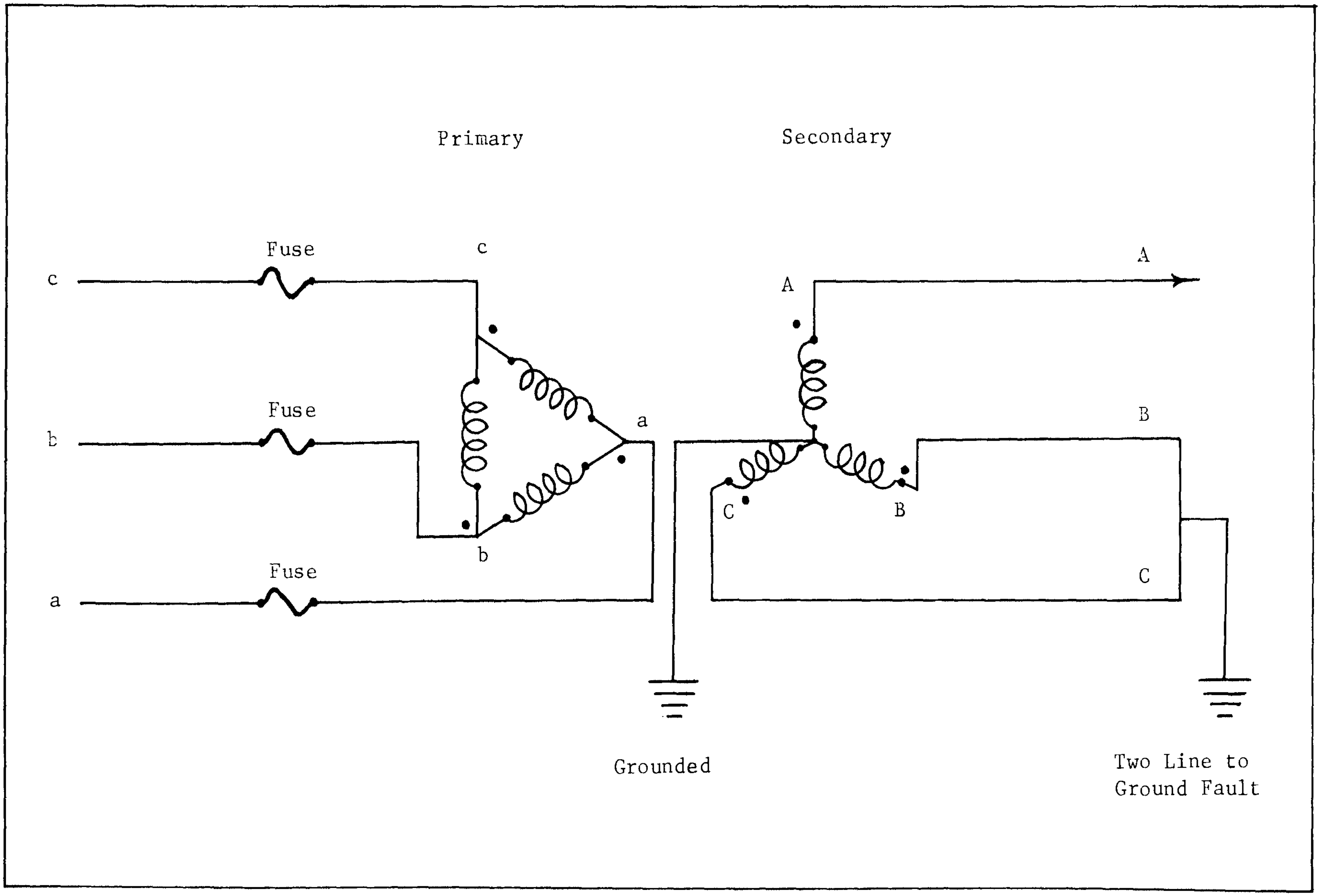



Figure 1. Configuration for Three-Phase Transformer

double unsymmetrical fault, occurring on opposite sides of the distribution transformer. The solution of the double fault condition was obtained by using the Alpha, Beta, and Zero components of the three-phase system. An equivalent circuit connecting the Alpha, Beta, and Zero networks together has been developed to satisfy both the fault conditions. The various system currents and voltages have been calculated using the equivalent circuit.

Once the fault current flowing through the grounding wire of the transformer is known, the melting time ($t_2$) for this fault current is computed from the characteristics of the particular ground wire used for the transformer. With the grounding wire opened, the zero sequence network is altered and a new equivalent circuit is formed. Thus, the final values of various system currents and voltages are again found.

Voltages resulting from the above double fault condition are found to be much above the normal voltage of the system and, consequently, the customers' appliances are damaged.

Thus, an analytical method for studying the double unsymmetrical fault conditions has been developed to determine the system conditions, and improvements in the protection scheme of such distribution systems have also been suggested to avoid repeating such fault conditions. An example problem has also been solved with the data supplied by the Central Electric Power Cooperative.

## II. REVIEW OF LITERATURE

In 1918, Dr. C. L. Fortescue[1] developed a method of symmetrical coordinates applied to the solution of polyphase networks. Unsymmetrical faults on transmission systems, which may consist of short circuits, impedance between lines, impedance from one line or two lines to ground, or open conductors, are studied by the method of symmetrical components.

In the method of symmetrical components, an unsymmetrical set of vector currents or voltages is resolved into three balanced sets of components.

The theory of symmetrical components has been used extensively for solving power system problems with single unsymmetrical faults. For cases of double faults, each fault was solved separately, and then by using the superposition theorem, the final values of the system currents and voltages were obtained. The results thus obtained were only applicable for simple system networks and the same technique, when applied to complicated networks, proved to be incorrect and appreciable error was generated. In the year 1931, a paper presented at an A.I.E.E. meeting indicated that approximately 20 percent of the faults on double-circuit overhead transmission lines on the same tower involved both circuits. Occasionally, two faults occurred simultaneously at points which were separated geographically as well as electrically, particularly on systems which were grounded through high impedances.

In the same year Miss Edith Clarke[2] developed a method dealing with simultaneous faults. This was published as "Simultaneous Faults on Three-Phase Systems" in the A.I.E.E. Transactions in September, 1931. In her paper, the method of symmetrical components was extended to apply to three-phase systems during simultaneous faults at two or more points of the system. A general equivalent circuit was developed to replace, in the positive sequence diagram, two simultaneous faults involving any combination of the six conductors. A double unbalance cannot be correctly represented by connecting the sequence networks at each of the two points of fault as they would be connected for single unbalances at those points. Miss Clarke's paper proved to be very useful in solving problems of double unbalances by using the method of equivalent circuits of various sequence networks.

Subsequently, the problems of double unbalances were also solved by Wagner and Evans[3] in 1933 and by Lyon[4] in 1937 in their books dealing with symmetrical components and its application to the analysis of unbalanced electrical circuits. In 1935, Kimbark[5] developed a practical way of solving the problem of double unbalances by setting up miniature experiments to represent the faulted networks.

Among all these studies, Miss Clarke's method presents a unique way of solving any combination of double unbalances in a three-phase power system. This same method was pursued for the problem initially, but failed because of the complicated system developments which make it difficult to obtain a solution using the symmetrical components theory,

In the year 1938, another method was developed by Miss Edith Clarke[6] to solve simultaneous faults titled "Modified Symmetrical Components" using Alpha, Beta, and Zero components. One year later Dr. Kimbark[7] developed another technique, quite similar to Alpha, Beta, and Zero components, and called them x, y, and z components, respectively, to study unsymmetrical system conditions. The use of Alpha, Beta, and Zero components gave simpler solutions than the use of symmetrical components in power systems when equal positive and negative sequence impedances are used for representing rotating machines. Thus, in an unsymmetrical circuit where the impedances of the two phases are equal, or two phases are symmetrical with respect to the third phase, Alpha, Beta and Zero components give an almost immediate solution to many systems. The theory of Alpha, Beta, and Zero components are also detailed in the book by Miss Clarke[8]. This theory of Alpha, Beta and Zero components is used to solve problems when it becomes difficult to use the symmetrical components theory and the same has been used to solve the double unbalanced fault condition in the present problem.

In 1941, Clarke, Peterson, and Light[9] determined the condition under which abnormal voltages of sufficient magnitude developed to damage equipment when one or two conductors opened in circuits supplying ungrounded transformers. Investigations were made by means of calculations and laboratory tests. But the results obtained by these tests cannot be used in the present problem because of sustained simultaneous faults and, as such, the theory of Alpha, Beta, and Zero components has been used to determine the value of overvoltages which damaged the customers' equipment during the conditions under study.

# III. THEORETICAL DEVELOPMENT OF THE MATHEMATICAL MODEL

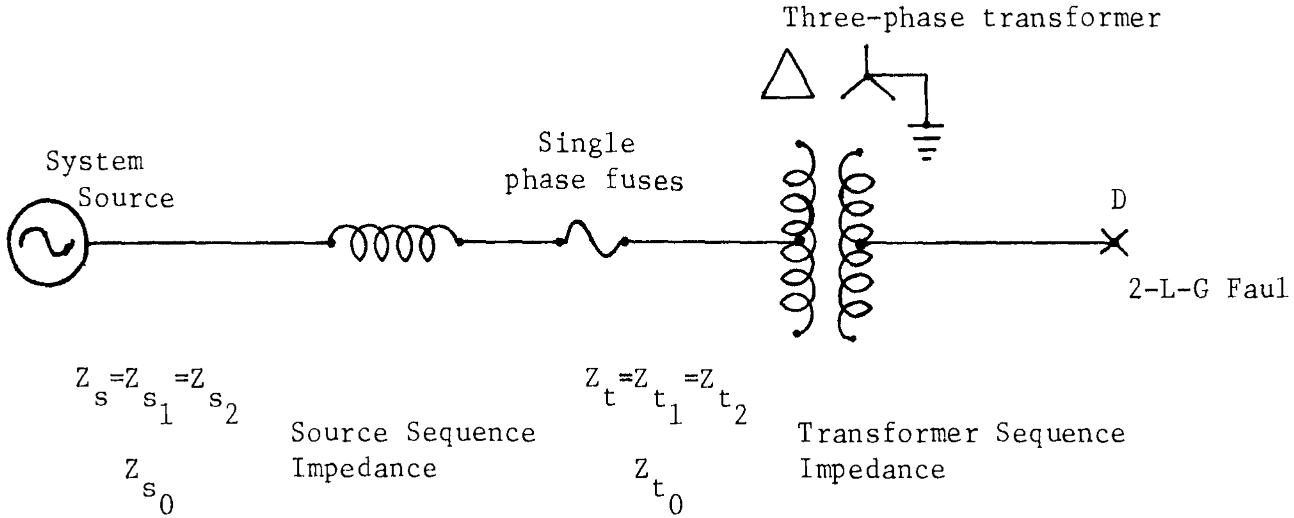
The distribution system of the Central Electric Power Cooperative, Jefferson City, can be represented by the one line diagram given in Figure 2. The positive sequence source impedance and the positive sequence transformer leakage reactance are equal to their negative sequence values respectively as per the system specifications. The problem will be solved in three stages as the various faults occur. Saturation and the magnetizing current have been neglected.

## A. Two Line to Ground Fault

At time $t = 0$, a two line to ground fault occurred on the secondary side of the distribution transformer on phases B and C at the point marked D in Figure 2. This portion of the problem is solved using symmetrical component theory as presented by Stevenson[10] in his book and detailed in Appendix A. We will apply Thevenin's theorem, which allows us to find the current in the fault by replacing the entire system by a single generator and series impedance, assuming the circuit to be lumped, linear and bilateral.

Now, let $I_A$, $I_B$, and $I_C$ be the currents flowing out of the original balanced system at the fault (D) from phases A, B, and C, respectively. The line-to-ground voltages at the fault will be designated $V_A$, $V_B$, and $V_C$. Before the fault occurs, the line-to-neutral voltage of phase A at the fault will be called $V_f$, which is a positive-sequence voltage since the system is assumed to be balanced before the fault.

Figure 3 shows the three sequence networks with the system impedances represented and the fault marked at point D. Since linearity has been assumed in drawing the sequence networks, each of the networks can be




$Z_{s_1}$ = Positive Sequence Source Impedance

$Z_{s_2}$ = Negative Sequence Source Impedance

$Z_{s_0}$ = Zero Sequence Source Impedance

$Z_{t_1}$ = Positive Sequence Transformer Leakage Impedance

$Z_{t_2}$ = Negative Sequence Transformer Leakage Impedance

$Z_{t_0}$ = Zero Sequence Transformer Leakage Impedance

Figure 2. Single Line Diagram

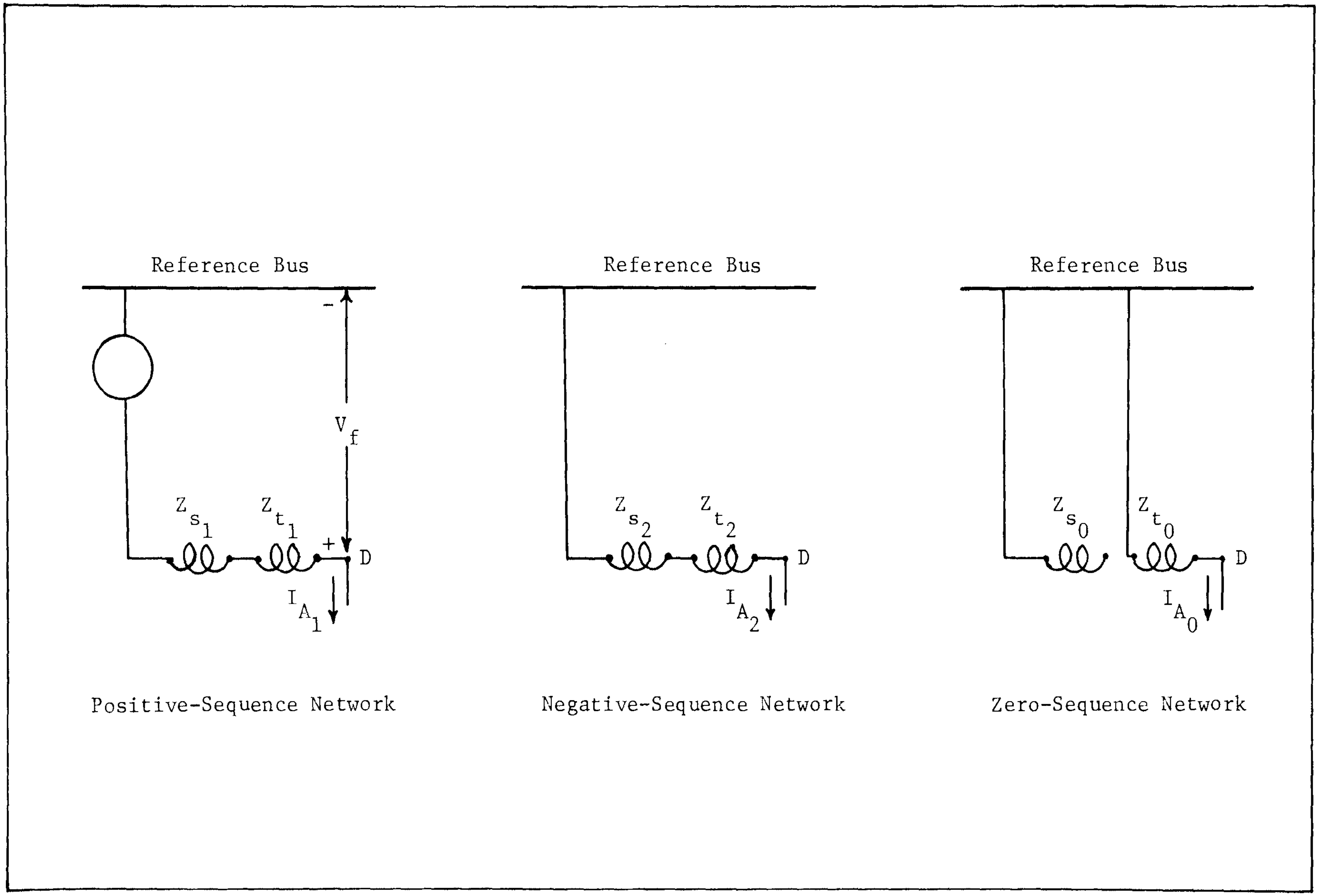



Figure 3. Sequence Networks

replaced by its Thevenin equivalent between the two terminals composed of its reference bus and the point of application of the fault. The Thevenin equivalent circuit of each sequence network has also been shown in Figure 4.

The internal voltage of the single generator of the equivalent circuit for the positive-sequence network is $V_f$, the prefault voltage to neutral at the point of application of the fault. The impedance $Z_1$ of the equivalent circuit is the impedance measured between point D and the reference bus of the positive-sequence network with all the internal emf's short circuited.

Since no negative- or zero-sequence currents are flowing before the fault occurs, the prefault voltage between point D and the reference bus is zero in the negative- and zero-sequence networks. Therefore, no emf's have been shown in the equivalent circuits of the negative- and zero-sequence networks. The impedances $Z_2$ and $Z_0$ are measured between point D and the reference bus in their respective networks.

Since $I_A$ is the current flowing from line A of the system into the fault, its components $I_{A_1}$, $I_{A_2}$, and $I_{A_0}$ flow out of their respective sequence networks and out of the equivalent circuits of the networks at D, as shown in Figure 3 and 4. Now the matrix equations for the symmetrical components of voltages at the fault can be written from the equivalent circuits as follows:

$$\begin{bmatrix} V_{A_0} \\ V_{A_1} \\ V_{A_2} \end{bmatrix} = \begin{bmatrix} 0 \\ V_f \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{A_0} \\ I_{A_1} \\ I_{A_2} \end{bmatrix} \qquad (3.1)$$

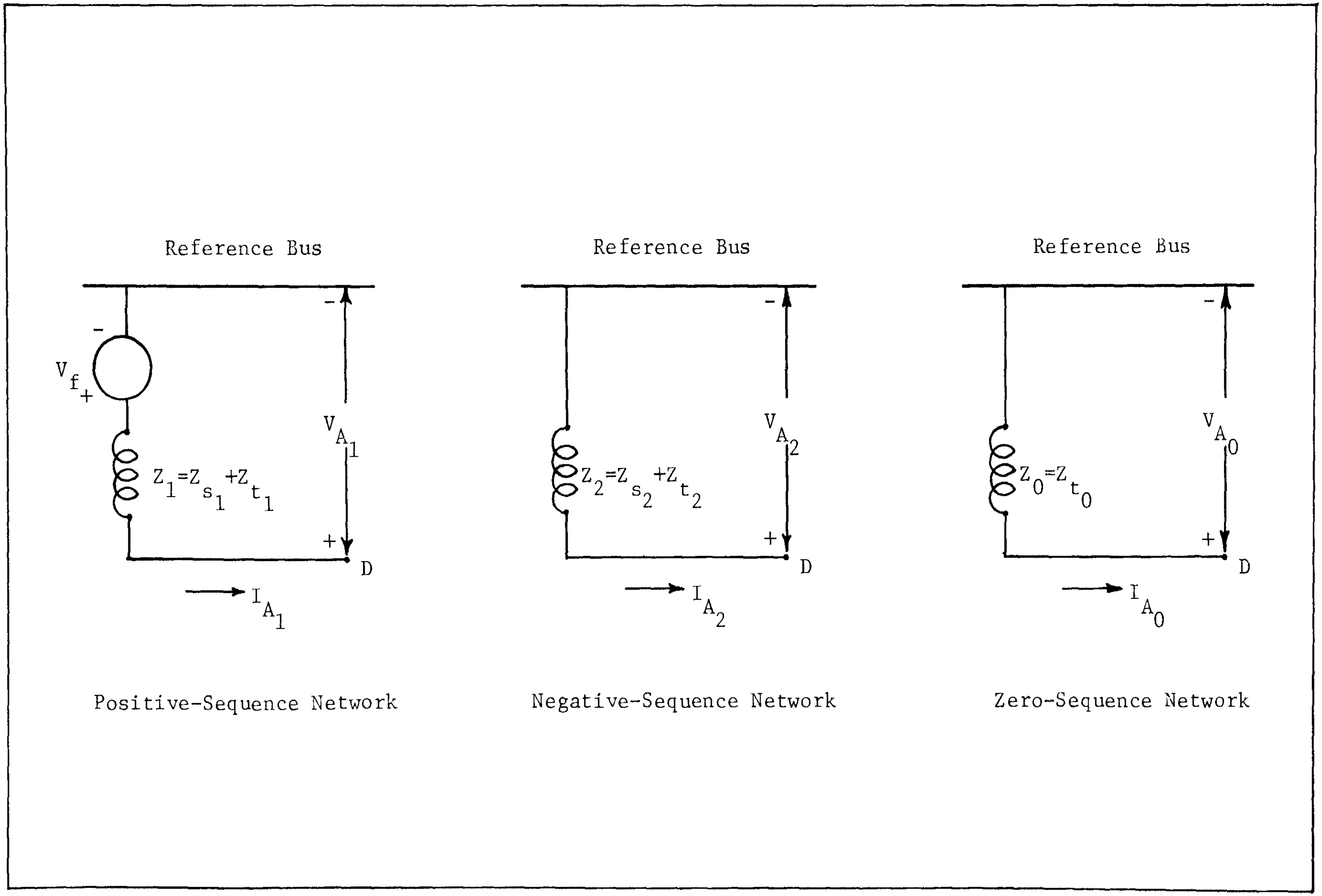



Figure 4. Thevenin Equivalent of Sequence Networks

or

$$V_{A_0} = -I_{A_0} Z_0$$

$$V_{A_1} = V_f - I_{A_1} Z_1$$

$$V_{A_2} = -I_{A_2} Z_2$$

Now, for a double line to ground fault (the faulted phases being B and C), the following relations exist at the fault:

$$V_B = V_C = 0$$

The contribution of current from phase A to the fault is $I_A = 0$ and the fault condition is represented as shown by the connection diagram of Figure 5.

Now, with $V_B = 0$ and $V_C = 0$, the symmetrical components of the voltage are given by

$$\begin{bmatrix} V_{A_0} \\ V_{A_1} \\ V_{A_2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_A \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} V_A \\ V_A \\ V_A \end{bmatrix} \tag{3.2}$$

therefore

$$V_{A_0} = V_{A_1} = V_{A_2} \tag{3.3}$$

Now, as $I_A = 0$

hence,

$$I_{A_0} + I_{A_1} + I_{A_2} = 0 \tag{3.4}$$

therefore,

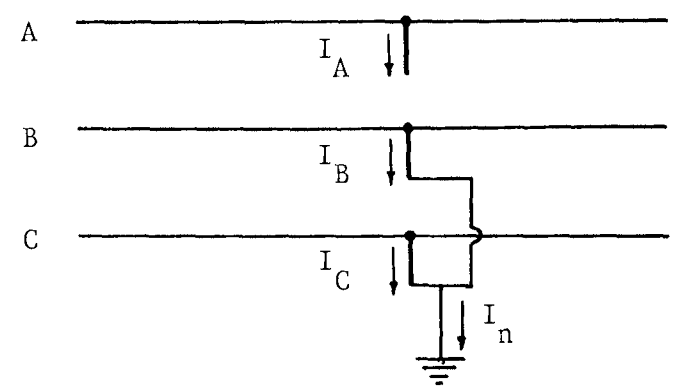



Figure 5. Hypothetical Stubs Diagram for a Double Line to Ground Fault

$$I_{A_1} = -(I_{A_0} + I_{A_2})$$

Now, from Equations (3.1) and (3.3),

$$I_{A_0} = -\frac{V_{A_0}}{Z_0} = -\frac{V_{A_1}}{Z_0}$$

$$I_{A_2} = -\frac{V_{A_2}}{Z_2} = -\frac{V_{A_1}}{Z_2}$$

Substituting these values in Equation (3.4) gives

$$I_{A_1} = -(I_{A_0} + I_{A_2}) = V_{A_1}\left(\frac{1}{Z_0} + \frac{1}{Z_2}\right) = V_{A_1}\frac{Z_0 + Z_2}{Z_0 Z_2}$$

Therefore,

$$V_{A_1} = I_{A_1}\frac{Z_0 Z_2}{Z_0 + Z_2}$$

Substituting this value of $V_{A_1}$ into Equation (3.1)

$$I_{A_1}\frac{Z_0 Z_2}{Z_0 + Z_2} = V_f - I_{A_1} Z_1$$

Therefore,

$$I_{A_1}\left(Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}\right) = V_f$$

and

$$I_{A_1} = \frac{V_f}{\left(Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}\right)} \qquad (3.5)$$

Equations (3.3) and (3.5) indicate that the three equivalent sequence networks should be connected in parallel at the fault point in order to simulate a double line to ground fault, and the same has been shown in Figure 6. The prefault current (load current), being very small, is not being taken into account here and has been neglected

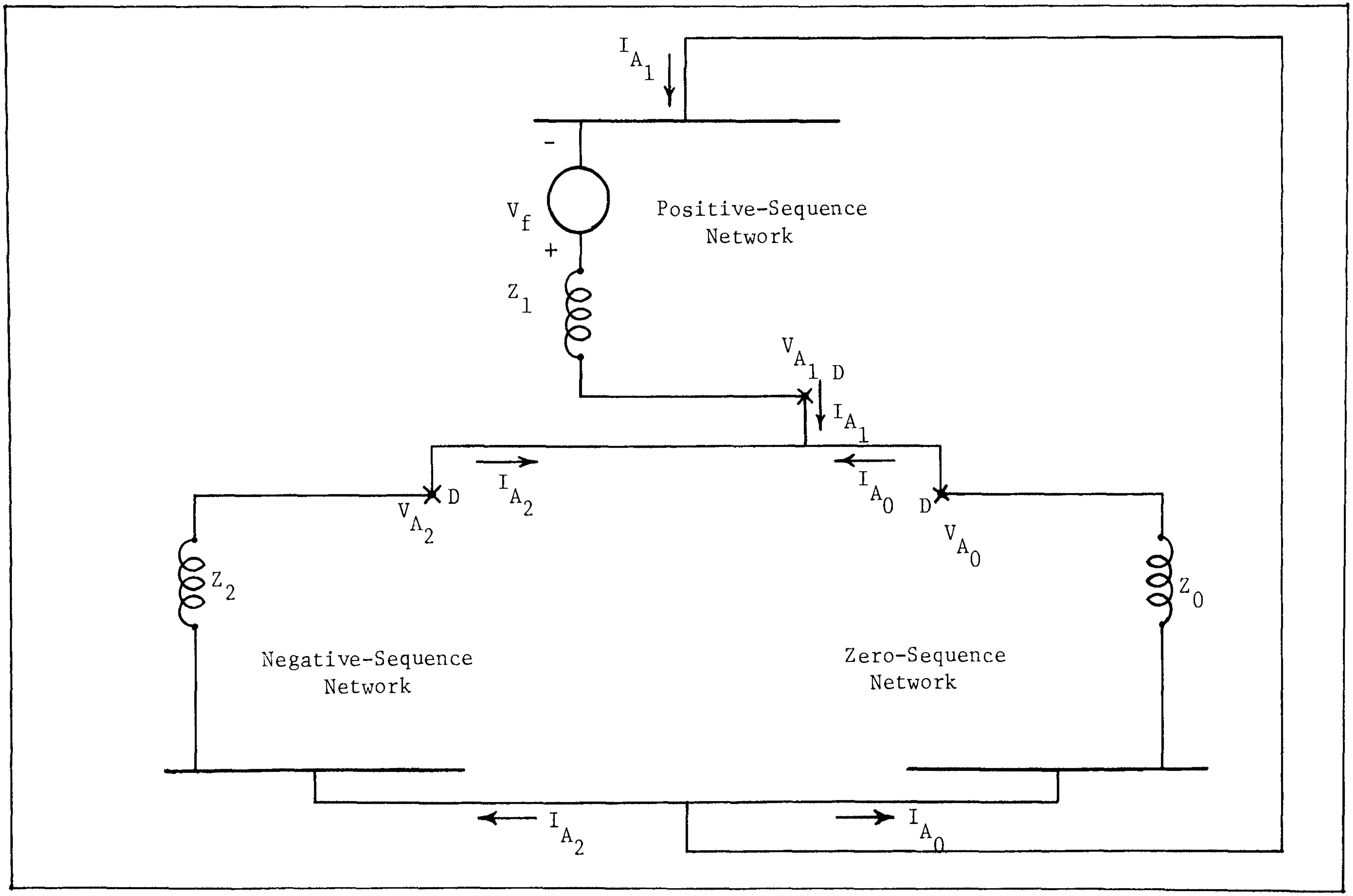



Figure 6. Connection of Sequence Networks for a Double Line to Ground Fault at D

altogether as it will have negligible effect on the fault current.

Now, knowing $V_f$, $Z_0$, $Z_1$, and $Z_2$ in per unit values, the positive sequence current $I_{A_1}$ can be calculated from Equation (3.5) and then $I_{A_2}$, $I_{A_0}$, $V_{A_1}$, $V_{A_2}$, and $V_{A_0}$ can all be found from various equations already derived. From the sequence values, the currents and voltages in all three phases can be found on the secondary side of the transformer by the symmetrical components relations given in Appendix A. Once the transformer secondary values of current and voltages are known, the primary currents and voltages can be calculated by taking the phase shift of the transformer into account.

With the primary phase a current known, the time ($t_1$) for phase a fuse to blow and clear can be determined. The phase a fuse blows earlier than the other two fuses of phases b and c because the current, $I_a$, is greater than $I_b$ and $I_c$. Once the fuse of phase a blows on the primary side of the transformer, there exists an open conductor on phase a. The system with an open conductor on phase a and a double line to ground fault on phases B and C presents a double unbalanced fault condition.

B. Simultaneous Faults on a Grounded Transformer

Now the system, having two faults, namely a two line-to-ground fault on the secondary side of the transformer and an open conductor on the primary side of the transformer, presents a condition with simultaneous faults. This simultaneous fault is a combination of a short circuit involving ground and an open conductor. Each exists on the opposite side of a distribution transformer, creating a phase shift problem because of the delta-wye connection of the transformer. Since each fault affects the voltages and currents resulting from the other, the simultaneous faults cannot be treated separately as presented earlier. In order to solve this problem, the phase shift of the delta-wye transformer must be taken into account.

A delta-wye transformer bank usually divides the zero sequence system into two parts which have no connection with each other in the zero sequence network. However, the positive and negative sequence equivalent circuits for three-phase power systems are based on equivalent wye-wye transformer connections. As such, the zero sequence quantities are unaffected on either side of the transformer but the positive and negative sequence quantities thus obtained are based on a wye-wye connection. Hence, a phase correction is needed for both positive and negative sequence quantities in order to get correct values based on the actual delta-wye connection of the intervening transformer.

To find the phase correction, let the transformer connection be given as in Figure 7. Let the secondary circuit of the transformer with phases A, B, and C be chosen as the reference circuit, and phase A as the reference phase. In the primary circuit, let the reference phase be designated a, and so chosen that the line-to-neutral voltage $V_a$ at no load without any fault is 90 degrees out of phase with the line to neutral voltage $V_A$. From the Figure, it is seen that $V_a$ lags $V_A$ by 90 degrees for the connection shown. Hence, the positive and negative sequence quantities of current and voltage based on equivalent wye-wye transformer connections with the secondary circuit as the reference, are multiplied by -j and j, respectively, in order to get the actual values of the positive and negative sequence quantities. The

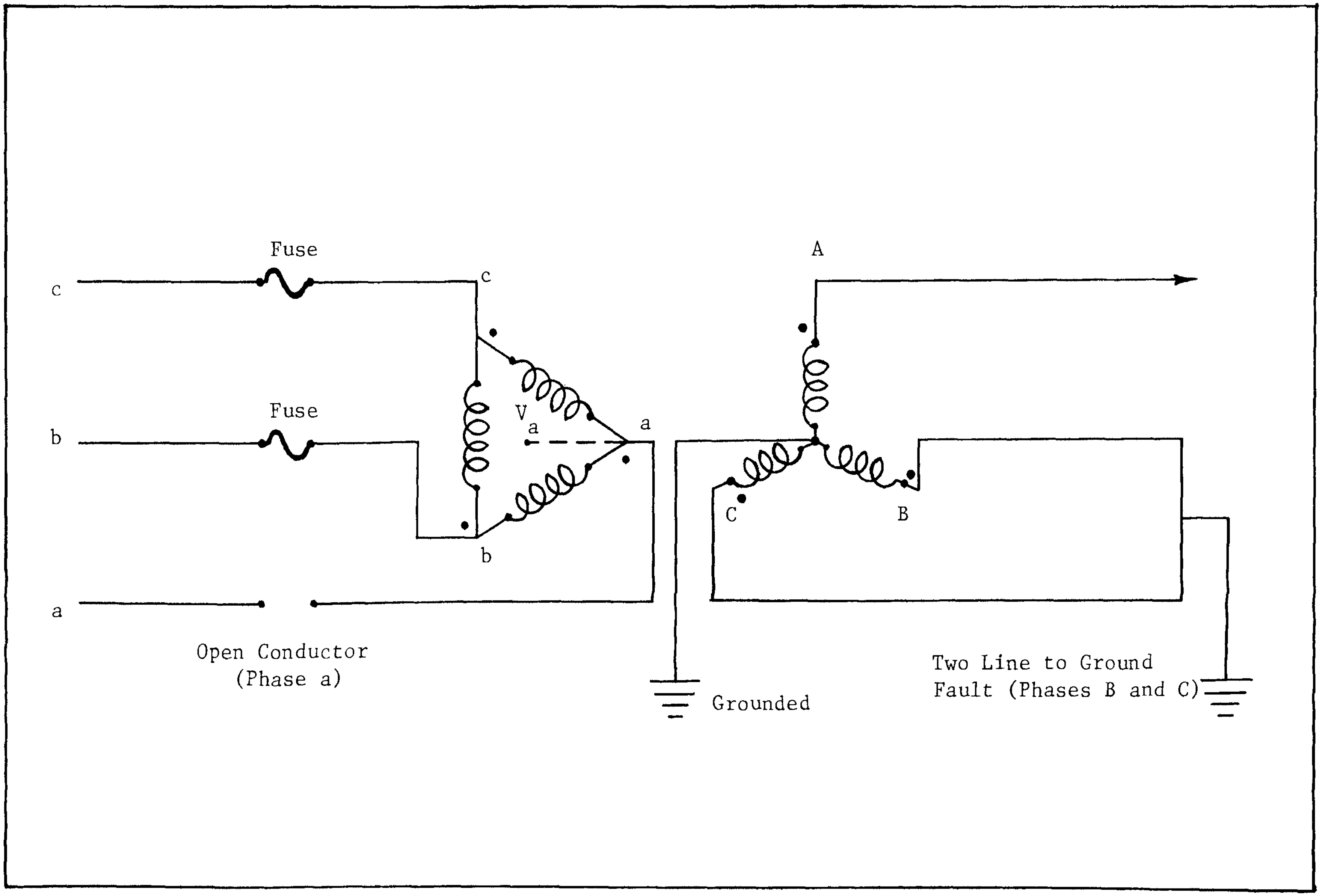



Figure 7. Phase Shift with a Delta-Wye Transformer

phase correction is thereby obtained for the positive and negative sequence quantities based on the delta-wye connection.

Now, in order to solve the double unbalanced fault condition the theory of Alpha, Beta, and Zero components of three phase system is applied and the same has been presented in Appendix B. These components can be easily applied as long as the positive- and negative-sequence impedances of the rotating machines are equal, which is true in the system under study. Before applying the theory of Alpha, Beta, and Zero components to the simultaneous faults, each fault condition will be discussed separately using the Alpha, Beta, and Zero theory. For a double line to ground on phases B and C, conditions at the fault as dealt earlier, are

$$V_B = V_C = 0 \; ; \quad I_A = 0$$

Now, from Appendix B, certain relations between line currents and phase voltages and their Alpha, Beta, and Zero components exist. All the equations have been derived in detail in Appendix B. The same numbers will be used for each equation as in Appendix B.

$$V_A = V_\alpha + V_0 \tag{B.1}$$

$$V_\alpha = \frac{2}{3} \left(V_A - \frac{V_B + V_C}{2}\right) \tag{B.4}$$

$$V_\beta = \frac{1}{\sqrt{3}} (V_B - V_C) \tag{B.5}$$

and

$$I_A = I_\alpha + I_0 \tag{B.7}$$

Applying the above fault conditions to these equations, the equations satisfying a two line to ground fault can be obtained. These are, with

$V_B = V_C = 0$ from Equation (B.4)

$$V_\alpha = \frac{2}{3} V_A$$

or

$$V_A = \frac{3}{2} V_\alpha$$

Now, from Equation (B.1), we have

$$\frac{3}{2} V_\alpha = V_\alpha + V_0$$

or

$$\frac{1}{2} V_\alpha = V_0$$

or

$$V_\alpha = 2V_0 \qquad (3.6)$$

Also, from Equation (B.5), with $V_B = V_C = 0$,

$$V_\beta = 0 \qquad (3.7)$$

and, from Equation (B.7), with $I_A = 0$,

$$I_\alpha = -I_0 \qquad (3.8)$$

In the circuit under study, the positive- and negative-sequence impedances are equal and, as such, the Alpha, Beta and Zero impedances are available from the sequence impedances,

$$Z_\alpha = Z_1$$

$$Z_\beta = Z_2 = Z_1$$

$$Z_0 = Z_0$$

The connection of the Alpha, Beta, and Zero networks that satisfy the fault condition equations, is shown in Figure 8. Here, $V_f$, is the prefault voltage of phase A to ground. In order to have a zero-sequence current equal to the Alpha current, all the zero-sequence impedances must be multiplied by two in the zero-sequence network.

The theory of Alpha, Beta, and Zero components is also applied to the open conductor on phase a on the primary side of the transformer. Figure 9 shows the configuration for one open conductor. Here let v be the series voltage drop and I' be the line current through the series impedance. Now the relations between $v_\alpha$, $v_\beta$, $v_0$, the components of the series voltage drop, and between $I_\alpha'$, $I_\beta'$, and $I_0'$, the components of line current, will be developed considering the fault conditions.

The conditions at the fault are

$$v_b = v_c = 0$$

and $$I_a' = 0$$

Now, from Appendix B, the various relations among the components and the phase values are given by the following equations:

$$v_a = v_\alpha + v_0 \tag{B.1}$$

$$v_\alpha = \frac{2}{3} \left(v_a - \frac{v_b + v_c}{2}\right) \tag{B.4}$$

$$v_\beta = \frac{1}{\sqrt{3}} (v_b - v_c) \tag{B.5}$$

and $$I_a' = I_\alpha' + I_0' \tag{B.7}$$

Now as $v_b = v_c = 0$

Therefore, from Equation (B.4),

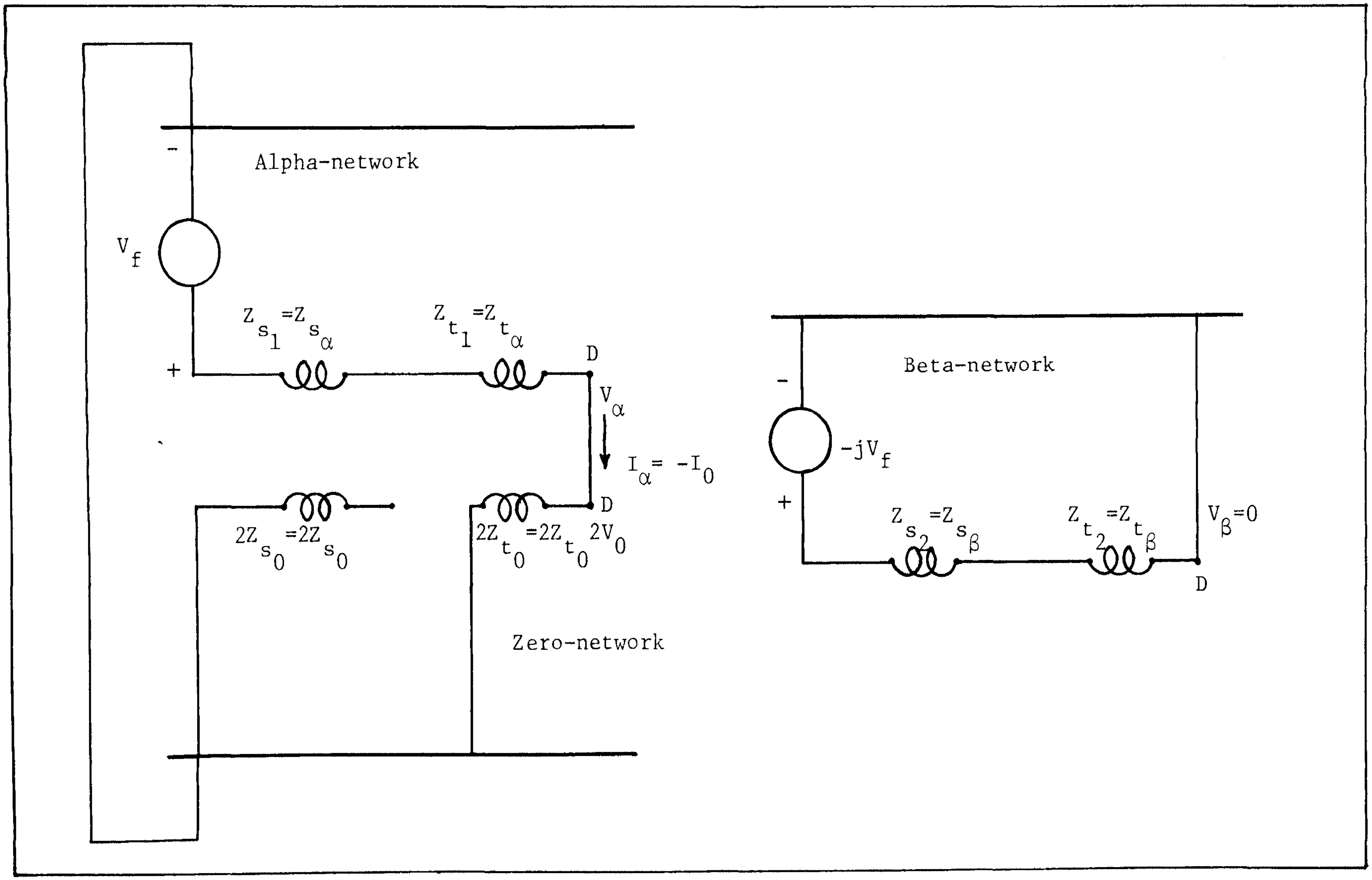



Figure 8. Direct Connections of the Alpha, Beta and Zero Networks for a Two Line to Ground Fault

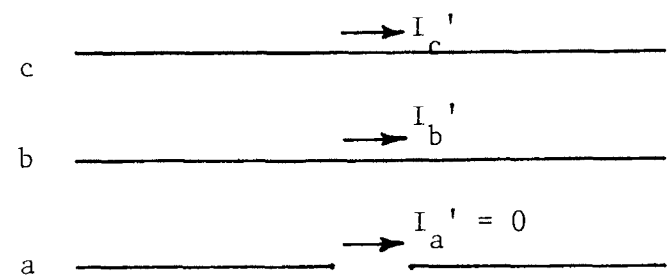



Figure 9. One Open Conductor (Phase a)

$$V_\alpha = \frac{2}{3} v_a$$

or $$v_a = \frac{3}{2} v_\alpha$$

Now, from Equation (B.1)

$$\frac{3}{2} v_\alpha = v_\alpha + v_o$$

or $$v_\alpha = 2v_o \qquad (3.9)$$

Also from Equation (B.5),

$$v_\beta = 0 \qquad (3.10)$$

and from Equation (B.7), with $I_a' = 0$:

$$I_\alpha' = -I_o' \qquad (3.11)$$

Direct connection of Alpha and Zero networks is made possible by multiplying the zero impedances by two as shown in Figure 10. In this figure, the currents in the zero network are zero but the voltages are twice the zero voltages. The Beta network is unaffected by one open conductor on phase a.

The relations between the Alpha, Beta and Zero components of voltage and current in terms of symmetrical components of the voltage and current are also given in Appendix B.

Now the relations derived are

$$V_\alpha = V_{a_1} + V_{a_2}$$

$$V_\beta = -j(V_{a_1} - V_{a_2})$$

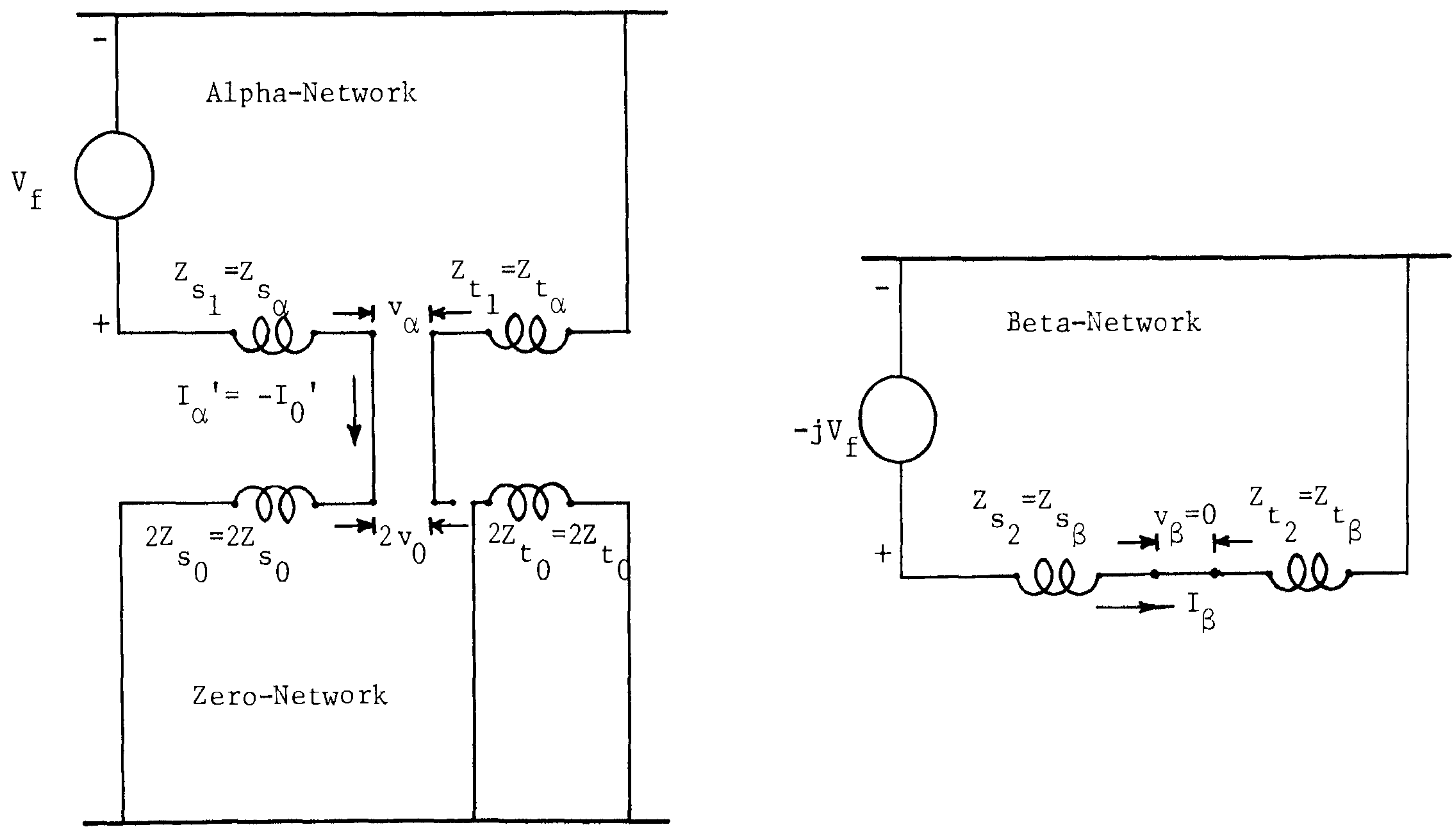



Figure 10. Connection of Alpha, Beta and Zero Equivalent Circuits for Open Conductor in Phase a

$$V_0 = V_{a_o}$$

$$I_\alpha = I_{a_1} + I_{a_2}$$

$$I_\beta = -j(I_{a_1} - I_{a_2})$$

$$I_0 = I_{a_o}$$

As earlier discussed, since a delta-wye transformer is involved in the system, a phase correction must be applied by turning the positive-sequence components of line current and voltage to neutral backward 90 degrees and the negative sequence components of current and voltage forward 90 degrees in passing through the transformer bank,

$$V_\alpha = (V_{a_1} + V_{a_2}) \text{ becomes } -j(V_{a_1} - V_{a_2}) = V_\beta''$$

$$V_\beta = -j(V_{a_1} - V_{a_2}) \text{ becomes } -(V_{a_1} + V_{a_2}) = -V_\alpha''$$

Similarly, the corresponding currents are

$$I_\alpha = (I_{a_1} + I_{a_2}) \text{ becomes } -j(I_{a_1} - I_{a_2}) = I_\beta''$$

$$I_\beta = -j(I_{a_1} - I_{a_2}) \text{ becomes } -(I_{a_1} + I_{a_2}) = -I_\alpha''$$

In order to apply phase correction to the present problem of simultaneous faults on opposite sides of a delta-wye transformer, let the location of the double line to ground fault on the secondary side be represented by D and that of an open conductor on the primary side by C as shown in Figure 11 and let the circuit at D be assumed to be the reference circuit.

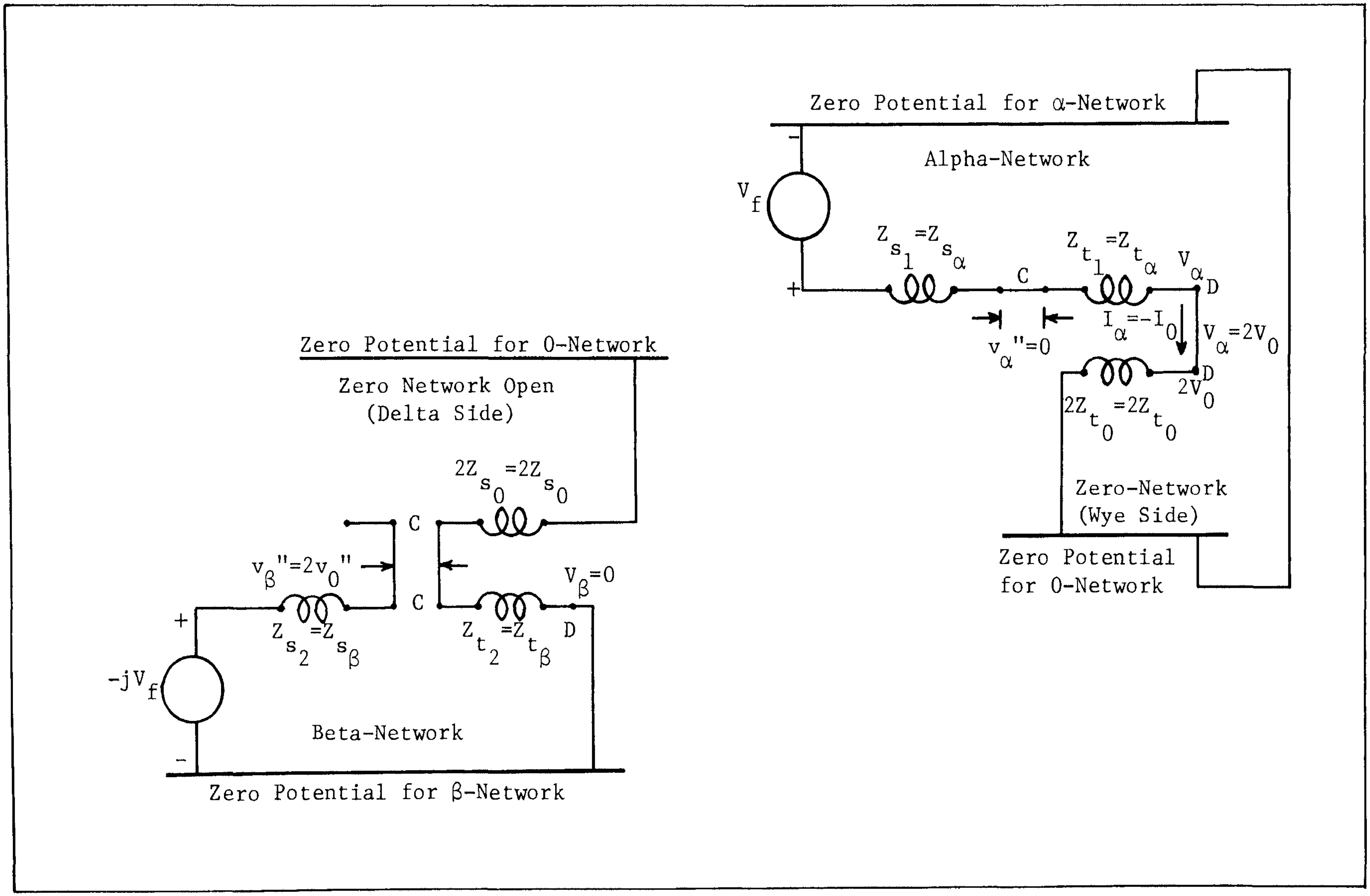



Figure 11. Equivalent Circuit for an Open Conductor at C and Two Line to Ground Fault at D with Grounded Transformer

For an open conductor at C, the existing relations as derived earlier are Equations (3.9), (3.10), and (3.11). For a two line to ground fault at D, the derived relations are Equations (3.6), (3.7), and (3.8).

Let the double primed symbols indicate the values of the components of the line current and series voltages at C (for an open conductor) referred to the circuit at D (secondary side of a delta-wye transformer), then applying the phase correction, as given earlier, to these values at C,

$$v_\alpha = v_\beta'' \quad , \quad I_\alpha' = I_\beta''$$

$$v_\beta = -v_\alpha'' \quad , \quad I_\beta' = -I_\alpha''$$

Substituting these values into Equations (3.9), (3.10), and (3.11),

$$v_\beta'' = 2v_o'' \tag{3.12}$$

$$v_\alpha'' = 0 \tag{3.13}$$

$$I_\beta'' = -I_o'' \tag{3.14}$$

Now, the Equations (3.6), (3.7), (3.8), (3.12), (3.13), and (3.14) describe the simultaneous faults based on an equivalent wye-wye transformer bank and Figure 11 is the equivalent circuit used for analytic calculations which satisfies all these equations. Because of infinite series impedances in the zero sequence network at C, the Zero and Beta networks are both open at C. The Alpha network is closed at C, since $v_\alpha'' = 0$.

As there is only one fault connection between the point D in the Alpha network and the zero-potential bus for the network, the generated voltage in the network can be replaced by the prefault voltage $V_f$ in phase a at the fault point before the dissemmetry occurred and the various components of the current and the voltages are calculated. The phase values are then obtained by using the relations of Appendix B.

The component impedances are equal to their sequence counterparts and have been used accordingly in Figure 11. Now, with the method developed for calculating the current, the current through the neutral wire of the transformer can be calculated and the time ($t_2$) in seconds for the ground wire to melt can finally be computed using available wire characteristics. The system is now an ungrounded delta-wye transformer which must now be treated differently.

C. Simultaneous Faults with Ungrounded Transformer

The system is still confronted with double faults, an open conductor in the primary circuit and a two line-to-ground fault in the secondary circuit of the delta-wye transformer. But, with the secondary neutral wire burned through, the transformer is ungrounded. The problem is again solved using Alpha, Beta and Zero components and the six equations (3.6), (3.7), (3.8), (3.12), (3.13), and (3.14). The equivalent circuit of Figure 11 changes to that of Figure 12 which shows the infinite impedance in the zero network, because of the melted neutral wire of the transformer. As no current can flow through the neutral wire,

$$I_0 = 0$$

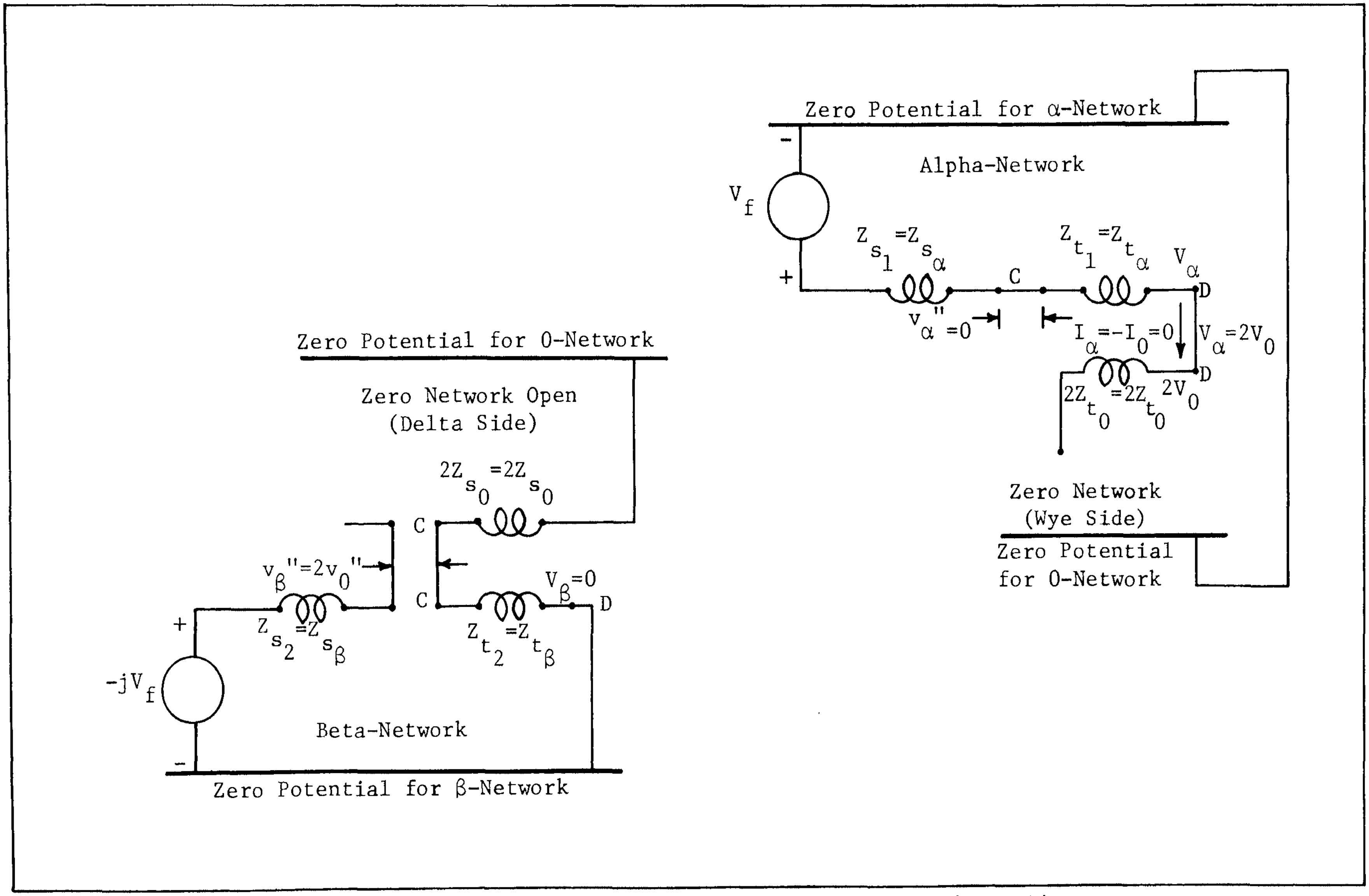



Figure 12. Equivalent Circuit for an Open Conductor at C and Two Line to Ground Fault at D with Ungrounded Transformer

Hence, from Equation (3.8),

$$I_\alpha = -I_0 = 0$$

Therefore, the voltage of the alpha network $V_\alpha = V_f$ the prefault voltage. Also from Equation (3.6),

$$V_\alpha = 2V_o$$

and from Appendix B, Equation (B.1)

$$V_A = V_\alpha + V_o = \frac{3}{2} V_\alpha = \frac{3}{2} V_f$$

It is evident from the above relation that the voltage of phase A increases abnormally with the burning of the neutral wire and this abnormal voltage of phase A can cause damage to the customers' electrical equipment.

# IV. APPLICATION OF THE MATHEMATICAL MODEL TO THE CENTRAL ELECTRIC SYSTEM

The system data was supplied by the Central Electric Power Cooperative, Jefferson City, Missouri for their distribution system and is shown in Table I. The problem is solved using the method developed in Chapter III. The magnetizing current and the saturation effects have been neglected. The load data, as given in the Table I, is estimated and it does not involve any rotating machinery. As such, the load current will have little effect on the total line current during the fault and has been neglected in our fault calculations.

The load is assumed to be at the base voltage, hence, the voltage of phase A before the fault is 1.00 p.u. Therefore, the base current for the secondary circuit is

$$I_{s.\ Base} = \frac{3750}{\sqrt{3} \times 13.2} = 164 \text{ amps.}$$

and the base current for the Primary circuit is

$$I_{P.\ Base} = \frac{3750}{\sqrt{3} \times 67} = 32.30 \text{ amps.}$$

Now, for a two line to ground fault (on phases B and C) in the secondary circuit, the positive sequence current $I_{A_1}$ of phase A is given by Equation (3.5)

$$I_{A_1} = \frac{V_f}{Z_1 + \dfrac{Z_0 Z_2}{Z_0 + Z_2}}$$

Now, $V_f$ = 1.00 p.u. (Pre-fault Voltage)

TABLE I. System Data

| S. No. | System Components | Data | Remarks |
|---|---|---|---|
| (i) | Voltage Rating of the Transformer | 67-13.2 KV | Delta-Wye with solidly grounded secondary. Voltage rating selected as the base |
| (ii) | KVA Rating of the Transformer | 3750 KVA | Selected as base |
| (iii) | System Base KVA | 3750 KVA | |
| (iv) | Leakage Reactance of the transformer | $Zt_1=Zt_2=Zt=0.0722$p.u.<br>$Zt_0=0.070$p.u. | Based on System Base KVA |
| (v) | System Impedance & Voltage | $Z_{s_1}=Z_{s_2}=Z_s=0.0269$p.u.<br>$Z_{s_0}$ is not a factor<br>V=1p.u. | Based on System Base KVA |
| (vi) | Transformer load at fault inception, load voltage and power factor | 1500 KW, V=1 p.u.<br>0.93 power factor | Estimated values, neglected in fault calculations. |
| (vii) | Type & characteristic of the fuses used on the primary side | P.E.-40E fuse kits | Total clearing time in seconds versus current in amperes characteristic shown in Figure 14. |
| (viii) | Size and type of ground wire used with the transformer secondary winding | 70.7 MCM Cu-wire | Total melting time in seconds against the current in amperes characteristic given in Figure 13. |

$$Z_1 = Z_{s_1} + Z_{t_1} = j\ 0.0991 \text{ p.u.}$$

$$Z_2 = Z_{s_2} + Z_{t_2} = j\ 0.0991 \text{ p.u.}$$

$$Z_0 = Z_{t_0} = j\ 0.070 \text{ p.u.}$$

$$I_{A_1} = -j\ 7.14 \text{ p.u.}$$

and

$$I_{A_2} = -I_{A_1} \times \frac{Z_0}{Z_0 + Z_2}$$

or

$$I_{A_2} = j\ 2.955 \text{ p.u.}$$

Similarly

$$I_{A_0} = -I_{A_1} \times \frac{Z_2}{Z_0 + Z_2}$$

or

$$I_{A_0} = j\ 4.185 \text{ p.u.}$$

Now the current in line A,

$$I_A = I_{A_0} + I_{A_1} + I_{A_2}$$

$$I_A = 0 \text{ p.u.}$$

The current magnitude through line A,

$$|I_A| = 0 \times 164 = 0 \text{ amps.}$$

The current in Line B,

$$I_B = I_{A_0} + a^2 I_{A_1} + aI_{A_2}$$

or

$$I_B = -8.735 + j\,6.278$$

Therefore

$$I_B = 10.75 \angle 144.3^o \text{ p.u.}$$

The current magnitude through line B,

$$|I_B| = 10.75 \times 164 = 1760 \text{ amps.}$$

Similarly the current in Line C,

$$I_C = I_{A_0} + aI_{A_1} + a^2 I_{A_2}$$

or

$$I_C = 8.735 + j\,6.278$$

Therefore $I_C = 10.75 \angle 35.7^o$ p.u.

The current magnitude through line C,

$$|I_C| = 10.75 \times 164 = 1760 \text{ amps.}$$

The current through the neutral of the transformer secondary winding,

$$I_{n_1} = 3I_{A_0} = j\,12.555 \text{ p.u.}$$

The current magnitude through the grounding wire is,

$$|I_{n_1}| = 12.555 \times 164 = 2060 \text{ amps.}$$

From the characteristics of the grounding wire in Figure 13, the melting time $t_2' = 24$ seconds for this value of current.

The sequence voltages at the fault are

$$|V_{A_1}| = |V_{A_2}| = |V_{A_0}| = |I_{A_1}| \times \left| \frac{Z_0 \; Z_2}{Z_0 + Z_2} \right| = 0.2933 \text{ p.u.}$$

The phase voltages to ground at the fault are:

$$|V_A| = |V_{A_0} + V_{A_1} + V_{A_2}| = 3|V_{A_0}|$$

or

$$|V_A| = 0.8799 \text{ p.u.}$$

$$V_B = V_C = 0$$

In order to account for phase shift, the positive and the negative sequence components of the line current and the phase voltages on the system side of the Delta-Wye transformer are shifted $-90^o$ and $+90^o$ respectively from the corresponding components on the load side. As such, on the system side,

$$I_{a_1} = -j \, (I_{A_1}) = 7.140 \text{ p.u.}$$

and

$$I_{a_2} = j \, (I_{A_2}) = 2.955 \text{ p.u.}$$

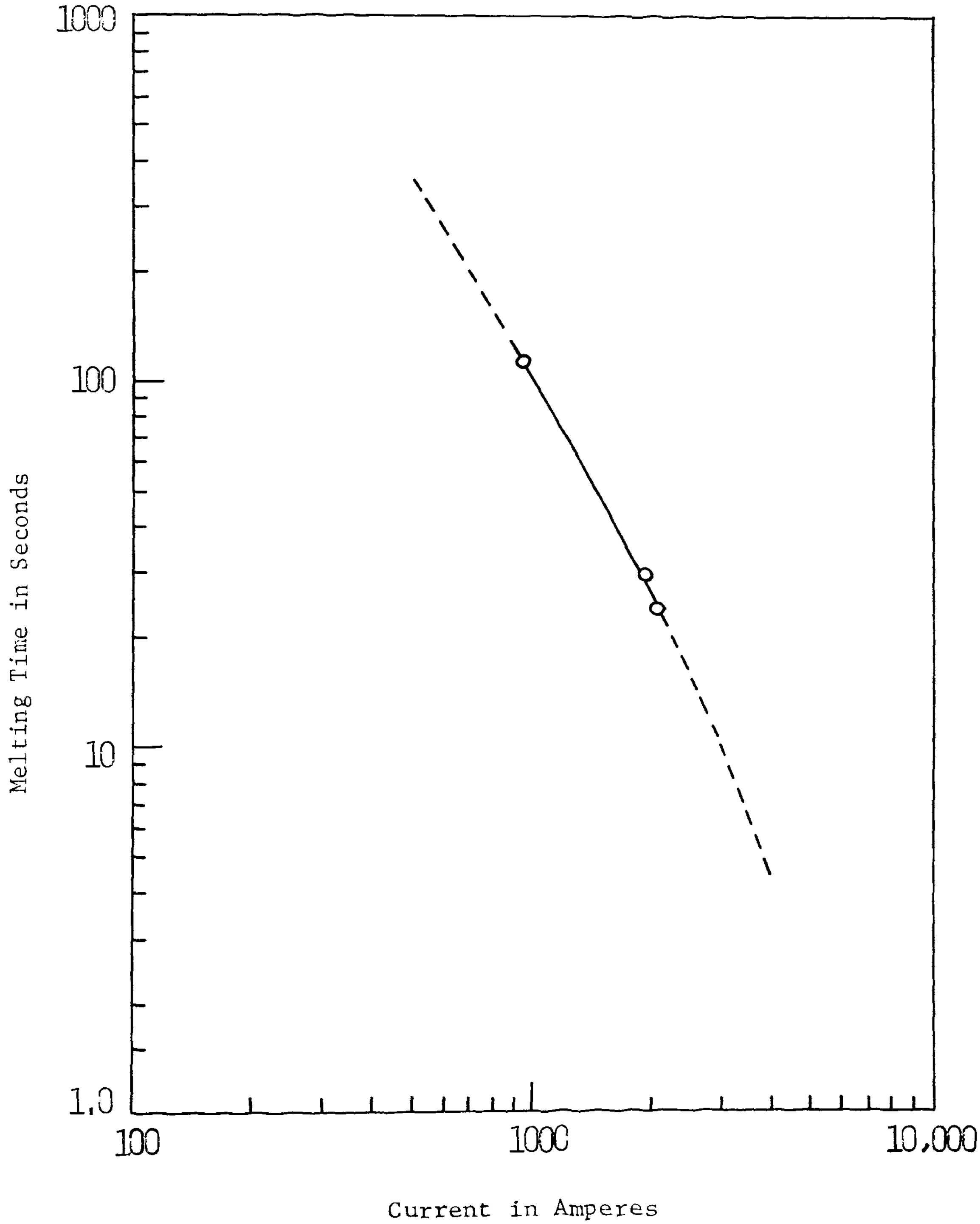



Figure 13. Grounding Wire Characteristics

$I_{a_0} = 0$ (since there is no zero sequence current)

The current through line a on the primary side is

$$I_a = I_{a_0} + I_{a_1} + I_{a_2}$$

or

$$I_a = 10.0\ 95 \angle 0^\circ \text{ p.u.}$$

The current magnitude through line a on the primary side,

$$|I_a| = 10.095 \times 32.30 = 325 \text{ amps.}$$

From the characteristics of the fuse in Figure 14, the time $t_1 = 1.70$ seconds. This time is for total clearing and includes the melting period of the fuse in line a.

Now the currents in phases b and c are calculated as follows:

$$I_{b_1} = a^2 I_{a_1} = -3.57 - j\ 6.180 \text{ p.u.}$$

$$I_{b_2} = aI_{a_2} = -1.4775 + j\ 2.555 \text{ p.u.}$$

$$I_{b_0} = 0 \text{ p.u.}$$

The current through line b on the primary side is:

$$I_b = I_{b_0} + I_{b_1} + I_{b_2}$$

or

$$I_b = -5.0475 - j\ 3.625$$

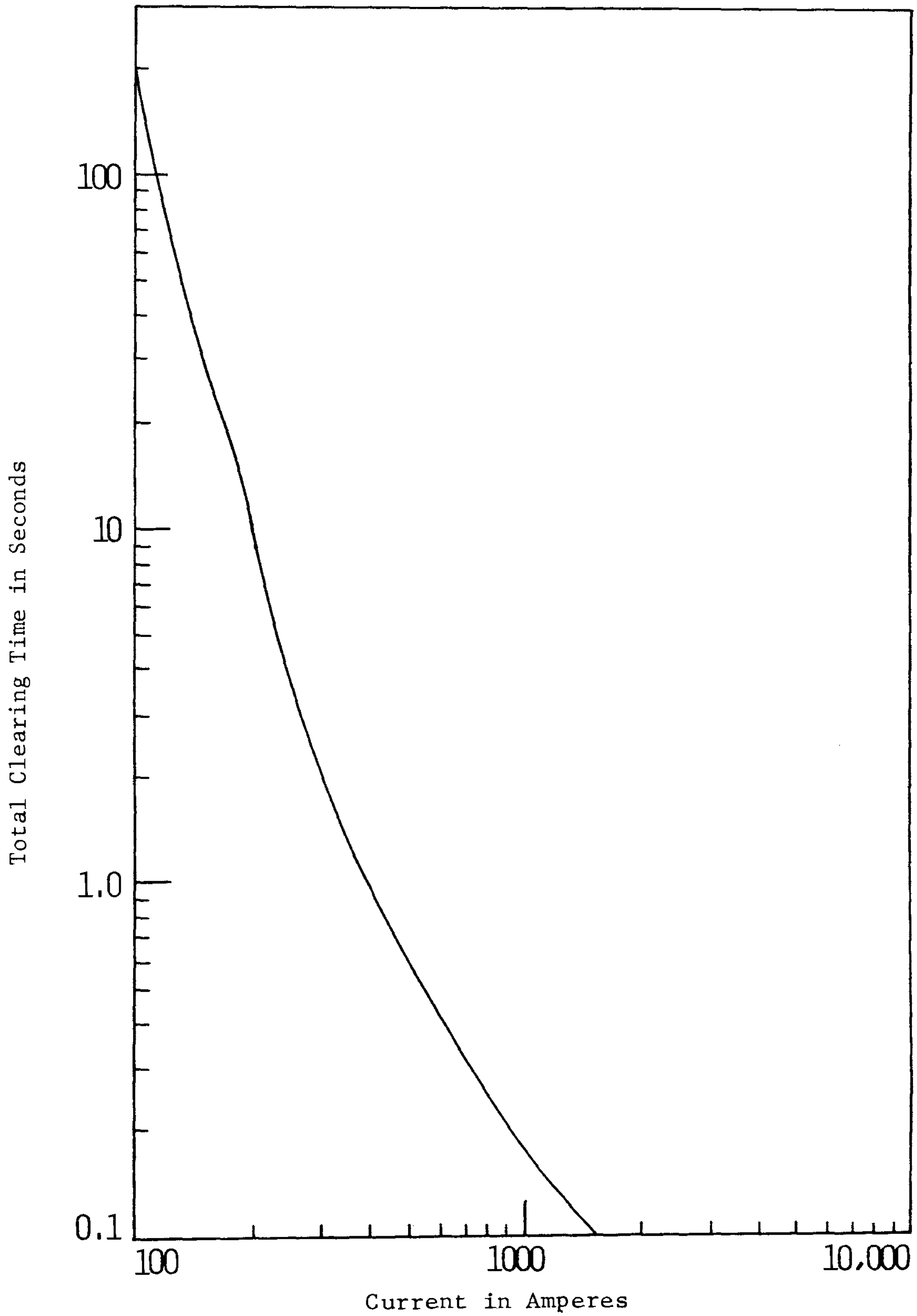



Figure 14. Fuse PE - 40 Characteristics

Therefore

$$I_b = 6.22 \angle 215.7^{o} \text{ p.u.}$$

Now the current magnitude through line b on the primary side is

$$|I_b| = 6.22 \times 32.30 = 201 \text{ amps.}$$

Now for line c:

$$I_{c_1} = aI_{a_1} = -3.57 + j\ 6.180 \text{ p.u.}$$

$$I_{c_2} = a^2 I_{a_2} = -1.4775 - j\ 2.555 \text{ p.u.}$$

$$I_{c_0} = 0 \text{ p.u.}$$

The current through line c on the primary side is:

$$I_c = I_{c_0} + I_{c_1} + I_{c_2}$$

$$I_c = -5.0475 + j\ 3.625 \text{ p.u.}$$

Therefore $I_c = 6.22 \angle 144.3^{o}$ p.u.

The current magnitude through line c on the primary side is,

$$|I_c| = 6.22 \times 32.30 = 201 \text{ amps.}$$

A check on the primary side of the transformer is that $(I_a + I_b + I_c)$ must be equal to zero. It was found to check properly.

The voltage components on the primary side can be calculated from the currents on the primary side and the system reactances of the sequence networks. Now

$$V_{a_1} = V_f - I_{a_1} Z_{s_1}$$

or

$$V_{a_1} = 1.00 - j\,0.2715 \text{ p.u.}$$

Similarly

$$V_{a_2} = - I_{a_2} Z_{s_2}$$

$$V_{a_2} = j\,0.0795 \text{ p.u.}$$

and

$$V_{a_0} = - I_{a_0} Z_{s_0}$$

$$V_{a_0} = 0 \text{ (since } I_{a_0} = 0)$$

The voltage of line a to ground

$$V_a = V_{a_1} + V_{a_2} = 1.00 - j\,0.1920 \text{ p.u.}$$

Therefore

$$V_a = 1.02 \angle{-10.9^{\circ}} \text{ p.u.}$$

The voltage of line b to ground:

$$V_b = a^2 V_{a_1} + aV_{a_2}$$

or

$$V_b = -\ 0.8038 - j\ 0.77$$

Therefore

$$V_b = 1.11 \angle 223.8^\circ \text{ p.u.}$$

And the voltage of line c to ground:

$$V_c = aV_{a_1} + a^2 V_{a_2}$$

or

$$V_c = -\ 0.1962 + j\ 0.962$$

Therefore

$$V_c = 0.982 \angle 101.5^\circ \text{ p.u.}$$

Thus all the voltages and currents are known for a two line to ground fault (phases B and C) on the secondary side of the transformer.

Now after $t_1$ = 1.70 seconds, the fuse of phase a is blown and thus creates an open conductor situation in phase a. With a double line to ground fault still existing and an open conductor, a simultaneous fault developes which will be solved using the theory of Alpha, Beta and Zero components (given in Appendix B) and the equivalent circuit of Figure 11 developed in the last chapter. As the time $t_2' = 24$ seconds has still to elapse, the grounding wire of the transformer still exists and the transformer is still grounded. However the figure $t_2' = 24$ is meaningless since the current through the grounding wire of the transformer will now change.

From Figure 11 on page 27,

$$I_\alpha = \frac{V_f}{Z_\alpha + 2Z_0}$$

or

$$I_\alpha = \frac{V_f}{Z_{s_\alpha} + Z_{t_\alpha} + 2Z_{t_0}}$$

$$Z_\alpha = Z_{s_\alpha} + Z_{t_\alpha} = Z_{s_1} + Z_{t_1} = Z_1$$

and

$$2Z_0 = 2Z_{t_0} = 2Z_{t_0} = 2Z_0$$

Substituting these values in the above expression,

$$I_\alpha = -j\ 4.18 \text{ p.u.}$$

For a two line to ground fault,

$$I_0 = -\ I_\alpha = j\ 4.18 \text{ p.u.}$$

Since the Beta network in Figure 11 is open, there will be no Beta current.

$$I_\beta = 0$$

From Appendix B, the current in line A on the secondary side:

$$I_A = I_\alpha + I_0$$

Therefore

$$I_A = 0 \text{ p.u.}$$

The current magnitude in line A,

$$|I_A| = 0 \times 164 = 0 \text{ amps.}$$

Similarly the current in line B on the secondary side:

$$I_B = -1/2\ I_\alpha + \sqrt{3}/2\ I_\beta + I_0$$

Therefore

$$I_B = 6.27 \angle 90^\circ \text{ p.u.}$$

The current magnitude in line B,

$$|I_B| = 6.27 \times 164 = 1025 \text{ amps.}$$

And the current in line C on the secondary side:

$$I_C = -1/2\ I_\alpha - \sqrt{3}/2\ I_\beta + I_0$$

Therefore

$$I_C = 6.27 \angle 90^\circ \text{ p.u.}$$

The current magnitude in line C,

$$|I_C| = 6.27 \times 164 = 1025 \text{ amps.}$$

Now the current through the grounding wire of the transformer,

$$I_{n_2} = I_B + I_C$$

Therefore

$$I_{n_2} = 12.54 \angle 90^o \text{ p.u.}$$

The current magnitude through the grounding wire of the transformer,

$$|I_{n_2}| = 12.54 \times 164 = 2055 \text{ amps.}$$

From the characteristics of the grounding wire in Figure 13, the time $t_2$ = 23.95 seconds for the wire to melt completely after the occurence of the fuse opening in phase a.

Now from Equation (3.6) we have

$$V_\alpha = 2V_0$$

From Appendix B, we have from Equation (B.1)

$$V_A = V_\alpha + V_0$$

Therefore

$$V_A = 3/2 \ V_\alpha$$

Now

$$V_\alpha = V_f - I_\alpha \times Z_\alpha$$

Therefore

$$V_\alpha = 0.585 \angle 0^\circ \text{ p.u.}$$

The voltage of line A to ground,

$$V_A = 3/2\ V_\alpha = 0.878 \angle 0^\circ \text{ p.u.}$$

The voltage of line B to ground,

$$V_B = -\ 1/2\ V_\alpha + \sqrt{3}/2\ V_\beta + V_0$$

Since

$$V_\alpha = 2V_0$$

and

$$V_\beta = 0$$

$$V_B = 0 \text{ p.u.}$$

Similarly the voltage of line C to ground,

$$V_C = -\ 1/2\ V_\alpha - \sqrt{3}/2\ V_\beta + V_0$$

Again since

$$V_\alpha = 2V_0$$

$$V_\beta = 0$$

$$V_C = 0 \text{ p.u.}$$

In order to get the line current and voltage to ground on the primary side, use is made of the equivalent circuit of Figure 11 on page 27. Here the double primed symbols indicate the values of the components of the line current and the voltage to ground on the primary side referred to the circuit at the secondary side of the delta-wye transformer. Whereas the single primed symbols of the components of the line current and the voltage to ground indicate the actual values on the primary side of the delta-wye transformer.

$$I_\alpha'' = I_\alpha = -j\ 4.18 \text{ p.u.}$$

Now from equations derived for phase shift corrections

$$I_\beta' = -\ I_\alpha'' = j\ 4.18 \text{ p.u.}$$

$$I_\alpha' = I_\beta''$$

Now from Equations (3.11) and (3.14) we have

$$I_\alpha' = -\ I_0'$$

$$I_\beta'' = -\ I_0''$$

Since the Beta and Zero networks are open circuited,

$$I_\beta'' = -\ I_0'' = 0$$

Also

$$I_\alpha' = I_\beta'' = 0$$

Therefore from Equation (3.11)

$$I_\alpha' = -I_0' = 0$$

and

$$I_\beta' = 4.18 \angle 90^\circ \text{ p.u.}$$

The current in line a on the primary side becomes

$$I_a = I_\alpha' + I_0' = 0 \text{ amps.}$$

Since line a is open, $I_a = 0$ checks as it should.

The current in line b on the primary side is:

$$I_b = -1/2\, I_\alpha' + \sqrt{3}/2\, I_\beta' + I_0'$$

or

$$I_b = j\, 3.62 = 3.62 \angle 90^\circ$$

Therefore the current magnitude in line b,

$$|I_b| = 3.62 \times 32.30 = 117 \text{ amps.}$$

The current in line c on the primary side is:

$$I_c = -1/2\, I_\alpha' - \sqrt{3}/2\, I_\beta' + I_0'$$

or

$$I_c = -j\, 3.62$$

Therefore

$$I_c = 3.62 \angle -90^{\circ} \text{ p.u.}$$

The current magnitude in line c,

$$|I_c| = 3.62 \times 32.30 = 117 \text{ amps.}$$

From the characteristics in Figure 14, the fuses of phase b and c will take $t_1' = 95$ seconds to blow. Since time $t_1'$ is greater than $t_2 = 23.95$ seconds, the grounding wire will burn out earlier than the fuses getting blown out and thus the transformer is still energized through phases b and c supplying the power.

In order to get the voltage to ground on the primary side, we have from the equivalent circuit of Figure 11 and the equations derived for phase shift corrections,

$$V_\alpha'' = V_f - I_\alpha'' \times Z_{s_\alpha}$$

Therefore

$$V_\alpha'' = 0.8875 \angle 0^{\circ} \text{ p.u.}$$

Since no current flows through the Beta network and Zero network, therefore

$$V_0'' = 0$$

Also

$$V_\beta'' = -jV_f = -j\,1.00$$

Now from the equations for phase shift correction,

$$V_\alpha' = V_\beta'' = -j\ 1.00 \text{ p.u.}$$

$$V_\beta' = -\ V_\alpha'' = -\ 0.8875 \text{ p.u.}$$

$$V_0' = V_0'' = 0 \text{ p.u.}$$

Now the voltage of line a to ground is:

$$V_a = V_\alpha' + V_0'$$

or

$$V_a = -j\ 1.00$$

Therefore

$$V_a = 1.00 \angle{-90^\circ} \text{ p.u.}$$

Now the voltage of line b to ground is:

$$V_b = -\ 1/2\ V_\alpha' + \sqrt{3}/2\ V_\beta' + V_0'$$

or

$$V_b = -\ 0.768 + j\ 0.50$$

Therefore

$$V_b = 0.917 \angle{147^\circ} \text{ p.u.}$$

Now the voltage of line c to ground is:

$$V_c = -\ 1/2\ V_\alpha' - \sqrt{3}/2\ V_\beta' + V_0'$$

or

$$V_c = 0.768 + j\ 0.50$$

Therefore

$$V_c = 0.917 \angle 33^{o} \text{ p.u.}$$

The sum of $V_a$, $V_b$ and $V_c$ must be equal to zero which checks as it should.

All the voltages and currents on both sides of the transformer are now known for the simultaneous fault with a grounded transformer.

After $t_2 = 23.95$ seconds has elapsed, the grounding wire of the transformer melts and the distribution transformer is now ungrounded and the equivalent circuit of Figure 12 on page 30 is now applicable.

Since the Zero network is now open,

$$I_{\alpha} = - I_0 = 0$$

Also the Beta network is open, there will be no Beta current, therefore,

$$I_{\beta} = 0$$

Now, as the Alpha, Beta and Zero components of the current are all equal to zero, the line currents on all the phases will also be zero on the secondary side as well as the primary side of the transformer.

Now the voltage to ground on the secondary side is calculated. From Appendix B, we have from Equation (B.1)

$$V_A = V_{\alpha} + V_0$$

Since

$$V_\alpha = 2V_0$$

Therefore

$$V_A = 3/2\ V_\alpha$$

Now since the Zero network is open, $I_\alpha = -\ I_0 = 0$

Therefore

$$V_\alpha = V_f = 1.00 \text{ p.u.}$$

Now as

$$V_A = 3/2\ V_\alpha$$

$$V_A = 1.5 \text{ p.u.}$$

The voltage of line B to ground is,

$$V_B = -\ 1/2\ V_\alpha + \sqrt{3}/2\ V_\beta + V_0$$

Since

$$V_\alpha = 2V_0$$

and

$$V_\beta = 0$$

Therefore

$$V_B = 0 \text{ p.u.}$$

The voltage of line C to ground is,

$$V_C = - 1/2\ V_\alpha - \sqrt{3}/2\ V_\beta + V_0$$

Since

$$V_\alpha = 2V_0$$

and

$$V_\beta = 0$$

Therefore

$$V_C = 0 \text{ p.u.}$$

Also the voltages to ground on the primary side is given by the equivalent circuit in Figure 12,

$$V_\alpha'' = V_f - I_\alpha''\ Z_{s_\alpha}$$

Since the zero network is open, $I_\alpha'' = - I_0'' = 0$

Therefore

$$V_\alpha'' = 1.00 \text{ p.u.}$$

Since no current flows through the Beta and Zero network, therefore,

$$V_0'' = 0$$

and also

$$V_\beta'' = -j\ V_f = -j\ 1.00 \text{ p.u.}$$

Now from the equations for phase shift correction,

$$V_\alpha' = V_\beta'' = -j\ 1.00 \text{ p.u.}$$

$$V_\beta' = -\ V_\alpha'' = -\ 1.00 \text{ p.u.}$$

$$V_0' = V_0'' = 0$$

The voltage of line a to ground is:

$$V_a = V_\alpha' + V_0'$$

or

$$V_a = -j\ 1.00 \text{ p.u.} = 1.00 \angle -90^\circ \text{ p.u.}$$

The voltage of line b to ground is:

$$V_b = -\ 1/2\ V_\alpha' + \sqrt{3}/2\ V_\beta' + V_0'$$

or

$$V_b = j\ 0.50 - 0.866 \text{ p.u.}$$

Therefore

$$V_b = 1.00 \angle 150^\circ \text{ p.u.}$$

Now the voltage of line c to ground is:

$$V_c = -\ 1/2\ V_\alpha' - \sqrt{3}/2\ V_\beta' + V_0'$$

or

$$V_c = j\ 0.50 + 0.866 \text{ p.u.}$$

Therefore

$$V_c = 1.00 \angle 30^\circ \text{ p.u.}$$

The sum of $V_a$, $V_b$ and $V_c$ is equal to zero. All the voltages and currents on both sides of the transformer are now known for the fault condition with an ungrounded transformer.

The various results have been tabulated in Table II. With the two line to ground fault on phases B and C on the secondary side and open conductor on phase a on the primary side with the transformer being ungrounded, overvoltage occurs on phase A to ground to 150 percent of normal, and indicates the posibility of damage to the customers supplied by phase A due to this overvoltage under the above fault conditions.

TABLE II. Results

| System Condition | Primary Currents & Voltages $I_a$ Amp. | $I_b$ Amp. | $I_c$ Amp. | $V_a$ p.u. | $V_b$ p.u. | $V_c$ p.u. | Fuse time in seconds Total clearing Time | Secondary Currents & Voltages $I_A$ Amp. | $I_B$ Amp. | $I_C$ Amp. | $I_n$ Amp. | $V_A$ p.u. | $V_B$ p.u. | $V_C$ p.u. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (i) Double Line-to-Ground Fault (Phases B & C) | 325 | 201 | 201 | 1.02 | 1.11 | 0.982 | $t_1$=1.7 secs. for phase a fuse | 0 | 1760 | 1760 | 2060 | 0.879 | 0 | 0 |
| (ii) Faults with grounded trans-former (Open Conductor phase a and double line-to-ground fault phases B & C) | 0 | 117 | 117 | 1.00 | 0.917 | 0.917 | $t_1$'=95 secs. for phases b & c fuses | 0 | 1025 | 1025 | 2055 | 0.878 | 0 | 0 |
| (iii) Faults with ungrounded trans-former (Open-Conductor phase a and double line to ground fault phases B & C) | 0 | 0 | 0 | 1.00 | 1.00 | 1.00 | | 0 | 0 | 0 | 0 | 1.5 | 0 | 0 |

Table II. Cont.

TABLE II. Results (cont.)

| Grounding wire-time in seconds to melt | Remarks (Effective time t in seconds ) |
|---|---|
| $t_2' = 24$ secs. | $0 \leq t \leq 1.7$ for phase a fuse to blow out. |
| $t_2 = 23.95$ secs. | $1.7 \leq t \leq 23.95$ for grounding wire to melt. |

# V. CONCLUSION

This study has provided a better understanding of the voltages and currents in double unsymmetrical fault conditions. The opening of phase a fuse on the primary side, due to excessive current in phase a, can be easily avoided by using a circuit breaker on the primary side which can open all the three phases simultaneously in case of a fault and thus protect the customers from overvoltage situations. The melting of the transformer neutral wire can be avoided if a grounding resistor is used with the grounding wire instead of solid grounding in order to limit the fault current. This would also help avoid the overvoltage situation that developed as well.

In summary, two separate modifications are suggested to effect a better protection scheme for the distribution system of the Central Power Electric Cooperative. These are given below:

1) A set of three fuses of suitable ratings should be installed on phases of the secondary circuit of the transformer so that, in case of a double fault like this one, the secondary circuit fuses open earlier than those of the primary circuit of the transformer. This protection scheme will help avoid damage to the customer and the transformer.

2) The fuses on the primary circuit of a transformer should be replaced by a circuit breaker or a combination of circuit breaker and fuses installed in order to avoid such single conductor opening in the system and damage to the customers' appliances due to the overvoltages.

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# VITA

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# APPENDICES

# APPENDIX A

## Symmetrical Components of a Three Phase System

Fundamental symmetrical component equations:

$$I_A = I_{A_0} + I_{A_1} + I_{A_2} \tag{A.1}$$

$$I_B = I_{A_0} + a^2 I_{A_1} + aI_{A_2} \tag{A.2}$$

$$I_C = I_{A_0} + aI_{A_1} + a^2 I_{A_2} \tag{A.3}$$

$$V_A = V_{A_0} + V_{A_1} + V_{A_2} \tag{A.4}$$

$$V_B = V_{A_0} + a^2 V_{A_1} + aV_{A_2} \tag{A.5}$$

$$V_C = V_{A_0} + aV_{A_1} + a^2 V_{A_2} \tag{A.6}$$

$$I_{A_0} = \frac{1}{3}(I_A + I_B + I_C) \tag{A.7}$$

$$I_{A_1} = \frac{1}{3}(I_A + aI_B + a^2 I_C) \tag{A.8}$$

$$I_{A_2} = \frac{1}{3}(I_A + a^2 I_B + aI_C) \tag{A.9}$$

$$V_{A_0} = \frac{1}{3}(V_A + V_B + V_C) \tag{A.10}$$

$$V_{A_1} = \frac{1}{3}(V_A + aV_B + a^2 V_C) \tag{A.11}$$

$$V_{A_2} = \frac{1}{3}(V_A + a^2 V_B + aV_C) \tag{A.12}$$

where,

$$a = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = 1.00 \angle 120^\circ \tag{A.13}$$

$$a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = 1.00 \angle 240^\circ = 1.00 \angle -120^\circ \tag{A.14}$$

Also,

$I_A$, $I_B$, and $I_C$ are the three line currents at any point of the system.

$V_A$, $V_B$, and $V_C$ are the three voltages to ground at any point of the system.

$I_{A_1}$, $I_{B_1}$, and $I_{C_1}$ are the positive sequence currents in the three conductors. By definition, these currents are equal in magnitude and $I_{A_1}$ leads $I_{B_1}$ by $120^o$ and $I_{C_1}$ by $240^o$.

$I_{A_2}$, $I_{B_2}$, and $I_{C_2}$ are the negative sequence currents in the three conductors. By definition, these currents are equal in magnitude and $I_{A_2}$ leads $I_{C_2}$ by $120^o$ and $I_{B_2}$ by $240^o$.

$I_{A_0}$, $I_{B_0}$, and $I_{C_0}$ are the zero sequence currents in the three conductors. By definitions, these currents are equal in magnitude and in phase.

Notation for components of voltages corresponds to that for the components of currents as described above.

## APPENDIX B

### Alpha, Beta and Zero Components of Three Phase System

With phase A as the reference phase in a three phase system, the Alpha, Beta and Zero components of current and voltage are defined as follows:

Alpha components in phases B and C are equal. Also, they are opposite in sign and of half the magnitude of the Alpha components of phase A.

Beta components in phases B and C are equal in magnitude and opposite in sign; also, they are zero in phase A.

Zero components are equal in the three phases.

Thus, the matrix equation for a set of three voltage vectors $V_A$, $V_B$, and $V_C$ of a three phase system can be written in terms of their Alpha, Beta, and Zero components as given below:

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -1/2 & \sqrt{3}/2 & 1 \\ -1/2 & -\sqrt{3}/2 & 1 \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \\ V_0 \end{bmatrix}$$

Also the three vectors $V_A$, $V_B$, and $V_C$ can be expressed in terms of their Alpha, Beta, and Zero components by equations:

$$V_A = V_\alpha + V_0 \tag{B.1}$$

$$V_B = -\frac{1}{2} V_\alpha + \frac{\sqrt{3}}{2} V_\beta + V_0 \tag{B.2}$$

$$V_C = -\frac{1}{2} V_\alpha - \frac{\sqrt{3}}{2} V_\beta + V_0 \tag{B.3}$$

The three voltage vectors $V_\alpha$, $V_\beta$, and $V_0$ are also expressed in terms of the system vectors $V_A$, $V_B$, and $V_C$ by solving the above three equations.

Subtracting one-half the sum of Equations (B.2) and (B.3) from Equation (B.1) and solving for $V_\alpha$ gives:

$$V_\alpha = \frac{2}{3} (V_A - \frac{V_B + V_C}{2}) \quad \text{(B.4)}$$

Subtracting Equation (B.3) from Equation (B.2) and solving for $V_\beta$ gives:

$$V_\beta = \frac{1}{\sqrt{3}} (V_B - V_C) \quad \text{(B.5)}$$

Adding Equations (B.1), (B.2), and (B.3) and solving for $V_0$ gives:

$$V_0 = \frac{1}{3} (V_A + V_B + V_C) \quad \text{(B.6)}$$

Also the matrix equation for the three voltage vectors $V_\alpha$, $V_\beta$ and $V_0$ can be written in terms of the system vectors $V_A$, $V_B$ and $V_C$ as given below:

$$\begin{bmatrix} V_\alpha \\ V_\beta \\ V_0 \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ 0 & \frac{1}{\sqrt{3}} & -1/\sqrt{3} \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}$$

The corresponding current equations are:

$$I_A = I_\alpha + I_0 \quad \text{(B.7)}$$

$$I_B = -\frac{1}{2} I_\alpha + \frac{\sqrt{3}}{2} I_\beta + I_0 \quad \text{(B.8)}$$

$$I_C = -\frac{1}{2} I_\alpha - \frac{\sqrt{3}}{2} I_\beta + I_0 \quad \text{(B.9)}$$

Also,

$$I_\alpha = \frac{2}{3} (I_A - \frac{I_B + I_C}{2}) \tag{B.10}$$

$$I_\beta = \frac{1}{\sqrt{3}} (I_B - I_C) \tag{B.11}$$

$$I_0 = \frac{1}{3} (I_A + I_B + I_C) \tag{B.12}$$

Substituting the above equations in the symmetrical components equations given in Appendix A, the relations between the symmetrical components and the Alpha, Beta, and Zero components of voltage and current are as given below:

$$V_\alpha = V_{A_1} + V_{A_2}$$

$$V_\beta = -j (V_{A_1} - V_{A_2})$$

$$V_0 = V_{A_0}$$

Also,

$$I_\alpha = I_{A_1} + I_{A_2}$$

$$I_\beta = -j (I_{A_1} - I_{A_2})$$

and

$$I_0 = I_{A_0}$$