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Compressibility effects on transient gas pipe flow

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COMPRESSIBILITY EFFECTS ON TRANSIENT GAS PIPE FLOW

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ABSTRACT (1)

Methods of iterative solution for the partial-differential equations that govern the transient flow of gases in pipelines are obtained by using the method of characteristics and linear finite-difference techniques. Solutions are developed for 1) a constant gas compressibility factor throughout transient conditions, and 2) a variable gas compressibility factor at constant temperature dependent upon pressures encountered during transient flow. Theoretical studies are made to compare results using both approaches for pipelines operating at various constant flowing temperatures. Results show greater differences between the two methods at lower values of flowing temperature due to the more rapidly changing compressibility factor as a function of variable pressure.

KEY WORDS: compressibility; gases; gas flow; pipe flow; pipelines; temperature; unsteady flow

ABSTRACT (2)

Equations are developed for transient gas pipe flow using both a constant and variable gas compressibility factor. Solution is by characteristics method using linear finite-differences. Results are used to compare the effects of compressibility factor at various constant flowing temperatures.

KEY WORDS: compressibility; gases; gas flow; pipe flow; pipelines; temperature; unsteady flow

TABLE OF CONTENTS

LIST OF FIGURES (1)

LIST OF FIGURES (2)

(For Thesis Only)

LIST OF TABLES

COMPRESSIBILITY EFFECTS ON TRANSIENT GAS PIPE FLOW

by Gerald F. Mouser, A. M. ASCE

INTRODUCTION

The partial-differential momentum and mass equations that describe transient gas pipe flow have been solved by various numerical methods (6,7,12). Among the techniques frequently employed in recent studies are implicit finite-differences and the method of characteristics (3,10,11). While these studies have been concerned with the numerical techniques and stability criteria required in transient flow solutions, few investigations have examined the effect of a variable gas compressibility factor on transient behavior.

The purpose of this study is to develop and apply the numerical equations of transient gas pipe flow based on 1) a constant gas compressibility factor evaluated at the average of inlet and outlet pressures at initial time, and 2) a gas compressibility factor completely dependent upon the gas pressures encountered in the pipeline during steady and unsteady-state flow. For both approaches, digital computer solutions of the partial-differential equations

which describe gas pipe flow are made possible by application of the characteristics transformation and linear finite-difference approximations. The theoretical results, based on both constant and variable gas compressibility, are compared graphically for various transient flow conditions with particular emphasis placed upon temperature effects.

BASIC DEVELOPMENT

Attractive forces existing between gas molecules cause the actual volume of a gas to deviate from those volumes predicted by the ideal gas law (8). To account for these variations in volume, the gas compressibility factor is applied as a correction term to the ideal gas equation of state. Thus, for methane and natural gas:

$$P\ V = z\ n\ R\ T \quad \text{.....(1)}$$

where P = absolute pressure of the gas; V = volume of the gas; z = dimensionless gas compressibility factor; n = number of moles of gas under consideration; R = universal gas constant; and T = absolute temperature of the gas.

As the value of the compressibility factor is essential in gas metering, charts and tables have been devised (4) to aid in its determination; but this procedure usually requires a lengthy calculation involving gas composition, pressure, and temperature. At constant flowing temperature, z has been correlated as a function of pressure for a gas of constant composition (12), that is

$$z = \frac{1}{1 + w\ \rho} \quad \text{.....(2)}$$

where $$w = \frac{R\ T}{144\ P_c\ 29\ G}\left(0.533\ \frac{T_c}{T} - 0.257\right) \quad \text{.....(3)}$$

In these equations ρ = mass density of the gas; G = gas specific gravity; P_c = critical pressure of the gas; and T_c = critical temperature of the gas. These relationships provide a good approximation for z within the normal range of operating pressures encountered in gas transmission systems.

Numerous limiting assumptions are required in the development of any steady or unsteady-state flow equations. In this study, the following assumptions are made:

1) Elevation changes in the pipeline are negligible.
2) Flow is isothermal and single phase.
3) Gas composition remains constant throughout each transient flow investigation.
4) Friction factor is constant.
5) All variations in flow parameters take place at inlet and outlet.
6) Pipeline is of constant cross-sectional area.

The first three assumptions are essential for the equation development in this investigation. While the remaining assumptions could be neglected without debasing the integrity of the development, they are accepted here in order to simplify the numerical solution.

<u>Variable Compressibility Factor</u>. One-dimensional gas pipe flow may be described by the following momentum balance:

$$\frac{g_c\ A\ R\ T}{29\ G}\ (\rho z)_x + M_t + \frac{1}{A}\ (M\ /\rho)_x + \frac{f\ M^2}{2\ D\ A\ \rho} = 0 \quad \ldots.(4)$$

in which g_c = gravitational conversion constant; A = cross-sectional area of pipe; M = mass flowrate of the gas (pounds mass per second); f = Moody friction factor; D = inside diameter of the pipe; and the subscripts x and t denote partial differentiation with respect to the independent variables, distance and time.

A continuity equation (or mass balance) may also be written for the flowing gas, as follows:

$$\rho_t + \frac{1}{A} M_x = 0 \quad \text{.....(5)}$$

The momentum and mass equations shown above form the basic solution to transient flow problems. A direct solution to this system of partial-differential equations may be accomplished by application of the method of characteristics and finite-difference techniques. Lister (5), Abbott (1), and Streeter and Wylie (9) have presented the basic approaches required for the methods, while others (10,11) have applied the characteristics transformation to various forms of the transient gas flow equations.

In order to incorporate the changing compressibility factor into the solution, Equation 2 was substituted for the value of z in Equation 4. The resulting expression was simplified, and Equations 4 and 5 were non-dimensionalized by defining the variables (M, ρ, t, and x) in dimensionless form as ratios of the original variables to constant or known pipeline values. The characteristics transformation

and first-order finite-differences were then applied, which resulted in the following set of grid-slope and characteristic equations:

$$(x_P - x_A) = (z + M/\rho)_A \, (t_P - t_A) \quad \ldots\ldots\ldots\ldots(6)$$

$$(z - M/\rho)_A \, (\rho_P - \rho_A) + (M_P - M_A) + (K \, M_A^2/\rho_A)(t_P - t_A) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

$$(x_P - x_B) = (- z + M/\rho)_B \, (t_P - t_B) \quad \ldots\ldots\ldots\ldots(8)$$

$$(- z - M/\rho)_B \, (\rho_P - \rho_B) + (M_P - M_B) + (K \, M_B^2/\rho_B)(t_P - t_B) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

All terms in the above equations are dimensionless, and the constant $K = fL/2D$. The subscript P denotes unknown conditions at the intersection of two grid-slope lines (defined by the simultaneous solution of Equations 6 and 8), while A and B denote known points on the positive and negative characteristic lines.

By using an iterative procedure, transient flow properties may be calculated, as functions of elapsed time and position in the pipeline, by the simultaneous solution of Equations 6-9. For this development, the method of specified time intervals (5,9) was used to provide for an orderly arrangement of position and time increments. Thus, M_P and ρ_P are calculated at the same positions along the pipeline for each increase in the value of elapsed time.

Constant Compressibility Factor. Equations 6-9 were developed to include a varying gas compressibility factor in the solution. If, for all values of pipeline position during the transient study, the value of z is assumed to be a constant evaluated at the average of inlet and outlet pressures at initial time, $(\rho z)_x = z \rho_x$ in Equation 4. If the method of characteristics, and the other procedures outlined above, are applied to Equation 4 (rewritten for a constant z and assuming the third term to be negligible) and Equation 5, the following set of simultaneous equations results:

$$(x_P - x_A) = \sqrt{z}\,(t_P - t_A) \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

$$\sqrt{z}\,(\rho_P - \rho_A) + (M_P - M_A) + (K\,M_A^2/\rho_A)(t_P - t_A) = 0 \quad \ldots\ldots (11)$$

$$(x_P - x_B) = -\sqrt{z}\,(t_P - t_B) \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

$$-\sqrt{z}\,(\rho_P - \rho_B) + (M_P - M_B) + (K\,M_B^2/\rho_B)(t_P - t_B) = 0 \quad \ldots (13)$$

Since the grid-slope relationships (Equations 10 and 12) have constant equal slopes ($z^{\frac{1}{2}}$), the iterative simultaneous solution of Equations 11 and 13 is based on a simple rectangular grid system, rather than the method of specified time intervals previously discussed.

Boundary conditions for both developments are provided by specifying one of the two flow properties (either pressure or flowrate) at both pipe inlet and outlet as a function of elapsed time. The value of the unknown flow condition at the grid-boundary intersection is then determined

by solution of the characteristic equation which correctly relates the distance and time intervals at the boundary.

COMPUTER TECHNIQUES

The non-dimensionalized iterative equations, for both a constant and variable gas compressibility factor, were incorporated into separate FORTRAN programs for solution on the IBM 360 Model 50 digital computer. Although both programs followed the same general format, the variable z solution involved longer iterative equations, and required a group of interpolations for application of the method of specified time intervals. Due to these lengthier calculations, the computer solution using a varying compressibility factor required about fifty percent more time than that of the constant z, rectangular grid method of solution.

To correctly apply an iterative method that propagates results from a set of initial and boundary conditions, stability must be maintained throughout the solution. In the characteristics method, stability is based on the interrelated position and time increments (Δx and Δt). Preliminary numerical investigations for the gas pipelines considered in this study showed that an increment size equal to one-fiftieth of total pipeline length allowed sufficiently stable results at reasonable values of computer time.

For both computer solutions, initial steady-state flow properties throughout the pipe system were determined by Weymouth's Equation (4):

$$Q = 3.22 \frac{T_s}{P_s} \left[\frac{D^5 (P_1^2 - P_2^2)}{f\ L\ G\ T\ z} \right]^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots(14)$$

Those terms not previously defined are: Q = flowrate of the gas in standard cubic feet per hour; T_s = standard temperature (degrees Rankine); P_s = standard pressure (pounds per square inch absolute); P_1, P_2 = inlet and outlet pressure, respectively (psia); and L = total length of the pipeline in miles. An average value of z, as calculated by Equation 2, was used in Weymouth's Equation; but, for all other calculations throughout the variable z solution, the gas compressibility factor was determined as a function of mass density at each point and time in question.

If a high value of Reynolds number is assumed for the flowing gas, the Moody friction factor, f, may be expressed entirely as a function of pipe diameter. Thus, from the work by Cullender and Smith (2)

$$f = 0.0175/D^{0.225} \qquad D < 4.277 \quad \ldots\ldots\ldots\ldots(15)$$

$$f = 0.01603/D^{0.164} \qquad D > 4.277 \quad \ldots\ldots\ldots\ldots(16)$$

where D is the inside diameter of the pipe in inches. The value of the friction factor, as calculated by either Equation 15 or 16, was used in Weymouth's steady-state equation and in all transient flow calculations for both methods of solution.

The transient flow calculations are performed by using the iterative characteristic equations previously developed. Values of time and position, at each increment on the pipeline, are related by the grid-slope equations. In order to assure that the interpolations used in the method of specified time intervals are able to converge on a new incremental point in the x-t plane, the smallest value of Δt is selected at each new time iteration as the step-time increment for the variable z solution. The constant z solution uses a constant step-time increment for all iterations.

As the transient pipe flow investigations for this study begin at steady-state operating conditions, flowing properties of the gas throughout the pipe system may be calculated from Weymouth's Equation. Transient behavior is induced by manipulating conditions at one or both boundaries of the system, and the resulting gas flow properties are calculated at evenly spaced intervals along the pipeline for increasing values of elapsed time. These properties are calculated in dimensionless form by the iterative boundary equations, and are converted to field units for output. Thus, the distribution of pressure and volume flowrate in the pipeline is obtained as a function of elapsed time for transient investigations performed with both a constant and variable gas compressibility factor.

DISCUSSION OF RESULTS

Three different transient gas pipe flow situations were devised to study the effect of the gas compressibility factor. Each situation involved a constant diameter gas pipeline initially operating under steady-state flow conditions, from which transient conditions were induced by regulating either the flowrate or the pressure at the pipe outlet. Upstream pressure was held constant for each investigation, thus satisfying the inlet boundary condition.

Constant values of gas composition (critical properties and gas gravity), pipeline length, and initial inlet and outlet pressures were selected for each of the three studies. (These values are presented in Table 1, along with other information pertaining to each investigation.) Two extreme values of flowing temperature were chosen for each of the three sets of pipe flow data, and transient outflow conditions were applied. The dynamic flow properties were then calculated, using both a constant and a variable gas compressibility factor, and the results compared graphically.

The first study examined pipe flow conditions induced by a sinusoidal varying outlet flowrate. Pressure and flowrate distributions in the pipeline were calculated for flowing temperatures of - 40°F and + 80°F; and the results, for both constant and variable gas compressibility, are compared in Figures 1-4.

Both methods of solution predict the same basic transient behavior for the investigations, but the graphical results show more deviation between the constant and variable z methods for properties predicted at the lower temperature value (Figures 1 and 3). This difference may be explained by noting that the gas compressibility factor changes more rapidly as a function of pressure for lower temperature ranges. Thus, as the pressures throughout the system fluctuate due to the varying outlet flowrate, the gas compressibility factor deviates more from the initial average value used in the constant z solution at the lower temperature range than at the higher temperature.

Pipeline shut-in conditions were simulated in the second transient flow study (Figures 5-8) by shutting off gas flow at the downstream face of the pipe system. This boundary condition causes a reduction of flowrates throughout the system (Figures 5 and 6), initially more pronounced at points near the outlet, but eventually affecting flow at the pipe inlet. As a result, pressures in the system increase (Figures 7 and 8), demonstrating the gas storage potential of the pipeline.

In the final study, transient pipe flow was induced by increasing the outlet pressure to a new constant value. Under this condition, gas flow in the system should eventually stabilize at a new constant value, slightly less than the original steady-state flowrate. The initial and final

flowrates (as predicted by Weymouth's Equation) are shown on the graphs (Figures 9 and 10); and, during the elapsed time presented, the calculated flowrates for both temperature studies show a tendency to approach the final theoretical flowrate value.

The results of the final two investigations again show greater differences between the constant and variable z solutions at the lower temperature value (Figures 5, 7 and 9). In general, at normal operating temperatures (+ 60°F and + 80°F) the results from both methods demonstrate satisfactory agreement.

TABLE 1.--DATA FOR TRANSIENT FLOW INVESTIGATIONS

Study Number	(1)	(2)	(3)
Inside diameter of pipe, in inches	15.5	13.125	21.50
Gas gravity	0.654	0.615	0.622
Critical pressure of gas, in pounds per square inch absolute	657.0	650.0	670.0
Initial upstream pressure, in pounds per square inch absolute	700.0	675.0	690.0
Initial downstream pressure, in pounds per square inch absolute	500.0	525.0	505.0
Standard pressure, in pounds per square inch absolute	14.7	14.7	14.7
Flowing temperatures used for study, in degrees Rankine	420.0 and 540.0	430.0 and 520.0	420.0 and 540.0
Critical temperature of gas, in degrees Rankine	351.6	350.0	350.0
Standard temperature, in degrees Rankine	520.0	520.0	520.0
Length of pipeline, in miles	100.0	100.0	100.0
Inlet boundary condition	Constant pressure, equal to initial inlet pressure		
Outlet boundary condition	Sinusoidal varying flowrate	Flowrate decreased to zero	Pressure increased to new constant value

Figure 1 Flowrates, Induced by Varying the Outlet Flowrate as a Sine Function for a Temperature of $-40^{\circ}F$, Versus Time.

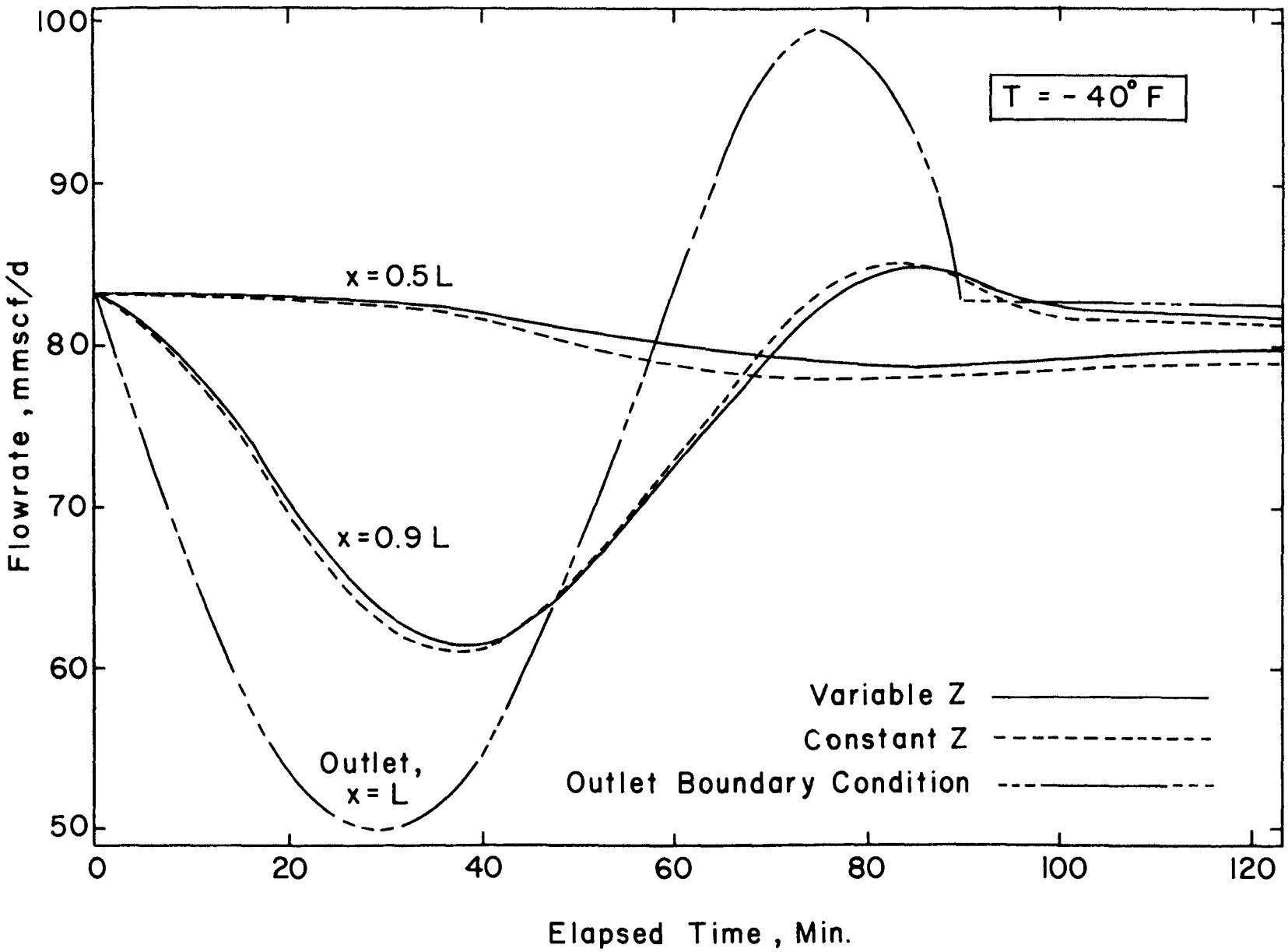
T = - 40° F
x = 0.5 L
x = 0.9 L
Outlet,
x = L
Variable Z
Constant Z
Outlet Boundary Condition
Flowrate, mmscf/d
Elapsed Time, Min.
100
90
80
70
60
50
0
20
40
60
80
100
120

Figure 1

Figure 2 Flowrates, Induced by Varying the Outlet Flowrate as a Sine Function for a Temperature of + 80°F, Versus Time.

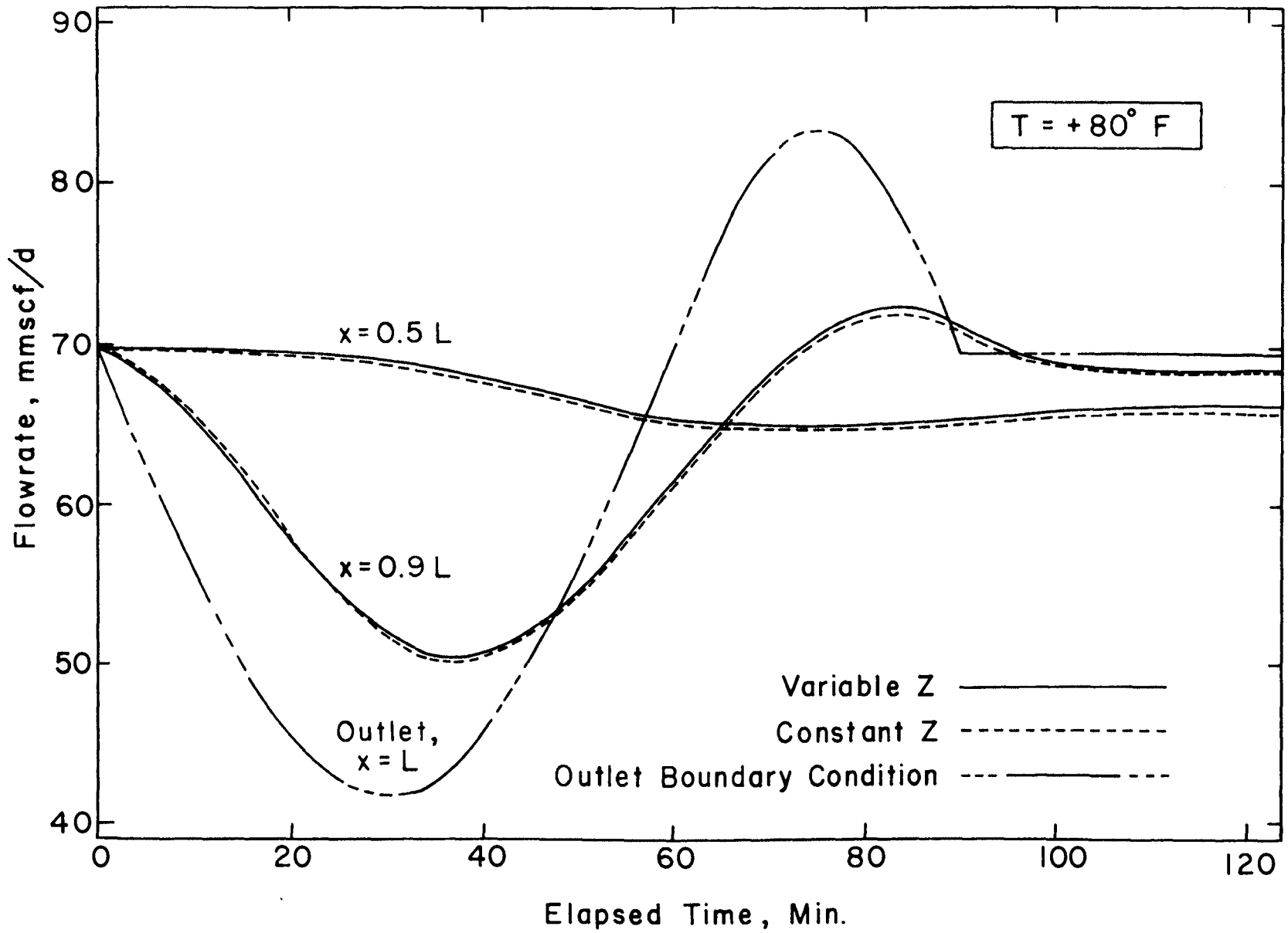
T = +80° F
x = 0.5 L
x = 0.9 L
Outlet,
x = L
Variable Z
Constant Z
Outlet Boundary Condition
Flowrate, mmscf/d
Elapsed Time, Min.
90
80
70
60
50
40
0
20
40
60
80
100
120

Figure 2

Figure 3 Pressures, Induced by Varying the Outlet Flowrate as a Sine Function for a Temperature of - 40°F, Versus Time.

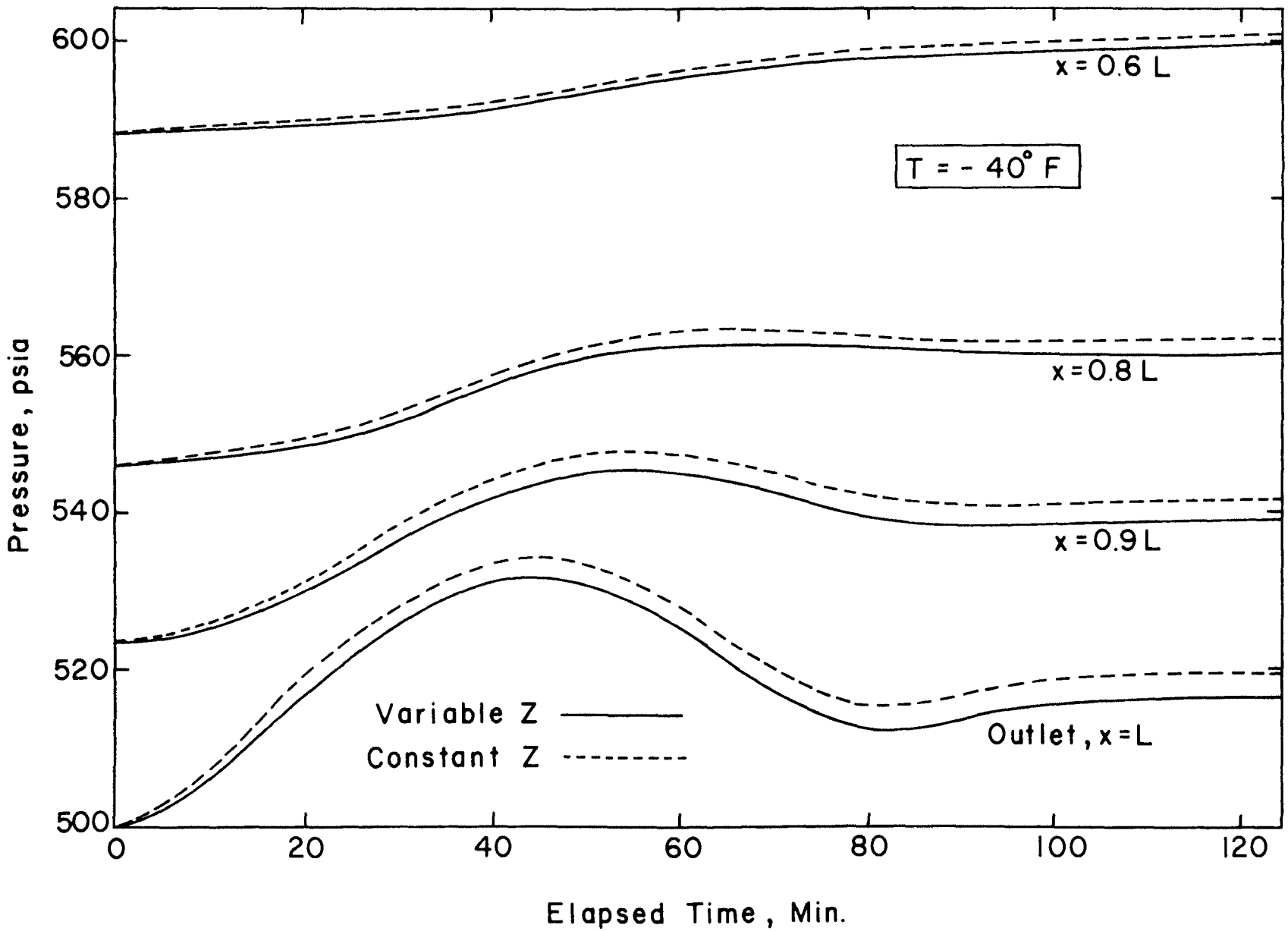
600
580
560
540
520
500
Pressure, psia
0
20
40
60
80
100
120
Elapsed Time, Min.
x = 0.6 L
T = - 40° F
x = 0.8 L
x = 0.9 L
Outlet, x = L
Variable Z
Constant Z

Figure 3

Figure 4 Pressures, Induced by Varying the Outlet Flowrate as a Sine Function for a Temperature of + 80°F, Versus Time.

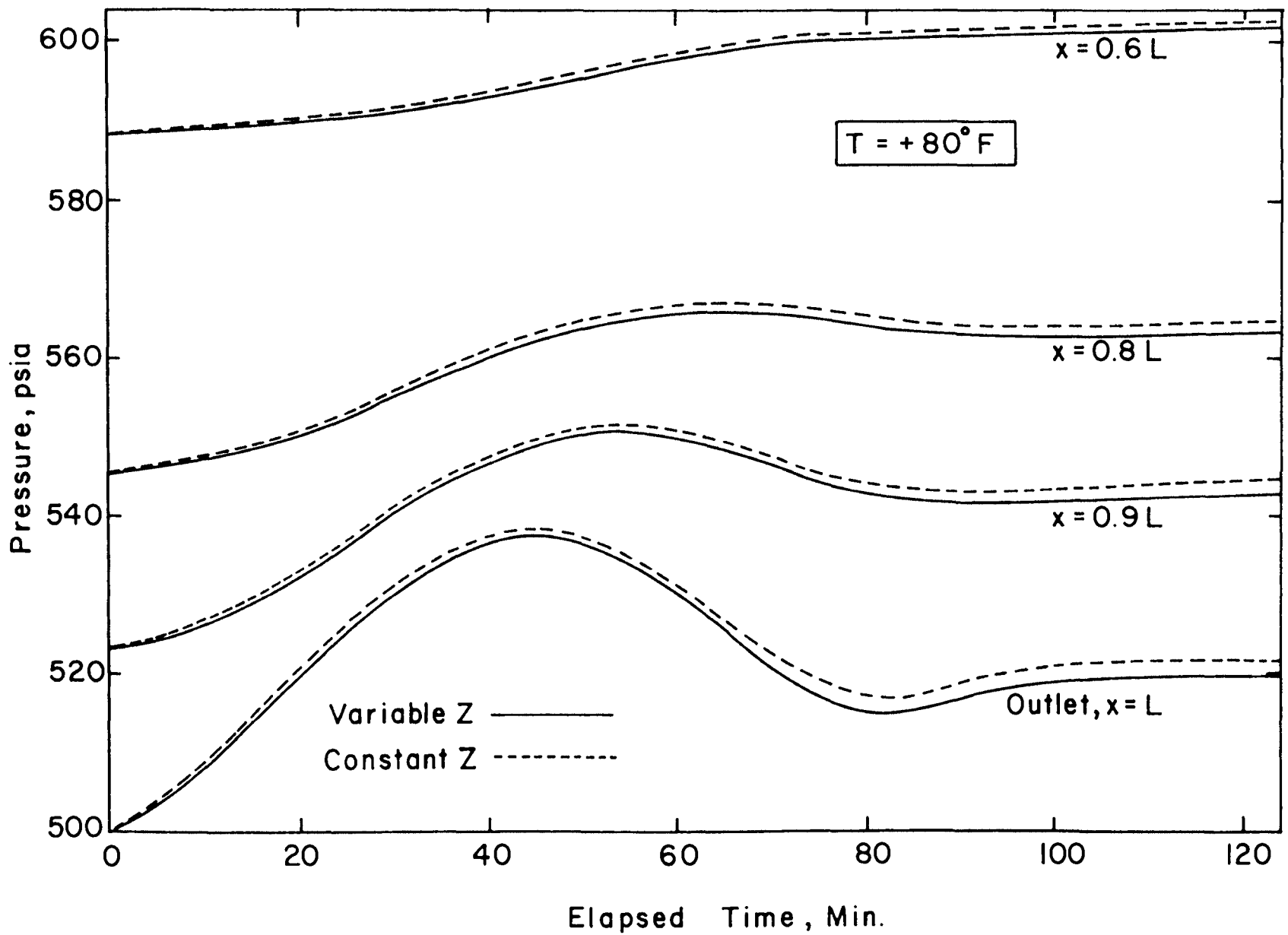
600
580
560
540
520
500
Pressure, psia
0
20
40
60
80
100
120
Elapsed Time, Min.
x = 0.6 L
T = + 80° F
x = 0.8 L
x = 0.9 L
Outlet, x = L
Variable Z
Constant Z

Figure 4

Figure 5 Flowrates Versus Time for Pipeline Shut-In Conditions at a Temperature of - 30°F.

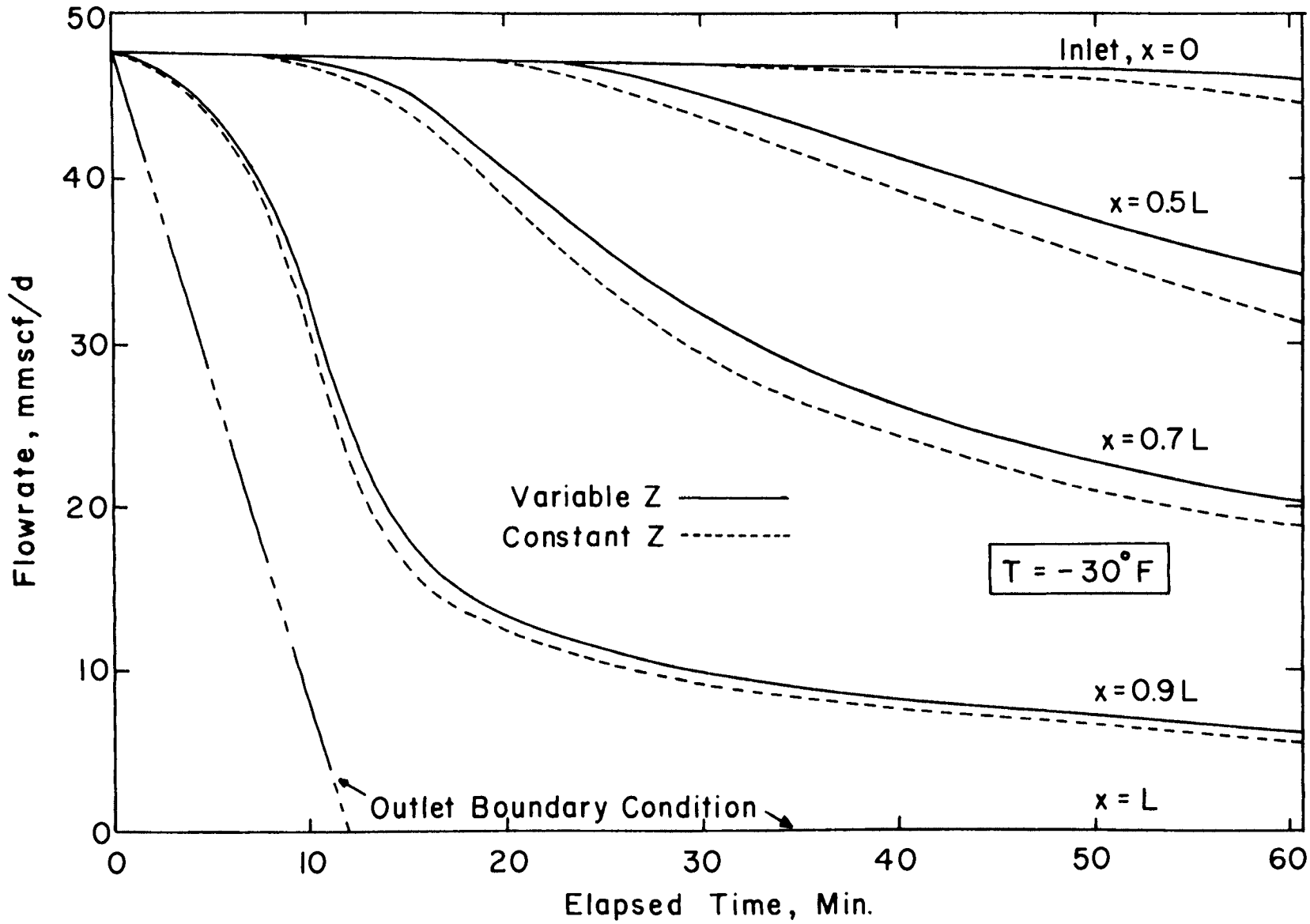
Inlet, x = 0
x = 0.5 L
x = 0.7 L
x = 0.9 L
x = L
Variable Z
Constant Z
T = -30°F
Outlet Boundary Condition
Flowrate, mmscf/d
Elapsed Time, Min.
0
10
20
30
40
50
60

Figure 5

Figure 6 Flowrates Versus Time for Pipeline Shut-In Conditions at a Temperature of + 60°F.

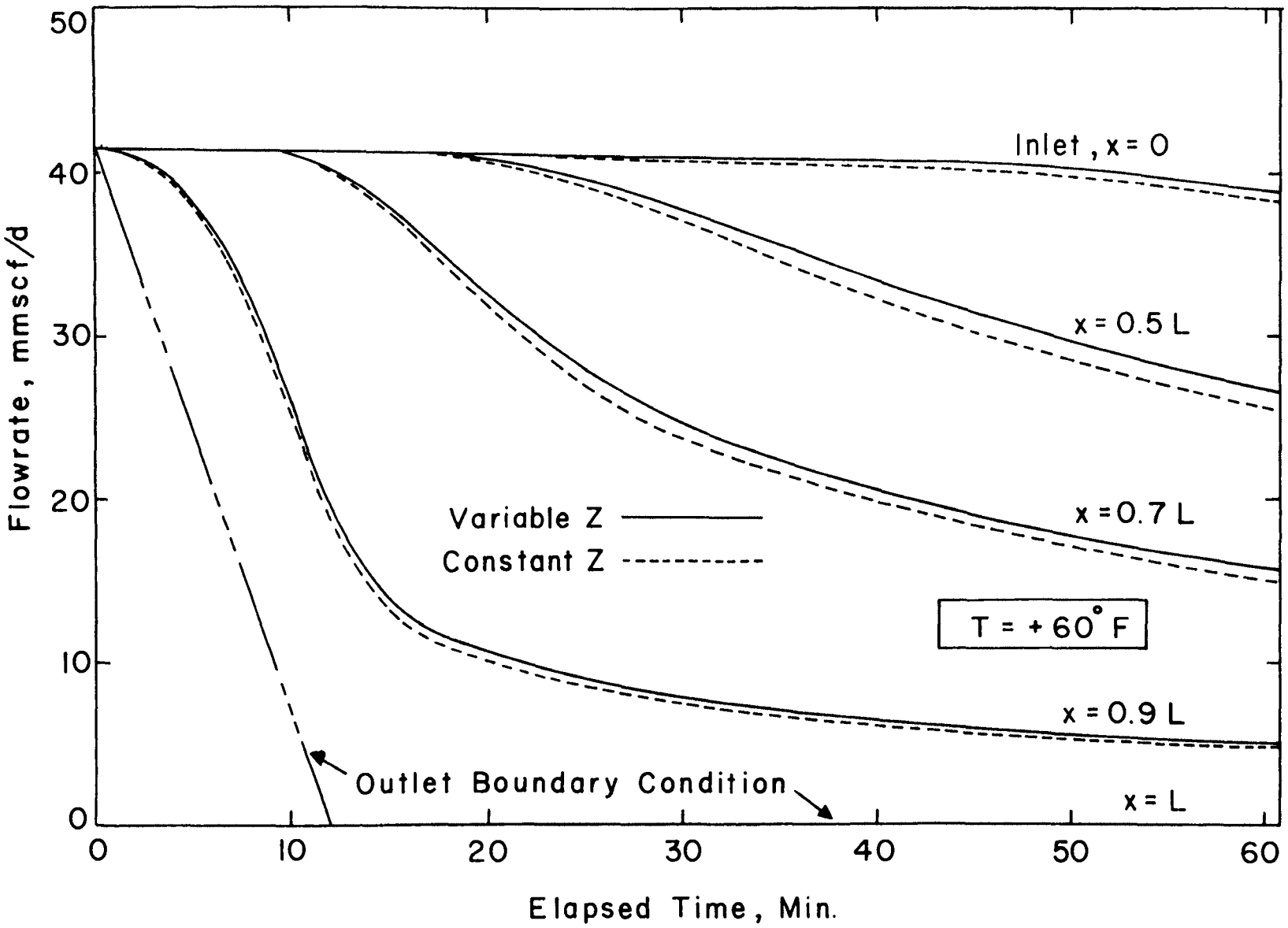
Flowrate, mmscf/d
Elapsed Time, Min.
Inlet, x = 0
x = 0.5 L
x = 0.7 L
x = 0.9 L
x = L
Variable Z
Constant Z
T = + 60° F
Outlet Boundary Condition
0
10
20
30
40
50
60

Figure 6

Figure 7 Pressures Versus Time for Pipeline Shut-In Conditions at a Temperature of - 30°F.

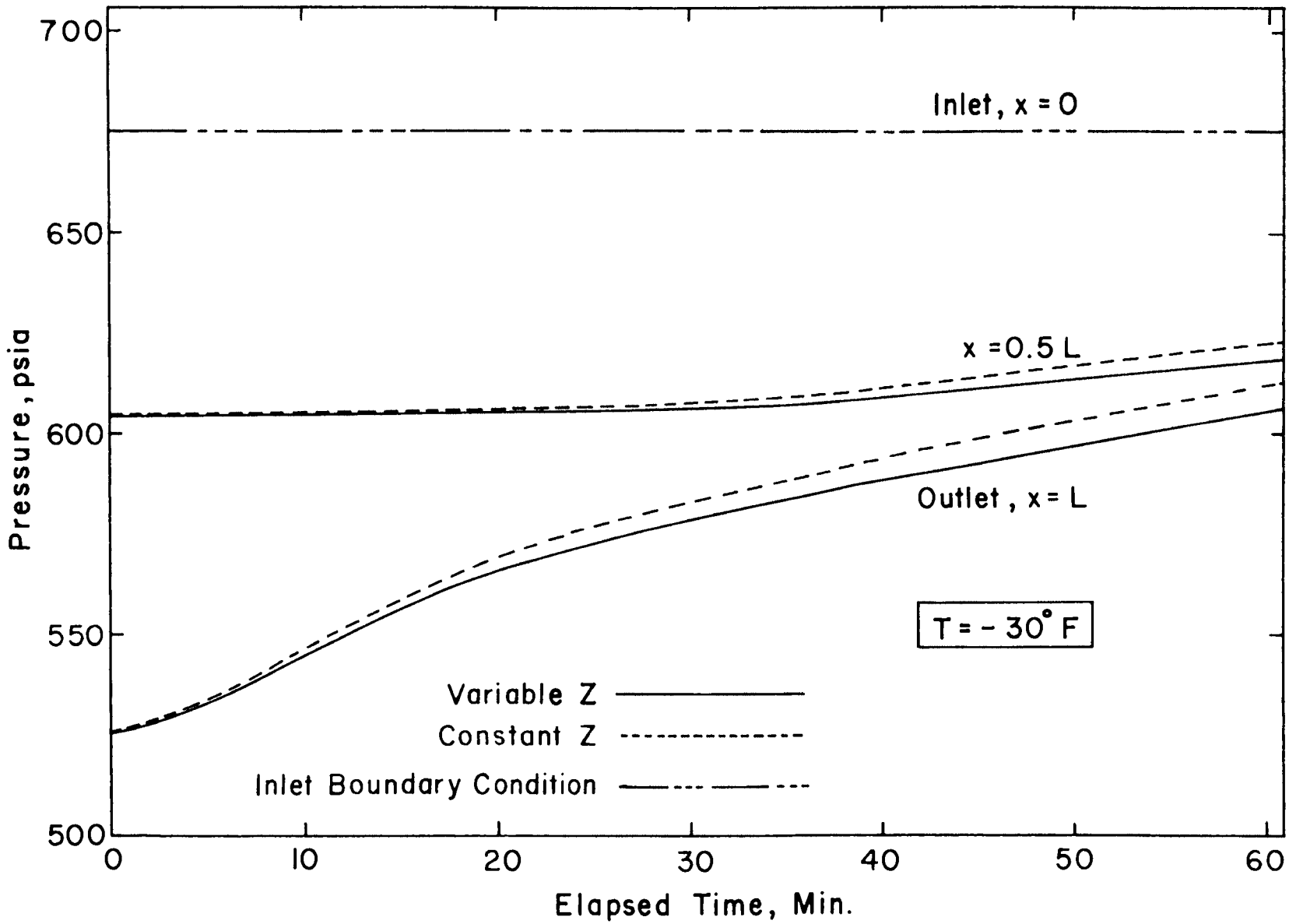
700
650
600
550
500
Pressure, psia
0
10
20
30
40
50
60
Elapsed Time, Min.
Inlet, x = 0
x = 0.5 L
Outlet, x = L
T = - 30° F
Variable Z
Constant Z
Inlet Boundary Condition

Figure 7

Figure 8 Pressures Versus Time for Pipeline Shut-In Conditions at a Temperature of + 60°F.

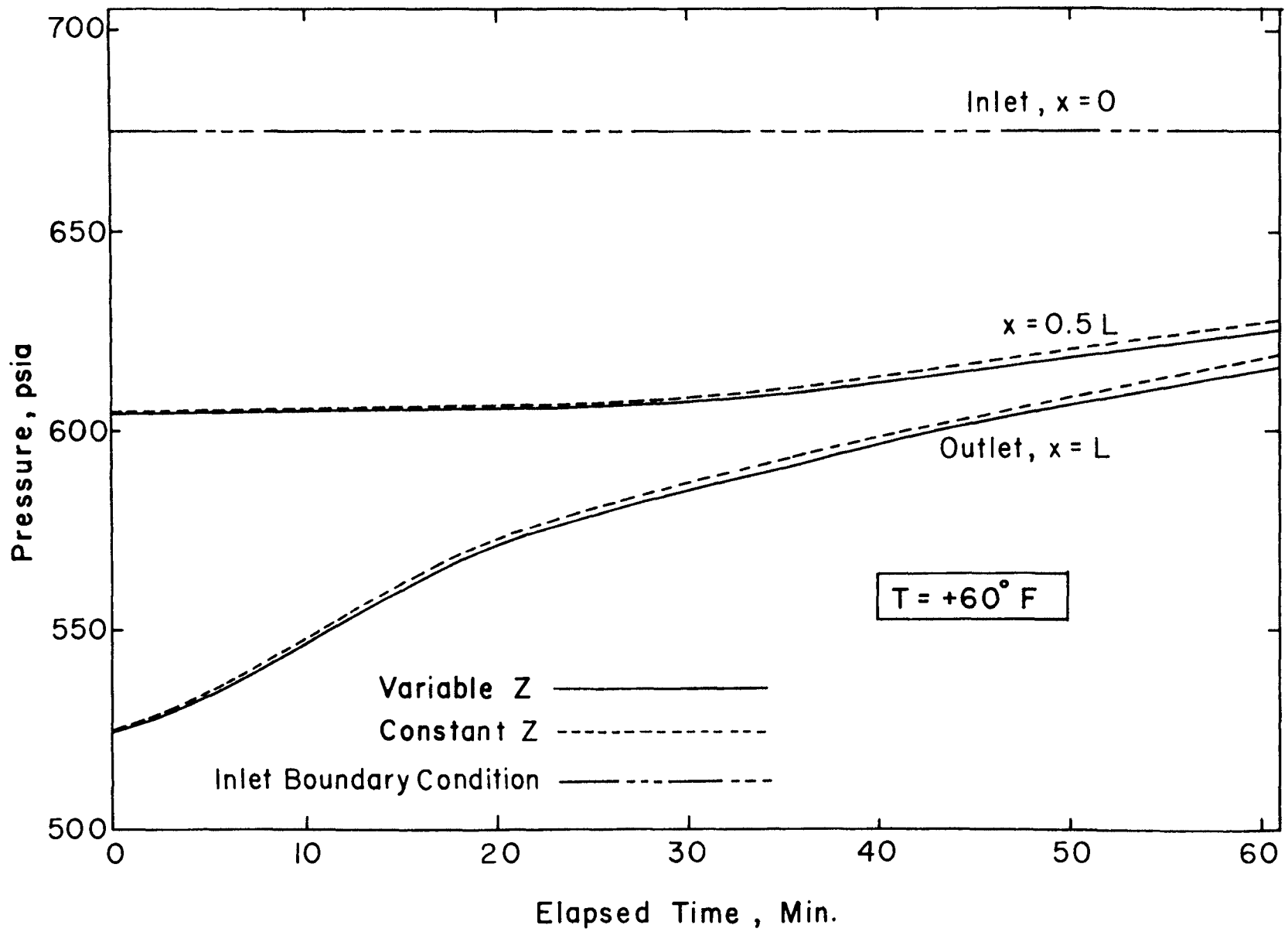
700
650
600
550
500
Pressure, psia
0
10
20
30
40
50
60
Elapsed Time, Min.
Inlet, x = 0
x = 0.5 L
Outlet, x = L
T = +60° F
Variable Z
Constant Z
Inlet Boundary Condition

Figure 8

Figure 9 Outlet Flowrate and Pressure Versus Time for Flowrate Convergence Study at a Temperature of - 40°F.

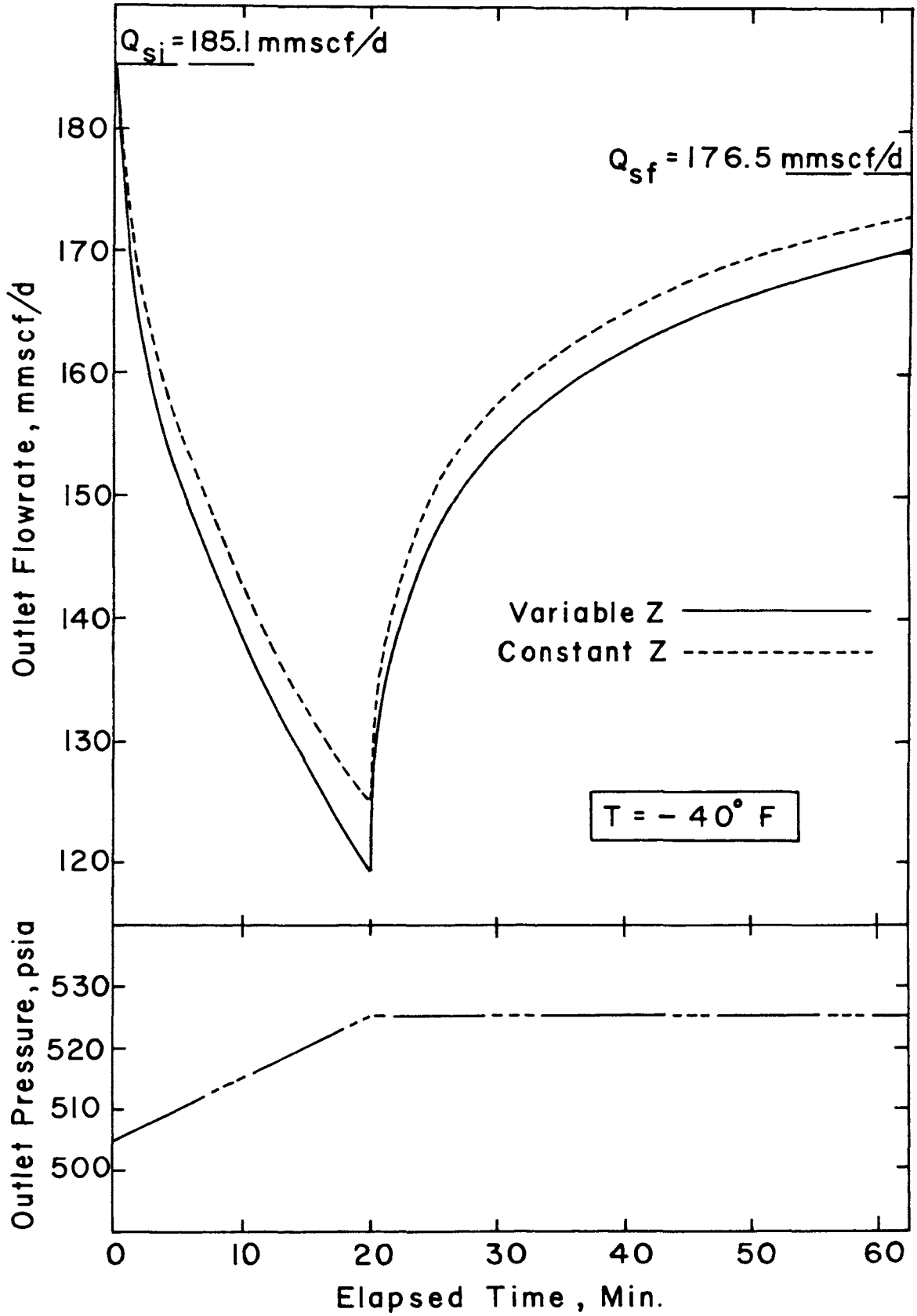
Q_{si} = 185.1 mmscf/d
Q_{sf} = 176.5 mmscf/d
Variable Z
Constant Z
T = - 40° F
Outlet Flowrate, mmscf/d
180
170
160
150
140
130
120
Outlet Pressure, psia
530
520
510
500
0
10
20
30
40
50
60
Elapsed Time, Min.

Figure 9

Figure 10 Outlet Flowrate and Pressure Versus Time for Flowrate Convergence Study at a Temperature of + 80°F.

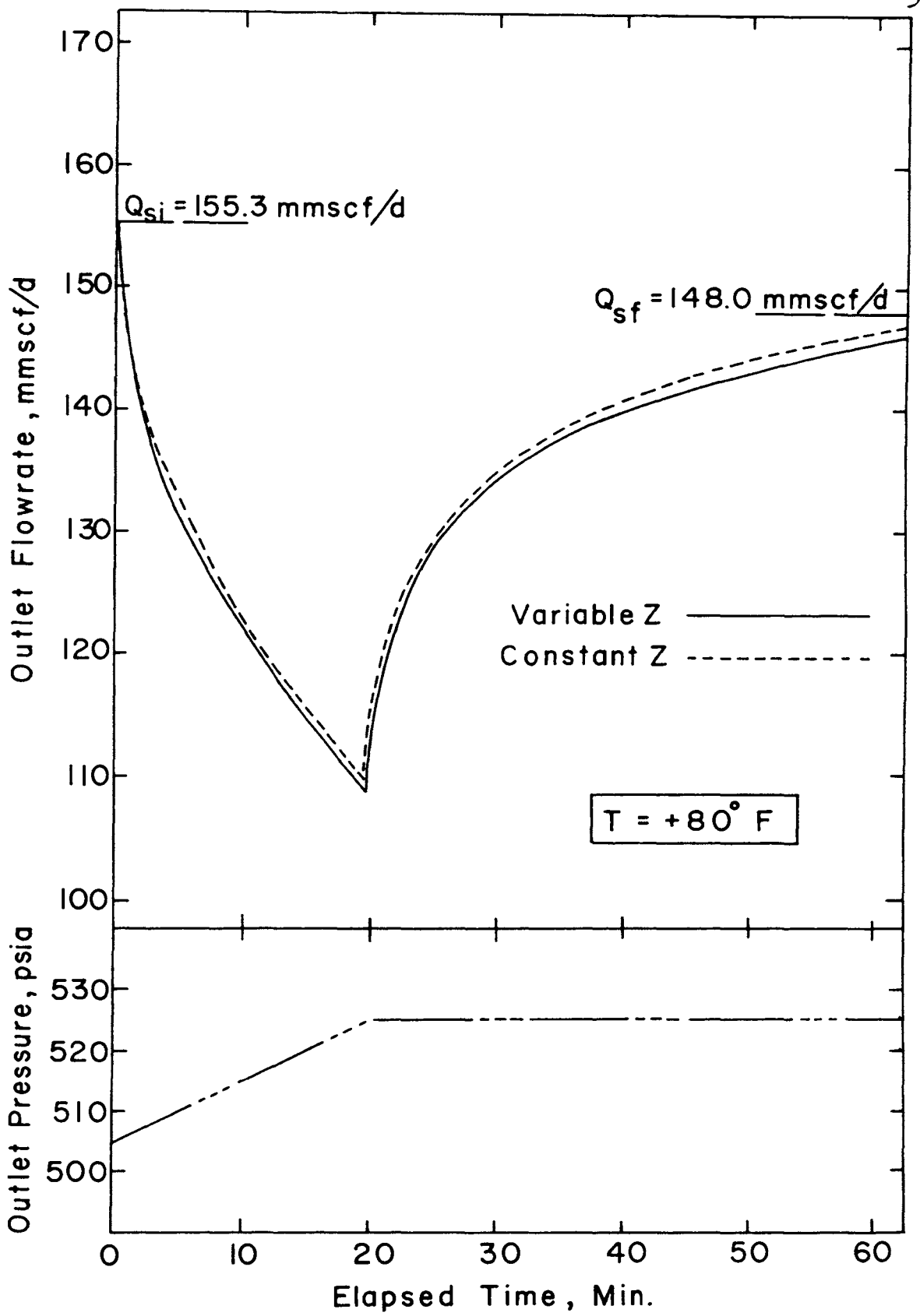
170
160
150
140
130
120
110
100
Outlet Flowrate, mmscf/d
Qsi = 155.3 mmscf/d
Qsf = 148.0 mmscf/d
Variable Z
Constant Z
T = +80° F
530
520
510
500
Outlet Pressure, psia
0
10
20
30
40
50
60
Elapsed Time, Min.

Figure 10

SUMMARY AND CONCLUSIONS

Solutions to the partial-differential equations which govern transient gas pipe flow have been obtained by the method of characteristics and linear finite-differences for both a constant and variable gas compressibility factor. Theoretical investigations were made of various constant flowing temperatures and transient conditions in order that the results for both the constant and variable z solutions could be compared.

Marked differences occur in the prediction of flow properties at extremely low flowing temperatures by the two approaches. This is a result of the variation of the compressibility factor with pressure changes in these temperature ranges. Deviations between the two methods are much less pronounced at normal pipeline operating temperatures.

Transient behavior in gas pipelines may readily be approximated by using an average constant value of the gas compressibility factor for most transmission conditions.

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The author would like to also express his appreciation to Danny L. Fread for his help in understanding the basic approach to numerical solutions by the method of characteristics.

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APPENDIX II.--NOTATION (1)

The following symbols are used in this paper:

A = cross-sectional area of pipe, point on x-t plane;

B = point on x-t plane;

D = inside diameter of pipeline;

f = Moody friction factor;

G = gas specific gravity;

g_c = gravitational conversion constant;

K = dimensionless constant = $0.5fL/D$;

L = total length of pipeline;

M = mass flowrate of gas;

n = number of moles of gas;

P = absolute pressure of gas, point on x-t plane;

P_1, P_2 = pressure of gas at inlet and outlet, respectively;

P_c = critical pressure of gas;

P_s = standard pressure;

Q = volume flowrate of gas;

R = universal gas constant;

T = temperature of gas;

T_c = critical temperature of gas;

T_s = standard temperature;

t = time;

V = volume of gas;

w = constant grouping of terms used in the calculation of z;

x = distance along the pipeline;

z = gas compressibility factor;

ρ = mass density of gas.

APPENDIX II.--NOTATION (2)

(for thesis only)

The following symbols are used in this paper:

A' = point on x-t plane;

B' = point on x-t plane;

C = point on x-t plane;

F = forces acting during gas flow;

L, L_1, L_2 = equation representation used in characteristics transformation;

M_{PI} = inlet mass flowrate of the gas;

S_+, S_- = representation for positive and negative characteristic grid lines;

v = average velocity of the gas;

v_s = velocity of sound in the gas;

λ = multiplier used in characteristic transformation;

ρ_I = initial inlet mass density of the gas;

ρ_{PI} = inlet mass density of the gas;

τ_o = shear stress at pipe wall.

APPENDIX III.--LITERATURE REVIEW

Numerical solutions to predict pipe flow conditions during transient fluid flow have become feasible through application of digital computer techniques. Studies of unsteady-state compressible fluid flow have been limited in number because transient gas flow is primarily a problem of an industrial nature. Until the recent organization of the Transient Flow Committee of the American Gas Association, few major research projects have dealt with the subject.

In 1951, Olds and Sage (7) presented a method which graphically integrated the partial-differential force and material balance equations describing gas flow. This procedure was reported to have yielded satisfactory results for various transient flow conditions. The investigation considered gas compressibility by using an equation of state to determine the specific weight of the gas as a function of temperature and pressure in the system.

A computer study of unsteady-state natural gas pipe flow was reported by Nelson and Powers (6) in 1958. The procedure involved a trial-and-error solution of the fundamental mass and momentum equations for compressible fluid flow. The gas compressibility factor was expressed as a linear function of reduced pressure at constant temperature conditions. Various pipeline storage and flow-rate depletion studies were made, but an excessive amount

of computer time was required due to the lengthy equations involved in the solution and the testing procedures required to insure correct application of boundary conditions.

In 1962, Taylor, Wood, and Powers (11) presented details of a computer program to simulate transient gas conditions. A direct solution was provided by application of the method of characteristics (1,5,9) and finite-difference techniques to the differential equations of gas flow. An ideal gas ($z = 1.0$) was assumed, thus simplifying the method considerably. Although correlation between computed results and field data were inconclusive, the investigation suggested a possible method to obtain a direct solution to the transient flow equations for an ideal gas.

An analytical approach to transient gas flow was reported in 1965 by Wilkinson, Holliday, and Batey (12). A recurring power series solution was applied to the equations of continuity and momentum. Gas flow properties at one end of a pipe section were determined, based on known flow properties at the other end of the section. A very useful equation was developed to calculate the gas compressibility factor as a function of mass density and critical gas properties. A study was made comparing solutions obtained by using a varying compressibility factor with those made by assuming a constant compressibility factor evaluated at mean line pressure. The report stated that the use of a

mean compressibility factor did not significantly degrade the results.

In early 1971, Distenfano (3) introduced a general digital-computer program designed to simulate the transient flow of an entire gas pipeline network. The program, referred to as PIPETRAN IV, is completely flexible so that pressure and flowrate changes for all phases of a pipeline system (pipe segments, compressor stations, storage facilities, junctions, etc.) may be incorporated into the solution. The method applies finite-differences directly to the partial-differential equations of mass and momentum conservation in order to determine the dynamic behavior of the system. The gas compressibility factor is calculated by an equation similar to that used by Wilkinson, Holliday, and Batey (14), but its value is assumed to be constant for the pipeline. PIPETRAN IV is reported to provide very speedy solutions and appears to be a valuable tool in transient gas flow investigations, provided temperature and pressure changes in the pipe system permit only small variations in the gas compressibility factor.

APPENDIX IV.--THEORETICAL DEVELOPMENT OF EQUATIONS

A basic momentum balance to describe gas flow may be written by considering an element of flowing gas in the pipeline.

$$\Sigma F = \frac{M v}{g_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Assuming one-dimensional flow, F = forces acting parallel to gas flow; M = mass flowrate of the gas in pounds mass per second; v = fluid velocity in feet per second; and g_c = the gravitational conversion constant in foot-pounds mass per pound force-second squared. The left side of this equation can be expanded to include differential pressure and shear forces acting in the direction of flow, while momentum changes with respect to time and position are included for the right hand term. Thus, for an incremental element of pipe

$$- g_c A P_x dx - g_c \pi D \tau_o dx = M_t dx + (M v)_x dx \quad \ldots (18)$$

where P = pressure of the flowing gas (pounds per square foot absolute); A = cross-sectional area of the pipe in square feet; τ_o = shear stress in pounds per square feet; D = inside diameter of the pipe (feet); and the subscripts x and t denote partial differentiation with respect to the independent variables, distance and time.

In order to eliminate variables involving gas velocity, shear stress, and pressure, the following relationships based upon the equations of continuity, friction loss, and gas state are substituted into Equation 18:

$$v = \frac{M}{\rho A} \quad \text{.....(19)}$$

$$\tau_o = \frac{f \rho v^2}{8 g_c} = \frac{f M^2}{8 g_c \rho A^2} \quad \text{.....(20)}$$

$$P = \frac{\rho z R T}{29 G} \quad \text{.....(21)}$$

In these equations, ρ = mass density of the gas in pounds mass per cubic foot; f = Moody friction factor; z = gas compressibility factor (dimensionless); R = universal gas constant (foot-pounds force per degrees Rankine-pound moles); T = absolute temperature of the gas in degrees Rankine; and G = gas specific gravity (dimensionless). Performing these substitutions and rearranging terms, the momentum equation becomes

$$\frac{g_c A R T}{29 G} (\rho z)_x + M_t + \frac{1}{A} (M^2/\rho)_x + \frac{f M^2}{2 D A \rho} = 0 \quad \text{....(22)}$$

A continuity equation may also be written to describe one-dimensional gas flow:

$$- (\rho v)_x = \rho_t \quad \text{.....(23)}$$

Noting that $\rho v = M/A$, and rearranging terms, Equation 23

may be rewritten as

$$\rho_t + \frac{1}{A} M_x = 0 \quad \text{.....(24)}$$

Equations 22 and 24 form the basic momentum and mass balance relationships used in this study. In order to provide simpler working equations, all terms will be defined or rearranged in dimensionless form. Thus, the following dimensionless ratios (11) are defined:

$$x' = x/L \quad \text{.....(25)}$$

$$t' = t\ v_s/L \quad \text{.....(26)}$$

$$M' = M/(A\ \rho_I\ v_s) \quad \text{.....(27)}$$

$$\rho' = \rho/\rho_I \quad \text{.....(28)}$$

$$K = 0.5\ f\ L/D \quad \text{.....(29)}$$

where, L = total length of the pipeline in feet; ρ_I = initial mass density of the gas at pipe inlet pressure; and v_s = $(g_c\ R\ T/29\ G)^{\frac{1}{2}}$ = the velocity of sound in a gas of given composition and temperature.

If v_s and the dimensionless quantities defined above are substituted into Equations 22 and 24, and all prime notation is removed, the dimensionless momentum and mass equations of gas flow become

$$(\rho\ z)_x + M_t + (M^2/\rho)_x + K\ M^2/\rho = 0 \quad \text{.....(30)}$$

and

$$\rho_t + M_x = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

An equation may be used to determine the gas compressibility factor as a function of mass density and the critical gas properties (12). Thus

$$z = \frac{1.0}{1.0 + w\,\rho} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

where both w and ρ are dimensionless quantities, and

$$w = \frac{R\ T}{144\ P_c\ 29\ G}\left(0.533\ \frac{T_c}{T} - 0.257\right)\rho_I \quad \ldots\ldots\ldots\ldots (33)$$

In Equation 33, P_c = critical gas pressure (psia); and T_c = critical gas temperature (degrees Rankine). If the value of z from Equation 32 is substituted into the momentum balance (Equation 30), all products differentiated, and the terms regrouped; the following equation results:

$$\left[\frac{1}{(1 + w\,\rho)^2} - \frac{M^2}{\rho^2}\right]\rho_x + M_t + \frac{2\ M}{\rho}\ M_x + \frac{K\ M^2}{\rho} = 0 \quad \ldots\ldots (34)$$

Equations 31 and 34 form a system of quasi-linear partial-differential equations with two dependent variables (ρ and M), and two independent variables (x and t). A numerical solution to this set of equations requires a transformation known as the method of characteristics (1,5,9).

If Equations 31 and 34 are written in the following forms:

$$L_1 = \left[z^2 - \frac{M^2}{\rho^2}\right]\rho_x + M_t + \frac{2\,M}{\rho}\,M_x + \frac{K\,M^2}{\rho} \quad \text{.....(35)}$$

$$L_2 = \rho_t + M_x \quad \text{.....(36)}$$

and combined by using an unknown multiplier, λ, then

$$L = L_1 + \lambda L_2 \quad \text{.....(37)}$$

The following equation results upon substituting and regrouping terms:

$$L = \lambda\left[\left[\frac{z^2 - M^2/\rho^2}{\lambda}\right]\rho_x + \rho_t\right] + \left[\left[\lambda + \frac{2\,M}{\rho}\right]M_x + M_t\right] + \frac{K\,M^2}{\rho} \quad \text{.....(38)}$$

If $\rho = \rho(x,t)$ and $M = M(x,t)$, then the total derivatives of ρ and M with respect to time may be written as

$$\frac{d\rho}{dt} = \rho_x \frac{dx}{dt} + \rho_t \quad \text{.....(39)}$$

$$\frac{dM}{dt} = M_x \frac{dx}{dt} + M_t \quad \text{.....(40)}$$

By comparison of Equations 38, 39 and 40 it follows that

$$\frac{dx}{dt} = \frac{z^2 - M^2/\rho^2}{\lambda} = \lambda + \frac{2\,M}{\rho} \quad \text{.....(41)}$$

$$L = \lambda\frac{d\rho}{dt} + \frac{dM}{dt} + \frac{K\,M^2}{\rho} = 0 \quad \text{.....(42)}$$

Equations 41 are solved simultaneously for λ, and the result is substituted into Equations 41 and 42. Thus

$$\lambda = \pm z - M/\rho \quad \text{.....(43)}$$

$$dx/dt = \pm z + M/\rho \quad \text{.....(44)}$$

$$(\pm z - M/\rho)\, d\rho + dM + (K M^2/\rho)\, dt = 0 \quad \text{.....(45)}$$

As pointed out by Lister (5), every solution of Equations 44 and 45 is a solution to the original system, Equations 31 and 34.

Equation 44 is defined as the characteristic grid-slope equation and may be either positive or negative depending upon the sign before z. A graphical description of the equations in the x-t plane is shown in Figure 11. Points A, B and P are related by the sloping grid lines (S_+ and S_-).

A solution to Equations 44 and 45 can be accomplished by using first-order finite-difference approximations defined by the formula

$$\int_{x_A}^{x_P} f(x)\, dx \approx f(x_A)(x_P - x_A) \quad \text{.....(46)}$$

Referring to the notation of Figure 11, and applying this linear approximation to Equations 44 and 45, there results:

$$(x_P - x_A) = (z + M/\rho)_A\, (t_P - t_A) \quad \text{.....(47)}$$

$$(z - M/\rho)_A \, (\rho_P - \rho_A) + (M_P - M_A) + (K \, M_A^2/\rho_A)(t_P - t_A) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(48)$$

$$(x_P - x_B) = (- z + M/\rho)_B \, (t_P - t_B) \quad \ldots\ldots(49)$$

$$(- z - M/\rho)_B \, (\rho_P - \rho_B) + (M_P - M_B) + (K \, M_B^2/\rho_B)(t_P - t_B) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(50)$$

If the variables are known at points A and B, Equations 47-50 represent a system of 4 equations with 4 unknowns (ρ_P, M_P, x_P and t_P). Thus as shown in Figure 12, from a set of known values at initial time ($t = t_o$), a solution can be marched out by using an iterative system of calculations based upon Equations 47-50.

In order to arrive at a solution that will predict gas properties at evenly spaced intervals along the pipeline for the same elapsed time condition, the method of specified time intervals (5,9) is used.

The pipeline is divided into any number of equally spaced increments, and a constant time interval (Δt) is chosen for each time iteration such that

$$(x_P - x_A) = \Delta x > (z + M/\rho)_A \, \Delta t \quad \ldots\ldots\ldots\ldots\ldots(51)$$

and $$(x_P - x_B) = \Delta x < (z - M/\rho)_B \, \Delta t \quad \ldots\ldots\ldots\ldots\ldots(52)$$

are satisfied for every position increment along the pipeline. Thus, the smallest value of Δt is chosen as the step time.

Referring to Figure 13, if the flow conditions at points A, B and C are known, an interpolation process can be used to locate and determine the values at points A' and B'. Time and position are related by the grid-slope equations, and if a linear relationship is assumed to exist between points, then

$$\Delta x_{A'} = \frac{\Delta t}{2}\left[(z + M/\rho)_A + (z + M/\rho)_C\right] \quad \text{.....(53)}$$

and

$$\Delta x_{B'} = \frac{\Delta t}{2}\left[(z - M/\rho)_C + (z - M/\rho)_B\right] \quad \text{.....(54)}$$

from which

$$M_{A'} = M_C - \Delta x_{A'}\,(M_C - M_A) \quad \text{.....(55)}$$

and

$$M_{B'} = M_C - \Delta x_{B'}\,(M_C - M_B) \quad \text{.....(56)}$$

Mass density and the gas compressibility factor are also determined in like manner.

If Equations 48 and 50 are written for these specified time conditions and are each solved for their unknown quantities (ρ_P and M_P), the final pair of iterative equations become

$$M_P = \left[\frac{M_A}{z_A - M_A/\rho_A} + \frac{M_B}{z_B + M_B/\rho_B} + \rho_A - \rho_B - \frac{K\,\Delta t\,M_A^2}{\rho_A\,(z_A - M_A/\rho_A)} - \frac{K\,\Delta t\,M_B^2}{\rho_B\,(z_B + M_B/\rho_B)}\right] \Bigg/ \left[\frac{1}{z_A - M_A/\rho_A} + \frac{1}{z_B + M_B/\rho_B}\right] \quad \text{.....(57)}$$

$$\rho_P = \frac{\left[\rho_B z_A + \rho_A z_B - \frac{K \Delta t M_A^2}{\rho_A} + \frac{K \Delta t M_B^2}{\rho_B}\right]}{\left[z_A - M_A/\rho_A + z_B + M_B/\rho_B\right]} \quad \ldots\ldots\ldots(58)$$

A and B are now defined as the points calculated by the specified time interpolations so that Δt is constant for all iterations along the pipeline corresponding to a particular value of elapsed time.

While the unknown flow properties may be calculated at each intersection of two grid lines by using Equations 57 and 58, a slightly different situation exists at each grid-boundary intersection. Only one grid-slope and characteristic equation is available at each boundary location (Figure 14), requiring that one of the two flow properties (mass flowrate or density) be known at that point either as constant or as some function of time. For instance, if pressure (density) is to be the controlling boundary condition at the pipe inlet, Equation 50 may be solved for the unknown value of mass flowrate (M_{PI}) in terms of the known mass density (ρ_{PI}) and the flow properties at point B (determined by applying specified time interval interpolations to the results of the previous time iteration). Thus

$$M_{PI} = M_B + (\rho_{PI} - \rho_B)(z_B + M_B/\rho_B) - (K \Delta t M_B^2/\rho_B) \quad \ldots.(59)$$

If inlet flowrate were controlled, Equation 50 could be solved for ρ_{PI}, and similarly the outlet boundary conditions could be treated by various manipulations of Equation 48.

The preceding development was required in order to include a changing gas compressibility factor (primarily as a function of gas density) in all equations. The resulting lengthy iterative equations and their numerous linear interpolations can be greatly simplified if gas compressibility is assumed constant throughout the pipe system during the transient investigation.

Equations 30 and 31, describing the dimensionless momentum and mass equations of gas flow, may be modified for a constant gas compressibility factor. Thus

$$z\ \rho_x + M_t + (M^2/\rho)_x + K\ M^2/\rho = 0 \quad \text{.....(60)}$$

$$\rho_t + M_x = 0 \quad \text{.....(61)}$$

Taylor, Wood, and Powers (11) have shown that the value of $(M^2/\rho)_x$ is negligible in comparison with the other terms in Equation 60. Thus, it is eliminated from this development. (This term was included in the general derivation because its reduced characteristic form, M/ρ, could easily be handled, along with the changing gas compressibility factor, by means of the linear interpolations. It is omitted here in order to avoid any type of interpolation.)

Again, applying the characteristics transformation

$$L_1 = z\ \rho_x + M_t + K\ M^2/\rho \quad \text{.....(62)}$$

$$L_2 = \rho_t + M_x \quad \text{.....(63)}$$

$$L = L_1 + \lambda L_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(64)$$

it follows that

$$L = \lambda (z\, \rho_x/\lambda + \rho_t) + (\lambda M_x + M_t) + K M^2/\rho = 0 \quad \ldots.(65)$$

If an approach, similar to that used in Equations 38-45, is followed here, the resulting characteristic and grid-slope equations become

$$dx/dt = \pm \sqrt{z} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(66)$$

$$\pm \sqrt{z}\, d\rho + dM + (K M^2/\rho)\, dt = 0 \quad \ldots\ldots\ldots\ldots(67)$$

The application of first-order finite-difference approximations to Equations 66 and 67 results in the following simplified grid-slope and characteristic equations:

$$(x_P - x_A) = \sqrt{z}\, (t_P - t_A) \quad \ldots\ldots\ldots\ldots\ldots(68)$$

$$\sqrt{z}\, (\rho_P - \rho_A) + (M_P - M_A) + (K M_A^2/\rho_A)(t_P - t_A) = 0 \quad \ldots.(69)$$

$$(x_P - x_B) = -\sqrt{z}\, (t_P - t_B) \quad \ldots\ldots\ldots\ldots\ldots(70)$$

$$-\sqrt{z}\, (\rho_P - \rho_B) + (M_P - M_B) + (K M_B^2/\rho_B)(t_P - t_B) = 0 \quad \ldots(71)$$

Since for this development an average constant value of z is assumed, Equations 68 and 70 may be written as one grid-slope equation

$$\Delta x = \sqrt{z}\, \Delta t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(72)$$

This slope equation applies to both the positive and negative sloping grid lines, depending upon the direction in which the equally spaced pipeline increments (Δx) are measured.

Thus, the time increment, Δt, is constant for each transient flow investigation, and a simple rectangular grid system results as shown in Figure 15. No interpolations are required as Equations 69 and 71 may be easily solved for the unknown flow properties at each predetermined grid intersection. Thus

$$M_P = \frac{1}{2}\left[M_A + M_B + \sqrt{z}\,(\rho_A - \rho_B) - \frac{K\,\Delta t\,M_A^2}{\rho_A} - \frac{K\,\Delta t\,M_B^2}{\rho_B}\right] \quad \ldots(73)$$

$$\rho_P = \frac{1}{2\sqrt{z}}\left[\sqrt{z}\,(\rho_A + \rho_B) + M_A - M_B - \frac{K\,\Delta t\,M_A^2}{\rho_A} + \frac{K\,\Delta t\,M_B^2}{\rho_B}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots(74)$$

where $\Delta t = \Delta x/\sqrt{z}$ = constant.

Boundary conditions are again treated by solving the original characteristic relationships, Equations 69 and 71, for the unknown flow property in terms of a known boundary value and the flow properties determined on the previous time iteration. Thus, controlling the inlet density results in

$$M_{PI} = M_B + \sqrt{z}\,(\rho_{PI} - \rho_B) - (K\,\Delta t\,M_B^2/\rho_B) \quad \ldots\ldots\ldots(75)$$

Other boundary equations follow by using the methods outlined previously.

Figure 11 Graphical Description of the Sloping Grid Lines in the x-t Plane.

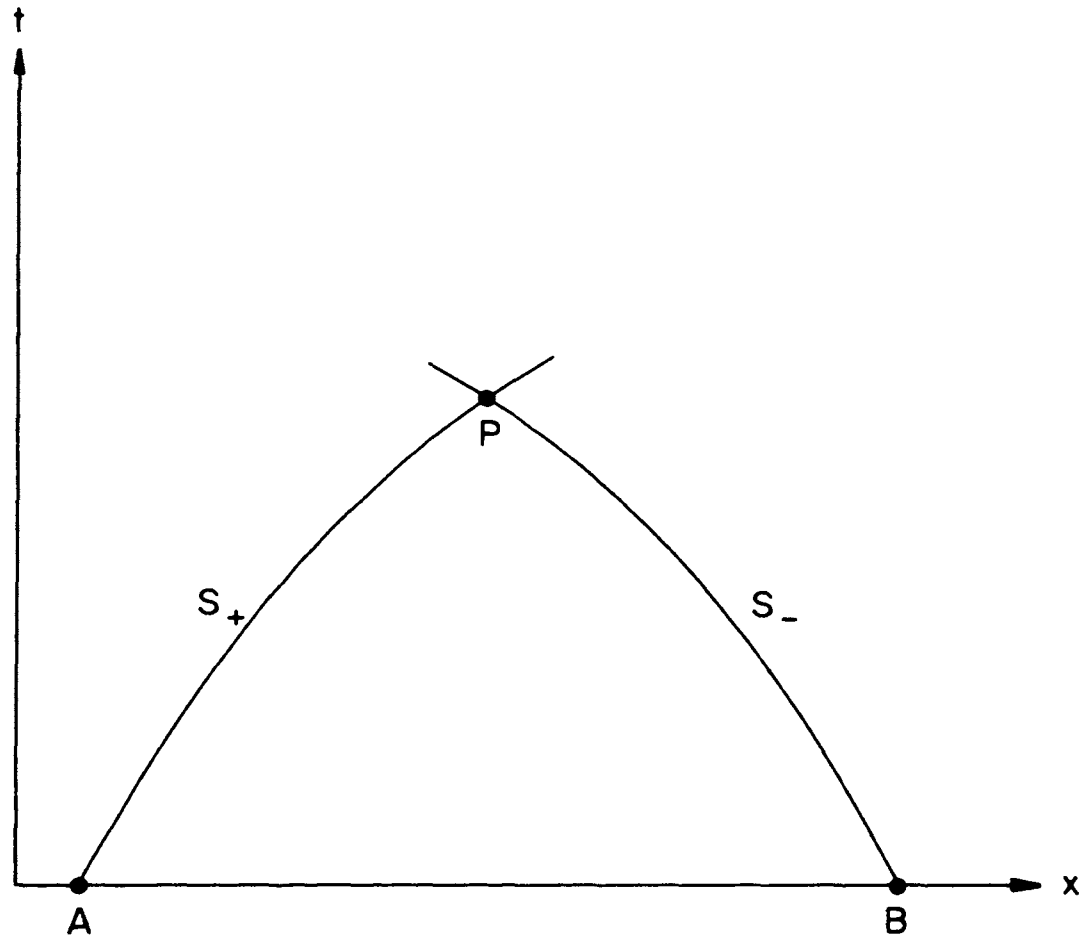

Figure 11

Figure 12 General Solution of Characteristic Equations in the x-t Plane.

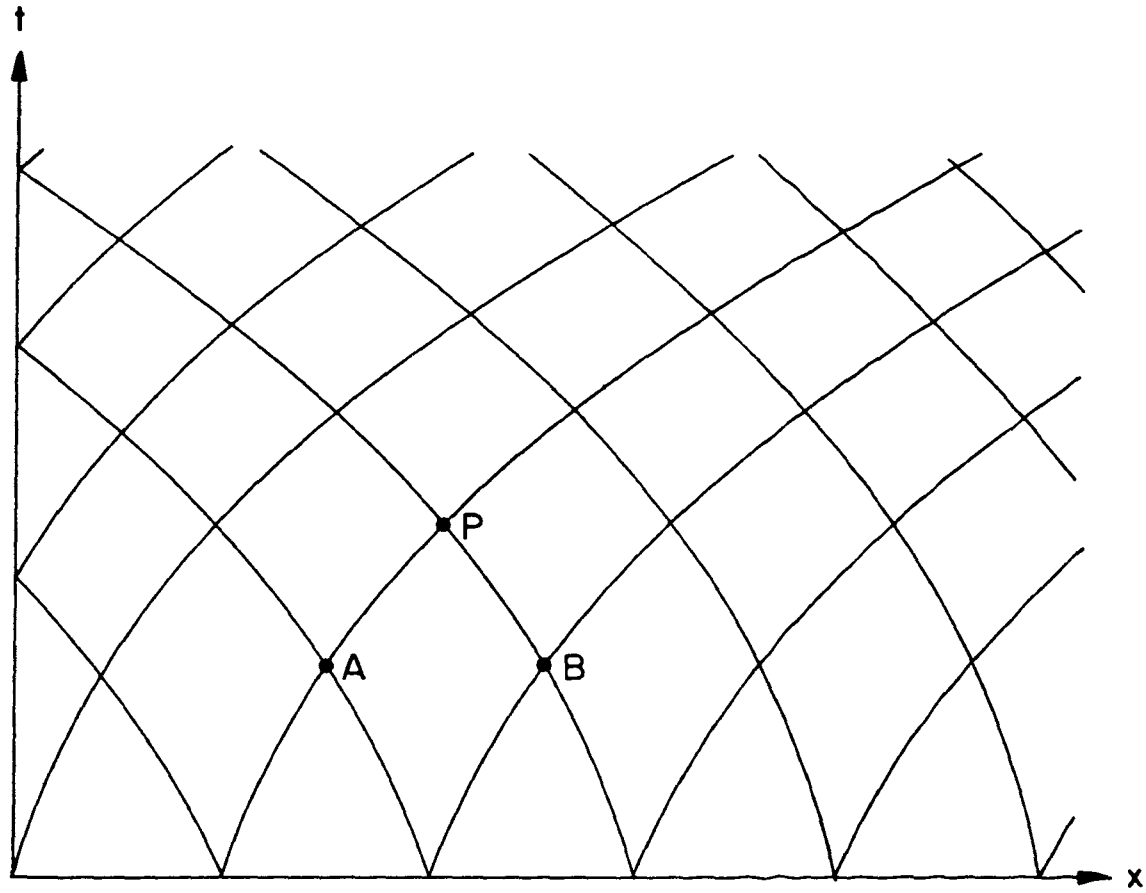

Figure 12

Figure 13 Method of Specified Time Intervals.

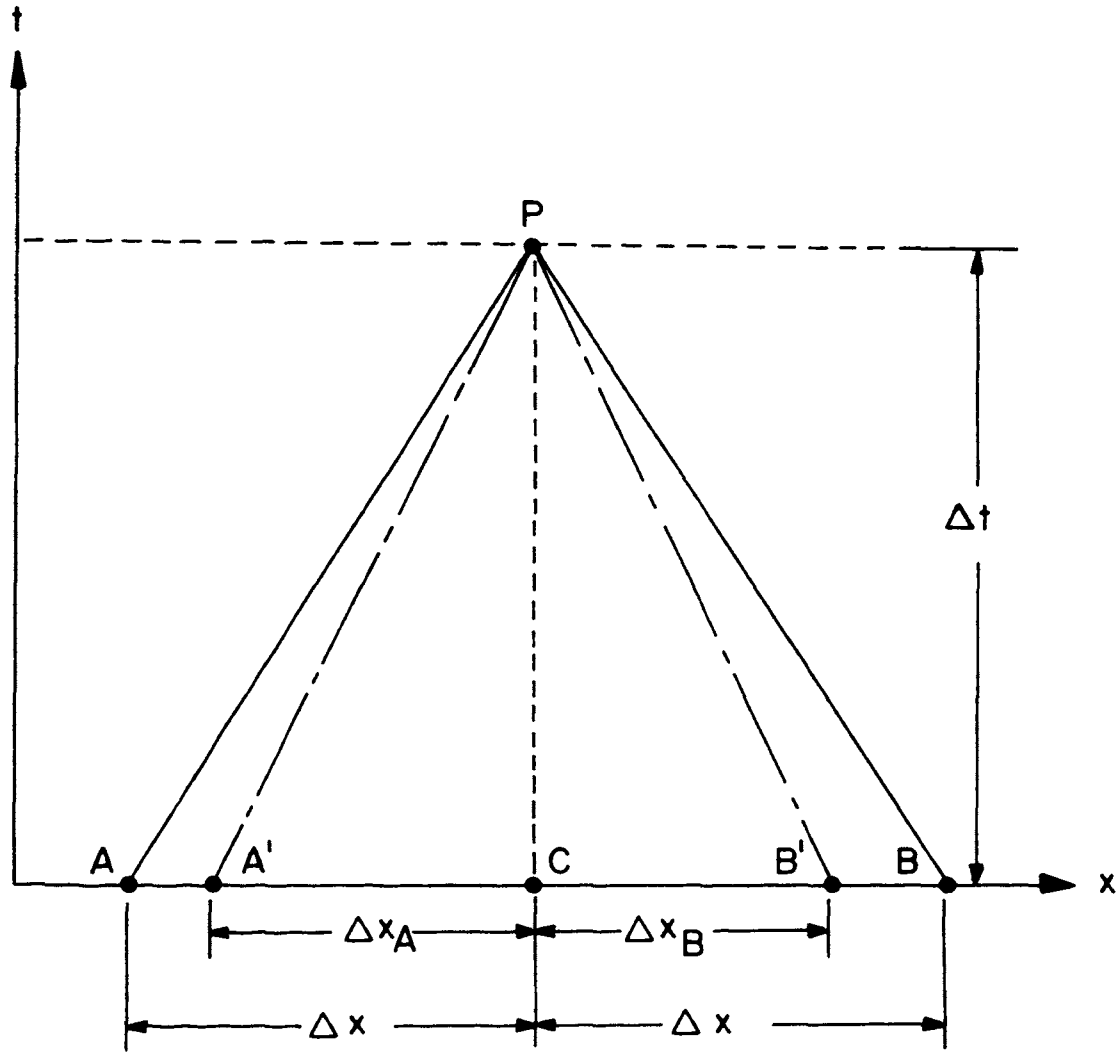

Figure 13

Figure 14 Conditions at Grid-Boundary Intersections.

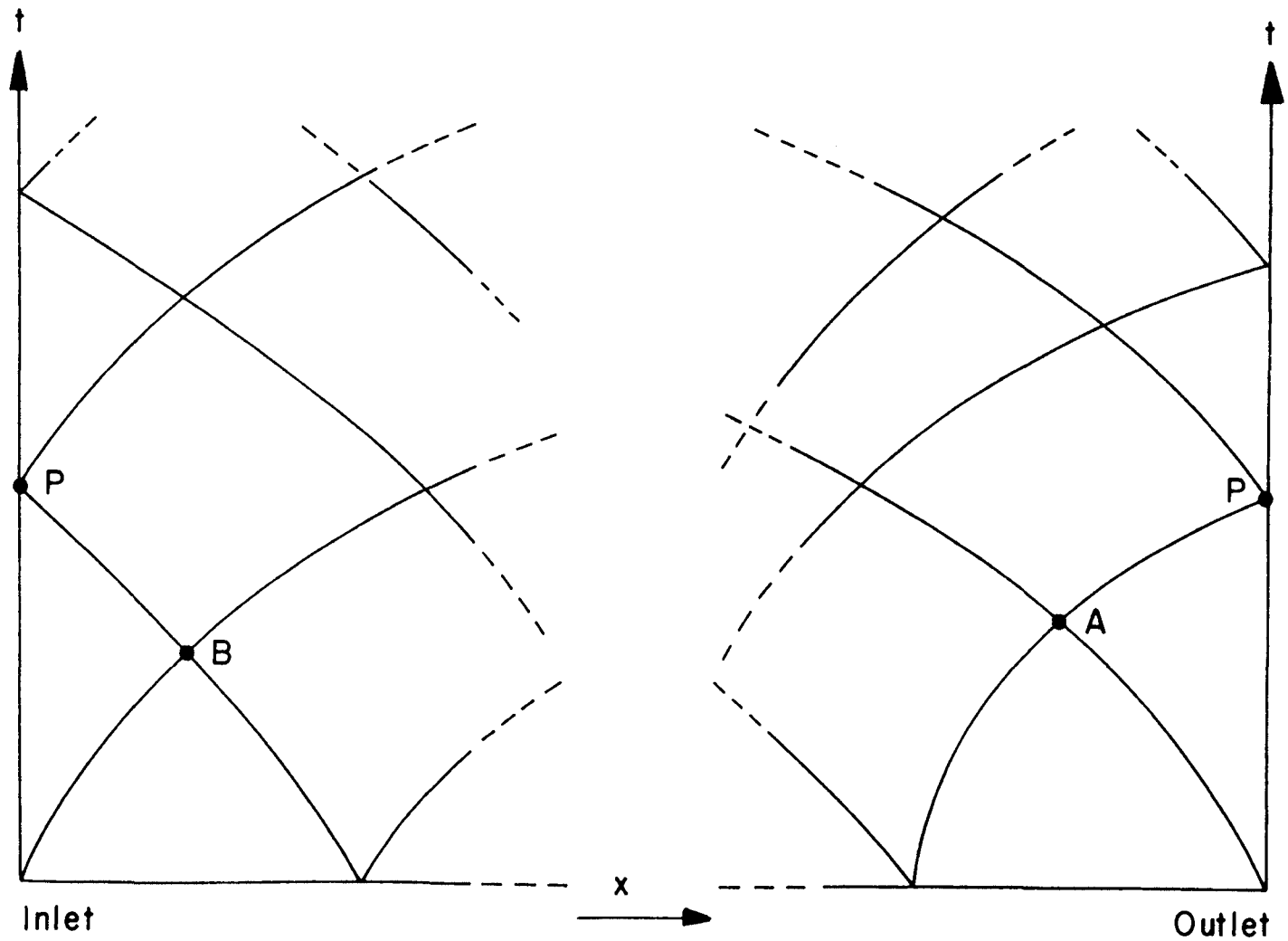

Figure 14

Figure 15 Rectangular Grid Solution of Characteristic Equations Assuming Constant Gas Compressibility.

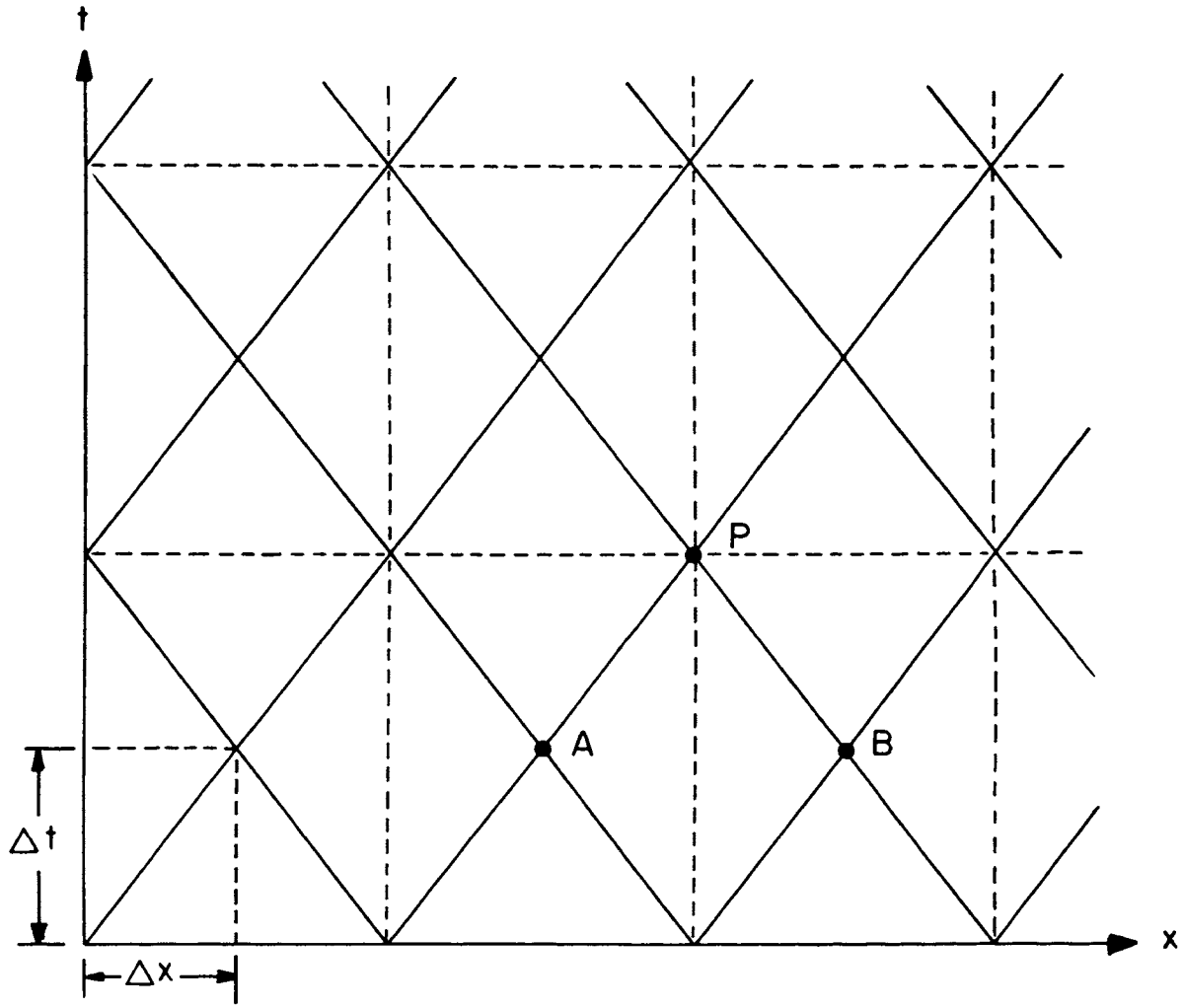

Figure 15

VITA

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