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Photoelastic investigation of notched corners in square apertures in flat plates subjected to tension

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Department:

PHOTOELASTIC INVESTIGATION OF NOTCHED CORNERS IN SQUARE APERTURES IN FLAT PLATES SUBJECTED TO TENSION

BY

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A

THESIS

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NOTATIONS

α	Amplitude of vibration
ω	Angular velocity
A	Light vector
V	Velocity of light
N	Index of refraction
d	Model thickness
f	Wave lengths
c	Photoelastic constant
σ	Normal stress
τ	Shear stress
S_t	Tensile Stress
S.C.	Stress concentration factor

INTRODUCTION

Photoelastic stress analysis is one of the most effective methods available to the engineer for solving problems of stress distribution. The effect of forces, such as pressure and tension may be studied using a scale model of a structural part. Special transparent models are subjected to forces proportional to those acting on the real object. The stress problem produced in the model indicates the stress distribution through a cross-section of the structural part.

Analysis of the photographic record of the stress analysis pattern provides data with a high degree of accuracy within a relatively short time. In simple cases, this data can be used either alone or as a check of mathematical solutions. They also yield reliable information on problems which are insolvable mathematically or at least involve complicated and tedious calculations. Examples include the stresses around notches and holes in bars, in riveted joints, spoked wheels, gear teeth and frame works with welded joints or in redundant members.

Other methods of stress analysis used in the design and testing of structural forms described briefly as follows:

BRITTLE COATING:

The structure is covered with a special coating designed to crack wherever strain exceeds a fixed

amount. This method indicates regions of dangerous stress and provides an over all picture of surface stresses. This method is mainly used to indicate the directions of the principal stresses so that electric strain gages may be accurately oriented.

ELECTRIC STRAIN GAGES:

These are the most versatile devices for use by the stress analyst. They are readily adaptable to all kinds of structures and materials. With the electronic equipment available today the variable resistance wire strain gage is without a doubt the most favorable means of measuring dynamic and transient surface strains. However, the wire is very susceptable to temperature changes whereby a thermocouple effect may be established.

Photoelastic stress analysis, however, has certain advantages:

(1) A more complete view of the stress pattern is obtained, instead of surface stresses at a limited number of points.

(2) Stresses in relatively inaccessible places can be studied such as points of stress concentrations.

(3) Tests can be quickly made and inexpensively before its structural counterpart is made.

HISTORY

In 1816 Sir David Brewster published an account of his findings that clear stressed glass when examined in polarized light exhibited colored patterns. However, the application of this discovery to engineering problems was not recognized before 1900, although the underlying theory was well developed by physicists such as Neumann, Maxwell, and Wertheim who formulated the concept that the optical retardation producing the color effects is proportional to the difference of the principal stresses existing in glass.

In 1920, Professor E. G. Coker of the University of London further advanced the theory by the introduction of cellulose models and the use of monochromatic light.

However, since 1940 the introduction of other methods of stress analysis caused a decrease in the development of photoelasticity as a method of stress determination.

OPTICS

There are two basic theories that attempt to explain the behavior of light. The corpuscular theory of light is based on the assumption that light is caused by the motion of light corpuscules emitted from a light source. This theory satisfactorily explains certain phenomena, such as reflection, refraction, and the photoelectric effect. However this theory does not satisfactorily explain the phenomena of polarization, as does the earlier electromagnetic wave theory.

According to the wave theory, light is assumed to be composed of electromagnetic waves. The amplitude of the wave motion is proportional to the electric and magnetic field strengths of the wave motion. Since both electric and magnetic fields are vector quantities and in light are definitely related, the wave motion may be represented by a single vector representing either electric or magnetic field strengths. The direction of the vector ray is normal to the path of propagation of light and in the plane of the wave motion.

A beam of light is considered as being composed of a number of rays of light, each ray travelling along its path independently of the other rays but subject to the same behavior.

If a beam of light is incident upon a transparant medium capable of transmitting light that is vibrating in

in one plane only, then only the ray of light in that plane and the components of other rays in that plane will be transmitted. This is called plane polarization of light.

The following sketch shows clearly the characteristics of electromagnetic wave motion light and the effect of polarization on the light rays.

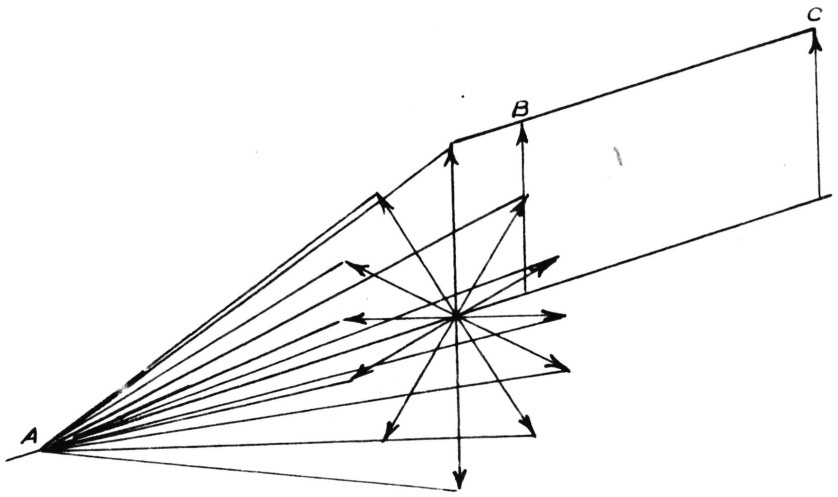

Fig. 1

S = Source of light

B = Polarizer

C = Ray of polarized light vibrating in one plane only.

The displacement of this beam of light can be expressed by the equation:

$$A = \alpha \sin \omega t$$

Where α is the amplitude of the vibration and ω is the circular frequency.

If the ray of light is passed through a second polarizer whose plane of polarization is at an angle θ with the first polarizer, the component of the ray of light that is passed by the second polarizer, known in photoelastic work as the analyzer, may be expressed by the equation:

$$A_1 = \alpha \cos\theta \sin \omega t$$

This indicates that as the analyzer is rotated from 0° to 90° that the light passing the analyzer will vary from a maximum at 0° to zero at 90°.

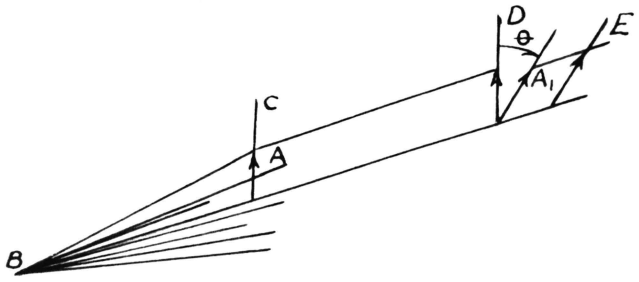

Fig. 2

B = Light source

C = First polarizing agent (polarizer)

D = Second polarizing agent (analyzer)

E = Observing screen

A = Amplitude of light passed by C

A_1 = Amplitude of light passed by D

If the light passing the first polarizer is incident on a double refracting material, oriented at an angle θ with the polarizer, the equation of the displacement of the light passed now becomes:

$$A_1 = \alpha \cos\theta \sin\omega t$$

and $$A_2 = \alpha \sin\theta \sin\omega t$$

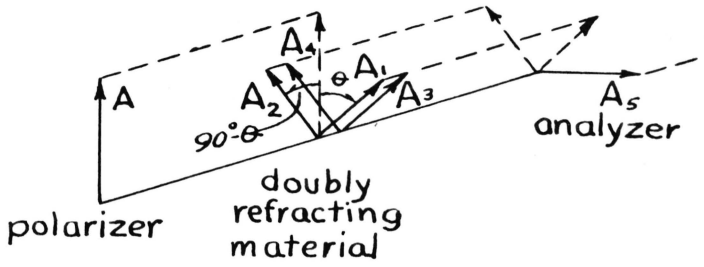

Diagram showing double-refracting material between polarizer and analyzer.

Fig. 3

A double refracting medium is one in which a beam of light is divided into two beams, one beam passing through the medium more rapidly than the other. This phenomena is due to the different indicies of refraction of the medium. Because of the different indicies of refraction of a material, the components of light emerging will have developed a phase difference so that:

$$A_3 = \alpha \cos\theta \sin \omega (t + \delta t) \quad (6)$$

and $$A_4 = \alpha \sin\theta \sin \omega t \quad (7)$$

The light passed by the analyzer then becomes:

$$A_5 = \alpha \sin 2\theta \sin \frac{\omega \delta t}{2} \cos (\omega t + \frac{\omega \delta t}{2}) \quad (8)$$

The time difference δt is a result of the difference in transmission velocities through a double refracting material. If v is the velocity of light in air and N_1 and N_2 are the indices of refraction then:

$$t_1 = N_1 d/v \quad (9)$$

and $$t_2 = N_2 d/v \quad (10)$$

where d is the model thickness

so that $$\delta t = \frac{d}{v} (N_1 - N_2)$$

$$\omega \delta t = \frac{\omega d}{v} (N_1 - N_2) = \frac{2\pi d}{\lambda} (N_1 - N_2) \quad (11)$$

where $$\lambda = \frac{2\pi v}{\omega} \quad (12)$$

If f is considered to be the number of wave lengths of relative retardations then:

$$\omega \delta t = 2\pi f = \frac{2\pi d}{\lambda} (N_1 - N_2) \quad (13)$$

$$f = \frac{d}{\lambda} (N_1 - N_2) \quad (14)$$

By experiment it has been found that the change in the refractive index of a material is proportional to the intensities of the principal stresses. The light polarized in the directions of the principal stresses is transmitted only on the planes of principal stress.

$$f = \frac{d}{C} (\sigma_1 - \sigma_2) \tag{15}$$

Where C is the photoelastic constant of the material which is a characteristic constant for each material so .

$$A_5 = \alpha \sin 2\theta \sin \frac{\pi d}{C} (\sigma_1 - \sigma_2) \cos (\omega t + \frac{2\pi d}{C} (\sigma_1 - \sigma_2) \tag{16}$$

As can be seen by equation (16) if the angle θ is equal to either zero or 90°, whereby the plane of polarization of the light is parallel to a principal stress axis direction, no light will pass theough the analyzer. This will produce black lines known as isoclinics. If the principal stress difference is such that:

$$\sin \frac{\pi d}{C} (\sigma_1 - \sigma_2) \tag{17}$$

will equal either zero or π or if the

$$\frac{\pi d}{C} (\sigma_1 - \sigma_2) \tag{18}$$

term is equal to any multiple integer of π no light will reach the screen. This condition will also produce black lines known as isochromatics if a monochromatic light source is used.

To remove the isoclinic lines from the screen, quarter wave plates must be introduced. These are transparent sheets of mica oriented at a 45° angle to the polarizer and of such thickness that they produce a retardation of one quarter of a wave length, causing circularly polarized light.

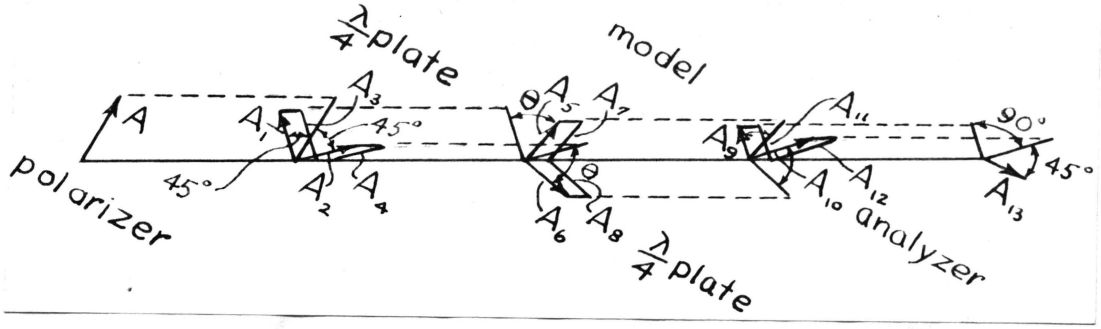

Fig. 4

From Fig. 4 it can be seen that the light that passes through the quarter wave plate and falls on the model will be:

$$A_3 = .706\ \alpha \sin(\omega t + 90°) \tag{19}$$

$$A_4 = .707\ \alpha \sin \omega t \tag{20}$$

These conponents of the circularly polarized light passing through the model are resolved into new components parallel to the principal stress directions so that:

$$A_5 = .707\ \alpha \sin(\omega t + 90° + \theta) \tag{21}$$

$$A_6 = .707\ \alpha \sin(\omega t + \theta) \tag{22}$$

Then those components entering the quarter wave plates are now:

$$A_7 = .707\ \alpha \sin(\omega t + 90° + \theta\ \omega\delta t) \tag{23}$$

$$= -.707\ \alpha \cos(\omega t + \theta + \omega\delta t) \tag{24}$$

$$A_8 = .707\ \alpha \sin(\omega t + \theta) \tag{25}$$

After passing through the next quarter wave plate

$$A_{11} = -.707\ \alpha \cos\theta \cos(\omega t + \theta + \omega\delta t + 90°) \tag{25}$$

$$= -.707\ \alpha \sin\theta \sin(\omega t + \theta\ 90°) \tag{26}$$

$$A_{12} = -.707\ \alpha \sin\theta \cos(\omega t + \theta\,\omega\delta t) + .707\ \alpha \cos\theta \sin(\omega t + \theta) \quad (27)$$

Then the light transmitted by the analyzer will be

$$A_{13} = \alpha \cos \frac{\omega\delta t}{2} \sin\left(\omega t + \frac{\omega\delta t}{2}\right) \quad (28)$$

Since the angle θ does not appear in the equation, the light passed by the analyzer is independent of the orientation of the principal stress directions of the model, which eliminates the isoclinics.

By various substitutions as previously made

$$A_{13} = \alpha \cos \frac{\pi d}{c}(\sigma_1 - \sigma_2) \sin \omega t + \frac{\pi d}{c}(\sigma_1 - \sigma_2) \quad (29)$$

STRESS

Since the photoelastic effects are related to the principal stresses it is advisable to show the relationship between stresses on the various planes that may be passed through any given point in a body. The relationships among the stresses on various plances passing through a point may be determined by applying the equations of equilibrium to a free body diagram of an element.

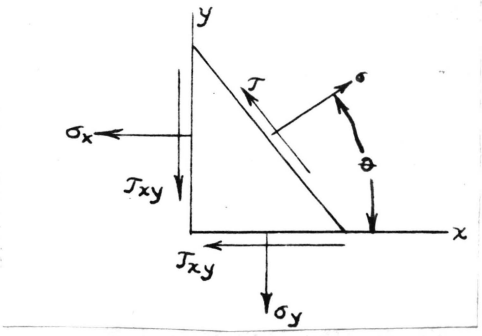

Fig.5

Stresses at a point in a two dimension system.

By the summation of forces it can be seen that

$$\sigma = \sigma_x \, \delta y \delta z \sin\theta + \tau_{xy} \delta y \delta z \cos\theta + \sigma_y \delta x dz \cos\theta + \tau_{xy} \, \delta x \, \delta z \sin\theta \qquad (1)$$

amd since $\delta y \delta z$ is the $\sin\theta$ and $\delta x \delta z$ is the $\cos\theta$

$$= \sigma_x \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta + \sigma_y \cos^2\theta \qquad (2)$$

$$= 1/2 \, (\sigma_x - \sigma_y) + 1/2 \, (\sigma_x - \sigma_y) \cos 2\theta + \tau_{xy} \sin 2\theta \qquad (3)$$

In a similar manner it can be found that

$$\tau = 1/2 \, (\sigma_x - \sigma_y) \sin 2\theta + \tau_{xy} \cos 2\theta \qquad (4)$$

By differentiating equation 3 and setting the result equal

to zero it will be found that

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad (5)$$

from this it can be seen that when σ is maximum and τ becomes zero, so that the algebraic maximum and minimum normal stresses lie on plances where there is no shearing stress, the normal stresses are then principal stresses. Since there are two possible values for the angle 2θ which are 180° apart there are two principal stresses lying on planes 90° from each other.

If the values of the principal stresses are found by substituting values from equation 5 into equation 3 and 4, the following relations are obtained:

$$\sigma_1 = 1/2\ (\sigma_x + \sigma_y) + 1/2\ \left[(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2\right]^{1/2} \qquad (6)$$

$$\sigma_2 = 1/2\ (\sigma_x + \sigma_y) - 1/2\ \left[(\sigma_x - \sigma_y)^2 + 4\tau_{x2y}^2\right]^{1/2} \qquad (7)$$

$$\tau_m = 1/2\ \left[(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}\right]^{1/2} \qquad (8)$$

$$= 1/2\ (\sigma_1 - \sigma_2) \qquad (9)$$

So that the maximum shearing stress at any point is equal to 1/2 the difference of the principal stresses.

By comparing equation (9) with equation (11) from the previous section on Optics it can be seen that isochromatics are regions of constant maximum shear stress.

PROBLEM

AND

PROCEDURE

The author's problem was the investigation of stress concentrations at notched corners of square apertures in flat plates subjected to tension. Previous work along this line had been done by R. H. Beaver. (1) He investigated the stress concentration at filleted corners

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(1) Beaver, R. H. Thesis. Missouri School of Mines 1951

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of square apertures in flat plates subjected to tension.

The problem was divided into two parts. For the first part a flat plate of constant length, width, thickness, and aperture size was used varying only the diameter of the notch. In the second part the notch remained constant at 0.125 inches and the plate varied in width and aperture size maintaining at the same time a constant ratio between the plate width and aperture size of 2.67. This ratio of plate width and aperture size was held constant as any variation would affect the stress concentration.

The models were constructed from clear catalin, a phenolformaldehyde condensate similar to bakelite, 6 inches long, 1 inch wide, and 7/32 inch thick. Models of this small size were used in order to obtain a maximum

amount of fringe lines without fracturing the material at the supports. It is interesting to note that the models failed at the 3/8 inch hole drilled so that the model could be supported in tension. This is because of the fact that the stress caused by the hole is 3 times the stress without the hole. (2).

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(2) Timoshenko, Theory of Elasticity p. 78, 1934

The equipment used was standard photoelastic apparatus consisting of polaroids, quarter wave plates, monochromatic light source and an attached camera. Photographs were made with 5 x 7 inch Super Ortho-Press Film at 1/25th of a second exposure. The photographs were enlarged to facilitate counting fringes.

In order to determine the maximum fringe order at the boundary, the fringes were plotted against the percent distance to the edge and the curve extrapolated.

The photoelastic constant of the material was determined by finding the change in fringe order caused by a variation of the load.

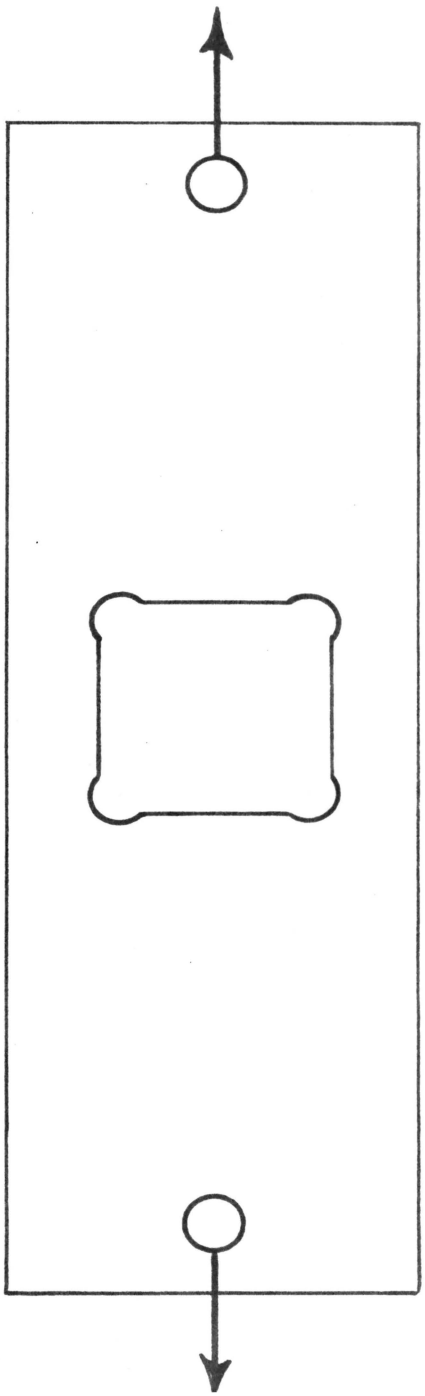


Figure 7

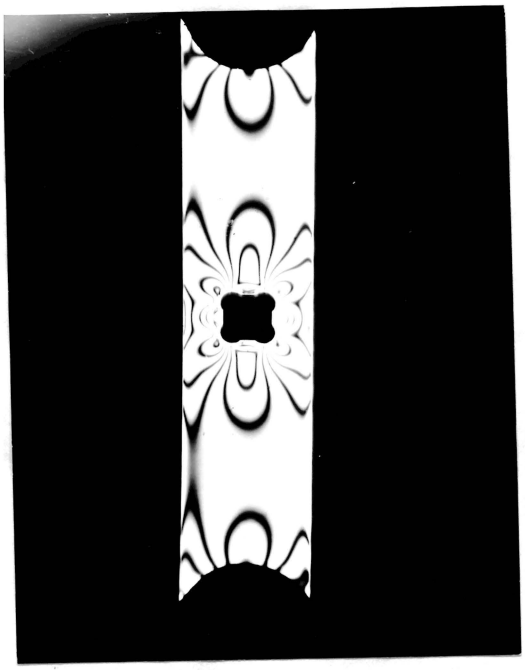


Figure 8

MODEL #1

Width = 1 inch

Length = 6 inches

Thickness = 0.2065 inches

Notch Diam. = 0.187 inches

Hole Size = 0.375 in. sq.

Load = 259 pounds

Fringe Order = 11.4

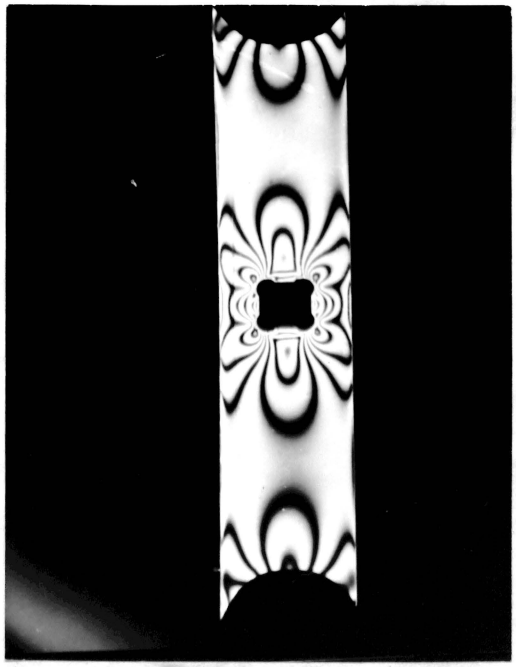


Figure 9

MODEL #2

Width = 1 inch

Length = 6 inches

Thickness = 0.2175 inches

Notch Diam. = 0.156 inches

Hole Size = 0.375 in. sq.

Load = 299 pounds

Fringe Order = 10.8

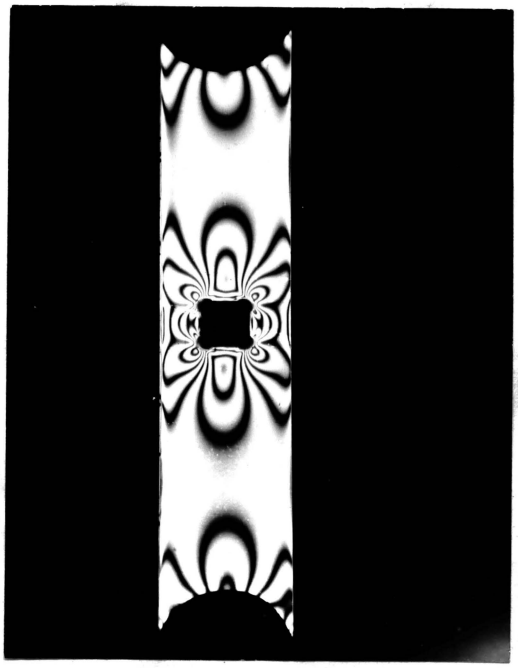


Figure 10

MODEL #3

Width = 1 inch

Length = 6 inches

Thickness = 0.2165 inches

Notch Diam. = 0.125 inches

Hole Size = 0.375 in. sq.

Load = 299 pounds

Fringe Order = 8.5

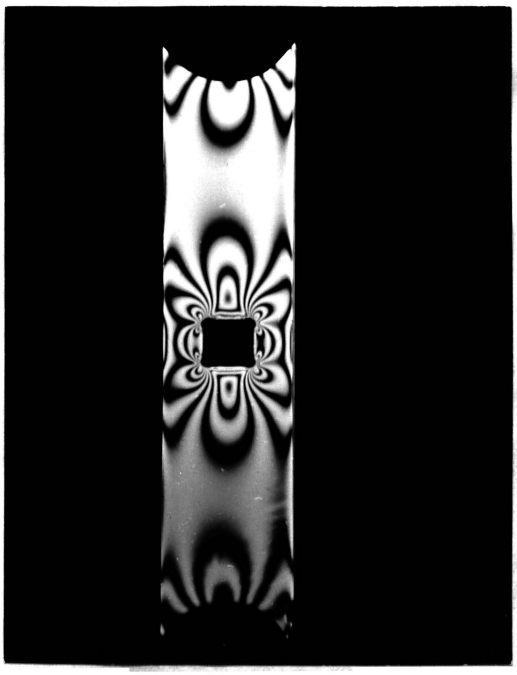


Figure 11

MODEL #4

Width = 1 inch

Length = 6 inches

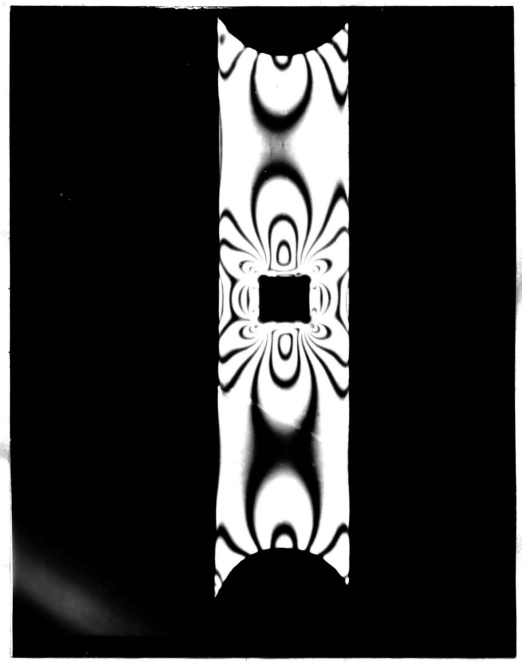
Thickness = 0.2145 inches

Notch Diam. = 0.093 inches

Hole Size = 0.375 in. sq.

Load = 299 pounds

Fringe Order = 9.2



MODEL #5

Width = 1 inch

Length = 6 inches

Thickness = 0.2065 inches

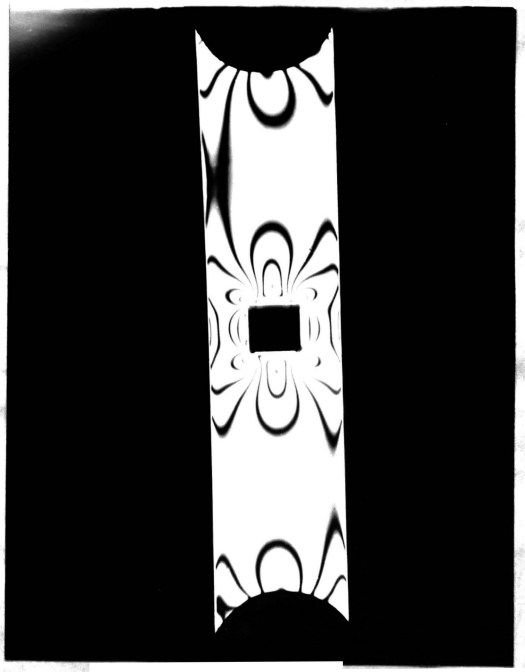
Notch Diam. = 0.076 inches

Hole Size = 0.375 in. sq.

Load = 299 pounds

Fringe Order = 9.2

Figure 12



MODEL #6

Width = 1 inch

Length = 6 inches

Thickness = 0.208 inches

Notch Diam. = 0.040 inches

Hole Size = 0.375 inches

Load = 299 pounds

Fringe Order = 10.2

Figure 13

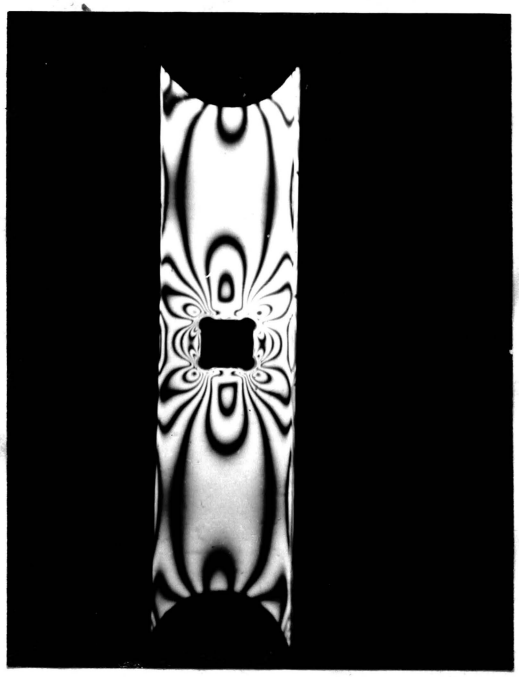


Figure 14

MODEL #7

Width = 1.068 inches

Thickness = 0.2075 inches

Notch Diam. = 0.125 inches

Hole Size = 0.4 in. sq.

Load = 299 pounds

Fringe Order = 8.4

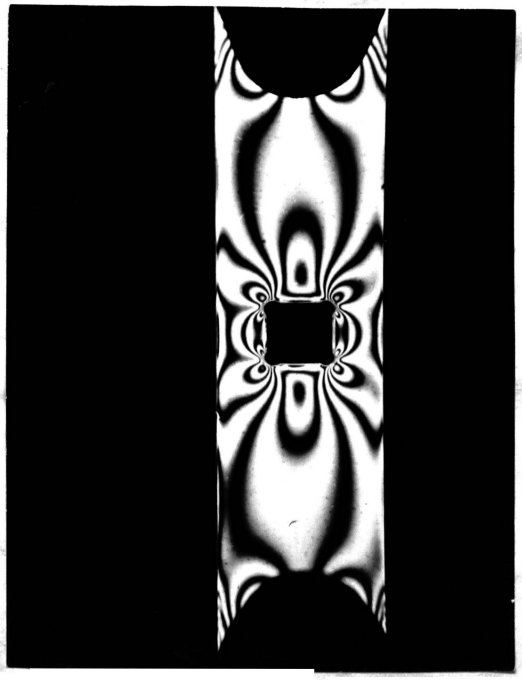


Figure 15

MODEL #8

Width = 1.333 inches

Thickness = 0.2045 inches

Notch Diam. = 0.125 inches

Hole Size = 0.5 in. sq.

Load = 299 pounds

Fringe Order = 7.4

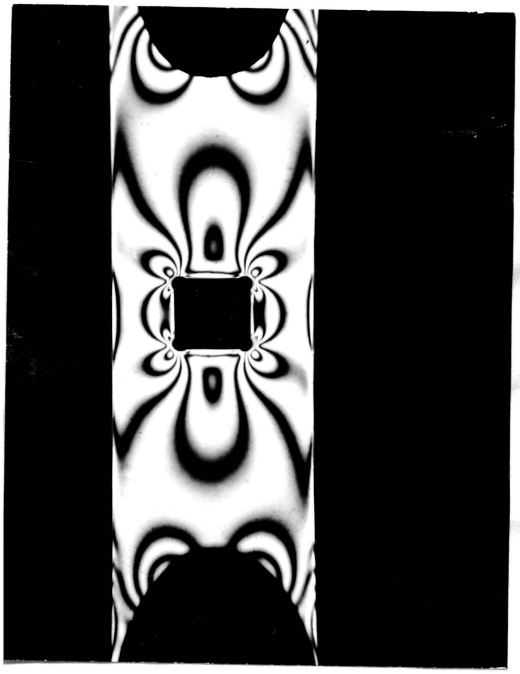


Figure 16

MODEL #9

Width = 1.600 inches

Thickness = 0.203 inches

Notch Diam. = 0.125 inches

Hole Size = 0.6 inches

Load = 299 pounds

Fringe Order = 5.6

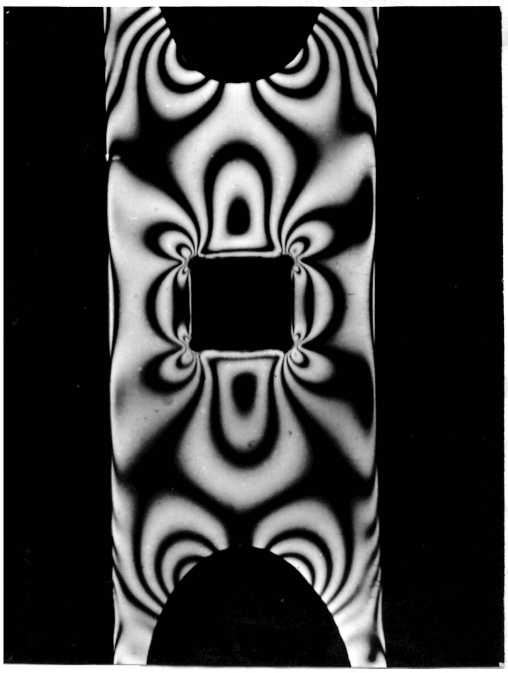


Figure 17

MODEL #10

Width = 2.00 inches

Thickness = 0.2193 inches

Notch Diam. = 0.125 inches

Hole Size = 0.75 inches

Load = 419 pounds

Fringe Order = 7.5

CALCULATIONS

The tensile stress in a model having a hole in it is equal to:

$$S_t' = \frac{(\text{Optical Constant}) \times (\text{Fringe Order}) \times 2}{(\text{Thickness of plate})}$$

Example: Model #6

$$S_t' = \frac{50 \times 10.2 \times 2}{.208} = 4910 \text{ psi.}$$

The tensile stress expected in a model

$$S_t = \frac{\text{Load}}{\text{Area}} = \frac{299}{0.625 \times 0.208} = 2310 \text{ psi.}$$

The stress concentration factor is equal to:

$$S.C. = S_t' / S_t = 4910 / 2310 = 2.12$$

DATA FOR FIRST TEST

Width of plate = 1 inch

Length of plate = 6 inches

Hole Size = 0.375 inches square

Optical constant = 50 psi / inch

TABLE 1

MODEL #	NOTCH DIAM.	LOAD	FRINGE ORDER	STRESS S_t'	STRESS S_t	S.C.
1	0.187	259	11.4	5520	2005	2.75
2	0.156	299	10.8	4960	2200	2.26
3	0.125	299	8.5	4080	2210	1.84
4	0.093	299	9.2	4290	2230	1.92
5	0.076	299	9.2	4320	2320	1.92
6	0.040	299	10.2	4910	2310	2.12

Notch diameter in inches

Load in pounds

Stress S_t' is the tensile stress determined from fringes

Stress S_t is the stress on a plate neglecting corner concentration.

S.C. is the stress concentration factor.

DATA FOR SECOND TEST

Length of plate = 6 inches

Width of plate / width of hole = 2.67

Notch diameter = 0.125 inches

Optical constant = 50 psi / inch

TABLE 2

MODEL #	HOLE SIZE	LOAD	FRINGE ORDER	STRESS S_t'	STRESS S_t	S.C.
3	0.375	299	8.5	4080	2210	1.84
7	0.400	299	8.4	4050	2610	1.88
8	0.500	299	7.4	3660	1755	2.06
9	0.600	299	5.6	2760	1470	1.88
10	0.750	419	7.5	3430	1525	2.25

Hole size in inches

Load in pounds

Stress S_t' is the tensile stress determined from fringes

Stress S_t is the stress on a plate neglecting corner concentration.

S.C. is the stress concentration factor.

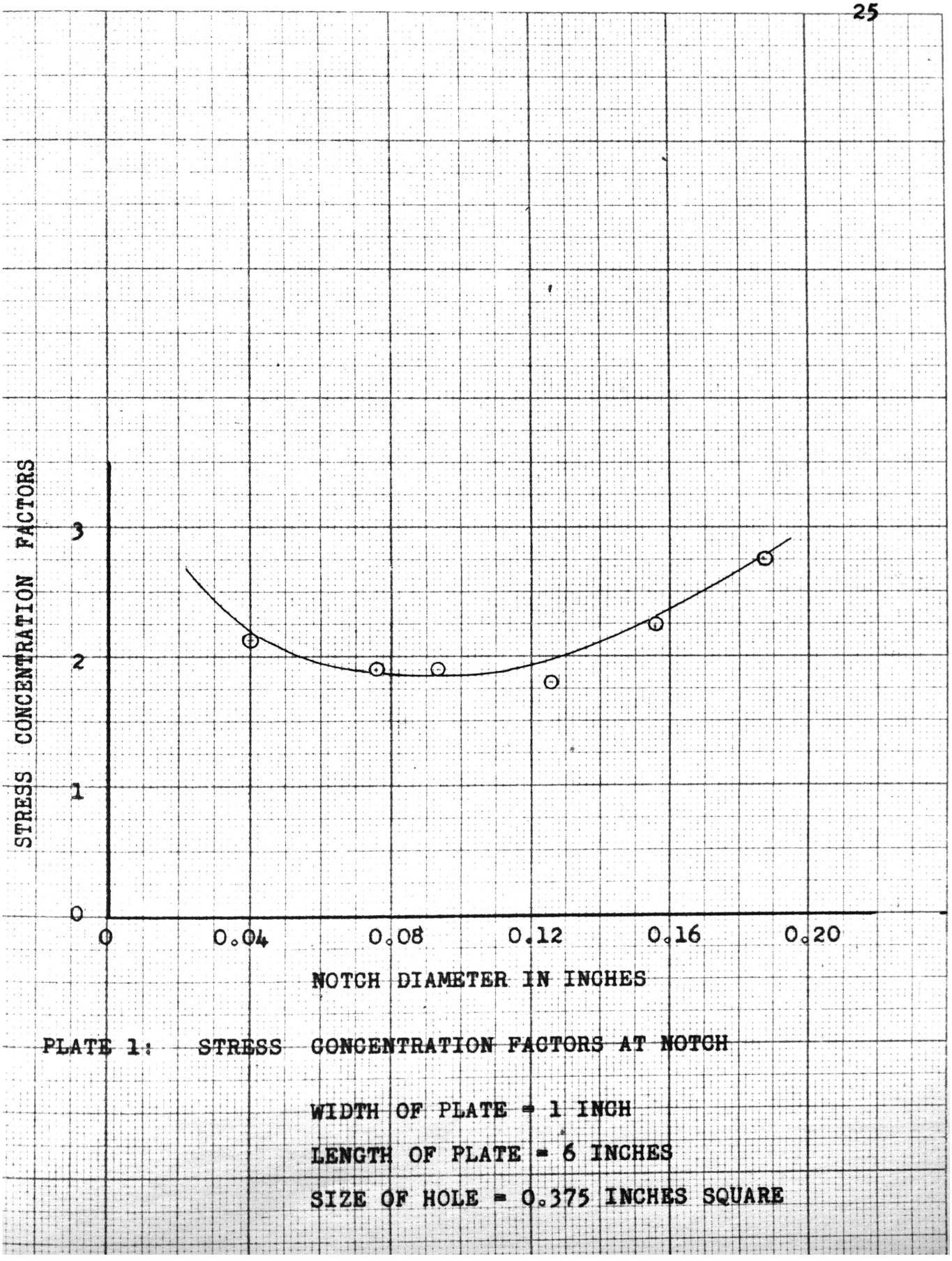
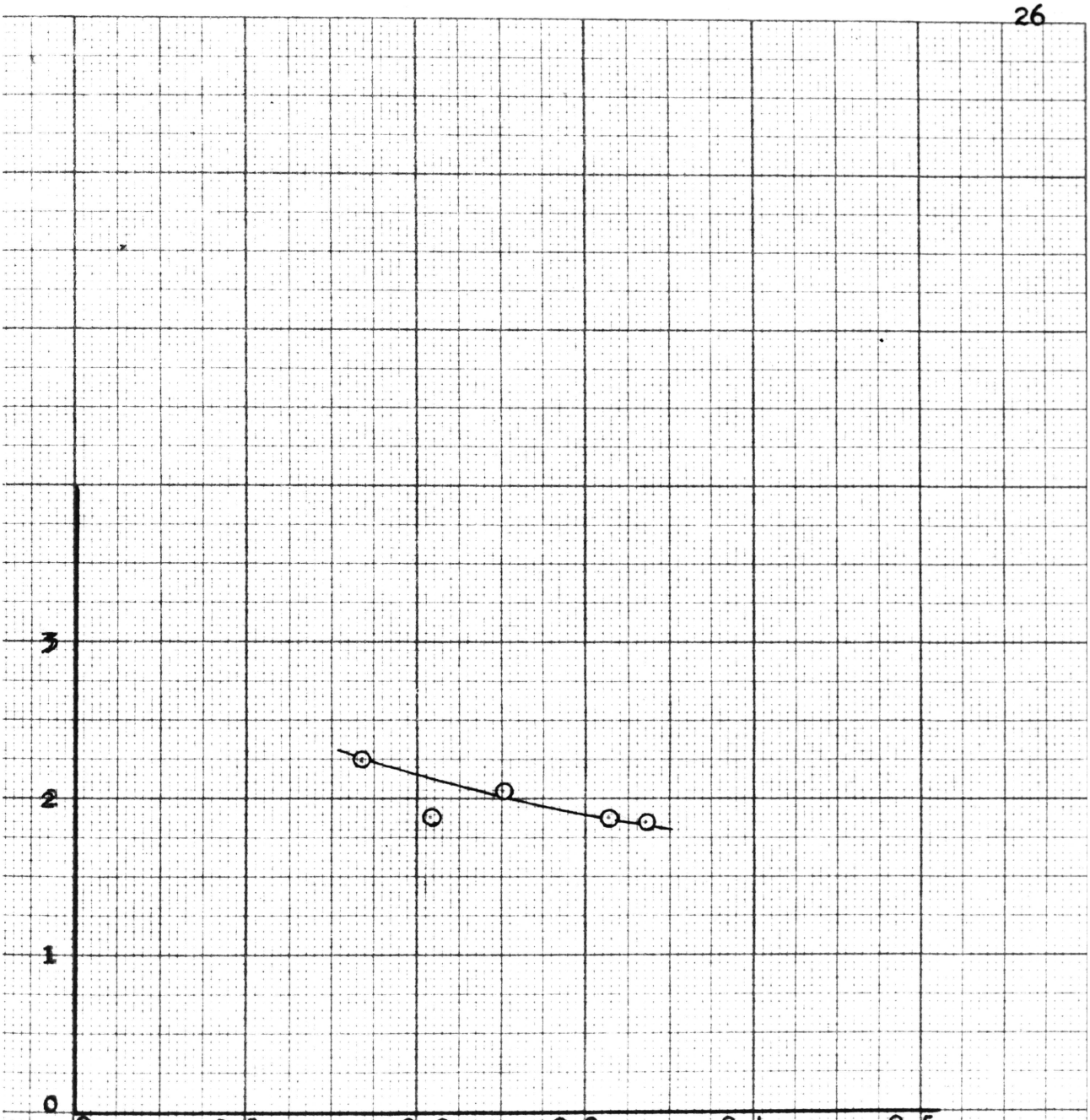


PLATE 1: STRESS CONCENTRATION FACTORS AT NOTCH

WIDTH OF PLATE = 1 INCH

LENGTH OF PLATE = 6 INCHES

SIZE OF HOLE = 0.375 INCHES SQUARE



LATE 2: STRESS CONCENTRATION FACTORS AT NOTCH

NOTCH DIAMETER = 0.125 INCHES

LENGTH OF PLATE = 6 INCHES

CONCLUSIONS

The results of the problem are rather interesting in that the notches of diameters 3/16 inch to 1/8 inch cause the point of maximum stress to move from the expected point at the hole corner to the vertical sides of the hole. This is caused by the larger notches creating another re-entrant corner where they enter the hole and has the effect of a knee frame. For notches of smaller diameters than 1/8 inch the points of maximum stress appear at the edges of the hole corners.

Plate #2 shows the effect of holding the notch diameter constant and varying the size of the hole. It can be seen that as the size of the hole is increased the stress concentration increases slightly.

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VITA

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