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The effect of the convection coefficient on the temperature amplitude in periodic heat flow

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THE EFFECT OF THE CONVECTION COEFFICIENT ON THE TEMPERATURE AMPLITUDE IN PERIODIC HEAT FLOW

BY

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A

THESIS

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INTRODUCTION

The problem to be investigated in this thesis is that of periodic heat flow in a semi-infinite solid. Periodic flow means that heat flow is a continuous function of time and repeats itself at regular intervals. A semi-infinite solid is one which is bounded by one and only one plane.

This subject has received considerable attention in the field of soil temperatures, both at the surface of the earth and at various depths. The surface of the earth is subjected to temperature changes which are nearly periodic. These temperature changes take place both daily and annually. A knowledge of these fluctuations is helpful in deciding such things as the depth at which water mains will be out of danger of freezing.

Its importance is not limited to problems on the earth's soil. The subject has also received attention in the fields of heat flow in cylinder walls. It is of interest in the field of temperature stresses where these stresses are set up by expansions and contractions of the material subjected to cyclic temperatures.

Because the analytical treatment becomes very involved in the more complex problems, these problems are usually attacked from the physical measurement standpoint.

This particular subject was chosen by the author because thus far all analytical treatment has been on the basis of heat flow taking place by conduction alone. It is the object of this paper to investigate the feasibility of considering the effect of both convection and conduction on the flow of heat.

REVIEW OF LITERATURE

An examination of the literature available on this subject reveals that it is possible to classify the work already done into two distinct groups. The first group is an analytical approach where the variables are determined by mathematics. The other group is the result of experimentation. Evidence and data are collected and developed until they are of value in some particular application of periodic heat flow.

On the analytical side, the equations for simple cases of periodic heat flow were derived first by H. S. Carslaw.[1]

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(1) H. S. Carslaw, Introduction to the Mathematical Theory of the Conduction of Heat in Solids, 2nd Ed., N. Y., Dover Publications, Am. Ed., 1945, pp. 47-50.

These equations have been re-derived in a somewhat simpler manner by L. R. Ingersoll.[2]

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(2) L. R. Ingersoll, O. J. Zobel, and A. C. Ingersoll, Heat Conduction With Engineering and Geological Applications, 1st Ed., N. Y., McGraw-Hill, 1948, pp. 45-47.

There are other publications by these same men deriving these same equations, but there is no significant change from the references given.

Most of the available literature on the practical application are found in the form of papers. However, some information in a condensed form may be found in a book by L. R. Ingersoll, O. J. Zobel, and A. C. Ingersoll.(3)

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(3) L. R. Ingersoll, O. J. Zobel, and A. C. Ingersoll, Heat Conduction With Engineering and Geological Applications, 1st Ed., N. Y., McGraw-Hill, 1948, pp. 47-57.

Information on the annual temperature wave is available in literature by Fitton and Brooks,(4) Tamura,(5) and Birge, Juday, and March.(6)

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(4) E. M. Fitton and C. F. Brooks, Monthly Weather Review, Vol. 59, 1931, pp. 6-16.

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(5) S. T. Tamura, Monthly Weather Review, Vol. 33, 1905, p. 296.

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(6) E. A. Birge, C. Juday, and H. W. March, Trans. Wisconsin Acad. Sci., Vol. 23, 1927, pp. 187-231.

The subject of periodic heat flow in cylinder walls has received attention by Callendar and Nicolson.(7)

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(7) H. L. Callendar and J. T. Nicolson, Proc. Inst. Civil Engrs. (London), Vol. 131, 1895, p. 147.

Some reference material is available on the subject of thermal stresses. Most of this material comes from Timoshenko.[8][9]

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(8) S. Timoshenko, Theory of Elasticity, N. Y., McGraw-Hill, 1934, p. 203.

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(9) S. Timoshenko and G. H. MacCullough, Elements of Strength of Materials, 2nd Ed., N. Y., D. Van Nostrand, 1940, p. 20.

In all the literature available from these sources, there was none that considered the flow of heat as taking place by the combined effects of conduction and convection.

TABLE OF UNITS

SYMBOL	UNITS	SIGNIFICANCE
θ	hour	time
t	F	temperature, degrees F.
T	R	temperature, degrees R.
Q	Btu	quantity of heat
x	ft	distance normal to surface
k	Btu $hr^{-1} ft^{-1} F^{-1}$	thermal conductivity
A	ft^2	area
q	Btu hr^{-1}	heat flow per unit time
C	Btu $lb^{-1} F^{-1}$	specific heat
ρ	lb ft^{-3}	density
α	ft^2 hr^{-1}	diffusivity
ω	hr^{-1}	frequency
S		constant
a		constant
b		constant
i	$\sqrt{-1}$	constant
γ		constant

DISCUSSION

Periodic heat flow occupies a unique position between steady and unsteady state heat flow. This is because all the heat that is transferred into the solid during the hot portion of the cycle is transferred out during the colder part with constant regularity.

This paper is interested in the application of a cyclic temperature function to the surface of a semi-infinite solid. In this particular case the conduction of heat will be in one direction, normal to the surface of the solid. Since the heat flow will take place in only one direction, the formulas will be derived with this in mind.

Since any study of periodic heat flow within a substance is primarily a study of conduction, it will be found to follow Fourier's law as set forth in his conduction equation. This law expressed mathematically is

$$\frac{dQ}{d\theta} = -kA\frac{dt}{dx} \qquad (1)$$

where dQ is the amount of heat flowing in differential time $d\theta$, A is the area of the section across which Q is flowing, $-dt/dx$ is the temperature gradient or the rate of change of temperature, t, with respect to the length of path x, and k is the proportionality factor known as the thermal conductivity of the material. The area, A, of the section is taken normal to the direction of the heat flow.[10]

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(10) W. H. McAdams, Heat Transmission, 2nd Ed., N. Y., McGraw-Hill, 1942, pp. 6-7.

In order to arrive at a more general equation for heat flow by conduction, it is necessary to use the following analytical reasoning. A small rectangular solid whose faces are parallel to the coordinate planes x, y, and z is assumed. The lengths of the respective sides of this differential cube will then be dx, dy, and dz.

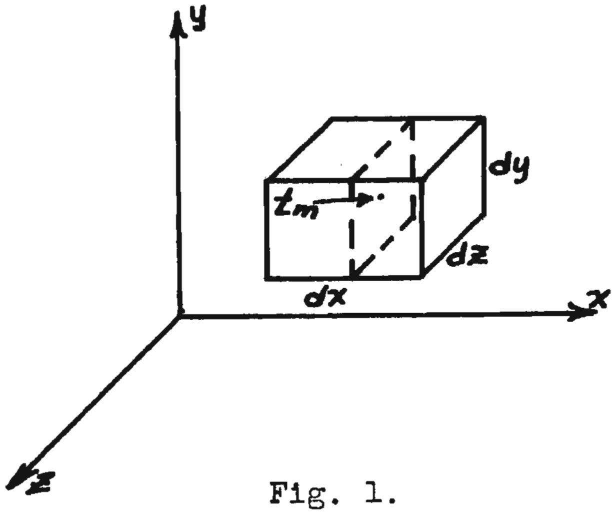

Fig. 1.

Since heat flowing in one direction is the main interest of this paper, the temperature at any given plane parallel to the surface upon which the cyclic temperature is being applied is assumed to be uniform at any time θ. Specifically, heat conduction may take place in the x direction which is normal to the plane bounding the semi-infinite solid. In this case, by the former reasoning, we can choose any plane parallel to one formed by the y-z axes as being one of uniform temperature at any time θ. Choosing such a plane half way through the existing cube, the temperature of this plane may be called t_m.

Now if heat is flowing in the positive x direction, as indicated by the arrow, the temperature on the left dy-dz face will necessarily be more than that on the right dy-dz face. The temperatures of these faces may be written

$$t_L = t_m - \left(\frac{\partial t}{\partial x}\right)\frac{dx}{2} \tag{2}$$

and

$$t_R = t_m + \left(\frac{\partial t}{\partial x}\right)\frac{dx}{2}\,. \tag{3}$$

Thus the actual temperature difference between the right and left face of this cube will be the difference between their respective temperatures.

$$t_L - t_R = t_m - \left(\frac{\partial t}{\partial x}\right)\frac{dx}{2} - t_m - \left(\frac{\partial t}{\partial x}\right)\frac{dx}{2}$$

$$\Delta t_{L-R} = -\left(\frac{\partial t}{\partial x}\right)dx \tag{4}$$

With the aid of the Fourier conduction equation, the heat flowing into the left hand face may be written

$$\left(\frac{dQ}{d\theta}\right)_L = -kA\frac{\partial t}{\partial x}.$$

Since $t = t_L$, $A = dydz$,

$$\left(\frac{dQ}{d\theta}\right)_L = -k\,dydz\,\frac{\partial}{\partial x}\left[t_m - \left(\frac{\partial t}{\partial x}\right)\frac{dx}{2}\right]. \tag{5}$$

The heat flowing out of the right hand side is

$$\left(\frac{dQ}{d\theta}\right)_R = -kA\frac{\partial t}{\partial x}.$$

Since $t = t_R$, $A = dydz$,

$$\left(\frac{dQ}{d\theta}\right)_R = -k\,dydz\,\frac{\partial}{\partial x}\left[t_m + \left(\frac{\partial t}{\partial x}\right)\frac{dx}{2}\right]. \tag{6}$$

The net amount of heat stored in the cube, in differential time $d\theta$, will then be the difference between the amount of heat flowing in and that flowing out.

$$\left(\frac{dQ}{d\theta}\right) = -k\,dydz\,\frac{\partial}{\partial x}\left[t_m - \left(\frac{\partial t}{\partial x}\right)\frac{dx}{2}\right] + k\,dydz\,\frac{\partial}{\partial x}\left[t_m + \left(\frac{\partial t}{\partial x}\right)\frac{dx}{2}\right]$$

$$\left(\frac{dQ}{d\theta}\right) = k\,dxdydz\,\frac{\partial^2 t}{\partial x^2} \tag{7}$$

It is known that the amount of heat, Q, that is stored in a solid body is dependent upon the density of the material ρ, the volume of the material V, its specific heat C, and the existing temperature difference across the body Δt. θ represents the time increment during which the heat is stored.

Written mathematically

$$q = \frac{Q}{\theta} = \frac{\rho c V \Delta t}{\theta},$$

or

$$\left(\frac{dQ}{d\theta}\right) = \rho c V \frac{\partial t}{\partial \theta} = \rho c \, dx dy dz \, \frac{\partial t}{\partial \theta}. \qquad (8)$$

Hence we have two expressions for the same quantity, namely the amount of heat stored by this differential cube. Equating these,

$$\rho c \, dx dy dz \, \frac{\partial t}{\partial \theta} = k \, dx dy dz \, \frac{\partial^2 t}{\partial x^2}.$$

$$\frac{\partial t}{\partial \theta} = \frac{k}{\rho c} \frac{\partial^2 t}{\partial x^2}$$

$$\frac{\partial t}{\partial \theta} = \alpha \frac{\partial^2 t}{\partial x^2} \qquad (9)$$

This is the general conduction equation which must be fulfilled no matter what the boundary conditions may be. The boundary condition in this paper will be taken as the cyclic temperature function which is being applied to the surface of the solid. The cyclic temperature function in this case will be taken as a simple, trigonometric one whose equation is

$$t = t_o \sin \omega \theta. \qquad (10)$$

at the surface where x equals zero. In this equation t_o represents the amplitude of the temperature wave, t represents the temperature at any time θ, and ω is the frequency of the cyclic function.

It should be made clear at this point that this particular boundary condition is chosen for reasons of simplicity, and that actually there are any number of cyclic functions that might have been chosen.[11] The particular function chosen lends itself well to the work of this paper. Some of the other functions might well be studied at some future date. In any case the reasoning would follow the same general procedure.

Since equation (9) is linear and homogeneous with constant coefficients, an equation of the following form may be chosen to be a solution for it:

$$t = S e^{a\theta + bx} \qquad (11)$$

where t is the instantaneous temperature at time θ; S, a, and b are constants; and x is the depth measured normal to the surface.

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(11) H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, (London) Oxford at the Clarendon Press, 1947, pp. 267-269.

Now the first and second partials of t with respect to x can be found by simple differentiation.

$$\frac{\partial t}{\partial x} = bS\,e^{a\theta + bx} \tag{12}$$

$$\frac{\partial^2 t}{\partial x^2} = b^2 S\,e^{a\theta + bx} \tag{13}$$

Also the first partial of temperature with respect to time can be found by the same means.

$$\frac{\partial t}{\partial \theta} = a\,S\,e^{a\theta + bx} \tag{14}$$

Now substituting equations (13) and (14) in their respective places in equation (9)

$$a\,S\,e^{a\theta + bx} = \alpha\,b^2 S\,e^{a\theta + bx},$$

and

$$a = \alpha\,b^2. \tag{15}$$

By this procedure one of the constants in the exponent can be found in terms of the other. Obtaining the exponent in equations (11) in terms of the constant a,

$$b = \pm\sqrt{\frac{a}{\alpha}}.$$

$$t = S\,e^{a\theta \pm \sqrt{\frac{a}{\alpha}}\,x} \tag{16}$$

Now since a is a constant, we can replace it with a new constant $\pm i\gamma$, where i is equal to the square root of minus one. This substitution will help in choosing the particular solution that will fit our boundary condition. By using Euhler's transformations we can obtain the equation in terms of trigonometric forms. Making the substitution

$$a = \pm i\gamma,$$

$$t = Se^{\pm i\gamma\theta \pm \sqrt{\pm i}\sqrt{\frac{\gamma}{\alpha}}x}. \tag{17}$$

The following are identities as far as i, the square root of minus one, is concerned.

$$\sqrt{+i} = \left(\frac{1+i}{\sqrt{2}}\right) \tag{18}$$

$$\sqrt{-i} = \left(\frac{1-i}{\sqrt{2}}\right) \tag{19}$$

With the incorporation of the identities in equations (18) and (19), equation (17) will now be

$$t = Se^{\pm i\gamma\theta \pm x\sqrt{\frac{\gamma}{\alpha}}\left(\frac{1+i}{\sqrt{2}}\right)} \text{ or } Se^{\pm i\gamma\theta \pm x\sqrt{\frac{\gamma}{\alpha}}\left(\frac{1-i}{\sqrt{2}}\right)},$$

$$t = S\left[e^{\pm i\gamma\theta}\right]\left[e^{\pm x\sqrt{\frac{\gamma}{2\alpha}}(1\pm i)}\right],$$

or

$$t = S\left[e^{\pm x\sqrt{\frac{\gamma}{2\alpha}}}\right]\left[e^{\pm i\left(\gamma\theta \pm x\sqrt{\frac{\gamma}{2\alpha}}\right)}\right]. \tag{20}$$

From this point there are many particular solutions for this equation. It should be remembered that in using the Euhler transformations for an equation of this type, a portion of the answer comes out real and another portion is imaginary. Since the temperature which is being solved for is real, the imaginary part of the equation will be dropped as having no value in these problems. In any case, it is imperative that when the sign of i, the square root of minus one, or its roots is chosen to be a certain value in any given portion of the solution, it must be maintained all the way through the solution.[12] With this in mind, the one

(12) L. R. Ingersoll, O. J. Zobel, and A. C. Ingersoll, Heat Conduction With Engineering and Geological Applications, 1st Ed., N. Y. McGraw-Hill, 1948, p. 47.

particular solution that fits the boundary conditions and is of value in giving results can be written

$$t = S e^{-x\sqrt{\frac{r}{2\alpha}}} \sin\left(r\theta - x\sqrt{\frac{r}{2\alpha}}\right). \qquad (21)$$

Substituting the boundary conditions,

$$t = t_0 e^{-x\sqrt{\frac{\omega}{2\alpha}}} \sin\left(\omega\theta - x\sqrt{\frac{\omega}{2\alpha}}\right) \qquad (22)$$

Now that the general solution has been found and the particular solution which fits the boundary conditions chosen, this equation may be applied to practical problems.

Although much work has been done on investigating the penetration of periodic temperature waves into the earth itself, very little work has been done in fields other than this. The review of literature on periodic heat flow showed that very little is available in fields outside that of the soil temperatures. There are many other fields in which this type of temperature distribution is present and could be studied. In almost all these fields the transfer of heat that takes place is not by conduction alone, but by a combination of conduction and convection. In some cases where the operating temperatures are high, the effect of radiation cannot be neglected. In these cases the transfer of heat is due to a combination of all three, conduction, convection, and radiation. It is the purpose of this paper to show that it is feasible to project the present knowledge far enough to be able to consider heat transfer taking place by the combined effects of conduction and convection.

In order to understand the methods used in this paper for extending the known equations to take care of the additional effect of convection, it will be necessary to look at the equations governing the flow of heat by convection and conduction.

$$q_{cond.} = -kA\frac{\Delta t}{\Delta x} \tag{23}$$

$$q_{conv.} = hA\Delta t \tag{24}$$

The term h in the convection equation is called the film coefficient. Although it is not proper to think of convection taking place in a film, it is very convenient to think of all the heat that is transferred by convection as having to pass through this film. Much work has been done in studying this film coefficient. Its use is universally accepted as far as the solution of problems in convection is concerned, although there are some objections to it on the grounds that it does not explain the physical means by which the heat is carried away by convection. However, it will be used here with its usual meaning.

On looking at the equations, (23) and (24), one will note that they are quite similar. Dimensionally the two equations are alike. The convection equation used the idea that all the heat must be transferred through a film whose resistance is h. There is no assigned thickness to this film and the value of h is dimensionally given as Btu hr^{-1} ft^{-2} F^{-1}. In other words, it tells the amount of heat that will pass through one square foot of the film if there is a temperature difference across the film of one degree Fahrenheit.

In the case of the conduction equation the value of k dimensionally is Btu hr^{-1} ft^{-1} F^{-1}. This unit k when divided by the depth of the material x gives the dimensional units Btu hr^{-1} ft^{-2} F^{-1}. Thus the dimensional value of the thermal conductivity divided by the length of path x is the same as that for the film coefficient h. If the formulas are re-written to group these two quantities which are dimensionally equal, the following would result:

$$q_{cond.} = -\left(\frac{k}{\Delta x}\right) A \Delta t \tag{25}$$

$$q_{conv.} = h A \Delta t \tag{26}$$

Therefore, for the same quantity of heat passing through the convection film as passes through the media, it can be seen that for a given square foot of surface having a film coefficient h, a certain temperature difference would be necessary. This same drop in temperature could be effected by a given thickness of some conducting material whose conductivity was k. Thus for a cross sectional area of one square foot there will be a given thickness of material which for purposes of temperature drop would be equivalent to the film coefficient. This can be shown by a clearing of the conduction and convection equations of their equalities

$$-\left(\frac{k}{\Delta x}\right) A \Delta t = h A \Delta t$$

$$-\left(\frac{k}{\Delta x}\right) = h \qquad (27)$$

It is through the use of this equivalent thickness of material that this paper will try to consider the effects of both conduction and convection on periodic heat flow.

As an example to clarify this point it might be well to take a practical example. A semi-infinite solid made of aluminum having a film coefficient of 130 Btu hr^{-1} ft^{-2} F^{-1} at the surface is assumed. Conductivity of aluminum can be taken as 130 Btu hr^{-1} ft^{-1} F^{-1}.[13] A sketch of this particular set up would look something like this:

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(13) L. S. Marks, Mechanical Engineers Handbook, 4th Ed., N. Y., McGraw-Hill, 1941, p. 392.

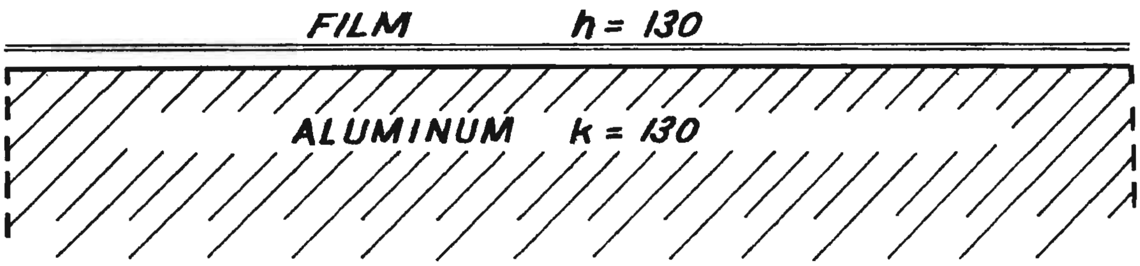

Fig. 2.

In order to replace the film coefficient with a certain thickness of the aluminum of which the semi-infinite solid is composed, the film coefficient and the thermal conductivity must be placed in equation (27). The result:

$$-\Delta x = \frac{k}{h}$$

$$-\Delta x = \frac{130}{130} = 1\ ft \quad \text{ABOVE SURFACE}$$

Thus it would take a one foot thickness of aluminum to be equal in resistance to the original film coefficient. A new picture of the semi-infinite solid may now be drawn which will result in the same properties as far as heat flow and temperature drop is concerned. Its new appearance would look something like this:

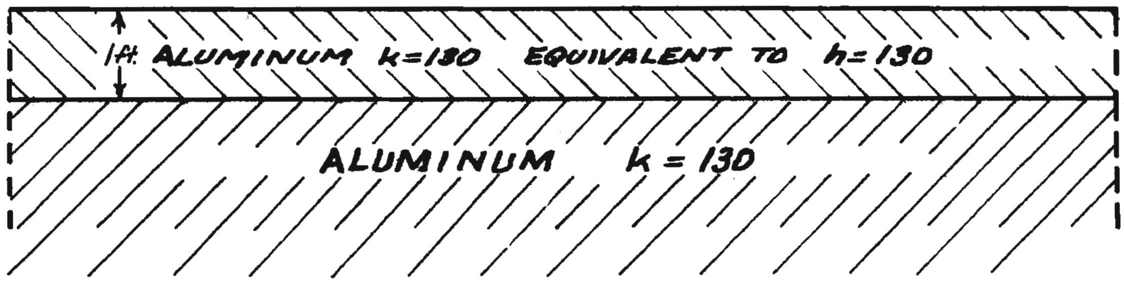

Fig. 3.

The cyclic temperature is applied to the top of the upper slab in Fig. 3 just as it was previously applied to the film in Fig. 2. The actual surface of the solid is now one foot below the surface to which the cyclic temperature is being applied. Thus having an equivalent system for solid and film, the effect of this one-foot, upper layer of aluminum shown in Fig. 3 can be studied. Its effect on the temperature distribution will be the same as that of the film; therefore, the heat flow taking place by the combined effects of conduction and convection can be equated to an equivalent system where only conduction is occurring.

Thus far in the paper it is assumed that the initial temperature function is being applied directly to the surface of the solid. If there is convection present, this becomes impossible. In the case where convection is present the convection film will come between the temperature function and the actual surface of the solid. The temperature function at the surface will still be cyclic in nature, but it will no longer be the same as that of the original function.

Now that it is possible to equate the convection film to a certain depth of the material, it is also possible to find the actual temperature function at the surface of the material. In this case the original function is applied to the surface of the medium representing the film, while the actual surface is at some depth below this virtual surface. If the temperature function is known, it will now be possible to find the amplitude of the disturbance at the real surface. This is because the formula for simple conduction is now able to take care of both convection and conduction.

It would now be proper to look at some of the practical aspects of the problem. One of these would be the actual effect of the value of h on the temperature amplitude at the surface of the material. It can be seen that as the film coefficient h increases, it has a smaller resistance to heat flow, or that as h increases, the same amount of heat will flow through the film with a smaller temperature difference. In terms of the equivalent system, as h increases, the thickness of material that is equivalent to it decreases. In the case of a decreasing h the inverse of the above statements would hold true.

It should be mentioned at this point that it is wise to construct the equivalent film coefficient of the same material composing the rest of the solid. If this is done, it will simplify the problem in that the various properties of the solid will be the same as those of the equivalent film. In this case only one set of material constants are needed.

Possibly the best way to look at these practical aspects would be to draw a series of curves depicting the desired variables to be studied. There are quite a number of possible curves that can be drawn for this type of heat flow. Each type of curve would have its own particular value. In this paper the temperature amplitude for each cycle will be plotted against the depth at which it occurs. This means that the part of the equation which is trigonometric in form will take on the value of unity. It is only reasonable to assume that the temperature amplitude will decrease as the depth increases. This is one of the ideas used in picking the possible particular solution that would give us valuable results. Again in the matter of frequencies it is reasonable to assume that the higher the frequency, the less will be the temperature fluctuation at some depth x. In other words, the material will act as a damper for the temperature oscillations.

After the curves are plotted, there is an interesting fact to be noticed as to the effect of the convection on the actual temperature amplitude at the surface. No matter what the frequency, any given value of h will be equivalent to only one depth of material. Now if the curves are plotted with depth as abscissa, and temperature amplitude as ordinate, then the value of h can be plotted as a line parallel to the ordinate going through the proper depth on the absissa.

If equation (22) is plotted on semi-log paper it will turn out a straight line. This fact is of value in that only two points are needed to locate the line. This enables the finding of points on Cartesian co-ordinates without making a long series of calculations. This plot on semi-log paper is not as effective in showing the effect on the heat flow of convection as is the plot on Cartesian co-ordinates.

The list of h values, as chosen for these curves, is representative of the values that it will usually take on. Actually it can take on values higher than those plotted, or lower, but this is the general range in which they will fall.

In the matter of frequencies, there are an unlimited number from which to choose. It is not meant that those in this paper are any more than a representative sample. It is hoped that those chosen are of value in showing the effect of convection to advantage.

The temperature function was chosen to have a low amplitude so that the effect of radiation on the flow of heat can be neglected.

Curves are drawn of temperature amplitude versus depth for copper, aluminum, steel, and soil.

TABLE I

PENETRATION OF THE CYCLIC TEMPERATURE IN STEEL OF DIFFUSIVITY $0.33\ FT^2\ HR^{-1}$

$k = 21$ Btu hr^{-1} ft^{-1} F^{-1}

$\rho = 492$ lb ft^{-3}

$C = 0.127$ Btu lb^{-1} F^{-1}

Applied temperature function: $t = 200 \sin\omega\theta$

	Temperature amplitude F			
x	$\omega = 30$	$\omega = 60$	$\omega = 120$	$\omega = 180$
0.1	102.00	77.00	52.10	38.30
0.2	51.90	29.75	13.60	7.32
0.3	26.50	11.38	3.54	1.41
0.4	13.50	4.36	0.92	0.27
0.5	6.87	1.68	0.24	0.05
0.6	3.51	0.65	0.06	----
0.7	1.79	0.25	0.02	----
0.8	0.91	0.10	----	----
0.9	0.46	0.04	----	----
1.0	0.24	0.01	----	----

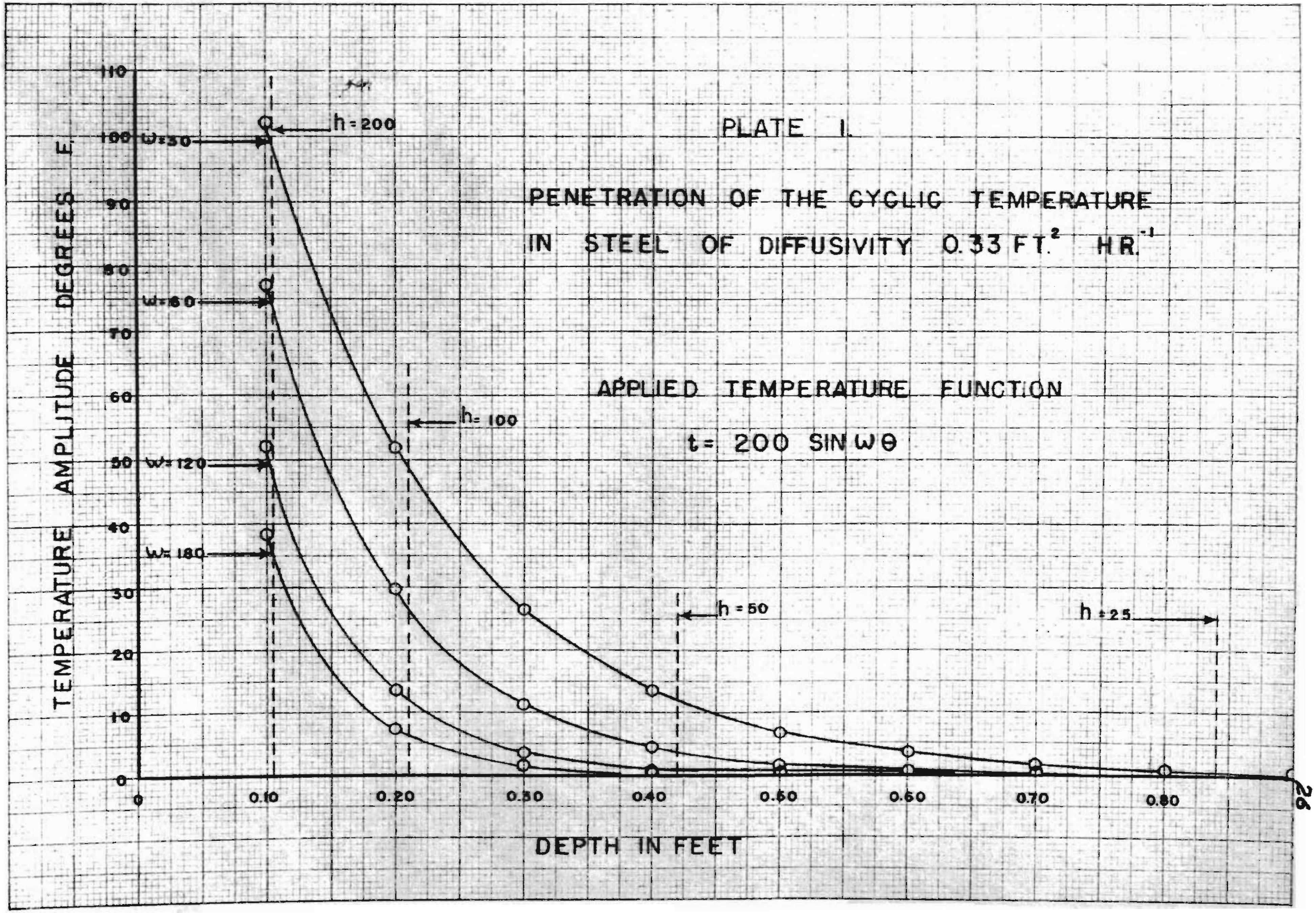

PLATE I.
PENETRATION OF THE CYCLIC TEMPERATURE
IN STEEL OF DIFFUSIVITY 0.33 FT.² HR.⁻¹
APPLIED TEMPERATURE FUNCTION
t = 200 SIN ω θ
h = 200
h = 100
h = 50
h = 25
ω = 30
ω = 60
ω = 120
ω = 180
TEMPERATURE AMPLITUDE DEGREES F.
DEPTH IN FEET
0
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
10
20
30
40
50
60
70
80
90
100
110

TABLE II

PENETRATION OF THE CYCLIC TEMPERATURE IN ALUMINUM OF DIFFUSIVITY 3.46 FT^2 HR^{-1}

$k = 130$ Btu hr^{-1} ft^{-1} F^{-1}

$\rho = 168$ lb ft^{-3}

$C = 0.224$ Btu lb^{-1} F^{-1}

Applied temperature function: $t = 200 \sin \omega \theta$

	Temperature amplitude F			
x	$\omega = 30$	$\omega = 60$	$\omega = 120$	$\omega = 180$
0.1	161.00	149.00	132.00	119.00
0.2	131.00	110.00	87.40	70.60
0.3	106.50	82.50	58.50	42.00
0.4	85.50	61.60	38.00	25.00
0.5	70.00	46.00	25.00	14.80
0.6	56.60	34.20	16.60	8.89
0.7	46.00	25.60	11.00	5.25
0.8	37.30	19.00	7.25	3.12
0.9	30.20	14.20	4.76	1.83
1.0	25.00	10.55	3.13	1.11

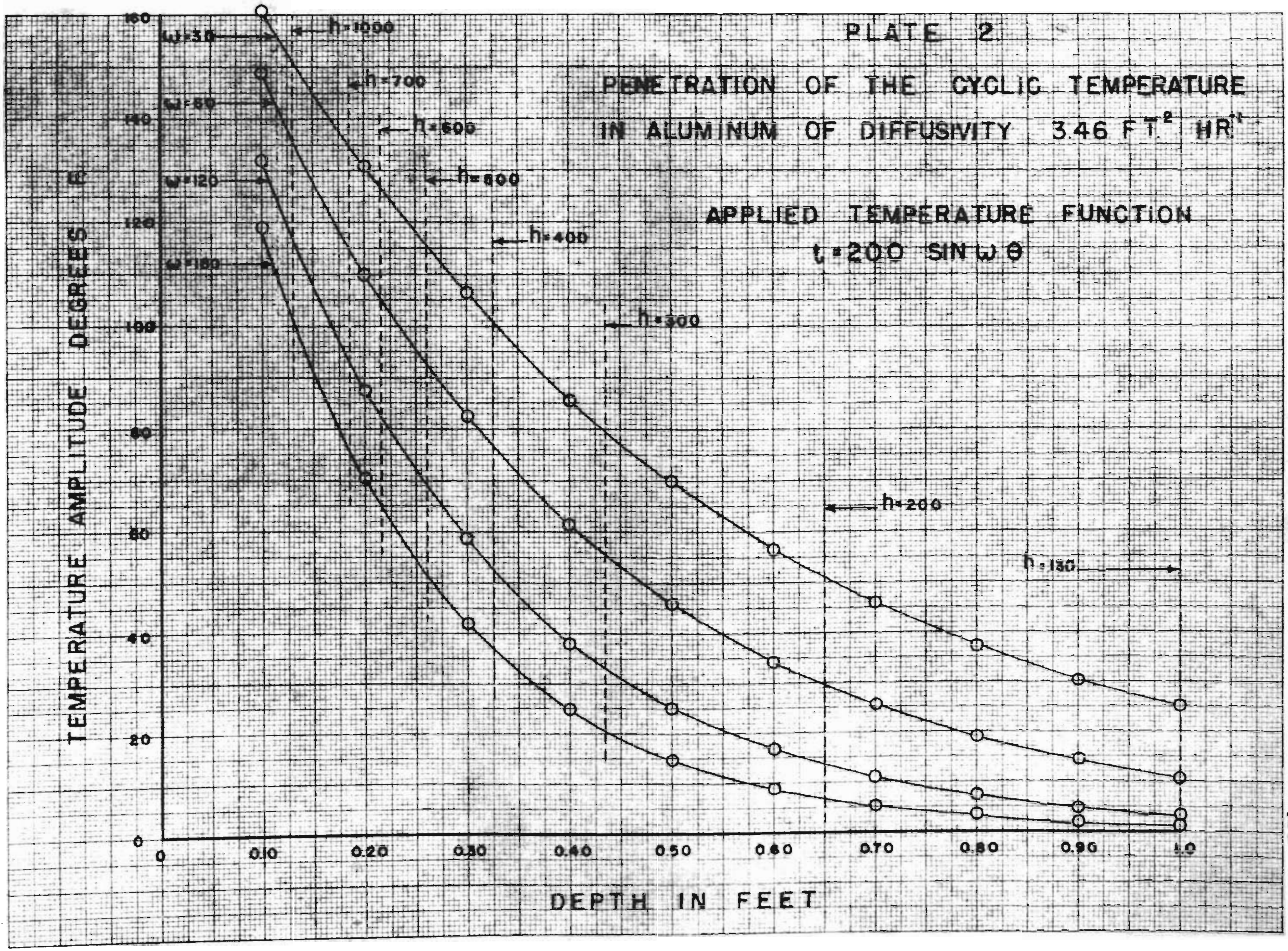
PLATE 2
PENETRATION OF THE CYCLIC TEMPERATURE
IN ALUMINUM OF DIFFUSIVITY 3.46 FT² HR⁻¹
APPLIED TEMPERATURE FUNCTION
t = 200 SIN ω θ
TEMPERATURE AMPLITUDE DEGREES F.
DEPTH IN FEET
ω=30
ω=60
ω=120
ω=180
h=1000
h=700
h=600
h=500
h=400
h=300
h=200
h=130
0
20
40
60
80
100
120
140
160
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.0

TABLE III

PENETRATION OF THE CYCLIC TEMPERATURE IN COPPER OF DIFFUSIVITY 4.25 FT^2 HR^{-1}

$k = 220$ Btu hr^{-1} ft^{-1} F^{-1}

$\rho = 558$ lb ft^{-3}

$C = .092$ Btu lb^{-1} F^{-1}

Applied temperature function: $t = 200 \sin\omega\theta$

	Temperature amplitude F			
x	$\omega = 30$	$\omega = 60$	$\omega = 120$	$\omega = 180$
0.1	165.80	153.10	137.50	126.00
0.2	137.30	117.40	94.40	79.90
0.3	113.90	90.00	64.80	50.30
0.4	94.45	69.00	44.50	31.70
0.5	78.10	53.00	30.50	20.10
0.6	65.00	40.50	21.00	12.70
0.7	53.80	31.10	14.50	8.00
0.8	44.40	23.80	9.85	5.10
0.9	36.90	18.25	6.80	3.20
1.0	30.60	14.00	4.65	2.00

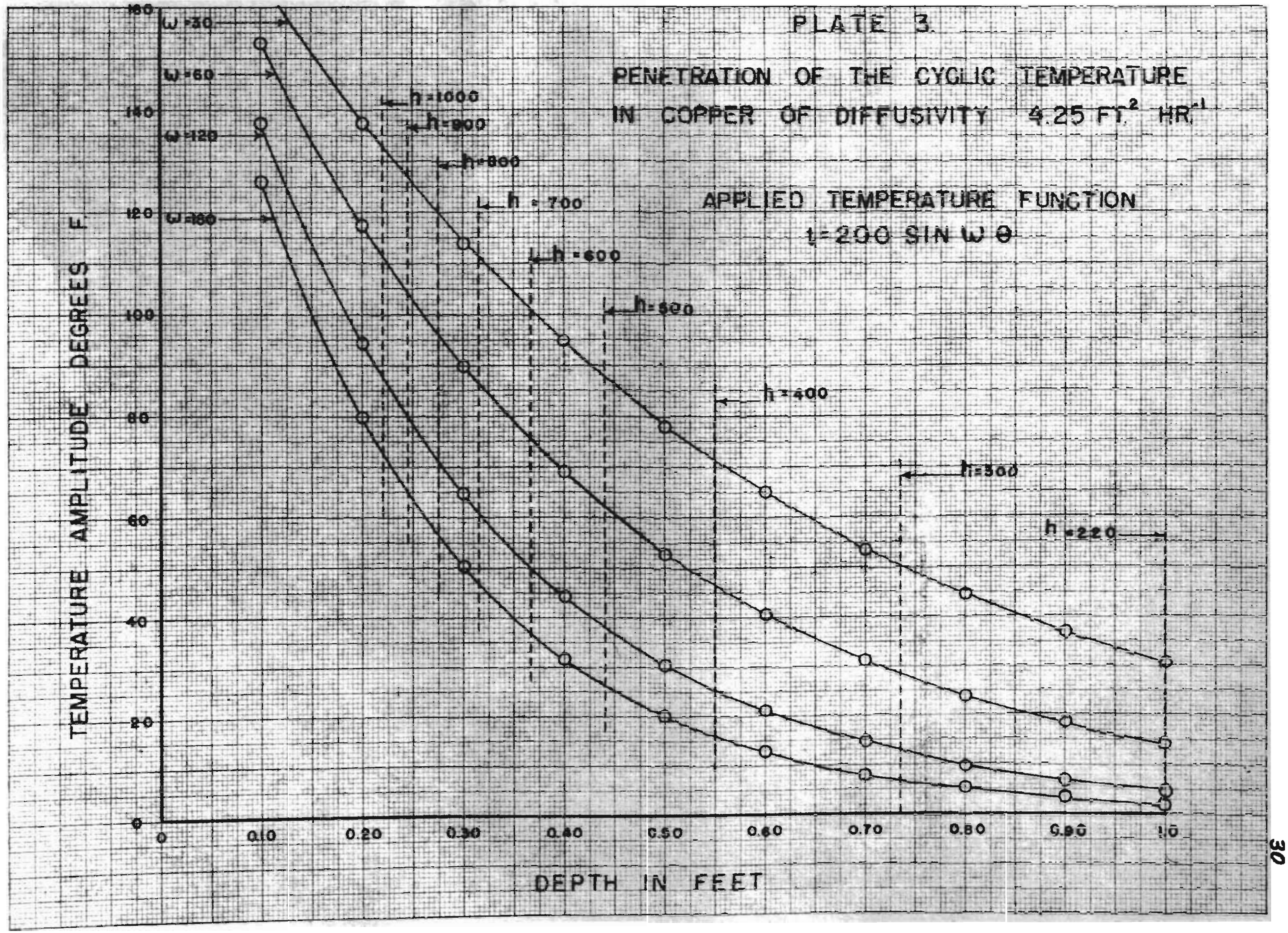
PLATE 3.
PENETRATION OF THE CYCLIC TEMPERATURE
IN COPPER OF DIFFUSIVITY 4.25 FT.² HR.⁻¹
APPLIED TEMPERATURE FUNCTION
t=200 SIN ω θ
TEMPERATURE AMPLITUDE DEGREES F.
DEPTH IN FEET
ω=30
ω=60
ω=120
ω=180
h=1000
h=900
h=800
h=700
h=600
h=500
h=400
h=300
h=220
0
20
40
60
80
100
120
140
160
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.0

TABLE IV

PENETRATION OF THE CYCLIC TEMPERATURE IN SOIL OF DIFFUSIVITY 0.055 FT^2 HR^{-1}

$k = 1.4$ Btu hr^{-1} ft^{-1} F^{-1}

$\rho = 126$ lb ft^{-3}

$C = 0.20$ Btu lb^{-1} F^{-1}

Applied temperature function: $t = 200 \sin \omega \theta$

	Temperature amplitude F			
x	$\omega = 30$	$\omega = 60$	$\omega = 120$	$\omega = 180$
0.05	87.80	61.60	38.50	27.00
0.10	38.50	19.00	7.35	3.64
0.15	16.80	5.90	1.41	0.25
0.20	7.35	1.82	0.27	0.07
0.25	3.22	0.55	0.05	----
0.30	1.40	0.17	----	----
0.35	0.59	----	----	----
0.40	0.27	----	----	----
0.45	0.12	----	----	----
0.50	----	----	----	----

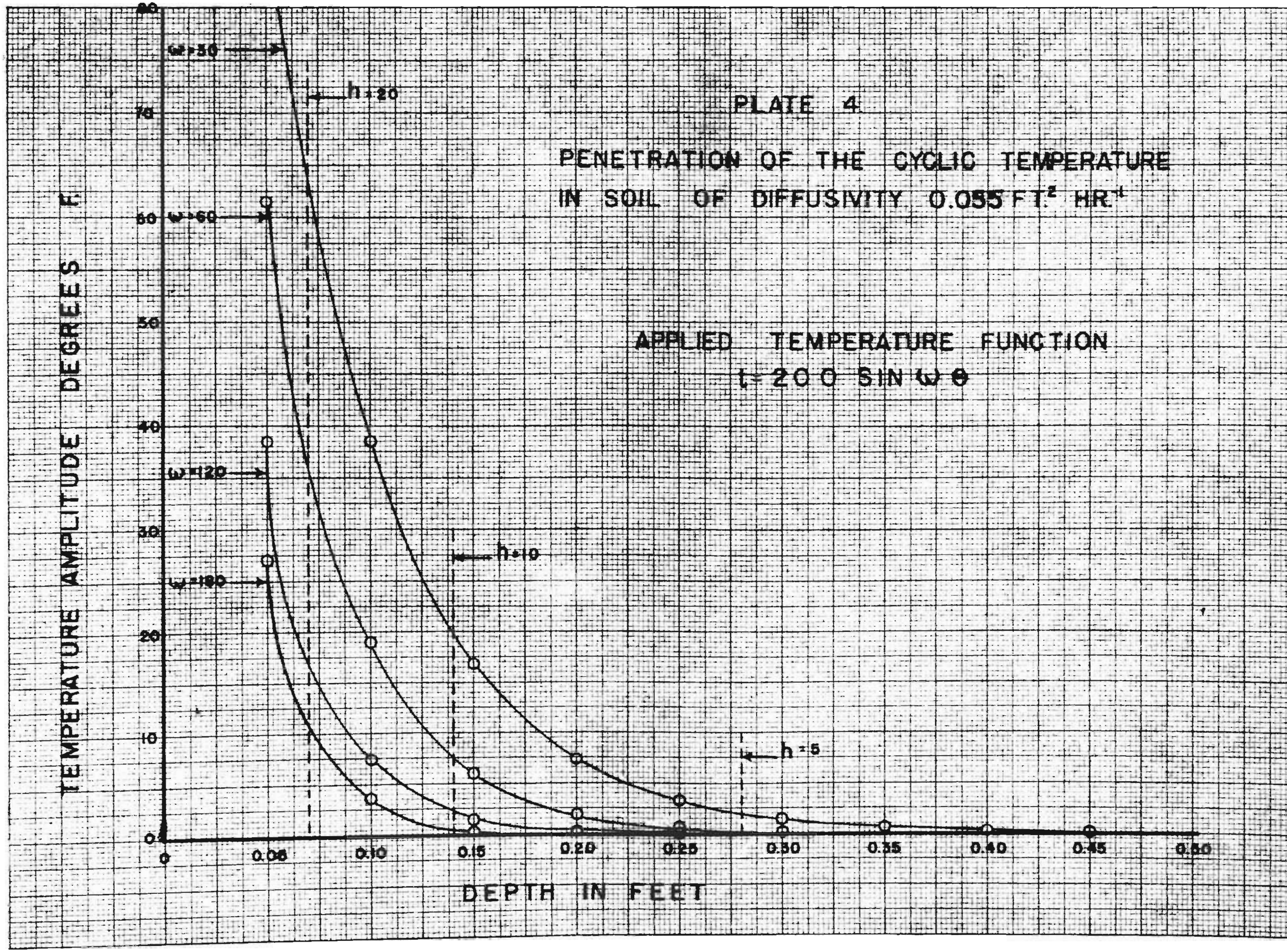
PLATE 4
PENETRATION OF THE CYCLIC TEMPERATURE IN SOIL OF DIFFUSIVITY 0.055 FT² HR⁻¹
APPLIED TEMPERATURE FUNCTION
t = 200 SIN ω θ
TEMPERATURE AMPLITUDE DEGREES F.
DEPTH IN FEET
ω = 30
ω = 60
ω = 120
ω = 180
h = 20
h = 10
h = 5
0
10
20
30
40
50
60
70
80
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50

CONCLUSIONS

From the results obtained in the investigation of periodic flow by the combined effect of convection and conduction, it can be seen that the equivalent system as set up in this paper is a simple means of evaluating the temperature amplitude at the surface of a semi-infinite solid.

This system as set up to take care of the convection film can be used in any type of heat transmission where the heat is transferred by means of conduction and convection. However, in many cases this system leads to more work than would a straight-forward approach to the problem. It is not recommended that this be used for all types of flow, because it fails to show in any physical sense the actual means by which the heat is transferred.

It should also be remembered that this system, as set up, is dealing with the temperature only since the temperature drop across the film can be set up as a temperature drop across an equivalent amount of conducting media.

From the curves drawn it can be seen that the effect of the convection film can be ascertained very easily in a graphical sense. Since the curves can be drawn without regard to the convection initially, they are not dependent in shape on this term. Thus it is quite simple to calculate the effect of convection or the effect of a change in convection if the curves for the material are known at the desired frequency. The equivalent film can be looked at in two separate ways. Convection can be thought of as increasing the effective depth of the solid, or it can be reasoned that the effective real surface so far as temperature fluctuations is concerned is buried within the solid by the effect of convection.

SUMMARY

The known analytical knowledge of the periodic flow of heat can be extended to take into consideration the combined effect of convection and conduction on this type of flow. The equivalent system as set up in this paper is a rather simple device and is quite satisfactory as far as the temperature distribution is concerned.

There are many problems left to be studied in this type of heat flow. In this paper the temperatures have been held purposely low so that the effect of radiation could be neglected. At higher temperatures the flow of heat is dependent on radiation to no small degree. The author believes that it would be possible to solve analytically for the effect of all three types of heat transfer on periodic flow. There are many other problems that can be attacked by analytical reasoning, and it is the hope of the author that at some future date the work of this paper can be continued and extended.

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