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# The lattice constant and coefficient of expansion of chromium 

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# THE LATTICE CONSTANT AND COEFFICIENT OF EXPANSION OF CHROMIUM 

By
CHAO-CHING, WENG

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THESIS
submitted to the faculty of the
SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI in partial fulfillment of the work required for the Degree of MASTER OF SCIENCE IN METALLURGICAL ENGINEERING Nola, Missouri

1953

Approved by

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## INT RODUCTION

The most important use of chromium, other than as an alloying element in the manufacture of stainless steel, is for electroplating; to form a coating on other metals for corrosion prevention in order to procure longer life, and to achleve a decorative effect.

The physical properties of ohromium are 1 mportant in the effectiveness of its uses, and its lattice constant, as well as coefficient of thermal expansion, seem worthy of exact determination.

A number of research workers, over a span of thirty years, have spent considerable effort in determining the aforementioned constants. However, their results do not check, and the degree of accuracy differs from person to person.

Chromium near $37^{\circ} \mathrm{C}$ was 1dentified by M. E. Fine as showing discontinuous changes of coefficient of expansion, Young's modulus, internal friction, electrical resistivity and thermoelectric power. Although the $X$-ray diffraction pattern gave no clue, a difference in the thermal expansity has been found. D. MacNair determined the expansity of chromium by means of an interferometric dilatometer, with the result that near $38^{\circ} \mathrm{C}$ the thermal expansity curve
(1) Fine, M. E., J. of Metals Tran. A. I.M.Eep. 189, 56, 1951.
(2) MacNair, D., Rev. of scientific Instruments, p. 12, 66, 1941.
went through an inflection point, corresponding to a minimum in the coefficient of expansion found by Fine. The purpose of this research 18 to check the results obtained by MaoNair and Fine concerning the expansion of ohromium, and to determine the exact lattice constant and thermal expansion of this metal by the Xwray powder method, using different samples, and at different temperatures within the range of $10^{\circ}$ to $50^{\circ} \mathrm{C}$.

## REVIEW OF LITERATURE

After the discovery of X-ray diffraction by crystal by M. von Laue, (3) Friedrich, and Knipping in 1912, the lattice constant of chromium could be easily determined. However, the degree of accuracy 18 dependent upon the purity of the sample, the method applied, and the kind of camera used.

Hull (4) used a molybdenum target to produce X-rays and to investigate the lattice constant of chromium. He got a figure of 2. 91 kX in 1919. In 1928, A. Westgren ${ }^{(5)}$ used a chromium target, and got a figure of 2.878 kX for the lattice constant. The results obtained by other investigators are shown in Table 1.

D1sch ${ }^{(6)}$ was the first investigator to publish data in 1921 concerning the linear thermal expansion of chromium. During the next year, Chevenard ${ }^{(7)}$ published data on the Inear thermal expansion of 98.3 percent pure chromium. Hidnert ${ }^{(8)}$ published data on the linear thermal expansion
(3) Laue, M., Ann. Physik, p. 41, 971, 1913 reprinted from an earlier publication in 1912.
(4) Hull, A. W., Phy s. Rev. p. 14, 540, 1919.
(5) Westgren, A., J. Iron \& Steel Institute, p. 117, 383, 1928.
(6) D1 sch, J., Z. Physik, p. 5, 173, 1921.
(7) Chevenard, P., Comptos rend. p. 174, 109, 1922.
(8) Hidnert, $P_{\text {. }} J_{1}$ of Research of the National Bureau of Standaras, p. 26, 81, 1941 .
coefficient of 99.3 percent and 98.7 percent pure electrolytic chromium in the years of 1931 and 1934. These curves are shown in Fig. 1.


Figure 1. The ourves of linear thermal expansion of chromium as obtained by different investigators.

TABLE 1
Figures of Lattice Constant of Cr Obtained by Different Investigators

| Name of research workers Name of periodicals | Target Temp. | Lattice Constant (kX) |
| :---: | :---: | :---: |
| A. W. Hull . . . . . . . . Phys.Rev. p.14, 540, 1919 | Mo. room | 2.91 |
| A. W. Hull . . . . . . . . Phys.Rev. p.17, 571, 1921 | Mo. room | 2.895 |
| Wm. C. Phebus \& F.C.Blake Phys.Rev. p. 25, 107, 1925 | Mo. room | 2.875 |
| Wh. C. Phebus \& F.C.Slake Phys.Rev. p. 25, 581, 1925 | Mo. room | 2.873 |
| R. A. Patterson . . . . Phys. Rev. p. 26, 56, 1925 | Mo. room | 2.872 |
| Cyril S. Smith . . . . Metal Ind. p. 28, 456, 1926 | Mo. room | 2. 860 |
|  | Mo. room | 2.872さ0.005 |
| A. We stgran, G.Phragmen, J.Iron \& Steel Institute, \& TroNegresco ..... p.117, 383, 1928 | Cr. room | 2.878 |
| Kumazo Sasaki \& Sinkiti Am.Electrochem. Soc.Trans., Sekito . . . . . . . . p. 59, 439, 1931 | Cr. room | $2.977 \pm 0.003$ |
| G. D. Preston . . . . . J.Iron \& Steel Institute, p.124, 139, 1931 | Cr. room | 2.8786\$0.0005 |
| E. Ro Jette - . . . . A.I. M.M. F.Tech, 522, 1934 | Cr. room | 2.8787 |
| L. Wright, H.Hirst, \& J.Riley . . . . . Farad. Soc. Trans., p. 31 , 1253,1935 | Cr. room | $\begin{aligned} & 2.8788 \\ & 2.8781 \end{aligned}$ |
| W. A. Wood . . . . . . Phil.Mag., p. 23, 984, 1937 | Cr. $\quad 18^{\circ} \mathrm{C}$ | 2.8796 |
| H. Sochtig . . . . . . . Ann.Phys., p.38, 97, 1940 | Cr. room | 2.846 |
| M. I. Fine, E.S.Greiner . . J.Metals A.I.M.E.Trans, \& W.C.Ellis p. 189, 56, 1951 | Cr. $20^{\circ} \mathrm{C}$ | $\begin{aligned} & 2.87900 \pm \\ & (2.8848 \&) \end{aligned}$ |

SAMPLES USED IN THIS INVESTIGATION

Four high purity ohromium samples were used in this investigation. Two of them were produced by electrolytic methods, and the other two by the 1odide method. The 10dide chromium samples (Battelle Memorial Institute) consisted of shiny crystals and were of two compositions: one sample being very low in metallics, and the other low in non-inetallics such as $H_{2}, N_{2}, C$, and $S$. The chemical analyses, as given by the manufacturers, differ somewhat from sample to sample, and are listed in Table II.

## TABLE II

## Compositions of the Four Different High Purity Chromium Samples

| Impurity Electrolytic | sintered Electrolytic | Iodide (low metallic) | Iodide (low non-metallic) |
| :---: | :---: | :---: | :---: |
| $\mathrm{H}_{2}$ | 0.0001 | 0.0009 | 0.0001 |
| 02 | 0.0088 | 0.014 | 0.001 |
| $\mathrm{N}_{2}$ | 0.019 | 0.013 | 0.001 |
| C | 0.005 |  | 0.001 |
| $s$ |  |  | 0.003 |
| sb | 0.01 |  |  |
| S1 |  | $<0.001$ | 0.001 |
| Fe |  | $<0.001$ | 0.005-0.1 |
| Cu |  |  | $<0.001$ |
| Mn |  | N. D. ${ }_{0}$ O. 001 | 0.001-0.01 |
| Mg |  | < 0.001 | < O.col Trace |
| Co |  | <0.001 | N. D. |
| N1 |  | N. D. | < 0.005 |
| T1 |  | N. D. | N. D. |
| Pb |  | N. D. | N. D. |
| Ca |  | <0.001 | < 0.001 Trace |
| Al |  | N. $\mathrm{D}_{0}<0 . \mathrm{CO1}$ | 0.0001-0.001 |
| W |  | N. $\mathrm{E}_{0}<0.1$ (2) | $0.001-0.01$ |
| Mo |  | N. D. | N. D. |
| Manu- Charles | Niagara | Battelle | Battelle |
| facturer:Hardy, Inc. | Falls | Memorial <br> Institute | Memorial <br> Institute |

(1). N.D. means not detected.
(2). Tungsten probably not present but standard used was only this sensitive.

## PRFPPARATION OF THE SAMPLES USED FOR X-RAY DIFFRACTION

1. Grinding and sieving

The purpose of using fine powder mounts in X-ray diffraction $1 s$ to obtain sharp, uniforing and unshifted diffraction lines on the films. Such lines are necessary for the exact detemination of lattice constants. For this reason, the first step in the preparation of a sample was to grind it as fine as possible with a mortar and pestle, and then sieve it. Because of plastic deformation, the metal, after grinding for several hours, was in the form of leallets. The powder was then sieved through a 325 mesh soreen, shaking the screen only slightly. Only the Ine powder that passed through the sieve was filled into a silica glass bulb (Fig. 2) to be heat treated.


Figure 2. A sketoh of an evacuated and sealed quartz glass bulb with Or. powder inside.
(I). Sealed part;
(2). Sample
2. Evacuation and Heat-treatment

The purpose of heat treatment is to get rid of hydrogen gas dissolved in the metal, and to release strains and deformation caused by grinding. However, the temperature should not be too high, as excessive grain growth, resulting Prom recrystallization, should be prevented. As chromium oxidizes quickly in the air at elevated temperatures, and changes its physical properties, the heating must be made in a vacuum. This can be done by connecting the silica glass bulb containing the chromium powder with a mechanical vacuum pump, which produces a vacuum as low as 1 micron of Hg. (Fig. 3) To remove the major part of the gases absorbed, the sample was heated during evacuation. After the best


Figure 3. Vacuum pump used for evacuating the silica glass bulb (a).
vacuum obtainable was reached, the silica glass bulb was sealed off with an oxygen-gas flame. The sealed part of the bulb was examined under a microscope to be sure that no leaks were in the sealed part. The bulb with the sample inside was then transferred to the furnace for heat treatment.

The electric tube-furnace had N1-Cr wire windings, and the temperature in the center of the furnace was measured by a chromel-alumel thermocouple and recorded automatically by a Brown potentiometer. The heating temperature In this experiment was $850^{\circ} \mathrm{C}$, and the samples were held for two and a half hours at this temperature. The arrangement for the heating of the samples is shown in Figure 4.


Figure 4. The schematic arrangement of the heat treatment system.
(a). Sealed glass bulb containing sample; (b). Thermocouple; (c). Electric tube furnace;
(d). Ni-Cr wire winding; (e). Porcelain boat.
3. The Powder Mount

At a temperature of $850^{\circ} \mathrm{C}$, the grains of the fine chromium powder did not stick together, which was a conven-

1ence in making the powder mounts to obtain $X$-ray diffraction patterns. For the preparation of the mounts, a thin lithium-boron glass hair was used, the constituents of the glass being 1.4 parts by weight of $\mathrm{BeCO}_{3}, 4.3$ parts by weight of $\mathrm{Li}_{2} \mathrm{CO}_{3}$ and 18.3 parts by weight of $\mathrm{B}(\mathrm{OH})_{3}$. The diameter of the glass hair was 0.08 mm , and its length about 10 mm . One end of the glass hair was placed in the groove at the tip of the adjustable sample holder, while the other end was supported by a cork, so that the hair was approximately parallel to the length of the groove. A drop of liquid glue (Dekadhese plastic cement) was then spread over the groove holding one end of the glass hair, and allowed to dry for 20 minutes. After that, the glass hair was cut with scissors to $5-6 \mathrm{~mm}$. in length. The glass hair then held its position firmly, even when heated above $70^{\circ} \mathrm{C}$. After mounting, the glass hair must be carefully oentered. Both mounting and centering were done under a microscope which must have a cross hair on 1ts objective lens (F1g. 5). Centering was achieved by adjusting the sorews of the sample holder until the axis of the glass hair exactiy coincided with the axis of rotation of the holder. After centering, the tip of the glass hair, about 2 mm . In length, was coated, by means of a soft copper wire of 0.1 mm . in diameter, with a small amount of non-drying glue (stop grease cello-seal diluted with oil). The layer of glue has to be thin and uniform. The tip of the hair carrying the glue was then dipped into the chromium powders,


Figure 5. (1). Set up for mounting, centering and coating of the glass hair 0.08 mm . In diameter;
(2). Long screw for adjusting the tip of the specimen holder; (3). Camera cover in the frame; (4). Aluminum foll for coating the glass hair w1th powder.
which was on the end of a thin triangle-shaped piece of aluminum foil. The fine powder adhered tightly to the glass hair especially after being rotated in the powder. The diameter of the whole sample, including the glass hair, was about 0.12 to 0.2 mm . (Fig. 6) The glass hair holding the powder rotates with the camera holder in the camera during the time of exposure by X -rays. It is important to get a thin, uniform, and well centered powder mount, because this reduces the width of the $X$-ray diffraction lines and their positions can be measured more accurately.


Figure 6. Powder mount 0.12 mm . in diameter.

## EQUIPMENT USED FOR X-RAY DIFFRACTION

1. The X-ray Camera

The precision camera used is shown in Fig. 7 and 8 . It was of a cylindrical type with a diameter of 64 mm. $X$-ray films of a size $3 \times 18 \mathrm{~cm}$. were used in this camera. The films were located in the asymmetric manner which has the following advantages:
(1). Both, all front and back diffraction lines, are simultaneously recorded on one film.
(2). The effective circumference of the film can be calculated from each film independently after its measurement. Thus errors due to incorrect knowledge of the diameter of the camera, and due to film shrinkage, are eliminated.
(3). No standard substances are necessary for camera calibration.
(4). The method is, therefore, absolute, as the absolute value of the lattice constants can be deter mined from the films themselves.
(5). Further, because of the careful construction of the camera and because of the very thin sample, the error due to eccentricity is largely minimized.
(6). The error due to absorption of X-rays 1 s greatly reduced, especially in the high back reflection region $\left(75^{\circ}-90^{\circ}\right)$ where the shift of the ines
due to absorption can be entirely neglected.


Figure 7. View of the outside of the precise camera.


Figure 8. The precision camera.
(Dimensions are given in mm .)
2. X-ray Machine and Temperature Control System X-ray Machine A crystalline substance can diffract X-rays and produce a sufficient number of diffraction lines, $1 f$ the wave length of the X-ray beam used 18 smaller than the lattice spacings of the crystaline substances. $E y$ changing the target of the tube, X-ray beams of different wave lengths can be obtained. The targets are made of different metals, such as chromium, molybdenum, iron, copper, nickel, and cobalt, and are cooled with water during the operation of the $X$-ray tube. The X-ray machine used in this laboratory was manufactured by the Picker X-ray Corp., and was designed for the production of soft radiation $X$-rays. The machine is equipped with a clock-switoh which shuts off the machine if the flow of cooling water to the target is by some means discontinued. This device protects the tar get of the X-ray tube from damage, as well as the whole machine.

Temperature Control system In the determination of lattice constants with highest precision, an exact know ledge of the temperature of the sample is important. It should be closely controlled, because small temperature fluctuations affect the magnitude of the lattice spacings. A amall change of temperature, even $0.1^{\circ} \mathrm{C}$, can influence the lattice constant in the fifth decimal place. Therefore, precision determination must be conducted with thermostats, as the room temperature may vary by some degrees during the occasional long exposures. One more application of the
thermostat is to change the temperature at certain intervals in order to determine the variation of lattice constants with temperature and to compute the thermal expansion coefficient of the substance. In the present investigation, the temperature of the specimens was controlled by a temperature control system which consists of three integral parts, namely, the thermostatic jacket in whioh the camera was placed, the thermostatic bath, and the cooling water bath.

The thermostatic jacket is a metallic container with a door, and having water channels of 12 mm . in diameter bored lengthwise through the container's walls. The first step was to lock the camera in the $j 1 g$, thus obtaining a good heat exchange (Fig. 9). Then the whole was placed in the jacket (Fig. 10), and secured to the bottom of the jacket by means of a set screw. The temperature of the jacket (with the camera inside) was maintained by a water stream of constant temperature, circulating from the thermostatic bath up through the channels in the walls of the jacket. The cooling water bath was only used when working below room temperatures. Two diametrically opposite holes, as inlets and outlets for $X$-rays were provided in the jacket. Two more holes were drilled on the right hand side of the jacket; one of these holes being used to insert the shaft for the rotation of the sample inside of the camera (connected by means of a fork with the cam of the camera shown in Fig. 9, (3) ) and the other hole for inserting a themometer into the thermostatic jacket. A thermometer

Figure 9. The camera locked in a metallic 11 g for fixing in the thermostatic jacket.
(1). Fluorescent camera screen on the exit port of X-rays; (2). The j1g; (3). The cam of the specimen-rotation shaft; (4). The set sorew for adjustment of the position of the specimen.


Figure 10. The camera placed in the thermostatic jacket with door removed.
(1). Calibrated thermometer; (2). Magnifying lens for reading the thermometer scale.
with an accuracy of $0.02^{\circ} C$ was used in order to measure the accurate temperature of the camera and sample inside the jacket. After the door of the jacket was closed (Fig. ll), the equipment was left until the inside of the jacket (whth the loaded camera) had reached the required temperature. Then 1t was left for an additional hour to be sure that the sample inside the camera had assumed the same temperature as shown by the thermometer.

The constant temperature in the jacket was maintained by a circulating thermostatic bath which was a five gallon porcelain water container. The temperature of the water


Figure 11. A view of the thermostatic jacket with the door olosed.
(1). Pulley for rotation of specimen;
(2). Rubber hoses for oiroulating water of a constant temperature into the jacket.
circulating through the jacket was controlled by an immer-sion-type, adjustable contact themoregulator connected with a Fishermserfass electronic relay. This bath was connected with the channels of the jacket by one-half inch inside diameter rubber hoses. During the work, the water of the bath was continuously pushed through the channels of the jacket by means of a centrifugal pump, whose speed could be regulated with a variable transformer. As the circulation of the water agitated the bath, no stirrer was necessary. The bath was heated by an adjustable 500 watt heater. For working above room temperature, the heater was plugged into the proper socket of the relay, and the thermo-regulator then maintained the desired temperature in the thermostatic jacket. Under these conditions, the alr of the room acted as a cooling medium. The themoregulator just mentioned had a sensitivity of 0.020 F , over a range of $50^{\circ}$ to $200^{\circ}$ F. Figure 12 shows the centrifugal pump (a), the thermoregulator (b), and the heater (c), on the cover of the thermostatic bath (B), and controling devices on the switchboard (E).

The exposures were required to be made below room temperature, a cooling water bath (Fig. 12 C) was used. The bath consists of a porcelain water container connected W1th an adjustable refrigerator (Fig. 12 D ) and was used to keep the temperature of the bath $B$ below that of the room. The temperature of the said bath was regulated by


Figure 12. The arrangement of the complete X-ray diffraotion unit.
(A). Thermostatio jacket with the camera inside;
(D). Thermostatic bath; (C). Cooling water bath;
(D). Refrigerator; (E). Switchboard;
(a) and (d). Centrifugal pumps;
(b). Thermo-regulator; (c). Heater.
a one-half inch copper coil, through which cooling water was driven by a second pump (Fig. 12 d ). The speed of this pump could also be regulated by a variable rheostat. For working below room temperatures, the pump was plugged into the other socket of the electronic relay and now air of the room acted as the heating medium. The thermo-regulator worked as well as it did for temperatures above room temper atures.

## 3. Comparator

The purpose of measuring positions of diffraction lines on the X-ray films is to calculate Bragg angles, and then the lattice constant of the sample. The comparator is the mechanical device used for measuring the $X$-ray films and is supplied with an exact micrometer screw.

The comparator (Fig. 13) used in this laboratory was manufactured by D. Mann of Lincoln, Massachusetts. It has a travelling carriage, and a microscope in a fixed frame mounted above the carriage. The microscope, of a low magnification, has an objective lens with cross hairs. The screw has a pitch of 1 mm. , and the miorometer drum, connected to the screw has 1000 divisions. By this device, the positions of the diffraction lines, if they are sharp enough, can be measured with an accuracy of 0.001 mm . The film has to be adjusted correctly on the carriage by means of two screws. An adjustment is correct when the center of the cross halrs were exactly on the central line of the film.


Figure 13. The Comparator.
$\left(\frac{1}{3}\right)$. Microscope; (2). Film for measuring;
(3). Micrometer drum.

## INDEXING AND MEASURING THE FILM

## 1. Theory

As mentioned above, the purpose of making and measuring the $X$-ray films is to find the Bragg angle $\theta$ of the diffraction lines and to calculate the exact lattice constant of the substance. The diffraction phenomena can be explained by the familiar Bragg equation:

$$
\begin{equation*}
\mathrm{N} \lambda=2 \mathrm{a} \sin \theta \tag{1}
\end{equation*}
$$

In this equation, $N$ (the order of diffraction), $\lambda$ (the wave length of the $X$-ray), and $\theta$ (Bragg angle of the diffraction spots) are used to find the value for $d$, which is the interplanar spacing of the substance under investigation; d has a close relationship to a (lattice constant), according to the equation:

$$
a=\frac{a}{\sqrt{h^{2}+k^{2}+1^{2}}} \cdots \cdots \cdot
$$

By substituting equations (2) for $\underline{d}$ in (3), Bragg formula for the calculation of lattice constants is obtained:

$$
\begin{equation*}
a=\frac{\lambda \sqrt{h^{2}+k^{2}+I^{2}}}{2 \sin \theta} \tag{3}
\end{equation*}
$$

In (3), the order of refraction is already multiplied by corresponding crystallographic indices. This equation clearly shows that for the precise determination of lattice constants, the accuracy of a depends only upon the angle $\theta$, as the correct Miller index is an integer, and
can be correctly obtained, while the wave length used is already known and determined with greatest precision. Bragg angle $\theta$ (or $Q$ ) used in equation (3) can be either under small angles in the front reflection or under large angles $\phi$ (or $\varphi$ ) in the back reflection (Fig. 14).


Figure 14. The Illm arrangement in the asymmetric method. $\ell$, the Bragg reflection angle; $\varphi$, the back reflection angle.

When $\theta$ is very large, i.e., nearly $90^{\circ}$, a small error in measuring gives only a small deviation in the value for $\sin \theta$, and will not affect the precision of the lattioe detemination too much. So interferences under large Bragg angles in the back reflection region are necessary for the determination of precise lattice constants. One more advantage in using large glacing angle $\theta$ is to reduce the
error due to absorption nearly to zero, and therefore, the shift of diffraction lines can be entirely neglected. Figure 14 shows that $\phi$, the back reflection angle, is complementary to $\theta$, which is the front reflection angle. The Bragg formula for the calculation of lattice constants from back reflection lines then becomes:

$$
\begin{equation*}
a=\frac{\lambda \sqrt{h^{2}+k^{2}+1^{2}}}{2 \operatorname{Cos} \phi} . \tag{4}
\end{equation*}
$$

where $\phi=90^{\circ}-\theta$
2. Indexing the Film and Selection of the Target The purpose of indexing the film is to find the Miller indices of the diffraction lines, in order to determine the lattice constants of the specimen. Besides this, it also helps to find the structure type and the atomic position of the sample. A convenient, graphical method, based on the principle of reciprocal lattice, (9) was used for indexing the cubic pattern in this research. In Figure 15 the cross section of the Debye-Scherrer camera is shown, indicating the path of the direct and reflected X-ray beams. From the fIgure, it follows directly:

$$
\begin{array}{rll}
\frac{X}{2 r} & =\sin \theta & \ldots . . . \\
\text { and } \quad \frac{P}{X} & =\sin \theta & \text {......(6) }
\end{array}
$$

Considering the inside wall of the camera as the sphere of reflection, the chord $X$ represents the reciprocal lattice
(9) Straumanis, M., Ze1t Kist, p. 104, 167, 1942


Figure 15. Path of the X-ray beam in a Debye-scherrer camera.

O, the powder sample;
$X$, Vector of the reciprocal lattice;
$P$, projection of the reciprocal lattice vector $X$ on the diameter of the camera;
$r$, radius of the camera.
vector of the three dimensional reciprocal lattice. Thus, $P$ is the projection of $X$ on the diameter of the camera.

From the Bragg equation,

$$
\lambda=2 \mathrm{~d} \sin \theta, \text { if } N=1
$$

It follows that:

$$
\begin{aligned}
& a^{2}=\frac{\lambda^{2} \sum h^{2}}{4 \sin ^{2}} \\
& \text { where } \quad \sum h^{2}=\left(h^{2}+k^{2}+1^{2}\right)
\end{aligned}
$$

By various combining of the equations (5) (6) and (7), one can obtain,

$$
p=\frac{r \lambda^{2}}{2 a^{2}} \sum h^{2}
$$

As $\frac{r \lambda^{2}}{2}$ is constant for the particular cubic sample and radiations, it follows from equation (8):

$$
\begin{equation*}
P=k \Sigma h^{2} \tag{9}
\end{equation*}
$$

Since the length of $P$, as obtained from other lines on the reflection circle, is proportional to $\Sigma h^{2}$, represssenting the integers $1,2,3,4$, etc., all differences of the projection $P$ of the subsequent reciprocal lattice vactors of a cubic substance are equal. There are no coresponding lines for $\Sigma h^{2}$, which cannot be split into integers of squares.

A reflection circle on a bigger scale, corresponding to the inside wall of a Debye-Scherrer camera, is shown in Figure 16. As an arbitrary camera, radius for copper radialLion $100 \mathrm{~mm} .{ }^{(10)}$ was chosen, and the radii for other radiallions are obtained as follows:

If the smallest $P$ is equal to $P_{m i n}$, resulting in $\Sigma h^{2}=1$, then

$$
\begin{equation*}
a=\lambda \sqrt{\frac{F}{P_{\min }}} \tag{10}
\end{equation*}
$$

Now, if $P$ is kept constant, the change in radiation from $\lambda_{1}$ to $\lambda_{2}$ will result in a changed radius $\left(r_{2}\right)$ of the reflection circle:

$$
\begin{equation*}
r_{2}=r_{1} \frac{\lambda_{1}^{2}}{\lambda_{2}^{2}} \tag{11}
\end{equation*}
$$

(IO) Straumanis, M., Am. Mineralogist, p. $37,48,1952$


Figure 16. The graphic method for indexing film and selecting radiation
(powder chromium)
where $r_{2}$, the radius of new radiation reflection circle.
$r_{1}$, the radius of $\mathrm{Cu}-\mathrm{K} \alpha_{1}$ radiation reflectron circle.
$\lambda_{2}$, the wave length of new radiation.
$\lambda_{1}$, the wave length of $\mathrm{Cu}-\mathrm{K} \alpha_{1}$ radiation.
If the radius of $C u-K \alpha_{1}$ radiation is taken as 100 mm ., then for other radiations are obtained as follows:
For Cu -K $\beta_{1}, \quad r_{2}=100 \times \frac{\lambda_{1}^{2}}{\lambda_{2}^{2}}=100 \times \frac{(1.5374)^{2}}{(1.3894)^{2}}=121.3 \mathrm{~mm}$.
For $\operatorname{Cr}-K \alpha_{1}, \quad r_{2}=100 \times \frac{\lambda_{1}^{2}}{\lambda_{2}^{2}}=100 \times \frac{(1.5374)^{2}}{(2.2850)^{2}}=45.3 \mathrm{~mm}$
For $\operatorname{Cr-X} B_{1}, \quad r_{2}=100 \times \frac{\lambda_{1}^{2}}{\lambda_{2}^{2}}=100 \times \frac{(1.5374)^{2}}{(2.0806)^{2}}=54.6 \mathrm{~mm}$
In Figure 16, the reflection circles for different radiations are drawn. A trial photograph was made with Cr radiation and so this reflection circle was taken as the base circle. The angles $2 \theta$, as measured from the film, are tabulated in Table III. They start from the origin 0 and are projected to a diameter of 200 mm . The distances from the origin 0 to the projection points are divided into the largest possible division (equal $P_{m i n}$ ), so that the end point of any one projection from the origin 0 is an integral multiple of $P_{m i n}$. This can be done by trial. The numerical number of a projection point is the $\Sigma h^{2}$ of the

## Table III

Values of the diffraction lines Cr- radiation

| No. of Ine | $2 \theta$ (degree). | K-radiation | $\underline{\mathrm{h}} \mathrm{k} \boldsymbol{l}$ |
| :---: | :---: | :---: | :---: |
| 2 | 61.527 | B | 110 |
| 2 | 68.458 | $\alpha$ | 110 |
| 4 | 105. 241 | $\alpha$ | 200 |
| 6 | 124. 661 | $\beta$ | 211 |
| 6 | 152.885 | $\alpha_{1}$ | 211 |
| 6 | 153.736 | $\alpha_{2}$ | 211 |

Miller indices of the corresponding reflection lines. Indexing of the film can also be made mathematically as shown in Table IV, but it is not so convenient as the graphioal method just mentioned. In selecting a proper radiation, the radiation whose reflection circle intersects closest to a $P$ division is the best. In Figure 16, the best radiation for chromium is copper and next, chromium. The glacing angle due to the copper radiation, however, is too large, i.e., nearly $90^{\circ}$, and is inside the collimator, and so cannot be seen on the film. Therefore, chromium was selected as the proper radiation in the measurements.

MATHEMATICAL INDEXING METHOD OF PURE CHROMIUM, CR-RADIATION


* 0.45007, calculated by the same method as in Table V.

3. Measurement of the Film

Diffraction lines on the film are measured by a comparator, as mentioned before. Each pair of lines was carefully measured three times in order to eliminate the visual error, and the average value used to calculate the Bragg angle $\theta$. How to calculate can be explained by a simple sketch as shown in Figure 17.


Figure 17. Schematic diagram of a simplified asymmetric powder pattern.

Suppose A, B, and C, D, are the corresponding pairs of lines, appearing in front-reflection and back-reflection regions. These rings are not circular, yet they are symmetrical to the outlet and inlet holes $I$ and $O$ on the film, although the symmetry lines of each pair of reflection may, or may not, coincide with the exact centers of the holes. The line $X-X^{l}$ in Figure 16 represents a millimeter scale, on which the film may be laid flat. Such a
film is measured as follows: The effective film circumference is equal to $(\overline{X D}+X C)-(\overline{X B}+\overline{X A})=20 I=360^{\circ}$; assuming that $(\bar{X} \bar{D}+\overline{X C})-(\bar{X} \bar{B}+\overline{X A})=200 \mathrm{~mm} .$, and 1 mm . on the film will be equal to 1.8 degree. As the angle measured is four times the glacing angle, so the factor $F=\frac{1.8}{4}=0.45$ will be obtained for conversion of the measured angular distances on the film into degree, in term of glacing angles. For example, if the distance between the lines of an interference pair is 10 mm ., then the glacing angle is $10 \times 0.45=4.5$ degrees. If the angle 1s in the front reflection region, then it is $\theta$ and is equal to 4. 5 degrees. If the interference lines are in the back reflection region, then the angle measured is $\phi=4.5^{\circ}$ degrees. The front reflection angle for the same pair then will be: $\theta=90-4.5=85.5$ degrees, because such a line lies in the back reflection region. The back reflection angle, for the interference $211 \alpha_{1}$, was used for the determination of lattice constants of chromium. An X-ray powder photograph of this metal is shown in Figure 18, and its measurement and calculation is given in Table $\overline{\text { g }}$.


Figure 18. X-ray powder photograph of chromium sample with ohromium radiation.

TABLE V
Record of Film Measurement and Calculation of Pure Chromium Chromium Radiation Room Temperature

Front Reflection


## 1. Lattice Constant

X-ray photographs to determine the exact value of the lattice constant of chromium were made with chromium powder, at ten degree intervals between $10^{\circ}$ and $50^{\circ} \mathrm{C}$. For each sample, three photographs were taken at every temperature, and the average value of the three lattice constants was considered to be the correct constant at that temperature. The lattice constants of four chromium samples are listed in Tables VI to $X$ and the changes of the lattice constants with the temperatures are shown in Figs. 19 to 22.

TABLE VI
Lattice Constants of Electrolytic Chromium at Different Temperatures

| Film <br> NO. | Temp $\left({ }^{\circ} \mathrm{C}\right)$ | $\begin{aligned} & \varphi_{211} \alpha_{1} \\ & \text { in degrees } \end{aligned}$ | Lattice Constants in kX | $\begin{gathered} \text { Lattice Constant } \\ \text { At in } k X \\ \text { (Average) } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| 788 | 13.0 | 13. 548 | 2.87870 |  |
| 789 | 13.0 | 13. 570 | 2.37897 | 2.87886 |
| 790 | 13.0 | 13. 567 | 2. 37891 |  |
| 791 | 20.0 | 13. 578 | 2. 87905 |  |
| 792 | 20.0 | 13. 580 | 2. 37908 | 2.37905 |
| 793 | 20.0 | 13.576 | 2. 87903 |  |
| 794 | 30.0 | 13. 584 | 2.87911 |  |
| 795 | 30.0 | 13. 593 | 2. 97923 | 2.37918 |
| 796 | 30.0 | 13. 590 | 2.87920 |  |
| 797 | 40.0 | 13. 591 | 2. 87920 |  |
| 798 | 40.0 | 13.607 | 2. 97941 | 2. 87928 |
| 799 | 40.0 | 13. 592 | 2. 37923 |  |
| 800 | 50.0 | 13. 593 | 2. 97923 |  |
| 801 | 50.0 | 13.625 | 2. 37962 | 2. 37947 |
| 802 | 50.0 | 13.620 | 2. 37950 |  |

## TABLE VII

Lattice Constants of sintered Electrolytic Chromium at Different Temperatures

| Film No. | Temp. <br> $\left({ }^{0} \mathrm{C}\right)$ | $Y_{211 \alpha_{1}}$ <br> in degrees | Lattice Constants in $k \times X$ | Lattice Constant A in kX (Average) |
| :---: | :---: | :---: | :---: | :---: |
| 811 | 10.0 | 13. 563 | 2.37888 |  |
| 810 | 10.0 | 13.554 | 2.37876 | 2. 87885 |
| 809 | 10.0 | 13. 566 | 2.87891 |  |
| 808 | 30.0 | 13.581 | 2. 37908 |  |
| 807 | 30.0 | 13. 568 | 2.87894 | 2.37905 |
| 806 | 30.0 | 13. 586 | 2.37914 |  |
| 805 | 50.0 | 13. 519 | 2. 37956 |  |
| 804 | 50.0 | 13. 615 | 2.37950 | 2.87951 |
| 803 | 50.0 | 13.613 | 2.87947 |  |

TABLE VIII

Lattice Constants of Iodide-Cr Low in Metallic Impurities at Different Temperatures

| F11m <br> No. | Temp. <br> $\left({ }^{0} \mathrm{C}\right)$ | $\varphi_{211} \alpha_{1}$ <br> in degrees | Lattice Constants in kX | Lattice Constant A in kX (Average) |
| :---: | :---: | :---: | :---: | :---: |
| 756 | 11.6 | 13. 569 | 2.97894 |  |
| 757 | 11.7 | 13.555 | 2.87876 | 2.37885 |
| 758 | 11.7 | 13.561 | 2.37885 |  |
| 753 | 13.0 | 13. 569 | 2.37894 |  |
| 754 | 13.0 | 13.543 | 2.87864 | 2.87874 |
| 755 | 13.0 | 13. 554 | 2.37876 |  |
| 759 | 20.0 | 13. 587 | 2.87917 |  |
| 760 | 20.0 | 13.554 | 2.97876 | 2.87898 |
| 761 | 20.0 | 13. 576 | 2.97903 |  |
| 765 | 30.0 | 13.591 | 2.37920 |  |
| 766 | 30.0 | 13.582 | 2.37911 | 2.87913 |
| 767 | 30.0 | 13.580 | 2.97908 |  |
| 762 | 40.0 | 13. 599 | 2.37929 |  |
| 763 | 40.0 | 13.573 | 2.97901 | 2.87916 |
| 764 | 40.0 | 13.587 | 2.87917 |  |
| 768 | 50.0 | 13. 505 | 2.97938 |  |
| 769 | 50.0 | 13.609 | 2.37944 | 2.87937 |
| 770 | 50.0 | 13.597 | 2.97929 |  |

## TABLE IX

Lattice Constants of Iodide-Cr Low in Non-Metallic Impurities at Different Temperatures

| $\begin{gathered} \text { Film } \\ \text { No. } \end{gathered}$ | Temp. $\left({ }^{\circ} \mathrm{C}\right)$ | $\varphi_{211} \alpha_{1}$ <br> in degrees | Lattice Constants in kX | Lattice Constant At in EX (Average) |
| :---: | :---: | :---: | :---: | :---: |
| 787 | 13.0 | 13.551 | 2.87873 |  |
| 786 | 13.0 | 13.553 | 2.37876 | 2.87876 |
| 785 | 13.0 | 13.556 | 2.87879 |  |
| 783 | 20.0 | 13.576 | 2.87903 |  |
| 782 | 20.0 | 13.579 | 2.87905 | 2.97902 |
| 781 | 20.0 | 13.570 | 2.87897 |  |
| 780 | 30.0 | 13.577 | 2.37903 |  |
| 779 | 30.5 | 13.588 | 2.87917 | 2.87909 |
| 778 | 30.0 | 13. 581 | 2. 87908 |  |
| 777 | 40.0 | 13. 594 | 2.87923 |  |
| 776 | 40.0 | 13. 591 | 2.87920 | 2.87921 |
| 775 | 40.0 | 13.590 | 2.87920 |  |
| 774 | 50.0 | 13. 505 | 2.37938 |  |
| 773 | 50.0 | 13.513 | 2.87947 | 2.87945 |
| 772 | 50.0 | 13.616 | 2.87950 |  |

TABLE X

## Lattice Constants of Four Chromium Samples at Different Temperatures

| Sample | $10^{\circ} \mathrm{C}$ | $20^{\circ} \mathrm{C}$ | $30^{\circ} \mathrm{C}$ | $40^{\circ} \mathrm{C}$ | $50^{\circ} \mathrm{C}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Electrolytic | 2.87886(130) | 2.87905 | 2.87918 | 2.87928 | 2.87947 |
| Sintered Eleotrolytic | 2. $37885\left(10^{\circ}\right)$ | - - | 2.37905 | - - | 2.87951 |
| Iodide (Low Metallic | 2.87898(120) | 2.87898 | 2.87913 | 2.87916 | 2.37937 |
| Iodide (Low NonMetallic) | 2.87876(130) | 2.87902 | 2.87909 | 2.87921 | 2.87945 |
| Average | 2.87886(120) | 2.87902 | 2.87911 | 2.37922 | 2.87945 |


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PFURE 22
2. Coefficient of Expansion

The coefficient of thermal expansion of chromium was calculated from the average lattice constants at different temperatures, using the following equation:

$$
\begin{equation*}
\alpha=\frac{a_{t_{2}}-a_{t_{1}}}{a_{t_{1}}\left(t_{2}-t_{1}\right)} \tag{12}
\end{equation*}
$$

$\alpha$, being the linear coefficient of thermal expansion $a_{t_{2}}$ being lattice constant at temperature $t_{2}$
$a_{t_{1}}$ being lattice constant at temperature $t_{1}$
The accurate coefficient of thermal lattice expansion is obtained by taking the average value ${ }^{(11)}$ of all possible combinations of lattice constants at $10^{\circ}, 20^{\circ}, 30^{\circ}, 40^{\circ}$, and $50^{\circ}$ C. This value is considered to be the most probable coefficient of thermal expansion of the respective samples between $10^{\circ}$ and $50^{\circ}$ C. The method of calculation is shown in the sample of sintered electrolytic chromium:

$$
\begin{aligned}
& a_{50}=2.87951, \quad a_{30}=2.37905, \quad a_{10}=2.87885 \\
& \alpha_{(50-30)}=\frac{a_{50}-a_{30}}{a_{30}(50-30)}=\frac{2.37951-2.87905}{2.87905 \times 20}=7.99 \times 10^{-6} \\
& \alpha_{(50-10)}=\frac{a_{50}-a_{10}}{a_{10}(50-10)}=\frac{2.37951-2.37885}{2.37885 \times 40}=5.73 \times 10^{-6} \\
& \alpha_{(30-10)}=\frac{a_{30^{-}} a_{10}}{a_{10}(30-10)}=\frac{2.87905-2.87885}{2.87885 \times 20}=3.47 \times 10^{-6}
\end{aligned}
$$

(11). Strauman1s, M., anorg Chem., p. 238, 175, 1938.

Average $\alpha_{(10-50)}=\frac{7}{3}(7.99+5.73+3.47)=5.73 \times 10^{-6}$
The average coefficients of thermal expansion obtained for four different chromium samples, ranged from 4.92 x $10^{-6}$ to $6.14 \times 10^{-6}$, between the temperature range of $10^{\circ}$ and $50^{\circ} \mathrm{C}$, as shown in Tables XI to $X V$. These ifgures are very close to the results obtained by Hidnert, (8) ranging from $5.2 \times 10^{-6}$ to $6.6 \times 10^{-5}$ in the temperature range of $20^{\circ}$ and $60^{\circ}$ C. Neither inflection points near $38^{\circ} \mathrm{C}$, as obtained by MacNair, ${ }^{(2)}$ nor a minimum in the coefficients of expansion near $37^{\circ} \mathrm{C}$, as found by Fine, ${ }^{(1)}$ were observed.

## TABLE XI

Coefficient of Thermal Expansion of Electrolytic Chromium Between $13^{\circ}$ and $50^{\circ} \mathrm{C}$. See Table VI.
$50-40$
10
20
30
37
$50=20$
$50-13$
$40-30$
40-20
40-13
10
20
27
30-20
30-13
20-13
Average
$50-30$
$50-20$

10
17
7
-
6.60
5.04
4.36
5.73
3.47
3.99
5. 40
4. 51
6.54
9. 43
5.56

TABLE XII

Coefficient of Thermal Expansion of Sintered Electrolytic Chromium Between $10^{\circ}-50^{\circ} \mathrm{C}$. See Table VII.

| Combinations | $\Delta t$ | $\alpha \cdot 10^{-6}$ |
| :---: | :---: | :---: |
| $50-30$ | 20 | 7.99 |
| $50-10$ | 40 | 5.73 |
| $30-10$ | 20 | 3.47 |
| Average | - | 5.73 |

## TABLE XIII

Coefficient of Thermal Expansion of Iodide-Chromium (Low Metallic) Between $12^{\circ}$ and $50^{\circ}$ C. See Table VIII.
Combinations $\Delta t \quad \alpha \cdot 10^{-5}$
$50=40$
$50=30$
$50=20$
$50=12$
$40=30$
$40=20$
$40=12$
$30=20$
$30=12$
$20=12$

10
20
30
50-12
40-30
38
10
40-20
20
28
10
30-12
20-12
18
8
Average
-
7.29
4.17
4. 51
5. 21
1.04
3.13
4.47
5. 21
6.37
7.81
4. 92

## TABLE XIV

Coefficient of Themal Expansion of Iodide-Chromium (Low Non-Metallio) Between $13^{\circ}$ and $50^{\circ} \mathrm{C}$. See Table IX.

| Combinations | $\Delta t$ | $\alpha \cdot 10^{-5}$ |
| :---: | :---: | :---: |
| $50-40$ | 10 | 8.34 |
| $50-30$ | 20 | 6.25 |
| $50-20$ | 30 | 4.98 |
| $50-13$ | 37 | 6.48 |
| $40-30$ | 10 | 4.17 |
| $40-20$ | 27 | 3.30 |
| $40-13$ | 10 | 2.79 |
| $30-20$ | 17 | 6.74 |
| $30-13$ | 7 | 12.90 |
| $20-13$ | - | 6.14 |

TABLE XV
Coefficient of Thermal Expansion of Four Different Kinds of Chromium Samples Between $10^{\circ}$ and $50^{\circ} \mathrm{C}$.

| Chromium Sample | Ave. coeff. of expansion $\alpha \times 10^{-6}$ |
| :--- | :--- |
| Electrolytic | 5.56 |
| Sintered electrolytio | 5.73 |
| Iodide (low metallics) | 4.92 |
| Iodide (low non-metallics) | 6.14 |
| Average | 5.59 |

The fluctuations of the lattice constants can now be determined by means of the expansion coefficient (Table XV) after the reduction to $20^{\circ}$ of the constants shown in Table X. The equation used was:

$$
\begin{array}{r}
\alpha=\frac{a_{t_{2}}-a_{t_{1}}}{a_{t_{1}}\left(t_{2}-t_{1}\right)}=\frac{a_{20}-a_{t_{1}}}{a_{t} \Delta t} \\
\text { or } a_{20}=a_{t}+\alpha a_{t} \Delta t \quad \ldots \tag{13}
\end{array}
$$

Sintered electrolytic chromium is taken as an example (See Table VII and Table XII):

$$
a_{t} \quad \alpha a_{t} \Delta t \quad a_{20^{\circ}} \text { in } k x
$$

For $10^{\circ} \mathrm{C}, a=2.87885+0.00016=2.87901$
For $30^{\circ} \mathrm{C}, a=2.87905+0.00016=2.87889$
For $50^{\circ} \mathrm{C}, a=2.87951+0.00050=2.87901$

$$
\begin{aligned}
\text { Average value } & =\frac{1}{3}(2.87901+2.87889+2.87901) \\
& =2.87897 \pm 0.00005
\end{aligned}
$$

Figures for the other three kinds of chromium samples are 11sted in Table XVI.
3. Correction of Refraction and The Precise Lattice Constants

From a measured glacing angle $\theta$, not the actual lattice constant, but a little smaller value obtained due to the deviation from Bragg's Law. These deviations are greater, the lower the order of the diffraction. The corrected Bragg Equation, as derived by Ewald, (12) may be
(12). Wien, W., Handb. d. Experimental Phys., p. 24, 94, 1930.
written as follows:

$$
\mathrm{N} \lambda=2 \mathrm{~d}\left(1-5.40 \rho \frac{\mathrm{~d}^{2}}{\mathrm{~N}^{2}} 10^{-6}\right) \sin \theta_{\mathrm{N}} \cdot \ldots(14)
$$

or, for more convenient use:

$$
\begin{equation*}
a_{0}=a_{N}\left(1+\frac{5.4 a^{2} \rho}{N^{2} \sum h^{2}} 10^{-6}\right) \tag{15}
\end{equation*}
$$

where: $a_{0}$, corrected constant
$a_{N}$, experimental constant
$N$, order of diffraction
$P$, density of the crystal
$\Sigma h^{2}$, quadratic sum of the Miller indexes
Equation (15) is valid only for the symmetrical reflecLion. The calculation for the refraction correction is as follows:
In this experiment, $a=2.87942, \quad \rho=7, N=1, \quad \sum h^{2}=6$

$$
\begin{aligned}
a_{0}= & 2.87942\left(1+\frac{5.4 \times(2.87942)^{2} \times 1}{1 \times 6} 10^{-6}\right. \\
& 2.87942(1+0.0000522) \\
& 2.87942+0.00015 \\
& 2.87957
\end{aligned}
$$

So 0.C0015 was taken as the correction for refraction. Figures after correction of refraction for four different kinds of samples are tabulated in Table XVI, and these figures represent the precise lattice constants of chromium at $20^{\circ}$ C. Moreover, as Barrett ${ }^{(13)}$ pointed out that the
(13). Barrett, C. S., Structure of Metals, McGraw-Hill, New York, p. 150, 1952.
refraction should be omitted in lattice constant determination, the correction for refraction in Table XVI is therefore added separately, so that the uncorrected figure is also available.

Table XVI shows that the precise lattice constant of electrolytic chromium at $20^{\circ} \mathrm{C}$ is $2.87914 \mathrm{kX} \pm 0.00005$, and of 1odide chromium at $20^{\circ} \mathrm{C}, 2.87908 \mathrm{kX} \mathrm{I} 0.00007$. These figures are very close to Fine's 2.87900 kX at $20^{\circ}$ C and to Wood's 2.8796 kX at $18^{\circ} \mathrm{C}$. As lodide chromium 18 purer than electrolytic chromium (see Table II), the lattice constant of the former is smaller. So the lattice constant of pure chromium becomes larger if impurities are present.

The Precise Lattice Constants of Four Different Chromium Samples


## DISCUSSION

1. Correction of Absorption

Using the equipment and procedures outlined above, X-ray films were taken of four different chromium samples, three films measured, and the lattice constants calculated. If smaller glacing angles in the front reflection region are used, then smaller lattice constants than the actual values are obtained, due to absorption. Consequently, in these cases it is necessary to make a correction for absorption in order to get the correct lattice constants. In the present investigation, a plot was made of lattice constants against different glacing angles, as shown in Figure 23. The relation between the lattice constant and angle of reflection was not linear, but was a curve, with the lattice constants changing rapidly at small angles, and remaining nearly horizontal from $75^{\circ}$ to $90^{\circ}$. The curve shows that the constant, obtained by extrapolation to $90^{\circ}$, falls completely within the range of fluctuations of the constants, as calculated from the diffraction lines (211) $\alpha_{1}$ and $\alpha_{2}$. Therefore, there was no need for an absorption correction for the lattice calculated.
2. Calculation of Atomic Weight

Though the lattice constants of chromium samples could be obtained with great accuracy, without using any standard substance, the accuracy of the absolute value of these constants is still not known. The absolute value of the


lattice constants can be tested by calculating the atomic weight, from the lattice constants obtained, and the density, of the respective substances. By comparing the resulting X-ray density atomic weight with its chemical weight, a conclusion concerning the precision of the X-ray method can be drawn. The atomic welght was calculated from the formula:

$$
A_{X}=K N_{s} V \frac{d}{N} \quad . \cdot . \cdot . \cdot(16)
$$

where

$$
\begin{aligned}
A_{x}= & X \text {-ray atomic weight (g/mol) } \\
\mathrm{K}= & 1.0002 \text { (factor resulting from the correc- } \\
& \text { tion of the molecular weight of calcite) } \\
\mathrm{N}_{\mathrm{s}}= & 6.0594 \times 10^{23} \text { (siegbahn's Avogadro number) } \\
\mathrm{V}= & \text { Volume of the unit cell ( } k x^{3} \times 10^{-24} \text { ) } \\
\mathrm{d}= & \text { Density ( } \mathrm{g} / \mathrm{cc} \text { ) } \\
\mathrm{N}= & \text { Number of atoms per unit cell. }
\end{aligned}
$$ Now, lattice constant of electrolytic chromium at $20^{\circ}=$ $2.87915 \mathrm{kX}, \mathrm{d}=7.03 \mathrm{~g} / \mathrm{co}, \mathrm{N}=2$. Therefore,

$$
\begin{aligned}
A_{x}= & 1.0002 \times 6.0594 \times 10^{23} \times(2.87915)^{3} \times \frac{7}{2} 03 \\
& 50.63 \mathrm{~g} \cdot
\end{aligned}
$$

The chemical atomic weight of chromium is 52.01 g. The difference between the two figures is probably due to the incorrect value for the specific gravity of chromium, concerning which until now no accurate data have been published. Otherwise, it would mean that there are vacancies in the chromium crystals. So the value of lattice constant of chromium can not be checked in this way, because of the insufficiently accurate density value.
3. Calculation of X-ray Density

The correct density of chromium, assuming that there are no imperfections in chromium crystals, can be calculated from the chemical atomic weight, and the precise lattice constant. For this X-ray density of chromium, the same equation (16) can be used:

$$
\begin{equation*}
d_{x}=\frac{A N}{K N_{s} V} \tag{17}
\end{equation*}
$$

where

$$
a_{x}=x \text {-ray density }(g / c c) \text { at } 20^{\circ} c
$$

$$
A=\text { The chemical atomic weight } 52.01 \mathrm{~g}
$$

$$
N=2
$$

$$
K=1.0002
$$

$$
N_{s}=6.0594 \times 10^{23}
$$

$$
V=(2.87915 \mathrm{kX})^{3} \times 10^{-24} \text { at } 20^{\circ} \mathrm{C}
$$

Therefore,

$$
\mathrm{d}=\frac{52.01 \times 2 \times 10^{24}}{1.0002 \times 6.0594 \times 10^{23} \times(2.87915)^{3}}=
$$

$$
7.1913 \mathrm{~g} / \mathrm{cc} \text { instead of } 7.03 \mathrm{~g} / \mathrm{cc}
$$

as obtained using rough density measurements.

## SUMMARY

1. Each of four different pure chromium samples was thoroughly ground, placed in a quartz tube, the tube evacuated, then sealed, and the whole treated at $850^{\circ} \mathrm{C}$ for two and a half hours.
2. The fine powder in the heat treated sample, which would pass a 325 mesh screen, was used for the preparation of a sample for X -ray diffraction.
3. The powder patterns were indexed and a target of proper wave length selected, so that sharp diffraction lines with the largest possible glacing angles could be obtained. This radiation was that of chromium.
4. Next, the diffraction lines obtained were used to calculate the lattice constants of the sample.
5. Due to the very thin powder sample and large glacing angle, the absorption correction was neglected.
6. A refraction correction of 0.00015 kX was calculated and oan be added to the final results if necessary (see Table XVI).
7. The lattice constants obtained at $20^{\circ} \mathrm{C}$ were 2.37914 kX $\pm 0.00005$ for eleotrolytic chromium, and 2.97908 kX $\pm 0.00007$ for lodide chromium (see Table XVI).
8. The average coefficients of expansion caloulated ranged from $4.92 \times 10^{-6}$ to $6.14 \times 10^{-5}$ within the temperature range of $10^{\circ}$ to $50^{\circ} \mathrm{C}$ (see Table XV).
9. The calculated atomic weight of chromium was 50.63 g., showing the incorrect density previously determined. 10. The X-ray density of chromium was $7.1913 \mathrm{~g} / \mathrm{cc}$ at $20^{\circ} \mathrm{C}$.

## APPENDIX

Film Measurements. The individual line readings and lattice constant calculations from all films measured during experiments are recorded below. Cr- radiation is used. Two sets of lines in the front reflection, namely (110) $\alpha$, (110) $B$, and two sets of lines in the back reflection, namely (21l) $\alpha_{1}$, $(211)^{\alpha_{2}}$, are measured and $(211) \alpha_{1}$ is used to calculate the lattice constant of chromium.

1. Electrolytic Chromium





| Film \#757 | $\begin{gathered} \text { Temperature } 11.7^{0} \mathrm{C} \\ 171.042+140.903=311.945 \\ 170.526+141.423=311.949 \end{gathered}$ | 311.947 | 30.139 |
| :---: | :---: | :---: | :---: |
| Line read | $\begin{aligned} & 93.963+17.858=111.821 \\ & 90.176+21.662=111.838 \end{aligned}$ | 111.830 |  |
| $F=0.44974$, | Circumference $\phi^{0}=13.555, \quad \operatorname{Cos} \phi=0$ | $\begin{aligned} & 200.117 \\ & 97215, \end{aligned}$ | $a=2.87876$ |
| Film \#758 | $\begin{gathered} \text { Temperature } 11.7^{0} \mathrm{C} \\ 172.785^{-t} 142.613=315.398 \\ 172.266+143.126=315.392 \end{gathered}$ | 315.395 | 30.172 |
| Line read | $\begin{aligned} & 95.690+19.447=115.137 \\ & 91.874+23.290=115.164 \end{aligned}$ | 115.151 |  |
|  | $\text { 5, } \quad \phi^{0}=13.561, \quad \text { Cos } \phi=0 \text {. }$ | $\begin{aligned} & 200.244 \\ & 97212, \end{aligned}$ | $a=2.87885$ |
| F11m \#753 | $\begin{array}{r} \text { Temperature } 13^{0} C \\ 175.664 \pm 145.507=311.171 \\ 175.170+146.007=311.177 \end{array}$ | 311.174 | 30.157 |
| Line read | $\begin{aligned} & 98.640+22.508=111.148 \\ & 94.810+26.353=111.163 \end{aligned}$ | 111.156 |  |
| $F=0.44996$, | Circumference <br> 6, $\phi^{0}=13.569, \operatorname{Cos} \phi=0$ | $\begin{aligned} & 200.018 \\ & 97209, \end{aligned}$ | $a=2.87894$ |
| F11m \#754 | $\begin{array}{r} \text { Temperature } 13^{0} \mathrm{C} \\ 173.187 \pm 143.079=316.266 \\ 172.675+143.582=316.257 \end{array}$ | 316.262 | 30.108 |
| Line read | $\begin{aligned} & 96.142+20.031=116.173 \\ & 92.340+23.857=116.197 \end{aligned}$ | 116.185 |  |
| $F=0.44983$, | Circumference <br> 3, $\phi^{0}=13.543, \operatorname{Cos} \phi=0$. | $\begin{aligned} & 200.077 \\ & 97219, \end{aligned}$ | $a=2.87864$ |
| F11m \#755 | $\begin{array}{r} \text { Temperature } 13^{0} \mathrm{C} \\ 171.695 \pm 141.574=313.269 \\ 171.191+142.084=313.275 \end{array}$ | 313.272 | 30.121 |
| Line read | $\begin{aligned} & 94.703 \& 18.558=113.261 \\ & 90.905+22.371=113.276 \end{aligned}$ | 113.269 |  |
|  | Circumference <br> 9, $\phi^{0}=13.554, \operatorname{Cos} \phi=0$ | $\begin{aligned} & 200.003 \\ & 37215, \end{aligned}$ | $a=2.87876$ |
| F11m \#759 | $\begin{array}{r} \text { Temperature } 20^{0} \mathrm{C} \\ 173.620+143.432=317.052 \\ 173.095+143.952=317.047 \end{array}$ | 317.050 | 30.188 |
| Line read | $\begin{aligned} & 96.584+20.496=117.080 \\ & 92.776+24.320=117.096 \end{aligned}$ | 117.088 |  |
|  | 9, $\phi^{0}=13.587$ Circumference ${ }^{\text {a }}$, Cos $\phi=0$. | $\begin{aligned} & 199.962 \\ & 27201, \end{aligned}$ | $a=2.87917$ |
| F11m \#760 | $\begin{array}{r} \text { Temperature } 200 \mathrm{C} \\ 173.407 \pm 143.274=316.681 \\ 172.926+143.761=316.687 \end{array}$ | 316.684 | 30.133 |
| Line read | $\begin{aligned} & 96.365+20.231=116.596 \\ & 92.564+24.051=116.615 \end{aligned}$ | 116.602 |  |
| $F=0.44982$, | Circumference <br> 2, $\phi^{0}=13.554, \quad \cos \phi=0$ | $\begin{aligned} & 200.082 \\ & 97215, \end{aligned}$ | $a=2.87876$ |



| Film \#768 | Temperature $50^{\circ} \mathrm{C}$ |  |  |
| :---: | :---: | :---: | :---: |
|  | 173.608 $\pm 143.445=317.053$ |  | 30.163 |
|  | $173.110+143.935=317.045$ | 317.049 |  |
| Line read | $96.725+20.775=117.500$ |  |  |
|  | $92.915+24.610=117.525$ | 117.513 |  |
|  | Circumference | 199.556 |  |
| $F=0.45105$, | 5, $\phi^{0}=13.605, \operatorname{Cos} \phi=0$ 。 | 97194, | $a=2.87938$ |
| Film \#769 | Temperature $50^{\circ} \mathrm{C}$ |  |  |
|  | 173.327 143.192 = 316.519 | 316.515 | 30.135 |
|  | $172.792+143.718=316.510$ | 316.515 | 30.135 |
| Line read | 96. $514+20.712=117.226$ |  |  |
|  | $92.731+24.502=117.233$ | 117.230 |  |
|  | Circumference | 199.285 |  |
| $F=0.45161$, | 1, $\phi^{0}=13.609, \operatorname{Cos} \phi=0$. | 97192, | $a=2.87944$ |
| F11m \#7ro | Temperature $50^{\circ} \mathrm{C}$ |  |  |
|  | 172.685 - 142.565 = 315.250 |  | 30.120 |
|  | 172.200 $+143.058=315.258$ | 315. 254 |  |
| Line read | $95.840+20.034=115.874$ |  |  |
|  | $92.042+23.852=115.894$ | 115.884 |  |
|  | Circumference | 199.370 |  |
| $F=0.45142$, | 2, $\phi^{0}=13.597, \operatorname{Cos} \phi=0$. | 97197, | $a=2.87929$ |
| IV. Iodide Chromium (Low Non-Metallic) |  |  |  |
| Film \#787 | Temperature $13^{\circ} \mathrm{C}$ |  |  |
|  | 173.338 ${ }^{\text {2 }} 143.303=316.641$ |  | 30.035 |
|  | $172.862+143.789=316.651$ | 316. 646 |  |
| Line read | 96. 4.88 ง 20.684 $=117.172$ |  |  |
|  | 92.688 24.490 $=117.178$ | 117.175 |  |
|  | Circumference | 199.4.71 |  |
| $F=0.45119$, | 9, $\phi^{0}=13.551, \operatorname{Cos} \phi=0$. | $97216,$ | $a=2.87873$ |
| F1lm \#786 | Temperature $13^{\circ} 0$ |  |  |
|  | 176.356 - 146.312 = 322.668 |  | 30.044 |
|  | $175.883+146.793=322.676$ | 322. 672 |  |
| Line read | $99.525+23.519=123.144$ |  |  |
|  | $95.761+27.103=123.164$ | 123.154 |  |
|  | Circunference | 199.518 |  |
| $F=0.45109$, | 9, $\phi^{0}=13.553, \operatorname{Cos} \phi=0$. | 97215, | $a=2.87876$ |
| Film \#785 | Temperature $13^{\circ} \mathrm{C}$ |  |  |
|  | $173.972 \pm 143.910=317.882$ |  | 30.062 |
|  | 173.506 + 144.384 = 317.890 | 317.886 |  |
| Line read | $97.122+21.169=118.291$ |  |  |
|  | $93.342+24.968=118.310$ | 118.301 |  |
|  | Circumference | 199. 585 |  |
| $F=0.45094$, | 4, $\phi^{0}=13.556, \operatorname{Cos} \phi=0$. | 97214, | $a=2.87879$ |



| F11m \#776 | Temperature $40^{\circ} \mathrm{C}$ |  | 30.137 |
| :---: | :---: | :---: | :---: |
|  | $171.461{ }^{2} 141.324=312.785$ | 312.787 |  |
| Line read | $171.000+147.788=312.788$ |  |  |
|  | $90.770+22.462=113.232$ | 113.227 |  |
| $F=0.4509$ | 0 O Circumference | 199.560 |  |
|  | , $\phi^{0}=13.591, \operatorname{Cos}$ |  | $a=2.87920$ |
| F1lm \#775 | Temperature $40^{\circ}$ | 318.050 | 30.146 |
|  | $174.095 \pm 143.949=318.044$ |  |  |
|  | $173.591+144.464=318.055$ |  |  |
| Line read | $97.175+21.223=118.398$ | 118.400 |  |
|  | $93.370+25.032=118.102$ |  |  |
| $F=0.4507$ | , $\phi^{0}=13.590, \operatorname{Cos} \phi$ | $\begin{aligned} & \text { 199. } 650 \\ & 972000, \end{aligned}$ |  |
| Film \#774 | Temperature $50^{\circ} \mathrm{C}$ | 319.902 | 30.146 |
|  | 175.025 T 144.879 = 319.904 |  |  |
|  | $174.51 .8+145.382=319.900$ |  |  |
| Line read | $98.150+-22.314=120.464$ | 120. 472 |  |
|  | $94.346+26.134=120.480$ |  |  |
| $F=0.45129$ | , $\phi^{0}=13.605, \operatorname{Cos} \phi=$ | $\begin{aligned} & 199.430 \\ & 97194, \end{aligned}$ | $a=2.37938$ |
| F1lm \#r73 | Temperature $50^{\circ} \mathrm{C}$ | 315.483 | 30.173 |
|  | $172.828 \pm 142.655=315.483$ |  |  |
|  | $172.324+143.158=315.182$ |  |  |
| Line read | $95.913+20.083=116.001$ | 115.996 |  |
|  | $92.096+23.895=115.991$ |  |  |
| $\mathrm{F}=0.4511$ | - Circumference | 199.487 |  |
|  | 6, $\phi^{0}=13.613, \operatorname{Cos} \phi=$ |  | $a=2.87947$ |
| F11m \#772 | Temperature $50^{\circ} \mathrm{C}$ | 315.415 | 30.188 |
|  | 172.799 $\ddagger 142.611=315.410$ |  |  |
|  | $172.353+143.0677=315.420$ |  |  |
| Line read | $95.868+20.011=115.879$ | 115.873 |  |
|  | $92.068+23.798=115.866$ |  |  |
| $F=0.45103$ | , $\phi^{0}=13.616, \cos \phi=0$ | $199.542$ | $a=2.87950$ |

## BIBLIOGRAPHY

1. Barrett, C. S., Structure of Metals, McGraw-Hill, New York, 150 pp., 1952.
2. Chevenard, P., Comptes rend., p. 174, 109, 1922.
3. Disch, J., Z. Physik, p. 5, 173, 1921.
4. Fine, M. E., J. of Metals Tran., AIME, p. 189, 56, 1951.
5. Hidnert, P., J. of Research of the National Bureau of standards, p. 26, 81, 1941.
6. Hull, A. W., Phy. Rev., p. 14, 540, 1919.
7. Laue, M., Ann. Physik, p. 41, 971, 1913.
8. MacNair, D., Rev. of Scientific Instruments, p. 12, 66, 1941.
9. Straumanis, M., Ze1t Krist, p. 104, 167, 1942.
10. Straumanis, M., Am. Mineralogist, p. 37, 48, 1952.
11. Straumanis, M., Anorg. Chem., p. 238, 175, 1938.
12. Wein, W., Handb. d. Experimental-Phys., p. 24, 94, 1930.
13. Westgren, $A_{0}, ~ J . ~ I r o n ~ a n d ~ S t e e l ~ I n s t i t u t e, ~ p . ~ l l 7, ~$ 383, 1928.


## VITA

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