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A multiple-plan evaluation model for small ungauged watersheds

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A MULTIPLE-PLAN EVALUATION MODEL FOR SMALL UNGAUGED WATERSHEDS

BY

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CIVIL ENGINEERING ABSTRACT

A Multiple-Plan Evaluation Model for Small Ungauged Watersheds, by James R. Dexter. A computer solution model is proposed for simulation of the effect of alternative measures for flood damage reduction. The goal of the model is to optimize the value of an objective function which will maximize the amount of net benefits returned by the project. The evaluation includes unit hydrograph synthesis, direct runoff construction, computation of average annual damages, and optimization of an objective function. A test application of the model is made on a small community affected by floods from a small ungauged stream.

A MULTIPLE-PLAN EVALUATION MODEL

FOR SMALL UNGAUGED WATERSHEDS

by James R. Dexter,[1] A.M. ASCE

KEY WORDS: flood control; systems engineering; economics; nonlinear programming; net benefit; unit hydrograph; optimization; nonstructural; computer.

ABSTRACT: A computer solution model is proposed for simulation of the effect of alternative measures for flood damage reduction. The goal of the model is to optimize the value of an objective function which will maximize the amount of net benefits returned by the project. System inputs and outputs are considered in commensurate (annual costs) units, i.e., the total social cost of flooding is made up of the cost of floods (outputs) and the cost of their prevention (inputs). The multiple-plan evaluation involves four areas of computation - unit hydrograph synthesis for the specified basin using Gray's Method, direct runoff construction for a series of specified frequency events, estimation of average annual flood for existing hydrologic conditions using a numerical method to integrate the damage-frequency function, and optimization of a non-linear objective function to minimize the sum of residual average annual flood damages plus equivalent annual costs to produce the effected flood damage reduction (net benefit criterion). A test application of the model is made on a small community affected by floods from a small, ungauged stream. Results of

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the test indicate the value of systems engineering in optimizing flood damage reduction measures considering not only engineered facilities, but nonstructural approaches as well.

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A MULTIPLE-PLAN EVALUATION MODEL FOR SMALL UNGAUGED WATERSHEDS

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INTRODUCTION

When considering alternative measures for flood damage reduction, the water resource planner must evaluate the benefits and costs of multiple plans with regards to their economic, social, and environmental implications. One may view the task of plan evaluation as the process of characterizing the components of the water resource system. The concept of the water resource system is shown in Figure 1. The characterization may be accomplished by assessing the quantitative and qualitative aspects of the system.

With regards to the economic effects of alternative plans for the system, planning has been concerned with the identification of the best or optimal plan whereby the optimum mix of desirable and undesirable outputs is achieved. This course implies that a prescribed criterion capable of measurement is available to express the results of the system outputs. Systems-analysis has been proposed by Hall and Dracup (7), to evaluate the performance of water resource systems with respect to an objective (criterion) function. Systems analysis also allows for the stochastic nature of hydrologic systems to be accounted for in the analysis. This thesis proposes the use of a mathematical programming technique to determine the optimal plan for system outputs, taking into account the stochastic nature of the inputs.

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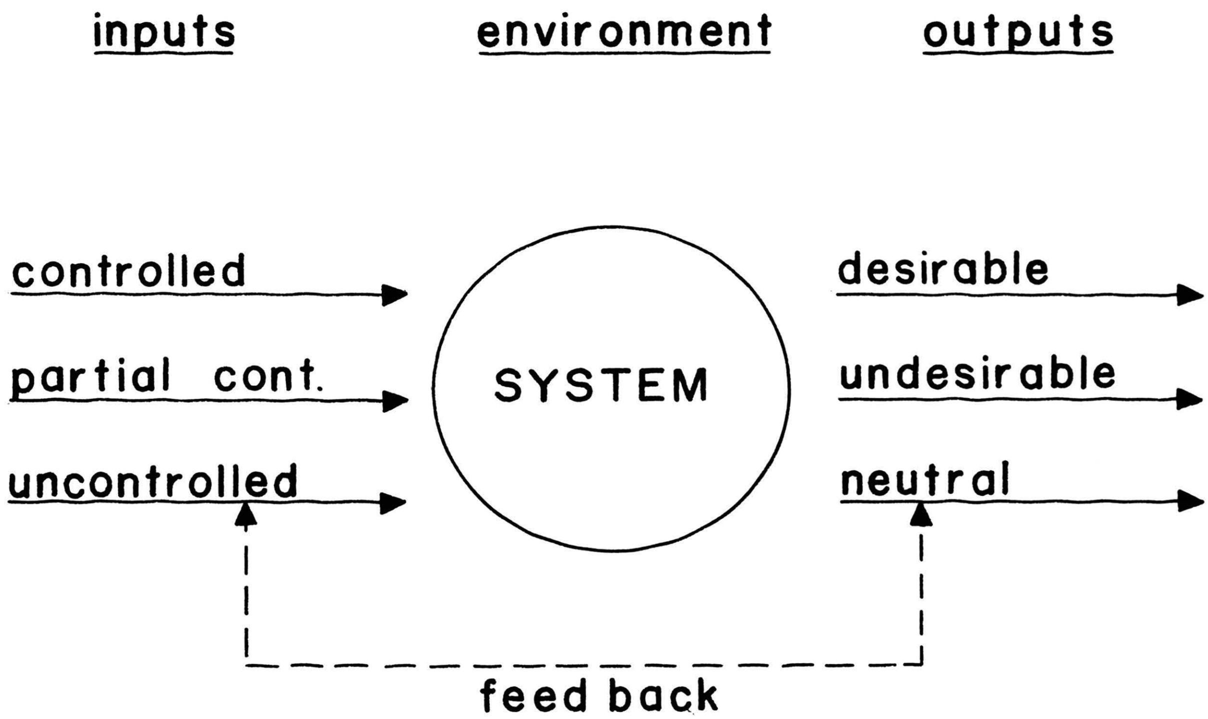
FIGURE 1. - SYSTEM CONCEPT



SYSTEM CONCEPT

Figure 1

During the process of characterizing potential improvements in the hydrologic system as it relates to flood damages incurred by man, traditional evaluation has been directed at structural measures to reduce the undesirable outputs of the system. More recently, emphasis has been increased on the need to evaluate nonstructural measures and structural measures alike to obtain the best policy for flood damage reduction. Structural measures are those engineering facilities (dams, channel modifications and levees) which are designed to control water. Nonstructural measures (flood plain regulations, flood-proofing and relocation) are designed to reduce business and residential use of flood plains and cause those who occupy these hazard areas to alter their use patterns in ways which will reduce damages. Noting the increasing rise of annual flood damage in the United States despite ever-increasing expenditures for flood control works, the Task Force on Federal Flood Control Policy (18) proposed consideration of flood plain management techniques as alternatives to civil works construction in a hope of trying to keep pace with accelerating destruction. Subsequently, numerous agencies have been challenged to evaluate these measures, particularly in response to Executive Order 11296(4). More recently, the National Water Commission (14) has emphasized the need to continue research on methods which assist the planner in making more equitable determinations.

In evaluating the desirable and undesirable system outputs where structural or nonstructural modifications to the system are proposed, measurement must be completed using commensurate units. The Water Resources Council (20) recommends that a test of efficiency should be applied among alternatives to determine the alternative, whether Federal or non-Federal, structural or nonstructural, that is the least cost means, considering all adverse effects, of achieving the specified objective. This thesis proposes a combination of structural and nonstructural measures for determining the best or optimum mix which will result in the maximization of net tangible system benefits as pertaining to flood damage reduction.

METAPHYSICAL DEVELOPMENT

The search for the optimum combination of system inputs to achieve maximum system output (flood damage reduction) may be viewed as the process of selecting the best mix from a production function which maximizes the value of some criterion. The criterion recommended for Federal water resource projects by the Water Resources Council (20) is the maximization of net tangible benefits (outputs minus imputs) of the system. For a hypothetical flood damage situation, the reduction in the expected value of the flood damages would be taken as the gross benefits (output). The commitment of resources (labor, material, water, etc.), to the achievement of the goal of flood damage reduction, would be taken as the costs (input). The gross benefit minus the cost would be the net benefit.

The development of a process for selecting the combination of flood control measures producing maximum net benefit has been described by James (9) as a search for the combination associated with the minimum net flood cost. The minimum flood cost approach considers floods as creating a number of costs. These costs include suffering the damages as they occur, building structural measures (such as channel modifications) to change the damage - flood flow relationships, regulating flood plain land use so that less damageable property is located there, or altering use patterns so that flood plain development is less susceptible to damage when in contact with water.

The estimation of spot flood damages as they occur is normally a measure of the depth of flooding at any given location in the flood plain. Total severity of the flood damage potential depends on the areal extent of the flooding. However, other important factors which effect total flood damage are duration of flooding, sediment deposition, flow velocity, and season in which flooding occurs. James (10) has proposed a mathematical relationship to relate flood peaks to the areal extent and depth of flooding. Bhavnagri and Bugliarello (2) have further shown how flood depth-damage relationships may be distributed to a series of flood plain locations. From knowledge of these flow-damage relationships, it is possible to relate damage suffered to the entire range of the system outputs, i.e., series of flood events. The direct runoff hydrograph is a method for describing the hydrologic response of a watershed and thus evaluate the damages incurred to the flood plain occupant.

Costs involved with flood damage reduction are project installation costs (such as construction), associated and induced costs, and operation and maintenance costs.

The function of the planner is to measure the cost of flooding and production costs to reduce flood damage. The combination of costs (relying upon changes in flood flow damage relationships), which results in the minimum total cost, is then deemed the optimum or most efficient system configuration. In reaching this efficiency goal, James (10) points out the following assumptions which the government planner must accept (as is done in this thesis):

"(1) The public viewpoint incorporates all costs and benefits to whomsoever they may accrue.

(2) The discount rate used may be lower then that used by private firms....

(3) When market prices lose their normative significance.... attempt to evaluate the true economic worth of each input and output.

(4) When analyzing projects producing products or outputs which are not marketable.... derive an equivalent market value through demand analysis."

Finally, an ideal market is assumed to exist to allow the combination of commensurate measures in an objective function. In this perfectly functioning market, it is assumed that people would join to produce a channel modification if the cost was less than the damage they incur; they would restrict urban development when the expected value of annual damages exceed the value of the more intensive land use - and they would flood-proof individual properties as long as the expense did not exceed the flood damage reduction.

MATHEMATICAL DEVELOPMENT

The key to determining optimal measures for flood damage reduction is the technique of evaluating flow-damage-probability relationships. The implied knowledge is the ability to predict peak discharge rates for the basin and correlate these to a specified frequency of occurrence.

Fundamental to this analysis is the study of the particular hydrologic characteristics of the watershed, the temporal and spatial rainfall patterns, and the antecedent moisture conditions, which affect the runoff hydrograph. A method presented by Gray (6) for small, ungaged watersheds in the Midwest to predict synthetic unit hydrographs, can be utilized in the development of a series of runoff hydrographs. A unit hydrograph, as defined by Sherman (16), is as follows: "If a given one-day rainfall produces a 1-inch depth of runoff over the given drainage area, the hydrograph showing the rates at which the runoff occurred can be considered a unit graph for that watershed."

Once this series of hydrographs is constructed for specified duration and recurrence intervals, flow damage relationships may be derived using methods proposed by James and others. A systems analysis approach to the problem of minimizing total flood costs is then taken by employing a gradient search technique to evaluate an objective function. In summary, the multiple plan evaluation involves four areas of computation - unit hydrograph synthesis for the specified basin, construction of direct runoff hydrographs for specified frequency events, estimation of average annual flood damages for existing hydrologic conditions, and optimization of an objective function to minimize the sum of residual average annual damages plus equivalent annual costs of flood damage reduction.

A discharge hydrograph for a given rainfall excess can be obtained for any watershed by applying the unit hydrograph theory. The unit hydrograph theory assumes that there is no temporal and spatial variation of rainfall. Gray's method for deriving synthetic unit hydrographs for Midwestern watersheds relies upon the construction of a dimensionless unit graph with ordinates expressed in percentage flow, based on a time increment equal to 1/4 the period of rise and the abscissa expressed as the ratio of any time, t, divided by the period of rise, P_r, the time from the beginning of runoff to the occurrence of the peak discharge. This results in a graph described by the equation:

$$\%\ Q_t\ (t/P_r) = \frac{25.0(\gamma')^q(e^{-\gamma' t/P_r})(t/P_r)^{q-1}}{\Gamma(q)} \tag{1}$$

Where $\%\ Q_t\ (t/P_r)$ = % flow evaluated at any given t/P_r value

γ' = dimensionless parameter equal to the product $P_r\gamma$

q = shape parameter

γ = scale parameter

Γ = gamma function

e = base of natural logarithms

Gray (6) developed correlations with the physiographic characteristics of 42 Midwestern watersheds to obtain the values of P_r and γ' used in this model. The storage factor P_f/γ' was correlated with the watershed characteristics $L/Sc^{.5}$ where L is the length of main channel of the watershed measured in miles and Sc is the average slope in percent. For Central Iowa, Missouri, and Wisconsin, Gray found

$$P_r/\gamma' = 9.27\ (L/Sc^{.5})^{.562} \tag{2}$$

Furthermore, he determined that q and γ' were linearly related as follows

$$q = 1 + \gamma' \tag{3}$$

Gray fitted the following linear equation to his data

$$\gamma' = 2.676 + .0139\ P_r \tag{4}$$

A more convenient form of this equation is

$$P_r/\gamma' = \frac{1}{\frac{2.676}{P_r} + .0139} \tag{5}$$

The dimensionless unit graph which results from eq. (1) allows the computation of discharge ordinates for the unit graph at times equal to a specified interval of the period of rise P_r. An interval equal to 1/4 the period of rise is utilized in eq. (1) as is done in Gray's paper. The unit hydrograph computed by this method will have a duration equal to 1/4 the period of rise. The use of this unit hydrograph and associated duration to compute runoff hydrographs results in runoff hydrographs of the same duration, unless lagging methods are used to change the duration.

Viessman, Harbaugh, and Knapp (19) explained a simple algorithm for the application of Gray's method to an ungauged area as follows

1. Determine L, S_c, and A for the watershed.
2. Determine the parameters P_r, γ', and q as follows:
   a. Using $L/Sc^{.5}$ solve for P_r/γ' using eq (2)
   b. With P_r/γ', determine P_r using eq (5)
   c. Compute $\gamma' = P_r / P_r/\gamma'$
   d. Use eq (3) to obtain q
3. Determine the ordinates for the dimensionless graph using eq (1).
4. Determine the unit hydrograph by
   a. Converting the volume of direct runoff under the dimensionless graph to 1 inch of precipitation excess over the entire watershed by equating the volume of the dimensionless graph to the unit hydrograph and solving for Σ FLOWS.
   b. Convert the dimensionless graph ordinates to unit hydrograph ordinates using the following equation

$$Q_u = \frac{\%\ Q/(t/P_r)}{100} \times \Sigma \text{ FLOWS}$$

   c. Translate the time base of the dimensionless graph (time/period of rise) to absolute time units. Runoff does not commence until the centroid of rainfall $P_r/8$.

Viessman, Harbaugh, and Knapp also demonstrated the adaptability of the algorithm to computer solution.

Linsley, Kohler and Paulhus (11) point out that once a unit hydrograph for a basin has been derived, the prediction of a direct runoff hydrograph of the same duration can be made by multiplying the ordinates of the unit hydrograph by the runoff excess as follows

$$q_n = Q\ U_n \tag{7}$$

Where q_n = ordinate of total runoff hydrograph at time n

Q = rainfall excess

U_n = unit hydrograph ordinate at time n

The estimation of the recurrence interval for the peak discharges obtained by the application of unit hydrograph theory requires the use of statistical methods. Correlation of discharges to frequency in small watersheds, which may often be ungauged, introduces additional problems in a point-frequency analysis. A method (17) is used to relate peak discharges to rainfall excess which is a function of recurrence interval and rainfall duration

$$i_{av} = f(R,\ t_r) \tag{8}$$

Where i_{av} = average rainfall in inches

R = recurrence interval expressed as the relative frequency in percent, i.e., $P = 1/R$

t_r = duration of rainfall in hours

The average rainfall may be viewed as rainfall excess once expected losses have been subtracted. These excesses can be employed with eq. (7) to compute the ordinate of the total runoff hydrograph.

The Hydrologic Engineering Center (8) has proposed that discharge frequency relationships (such as previously described) and discharge-drainage relationships can be evaluated in a hydrologic system simulation model. Once these relationships are known, it is possible to simulate the hydrologic response and corresponding flood damage for a series of reconstituted flood events. The known frequency and damage relationships may be used to estimate the expected value of flood damage by using a method shown by Franzini (5)

$$D_{av} = \left(\frac{D_1 + D_2}{2} \times (P_2 - P_1)\right) +$$

$$\left(\frac{D_2 + D_3}{2} \times (P_3 - P_2)\right) + \ldots$$

$$+\left(\frac{D_{n-1} + D_n}{2} \times (P_n - P_{n-1})\right) \qquad (9)$$

Where D_{av} = expected value of flood damage

D_n = damage caused by a flood of probability, P_n

Relationships between discharge and damage are derived from knowledge of the flood plain topography (rating curve showing elevation vs discharge) and relationships linking depth of flooding to damage. James (10) showed that the primary measure of spot flood severity is depth of flooding. Thus, assuming shallow flooding with minimal effect from velocity of flow, duration, and sediment deposition, the flood damage to yards, buildings, and contents increases linearly with depth

$$C_d = K_d \; M_s \; d \qquad (10)$$

Where C_d = direct flood damage in dollars

K_d = is a coefficient determined by analysis of the direct damage caused to similar property by historical floods

M_s = the market value of the structures in dollars

d = the depth of flooding in feet

Bhavnagri and Bugliarello studied this relationship in greater detail. They proposed a method whereby the flood depth-damage relationship of a structure could be written as the product of the unit damage function and the individual characteristic damage coefficient within small contour intervals. The sum of all the products for establishments within a contour interval denotes the damage potential of that interval and a profile of the contour characteristic damages per unit height indicates the damage potential for varying topography within the flood plain. Hence, the total damage within the flood plain caused by flooding of all intervals to the i th interval is

$$D_i = \sum_{j=1}^{i} B_j \, \sigma(S_i - \zeta_j) \quad i = 1, 2, m \tag{11}$$

Where D_i = damage to the i th interval

i, j = contour intervals between datum and highest elevation m

B_j = contour characteristic damage for the step number j, in dollars

σ = unit damage function, dimensionless

S_i = flood level at any given location within the i th contour interval

ζ_j = elevation of the j th step of the flood plain measured above the same datum as the flood level

The selection of optimal amounts of flood damage reduction by various methods may be viewed as a resource allocation problem. The resources to be allocated may include a set of inputs such as earth, concrete, and natural streamflow. These inputs are expected to produce an output - reduced flood damage. Engineering analysis shows the combination of inputs that may be expected to produce outputs. The relationship used to express the ability of given inputs to produce a specified output is called a production function. It is useful to determine the plans which are impractical or wasteful. In order to go further in deciding which plan along a production function locus should be selected, a criterion (objective function) must be used for evaluating worth. Such an objective function is described by James as

$$U(x,y) = \sum_{j=1}^{n} B_j\, y_j - \sum_{i=1}^{m} C_i\, X_i \qquad (12)$$

Where $U(x,y)$ = the utility or value of net benefit

x = input (a vector)

y = output (a vector)

B_j = unit benefit associated with the corresponding coordinates of the output vector

C_i = unit costs associated with the corresponding coordinates of the input vector

Mass, et al, (12) have described the optimality conditions that confers the highest possible value on the net benefit function. Simply stated, the goal is to maximize $U(x,y)$ subject to the constraint that the plan is on the locus of the production function $f(x,y) = 0$, i.e., the proposed system must be technically feasible before considering optimality.

A more convenient form of this utility function is obtained by viewing the function as having a number of component social costs which are to be minimized in the summation. Thus, the maximum net benefit value as shown in Figure 2, is identical to the minimum social cost value as follows

Proof

$$NB = (-C) + (D^{*} - D) \qquad\qquad TC = C + D$$

$$\frac{\partial NB}{\partial S} = \frac{-\partial f(C)}{\partial S} + \left(\frac{-\partial f(D)}{\partial S}\right) \qquad\qquad \frac{\partial TC}{\partial S} = \frac{\partial f(C)}{\partial S} + \frac{\partial f(D)}{\partial S}$$

Maximizing NB by setting first derivative equal to zero and multiplying by -1

$$\frac{\partial NB}{\partial S} = 0 = \frac{\partial f(C)}{\partial S} + \frac{\partial f(D)}{\partial S}$$

which is identical to right side

where NB = net benefits

TC = total costs

f(C) = cost function

f(D) = damage function

D* = expected value of damages under existing conditions (constant)

S = scale of project

For the purposes of this model, a non-linear program is invoked to solve for the optimal value of the net benefit function. The objective function may be viewed as a total cost of three components, the sum of which is to be minimized as follows

$$\text{Minimize } U = C_1 + C_2 + C_3 \tag{13}$$

Where C_1 = cost of flooding

C_2 = cost of structural measures

C_3 = cost of flood proofing

Subject to

$$C_1 \geq 0, \; C_2 \geq 0, \; C_3 \geq 0$$

Cline (3) has described the nature of these costs in the application of the University of Kentucky Flood Control Planning Program.

FIGURE 2. - MAXIMIZATION OF NET BENEFITS

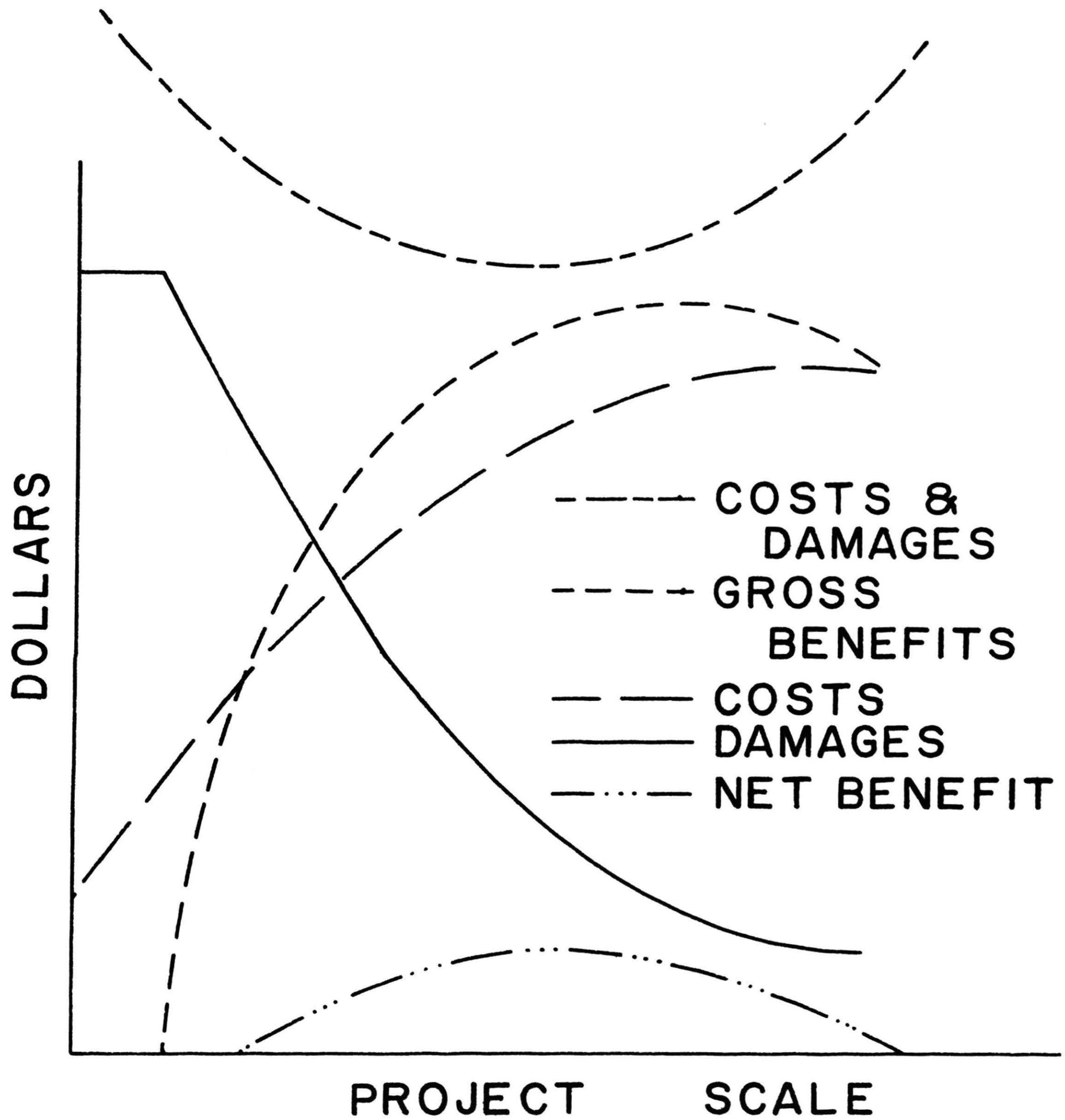

DOLLARS
COSTS &
DAMAGES
GROSS
BENEFITS
COSTS
DAMAGES
NET BENEFIT
PROJECT SCALE

Figure 2

$$C_1 = C_d M_s d \tag{14}$$

Where C_d = average damage inflicted by shallow flooding in dollars per foot of flood depth per dollar of structure market value

M_s = market value of structures in dollars per acre

d = average depth of flooding in feet

$$C_2 = C_a + O \tag{15}$$

Where C_a = first cost of construction including labor, materials, engineering and design, supervision and administration, plus a contingency factor; usually expressed as an average annual value

O = annual operation, maintenance and replacement costs of facilities

$$C_3 = C_p M_s h \tag{16}$$

Where C_p = installation cost of the measures per dollar of market value per foot of flood depth

M_s = market value of the structures

h = flood proofing design depth

Solution of the non-linear program is accomplished by a gradient search procedure in the model as described by Beard (1). This procedure relies on the Newton-Raphson Method for identifying the extremes of an unconstrained function. This is a scheme for beginning with an estimate of the optimizing value of a variable(s) and proceeding through successive approximations to converge on the optimum value of the variable(s). The approximation is a recursive relationship described by McMillan (13) for the single variable case as

$$x_{n+1} = x_n - f'(x_n)/f''(x_n) \tag{17}$$

Where x_{n+1} = the new approximation of variable x

x_n = the previous approximation of variable x

$f'(x_n)$ = the first derivative of f(x), evaluated at x_n

$f''(x_n)$ = the second derivative of f(x), evaluated at x_n

PROGRAM DEVELOPMENT

The previously described mathematical development for analysis of optimal flood damage reduction measures has been assembled into a computer program to facilitate rapid solution. The program consists of a main program and five sub-routines. This modular construction permits the re-use of the smaller sub-routines which perform special functions repeatedly during the main program execution. Four of the sub-routines perform the calculations corresponding to the four areas of computation described, i.e., unit hydrograph synthesis, runoff hydrograph computation, calculation of average annual flood damages, and optimization of flood damage reduction according to the net benefit criterion. The fifth sub-routine in the program computes the value of the objective function during the gradient search procedure. The program was coded and executed on an IBM 360 computer network and requires less than two seconds of computer time to process. A description of the program network is given in Figure 3. The program is capable of optimizing the amounts of channel modification and flood proofing needed to produce maximum net benefits. This is accomplished by relating incremental costs of these measures to the discharge amount which each one provides for flood damage reduction. The program does not evaluate the effect of reservoirs in the hydrologic system nor are individual reaches combined and routed to the control point.

The algorithm described on page 11 is the basis for sub-routine UHG - this sub-routine computes the basin unit hydrograph by Gray's method. Sub-routine RUNHYD computes runoff hydrographs and selects the peak discharges for specified storms of known recurrence intervals.

The average annual damages are computed as described on page 13 in sub-routine FLDDAM. The average annual damage is the area under the damage-frequency curve. The program relates stage damage information to frequency data and computes average annual damages by summing the multiples of the frequency range centered around that frequency for which the damage was related. Ten floods are used to describe the discharge (stage) damage relationship. Since depth damage relationships are more applicable to urban structures, only flood proofing costs and flood damages for urban structures were handled by the program. The technique used to determine the optimum mix of damage reduction measures is the gradient search method. Sub-routine OPTIM computes the optimal value of system variables by employing the Newton-Raphson procedure to evaluate the best direction for improvement of the objective function. Sub-routine CRITNV computes the value of the objective function for changes in the variables.

Input data include:

a) basin characteristics - area, average slope, and length of main channel

b) average precipitation intensities for specified duration and recurrence intervals

FIGURE 3. - COMPUTER MACRO-FLOW CHART

COMPUTER MACRO-FLOW CHART

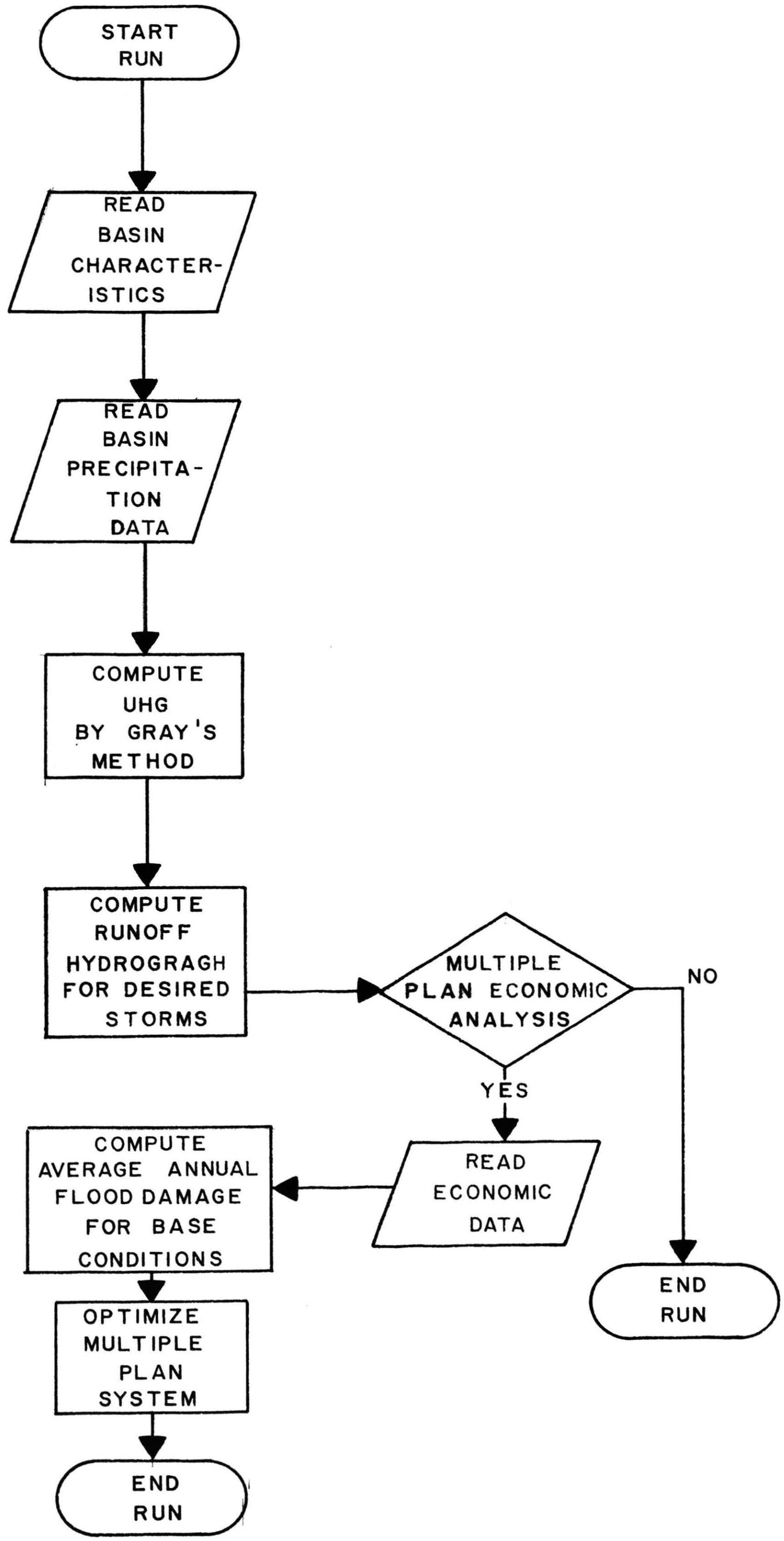


Figure 3

c) discharge-damage relationships for existing hydrologic conditions

d) discharge-cost relationships for types of flood damage reduction plans, i.e., channel modification, flood proofing, and land use regulation

e) initial values of channel modification and flood proofing to be used in the optimum search procedure

f) appropriate discount factors to convert the first cost of damage reduction to an average annual cost

Output data from the program include:

a) the optimum amount of discharge to be contained by channel modification, flood proofing, or other forms of flood damage reduction

b) total system capital and annual cost

c) average annual damages under existing conditions

d) average annual damages under modified conditions

e) damage reduction (benefits)

f) benefits minus costs (maximum net benefit)

APPLICATION TO A WATER RESOURCE PROBLEM

An application of the model was made to determine its usefulness in solving a real water resource problem. Information on a small community susceptible to flooding from a small, ungauged stream, which disects the town, was gathered as shown in Table 1.

A functional relationship between average rainfall intensity, recurrence interval and rainfall duration was derived for the basin. The algorithm for the development of this relationship is as follows:

1. Select the values of rainfall (inches) versus duration (hours) using TP 40 (15) to obtain intensity (in/hr) for the 2, 10, 25, 50, and 100-year events, as shown in Table 2.
2. Graph intensity versus duration for the various events listed above, using full logrithmic paper.
3. Linearize a family of curves by trial and error using a constant as shown in Figure 4.
4. Plot the intercept of the family of curves versus recurrence interval on semi-logrithmic paper to obtain $B = f\ (\ln R)$ shown in Figure 5.
5. Substitute $B = f(\ln R)$ into $i_{av} = B\ (t_r + a)^{-m}$ to obtain $i_{av} = f\ (R,\ t_r)$

The resulting equation for i_{av} is

$$i_{av} = (\ln R^{-.384} + 1.45)\ (t_r + .05)^{-.646} \tag{18}$$

Where i_{av} = average rainfall in inches

R = recurrence interval expressed as reciprocal of probability in percent

t_r = rainfall duration in hours

Using equation (18), average rainfalls were derived for specified duration and recurrence intervals. An assumed rainfall loss (inches), based on soil types and vegetal cover, was subtracted from the total rainfall to obtain the rainfall excess used to compute the runoff hydrographs of the same duration as the unit graph derived by Gray's Method. The data were assumed to fit log-normal frequency distribution. A graph of peak discharge versus frequency is shown in Figure 6. Stage-damage-probability relationships were based on this relationship. Each time a modification in the stage-probability relationship occurred due to a channel modification or flood proofing, a corresponding transformation in the damage-probability relationship was made to enable a new computation of annual flood damages.

The peak discharges were used to estimate average annual flood damages from empirical stage-damage relationships shown in Table 1. Incremental costs of alternative flood protection measures were estimated as shown in Table 3. Table 4 summarizes the output data obtained by the computer solution of the model.

DISCUSSION AND CONCLUSIONS

The model appears to have valuable analytical capability for planning local flood protection works. However, reservoir systems are not able to be evaluated by the model. The reason for this restriction is the obvious lack of ability to perform hydrologic routing of floods in this model. The addition of that capability would permit investigation of the effect of storage on attenuation of peak flood flows and would allow analysis of more than one control point in the system. The model complexity would greatly increase as each additional channel modification is introduced.

The results of the test application, as shown in Table 4, reveal that for the concept of one control point or index station, the

FIGURE 4. - RAINFALL INTENSITY VERSUS DURATION

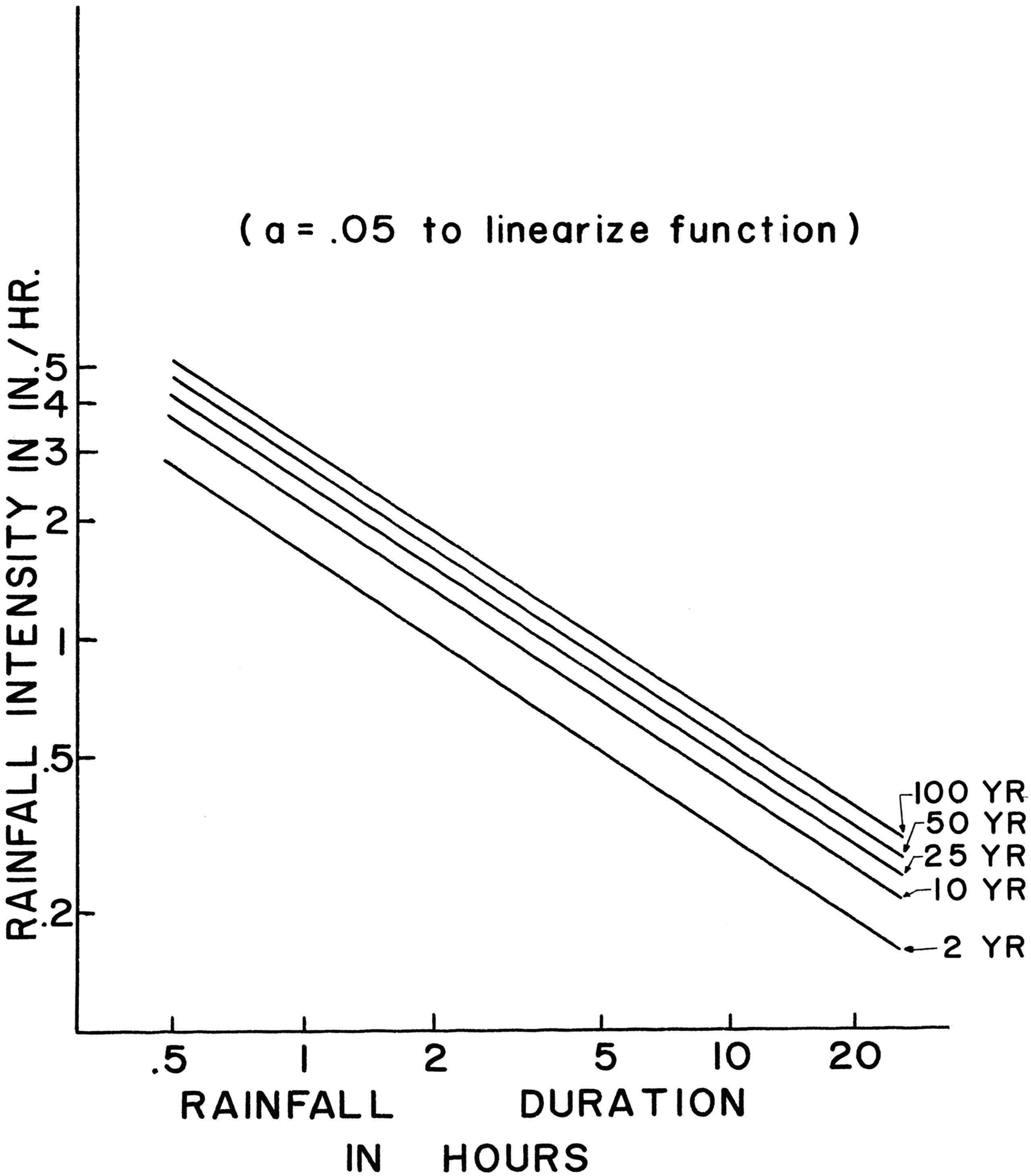

(a = .05 to linearize function)
RAINFALL INTENSITY IN IN./HR.
5
4
3
2
1
.5
.2
100 YR
50 YR
25 YR
10 YR
2 YR
.5
1
2
5
10
20
RAINFALL DURATION
IN HOURS

Figure 4

FIGURE 5. - "B" INTERCEPT VERSUS RECURRENCE INTERVAL

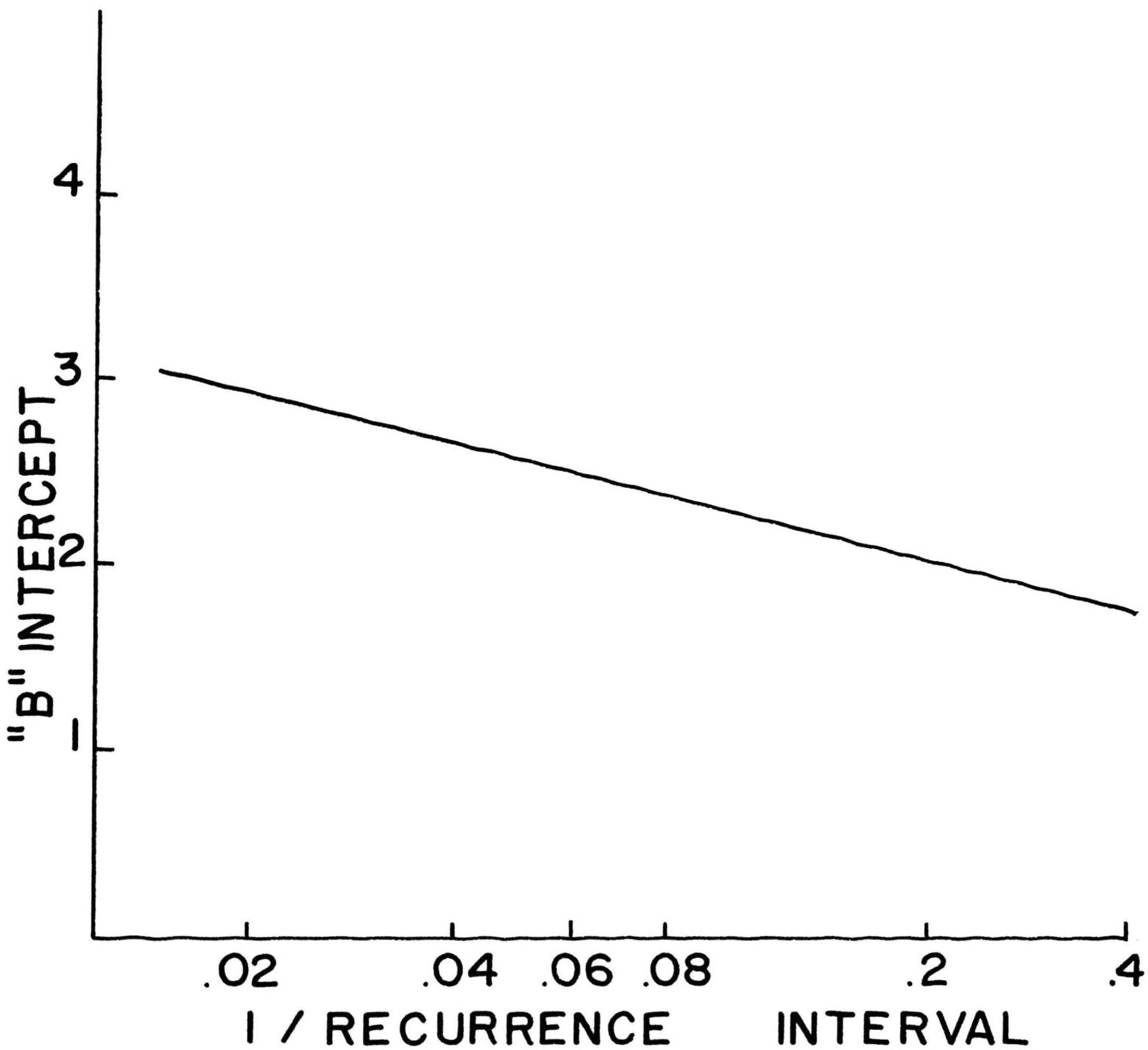


Figure 5

FIGURE 6. - FLOOD DISCHARGE VERSUS EXCEEDENCE INTERVAL

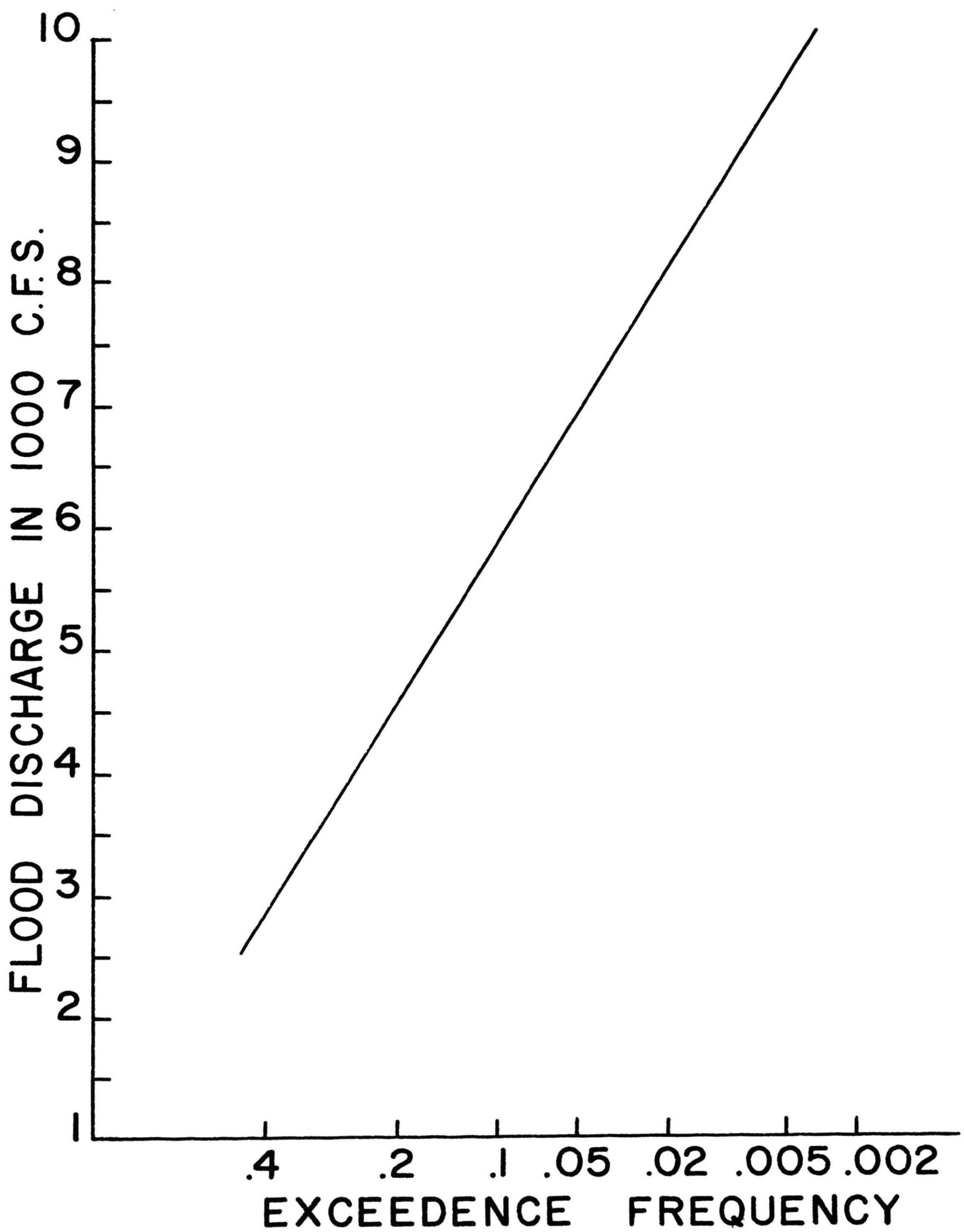

10
9
8
7
6
5
4
3
2
1
FLOOD DISCHARGE IN 1000 C.F.S.
.4
.2
.1
.05
.02
.005
.002
EXCEEDENCE FREQUENCY

Figure 6

TABLE 1. - COMMUNITY DISCHARGE-STAGE-DAMAGE RELATIONSHIP

a. Structural Damage

Flood Discharge (CFS)	Elevation (MSL)					Accumulated Damage ($)
	440	442	444	446	448	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
4,680	0	6,900	0	0	0	6,900
7,260	0	8,970	96,600	0		105,570
10,500	0	10,120	125,580	72,450	0	208,150
19,600	0	11,040	141,680	94,185	10,350	257,255

b. Content Damage

Flood Discharge (CFS)	Elevation (MSL)					Accumulated Damage ($)
	440	442	444	446	448	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
4,680	0	8,338	0	0	0	8,383
7,260	0	3,910	40,250	0	0	44,160
10,500	0	4,485	54,740	0	0	89,413
19,600	0	4,772	62,790	41,055	4,312	112,929

TABLE 2. - RAINFALL (IN) & RAINFALL INTENSITY (IN/HR) FOR TEST APPLICATION

a. RAINFALL (IN)

DURATION (HOURS)	RECURRENCE INTERVAL 2	10	25	50	100
(1)	(2)	(3)	(4)	(5)	(6)
.5	1.38	1.80	2.05	2.30	2.50
1	1.58	2.26	2.58	2.90	3.20
2	1.90	2.75	3.25	3.40	3.75
3	2.25	3.10	3.45	3.85	4.20
6	2.60	3.60	4.20	4.60	5.00
12	3.05	4.25	4.80	5.40	5.90
24	3.50	4.90	5.70	6.30	6.80

b. RAINFALL INTENSITY (IN/HR)

DURATION (HOURS)	2	10	25	50	100
(1)	(2)	(3)	(4)	(5)	(6)
.5	2.76	3.60	4.10	4.60	5.00
1	1.58	2.26	2.58	2.90	3.20
2	.95	1.37	1.62	1.70	1.87
3	.75	1.03	1.15	1.28	1.40
6	.43	.60	.70	.77	.83
12	.25	.35	.40	.45	.49
24	.14	.20	.24	.26	.28

TABLE 3. - FIRST COST FOR FLOOD DAMAGE REDUCTION

DISCHARGE (CFS)	CHANNEL MODIFICATION ($)	FLOOD PROOFING AND RELOCATION ($)
(1)	(2)	(3)
2,960	--	--
4,680	12,800	20,000
5,785	15,800	152,800
7,260	571,900	330,000
7,710	653,300	417,500
9,870	1,045,000	837,500
10,500	1,161,600	960,000
11,850	1,280,100	1,051,700
14,000	1,453,200	1,197,800
16,240	1,750,700	1,350,000

TABLE 4. - OPTIMIZATION RESULTS

a. Initial Values

Channel Size (CFS)	Flood Proofing (MSL)	Average Annual Damage ($)
(1)	(2)	(3)
6,000	--	20,400

b. Final Values

Channel Size (CFS)	Flood Proofing (MSL)	Average Annual Damage ($)
(1)	(2)	(3)
3,400	440.3	830

c. Damage Reduction

Annual Damage Reduction	Annual Net Benefit	Design Frequency (%)
(1)	(2)	(3)
19,500	19,100	.04

evaluation of modifications to the damage-probability relationships can be successfully handled by the model. Changes to channel storage were neglected to allow only consideration of the changes in damage-probability relationships. It was shown that a systems analysis approach to flood damage reduction is practical when considering the stochastic nature of the inputs. For example, changes in channel geometries affected the stage-damage-probability relationships by reducing the stage and consequently the damage (though not the peak discharge) for specified probabilities of occurrence. The assumption herein is that the effect of changed channel geometries, at the control point, will have a negligible effect upon the peak discharge of the basin runoff hydrograph. Where the length of stream is a small percentage of the total basin main stream length, as is the case in the test application, this assumption appears reasonable. The affected damage reductions, which are expressed as the expected value of benefit, can be comparably measured with the annual cost of production of the changed channel geometries. The results are analogous if one considers the output with a change in land use patterns; e.g., the effect of flood proofing individual family units may be measured by a process similar to that used for channel modifications. By assimilating all social costs of flooding, i.e., the costs of production and residual flood damages in one utility function, it was shown that meaningful results could be obtained towards the task of optimizing the amounts of project input and output according to the net benefit criterion.

The consideration of nonstructural measures along with structural measures for flood damage reduction was shown to be practical in this model. The term "nonstructural" as used in the literature refers to methods other than those involving engineered facilities (e.g., reservoirs and channel modifications) to reduce flood damages. In the context of the model application, flood proofing and relocation were the selected approach. Flood proofing of framed structures has been shown feasible provided they are elevated no higher than three feet onto new foundations. On that basis, relocation must be employed where flood depths are expected to exceed three feet.

The concept of minimizing the total social cost of flooding, as the objection function of net benefits, was shown to facilitate the comparison of flood proofing with channel modification as a method of flood damage reduction. As a practical consideration, flood proofing and relocation were considered as a combination measure in the test application. The program development takes into account the change in the damage-probability relationships caused by measures such as only when relocation is considered as an independent variable in the optimization process. Relocation of flood prone structures is the application of the philosophy which looks at controlling society's use of land as it pertains to flood hazards as opposed to controlling flood hazards as it pertains to society. The removal of damageable structures, in effect, reduces the expected value of damages at or below the design elevation from the total value of expected damage for the entire range of floods. This amount of damage reduction may then be creditable as a benefit to the project. The situation is somewhat analogous to the damage reduction produced by a levee. The value of expected damages prevented below the levee design elevation is creditable as benefits. In another application of this model, therefore, levees and/or relocation could be individually evaluated.

In summary, the use of systems engineering was shown to be valuable in the optimization of measures to produce flood damage reduction. By viewing the selection problem of system modifications from the utility function viewpoint where inputs and outputs were compared in commensurate units, i.e., flood costs, progress can be made towards determining optimal flood control systems according to the net benefit criterion. This approach also facilitates the comparison of nonstructural approaches to reduce flood damages. It was shown that many such measures have analogous effects on damage-probability relationships as those of channel modifications and levees. Therefore, the same transformation of probability distributions will occur in the model of system effects. Knowledge gained from the limited application of such simple models as described herein should enhance the water resource planner's ability to deal with more complex systems.

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APPENDIX II. - NOTATIONS

The following symbols are used within this paper:

A = area in square miles

a = constant used to linearize relationship - rainfall intensity versus duration

B_j = unit benefit associated with the corresponding coordinate, j, of the output vector

C = cost in dollars

C_a = first cost of construction expressed as an average annual value in dollars

C_d = average damage inflicted by shallow flooding in dollars

C_i = unit cost associated with the corresponding coordinates, i, of the input vector

C_p = installation cost

C_1 = annual cost of flooding in dollars

C_2 = annual cost of structural measures for flood damage reduction in dollars

C_3 = annual cost of nonstructural measures for flood damage reduction in dollars

D = flood damage in dollars

D_{av} = expected value of flood damage in dollars

$D_1, D_2, \ldots, D_n$ = flood damage caused by the corresponding n th flood in dollars

D_i = flood damage at the i th contour interval in dollars

$D*$ = expected value of flood damage under existing conditions in dollars

d = average depth of flooding in feet

e = base of natural logarithm approximately equal to 2.71828

Γ = gamma function

γ = scale parameter

γ' = dimensionless parameter equal to the product $P_r q$

h = height of flood proofing in feet

i_{av} = average rainfall in inches

K_d = flood damage coefficient determined by analysis of the direct damage caused to similar property by historical floods

L = length of main stream in miles

M_s = market value of flood prone structures in dollars

NB = net benefits (total benefit - total cost) in dollars

O = annual cost of operation and maintenance of flood control facilities in dollars

P_r = period of rise; the time from the beginning of runoff to the occurrence of peak discharge

$P_1, P_2, \ldots, P_n$ = probability that a flood will occur in any given year expressed as a percentage

Q = rainfall excess in inches

Q_t = flow at any given time, t

q = phase parameter

q_n = flow associated with the n th time period

R = recurrence interval expressed in years; reciprocal of probability

S = scale of project development in dollars

S_c = stream slope expressed as a percent

S_i = flood level at any given location within i th contour

σ = unit damage function

TC = total cost in dollars

t = time in minutes

t_r = rainfall duration in hours

U = the utility or value of net benefit function in dollars

U_n = unit hydrograph ordinate at time period n

$X_1, X_2, \ldots, X_n$ = approximations of variable evaluated at the n th iteration of the Newton-Raphson procedure

x = input vector

y = output vector

Z = value of objective function

ζ elevation of the j th step of flood plain in feet

APPENDIX III. - VITA

James Richard Dexter

Mr. Dexter, who is a native of Syracuse, New York, was born on 2 July 1948. He was graduated from West Genesee High School in 1966, where upon he enrolled in the University of Missouri at Rolla. Aside from temporary summer employment in Arizona and field training in Wyoming, he has lived entirely within the States of New York and Missouri. He is presently employed with the U. S. Army Corps of Engineers in St. Louis, Missouri, where he and his wife, Carolyn, make their home. They enjoy a number of outdoor recreational activities, including backpacking, float fishing, outdoor photography, bicycling and tennis. He is a member of the American Society of Civil Engineering and American Water Resources Association.

Mr. Dexter's professional training and experience were initiated with undergraduate studies in the major field of geological engineering. Upon graduation in December 1970, with a Bachelor of Science degree in that field, he accepted employment with the St. Louis District, Corps of Engineers, where he has principally worked in the Urban Studies Section of the Planning Branch. In this position, he has been involved with the planning of urban-oriented water resource projects within the civil works program of the Corps of Engineers. His present capacity as planning coordinator for an urban flood control project in the St. Louis Metropolitan area requires him to oversee and participate in studies in a number of fields, including hydrology, economics, and environmental impact assessments. His particular area of interest is studying the economic considerations involved with the combination of "structural" and "nonstructural" measures to achieve flood damage reduction. This interest has followed two courses of study. One course is on-the-job research on the application of computer models to evaluate alternatives for flood control; this application includes on-going experimentation with the Hydrologic Engineering Center in Davis, California. The second course is graduate study in the area of water resources engineering at the University of Missouri-Rolla Graduate Engineering Center, located in St. Louis.