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INVESTIGATION OF TARGET DETECTION

IN NONCOHERENT SYSTEMS

WITH COLORED NOISE

By

EDWARD WINDSOR BAILEY, 1937
22-18

A

THESIS

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ABSTRACT

Design efforts concerning the problem of detecting moving ground targets from an airborne platform with a noncoherent radar have been concentrated in the area of video filter design. The filter formulation generally follows an empirical path with no generally acceptable criterion for an optimum processor. This Thesis considers several problem formulations which are based on a Neyman-Pearson detection criteria. A square-law second detector is assumed and the resulting likelihood ratio shown to be too complex for closed form solution. The problem is reformulated in terms of sequences using complex random variable representations and the likelihood ratio is investigated. A test statistic is derived and discussed in terms of a practical implementation. A suboptimum receiver is implemented in the video frequency region and compared with existing MTI processors by using computer simulation programs. A clutter rejection video filter shaped in accordance with the optimum receiver derivation is shown to have some advantage over conventional shaping with which it is compared.

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CHAPTER I

INTRODUCTION

A frequently occurring problem in the areas of both radar and sonar is the detection of a slowly moving target in the presence of strong, highly correlated noise arising from the target surroundings. A broad range of approaches have been investigated relative to defining "optimum" techniques. Generally, the investigations presented in the literature have been concerned with coherent systems. However, for the more simply implemented class of noncoherent systems, a need exists to examine the problem from the aspect of deriving maximum performance within the operational constraints.

The purpose of this Thesis is to explore the problem of moving target detection in a clutter background from an airborne platform. The radar system is assumed noncoherent and initially constrained by existing operating characteristics of fixed transmitted signal form, antenna parameters, and receiver class. A summary of signal conditions and general discussion of the processing techniques previously studied are presented. A basic system is outlined and the signal and clutter statistics formulated through a square-law device. A general likelihood ratio is then developed for consideration in the search for a processing technique and detector statistic.

The mathematical complexity of the likelihood ratio at the output of a square-law device, even for a short processing sequence, is shown to preclude a closed form solution and alternate formulations are derived in the system prior to second detection. Complex variables are used in the development of the likelihood ratio for a sequence of signals in colored noise, and a closed form solution is proposed. A receiver test

statistic is derived and shown, under certain assumptions, to be similar to the optimum receiver for multiple observations of a single pulse in colored noise.

Due to the complexity of the optimum receiver implementation, a suboptimum form is suggested for use in the video section of the radar system. A filter is formulated directly from the optimum receiver and is evaluated in comparison with existing conventional video processing techniques. A previously developed computer simulation is used in generating the input signals for the processor operation, and the suboptimum form is shown in some cases to be superior to either of the conventional processors with which it is compared.

CHAPTER II

SUMMARY OF SIGNAL CONDITIONS

The basic problem is the extraction and detection of a weak desired "target" from a generally strong noise signal. The noise, in this case, will be considered to be the energy reflected from a multitude of scatters located at random in the target area. Such noise will be designated "clutter" to differentiate it from the thermally generated noise originating in the radar receiver.

The most challenging circumstances are those in which the clutter power at the output of the radar receiver is many times greater than the signal power. It will be the heavy clutter problem considered in this Thesis. Furthermore, the clutter power will be assumed much greater than the "thermal" noise of the system, though the signal will not be so constrained.

As indicated above, the source of the clutter is chaotic reflections from randomly located scatters in the target area. Typical of the clutter source is a foliage-covered stretch of terrain, generally large in extent compared with the expected site of the desired target. The clutter voltage obtained in the radar receiver results from the superposition of a large number of terms originating from the fields of a large number of individual elemental scattering objects. The clutter can therefore be regarded as a random process at the receiver input. Due to the number of scattering elements, the Central Limit Theorem allows that it may be modeled as a Gaussian process. However, due to the periodic and finite duration of the illuminating radar signal, as well as the localized characteristics of the clutter elements, the process is

nonstationary. The difficulty of nonstationarity of the process is avoided by consideration of the problem being investigated. Attention is restricted to a single target located in a resolution cell of the radar system. The range resolution cell is periodically illuminated, the return containing the desired target as well as the clutter or noise. If a processor is sought to detect the target in that specific range resolution cell, then the characteristics of the clutter outside that target area will not change the detection problem. If the clutter characteristics outside the target area are assumed to be identical to those in the target area, the statistical properties of the clutter are continued throughout the repetition period and can then be considered stationary. More extensive discussions are contained in the literature, specifically (1).

Inasmuch as the desired signal originates from a target moving with respect to the background generating the clutter, the frequency domain separation of the desired signal from the clutter spectrum based on the target doppler frequency "shift" suggests standard filtering techniques. As stated in Chapter I, substantial literature exists on investigations of processing techniques. The work on statistical decision theoretic optimality criteria contained in (1), which considers two element sequence length processings, and the interesting approach discussed in (2), are generally representative. Also pointed out, these works deal with coherent systems and predict optimum receivers consisting of predetection filters in the presence of correlated noise.

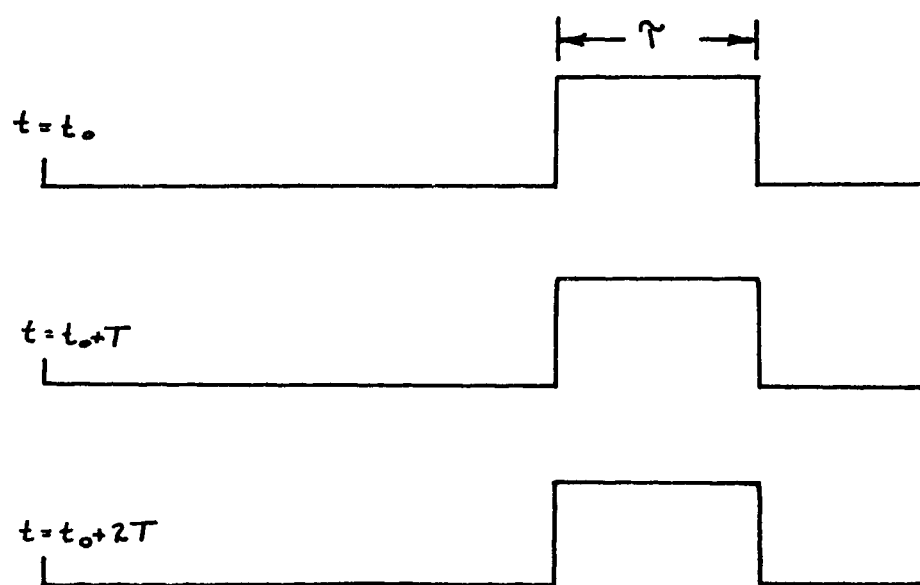
A somewhat more comprehensive investigation of the target detection in correlated noise background includes not only the design of an "optimum"

processor but the unified consideration of an optimum transmitted signal waveform. Works which have considered the problem from such a viewpoint are represented by (3), (4), and (5).

The detection of a ground moving target from an airborne platform is substantially more complicated, particularly for a high performance aircraft. The problem has been carefully presented in such works as (6), (7), and (8). As discussed therein, the clutter spectrum in the receiver is spread due to the platform motion and the finite radar beamwidth, as well as various system instabilities. A significant consideration is the fact that the spread of the clutter spectrum is a function of the scanning antenna pointing angle relative to the aircraft ground track or velocity vector. This fact generally calls for some adaptive techniques in processing, in that a target at a given doppler frequency, perhaps detectable with the antenna near ground track, may be completely submerged in the clutter spectrum as the antenna scans in the azimuth plane.

This Thesis, however, is concerned with investigating the clutter rejection or moving target detection in a noncoherent reception system to differentiate it from the attempts at systems called coherent-on-reception using a noncoherent transmission device. The noncoherent reception system generally utilizes processing in the video section, or after second detection. At the second detector output, the desired signal is the intermodulation or interference term between the clutter and the target returns. Hereafter, when desired signal is mentioned after the second detector, it shall be taken to refer to that intermodulation term which has acquired the moving target doppler shift.

For the noncoherent system a variety of filtering techniques have been investigated in the literature including analog mechanization (7), digital cancellers (6), and fast Fourier transform filtering techniques (9), each of which attempts to reject the clutter spectrum while passing the desired signals. The second detector input for the system in which such processors are utilized is the signal train, as shown in Figure 1, where the pulses represent a pulse modulated carrier of frequency generally in the region 9000 MHz to 20 GHz. The pulse duration may be on the order of 0.1 to 1.0 microseconds. The spectrum, prior to detection, for such a signal train is shown in Figure 2 a. for a coherent system. The spectral lines are spread due to the finite observation time of any target area, as determined by physical parameters of the beamwidth and scan rate. Figure 2 b. shows a typical video spectrum for the problem to illustrate the processing techniques presently used. The clutter spectrum, as mentioned earlier, is spread depending upon the system characteristics and antenna pointing angle. Figure 3 shows the application of the range-gated filter, and delay line frequency responses on the video spectrum. Figure 4 shows block diagrams of two types of video processing. The general technique has been to shape the filter response to obtain a maximum ratio of peak signal power to average clutter plus noise power at the filter output. Also in the general case, some threshold level is set which the signal plus clutter plus noise must exceed to obtain detection. The threshold level is determined by some false alarm number and the clutter plus noise power at the output. Investigations have also been made in the area of utilizing adaptive thresholds based on mean clutter plus noise at the filter output.



T = interpulse period

τ = radar illuminating pulse width (on the order of 1 microsecond)

FIGURE 1. System pulse train

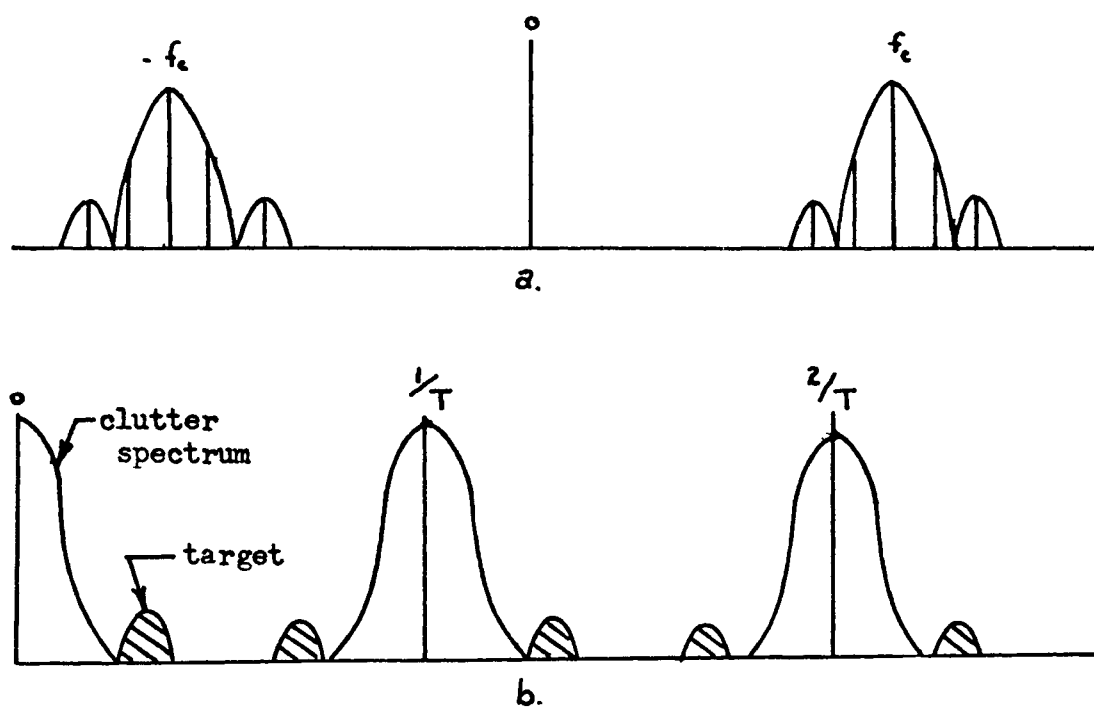


FIGURE 2. (a) spectrum of the rectangular pulse train; (b) video spectrum positive frequency

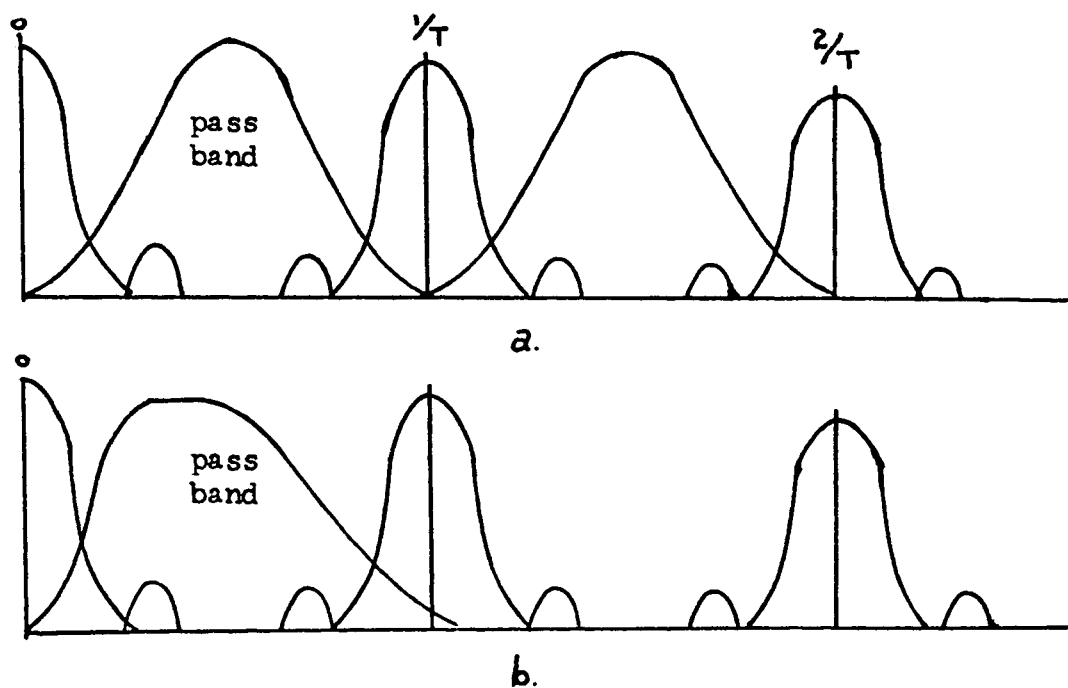
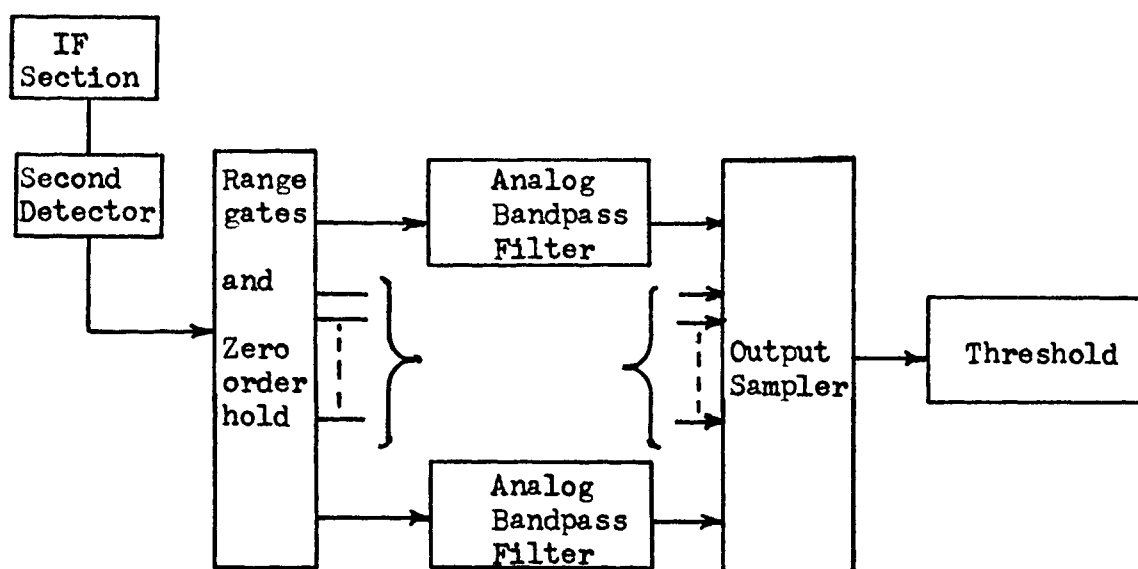
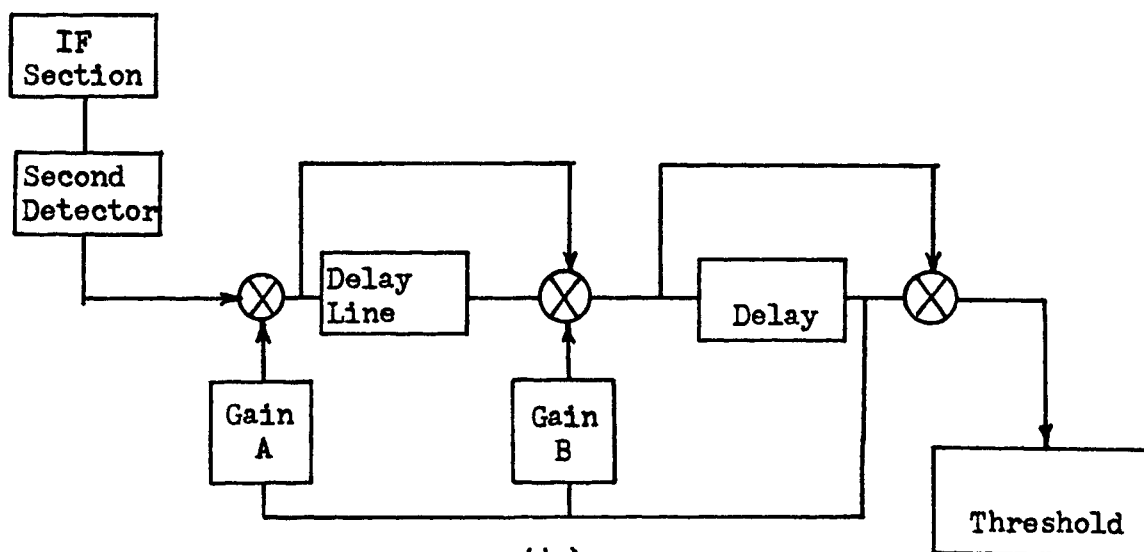


FIGURE 3. (a) video spectrum with periodic filter (delay line) superimposed;
(b) video spectrum with analog bandpass filter superimposed



(a)



(b)

FIGURE 4. (a) range-gate and filter mechanization; (b) dual delay cancellation mechanization

However, in all cases the filter (processor) output provides a composite signal plus clutter plus noise waveform which, for some clutter and target velocity conditions, results in a peak signal power to clutter plus noise power ratio on the order of unity. At the filter output the problem is still one in which the signal must be detected in the presence of a highly correlated clutter residue whose autocorrelation function extends substantially beyond a single interpulse period.

The general scope of this effort is to examine the use of statistical detection theory to improve the signal processor in detecting the presence of a moving target. In Chapter III the statistics of the clutter and signal will be investigated to illustrate the difficulty in defining the second detector output distributions. It will be shown that in the simple case of processing a sequence of only two pulses that the joint probability density functions needed are difficult to characterize and the resulting likelihood ratio is considered. In Chapter IV the likelihood ratio is reformulated prior to second detection.

CHAPTER III

SUMMARY OF SYSTEM DETECTOR FORMULATION AND STATISTICS

Prior to proceeding to the presentation of system statistics, a brief discussion of the detection problem is provided. Detection is used in this section to mean the decision or testing device. In this Thesis the detection problem represents one in which the area of statistical inference concerning hypotheses testing is utilized to implement a binary decision rule. The hypotheses for test will be to determine whether the received data consists of the noise alone, the null hypothesis, or consists of signal plus noise, the alternate hypothesis. The detectors considered in this work will be of the class where some functional of the received data is compared with a detection threshold, initially assumed fixed, and the result of the comparison utilized in the decision rule. For example, the hypotheses may be written as

$$H_0 \text{ (signal absent) : received data = noise}$$

$$H_1 \text{ (signal present): received data = signal + noise}$$

If T^1 is defined as some fixed threshold, the decision rule would be written as

$$\text{If } T \geq T^1 \quad \text{decide signal present (accept } H_1, \text{ reject } H_0)$$

$$\text{If } T < T^1 \quad \text{decide noise alone (accept } H_0, \text{ reject } H_1)$$

where T is some function of the received data.

A detector operating in the manner described above can make basically two types of errors which in the context of the radar or

communication problems can be expressed as:

TYPE I - The detector decides signal is present when in reality it is absent.

TYPE II - The detector decides signal is absent when in reality it is present.

The TYPE I error is commonly referred to as a false alarm, the probability of such an error being designated α and referred to as the Probability of False Alarm. Similarly, the probability of a TYPE II error is referred to as the Probability of False Dismissal. The problem constraints will generally establish limits on the formulation of the final decision rule. The various aspects of selecting decision rules are discussed at length in most texts on the theory of statistics--for example, Chapter 12 of (12). The optimality criteria investigated initially in this paper is that of Neyman-Pearson.

The Neyman-Pearson criterion is based on maintaining a fixed probability of false alarm (α) while maximizing the probability of detection. The probability of detection corresponds to the power of the test or to $1 - (\text{probability of TYPE II error})$. The Neyman-Pearson approach relies on knowledge of the exact nature of the probability functions and furthermore, the selection of the threshold level depends upon a parameter (s) of the distribution functions. Such a detector is generally called parametric in nature. The Neyman-Pearson type detector forms for Gaussian noise with various known parameters and signal characteristics are discussed in (12), and more briefly in (13). However, the likelihood ratio formulation under conditions where the individual observations are not independent or non-Gaussian is very difficult to calculate for the general case and may be even more complicated to implement. For this reason, the likelihood formulation

following the second detector is limited to a processing sequence length of "two", and it is shown that even for such basic constraints the expressions for the colored noise environment are intractable.

A. SUMMARY OF LIKELIHOOD RATIO DEVELOPMENT

The purpose of this section is to develop the likelihood ratio formulation and terminology which will be used throughout the remainder of this work. For this purpose, let $f(t)$ be the received waveform for which the processing is to be designed. Let $f(t)$ be represented as

$$f(t) = m(t) + n_c(t)$$

where $m(t)$ and $n_c(t)$ are defined as follows:

$$m(t) = \text{desired signal for detection}$$

$$n_c(t) = \text{colored noise}$$

In formulating the likelihood ratio, the following definitions are utilized:

$$p_f(m) = \begin{array}{l} \text{a posteriori probability density function of} \\ \text{signal } (m) \text{ being present given that } (f) \text{ has} \\ \text{been received} \end{array}$$

$$p_m(f) = \begin{array}{l} \text{conditional probability density function on} \\ \text{receiving } (f) \text{ given } (m) \text{ is present} \end{array}$$

$$p_o(f) = \begin{array}{l} \text{conditional probability density function of } (f) \\ \text{given that signal is absent} \end{array}$$

The likelihood ratio is defined by (1) as

$$\Lambda = \frac{p_m(f)}{p_o(f)} .$$

The likelihood ratio is used in the decision process. However, the amplitude of the signal may be described as a statistical quantity and not a "sure signal". The likelihood function must then be modified from the simple detection case as follows. Let $p[m(g)]$ = the probability density function of the signal envelope having the value (g) . Then the new likelihood ratio $\hat{\Lambda}$ can be expressed as

$$\hat{\Lambda} = \int p[m(g)] \Lambda dg .$$

Then the derivation of a suitable likelihood ratio can proceed by first developing Λ and modifying to the form of $\hat{\Lambda}$ shown.

In the following development of a likelihood ratio, the input waveform is expressed as a sequence where the elements of the sequence are samples from processes described by the appropriate univariate p.d.f.

The samples are taken at times $t_l = t_1 + (l-1) \Delta t$ where Δt is taken to be the radar interpulse period, T . The length of the sequence shall be $(l = 1, 2, \dots, L)$, and various values of L would be examined depending upon processing techniques evaluated.

Let the received function $f(t)$ be designated f_l and be given by:

$$f_l = m_l + n_l$$

where the n_l are sequence elements taken from a colored noise process. The probability density for the colored noise sample values $n_1, n_2, n_3, \dots, n_L$ is written as:

$$p(n) = p(n_1, n_2, n_3, \dots, n_L)$$

where $p(n)$ is the joint probability density function of dimension equal to the length of the sequence. In a similar manner, the p.d.f. of signal could be formed. For preliminary development, the factor Λ will be formulated for a "sure signal" case and then modified as required to provide the necessary function $\hat{\Lambda}$.

Rewriting

$$\Lambda = \frac{p_m(f)}{p_o(f)}$$

the function $p_o(f)$ can be written directly since $f_1 = n_1$ and $p_o(f)$ is

$$p_o(f) = p_o^n(f_1, f_2, f_3, \dots, f_L)$$

where the superscript n is used to signify the p.d.f. is the form of that for the noise component. The function $p_m(f)$ can likewise be written directly assuming the sure signal form since $n_1 = f_1 - m_1$ and is

$$p_m(f) = p^n(f_1 - m_1, f_2 - m_2, \dots, f_L - m_L)$$

and the desired quantity Λ is written as

$$\Lambda = \frac{p^n(f_1 - m_1, f_2 - m_2, \dots, f_L - m_L)}{p^n(f_1, f_2, f_3, \dots, f_L)}$$

But the signal m_L has parameters known only in the statistical sense. These parameters may be amplitude (g) and phase (ϕ). The desired likelihood ratio is $\Lambda(g, \phi) = p_m(g, \phi) / \Lambda$ and the function $p_m(g, \phi) = p[m(g, \phi)]$ is the p.d.f. that signal will have amplitude in the interval g to $g + dg$ and random phase in the interval ϕ to $\phi + d\phi$.

Rewriting the function $\Lambda(g, \phi)$ where m has the amplitude g and phase ϕ ,

$$\Lambda(g, \phi) = p_m(g_1 \text{---} g_L; \phi_1 \text{---} \phi_L) \Lambda.$$

The desired likelihood ratio $\hat{\Lambda}$ is expressed as

$$\hat{\Lambda} = \int_{g_1} \int_{g_L} \int_{\phi_1} \int_{\phi_L} \frac{p_m(g_1 \text{---} g_L; \phi_1 \text{---} \phi_L) p^n(f_1 - g_1, f_2 - g_2, \text{---} f_L - g_L)}{p^n(f_1, f_2 \text{---} f_L)} \times \\ dg_1 \text{---} dg_L \quad d\phi_1 \text{---} d\phi_L$$

In order to proceed with the representation of a useful likelihood ratio, the joint probability density functions for the colored noise and the desired signal must be derived. The derivation is presented in the following sections.

B. RECEIVED DATA STATISTICS AT THE OUTPUT OF A QUADRATIC DETECTOR

As a preliminary step in the investigation of moving target indication (MTI) in a noncoherent system, the statistics of the received data will be considered for a system as represented in Figure 5. The resulting probability distribution functions will be utilized in attempting to define some processor and detector optimized for the specific output statistics.

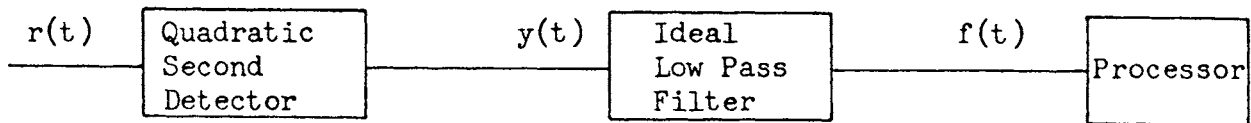


FIGURE 5. SYSTEM DESCRIPTION

In Figure 5 the input or received data is represented by $r(t)$ where

$$r(t) = s(t) + c(t) + n_o(t) \quad (3.1)$$

The term $s(t)$ is the signal whose presence is to be detected, resulting from a point target moving with respect to the background clutter. To be more general in the problem formulation and to more nearly equate to the physical circumstances, the assumption of a point target should be relaxed. This results in a complex target reflection characteristic similar in origin to that postulated for clutter. A complex shape would result in a reflected signal due to superposition of fields from a large number of "specular points" distributed over the surface of the target in a manner which may be assumed random with respect to the illuminating radar. With very minor alterations in the aspect angle of the target relative to the radar location, the reflected signal may undergo wide variation as discussed in (7). This "scintillating target" model would demand the use amplitude fluctuation statistics. However, the point target assumption is retained for this work. Let $s(t)$ be represented as:

$$s(t) = S \cos[2\pi(f_c + f_d) t + \theta] \quad (3.2)$$

where f_c = the center or carrier frequency of the radar system

f_d = target doppler frequency shift

S = amplitude of return calculated from radar equation (target reflectivity is not assumed to be a fluctuating quantity)

θ = random phase term arising from illumination by a noncoherent radar and uniformly distributed over $0 \leq \theta \leq 2\pi$

Because of the origin of clutter echo in the system, it may be considered a Gaussian random process. Furthermore, assuming the clutter originates from a waveform illuminating a uniformly distributed reflecting background and that the spectral width is narrow compared to the center frequency f_c , the random process may be considered a narrow-band random process. It is known that the envelope and phase probability distributions of such a process can be represented as (11):

$$p(V_t, \Phi_t) = \begin{cases} \frac{V_t}{2\pi\sigma_x^2} \exp \left[\frac{-V_t^2}{2\sigma_x^2} \right] & \text{for } V_t \geq 0 \\ & 0 \leq \Phi \leq 2\pi \\ 0 & \text{otherwise} \end{cases} \quad (3.3)$$

$$p(V_t) = \begin{cases} \frac{V_t}{\sigma_x^2} \exp \left[\frac{-V_t^2}{2\sigma_x^2} \right] & \text{for } V_t \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (3.4)$$

where σ_x^2 = variance of the input process and where V_t and Φ_t are represented in polar coordinates according to

$$c(t) = V(t) \cos \left[\omega_c t + \Phi(t) \right] . \quad (3.5)$$

In Equation (3.1), $n_o(t)$ represents the noise component of the input originating from thermal noise considerations. The noise $n_o(t)$ is assumed to be a sample function from a Gaussian random process having a "white" spectral density. The noise will be assumed described by the following univariate probability density function having zero mean and variance σ_n^2 :

$$p(n_o) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp \left[\frac{-n^2}{2\sigma_n^2} \right] \quad (3.6)$$

The variance or mean squared value of $n(t)$ is determined for the system from Nyquist theorem. However, in the initial formulation of this problem the contribution to the received waveform from $n(t)$ is assumed negligible and the received function is represented as

$$r(t) = s(t) + c(t). \quad (3.7)$$

For the initial formulation, a likelihood ratio at the output of the second detector will be sought for optimum processing in the presence of colored noise. The likelihood ratio and resulting decision statistic will then be considered relative to some more common processing devices. The first step in likelihood ratio definition will be to determine probability density functions at the second detector output. From Equation (3.7), the received waveform is $r(t) = s(t) + c(t)$ and at the output of the square law device is

$$y(t) = r^2(t) = s^2(t) + c^2(t) + 2 s(t) c(t). \quad (3.8)$$

The problem being considered in this Thesis is specifically one where the terms of $r(t)$ representing clutter are much stronger than the signal term, $c(t) \gg s(t)$. Therefore, the term $s^2(t)$ in Equation (3.8), since it only adds slightly to clutter components around DC, will be neglected. At the output of the detector (i.e. filter output) the terms will be reidentified in terms of "desired signal" and "colored noise".

The term $c^2(t)$ represents the noise background in which it is necessary to detect the moving target. The $c^2(t)$, after passing through the ideal lowpass filter, essentially represents the spectrum of the narrow-band Gaussian process translated to DC. This spectrum at the filter output is defined as resulting from colored noise process sample function $n_c(t)$:

$$f(t) = [c^2(t)]_{LF} + 2[s(t)c(t)]_{LF}. \quad (3.9)$$

where the subscript LF indicates the low frequency portion of the components following lowpass filtering.

$$n_c(t) = [c^2(t)]_{LF}$$

The term of $f(t)$, represented by $2[s(t)c(t)]_{LF}$, is essentially the input signal modulated by the random process representing $c(t)$ and will result in a new spectrum displaced from DC by the target doppler. The shifted spectrum or component represented by $[s(t)c(t)]_{LF}$ will be defined as the desired signal, $m(t)$, at the square law detector output,

$$m(t) = 2[s(t)c(t)]_{LF}.$$

First, evaluating the clutter components in Equation (3.8) using Equation (3.5)

$$c^2(t) = \frac{v^2(t)}{2} + \frac{v^2(t)}{2} \left(\cos [2 \omega_c t + 2 \Phi(t)] \right).$$

But the output from the filter due to clutter is

$$f_c(t) = \frac{v^2(t)}{2} \quad (f_c = \text{component of } f \text{ due to clutter}) \quad (3.10)$$

where the filter utilized is an ideal filter. The assumption of the ideal filter is considered justified because the spectral width of the low frequency components is much less than the input center frequency. But from Equations (3.3) and (3.4), the probability density function (p.d.f.) at the clutter input is known. Since the filter output is given by Equation (3.9), a transformation of variables yields the detector p.d.f. at the output in the presence of clutter only as follows:

$$p [f_c(t)] = \frac{p(v_t)}{\frac{d(f_c)}{dv_t}}$$

$$p [f_c(t)] = \text{p.d.f. of the clutter component at the filter output}$$

$$p [f_c(t)] = \frac{1}{\sigma_x^2} \exp (-f_c / \sigma_x^2), f_c \geq 0 \quad (3.11)$$

Next, evaluate the desired signal component in $f(t)$. From Equations (3.2) and (3.5)

$$m(t) = 2 \left\{ SV(t) \left[\cos \left(\omega_c t + \theta + \Phi(t) \right) \right] \left[\cos \left((\omega_c + \omega_d)t + \theta \right) \right] \right\}_{LF}$$

The random phase θ is identical in the cross product inasmuch as both signal and clutter are within an interval illuminated by a given pulse. The desired signal $m(t)$ then reduces to

$$m(t) = SV(t) \cos \left[\omega_d t + \Phi(t) \right]$$

where only the low frequency portion has been retained and the terms $V(t)$ and $\Phi(t)$ are the random variables of amplitude and phase of the narrow-band Gaussian noise process which represents the colored noise (clutter). Letting $g = SV(t)$ then $m(t) = g \cos \left[\omega_d t + \Phi(t) \right]$.

The probability density function of the desired signal must be found. It is known for the input narrow-band Gaussian process that the amplitude and phase are independent random variables (11). The square-law device input amplitude p.d.f. has been expressed by Equation (3.4) to be

$$p(V_t) = \begin{cases} \frac{V_t}{\sigma_x^2} \exp \left[-\frac{V_t^2}{2\sigma_x^2} \right] & \text{for } V_t \geq 0. \\ 0 & \text{otherwise} . \end{cases}$$

By a simple transformation of variable using $g = SV_t$ as noted above, the p.d.f. can be expressed at the output of the square-law second detector to be

$$p(g) = \begin{cases} \frac{g}{\sigma_x^2 S^2} \exp \left[-\frac{g^2}{2S^2 \sigma_x^2} \right] & \text{for } g \geq 0. \\ 0 & \text{otherwise} . \end{cases} \quad (3.12)$$

The p.d.f. of the phase term $\Phi(t)$ can be expressed as the following

$$p(\Phi_t) = \begin{cases} \frac{1}{2\pi} & \text{if } 0 \leq \Phi_t \leq 2\pi \\ 0 & \text{otherwise} \end{cases} \quad (3.13)$$

In this section, the received data has been considered and defined in terms of the input and output of an ideal square-law detector. The univariate p.d.f.'s for the desired signal and noise at the detector input and output were defined. In the following section, the joint probability density functions will be formulated and used in conjunction with the likelihood ratio equations of Section A to attempt definition of the optimum processor. Due to the complexity of the higher order joint p.d.f., the processing sequence length will be limited to two pulses.

C. LIKELIHOOD RATIO BASED ON SEQUENCE LENGTH OF TWO

Let it be assumed that the sequence length available for processing at the square-law detector output is $L = 2$. Therefore, only the second-order joint probability density functions are required for the quantities $p_m(g, \Phi)$, $p_o(f)$, and $p_m(f)$ where the functions are as described in Section A.

Evaluate first the joint probability density function for $p_m(g, \Phi)$. The quantity $p_m(g)$ was expressed by Equation (3.12). Therefore,

$$p_m(g, \Phi) = \frac{g}{2\pi\sigma_x^2 S^2} \exp \left[\frac{-g^2}{2\sigma_x^2 S^2} \right]$$

where $\sigma = \sigma_x^2 S^2$ and the quantity S is determined from the radar system parameters and the target range. The second order joint density function of the envelope can be written as in (1), where

$$p_m(g_1, g_2) = \frac{g_1 g_2}{2\sigma^4(1-r^2)} \exp\left[\frac{-g_1^2 - g_2^2}{\sigma^2 2(1-r^2)}\right] I_0\left[\frac{r g_1 g_2}{\sigma^2(1-r^2)}\right] \quad (3.14)$$

where r is the normalized correlation coefficient and I_0 is the zero order modified Bessel function of the first kind.

Next, the joint density function ($L = 2$) for the quantity $p_o(f)$ will be completed and the variables transformed to provide $p_m(f)$. But $p_o(f)$ has been identified as equal to $p(n)$ or the probability density function of the colored noise (clutter) at the second detector output. The quantity $p(n)$ has been written in Equation (3.6). Examine the general joint probability function. As discussed herein, the clutter at the input to the second detector can be represented as a narrow-band Gaussian random process. The general form of the multivariate Gaussian distribution, (11), is

$$p(c_1, c_2, \dots, c_n) = \frac{1}{(2\pi)^{n/2} |\underline{R}|^{1/2}} \exp\left[-\frac{1}{2} \sum \sum Q_{nm}(c_n, c_m)\right]$$

where \underline{Q} is the inverse of the covariance matrix \underline{R} and $|\underline{R}|$ is the determinant of the matrix \underline{R} . But from Equation (3.5), we can write c_n as: $V(t_n) [\cos \{\omega_c t_n + \Phi(t_n)\}]$ and since the square law detector output variable is expressed as

$$n_c(t) = \frac{V(t)^2}{2}$$

a transformation of variables will be attempted to represent the output joint probability density function as:

$$p[n_c(t)] = \frac{p(V_t)}{dn_c(t)/dV_t}.$$

It is known that the density function of the envelope at the input has been derived as being (11)

$$p(V_t) = \frac{V_t}{\sigma_x^2} \exp \left[-\frac{V_t^2}{2\sigma_x^2} \right] \quad (0 \leq V_t \leq \infty) \quad (3.15)$$

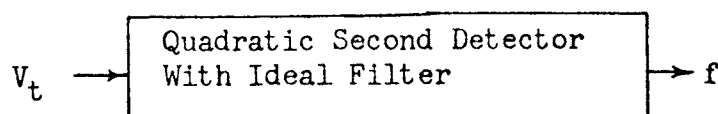
$$p(V_t) = 0 \quad (V_t < 0)$$

The joint density functions can therefore be expressed as follows:

$p_I(V_1, V_2, \dots, V_n)$ = Joint probability density function at the quadratic device input.

$p_f(f_1, f_2, \dots, f_n)$ = Joint probability density function at the filter output.

The system thus described is symbolized as



The device output can be expressed as $f = 1/2 V_t^2$ and where f and V_t are related by a one-to-one mapping due to (3.15) above; i.e., $f = 1/2 V_t^2$ and $V_{tn} = + \sqrt{2 f_n}$ where the negative root is not allowed under the requirements of Equation (3.15) above. For this reason, each point in the input variable space corresponds to one, and only one, point in the output variable space. Then the functional relation between input and output variable can be expressed as:

$$f_n = (1/2) V_{tn}^2$$

$$V_{tn} = + \sqrt{2 f_n}$$

but

$$\int_{\text{Input}} \dots \int p_I(V_{t1}, \dots, V_{tn}) dV_{t1} \dots dV_{tn}$$

$$= \int_{\text{Output}} \dots \int p_f(f_1, \dots, f_n) df_1 \dots df_n$$

where J is the Jacobian of the transformation. In this problem, the Jacobian is expressed as a diagonal matrix with factors $\sqrt{2f_n}^{-1}$ as elements, and therefore $|J| = (\text{determinate of } J)$,

$$J = \prod_{n=1}^n (2f_n)^{-1/2}.$$

Rewriting the joint density transformed to the filter output

$$\int \cdots \int p_I (v_{t1} \cdots v_{tn}) dv_{t1} \cdots dv_{tn} = \int \cdots \int p_I (\sqrt{2f_1} \cdots \sqrt{2f_n}) |J| df_1 \cdots df_n \quad (3.16)$$

where from (1) the bivariate joint probability density function is expressed as

$$p (V_1, V_2) = \frac{V_1 V_2}{\sigma_x^4 (1-p^2)} \exp \left[- \frac{V_1^2 + V_2^2}{\sigma_x^2 2(1-p^2)} \right] I_0 \left[\frac{p V_1 V_2}{\sigma_x^2 (1-p^2)} \right] \quad (3.17)$$

where p is the correlation coefficient and the integration has been carried out over the random phase. Then, using Equation (3.17) and substituting from Equation (3.16)

$$p(f_1, f_2) = \frac{1}{2 \sigma_x^4 (1-p^2)} \exp \left[- \frac{f_1 + f_2}{\sigma_x^2 2(1-p^2)} \right] I_0 \left[\frac{p \sqrt{f_1 f_2}}{\sigma_x^2 (1-p^2)} \right] . \quad (3.18)$$

Re-identifying in terms of the noise output

$$p(n_1, n_2) = \frac{1}{2 \sigma_x^4 (1-p^2)} \exp \left[- \frac{n_1 + n_2}{2 \sigma_x^2 (1-p^2)} \right] I_0 \left[\frac{p \sqrt{n_1 n_2}}{\sigma_x^2 (1-p^2)} \right] \quad (3.19)$$

where $p (n_1, n_2)$ = the bivariate joint p.d.f. of the colored noise at the filter output.

In summary, $p(n)$ is expressed as shown in Equation (3.19); but, $p_o(f)$ is equivalent to $p(n)$. Therefore, in deriving the likelihood ratio, Equations from Section A can be used directly for the sequence length of two to express the interim ratio as

$$\Lambda = \frac{p_m^n(f)}{p_o^n(f)} = \exp \left[\frac{m_1 + m_2}{2 \sigma_x^2 (1-p^2)} \right] \frac{I_0 \left[\frac{p \sqrt{(f_1 - m_1)(f_2 - m_2)}}{\sigma_x^2 (1-p^2)} \right]}{I_0 \left[\frac{p \sqrt{f_1 f_2}}{\sigma_x^2 (1-p^2)} \right]} \quad (3.20)$$

But from Section A, the expression for Λ is a function of the random variable $\Phi(t)$ and time where

$$m_1 = SV_1 \cos \left[\omega_d t_1 + \Phi_1 \right]$$

$$m_2 = SV_2 \cos \left[\omega_d t_2 + \Phi_2 \right] .$$

For the problem being considered, the sampling interval $t_2 - t_1$ is fixed. The ratio Λ must be integrated over the phase random variable inasmuch as the phase is not useful in the noncoherent detection problem.

Rewriting the likelihood ratio

$$\Lambda(g, \Phi) = \exp \left\{ \frac{g_1 \cos \left[\omega_d t_1 + \Phi_1 \right] + g_2 \cos \left[\omega_d t_2 + \Phi_2 \right]}{2 \sigma_x^2 (1-p^2)} \right\} \times$$

$$\begin{aligned}
 & I_o \left[\frac{p \sqrt{f_1 - g_1} \cos \left[\omega_d t_1 + \Phi_1 \right] \sqrt{f_2 - g_2} \cos \left[\omega_d t_2 + \Phi_2 \right]}{\sigma_x^2 (1-p^2)} \right] \\
 & \qquad \qquad \qquad (3.21) \\
 & I_o \left[\frac{p \sqrt{f_1} \sqrt{f_2}}{\sigma_x^2 (1-p^2)} \right]
 \end{aligned}$$

Equation (3.21) could not be integrated to provide a closed form solution. One remark is worthy of note. The complication of the random phase component is generally avoided in the problem when formulated in the frequency domain. The random phase component is usually ignored by arguing that its impact is a spreading of the clutter spectrum which can result in no more than an error of 1/2 in a filter signal to noise ratio at the output. An alternate formulation will now be undertaken to determine the characteristics of the optimum receiver under more general conditions than those utilized in this Chapter.

CHAPTER IV

FORMULATION OF THE LIKELIHOOD RATIO PRIOR TO "SECOND DETECTION"

The preceeding Chapter demonstrates the mathematical complexity in dealing with even low order statistics after nonlinear detection and did not yield a satisfactory formulation of a test statistic on which to base a processor design. In this Chapter, a specific demodulation technique will not be assumed, and the investigation of optimum processing will be conducted in more general terms. In order to accomplish this goal, the noncoherent system characteristics must be reformulated.

In selecting a model for the problem, several factors must be considered. In the noncoherent system, there is no correlation between successive pulses due to the random starting phase of the carrier within each pulse. However, it is known that the amplitudes of successive pulses are statistically related. As suggested earlier, the clutter noise process can be assumed to be a narrow-band Gaussian random process. This model is also used by Van Trees, (18), and Helstrom, (19). It has further been shown (1) that by representing the Gaussian process in terms of complex random variables with suitably defined covariance functions, the expected statistical relations between envelope and phase of the narrow-band process can be derived. The general model for the received data will now be derived in terms of complex random variables.

A. PROCESS FORMULATION IN COMPLEX VARIABLES

Let the narrow-band process be represented by the following set of equations as proposed by (18) where $\text{Re} [.]$ indicates the "real part of":

$$n(t) = \text{Re} \left[\tilde{n}(t) \exp \left[j \omega_c t \right] \right] \quad (4.0)$$

$$\tilde{n}(t) = |\tilde{n}(t)| \exp \left[j \phi_n(t) \right] \quad (4.1)$$

where ω_c is the carrier radian frequency, and the symbol \sim indicates a complex function. If the covariance function is used as defined by Helstrom (19), and Van Trees (18), the following relations result

$$\tilde{K}(t,u) = E \left[\tilde{n}(t) \tilde{n}^*(u) \right] \quad (4.2)$$

and

$$E \left[\tilde{n}(t) \tilde{n}(u) \right] = 0 \quad (4.3)$$

where $E[\star]$ is the expected value of the quantity in brackets and $*$ indicates the conjugate. The complex covariance function $\tilde{K}(t,u)$ has the desired properties for the model. For completeness, if $\tilde{n}(t) = X(t_1) + jY(t_1)$, the relation between the quadrature components is expressed as

$$\begin{aligned} E \left[X(t_1)X(t_2) \right] &= E \left[Y(t_1)Y(t_2) \right] = \text{Re} \left\{ \tilde{K}(t_1, t_2) \right\} \\ E \left[Y(t_1)X(t_2) \right] &= -E \left[X(t_1)Y(t_2) \right] = \text{Im} \left\{ \tilde{K}(t_1, t_2) \right\} \end{aligned} \quad (4.4)$$

The multivariate Gaussian density function for the process can then be represented by

$$p \left[\tilde{n}(t_1), \tilde{n}^*(t_1), \dots, \tilde{n}(t_N), \tilde{n}^*(t_N) \right] = \frac{1}{(4\pi)^N \det \underline{\tilde{K}}(\tau)} \exp \left\{ -\frac{1}{2} \sum_{ij} \tilde{q}_{ij} \tilde{n}_i \tilde{n}_j^* \right\}$$

where $\det \underline{\tilde{K}}(\tau)$ is the determinant of the covariance matrix $\underline{\tilde{K}}$ for a

stationary process and the \tilde{q}_{ij} are elements of the matrix \tilde{Q} which is the inverse of $\tilde{K}(\tau)$. The density function may also be expressed in vector-matrix form as

$$p \left[\tilde{n}(t_1), \tilde{n}^*(t_1) \dots \tilde{n}(t_N), \tilde{n}^*(t_N) \right] = \frac{1}{(4\pi)^N \det \tilde{K}(\tau)} \exp \left\{ -\frac{1}{2} \underline{\tilde{n}}^* \underline{\tilde{Q}} \underline{\tilde{n}} \right\} \quad (4.5)$$

where the lower case letters with underbar indicate a column vector, and the * superscript by a vector or matrix indicates conjugate transpose.

Representing the problem by $f(t)$ as received data, $s(t)$ as desired signal, and noise as $n(t)$, the basic system hypotheses can be written

Hypothesis H_1 corresponds to $f(t) = s(t) + n(t)$

Hypothesis H_0 corresponds to $f(t) = n(t)$

In order to write the likelihood ratio, the received data and desired signal must be represented as narrow-band signals by

$$\begin{aligned} f(t) &= R_e \left[\tilde{f}(t) \exp(j\omega_c t) \right] \\ s(t) &= R_e \left[\tilde{s}(t) \exp(j\omega_c t) \right] \end{aligned} \quad (4.6)$$

where

$$\tilde{s}(t_i) = |\tilde{s}(t_i)| \exp(j\theta_i) \quad (4.7)$$

and the θ_i represents the random phase for each signal pulse.

B. LIKELIHOOD RATIO REPRESENTATION BY TIME SAMPLES

In this investigation of the noncoherent detection problem, the received data will be treated as a sequence shown in Figure 6. As discussed in Chapter II, the nonstationarity of the process can be overcome, for simplifying the mathematics, by assuming the noise process to be continued throughout the interpulse period. However, in the problem represented in Figure 6, care must be taken in receiver formulation to avoid a system which relies on the conceptual artifice mentioned above. To guard against this occurrence, the p.d.f. for the likelihood ratio will now be formulated in terms of sequence elements. Using previously defined terminology for the likelihood ratio (Section A of Chapter III) and continuing use of the complex variables, Λ may be written as

$$\Lambda(\theta_1 \text{-----} \theta_N) = \frac{\exp \left\{ -\frac{1}{2} [F^* - S^*] Q [F - S] \right\}}{\exp \left\{ -\frac{1}{2} [F^* Q F] \right\}} \quad (4.8)$$

The capital letters are taken to indicate column vectors or matrices as defined below

$$F = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_N \end{bmatrix} \quad S = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_N \end{bmatrix} \quad Q = \begin{bmatrix} q_{11} & q_{12} \text{-----} q_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ q_{N1} \text{-----} & \text{-----} & q_{NN} \end{bmatrix} \quad (4.9)$$

where F is the received data made up of samples $(f_1 \text{-----} f_N)$, S is the

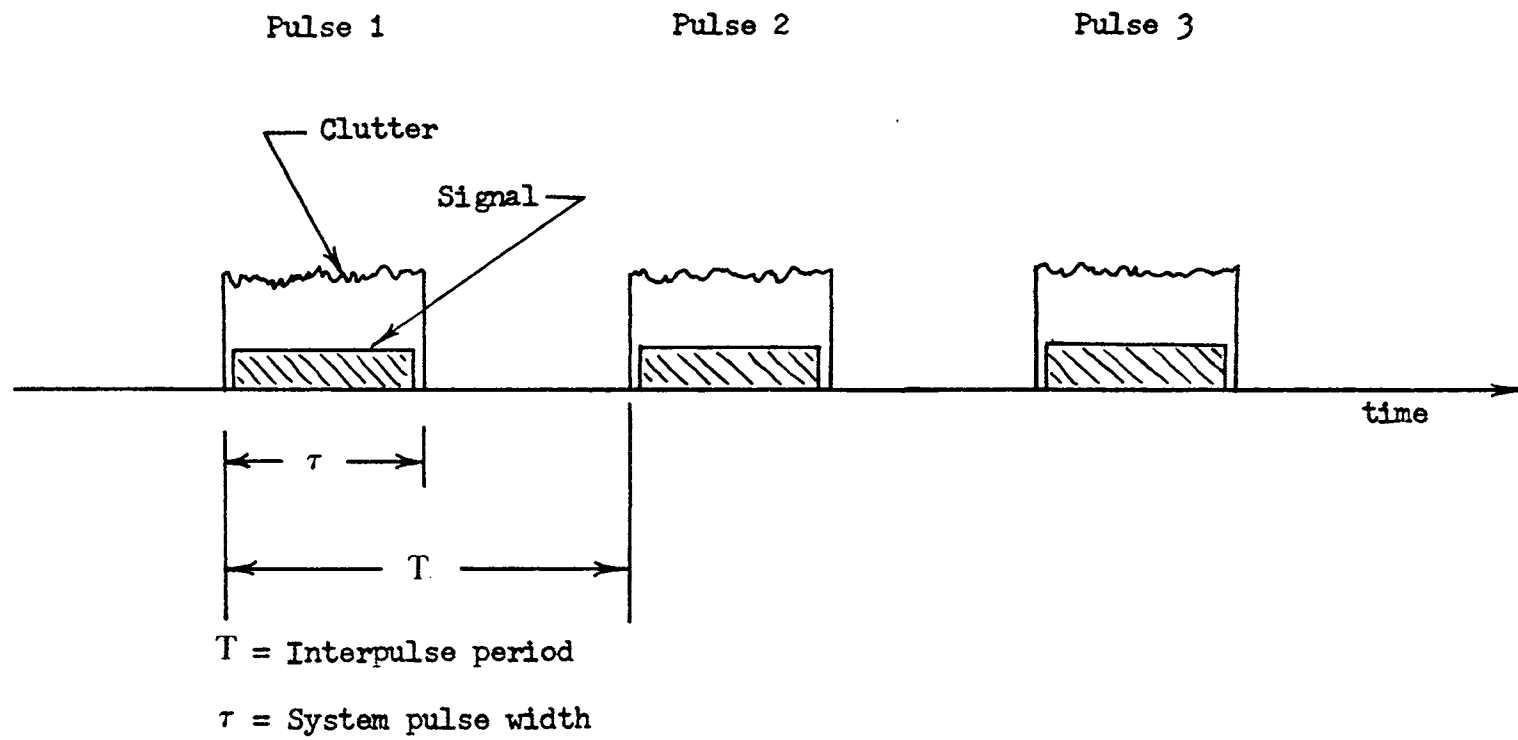


Figure 6 Clutter and signal pulse sequence

desired signal made up of sample (s_1 ----- s_N) occurring at times

$$t_n = (t_0 + nT) \quad (n = 0, 1, 2, \text{-----} N-1) \quad , \quad (4.10)$$

a sample being taken at the peak of the envelope of each pulse. The matrix Q is the inverse of the covariance matrix K . The covariance matrix K has elements defined on the basis of Equations (4.2), (4.3), and (4.4), except we shall only be interested in covariance coefficients from pulses T seconds apart in time. Then, recalling that each element of the signal vector, S , is made up of terms of the form

$$s'_i = |s'_i| \exp(j \omega_d t_n) \exp(j \theta_i) = s_i \exp(j \theta_i) \quad (4.11)$$

where θ is the random phase angle uniformly distributed in accordance with Equation (3.13), it can be seen that likelihood ratio of Equation (4.8) will be difficult to integrate unless the exponential in θ_i can be reduced to a product of the form $\prod_{i=1}^N \exp h(\theta_i)$. If the noise process $n(t)$ were white noise, the Q would be a diagonal matrix and the integration could be carried out directly. Since the noise is assumed colored in this problem due to its physical origin, it will have a covariance matrix $K(\tau)$ which is positive definite, Hermitian. Since the covariance matrix is Hermitian, it can be diagonalized. This step is undertaken next.

The numerator of the right hand term of Equation (4.8) can be written as

$$\exp \left\{ -\frac{1}{2} \left[F^* Q F - 2 R_{\theta} (F^* Q S) + S^* Q S \right] \right\} \quad (4.12)$$

Techniques of matrix algebra such as found in (22) and (23) are utilized to develop the following unitary transformations

$$\left. \begin{aligned} F &= UY \\ Y &= U^1 F \end{aligned} \quad \begin{aligned} S &= UZ \\ Z &= U^1 S \end{aligned} \right\} \quad (4.13)$$

where U is the unitary transformation derived from the characteristics of the matrix Q. Substituting Equations (4.13) into the bracketed term of Equation (4.12)

$$\begin{aligned} F^* Q F - 2R_e(F^* Q S) + S^* Q S &= Y^* U^* Q U Y - 2R_e[Y^* U^* Q U S] + Z^* U^* Q U Z \\ F^* Q F - 2R_e(F^* Q S) + S^* Q S &= Y^* D Y - 2R_e[Z^* D Y] + Z^* D Z \end{aligned} \quad (4.14)$$

where D is a diagonal matrix whose elements are the eigenvalues of Q and represented by $(\mu_1 \mu_2 \dots \mu_n)$. The eigenvalues are real and positive due to the positive definite Hermitian nature of K and hence Q. Noting that $F^* Q F = Y^* D Y$, and substituting Equation (4.14) in Equations (4.12) and (4.8), the likelihood ratio reduces to

$$\Lambda(\theta_1, \theta_2, \dots, \theta_n) = \exp \left\{ -\frac{1}{2} \left[Z^* D Z - 2R_e Z^* D Y \right] \right\} \quad (4.15)$$

The term $Z^* D Z$ related to the desired signal is Hermitian in form and may be written $Z^* D Z = \sum \mu_i |z_i|^2$ where z_i represents the elements of the vector defined by Equation (4.13).

The diagonalization of the complex inverse covariance matrix can be accomplished by the unitary transformation due to the positive definite Hermitian nature of Q suggested earlier. The columns of the matrix are the normalized eigenvectors of the inverse covariance matrix.

The matrix U is represented

$$U \triangleq \begin{bmatrix} \phi_{11} & \cdots & \phi_{1N} \\ \vdots & & \vdots \\ \phi_{N1} & \cdots & \phi_{NN} \end{bmatrix} = \begin{bmatrix} \Phi^1 & \Phi^2 & \cdots & \Phi^N \end{bmatrix} \quad (4.16)$$

where the Φ^n are the normalized eigenvectors with the superscript indicating column position in the matrix U . Since U is unitary, the inverse $\{U^{-1}\}$ is simply the conjugate transpose of U or

$$U^{-1} = U^* \triangleq \begin{bmatrix} \phi_{11}^* & \cdots & \phi_{N1}^* \\ \vdots & & \vdots \\ \phi_{1N}^* & \cdots & \phi_{NN}^* \end{bmatrix} = \begin{bmatrix} \Phi^{1*} \\ \vdots \\ \Phi^{N*} \end{bmatrix} \quad (4.17)$$

The elements of a given eigenvector Φ^i are written as ϕ_{mi} where, as before, the upper case Phi represents a vector quantity while lower case Phi represents particular elements of that vector. It should be noted that the elements of the matrix U^* have retained the subscript order of the original eigenvector definition rather than row/column ordering of the U^* matrix location in order that the eigenvectors may be more easily identified as the work progresses.

It should be noted that since U diagonalizes the matrix Q , it also diagonalizes the covariance matrix K . It can be shown to be true by premultiplying $U^*QU = D$ by U , then K , post-multiplying by D^{-1} , pre-multiplying by U^* to yield

$$U^* KU = D^{-1} .$$

The elements of the inverse of the diagonal matrix are $\lambda_i = 1/\mu_i$ which is consistent with the fact that the eigenvalues of a square matrix are the inverse of the eigenvalues of the inverse matrix.

Returning now to the evaluation of the likelihood ratio, in order to separate the various θ_i , the second term in the exponent of Equation (4.15) is rewritten

$$-2R_e Z^* D Y = -2R_e \sum_{i=1} \sum_{k=1} \mu_i y_i \phi_{ki} s_k^*$$

where the term $\sum_k \phi_{ki} s_k^*$ is equal to the quantity z_i^* or the i^{th} component of the Z vector. Similarly, y_i is the i^{th} component of the Y vector. The desired form is obtained by interchanging the order of summation

$$-2R_e Z^* D Y = -2R_e \sum_k \sum_i \mu_i y_i \phi_{ki} s_k^*$$

and letting

$$M_k \exp(j \gamma) = \sum_i \mu_i y_i \phi_{ki} s_k^*$$

$$M_k \exp(j(\gamma_k + \theta_k)) = \sum_i \mu_i y_i \phi_{ki} r_k^* \exp(-j \theta_k)$$

where the relation $s_k = r_k \exp(j \theta_k)$ has been used. It should be noted the doppler shift has been left in association with the term r_k . Then

$$M_k = \left| \sum_{i=1}^N \mu_i y_i \phi_{ki} r_k^* \right| \quad (4.18)$$

and Equation (4.15) may be rewritten as

$$\Lambda(\theta_1, \dots, \theta_N) = \exp \left\{ -\frac{1}{2} \sum_i \mu_i |z_i|^2 \right\} \exp \left\{ \sum_k M_k \cos(\gamma_k + \theta_k) \right\}$$

Integrating over the random variables

$$\Lambda = \exp \left\{ \sum_i -\frac{1}{2} \mu_i |z_i|^2 \right\} \int_{\theta_1} \dots \int_{\theta_N} \frac{1}{2\pi} \exp \left\{ \sum_k M_k \cos[\gamma_k + \theta_k] \right\} d\theta_1 \dots d\theta_N \quad (4.19)$$

$$\Lambda = \exp \left\{ -\frac{1}{2} \sum_i \mu_i |z_i|^2 \right\} \prod_{k=1}^N I_0[M_k] \quad (4.20)$$

In the integration, it has been assumed that the term $|z_i|^2$ is dominated by the signal magnitude squared elements. Similar assumptions are made in the derivations in both (1) and (24) concerning the random phase in the desired signal terms. That it is an acceptable approximation is seen by noting that the remaining terms of $|z_i|^2$, other than signal magnitude squared, are of the form $s_m s_n \exp[j(\theta_m - \theta_n)]$. Based on the assumed uniform distribution of the phase elements, the expected value of such terms approaches zero. It is, therefore, considered acceptable to assume the contribution of such terms to the value of $|z_i|^2$ to be negligible. Using the assumption, and taking the logarithm of both sides of Equation (4.20), the final expression is

$$\ln \Lambda = \left\{ -\frac{1}{2} \sum_{i=1}^N \mu_i |z_i|^2 \right\} + \sum_{k=1}^N \ln I_0[M_k] \quad (4.21)$$

The receiver must then form M_k and, as in Appendix A, pass the output of the processor into a $\ln I_0[\star]$ detector. The outputs of the detector are then summed over the index k of the desired signals, assuming

that over the input sequence the signal represents a target having a given doppler shift. If the doppler shift is an unknown parameter, for example, uniformly distributed over ω_d to $\omega_d + \Delta\omega_d$, then Equation (4.18) would be rewritten as

$$\Lambda(\omega_d) = \exp \left\{ -\frac{1}{2} \sum_i \mu_i |z_i|^2 \right\} \prod_{k=1}^N \langle I_o[M_k] \rangle_{\omega_d} \quad (4.22)$$

where the $\langle \star \rangle_{\omega_d}$ is taken to indicate the statistical average over the parameter ω_d . However, the average would again result in a form similar to that encountered in Chapter III herein, and no obvious closed form solution exists. To avoid this difficulty, it will be assumed that ω_d is a known parameter. Then, rewriting Equation (4.21), and using the definitions of Equation (4.17), the likelihood ratio becomes

$$\ln \Lambda = B + \sum_{k=1}^N \ln I_o \left\{ \left| \sum_i \mu_i \left[\left(\sum_{m=1}^N \phi_{mi}^* f_m \right) \left(\phi_{ki} r_k^* \right) \right] \right| \right\} \quad (4.23)$$

where the index m is over the input received data sequence and the k designates the desired signal elements at the sequence times denoted by Equation (4.10) where r_k is the signal at time t_k . The term B is the bias term arising from the likelihood ratio development.

Equation (4.23) then describes the structure for a threshold receiver operating on discrete samples of the received data in a noncoherent system. Considering the argument of $\ln I_o[\star]$, the receiver must perform the operation defined by

$$\left| \sum \left[\mu_i \left(\sum_{m=1}^N \phi_{mi}^* f_m \right) \left(\phi_{ki} r_k^* \right) \right] \right| \quad (4.24)$$

This can be interpreted in several general forms, depending upon the order in which the elements of the statistic are viewed. As written above, the receiver is of a form which may be considered the discrete-time analog to "whitening" of the received data and correlation with a "whitened" replica of the desired signal. Similarly, if in Equation (4.24), the signal term r_k is moved outside the summation over the index i , the receiver represents a discrete filter operating solely on the received data, the output of which is "correlated" with the desired signal prior to envelope detection.

C. INVESTIGATION OF THE SUBOPTIMUM RECEIVER

These receiver operations are carried out at the system carrier frequency or prior to second detection. The operations are discrete or digital in nature requiring substantial mechanization and difficulty in visualization. Generally, a continuous system is often simpler than the discrete counterpart, and for that reason the discrete formulation will be investigated under the limiting assumptions of continuous received data. A formal discussion concerning passage from the discrete sample case to the continuous data representation can be found in (24).¹ However, a more heuristic argument suffices here to justify the investigation of the continuous data representation. It can be shown that a pulse radar system can be represented by a continuous wave (CW) illumination system so long as it is recalled that the representation is accurate only over the frequency domain limits between $-PRF/2$ and $+PRF/2$ where PRF , as before, is the pulse repetition frequency.

Appendix A of this Thesis assumes the clutter or colored noise process to be extended beyond the limits of the signal in the observation

¹ Pages 822-824

interval and formulates a receiver using the Karhunen-Loève expansion. Assumptions are made which allow the definition of an optimum receiver using Fourier Transform techniques. Equations (A-17) and (A-18) represent a simple interpretation of such a receiver and correspond with the Optimum Receiver derived in (18) under the restrictions of a given range point target with some doppler shift in a reverberation background. It can be noted that the discrete formulation shown in Equation (4.24) requires the received data sequence be multiplied by the weighting function representing the eigenvectors of the covariance matrix. This is analogous to the weighting of the received data by the eigenfunctions to provide the expansion coefficients of Equation (A-4) in the continuous representation of Appendix A.

At this point, several avenues of investigation are open. The performance of the receiver could be evaluated utilizing the quantity d_0^2 defined by Helstrom (19)¹ and utilized by Van Trees (5). Similarly, Equation (4.22) could be expanded in some appropriate series form, the averages considered in the manner of (24)², and the performance of the resulting system evaluated. However, the receiver of Equations (A-15) to (A-18) offers an interesting interpretation in comparison with the video domain systems of Chapter II herein. Therefore, since the optimum receiver represents a complex mechanization problem, the processor of Appendix A will be modified to the suboptimum form of Figure 7, applied directly to the video section of the radar and compared with processors shown in Figure 4.

Another factor which causes complication of the IF processing described in the derivation of Sections A and B is a characteristic not

¹ Pages 149-156

² Pages 845-848

specifically defined to this point. It was mentioned that the spectral spread of the clutter return is a function of the radar system azimuth scan angle and, for similar reasons, the center of the clutter spectrum shifts with the antenna pointing angle. The resulting impact on the IF processing is to require that any equivalent filtering operations be mechanized to track the center doppler frequency of system antenna bore-sight angle. This, of course, could be accomplished by utilizing an analog signal from the antenna scan loop.

A comparison of the processors in the video frequency region can be easily implemented by using a computer simulation of the radar system and allowing the processors of Figure 4, and that of Figure 7, to operate on the simulated outputs. Several simulation techniques are available, one of the most useful of which is the time domain simulation presented in (25). For this Thesis, however, a frequency domain simulation represented less computer time and was selected for use. The author worked with Mr. R. P. Brueggemann in the formulation of a frequency domain simulation which has been used in other study programs. The simulation was mechanized to provide the signal to clutter plus noise power ratio at the processor output, and to compute cumulative probability of false alarm based on human observation of a radar display. But rather than select the decision criteria associated with the arbitrary probability of false alarm, the outputs of the processors were compared directly. A block diagram and brief discussion of the simulation are presented in Appendix B of this Thesis.

In utilizing the receiver of Figure A-1, it must be formulated at video frequencies. Since the simulation program developed the clutter spectrum at the output of a square-law second detector, the inverse of

that spectrum was used directly to represent the first filter element. The factors of physical or practical realizability of such a device were not studied. The element of Figure A-1, representing the matched filter based on desired signal, must be interpreted in terms of the desired signal in the video frequency region. In Section B of Chapter III, the desired signal was identified as the intermodulation terms between the clutter and signal. It is to this intermodulation term that the filter must be matched.

However, as discussed in Appendix B, the signal matched filter element was omitted for direct comparison as a clutter rejection device. The curves shown in Appendix B, Figure B-2 and Figure B-3, are the result of the simulation depicting the ratio of signal power plus clutter power plus noise power to clutter power plus noise power at the output of the individual device versus the antenna scan angle in degrees. It should be noted that the antenna scan angle is analogous to the clutter spectral spread by virtue of the problem as discussed in Chapter II herein.

Furthermore, the simulation was run for a specific radar system antenna beamwidth. To select a much more narrow beamwidth, or more broad beamwidth, could be expected not only to alter the curve shape but also the relative position. However, it appears from the definition of the suboptimum filter that the shape of a video domain device should take into account the spectral shape rather than simply selecting the steepest slope response. Additional conclusions are drawn in the following chapter.



Figure 7 Suboptimum receiver implementation

Note: Notation of this Figure defined in Appendix A.

CHAPTER V

CONCLUSION

In this Thesis, the problem of detecting a moving target in the presence of heavy ground clutter, represented as colored noise, was investigated for a noncoherent radar system. A survey of the techniques presently utilized, based on empirical results in the video frequency region of radar systems, was presented for reference. The problem of formulating an optimum receiver, both in the video frequency region after square-law detection and in the general case of operating on the received data, was considered. In the former system, a closed form solution for optimum video processing was not available; however, for the system operating at the "carrier" frequency, a general implementation was defined. In both cases, the optimum system was based on the Neyman-Pearson likelihood ratio criterion.

The optimum receiver described in Chapter IV in terms of discrete time notation appears to be the most practical system to implement as opposed to the continuous filter approach, within the constraints of a pulsed radar system. Also, as suggested in Chapter IV, the performance of the discrete time formulation can best be evaluated utilizing a computer simulation of the general type discussed in (25). The practical system implementation at the system IF represents a significant increase in hardware complexity over the processing techniques implemented at video frequencies, and for this reason the optimum form was evaluated in a suboptimum application by employing the continuous filter shaping of Appendix A to the video section.

The comparison of the suboptimum receiver with the range gate and filter (RGF), and the shaped double delay canceller (SDDC), shows the

suboptimum receiver under some conditions to provide generally better performance. The filter shaping, based on the inverse of the clutter spectrum, appears to be a sound approach in the selection of a video frequency processor. Additional study and comparative evaluation is required over a more broad range of target velocities. Similarly, the evaluation should be extended to the angles near ground track and also to include comparative data on probability of false alarm. It was not possible to show a clear advantage for one device over the other in the application simulated. However, the suboptimum filter did demonstrate an essentially constant clutter plus noise power at the device output more than did either the RGF or SDDC over the range of clutter spread characteristics considered. This factor indicates a more nearly constant false alarm rate without the use of an adaptive threshold. However, generalizations concerning the superiority of the RGF or SDDC are meaningless without a careful examination of radar parameters, such as beamwidth and scan rate, and application, such as aircraft speeds and altitudes.

Several areas for future study are apparent in the derivations of Chapter IV and are worthy of mention in this concluding Chapter:

a.) The basic performance of the receiver of Chapter IV, Equation (4.23), in terms of probability of false alarm and of detection, should be studied. The most profitable evaluation would be by computer simulation due to the lack of a proven analytical model for clutter.

b.) The receiver of Chapter IV assumed a given signal amplitude and should be reconsidered in light of some statistical amplitude characteristics representing the expected target scintillation function.

Similarly, the implementation should be reviewed by averaging over the expected range of target doppler frequencies of interest.

c.) Efforts should be devoted toward definition of adequate clutter models along the lines being pursued by Van Trees with comparisons to results of simulation models such as (25).

APPENDIX A

INVESTIGATION OF NONCOHERENT DETECTION

ASSUMING NOISE IS AVAILABLE BEYOND TIME EXTENT OF TARGET

In Chapter IV of this Thesis, a discrete formulation of the likelihood ratio was utilized in developing the optimum receiver. Where continuous data is assumed relative to the noise sample, a more effective representation is available and is discussed in this Appendix with application to the noncoherent radar problem. In this Appendix, the symbol \sim is used to indicate a complex quantity.

The representation of the random process desired is one in which the representation utilizes an orthonormal set of coordinates having coefficients which are statistically independent. The Cardinal Series or Shannon Sampling Theorem expansion would provide a set of orthogonal coordinates; however, in this system problem involving colored noise and an availability of samples constrained by system parameters, the coefficients of the series expansion would not necessarily be statistically independent. The Sampling Theorem, therefore, does not provide an attractive representation. More useful for Gaussian processes is the Karhunen-Loève expansion which is discussed in (11), (14), (16), (19), and (21). The general procedure for utilizing such a representation is to find the coefficients n_i by which the random noise process may be expressed as

$$\tilde{n}(t) = \sum_{i=1}^N \tilde{n}_i \tilde{\psi}_i(t) \quad (\text{A.1})$$

where \tilde{n}_i is

$$n_i = \int_{t_1}^{t_2} \tilde{n}(t) \tilde{\psi}_i^*(t) dt \quad (\text{A.2})$$

and the factors $\tilde{\psi}_i(t)$ are determined from the integral equation

$$\lambda_i \tilde{\psi}_i(t) = \int_{t_1}^{t_2} \tilde{K}_n(t,u) \tilde{\psi}_i(u) du \quad . \quad t_1 < t < t_2 \quad (A-3)$$

The expansion provides a series representation of the random process $n(t)$ over the finite observation interval t_1 to t_2 . The λ_i are the eigenvalues and the $\tilde{\psi}_i(t)$ the eigenfunctions of the integral Equation (A-3). Since the complex covariance function $\tilde{K}_n(t,u)$ is positive definite Hermitian, the eigenvalues will be positive and real, and the $\tilde{\psi}_i(t)$ form an orthonormal set. The complex covariance function is positive real Hermitian since the narrow-band power spectrum of the noise process is assumed real. The desired orthonormal functions $\tilde{\psi}_i(t)$ are then found by solving the integral Equation (A-3). General properties of integral equations and their solution may be found in applicable mathematics texts but are briefly summarized in (14), (19), or (21). Van Trees, in (14), also clearly discusses the meaning of utilizing the open observation interval of Equation (A-3), and the advantages of including a white noise component in the noise process as well as conditions on $\tilde{K}_n(t,u)$ under which the $\tilde{\psi}_i(t)$ represent a complete orthonormal set.

Let the quantity $\tilde{x}(t)$ be the complex Gaussian noise process which may include some white noise component. The representation of the process using Equations (A-1) and (A-2) is

$$x(t) = \lim_{M \rightarrow \infty} \sum_{i=1}^M \tilde{x}_i \tilde{\psi}_i(t) \quad (A-4)$$

$$\tilde{x}_i = \int_{t_1}^{t_2} \tilde{x}(t) \tilde{\psi}_i^*(t) dt.$$

The expansion of Equation (A-4) is taken to converge to $x(t)$ in the mean-square sense where l.i.m. denotes "limit in the mean" defined

$$\lim_{M \rightarrow \infty} E \left[\left\{ \tilde{x}(t) - \sum_{i=1}^M \tilde{x}_i \tilde{\psi}_i(t) \right\}^2 \right] = 0 \quad t_1 < t < t_2$$

For convenience of notation, the limit operation is omitted in the formulation of Λ and will be reinserted and limits taken in evaluation of the receiver implementation. It should further be noted that where the process $x(t)$ includes both colored and white noise, the eigenvalue of Equation (A-3) is the eigenvalue associated with the colored noise process. However, the eigenvalue representing the variance of the total noise process in the likelihood ratio must be

$$\lambda_i^T = \frac{N_0}{2} + \lambda_i^c$$

where λ_i^c is the eigenvalue of the complex covariance function of the colored noise and N_0 represents the white noise component (14).

The received waveform is designated

$$\tilde{f}(t) = \tilde{x}(t) + \tilde{s}(t)$$

and the term $\tilde{f}(t)$ is likewise represented as

$$\tilde{f}_i = \int_{t_1}^{t_2} \tilde{f}(t) \tilde{\psi}_i^*(t) dt.$$

The signal $s(t)$, however, must be expressed as

$$\tilde{s}(t) = \sum_{k=1}^N \tilde{s}_k \exp [j \theta_k] = \sum_{k=1}^N s_k \exp [j \omega_d t_k] \exp [j \theta_k] \quad (\text{A-5})$$

where each \tilde{s}_k is a pulse of the carrier frequency shifted by the target doppler frequency ω_d , and of duration τ . Associated with each \tilde{s}_k is some random phase angle θ_k . The pulses are located within the observation interval $t_1 < t < t_2$ at times

$$t_k = t_o + kT \quad (k = 1, 2, \dots, N)$$

where T is the interpulse period, and $NT < t_2 - t_1$.

The transform of the signal to the orthogonal coordinates determined from the noise random process is written as

$$\tilde{s}_i = \int_{t_1}^{t_2} \left(\sum_{k=1}^N \tilde{s}_k \exp [j \theta_k] \right) \tilde{\psi}_i^*(t) dt$$

$$\tilde{s}_i = \sum_{k=1}^N \exp [j \theta_k] \int_{t_1}^{t_2} \tilde{s}_k \tilde{\psi}_i^*(t) dt$$

$$\tilde{s}_i = \sum_{k=1}^N \tilde{s}_{ik} \exp [j \theta_k] \quad (\text{A-6})$$

where

$$\tilde{s}_{ik} = \int_{t_1}^{t_2} \tilde{s}_k \tilde{\psi}_i^*(t) dt. \quad (\text{A-7})$$

As a result of the orthonormal representation and the statistical independence of the coefficients, the likelihood ratio may be written from Equation (4.8) as

$$\Lambda(\theta_1 \dots \theta_N) = \frac{\exp \left\{ -\frac{1}{2} \sum_{i=1}^M \frac{1}{\lambda_i} \left| \tilde{f}_i - \tilde{s}_i \right|^2 \right\}}{\exp \left\{ -\frac{1}{2} \sum_{i=1}^M \frac{1}{\lambda_i} \left| \tilde{f}_i \right|^2 \right\}}$$

which reduces to

$$\Lambda(\theta_1 \dots \theta_N) = \exp \left\{ -\frac{1}{2} \sum_{i=1}^M \frac{1}{\lambda_i} \left(\left| \tilde{s}_i \right|^2 - 2 \operatorname{Re} [\tilde{f}_i^* \tilde{s}_i] \right) \right\}.$$

Using Equation (A-6) and evaluating the real part of the second term in the exponential

$$\Lambda(\theta_1 \dots \theta_N) = \exp \left\{ -\frac{1}{2} \sum_{i=1}^M \frac{1}{\lambda_i} \left| \tilde{s}_i \right|^2 + \sum_{i=1}^M \operatorname{Re} \left[\frac{1}{\lambda_i} \tilde{f}_i^* \sum_{k=1}^N \exp(j \theta_k) \tilde{s}_{ik} \right] \right\}$$

and reversing the order of summation, the likelihood ratio becomes

$$\Lambda(\theta_1 \dots \theta_N) = \exp \left\{ -\frac{1}{2} \sum_{i=1}^M \frac{1}{\lambda_i} \left| \tilde{s}_i \right|^2 + \sum_{k=1}^N \operatorname{Re} \sum_{i=1}^M \frac{1}{\lambda_i} \tilde{f}_i^* \exp(j \theta_k) \tilde{s}_{ik} \right\}.$$

Letting

$$M_k \exp(j \alpha_k) = \sum_{i=1}^M \frac{\tilde{f}_i^* \tilde{s}_{ik}}{\lambda_i}$$

the likelihood ratio becomes

$$\Lambda(\theta_1 \dots \theta_N) = \exp \left\{ -\frac{1}{2} \sum_{i=1}^M \frac{1}{\lambda_i} |\tilde{s}_i|^2 + \sum_{k=1}^N M_k \cos(\alpha_k + \theta_k) \right\} \quad (A-8)$$

or

$$\Lambda(\theta_1 \dots \theta_N) = \exp \left\{ W \right\} \prod_{k=1}^N \exp \left\{ M_k \cos(\alpha_k + \theta_k) \right\} . \quad (A-9)$$

Using the orthonormal expansion thus reduces the integration over the random phase terms to a product of N single integrations. Using the identity

$$I_0(x) = \int_0^{2\pi} \exp \left[x \cos(\theta - \alpha) \right] \frac{d\theta}{2\pi}$$

Equation (A-9) can be integrated as shown below

$$\Lambda = \exp \left\{ W \right\} \prod_{k=1}^N I_0(M_k) . \quad (A-10)$$

The form of the receiver or processor for a sequence of doppler shifted target pulses in colored noise is similar to the receiver derived by Helstrom (19) for detection of a single pulse of a noncoherent system in colored noise for N successive observations.

DISCUSSION OF RECEIVER IMPLEMENTATION

As suggested above, the likelihood ratio of Equation (A-10) is very

similar to that discussed in (19). In order to consider the differences demanded by Equation (A-10), a brief summary of the implementation will be provided. Since it is only necessary to compare the likelihood ratio or some monotone increasing function thereof with a threshold to make decisions on the hypothesis and alternative, the logarithm of Equation (A-10) is taken. The eigenvalue utilized in the following discussion is λ_i^T , the eigenvalue associated with white and colored noise.

$$\ln \Lambda = \sum_{k=1}^N \ln I_o(M_k) + \sum_{k=1}^N \sum_{i=1}^M |\tilde{s}_{ik}|^2 \frac{1}{\lambda_i^T} \quad (\text{A-11})$$

The term W has been rewritten from the appropriate terms of Equations (A-8) and (A-6). During the observation interval, the receiver forms the quantity

$$M_k = \left| \sum_{i=1}^M \frac{1}{\lambda_i^T} \tilde{f}_i^* \tilde{s}_{ik} \right| \quad (\text{A-12})$$

for each signal input index k and applies M_k to a detector having a characteristic $\ln I_o(M_k)$. A summation of the detector outputs over $k=1$ to $k=N$ is implemented and the result compared with a threshold determined in part by the second term on the right hand side of Equation (A-11). As can be seen, the threshold depends on the expected amplitude of the signal through the relations of Equations (A-11), (A-7), and (A-5) and, therefore, does not provide a uniformly most powerful test relative to signal amplitudes. A major difference in this problem from that derived by Helstrom is the dependence of the statistic of Equation (A-12) upon the target doppler frequency. The implementation of the statistic M_k must

provide for the processing and detection of targets over the doppler range of interest.

The actual implementation of the statistic M_k must now be investigated. Rewriting Equation (A-12), and passing to the limit as defined by Equation (A-4), the result is

$$M_k = \left| \sum_{i=1}^{\infty} \frac{1}{\lambda_i} \tilde{f}_i^* \tilde{s}_{ik} \right|. \quad (\text{A-13})$$

Define the quantity within the magnitude sign as a new functional

$$g_k(f_i; \omega_d) = \sum_{i=1}^{\infty} \frac{\tilde{f}_i \tilde{s}_{ik}^*}{\lambda_i}.$$

Substituting the equation defining f_i

$$g_k(f_i; \omega_d) = \sum_{i=1}^{\infty} \left(\frac{1}{\lambda_i} \int_{t_1}^{t_2} \tilde{f}(t) \tilde{\psi}_i^*(t) dt \right) \tilde{s}_{ik}^* = \int_{t_1}^{t_2} \tilde{f}(t) \sum_{i=1}^{\infty} \frac{\tilde{s}_{ik}^* \tilde{\psi}_i^*(t)}{\lambda_i} dt$$

and letting

$$h_k^*(t; \omega_d) = \sum_{i=1}^{\infty} \frac{s_{ik}^* \psi_i^*(t)}{\lambda_i} \quad (\text{A-14})$$

the test functional is represented by the form

$$M_k = \left| \int_{t_1}^{t_2} f(t) h^*(t; \omega_d) dt \right| . \quad (A-15)$$

The statistic is then derived from the envelope of a matched filter operation on the received data. From (14), (16), or (19), the function $h(t_i; \omega_d)$ is the solution to the integral equation

$$\int_{t_1}^{t_2} \tilde{K}_n(t, u) \tilde{h}(u; \omega_d) du = \tilde{s}(t) \quad (A-16)$$

where the cited references treat the conditions for existence of the solution. However, if the colored noise process is stationary, and the observation interval is allowed to become very long, $t_2 - t_1 \rightarrow \infty$, then Equation (A-16) can be investigated by Fourier Transform techniques. Then, since Equation (A-16) is a generalized convolution, it can be written in terms of transforms as

$$F[\tilde{h}(u; \omega_d)] = \frac{F[\tilde{s}(t)]}{F[\tilde{K}_n(\tau)]}$$

$$H(\omega) = \frac{S(\omega)}{2 \Psi(\omega)} \quad (A-17)$$

where $\Psi(\omega)$ is the spectral density of the noise process, which is assumed to include white noise to avoid the inconsistency of singular detection. The factor of 1/2 arises from the definition of the complex covariance. The impulse response of the filter is then

$$h(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H(\omega) e^{+j\omega\tau} d\omega \quad (\text{A-18})$$

where the $\Psi(\omega)$ is assumed to be such that $S(\omega)/\Psi(\omega)$ approaches zero rapidly as $\omega \rightarrow \pm\infty$ and the inverse transform exists. The receiver then takes the form shown in Figure A-1 herein.

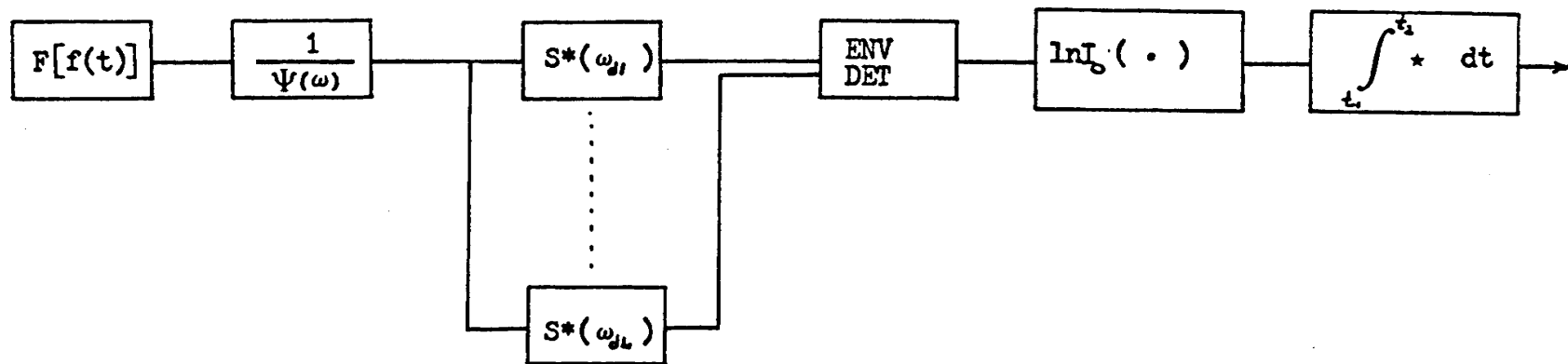


Figure A-1 Optimum receiver implementation

APPENDIX B

REVIEW OF THE SIMULATION FOR VIDEO PROCESSOR EVALUATION

A computer simulation in the frequency domain was developed for use in analyzing and predicting the performance of air-to-ground MTI processors during a study program at McDonnell Aircraft Company. Operation of the MTI processor was assumed with a noncoherent radar system. A mathematical model of radar ground clutter at IF was derived by projection of the antenna pattern on iso-doppler contours. Techniques were utilized whereby such factors as radar system instabilities and modulation due to antenna scanning were incorporated in the model. The video detector characteristic was modeled in the frequency domain and representations for clutter, signal and noise at video developed. Though methods were implemented to determine the probability of detection of a moving target immersed in a background of clutter, that feature was not utilized for this Thesis. Figure B-1 illustrates a simplified block diagram of the simulation. A detailed description of the derivation in formulating the simulation is not presented herein as future publication is anticipated.

In order to compare the suboptimum receiver formulation of Chapter IV, Section C, with existing processing techniques, systems as depicted by Figure 4 were simulated in the frequency domain. The transfer function of the "range gate and filter" (RGF) was approximated by

$$H_1(f) = \frac{1}{1 + \left(\frac{f_{o1}}{f}\right)^4} \quad (B-1)$$

in cascade with

$$H_2(f) = \frac{1}{1 + \left(\frac{f}{f_{o2}}\right)^2} \quad (B-2)$$

to provide the desired passband. The term f_{o1} designates the high-pass filter corner location and was varied with the predicted clutter spectral spread as a function of antenna scan angle. The resultant filter implementation was

$$\left| H_{RGF}(f) \right|^2 = \frac{1}{\left[1 + (f_{o1}/f)^8 \right] \left[1 + (f/f_{o2})^4 \right]}. \quad (B-3)$$

The transfer function of the processor of Figure 4-(b) was

$$\left| H_c(\omega) \right|^2 = \frac{(\gamma - 2 \cos \omega T)^2}{a_0 + a_1 \cos \omega T + a_2 \cos 2\omega T} \quad (B-4)$$

where the factors a_0 , a_1 , and a_2 are functions of the gains A and B, and T is the interpulse period. Similar to varying the corner frequency of the RGF, the feedback gains A and B were varied as a function of the antenna pointing angle to shape the filter as a function of the clutter doppler spread.

The implementation of the suboptimum system was not so direct as that of the RGF and SDDC. Based on Figure 7, the suboptimum processor must provide first a transfer function

$$\left| H_{so}(\omega) \right|^2 = \frac{1}{\Psi_T(\omega)} \quad (B-5)$$

where $\Psi_T(\omega)$ is the power spectral density of the total noise. The system total noise is composed of both white noise and the clutter originated colored noise. The autocorrelation function of the total noise is, therefore, of the form

$$R_T(\tau) = R_w(\tau) + R_c(\tau)$$

$$R_T(\tau) = \frac{N_o}{2} \delta(\tau) + R_c(\tau)$$

with the power spectral density (P.S.D.)

$$\Psi_T(\omega) = \Psi_w(\omega) + \Psi_c(\omega). \quad (\text{B-6})$$

The simulation computes the clutter as a function frequency and similarly distributes thermal noise power over the region $-\text{PRF}$ to $+\text{PRF}$. In order to prevent unrealistic gains from the device, the transfer function was modified to provide unity gain where the noise power in an interval f to $f = \Delta f$ was approximately equal to the thermal noise in that interval. The simulation provided clutter and thermal noise power as follows:

$$\Psi_c(\omega_i) = \text{Clutter power in the interval } \omega_i \text{ to } \omega_i + \Delta\omega = \text{SCP}(\text{I})$$

$$\Psi_w(\omega_i) = \text{Thermal power in the interval } \omega_i \text{ to } \omega_i + \Delta\omega = \text{PN}(\text{I})$$

The suboptimum receiver was simulated by letting

$$\left| H(\omega_i) \right|^2 = \frac{\text{PN}(\text{I})}{\text{SCP}(\text{I}) + \text{PN}(\text{I})} \quad (\text{B-7})$$

and the filter element simulating an element matched to some signal doppler was deleted to allow a direct comparison as a clutter rejection device.

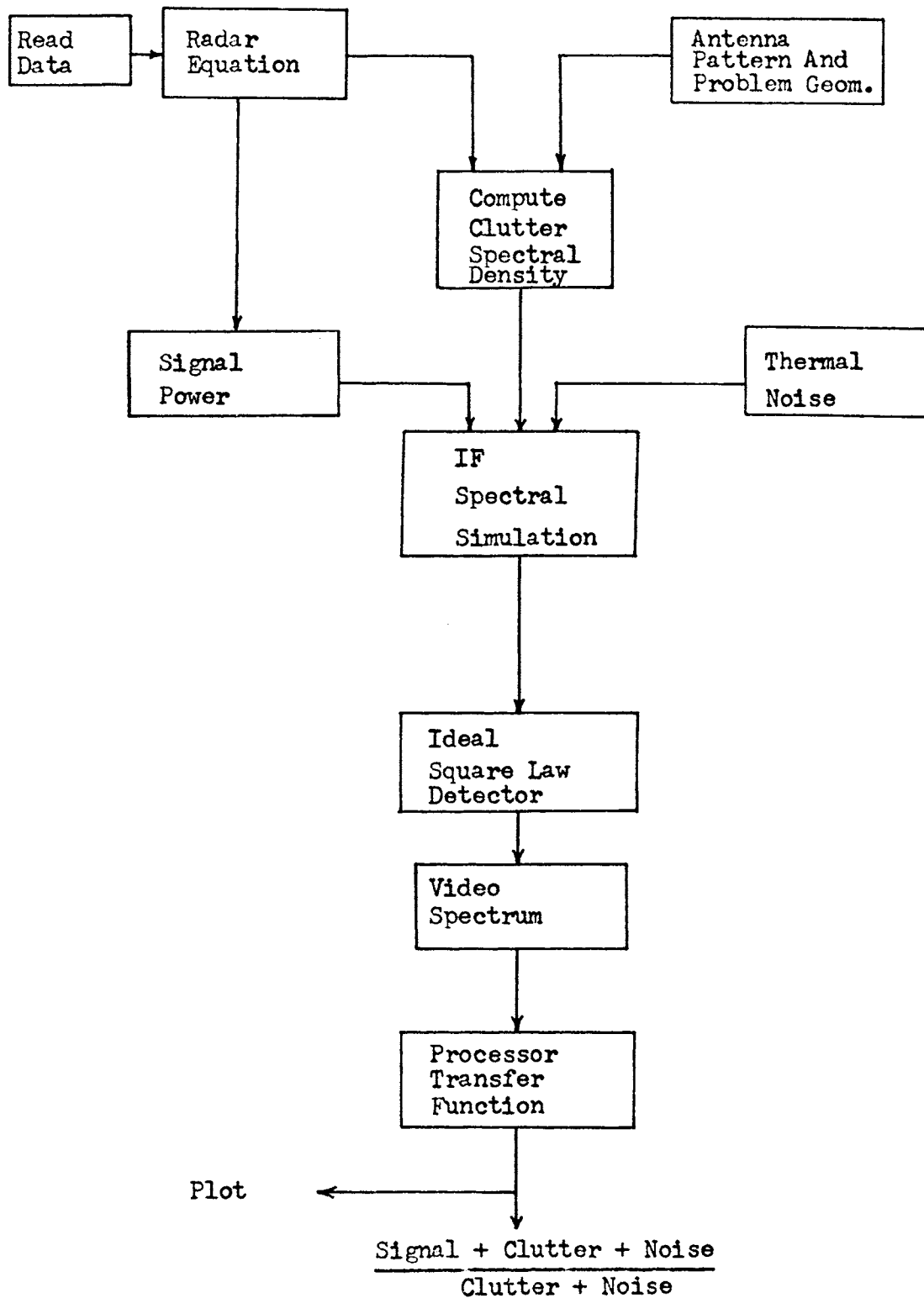


Figure B-1 Simulation block diagram

- RGF (RANGE GATE AND FILTER)
- SO (SUBOPTIMUM)
- SDDC (SHAPED DOUBLE DELAY CANCELLER)

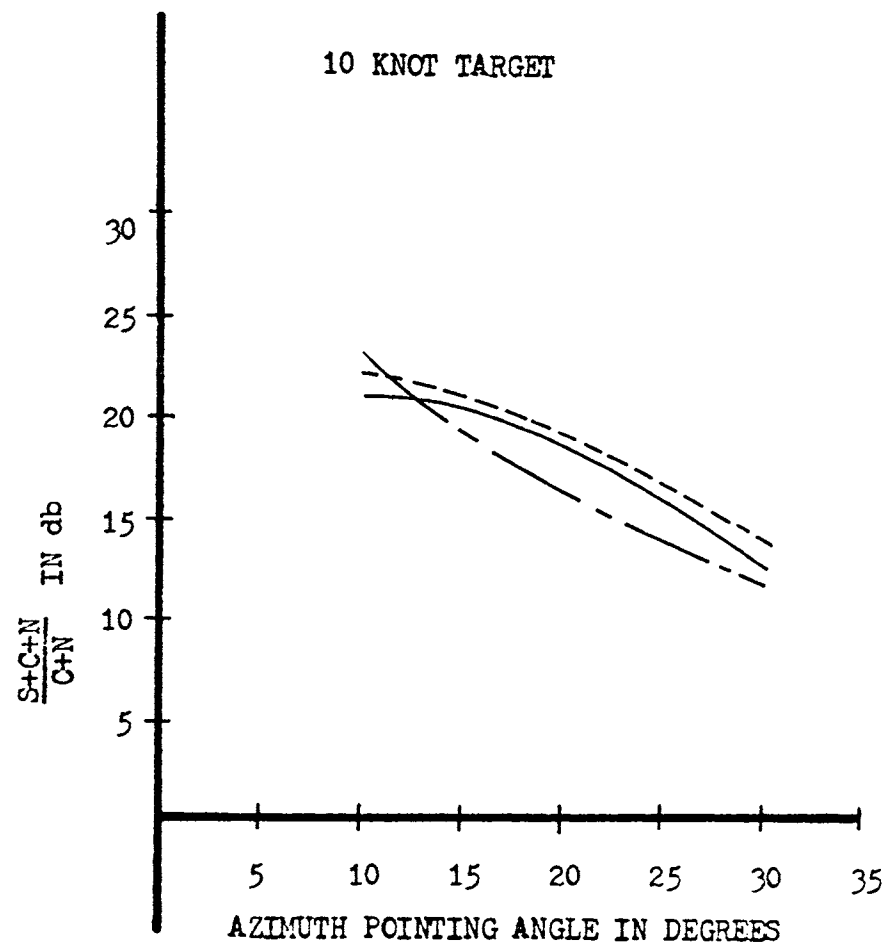
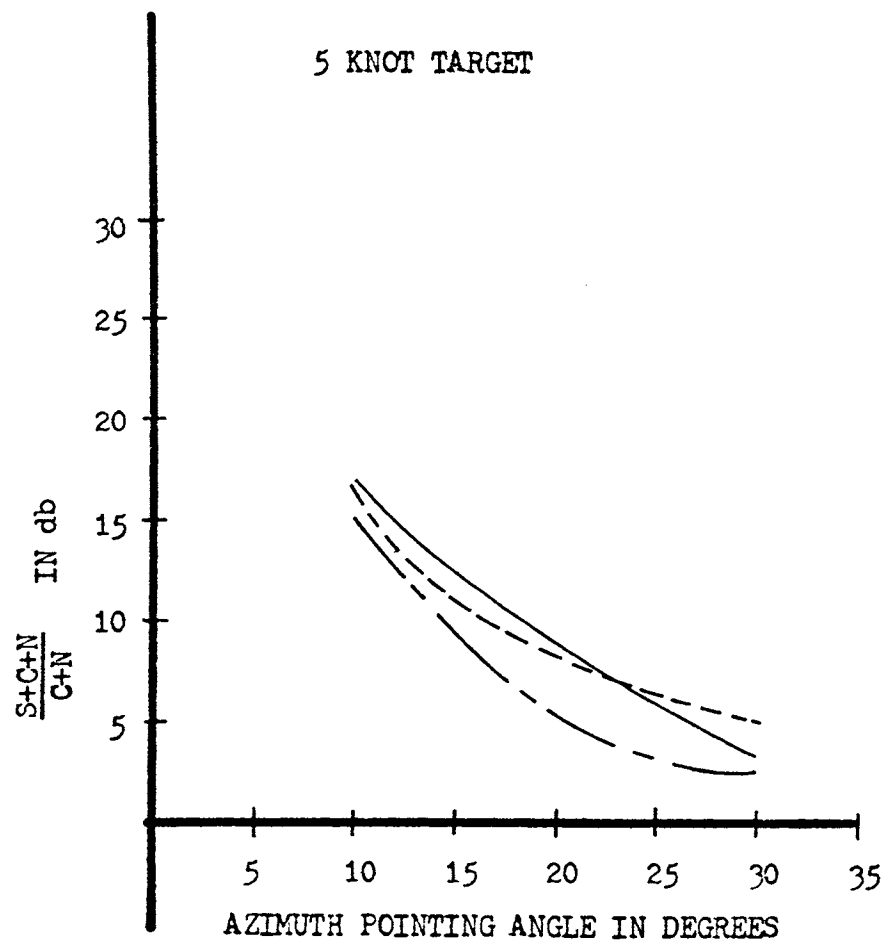


FIGURE B-2 FILTER RESPONSE PLOT; 5 AND 10 KNOT TARGETS

- - - - RGF (RANGE GATE AND FILTER)
 ——— SO (SUBOPTIMUM)
 — - - SDDC (SHAPED DOUBLE DELAY CANCELLER)

20 KNOT TARGET

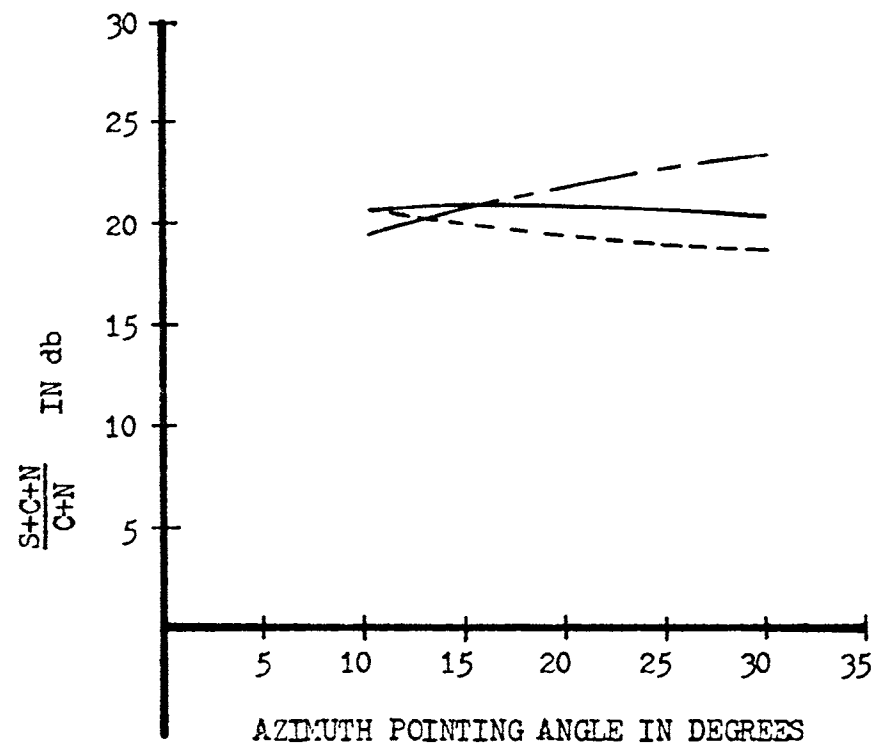


FIGURE B-3 FILTER RESPONSE PLOT; 20 KNOT TARGET

BIBLIOGRAPHY

1. L. A. Wainstein and V. D. Zubakov, Extraction of Signals from Noise, New Jersey; Prentice-Hall, 1962.
2. Jack Capon, "Optimum Weighting Functions for the Detection of Sampled Signals in Noise," IRE Trans. on Information Theory, Vol. IT-10, pp. 152-159, April, 1964.
3. L. J. Spafford, "Optimum Radar Signal Processing in Clutter", IEEE Trans. Information Theory, Vol. IT-14, pp. 734-743, September, 1968.
4. D. F. Delong, Jr. and E. M. Hofstetter, "On the Design of Optimum Radar Waveforms for Clutter Rejection", IEEE Trans. Information Theory, Vol. IT-13, pp. 454-463, July, 1967.
5. H. L. Van Trees, "Optimum Signal Design and Processing for Reverberation-Limited Environments", IEEE Trans. on Military Electronics, Vol. MIL-9, pp. 212-229, July, 1965.
6. R. S. Berkowitz, Modern Radar, New York; Wiley and Sons, 1965.
7. M. I. Skolnik, Introduction to Radar Systems, New York, McGraw-Hill, 1962.
8. D. K. Barton, Radar Systems Analysis, New Jersey; Prentice-Hall, 1964.
9. E. B. Anders, et al., "Digital Filters", National Aeronautics and Space Administration Contractor Report, NASA CR-136, 1964.
10. P. M. Woodward, Probability and Information Theory, with Applications to Radar, New York; Pergamon Press, 1964.
11. W. B. Davenport, Jr. and W. L. Root, Introduction to Random Signals and Noise, New York; McGraw-Hill, 1960.
12. A. M. Mood and F. A. Graybill, Introduction to the Theory of Statistics, New York; McGraw-Hill, 1963.
13. J. J. Komo, "Nonparametric Detection of Signals Causing Arbitrary Changes in the Accompanying Noise Backgrounds", Dissertation for Doctor of Philosophy, University of Missouri at Rolla, 1966.
14. H. L. Van Trees, Detection, Estimation, and Modulation Theory (Part 1), New York; John Wiley and Sons, Inc., 1968.
15. A. M. Yaglom, An Introduction to the Theory of Stationary Random Functions, New Jersey; Prentice-Hall, 1962.
16. A. V. Balalcrishnan, Communication Theory, New York; McGraw-Hill, 1968.

17. E. A. Feustel, "Application of Rank Statistics to Detection", Technical Report No. 18, Department of Electrical Engineering, Princeton University, 1967.
18. H. L. Van Trees, Detection, Estimation, and Modulation Theory (Part II), Notes to be published.
19. C. Helstrom, Statistical Theory of Signal Detection, London; Pergamon, 1960.
20. W. McC. Siebert, "A Radar Detection Philosophy", I.R.E., Transion Information Theory, Vol. IT-2, pp. 204-221, September, 1956.
21. Ivan Selin, Detection Theory, New Jersey; Princeton University Press, 1965.
22. E. A. Guillemin, The Mathematics of Circuit Analysis, Massachusetts; the M.I.T. Press, 1949.
23. K. Ogata, State Space Analysis of Control Systems, New Jersey; Prentice-Hall, 1967.
24. D. Middleton, An Introduction to Statistical Communication Theory, New York; McGraw-Hill, 1960.
25. R. P. Brueggemann and R. B. Rutledge, "A Digital Simulation of Clutter Rejection MTI Radars", IEEE Electromagnetic Compatibility Symposium Record, pp. 281-294, 1967.

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