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Some aspects of transient flow behavior in artesian aquifers and in hydrocarbon reservoirs surrounded by artesian aquifers

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SOME ASPECTS OF TRANSIENT FLOW BEHAVIOR IN ARTESIAN AQUIFERS AND IN HYDROCARBON RESERVOIRS SURROUNDED BY ARTESIAN AQUIFERS

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ABSTRACT

An analysis of the principal theories concerning the transient movement of water in artesian aquifers and from artesian aquifers into hydrocarbon reservoirs is presented. Various analytical methods which can be applied to depict the behavior of these two types of aquifer systems are compared.

Most aquifer studies to date have been based upon the Theis Non-Equilibrium Formula. This equation has been used to calculate the coefficients of storage and transmissibility for aquifers; to evaluate well performance; and to investigate problems concerning recharge, movement, and discharge of aquifer water. However, one problem which still remains is that of accurately predicting the recharge volumes which supply the reservoir drawdown area under the effects of pressure decline.

The present study suggests a possible means of accurately determining recharge volumes by the application of the Van Everdingen-Hurst Laplacian solution to the radial diffusivity equation for transient water movement. This solution would be based upon the withdrawal rates, cumulative withdrawals, and aquifer pressure profiles.

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I. INTRODUCTION

At the present time, much concern is being shown regarding the serious depletion of fresh water in almost all parts of the country. Aquifer depletion has already become critical in some of the more densely populated sections along the east coast. In considering the potentialities for growth and industrial expansion in these regions, it is of the utmost importance that accurate data and techniques are available to provide for the explicit prediction of the quantities of aquifer water which will be available at various times in the future, and the rates at which these underground aquifers can be produced without seriously reducing the recharge volumes or sustaining pressures.

One of the main emphases of current hydrologic research is the accurate evaluation of the availability of aquifer water. The worth of an aquifer as a source of surface water supply rests primarily on its ability to transmit and store recharge water. Therefore, the Coefficient of Storage (the ability of the aquifer to store recharge volumes of water) and the Coefficient of Transmissibility (the index reflecting the ability of the reservoir to discharge water) form the bases for most quantitative aquifer studies.

In the past most of the research concerning aquifer water movement and surface water supply has been undertaken by hydrologists, geologists, and civil engineers working either individually or in cooperation with some state or federal agency such as the U. S. Geological Survey. In the last few years, however, the development of underground gas storage pools in aquifers, the increased demand for large quantities of water for water-flooding projects, and the problem of large scale salt water disposal has caused many petroleum and natural gas companies to become actively engaged in aquifer research. As a result of this new interest, considerable work has been undertaken recently by petroleum and natural gas engineers in the analysis of the underground movement of water in contact with hydrocarbon reservoirs.

The quantitative research work concerning aquifer water movement presented to date by the hydrologist generally deals with the amounts of water entering the wellbore and the resulting pressure drawdown in the reservoir at various producing rates. On the other hand, the petroleum industry has primarily concerned itself heretofore with the amount and rate of water influx into oil and gas reservoirs, and, indirectly, with the transient flow behavior in aquifers.

These two different approaches to the definition of aquifer water movement by the hydrologists and by the petroleum industry have indicated that perhaps some of the techniques and methods developed by both groups could be combined into an improved approach to the problem of underground water movement.

The purpose of this study is threefold:

1. To review the technology concerning water movement in artesian aquifers and from artesian aquifers into hydrocarbon reservoirs;

2. To point out the basic similarities and differences between these two approaches to aquifer studies; and

3. To investigate the feasibility of adapting the Van Everdingen-Hurst Laplacian solution to the diffusivity equation (along with digital computing techniques) to predict the deliverability of radial municipal water supply reservoirs.

II. LITERATURE REVIEW

A. Aquifer Performance

The flow of aquifer water may occur as a steady-state or as an unsteady-state phenomenon. If the former is in effect, all water produced from the aquifer is assumed immediately replaced by equal amounts flowing in from some contiguous source. In unsteady-state flow, however, the volume of water produced from the aquifer system is not equal to the recharge volume. If recharge is greater than production (or through-put), the aquifer will be recharged; if outflow is greater than recharge, the aquifer will be progressively depleted.

The movement of aquifer water is principally in a lateral direction through permeable strata and where these strata lie between relatively impermeable strata, the latter tend to form more or less effective confining beds. If the formations are tilted or deformed, the water may flow through the permeable strata for long distances away from the intake area. Generally water moves down dip for some distance away from the intake area, but at later times it may move either up or down dip depending upon the deformation of the strata.

If the loss in head due to flow resistance is less than the net descent of the water-bearing formation, then the water is under artesian pressure in the sense that it will rise in producing wells to some level above the top of the formation itself. If the loss in head is less than the descent of the land surface then the artesian pressure may be sufficient to cause the wells to overflow at the surface.

If water above the overlying confining bed of the artesian aquifer is under a greater head than the water in the confined aquifer itself, there will be some degree of percolation or leakage into the artesian aquifer unless the confining bed is strictly impermeable. In many artesian systems, such movement or leakage of water in either direction through the confining beds is an important factor in the recharge and discharge of the aquifer and in the prediction of the extent of the "cone of depression" around a discharging well. Hantush and Jacob(16) are among those authors who have recently considered the problem of leaky artesian aquifers.

An aquifer is considered to be of infinite areal extent when its exterior radius is so large relative to its interior radius (represented by a wellbore or the outer boundary of a hydrocarbon reservoir) that the water movement in the vicinity of this exterior boundary is negligible during the time period under consideration (e.g., usually 20 years). If the radius of water movement (radius of drainage) reaches the exterior boundary of the aquifer during the above time period, then the aquifer is considered to be of finite extent.

The pressure decline within an aquifer is, in almost all cases, an unsteady-state phenomenon in that pressures within the porous strata at a given point are time-dependent.

As the pressure changes on each element of the water bearing aquifer, the amount of water leaving each element is not the same as the amount entering that element due to the compressibility of the water.

For an artesian aquifer the head of water may decrease due to production from various wells, but the aquifer remains saturated before, during, and after this decrease in head. The confining impermeable beds can be considered as fluid in the sense that they have no ability to absorb or dissipate changes in forces external to or within the aquifer. Since no dewatering or filling of the pore space is involved in the case of an artesian aquifer, the water released from or taken into storage (by production or recharge) can be attributed only to the compressibility of the aquifer material and of the water. The volume of water (measured at the surface) released or stored divided by the product of the head change and the area of aquifer surface over which the head change (either a decrease or an increase) is effective determines the Storage Coefficient (S) of the aquifer (See Fig. 7a, Definitions.). Normal values of S for artesian aquifers are from 0.00001 to 0.001.

In elastic artesian aquifers the Coefficient of Transmissibility (See Fig. 7b, Definitions.) is assumed to decrease due to the compaction of the aquifer upon release of pressure due to water production. In such aquifers, a specific amount of water is discharged instantaneously from storage as the pressure falls.

1. Equilibrium Formula

Prior to 1935, when Theis(1) viewed the problem of aquifer water movement as essentially an unsteady-state relationship, it was assumed that equilibrium was attained in the producing portion of the aquifer and that water levels did not fall. This equilibrium approach was thus based on the assumption that the aquifer system had been pumped long enough so that steady-state conditions existed in the reservoir. Theim(2) developed his Equilibrium Formula through a modification of Dupuit's(3) analysis of Darcy's Law(4). This work provided for the determination of a coefficient of transmissibility, $\underline{T}$, which, in turn, provided the rate of discharge, Q, and the drawdowns, $\bar{s}_1$ and $\bar{s}_2$, for two observation wells located known distances, r_1 and r_2, from the discharging well. Theim's equation in standard hydrologic units is:

$$\underline{T} = \frac{Q \, Ln[r_2/r_1]}{2\pi \, [\bar{s}_1 - \bar{s}_2]} \qquad (1.1)$$

where:

Q = Rate of discharge, cubic feet per day.

$\underline{T}$ = Coefficient of transmissibility, cubic feet of water per day per foot of aquifer width. (See Fig. 7b, Definitions.)

r_1 & r_2 = Distances from the discharge well to the two observation wells, feet.

$\bar{s}_1$ & $\bar{s}_2$ = Water level drawdown in the two observation wells, feet.

Ln = Naperian logarithm (base e).

Theim's formula is based on the following assumptions, and its use is dependent upon the degree to which these assumptions represent actual field conditions:

1. The aquifer is homogeneous and isotropic, and of infinite areal extent.

2. The discharge well fully penetrates and receives water from the entire thickness of the aquifer.

3. The Coefficient of Transmissibility is constant at all times and at all places.

4. There is laminar flow to the discharge well.

5. Pumping has continued at an uniform rate long enough for equilibrium or steady-state flow to exist in the hydraulic system.

Wenzel[5] showed that the equilibrium formulas of Slichter[6], Turneaure and Russell[7], and Muskat[8] were but modified forms of Theim's method and were subject to the same limiting assumptions. These equilibrium formulas depend upon the determination of R, defined as the distance from the discharge well at which the drawdown of the water level is negligible. These formulas also assume that a condition of equilibrium exists over the entire area of influence, that is, from the discharge well to the distance, R. The assumed radius, R, can be used when the required two observation wells are not available and the two points necessary for Theim's method are:

1. The assumed radius of negligible drawdown, R.

2. The radius of the discharge well, r, and its water level, $\bar{s}$, which can be measured.

2. Equilibrium Radius

Observations of the behavior of the water level around pumped wells made by the U. S. Geological Survey show that the form of the cone of depression (See definitions.) reaches essential stability in a small area around a pumped well in a relatively short time after pumping begins. However, the area of essential stability expands very slowly, and a considerable period of pumping is necessary for the cone to reach an approximate equilibrium form very far from the pumped well.

Several investigators have given arbitrary values to be used for R - Slichter[6], 600 feet; Muskat[8], 500 feet; and Tolman[9], 1000 feet. All three of these assumed values for R are for artesian aquifers. Leggette[10], however, observed appreciable fluctuations of water levels in wells caused by shutting down of pumped wells as much as seven miles distant from the observation wells. This contrasts appreciably with other authors' values.

The basic assumption of these formulas of the equilibrium type is for practical purposes valid for only a small area around a discharging well in which equilibrium may be reached. The extent of the cone of depression is of practical significance in determining the spacing of wells and in the solving of many important legal controversies. Because empirical values for R, mainly intended for use in areas of known permeability, appear so frequently in the literature, it is often incorrectly assumed that the cone of depression can not exceed these values.

The formulas so far described are based on the assumption that the hydraulic system can attain a state of equilibrium - a condition that is reached only approximately near the discharging well. The factor of time is included in these formulas only in the sense that the well is assumed to have been discharging long enough to produce a state of equilibrium.

3. <u>Non-Equilibrium Equation</u>

In 1935 Theis[1] showed that the unsteady-state flow of underground water into a radial sink area, such as a well-bore or hydrocarbon reservoir, is governed by the Diffusivity Equation.

$$\frac{\partial^2 \bar{s}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{s}}{\partial r} = \frac{S}{T}\frac{\partial \bar{s}}{\partial t} \qquad (1.2)$$

where:

S = The Coefficient of Storage.

$\bar{s}$ = Drawdown at any point r around a well.

r = Distance from the discharge well.

t = Time of pumping.

The assumptions imposed on the derivation to Equation (1.2) are:

1. Darcy's Law (steady-state) applies.
2. The higher order $\left[\frac{\partial \bar{s}}{\partial r}\right]^2$ term is negligible.
3. A single fluid is present that occupies the entire pore volume.
4. The reservoir is horizontal, homogeneous, uniform in thickness, and of infinite radial extent.

5. The compressibility and viscosity of the fluid remains constant at all pressures.

6. The fluid density obeys the equation,

$$\rho = \rho_o e^{-c(P_o - P)} \tag{1.2a}$$

where:

ρ_o = Density of fluid.

c = Compressibility of fluid.

The development of the non-equilibrium equation based on Equation (1.2) was a major advance in hydrology. Theis' solution to the differential equation for the radial flow of water in an elastic artesian aquifer for a constant discharge rate is given by:

$$\bar{s} = \frac{114.6\ Q}{\underline{T}} \int_{\frac{1.87\ r^2 S}{\underline{T}\ t}}^{\infty} \frac{e^{-u}}{u}\, du \tag{1.3}$$

where:

Q = Discharge of the well, gallons per minute.

t = Time since pumping started, days.

$u = \frac{1.87\ r^2 S}{Tt}$

The integral expression in Equation (1.3) cannot be integrated directly, but can be approximated by the series[5]:

$$\int_{\frac{1.87\ r^2 S}{\underline{T}t}}^{\infty} \frac{e^{-u}}{u}\, du = W(u) = -.577216 - Ln(u) + u - \frac{u^2}{2(2!)} + \frac{u^3}{3(3!)} \cdots \frac{u^n}{n(n!)} \tag{1.4}$$

The exponential integral (1.4) is written symbolically as W(u) which is read "well function of u". Values of W(u) have been tabulated by Wenzel[5] and the non-equilibrium formula can be solved for T by a type-curve matching process[60].

The non-equilibrium solution (1.3) is based on the assumptions that:

1. The aquifer is homogeneous and isotropic.
2. The aquifer has infinite areal extent.
3. The discharge or recharge well fully penetrates the formation and receives water from the entire thickness of the aquifer.
4. The coefficient of transmissibility is constant at all times and at all places.
5. The well has an infinitesimal (reasonably small) diameter.
6. Water removed from storage is discharged instantaneously with the decline in head.

These restrictions have been found to take on varying degrees of significance in practice. Because the non-equilibrium formula assumes that the transmissibility of the aquifer does not change during the discharge period it can be strictly applied only to artesian conditions. The effect of aquifer heterogeneity on various solutions to Equation (1.3) is not definitely known. Stallman[12] and other authors have used the theory of images to analyze the effect of various types of finite boundaries on the solution to the non-equilibrium formula. Jacob[13] reviewed the works of Muskat[36] and Wenzel[5] concerning the effect of a discharge well tapping less than the full thickness of an aquifer and concluded that corrections must be made to the water levels observed during field tests before an accurate coefficient of transmissibility can be calculated from Equation (1.3). Muskat[14] showed that the error in drawdown level calculated by Equation (1.3) (which assumes a vanishing wellbore diameter rather than a wellbore of finite diameter) is insignificant except at very small pumping times or very short distances from the wellbore.

Jacob[15] recognized that the series of terms beyond Ln(u) in Equation (1.4) was not significant when (u) becomes small (i.e., when t increases or r decreases) and that this series could be truncated without adding significant error to Theis' equation. Jacob's Modified Equation is:

$$\bar{s} = \frac{Q}{4\pi \underline{T}} \left[\text{Ln} \left(\frac{4\underline{T}t}{r^2 \underline{s}} \right) - .5772 \right] \qquad (1.5)$$

or in standard hydrologic units:

$$\bar{s} = \frac{264\ Q}{\underline{T}} \left[\text{Log}\ \frac{0.3\underline{T}\ t}{r^2 \underline{s}} \right] \qquad (1.6)$$

Jacob realized that after equilibrium was attained, Equation (1.6) could be solved by graphical means to find the storage coefficient and the coefficient of transmissibility.

4. Cone of Depression

In nature, the hydraulic system within any aquifer is considered to be in balance. If further discharge is imposed by, say, a new well on this balanced system, then before equilibrium can be re-established, the water level must fall throughout the aquifer to such an extent that the natural discharge from the aquifer (measured prior to the new well) is decreased by an amount equal to the new quantity imposed on the system by the discharge well, or in a like manner, the recharge volume increased by the same magnitude. Until this equilibrium is re-established water will be withdrawn from storage in the aquifer. Conversely, balance cannot be re-established until sufficient water is withdrawn by the well from storage to depress the piezometric surface of the aquifer sufficient to change the natural discharge or recharge by the proper amount.

In an ideal aquifer of infinite extent the most important variable describing the growth of the cone of depression is considered to be time. The rate of lateral growth of the cone of depression with time during the non-equilibrium period of flow depends only on the physical properties of the reservoir and is independent of the discharge rate of the well(1). In artesian aquifers, the cone grows laterally much faster than it does in water-table aquifers. (See Fig. 7C, p. 60) This is due to the quantity of water removed from storage in an artesian aquifer by compaction of the strata and the expansion of the water in the aquifer upon decline in pressure is much less than the quantity of water that would be removed by the dewatering of the aquifer pore space under the same pressure decline in a water-table aquifer. The cone of depression for fine-grained sand aquifers appears to approximate the cone of ideal aquifers.

The expansion of the cone of depression around a discharging well is limited only when the exterior boundary of the aquifer is reached. The expansion of the area from which water is diverted, however, will occur until the recharge to this area is equal to the discharge from the well. The time at which stabilization of the cone is achieved is independent of both the rate of discharge and the continuity of discharge of the well.

5. Varying Discharge Rate

A common hydraulic problem is that of determining the effects of pumping at different rates on the ultimate drawdown or change in water level within the immediate area of the well. The rate at which water is pumped from a well or from a reservoir commonly varies with the seasonal surface requirements. In many cases the pumping rate, as recorded in terms of daily or monthly discharge, is found to change continuously. With this variation in pumping rate, the methods previously described cannot be applied without tedious modifications. Stallman(17) introduced a method of approximating this varying discharge rate by a series of graphical steps. The analysis of each step is subsequently undertaken using the conventional equations. A type curve for analyzing the observed drawdowns caused by this stepped pumping rate can be constructed by the use of the Theis non-equilibrium formula. This development proceeds as follows:

The drawdown, $\bar{s}$, at any distance, r, from the pumped well, at any time, t, is:

$$\bar{s} = \bar{s}_1 + \bar{s}_2 + \bar{s}_3 + \ldots \bar{s}_n . \quad (1.7a)$$

Applying the non-equilibrium formula Equation (1.3) to define each of the drawdown components given in Equation (1.7a) yields

$$\bar{s} = \frac{114.6}{\underline{T}}\left[\Delta Q_1 W(u)_1 + \Delta Q_2 W(u)_2 + \ldots \Delta Q_n W(u)_n\right]. \qquad (1.7b)$$

The corresponding u values are:

$$u_1 = \frac{1.87\ r^2 S}{\underline{T}(t - t_1)}\ ;\ u_2 = \frac{1.87\ r^2 S}{\underline{T}(t - t_2)}\ ;\ \ldots\ u_n = \frac{1.87\ r^2 S}{\underline{T}(t - t_n)} \qquad (1.7c)$$

Thus:

$$u_2 = u_1 \frac{t - t_1}{t - t_2}\ ;\ u_3 = u_1 \frac{t - t_1}{t - t_3}\ ;\ u_n = u_1 \frac{t - t_1}{t - t_n}. \qquad (1.7d)$$

In this manner a family of curves can be constructed with (1/t) and ($1.87\ r^2S/\underline{T}$) as the independent variables and $\sum_1^n \Delta QW(u)$ as the dependent variable. The drawdown at any time t at any radius r can be found by superimposing the field-data plot of log $\bar{s}$ versus log (1/t) on this family of type curves (plotted as log $\sum_1^n \Delta QW(u)$ versus log (1/t) and shifting the field-data plot until its curvature is identical with an underlying type curve). This serves to identify the data curve with a specific ($1.87\ r^2S/\underline{T}$). Values for $\bar{s}$ and $\sum_1^n \Delta QW(u)$ are then read from the graphs and can be entered in Equation (1.7b) to solve for $\underline{T}$. The computed value of $\underline{T}$ can then be used with the value of ($1.87\ r^2S/\underline{T}$) to solve for S by Equation (1.3).

6. Jacob and Lohman Solution

Jacob and Lohman(18) obtained a solution to the diffusivity equation for finding the coefficients S and $\underline{T}$ from a test where the drawdown, $\bar{s}$, was held constant by varying the discharge to the well. In this analysis, S and $\underline{T}$ are assumed to be constant and the aquifer is assumed infinite in areal extent. The flow rate for this analysis is found to be:

$$Q = 2\pi \underline{T} S_w G(\alpha) \tag{1.8a}$$

where $\bar{s}$ = constant = S_w:

and:

$$G(\alpha) = \frac{4\alpha}{\pi} \int_0^\infty x e^{-\alpha x^2} \frac{\pi}{2} + \text{Tan}^{-1} \frac{Y_o(x)}{J_o(x)} \, dx \tag{1.8b}$$

where:

$$\alpha = \frac{\underline{T}\ t}{r_w^2 S} \quad . \tag{1.8c}$$

Equations (1.8a and 1.8c) can then be rewritten in standard hydrologic units as:

$$Q = \frac{\underline{T}\ S_w\ G(\alpha)}{229} \tag{1.9a}$$

and:

$$\alpha = \frac{.134\ \underline{T}\ t}{r_w^2\ S} \tag{1.9b}$$

where:

S_w = Constant drawdown in the discharge well, feet.

r_w = Effective radius of the discharge well, feet.

$J_o(x)$ = Bessel function of zero order, first kind.

$Y_o(x)$ = Bessel function of zero order, second kind.

This equation cannot be integrated directly and is often solved by numerical methods.

Jacob and Lohman(18) also showed that for large values of t, $G(\alpha)$ can be replaced by:

$$G(\alpha) = \frac{2}{W(u)} \qquad (1.10a)$$

and, since W(u) is equal to:

$$W(u) = 2.30 \text{ Log } \frac{2.25 \, \underline{T} \, t}{r_w^2 \, S} \qquad (1.10b)$$

then Equation (1.8a) becomes:

$$Q = \frac{\dfrac{4\pi \, \underline{T} \, s_w}{2.30}}{\text{Log}\left[\dfrac{2.25 \, \underline{T} \, t}{r_w^2 \, S}\right]} \qquad (1.11)$$

Equation (1.11) is the equation of a straight line such that (s_w/Q) plotted against log (t/r_w^2) has a slope of:

$$\text{Slope} = \frac{\Delta (s_w/Q)}{\Delta \text{ Log}(t/r_w^2)} = \frac{2.30}{4\pi \, \underline{T}} \qquad (1.12a)$$

Once the slope of the line is determined, the coefficient of transmissibility can be computed from the relation:

$$\underline{T} = \frac{2.30 \, \Delta \, (\text{Log } t/r_w^2)}{4\pi \, \Delta \, (s_w/Q)} \qquad (1.12b)$$

The coefficient of storage can then be found by substituting the value of $\underline{T}$ from (1.12b) and the coordinates of any point on the straight-line plot into (1.11).

B. Aquifer-Petroleum Reservoir Performance

Aquifers which surround many oil and gas reservoirs have the ability to supply water to such reservoirs as oil and gas are withdrawn. This water-influx (called natural water drive) provides one of the most effective driving mechanisms for the production of oil and gas. Oil replacement by water from the aquifer may occur under the influence of various factors operating singly or in combination: by volumetric water expansion as a result of field-pressure decline, by hydraulic flow as a result of water infiltration at the outcrops of the reservoir rock, or by artificial injection of water into the oil-bearing horizon. The pressure behavior of the reservoir under water drive is dependent upon the rate of hydrocarbon withdrawal and upon the rate of water encroachment. When exact volumetric balance exists between water influx and hydrocarbon withdrawals, field pressure is maintained.

Available methods for estimating water-influx into hydrocarbon reservoirs which can be applied to the problem of water-drive reservoirs include the steady-state method of Schilthuis(43), the Hurst(44) modified steady-state method, and the various unsteady-state methods such as those of Van Everdingen-Hurst(19), Hurst(20), and Carter-Tracy(22).

There are two basic approaches by which the water-influx into a radial sink area can be evaluated; the constant terminal pressure approach and the constant terminal rate approach. In the constant terminal pressure case the pressure at all points in the formation is constant and equal to unity at time zero. When the well or reservoir is opened, the pressure at the well or reservoir boundary, $r_D = 1$, immediately drops to zero and remains zero for the duration of the production history. The cumulative amount of water flowing across the well or reservoir boundary is then computed as a function of time. On the other hand, in the constant terminal rate case it is likewise assumed that initially the pressure everywhere in the formation is constant but that from time zero onward the fluid is withdrawn from the well bore or reservoir boundary at a unit rate. The resulting pressure drop is then computed as a function of the time.

1. Van Everdingen and Hurst Methods

Van Everdingen and Hurst[19] have presented a solution to the diffusivity equation (1.2) for the unsteady-state isothermal flow of a slightly compressible fluid encroaching into a homogeneous reservoir sink. Their solution, developed by the application of Laplace transforms, yields an exact determination of the aquifer water encroachment across the aquifer-hydrocarbon reservoir boundary under the assumption that such encroachment is of a steady-rate (viz., constant terminal rate) nature.

The pressure drop is given by $P_D = P_D(r_D, t_D)$ and at the hydrocarbon reservoir boundary where $r_D = 1$:

$$\left[\frac{\partial P_D}{\partial r_D}\right]_{r_D=1} = -1 \qquad (1.13)$$

The minus sign is introduced to compensate for the pressure gradient direction relative to the radius of the reservoir. If the cumulative pressure drop is expressed as ΔP, then:

$$\Delta P = q(t_D)\, P_D(r_D, t_D) \qquad (1.14)$$

where $q(t_D)$ is a constant relating the cumulative pressure drop with the pressure change for a unit rate of production. By applying Darcy's equation[4] for the rate of fluid flow into the well or reservoir per unit sand thickness, H:

$$q_{(T)} = \frac{-2\pi K q(t_D)}{\mu} \left[\frac{\partial P_D(r_D, t_D)}{\partial r_D} \right]_{r_D=1} \qquad (1.15)$$

which simplifies to:

$$q_{(t_D)} = \frac{q(T)\mu}{2\pi K} \;. \qquad (1.16)$$

The ΔP at the reservoir radius (or well radius) $r_D = 1$ for any constant rate of production is given by:

$$\Delta P = \frac{q(T)\mu}{2\pi K} P_{(t_D)} \;. \qquad (1.17)$$

Since the diffusivity equation is linear, the Duhamel Superposition Theorem can be applied as a sequence of constant terminal pressures or constant terminal rates in such a way that the production or pressure history at the aquifer-hydrocarbon reservoir boundary ($r_D = 1$) is reproduced.

The cumulative water produced at time (t_D) by a pressure drop ΔP_o, operative since time zero, is expressed by:

$$Q_{(T)} = 2\pi\phi c \, r_b^2 \Delta P_o \, Q_{(t_D)} \;. \qquad (1.18)$$

Then considering the pressure drop ΔP_1, which occurs at time (t_{D1}), and treating this as a separate entity acting since time (t_{D1}), the cumulative water produced by this increment of pressure drop is:

$$Q_{(T)} = 2\pi\phi c \, r_b^2 \Delta P_1 \, Q_{(t_D - t_{D1})} \;. \qquad (1.19)$$

By superimposing all the effects of pressure changes and by taking very small incremental pressure drops, the total water influx in time (t_D) is expressed as:

$$Q_{(T)} = 2\pi\phi c\ r_b^{\ 2} \int_0^{t_D} \frac{\partial \Delta P}{\partial t_D'} Q_{(t_D - t_D')}\ dt_D' \qquad (1.20)$$

Hurst[20] also presented a solution to the diffusivity equation derived by the application of a Fourier-Bessel series for an unsteady-state of water encroachment across an aquifer-hydrocarbon reservoir boundary (viz., constant terminal pressure case). Due to the similarity of Hurst's solution with that of Van Everdingen-Hurst, the Hurst method will not be discussed.

Van Everdingen and Hurst[19] developed a constant terminal pressure solution to the diffusivity equation by Laplace transforms which is similar to Hurst's[20] solution except for the nomenclature. By considering variable rates of fluid production and reproducing these rates as a series of constant steps, the pressure drop at the wellbore or reservoir boundary (r_D=1) in time (t_D), for the initial rate (q_o) can be found from:

$$\Delta P = q_o P_{(t_D)} + [q_1(t_{D1}) - q_o]\ P_{(t_D - t_{D1})} + \ldots [q_{(t_{Dn})} - q_{(t_{Dn-1})}]\ P_{(t_{Dn} - t_{Dn-1})} \ . \qquad (1.21)$$

If the increments are infinitesimal:

$$\Delta P = q_o\ P_{(t_D)} + \int_0^{t_D} \frac{dq(t_D')}{dt_D'}\ P_{(t_D - t_D')}\ dt_D' \qquad (1.22)$$

If $q_o = 0$:

$$\Delta P = \int_0^{t_D} q_{(t_D')}\ P'_{(t_D - t_D')}\ dt_D' \qquad (1.23)$$

where $P'_{(t_D)}$ is the derivative of $P_{(t_D)}$ with respect to (t_D).

2. <u>The Wilson-Carlile Approximation</u>

Wilson and Carlile(21) have reproduced the results of Van Everdingen-Hurst and of Hurst for the constant terminal pressure case with a simple logarithmic time function. This simplified approach eliminated the time consuming application of dimensionless rate functions in a series summation by expressing the water encroachment as a simple function of time alone. This approximation, while reducing the complexity of the solution by about 80%, reproduces the previous solutions(19)(20) to within 99%. The Wilson and Carlile approximation is given by:

$$W_e = 2\pi \phi c_w\ H\ r_b^2 \theta \sum_{j=0}^{i} (\Delta P)_j\ A_n [t_{D_i} - t_{D_j}]^{B_n} \qquad (1.24)$$

where:

W_e = The cumulative volume of water encroaching, bbl.

H = The net sink formation thickness, feet.

θ = The fraction of the periphery of the sink subjected to influx of water, fraction.

A_n, B_n = Coefficients of approximation.

t_D = Dimensionless time at which W_e volumes of water have encroached.

c_w = Compressibility of water, 1/psi.

ϕ = Porosity, fraction.

ΔP = Pressure change, psi.

3. Other Methods

Carter and Tracy[22] developed a method for calculating water influx which eliminated the superposition calculations of Van Everdingen and Hurst[19]. Their method is somewhat similar to that of Hurst[18] except that over finite time intervals the water influx rates are assumed constant rather than assuming constant oil production rates. By combining the Hurst approach with the material balance equation of Schilthuis[43] they developed a method which lends itself to easy solution by hand calculations.

Chatas[23][24][25] in a series of three articles summarized the work of Van Everdingen and Hurst and further extended their results for higher and lower ranges of dimensionless time (t_D).

Many investigators have used the response of the "dimensionless aquifer" to a unit pressure drop, or a unit fluid-withdrawal volume to calculate the performance of an aquifer in supplying water-influx to an oil reservoir. In the past, these response functions have been calculated with the aid of the Laplace transform. With the development of high speed computers, these response functions have been solved for by finite-difference methods.

Mueller[26] expanded the so-called V.T.M. method originally proposed by Hurst[18] and Van Everdingen-Hurst[19], and later expanded upon by Van Everdingen, Timmerman, and McMahan[27] to apply to the transient response of non-homogeneous aquifers.

In the V.T.M. method, a material balance is made on the fluids entering and leaving the reservoir. In this balance, the water-influx term is the product of the water-influx from an arbitrarily selected dimensionless aquifer system and a conversion number. If the correct dimensionless aquifer has been chosen, then the conversion number will remain constant over the history of the reservoir. If such a condition exists, then the function associated with the particular dimensionless system and the derived conversion number can be used to predict the future performance of the reservoir. These functions are referred to as "response functions".

The response function required for the solution of the constant-terminal-rate problem is a relationship between dimensionless pressure drop (P_D) and the dimensionless time (t_D). For the constant terminal pressure case a function relating dimensionless flow rate (Q_D) to dimensionless time (t_D) is required.

The shape of these various response functions depends upon the geometry of the system, the conditions imposed at the inner and outer boundaries, and the ratio of the outer boundary to the inner boundary. Since the system is assumed homogeneous, the character of the particular dimensionless function depends mainly upon the radius ratio of the aquifer system.

4. <u>Mortada's Work</u>

Mortada(28) discussed the problem of oilfield interference in water-drive reservoirs. He considered the problem of multiple oilfields located in a common aquifer and the effects of pressure drop in the various oil fields on the rate of water-influx into the reservoirs.

In his paper, Mortada presented solutions to the diffusivity equation (1.2) for values of dimensionless time (t_D) and dimensionless radius (r_D) for the constant rate case which are normally required for field analyses with the following boundary conditions:

1. $P_D(r_D, 0) = 0$ (uniform initial aquifer pressure)
2. $P_D(r_D, t_D) \rightarrow 0$ as $r_D \rightarrow \infty$ (extensive aquifer)
3. $\left[\dfrac{\partial P_D}{\partial r_D}\right]_{r_D=1} = -1$ (constant rate of water influx).

Mortada's values for the dimensionless pressure $P_D(r_D, t_D)$ were obtained by several methods.

For $t_D \leq .01$ the relationship

$$P_D(r_D, t_D) = \frac{2\sqrt{t_D}}{\sqrt{r_D}} \text{ ierfc } \frac{r_D - 1}{2\sqrt{t_D}} \tag{1.25}$$

was solved, where:

$$\text{ierfc}(\bar{x}) = \frac{e^{-\bar{x}^2}}{\sqrt{\pi}} - \bar{x}\ \text{erfc}(\bar{x}) \tag{1.26}$$

which can readily be found in mathematical function tables[29]

For $t_D \geq 500$:

$$P_D(r_D, t_D) = \frac{1}{2}\left[-Ei\left(\frac{-r_D^2}{4t_D}\right)\right] \tag{1.27}$$

which was derived from the continuous point sink solution of Lord Kelvin, where:

$$-Ei\left[\frac{-r_D^2}{4t_D}\right] = \int_{\frac{r_D^2}{4t_D}}^{\infty} \frac{e^{-\bar{x}}}{\bar{x}}\, d\bar{x}\ . \tag{1.28}$$

To bridge the gap between $t_D \leq .01$ and $t_D \geq 500$, a digital computer was used to solve a set of finite-difference equations based upon the diffusivity equation. This technique provided values of $P_D(r_D, t_D)$ which showed no change in the third decimal place as the values of Δt_D were chosen progressively smaller.

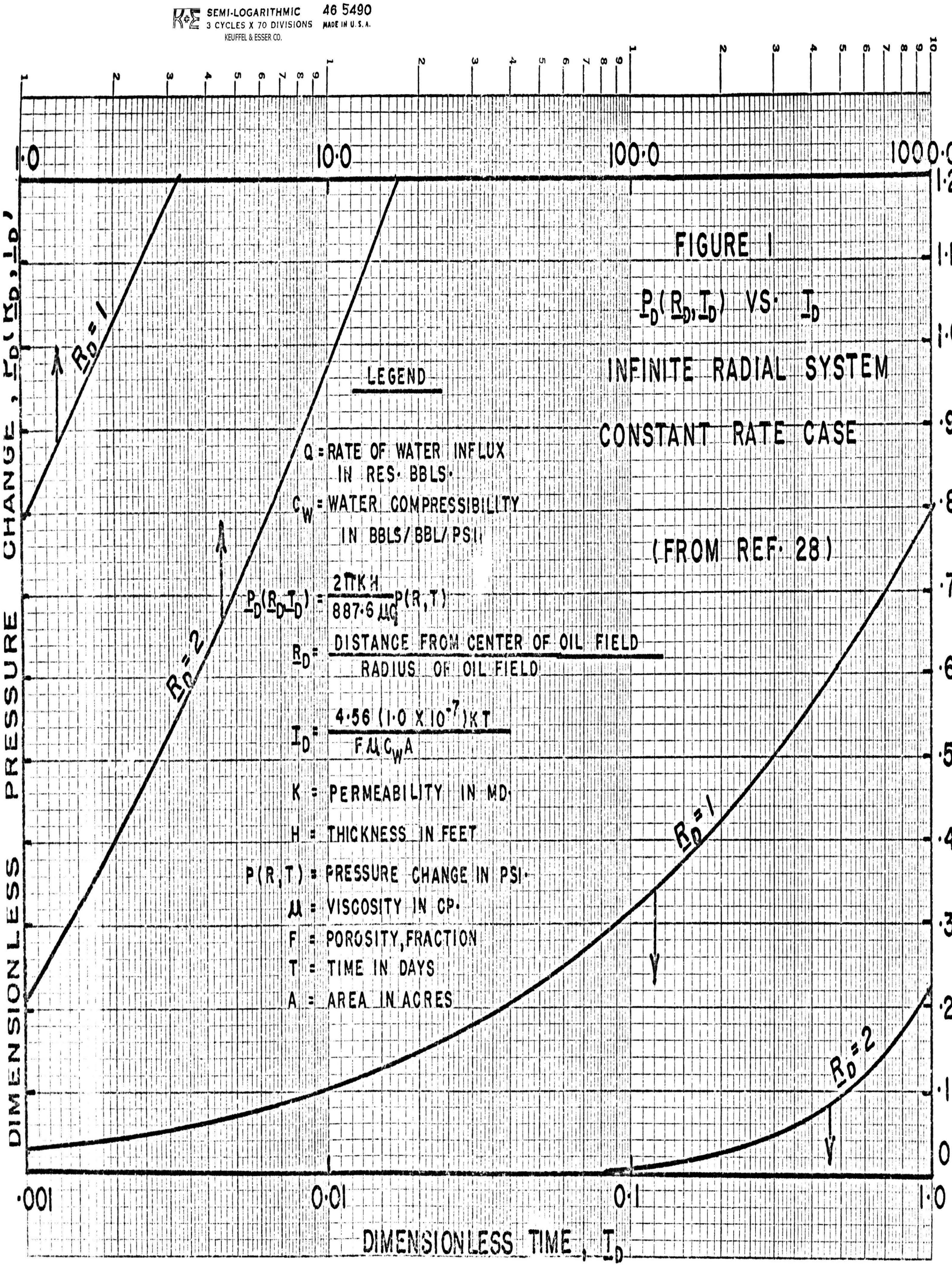
Flow equations are used in petroleum engineering to study the behavior of individual wells and reservoirs. For individual wells, the pressure response at the wellbore face is the major point of interest; whereas, in the case of reservoirs, the pressure response at the aquifer-hydrocarbon interface is sought. To aid in these studies, the flow equations have been solved in terms of the behavior at these two respective boundaries

5. Theis and Mortada

Only limited work has been published concerning the pressure conditions away from these inner boundaries, (i.e., within the reservoir or aquifer). Theis[1] and Mortada[28] are among the few who have reported on this problem. The Theis approach employs the exponential integral and is valid for pressure conditions that occur some distance away from the wellbore. It is derived from the concept of a point source, as opposed to a flow across a finite area. The Mortada results, on the other hand, are valid at all points within the reservoir or aquifer. They are presented in terms of dimensionless ratios of the radius where the pressure is desired to the radius where the flow rate is measured. Their main use in the past has been in aquifer studies. Mortada's results are presented in the form of graphs which are limited to a maximum radius ratio of 64 (See Fig. 1-3). These graphical results are cumbersome to interpolate at non-integral radius ratios, and therefore it often becomes necessary to use the analytical expressions given by Mortada: equations (1.25) to (1.28).

6. Interrelationship of Solutions

The solutions of Mortada and Theis are both based on the diffusivity equation (1.2) as applied to the case of an infinite radial system subject to a constant terminal rate. The diffusivity equation is obtained by combining the material balance equation with Darcy's flow equation. The assumptions involved in the use of this equation and thus imposed on these two solutions are:


K+E SEMI-LOGARITHMIC 46 5490
3 CYCLES X 70 DIVISIONS MADE IN U.S.A.
KEUFFEL & ESSER CO.
1·0
10·0
100·0
1000·0
FIGURE 1
P_D(R_D, T_D) VS· T_D
INFINITE RADIAL SYSTEM
CONSTANT RATE CASE
(FROM REF· 28)
LEGEND
Q = RATE OF WATER INFLUX IN RES· BBLS·
C_W = WATER COMPRESSIBILITY IN BBLS/BBL/PSI·
P_D(R_D, T_D) = 2πKH / 887·6 μQ P(R,T)
R_D = DISTANCE FROM CENTER OF OIL FIELD / RADIUS OF OIL FIELD
T_D = 4·56 (1·0 X 10^-7) KT / FμC_W A
K = PERMEABILITY IN MD·
H = THICKNESS IN FEET
P(R,T) = PRESSURE CHANGE IN PSI·
μ = VISCOSITY IN CP·
F = POROSITY, FRACTION
T = TIME IN DAYS
A = AREA IN ACRES
R_D = 1
R_D = 2
R_D = 1
R_D = 2
1·2
1·1
1·0
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
DIMENSIONLESS PRESSURE CHANGE, P_D(R_D, T_D)
·001
0·01
0·1
1·0
DIMENSIONLESS TIME, T_D

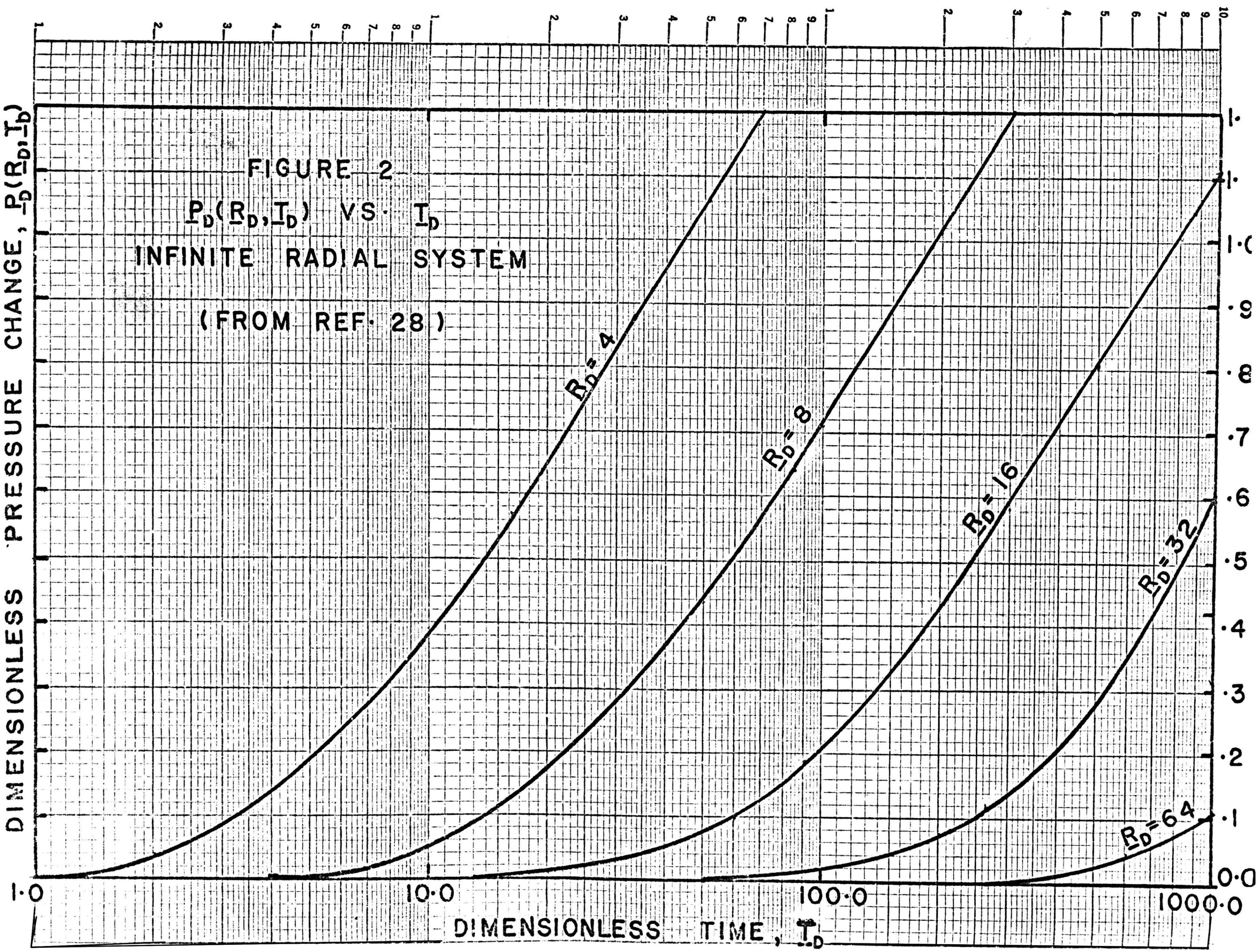


FIGURE 2
$P_D(R_D, T_D)$ VS. T_D
INFINITE RADIAL SYSTEM
(FROM REF. 28)
$R_D = 4$
$R_D = 8$
$R_D = 16$
$R_D = 32$
$R_D = 64$
DIMENSIONLESS PRESSURE CHANGE, $P_D(R_D, T_D)$
DIMENSIONLESS TIME, T_D
1.0
10.0
100.0
1000.0

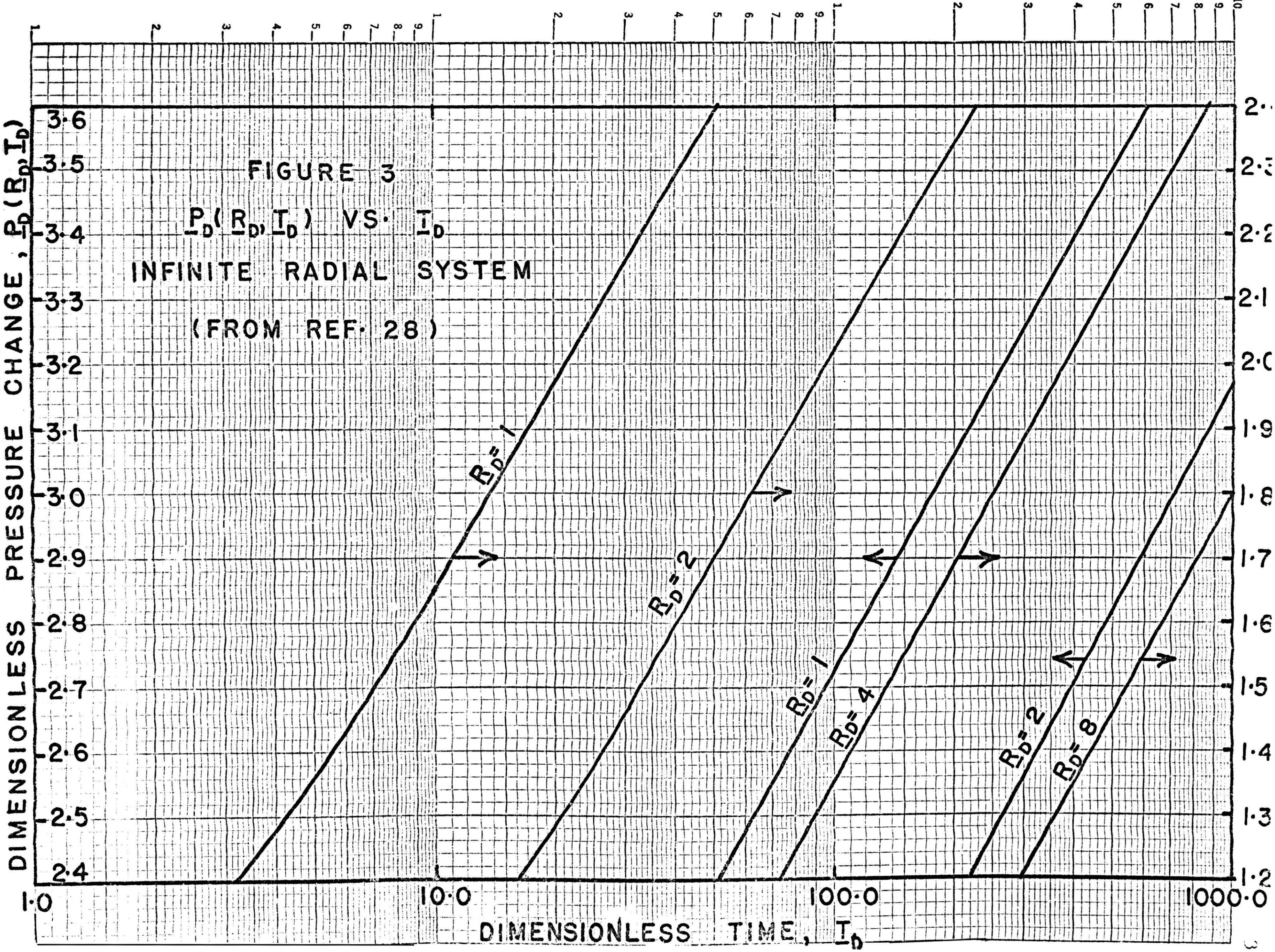
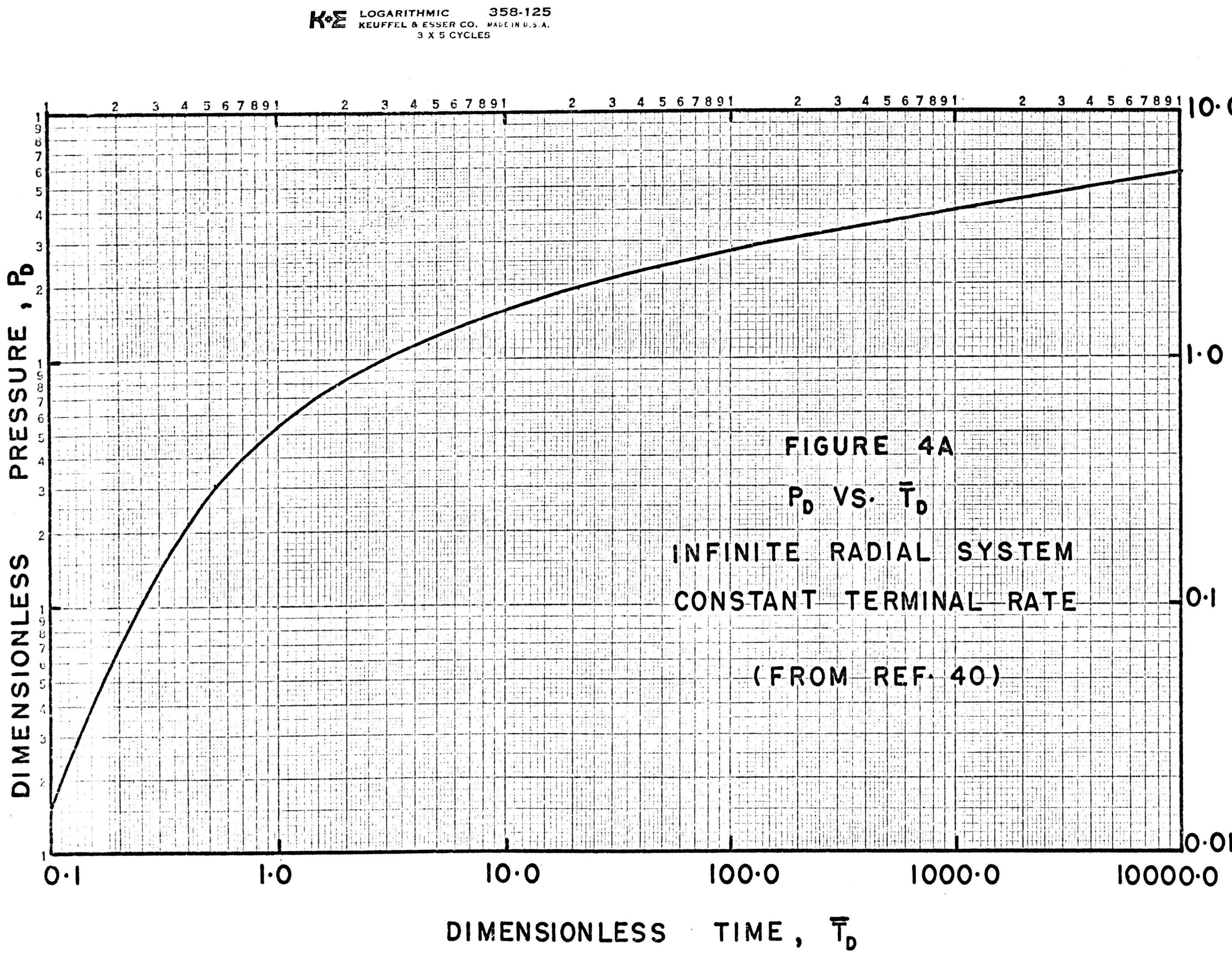
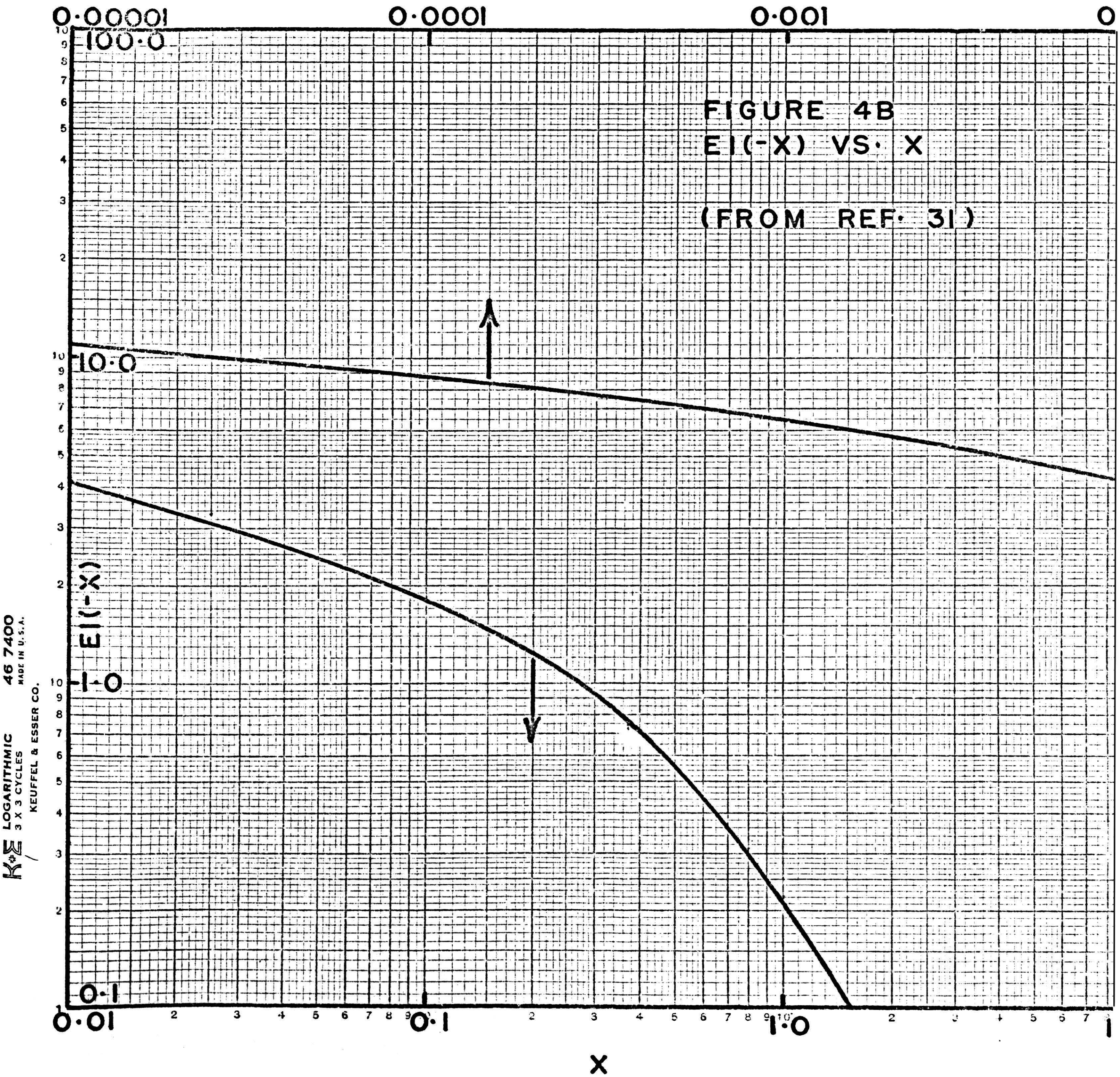
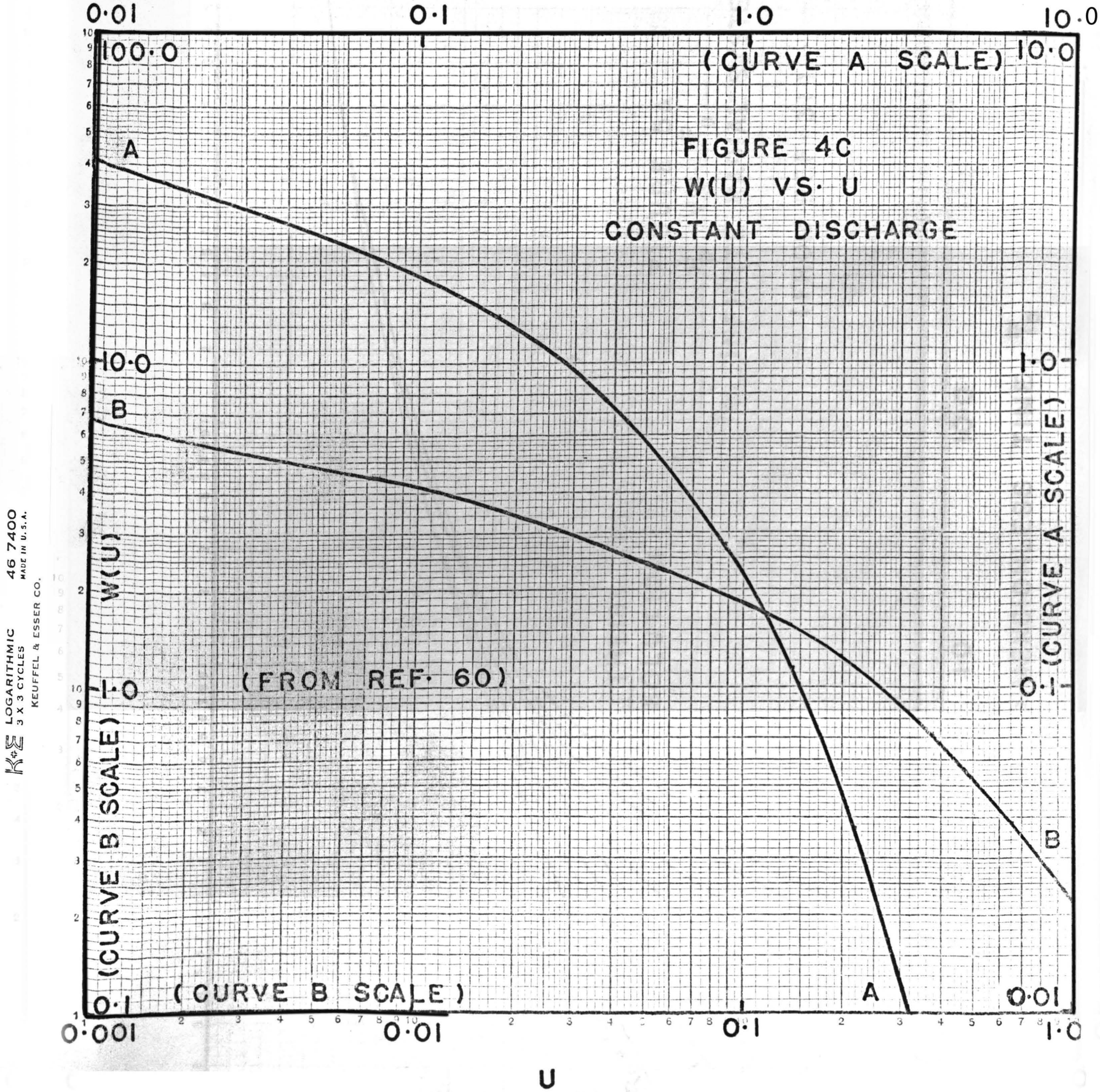


FIGURE 3
$P_D(R_D, T_D)$ VS. T_D
INFINITE RADIAL SYSTEM
(FROM REF. 28)


K+E LOGARITHMIC 358-125
KEUFFEL & ESSER CO. MADE IN U.S.A.
3 X 5 CYCLES
FIGURE 4A
P_D VS. $\bar{T}_D$
INFINITE RADIAL SYSTEM
CONSTANT TERMINAL RATE
(FROM REF. 40)
DIMENSIONLESS PRESSURE, P_D
DIMENSIONLESS TIME, $\bar{T}_D$
0.1
1.0
10.0
100.0
1000.0
10000.0
10.0
1.0
0.1
0.01


0·00001
0·0001
0·001
0
FIGURE 4B
EI(-X) VS· X
(FROM REF· 31)
100·0
10·0
1·0
0·1
EI(-X)
0·01
0·1
1·0
1
X
K&E LOGARITHMIC 46 7400
3 X 3 CYCLES MADE IN U.S.A.
KEUFFEL & ESSER CO.



0·01
0·1
1·0
10·0
100·0
(CURVE A SCALE)
10·0
FIGURE 4C
W(U) VS· U
CONSTANT DISCHARGE
A
10·0
1·0
B
(CURVE A SCALE)
W(U)
(FROM REF· 60)
1·0
0·1
(CURVE B SCALE)
B
A
0·1
(CURVE B SCALE)
0·01
0·001
0·01
0·1
1·0
U
K&E LOGARITHMIC 46 7400
3 X 3 CYCLES MADE IN U.S.A.
KEUFFEL & ESSER CO.

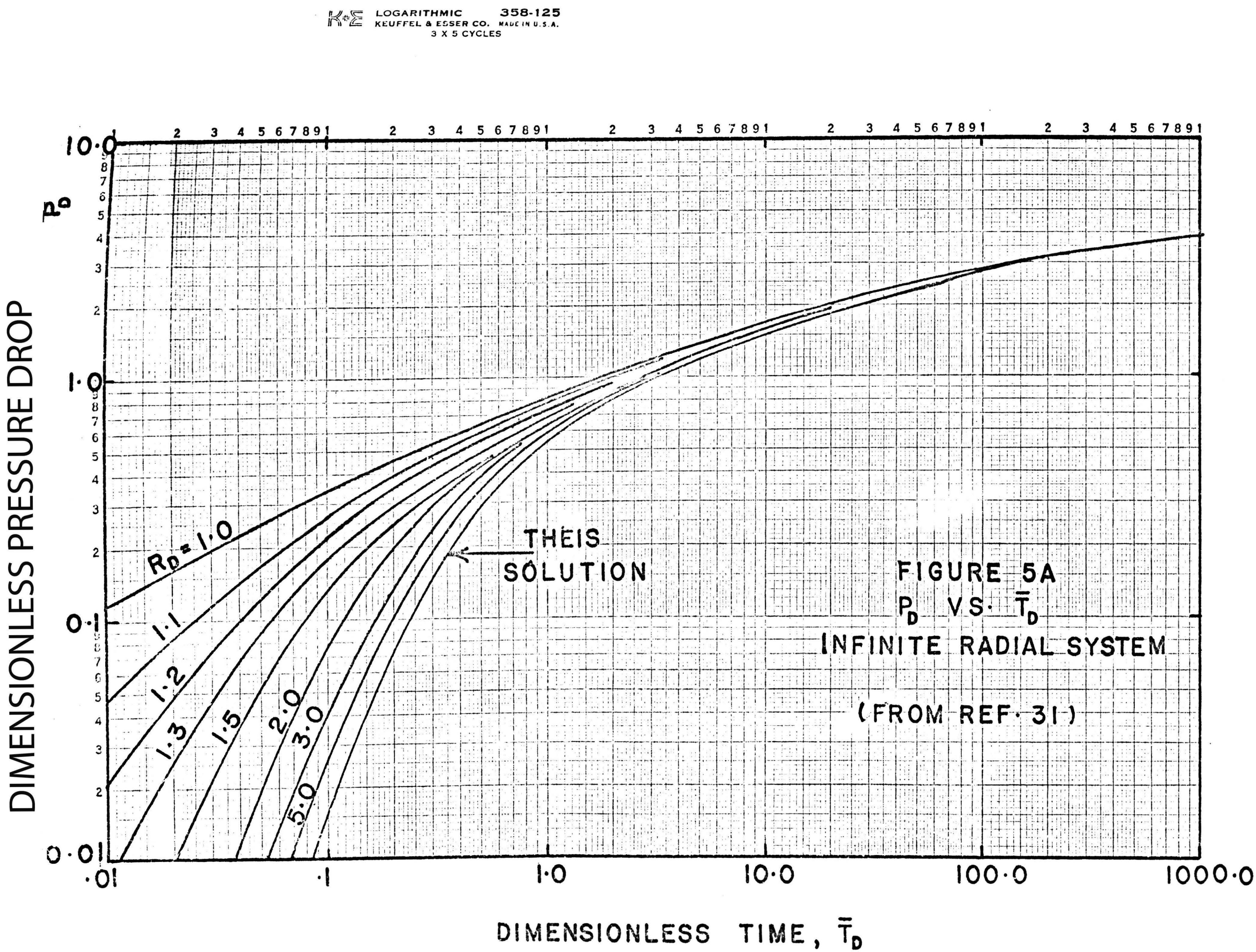


FIGURE 5A
P_D VS. $\bar{T}_D$
INFINITE RADIAL SYSTEM

(FROM REF. 31)

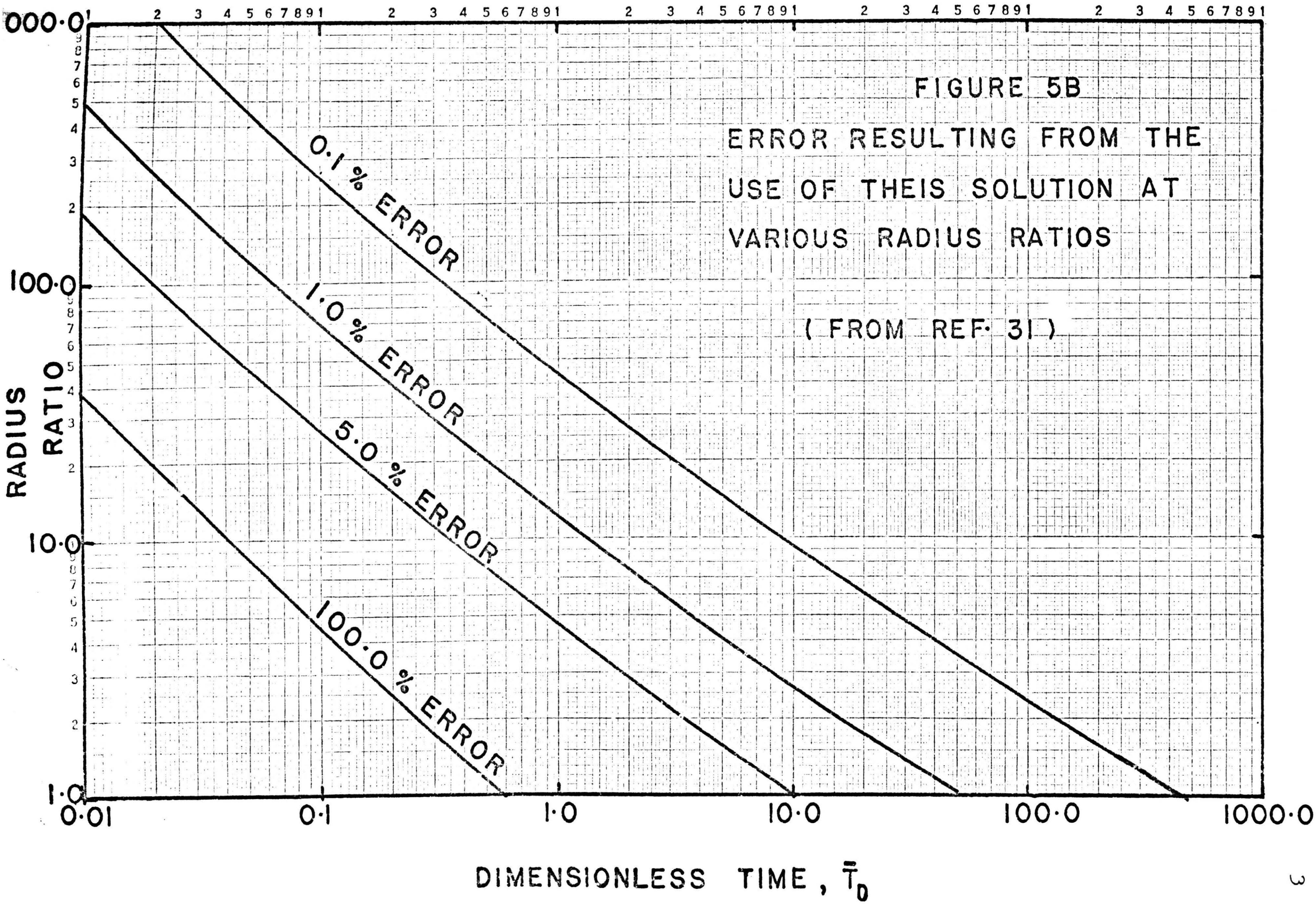


FIGURE 5B
ERROR RESULTING FROM THE USE OF THEIS SOLUTION AT VARIOUS RADIUS RATIOS
(FROM REF· 31)
0·1 % ERROR
1·0 % ERROR
5·0 % ERROR
100·0 % ERROR
RADIUS RATIO
1000·0
100·0
10·0
1·0
0·01
0·1
1·0
10·0
100·0
1000·0
DIMENSIONLESS TIME, $\bar{T}_D$

1. A single fluid is present that occupies the entire pore volume.

2. The reservoir is horizontal, homogeneous, uniform in thickness, and of infinite radial extent.

3. The compressibility and viscosity of the fluid remain constant at all pressures.

4. The fluid density obeys the equation,

$$\rho = \rho_o e^{-c(P_o - P)} . \qquad (1.2a)$$

The diffusivity equation for the conditions stated above can be written in cylindrical coordinates (and in oil field terms) as:

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{\phi \mu c}{K}\frac{\partial P}{\partial t} . \qquad (1.29)$$

When (1.29) is compared to the Theis solution (1.2), it can be seen that p in (1.29) has replaced s in Theis' equation and $\frac{\phi \mu c}{K}$ has taken the place of S/T in (1.2). Actually the only change which has occurred is that the drawdown measured in feet of water has been converted to an equivalent pressure term. In order to obtain a dimensionless equation to facilitate one solution which can be used for application of different porosity, permeability, and fluid properties, Mortada, Van Everdingen-Hurst, and Driscoll[32] have employed the following transformations:

Dimensionless pressure:

$$P_D = \frac{2\pi\ K\ H[P_2 - P_1]}{q\ \mu} . \qquad (1.30a)$$

Dimensionless radius:

$$r_D = \frac{r}{r_w}$$

or: (1.30b)

$$r_D = \frac{r}{r_b} .$$

Dimensionless time:

$$t_D = \frac{Kt}{\phi \mu c r_w^2}$$

or: (1.31)

$$t_D = \frac{Kt}{\phi \mu c r_b^2}$$

Substituting these dimensionless parameters into (1.29) yields:

$$\frac{\partial^2 P_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial P_D}{\partial r_D} = \frac{\partial P_D}{\partial t_D} . \qquad (1.32)$$

One solution of this equation has been given by Mortada(28) in which he presented dimensionless pressure drop as a function of dimensionless time. His graphical results are presented in Figures (1) to (3).

If the definition of dimensionless time in (1.31) is based upon any radius in the infinite system, we than have:

$$\bar{t}_D = \frac{Kt}{\phi \mu c r^2} . \qquad (1.33)$$

The dimensionless time of Mortada (t_D) is related to that of Equation (1.33) by:

$$\bar{t}_D = \frac{t_D}{r_D^2} . \qquad (1.34)$$

Table 1

Definition of Terms in Theis Solution

Term	Definition	$w = 2\pi$ $x = 1.0$	$w = 7.082 \times 10^{-3}$ $x = 6.331 \times 10^{-3}$	$w = 8.953 \times 10^{-5}$ $x = 4.386 \times 10^{-6}$
K	Permeability	sq. cm.	md.	md.
H	Thickness	cm.	ft.	ft.
P	Pressure Drop	dynes/sq. cm.	psi.	ft. water
q	Flow Rate	cc./sec.	bbl./day	gal./min.
μ	Viscosity	poise	cp.	cp.
t	Time	sec.	days	min.
ϕ	Porosity			
c	Compressibility	$\overline{\text{dynes/sq. cm.}}$	$\overline{\text{psi.}}$	$\overline{\text{psi.}}$
r	Distance	cm.	ft.	ft.

(Ref. 40)

From Table 2 it can be seen that when (1.34) is compared to the Theis equation (1.2) in the following manner:

$$\bar{t}_D = \frac{1}{4X} \tag{1.35}$$

and:

$$\Delta P_D = \frac{E_i[-X]}{2} \tag{1.36}$$

Equation (1.37) gives the relationship between the methods of analysis used by hydrologists and those commonly used in oil field work. In oil field terms, the Theis co-ordinates are:

$$P_D = \frac{wKH\Delta P}{q\mu} \quad ; \quad \bar{t}_D = \frac{XKt}{\phi\mu c r^2} \tag{1.37}$$

where w and x take on different values according to the dimensions selected (See Table 1.).

The definitions of the dependent and independent variables of Theis are compared with those of Mortada, and of Van Everdingen-Hurst in Table 2. The Theis solution of the exponential integral is shown in Figure (4b).

Table 2

Comparison of Dependent and Independent Variables

Method	Dimensionless Independent Variable	Dimensionless Dependent Variable
Theis	$X = \frac{r^2\phi\mu c}{4Kt}$	$E_i\ (-X)$
Mortada & V.E.-Hurst	$t_D = \frac{Kt}{\phi\mu c r_w^2}$	ΔP_D

Mortada's work shows that for reasonable values of real time (t), the Theis point-source solution represents the behavior of a slightly compressible system for all radial distances greater than about 30 times the radius of the pumping well, or in most cases about 15 feet or more away from the wellbore.

If the properties of the aquifer are known, the Theis curve (Fig. 4a) can be used directly to predict the behavior of the system 15 feet or more away from the wellbore. Mortada's solution can then be used to give the aquifer behavior in the vicinity of the well (less than 15 feet away from the wellbore).

Mueller and Witherspoon(31) adjusted the Theis results of Figure (4b) in accordance with the definitions of (1.35) and (1.36). They also modified Mortada's solutions of Figures (1) to (3) by means of Equation (1.34). Their results are a family of curves shown in Figure (5a) which converge on Theis' solution. Radius ratios not given in Mortada's work have been obtained from the results given by Mueller(26). Figure (5a) shows that the Theis solution can be used for radius ratios greater than 20 for practical times ($t_D \geq .1$). Figure (5b) shows the percent error that would result by using the Theis solution for various radius ratios instead of the Mortada solution.

At early times and at short distances from the inner boundary the "point source" solutions are invalid. The error introduced by the Theis solution (See Fig. 5b) may be negligible in most reservoir problems, but in the calculation of interference effects in an aquifer the error introduced can be appreciable.

Van Everdingen and Hurst[19] have presented results of the dimensionless pressure drop at the wellbore interface ($r_D=r/r_w=1$) as a function of dimensionless time defined in the same manner as by Mueller. More recently Driscoll[32] used the concept of dimensionless pressure vs. dimensionless time at various radius ratios for finite systems.

Driscoll discusses the use of well interference and pressure build-up data for the determination of water influx. His work also considers the problem of an effective compressibility in water reservoirs and gives a relationship for the adjusted rock compressibility after overburden pressures have been considered.

Although there are many different types of aquifer tests, the constant terminal rate discharge tests followed by pressure build-up tests are of particular interest to the engineers who develop natural gas storage pools in aquifers or those who work with producing gas and oil fields subject to water-drive or encroachment.

7. Radius of Drainage

Because of the apparent constancy of the pressure at various distances out in the reservoir, many authors have discussed what is termed a "radius of drainage". This drainage radius is usually defined as that distance beyond which the pressure change is only 1% of the change in

Figure 6

Van Poolen's Summary of Various Radius-of-Drainage and Stabilization-time Equations

(Ref. 33)

Reference		(Ref. 34) P.E. Jones	(35) Tek, Grove, and Poettmann	(36) Muskat	(37) Brownscombe Kern M.D.H.	(25) Chatas radial	(38) Hutchinson Kern	(64) Hurst	(33) Present
Radius of drainage	c-g-s units	$4\sqrt{\frac{Kt}{\phi\mu c}}$	$4.29\sqrt{\frac{Kt}{\phi\mu c}}$	$2\sqrt{\frac{Kt}{\phi\mu c}}$	$1.784\sqrt{\frac{Kt}{\phi\mu c}}$	$2\sqrt{\frac{Kt}{\phi\mu c}}$	$1.5\sqrt{\frac{Kt}{\phi\mu c}}$	$2.6408\sqrt{\frac{Kt}{\phi\mu c}}$	$2\sqrt{\frac{Kt}{\phi\mu c}}$
	Field units	$\sqrt{\frac{Kt}{10\phi\mu c}}$	$\sqrt{\frac{Kt}{9\phi\mu c}}$	$\sqrt{\frac{Kt}{40\phi\mu c}}$	$\sqrt{\frac{Kt}{50\phi\mu c}}$	$\sqrt{\frac{Kt}{40\phi\mu c}}$	$\sqrt{\frac{Kt}{70\phi\mu c}}$	$\sqrt{\frac{Kt}{22.5\phi\mu c}}$	$\sqrt{\frac{Kt}{39.2\phi\mu c}}$
Stabilization time	c-g-s units	$\frac{\phi\mu c r^2}{16K}$	$\frac{\phi\mu c r^2}{18.45K}$	$\frac{\phi\mu c r^2}{4K}$	$\frac{\phi\mu c r^2}{3.18K}$	$\frac{\phi\mu c r^2}{4K}$	$\frac{\phi\mu c r^2}{2.25K}$	$\frac{\phi\mu c r^2}{6.97K}$	$\frac{\phi\mu c r^2}{4K}$
	Field units	$\frac{10\phi\mu c r^2}{K}$	$\frac{9\phi\mu c r^2}{K}$	$\frac{40\phi\mu c r^2}{K}$	$\frac{50\phi\mu c r^2}{K}$	$\frac{40\phi\mu c r^2}{K}$	$\frac{70\phi\mu c r^2}{K}$	$\frac{22.5\phi\mu c r^2}{K}$	$\frac{39.2\phi\mu c r^2}{K}$

pressure in effect at the wellbore. Some authors, however, have described this radius as that point across which only 1% of the flow occurs when 100% flow is being experienced at the wellbore.

Van Poolen[33] has summarized the works of such authors as Jones[34], Tek[35], Muskat[36], Brownscombe and Kern[37], Chatas[25], Hutchinson and Kern[38], and Hurst, Haynie, and Walker[39]. Van Poolen's table for the various radius of drainage equations developed by each of the above authors is given in Figure (6).

8. Underground Gas Storage in Aquifers

The use of aquifers for underground storage of gas has become extremely important to the natural gas industry. A critical problem in assessing the feasibility of a specific aquifer for such gas storage use is the determination of the permeability of the caprock over the proposed storage aquifer.

Witherspoon, Mueller, and Donovan[40] evaluated the underground gas-storage conditions in aquifers by investigations of groundwater hydrology. Their work presents a finite-difference model which divides the aquifer-caprock system into layers with each layer further subdivided into a group of nested annular rings. The radii of these rings were chosen so as to increase in a geometric progression such that small radial distances could be used around the wellbore and progressively larger radii for greater distances away from the well. A sufficient number of annular rings are used so that the pressure transient created by the fluid withdrawal is assumed to not reach the outer boundaries of the system. The results obtained are therefore the same as would be obtained with an infinite radial system.

In this approach fluid is produced at a constant rate from the innermost ring of the aquifer without production from the innermost rings of the caprock. A material balance is made for every ring in the system at finite time steps. A point-by-point iteration scheme is then used to solve these material balance equations. In this manner, the transient behavior of the whole system can be numerically solved with the digitalized program. This procedure provides great detail on the pressure behavior at all parts of the system.

Evrenos and Rejda[41] have found that hydrological testing of an aquifer considered for natural gas storage and computerized evaluation of field data is the most practical method of determining the tightness of the caprock, aquifer geometry, and the coefficients of transmissibility, storage, and leakage.

Their program can be used to 1) compare the actual pressure performance of aquifer systems with the calculated pressure behavior based on analytical analogs permitting the selection of the analog which best fits the test data; and, to 2) predict the pressure response of an aquifer discharging through one or more wells in order to help design proper test procedures and to monitor test activities in the field.

Whenever a field data processing and evaluation run is made to determine aquifer description from field data, the procedure calculates pressures for each observation point according to the specified analogs, the input parameters, and the actual data point times; compares observed pressures to calculated pressures; optimizes the fit between field data and analog responses by varying certain parameters; and selects the analog which best fits field data.

One of Evrenos and Rejda's five analogs describes the pressure behavior of a homogeneous aquifer of infinite radial extent without leakage through the aquiclude.

The research sponsored at the University of Michigan from 1959 to 1961 by the American Gas Association was concerned with the prediction of water movement into and out of aquifers during gas storage cycles. This research led to a wealth of information dealing primarily with the movement of water in contact with natural gas reservoirs. The work of L. Katz, Tek, Coats, M. Katz, Jones, and Miller[42] published as a result of this research, and the later works of Katz, Vary, Elenbaas, Tek, Grove, Poettmann, Yoo, Coats, and White[45 - 52] form one of the bases of the growing research effort in the field of underground storage of natural gas in water reservoirs and the effects of water movements within these storage reservoirs.

III. DISCUSSION

The research work concerning the unsteady-state movement of water in aquifers and the work dealing with the unsteady-state water encroachment into hydrocarbon reservoirs stem originally from the same equation, namely, the Diffusivity Equation. For aquifer studies the hydrologists choose to express this equation in terms of drawdown, while the petroleum engineers, in dealing with hydrocarbon reservoirs, prefer pressure as the dependent variable. The grouping of terms on the right-hand side of the diffusivity equation also differs depending upon the specific investigation desired. Aquifer investigations usually use S/T, while petroleum literature employs $\phi \mu c/K$. In any case, the various groups are themselves inter-related so that the equation is essentially the same in both approaches. The assumptions necessary for the application of the diffusivity equation (point-source solution) to the unsteady-state flow of water are generally the same in both approaches.

The major difference between the two approaches is that the point in the aquifer-hydrocarbon reservoir system (or simple aquifer system) at which the evaluation of the water movement is made differs between the two types of solutions. Hydrologists have made most of their studies at the wellbore of the pumping or flowing well or at other observation wells within the drawdown area of the reservoir. They have developed numerous methods for finding the drawdown profile within the reservoir, the coefficients of storage and transmissibility, the permeability of the formation, and the quantities of water produced as a result of a given drop in head. Petroleum and natural gas engineers, however, have devoted the bulk of their research to the prediction of the quantity of aquifer water which can encroach into the hydrocarbon portion of the reservoir under a specified decrease in reservoir pressure (viz. reservoir fluid withdrawal).

These two different approaches are best characterized by the works of Theis[1] and of Van Everdingen-Hurst[19]. Besides attacking the problem of water influx from two different directions, these methods also differ in their degree of accuracy and their range of application. The Theis method employs an exponential integral and is derived from the concept of a point-source solution and, at best, gives only an approximate solution in the vicinity of the wellbore. Van Everdingen-Hurst have derived their solutions from the concept of a flow across a finite area using Laplace transforms and by forming a ratio of the radius where the pressure is desired to the radius where the flow rate is measured. Their development is an exact expression for the water-influx into a reservoir.

Mortada extended the work of Theis to cover the entire reservoir and presents a method which can be used to find the pressure distribution within the surrounding aquifer. Mueller and Witherspoon have compared the methods of Theis and Mortada and have found that there are similar groupings of the several variables in both methods.

At the present time there is some question as to just what the quantitative definition of the "radius of drainage" should be. Several authors have offered equations for a drainage radius, derived by different techniques, all of which give somewhat different results. All these authors do agree, however, that this drainage radius is a function of time alone for any one combination of reservoir properties.

During the course of this study it has become evident that the development of solutions for aquifer performance on high-speed digital computers has had a profound effect on the research activities of the petroleum industry. Some very important aquifer studies employing computer techniques have been presented by hydrologists who have either been employed by petroleum companies or have dealt with aquifer problems as related to petroleum or natural gas reservoirs.

As a result of this investigation, this investigator feels that enough similarities exist between the two approaches, at least in the basic assumptions made, to allow the water-influx method to be applied to aquifer problems. If digital computer techniques are employed in developing this new method then it should be possible to reduce, if not eliminate, the graphical work now required in the solution of various aquifer problems.

IV. SUGGESTED PROCEDURE FOR FUTURE RESEARCH

This study suggests that the unsteady-state flow of water in infinite radial reservoirs be investigated from the point of view of the amounts of aquifer water encroaching across an imaginary aquifer-reservoir boundary. This investigation could be performed by the application of the Van Everdingen-Hurst method (with the Wilson-Carlile simplification) for water influx determination. By using this type of approach to reservoir problems, it is proposed that solutions to various aquifer problems could be achieved which would not require supporting data from observation wells but, instead, would use only discharge well measurements and past production data.

Four possible objectives are suggested for future research into this area:

1. An attempt should be made to combine the present drawdown formulas with the various water-influx equations to determine the radius, R, at which the cone of depression will stabilize. The results of this study should then be compared to the values of R determined by the methods already presented by several authors.

2. It is proposed that a method could be developed which would permit the reservoir pressure and drawdown levels to be calculated for different

pumping rates at various times through the application of the Van Everdingen-Hurst Laplacian solutions at the imaginary aquifer-reservoir boundary.

3. By calculating the quantities of aquifer water encroaching into the reservoir for various pumping rates (i.e., for different pressures and different drawdown profiles) it is proposed that the maximum future pumping rate of aquifer bodies can be determined.

4. By examining the various parameters and constants involved in the Theis non-equilibrium and the water-influx approaches, the coefficient of storage, S, and the coefficient of transmissibility, T, can be determined by a method based on the water encroaching into the reservoir, past production data, and discharge well data rather than by the current graphical techniques.

It is proposed that these four objectives can be best attained by mathematical models with the bulk of the work being accomplished by computer application. It is anticipated that, resulting from this research, the current solutions to aquifer problems could be significantly improved and that a new approach to various types of solutions could be achieved.

V. CONCLUSIONS

Two different approaches to the problem of underground water movement exist at the present time. Both of these approaches start originally with the same basic equation (i.e., the diffusivity equation) but the resulting methods attack the problem of aquifer water movement from two different directions.

The hydrologists have developed methods by which the quantities of water entering the wellbore and the resulting pressure profile in the surrounding aquifer can be estimated. On the other hand, the petroleum and natural gas engineers have been mainly concerned with the quantities of water encroaching across the aquifer-hydrocarbon reservoir boundary and have developed their solutions to reflect this transient behavior.

Future research designed to develop new methods for calculating the water-influx into reservoirs, the aquifer pressure profiles, the radius of drainage, and the coefficients of storage and transmissibility, can be performed based upon the quantities of water encroaching across an imaginary aquifer-reservoir boundary. This research can be undertaken mathematically with the application of digital computing techniques.

NOMENCLATURE

A = area, acres

A_n = coefficient of approximation

B_n = coefficient of approximation

c = compressibility of fluid, vol/vol/psi.

c_w = compressibility of water, vol/vol/psi.

e = constant (2.71828)

$Ei(X)$ = exponential integral of the argument (X)

erfc = complementary error function, p. 27.

$G(\alpha)$ = function, defined in (1.8), p. 17.

H = net sand thickness, feet

ierfc = integral of the complementary error function, defined by (1.26), p. 26.

$J_o(x)$ = bessel function of zero order, first kind

K = permeability, darcies or millidarcies

Ln = naperian logarithm, base e

Log = common logarithm, base 10

m = height of aquifer prism, feet, Fig. 7, p. 58.

P = pressure, psi.

$\overline{P}$ = permeability coefficient (defined by Meinzer, p. 5'

P_D = dimensionless pressure, defined in (1.30a), p. 37.

$\underline{P}_D$ = dimensionless pressure of Mortada, Fig. 1, p. 29.

q = discharge rate, cubic feet per day

$q(T)$ = discharge rate over real time T, cubic feet/day

$q(t_D)$ = discharge rate to cumulative pressure drop constan· defined by (1.16), p. 21.

$Q(T)$ = cumulative discharge in time t_{D_n} by ΔP_n

Q_D = dimensionless flow rate, definition varies

R = equilibrium radius of drainage, feet

r = distance from the wellbore to some point in reservoir, feet

r_b = radius of the reservoir, feet

r_D = dimensionless radius, defined in (1.30b), p. 38.

r_w = radius of the discharge well, feet

$\underline{r}_D$ = dimensionless radius used with Mortada's $\underline{t}_D$ and $\underline{P}_D$ defined in Fig. 1, p. 29.

S = coefficient of storage, fraction

$\bar{s}$ = drawdown at some distance r from the wellbore, fee

S_w = constant drawdown in discharge well, feet

$\underline{T}$ = coefficient of transmissibility, cubic feet/day/ft

t = time, days

Tan^{-1} = angle whose tangent is

t_D = dimensionless time, defined in (1.31), p. 38.

$\underline{t}_D$ = dimensionless time of Mortada, Fig. 1, p. 29.

$\bar{t}_D$ = dimensionless time based on any radius r, defined in (1.33), p. 38.

u = $(r^2 S/ \underline{T}t)$ 1.87

w = constant depending upon units used, Table 1, p. 39.

W_e = cumulative volume of encroaching water, barrels

W(u) = well function of u, defined in (1.4), p. 11.

X = Theis dimensionless independent variable, Table 2,

x = constant depending on units used, Table 1, p. 39.

$Y_o(x)$ = bessel function of zero order, second kind

Δ = change in quantity

π = constant (3.141596)

μ = viscosity, centipoise

θ = fraction of the periphery of the sink subjected to influx of water, fraction

ϕ = porosity, fraction

ρ = density, pound/cubic foot

Σ = summation of terms

∂ = differential of

$\int$ = integral of

∞ = infinity

$\alpha = \dfrac{Tt}{r_w^2 S}$

DEFINITIONS

Artesian Aquifer:

An aquifer which is confined by beds of relatively impermeable material on both the top and bottom. These beds are assumed to be fluid in the sense that they have no ability to absorb or dissipate changes in forces external to or within the aquifer. (See Fig. 7a.)

Aquiclude:

The impermeable bed of material overlying the aquifer, often termed the caprock in petroleum literature. (Fig.7b)

Piezometric Surface:

The level of water sustained by the aquifer pressure. The natural head of a water well. (Fig. 7a)

Cone of Depression:

The cone-shaped region of drawdown of the piezometric surface surrounding a pumping water well. (Fig. 7c)

Coefficient of Storage:

The volume of water released or taken into storage per unit surface area of an aquifer per unit change of the component of head normal to that surface. Fig. 7a shows a prism of height, m, which can be used to define this coefficient. This prism extends vertically from top to bottom of the aquifer and laterally so that its cross-sectional area is coextensive with the aquifer-surface area over which the head change occurs. The volume of water released from storage in this prism, m, for any head change $\overline{X}$, divided by the product of the prism's cross-sectional area and the change in head, $\overline{X}$, results in a dimensionless number, S, which is the coefficient of storage.

Coefficient of Permeability:

This coefficient, $\overline{P}$, is a measure of a material's capacity to transmit water. As expressed by Meinzer, it is the rate of flow of water in gallons per day through a cross-sectional area of 1 square foot under a hydraulic gradient of 1 foot per foot at a temperature of 60°F. (Fig. 7b)

Coefficient of Transmissibility:

Theis introduced this coefficient, $\underline{T}$, which is expressed as the rate of flow of water, at the prevailing water temperature, in gallons per day, through a vertical strip of aquifer 1 foot wide extending the full saturated height of an aquifer under a hydraulic gradient of 100 percent. A hydraulic gradient of 100 percent means a 1-foot drop in the head in 1 foot of flow distance in the aquifer. (Fig. 7b)

Water Table Aquifer:

An aquifer which is not bounded above by an impermeable bed, but instead is bounded only by the surface of the ground. This type of aquifer has a water table which is the upper limit of free water existing in the formation. (Fig. 7c)

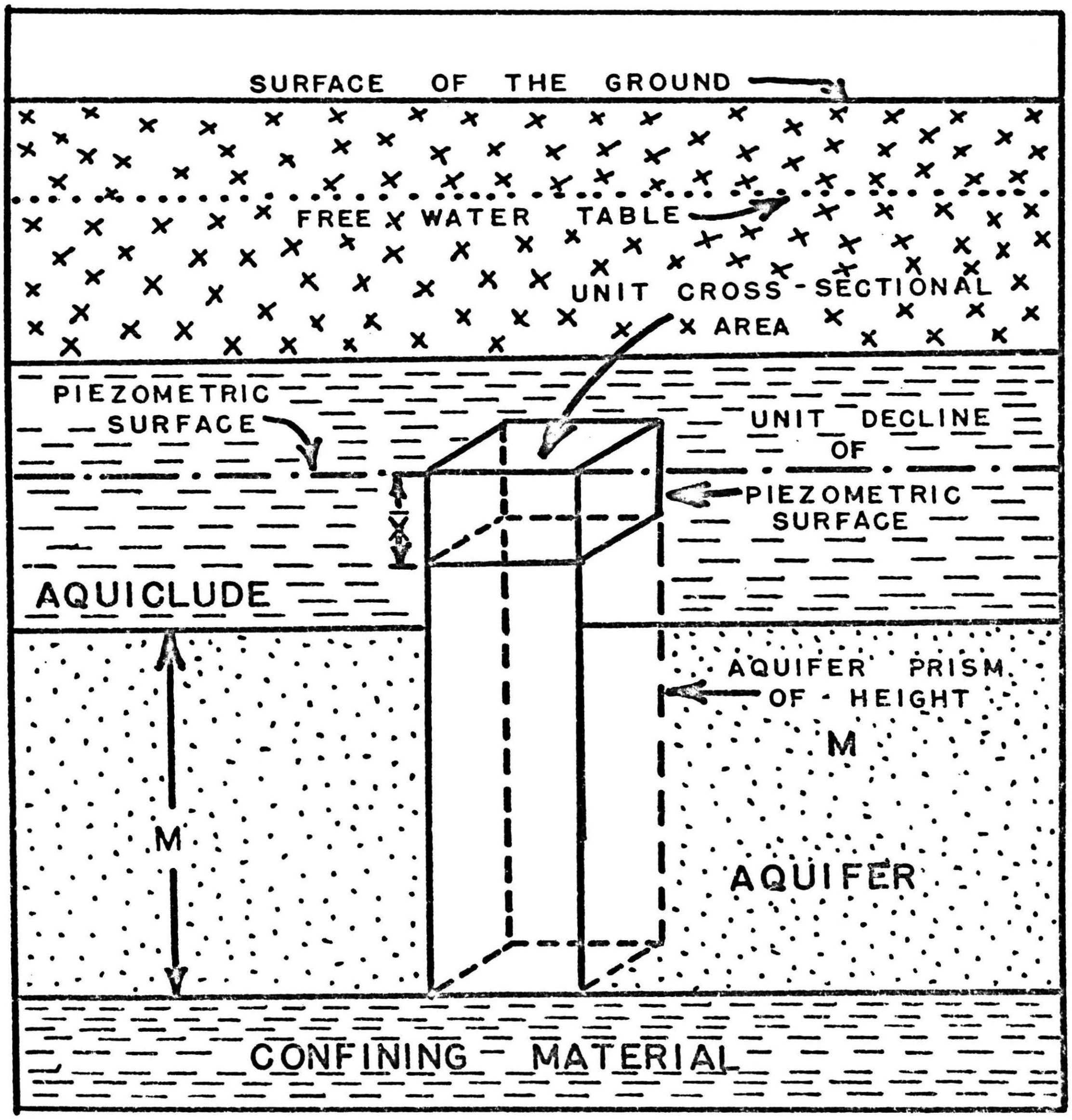


FIGURE 7A

ARTESIAN AQUIFER

AND

STORAGE COEFFICIENT

(FROM REF. 60)

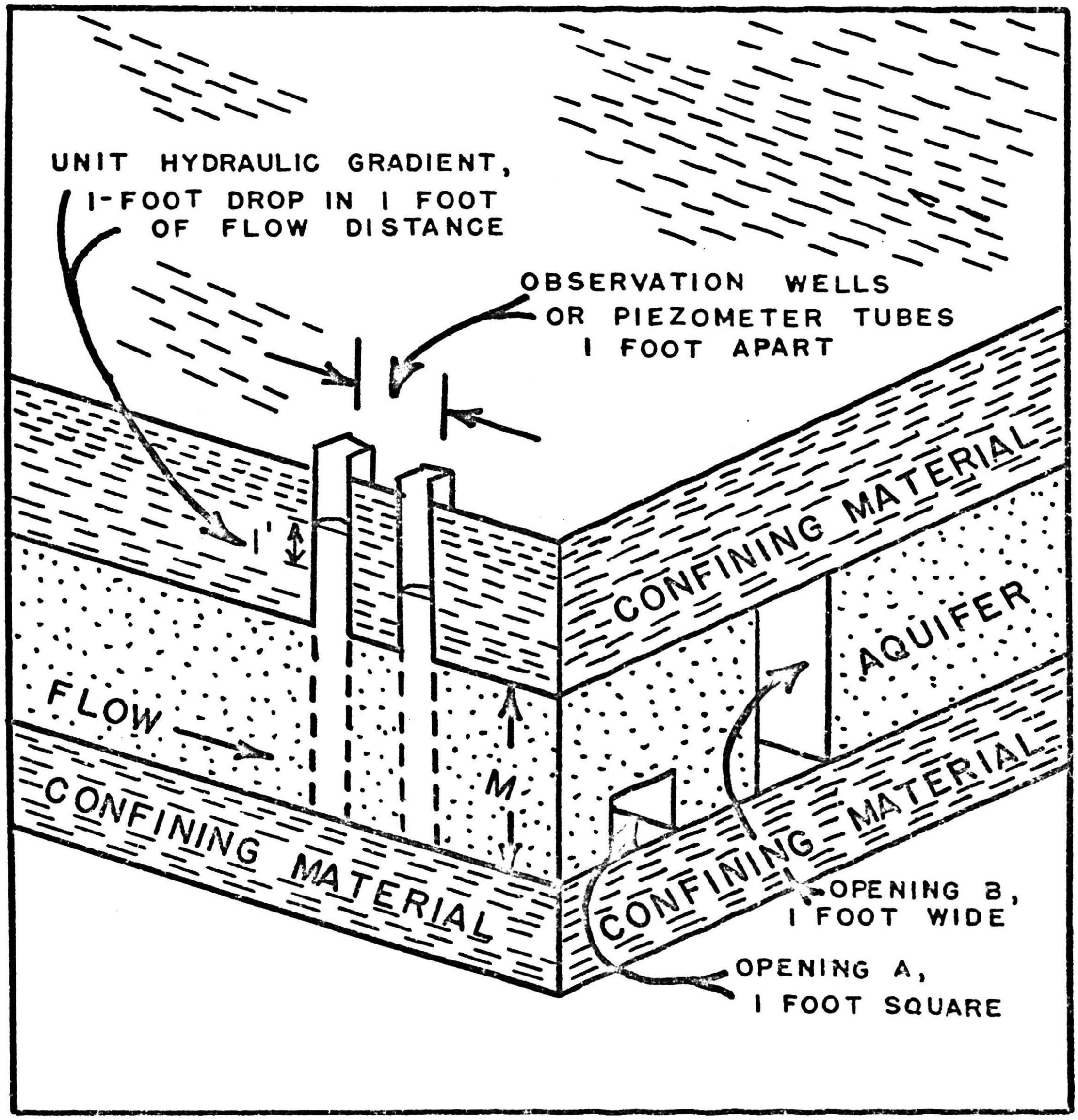


FIGURE 7B

COEFFICIENTS OF TRANSMISSIBILITY AND PERMEABILITY

(FROM REF. 60)

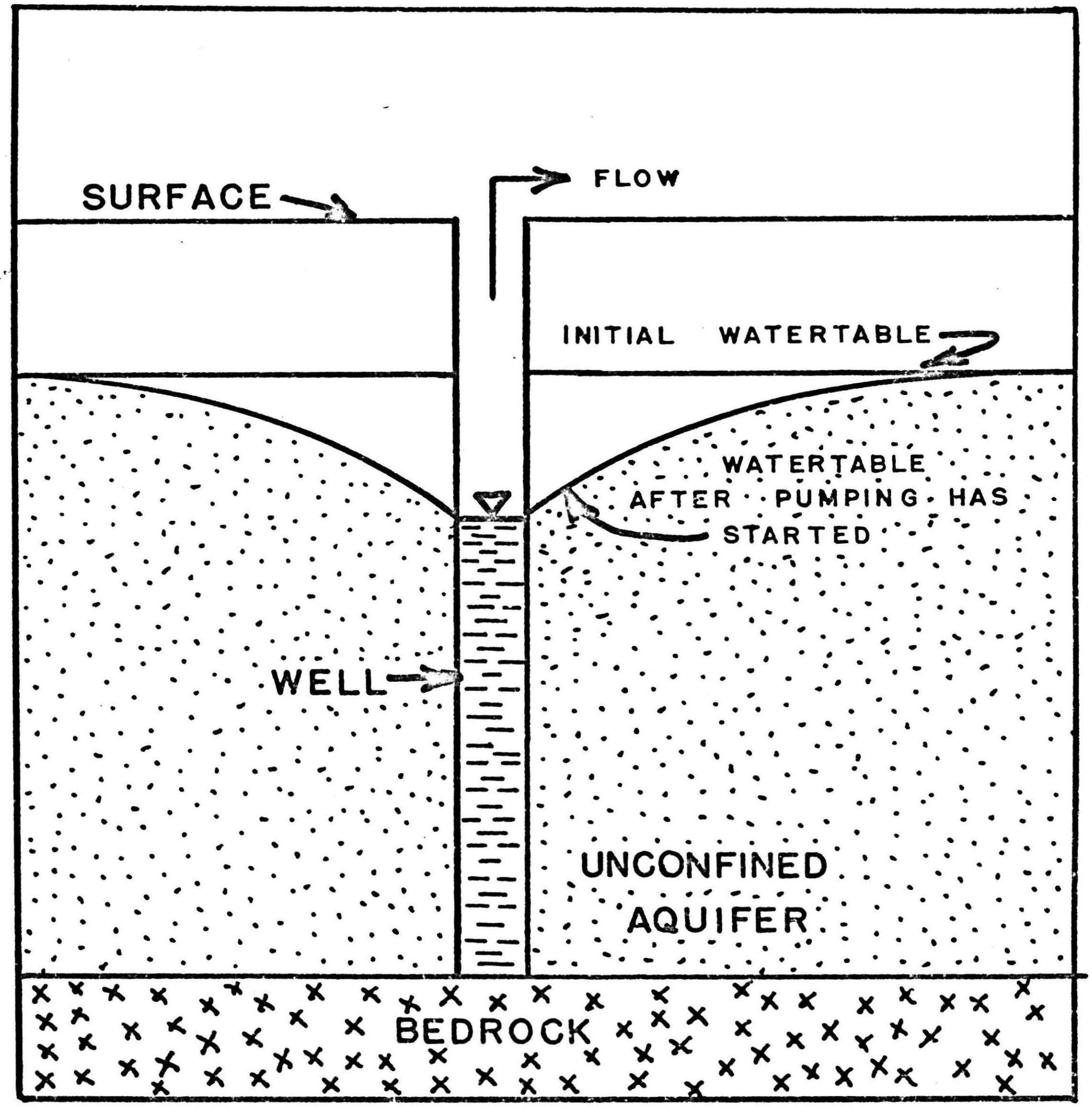


FIGURE 7C

WATER TABLE AQUIFER
AND
CONE OF DEPRESSION

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