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MSM
HISTORICAL
COLLECTION

A COMPARISON OF THE USE OF POLAROID AND COLORED FILTERS
IN THE MULTIPLEX AEROPROJECTOR

BY

ROBERT GREIG LIVINGSTON

- - - - -

A

THESIS

submitted to the faculty of the
SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI
in partial fulfillment of the work required for the
Degree of
MASTER OF SCIENCE IN CIVIL ENGINEERING
Rolla, Missouri
1947

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MSM
HISTORICAL
COLLECTION

Approved By:

E. V. Carlton
Professor of Civil Engineering

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PREFACE

The purpose of this thesis is the drawing of a comparison between the use of the colored filters, which are normally employed in the multiplex aeropjector by various mapping agencies throughout the country, and the use therein of polaroid filters.

The filters in the aeropjector serve to break the projection of a stereoscopic pair of photographs into two impressions which are viewed separately by each eye of the observer, thus providing the main requirement for the observance of a three-dimensional effect. This stereoscopic effect is utilized in the multiplex plotting machine for the delineation of contours.

The colored filters achieve this stereoscopic effect by the projection of adjacent photos in an aerial flight in complementary colors--one picture being projected through a red filter, and the other picture through a blue-green filter. The resulting superimposition of the stereo pair of photographs is called an anaglyph model.

The polaroid filters provide the same stereoscopic effect by the projection of polarized light through the negatives of a stereo pair of photographs, each picture being projected through a polaroid filter whose optical axis is perpendicular to its mate. The resulting superimposition of impressions in opposite planes of polarization will be referred to herein as the polaroid model.

It is necessary, in obtaining the stereoscopic effect, to view each model through a pair of spectacles whose lenses correspond in color, or optical plane, to the filters in the projectors. The separate images thus projected to each eye are fused together in the brain to produce a single, three-dimensional impression.

The colored filters now generally used in the multiplex aeropjector are quite suitable for most mapping, but it is a well-known fact that they appreciably reduce the light which comes through the lenses of the projectors and falls on the plotting table. The polaroid filters would appear to be superior to the colored filters on this particular point, but whether or not they would be as satisfactory in other respects remains to be discovered from subsequent experiments.

In order to form a suitable foundation for the research to follow, it has been deemed worthwhile to include several chapters at the beginning of this thesis which deal with such subjects as the fundamentals of photogrammetry, the use of the multiplex aeropjector, the principles of stereoscopy, and the theory of light polarization.

It is apparent that very little material has been published on the use of polaroid filters in the multiplex machine. A survey of current research with polaroid filters was made through reference to a number of mapping agencies, both governmental and civilian. The only man who has been working on this problem is Mr. H. H. Bradford, of the

Tennessee Valley Authority. To date, the T. V. A. has not released his findings for publication. Mr. Bradford, however, has indicated to the writer that he believes the polaroid model has definite advantages over the anaglyph model.

R. G. Livingston,
January, 1947.

CONTENTS

	<u>Page</u>
Acknowledgment.....	ii
Preface.....	iii
List of illustrations.....	vii
List of tables.....	ix
Introduction.....	1
Fundamental problems of aerial photography.....	7
Control.....	18
Stereoscopy.....	28
The multiplex aeropjector.....	42
Polarization of light.....	56
Preparations for research.....	66
Experiments with polaroid and colored filters.....	71
Conclusion.....	112
Bibliography.....	119

LIST OF ILLUSTRATIONS

<u>Fig.</u>		<u>Page</u>
1.	Aerial cameras.....	3
2.	Overlap.....	5
3.	Determination of scale.....	8
4.	Effect of topographic relief.....	13
5.	Difference in parallax.....	15
6.	Control points.....	24
7.	Drawing of rays through points.....	24
8.	Radial line intersection.....	26
9.	Abrams folding-type lens stereoscope.....	29
10.	Cross-section of human eye.....	29
11.	Minimum angle of depth perception.....	34
12.	Simple stereogram.....	34
13.	Optical illusion of depth.....	38
14.	Stereograms showing correct and pseudoscopic views.....	38
15.	Multiplex reduction printer.....	46
16.	Multiplex plotting machine.....	48
17.	Principles of multiplex orientation.....	50
18.	Multiplex vertical projectors.....	52
19.	Typical relation of aerial negative, diapositive plate, and projected image in multiplex process.....	53
20.	Multiplex tracing table.....	54
21.	Production of polarized light.....	59
22.	Crossed polaroid plates.....	59

<u>Fig.</u>		<u>Page</u>
23.	Principle of polarized light projection of three-dimensional pictures.....	64
24.	Multiplex machine used in research.....	67
25.	Filters and platens.....	69
26.	Weston illumination meter.....	77
27.	Orientation of polaroid filters.....	77
28.	Angles of vision for stereoscopic perception in the anaglyph model.....	86
29.	Angles of vision for stereoscopic perception in the polaroid model.....	87
30.	Model for map-accuracy test.....	93

LIST OF TABLES

<u>Table</u>		<u>Page</u>
1.	Light intensities.....	74
2.	Angles of vision for stereoscopic perception....	85
3.	Multiplex conversion chart.....	95
4.	Recorded elevations on map-accuracy test.....	96

CHAPTER I

INTRODUCTION

1. Photogrammetry:--Photogrammetry may be defined as that science which consists in utilizing photography for making reliable measurements. Applications of photogrammetry are found in a multiplicity of fields, including surgery, astronomy, engineering, architecture, geography, agriculture, criminology, archeology, military operations, aerotechnology, surveying, and mapping. At the present time, the field of mapping overshadows all others in importance and volume.

2. Aerial Photography:--With the advent of automatic cameras mounted in airplanes, it has become possible to photograph huge areas within a short space of time. This method of surveying furnishes one of the most rapid and economical means for making maps, and is well adapted to large areas of irregular tracts where the terrain is rugged or inaccessible; also, the field work is completed in a shorter time, and the photographs furnish a permanent record of the terrain.

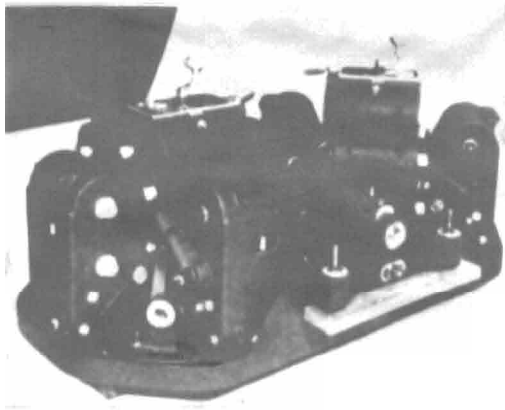
Aerial photographic surveying has now progressed to a point where it can safely be said that no survey of any size will be made without the aid of the aerial photograph. Much has been accomplished since the first World War to promote this type of surveying. Many improvements in cameras and plotting instruments have been made and are still in the process of making.

3. Aerial Cameras:--Most aerial cameras are equipped to use film. One type of camera uses plates, but the necessary weight of plates limits the number of photographs that can be made in one flight. Special high-speed film must be used because the exposure time varies from $1/500$ of a second near the ground to $1/100$ to $1/50$ of a second at higher altitudes. Film strips usually range from 75 to 500 feet in length. The exposures are 7 inches by 9 inches or 9 inches by 9 inches for single-lens cameras.

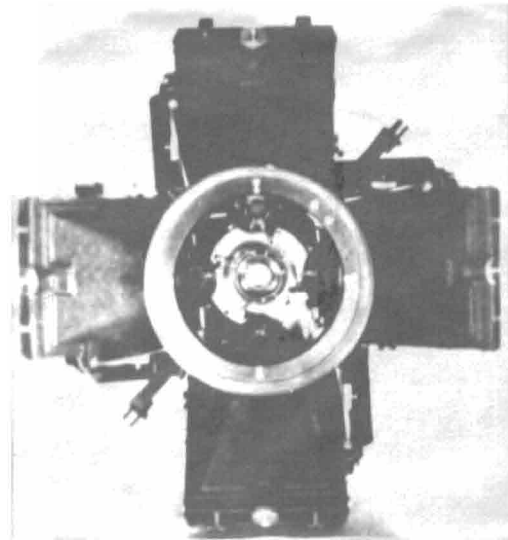
Much surveying has been done with the single-lens camera, taking only one photograph at an exposure. After several years of experiment in this country, a five-lens camera was developed which takes five pictures simultaneously (one vertical and four obliques). The advantages of the five-lens camera (See Fig. 1) are that fewer flight strips and fewer ground control points are necessary. On the other hand, a shorter focal length must be used; also, the plane must be flown at higher altitudes and, therefore, some of the detail is lost. Another type of multiple-lens camera has been developed which uses nine lenses, and takes nine photographs at one exposure.

4. Aerial Photographs:--In general, aerial photographs may be classed as:

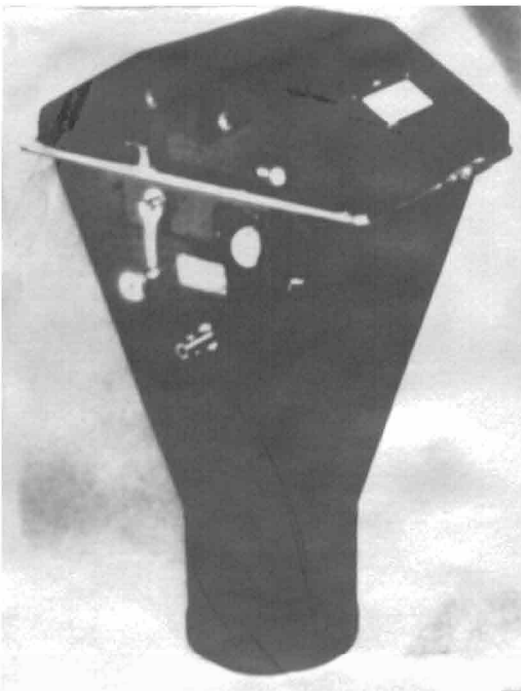
- a. Verticals--those photographs taken with the axis of the camera in a vertical position and taken directly beneath the plane;
- b. Obliques--those photographs taken at an inclined angle;



A. Fairchild Tandem T-3A Camera.



B. Five-Lens Camera.



C. Fairchild K-7A Aerial Camera.



D. Five-Lens Camera.

Fig. 1 - Aerial Cameras.

c. Composites--a combination of the other two, such as those taken with the multiple-lens camera. In this case, the center picture is a vertical and the others are obliques. The obliques are rectified into verticals afterward.

5. Overlap:--Overlap means that photographs are taken so that certain portions of the terrain appear in several pictures. To cover a large area, the photographs are taken in flight strips so that they are overlapped about 60% in the direction of flight (called end lap) and about 30% between adjacent flight strips (called side lap).

The reasons for overlap are:

- a. To orient prints to form continuous flight strips;
- b. To permit the use of a stereoscopic instrument for the drawing of contours on the map. Only the overlapped portion is available for this work;
- c. The central portion of the print is less distorted than the outer edges. With overlapped photographs, the outer portion can be discarded.

In Figure 2, the photographs are set over to illustrate overlap, but in actual practice the edges of a flight strip should be as near to a straight line as possible. It requires very good flying and proper equipment to accomplish a flight mission which gives straight photographic strips and pictures that are free from tilt.

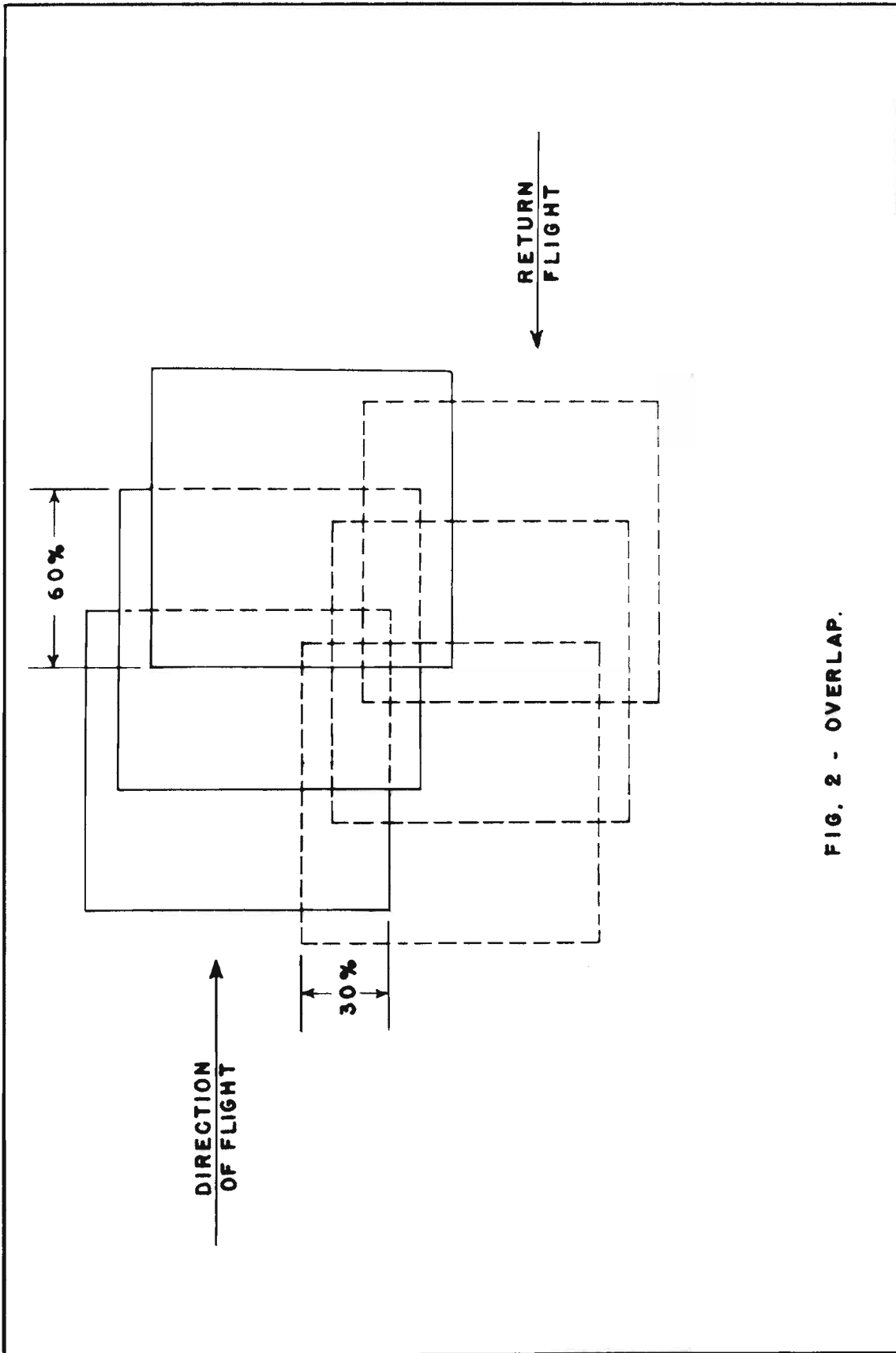


FIG. 2 - OVERLAP.

6. Planning the Photography:--Systematic aerial photography is always planned in advance. Due consideration is given to the time of day and to the season of the year when the area in question will show to the best advantage. Generally, a flight map is prepared from an existing map of the area as a guide for the pilot, showing individual flight lines in relation to landmarks visible from the air. The correct altitude for obtaining photos of the desired scale with the camera to be used is calculated, allowance is made for the proper overlap between adjacent flight lines, and from all of these data, the spacing of the flight lines is computed and drawn on the flight diagram.

7. Map Making from Aerial Photos:--There are four parts to the making of a map from aerial photographs:

- a. The flying,
- b. The photography,
- c. The ground control,
- d. The construction of the map.

The reproduction in quantity of the map could well be considered as the fifth part in the mapping process, as this lithographic operation is ordinarily performed by a different section or group than the one constructing the map.

The flying and photography involved in making the map are not the duties of the engineer, as a rule, but it is to his advantage to have a knowledge of the processes by which good photographs are obtained for his map construction.

CHAPTER II

FUNDAMENTAL PROBLEMS OF AERIAL PHOTOGRAPHY

1. Scale of the Photograph:--Photographs are perspective views that are formed by the light rays which form a cone as projected from the ground to the lens of the camera. Passing through the lens, they form a smaller cone which is intercepted by the negative in the camera. Figure 3 represents the ideal condition, with the ground level and the plate or negative in a horizontal position and parallel to the ground plane.

The scale of a photograph is the ratio of a distance on the photograph to a similar distance on the ground. It also represents the ratio of the focal length of the camera to the elevation of the plane above the ground; for example, a camera with a 12" focal length in a plane flying at an altitude of 5000 feet would produce a picture having a scale of 1/5000. This ratio is called a representative fraction, and is sometimes known as an "R. F."

In Figure 3, the scale of a photograph is determined by a simple proportion, $S = AB/ab = H/f$. If (H), the altitude of the plane, and (f), the focal length of the camera, are given in feet and inches, respectively, the scale will be in feet per inch. The altitude of flight may be anything from 5000 feet to 5 miles. Flights of less than 5000 feet are usually difficult to control, because of the uncertain air conditions near the ground. Flights at heights higher than 5 miles produce pictures that are lacking in sufficiently clear terrain detail.

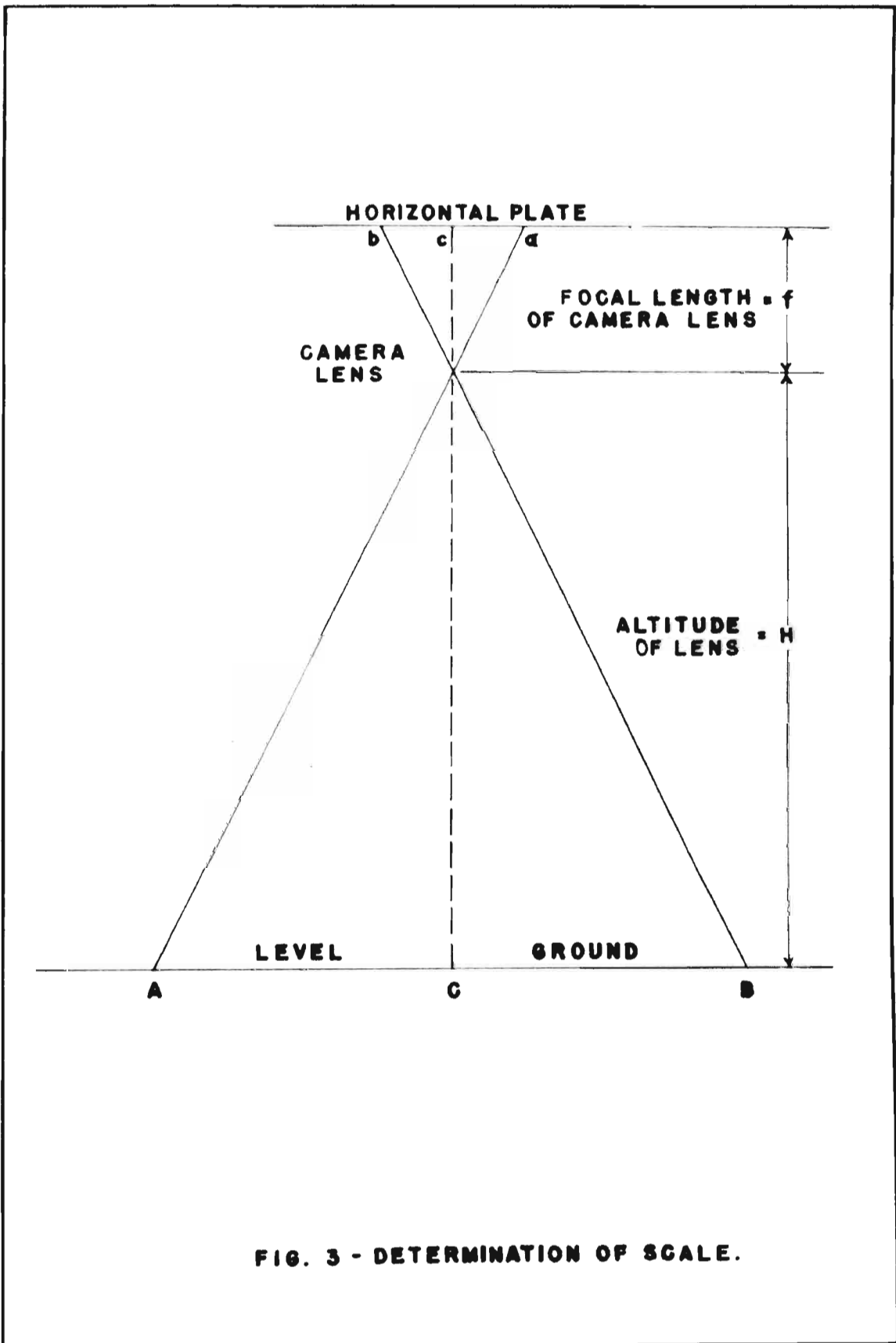


FIG. 3 - DETERMINATION OF SCALE.

Following are typical problems concerning photographic scales:

- a. An airplane with a camera having a focal length of 8 inches flies level over level terrain at an altitude of 12,800 feet above the ground.

(1) How many feet on the ground does one inch on the photograph represent?

$$d = H/f = 12,800/8 = 1600 \text{ ft.}$$

(2) What is the representative fraction of the photograph?

$$\text{R. F.} = 8"/(12,800' \times 12") = 1/19,200$$

- b. Two points on an untilted photograph with the same elevation measure 2.35 inches apart. The same distance on a map whose R. F. is 1/20,000 is 3.67 inches.

(1) What ground distance in feet does one inch on the photograph represent?

$$d = (3.67 \times 20,000)/(2.35 \times 12) = 2600 \text{ ft.}$$

(2) What is the R. F. of the photograph?

$$\text{R. F.} = 2.35/(3.67 \times 20,000) = 1/31,200$$

2. Area Covered by Photographs:--If the scale of the photograph is known and also the length and width of the exposed portion, the area of the ground covered in one photograph is determined by the equation: $\text{Area} = (\text{length} \times \text{scale fraction}) \times (\text{width} \times \text{scale fraction})$.

As an example of the above, assume that the altitude (H) = 10,000 feet, and the focal length of the lens (f) = 10 inches.

Then Scale Fraction = $10,000/10 = 1000$ ft. per in.

Let the size of the exposed negative be 7 inches by 9 inches; the ground area is then found by the following:

$$\text{Area} = (7 \times 1000) \times (9 \times 1000) = 63,000,000 \text{ sq. ft.}$$

Reducing this to acres and then to square miles:

$$\text{Acres} = 63,000,000/43,560 = 1446, \text{ and}$$

$$\text{Square Miles} = 1446/640 = 2.26.$$

This means that each exposure covers an area of approximately 2.26 square miles of territory.

3. Time Interval of Exposure:--Now assume that we want to determine the time interval between exposures in making a flight strip (using the information derived in Paragraph 2 and assuming a ground speed of 100 miles per hour).

Using an end lap of 60% and the 7 inch x 9 inch photograph, we should cover, in the direction of flight, a distance of 2800 feet between exposures, as calculated from the formula:

$$d = 7 \times 1000 \times 0.40 = 2800 \text{ ft.},$$

where d = distance between camera stations; 7 inches = dimension of negative along line of flight; 1000 feet per inch = scale of map; and $0.40 = 100\% - 60\%$.

Since the speed of the plane is 100 miles per hour, this reduces to velocity in feet per second as follows:

$$V = (5280 \times 100)/(60 \times 60) = 146.7 \text{ feet per second.}$$

Then, using the calculated distance between exposures and the velocity of the plane,

$$I = 2800/146.7 = 19.1 \text{ seconds (say 19 seconds)}$$

This means that exposures should be made every 19

seconds during the flight.

4. Number of Pictures to Cover a Given Area:--Suppose that we want to map a quadrangle measuring 15 miles on a side using aerial photos having a scale of 1000 feet per inch. We have a single-lens camera which takes pictures 7" by 9". The overlap between photos in a flight is to be 60% and between flights 30%, with one extra flight across each end of the area for better control. The problem is to calculate how many photographs will be required to cover the given area.

The distance progressed per photograph along the flight line will depend on the width of the photo, the 60% overlap in the line of flight, and the scale of the photograph:

$$D_1 = (100\% - 60\%) \times 7 \times 1000 = 2800 \text{ ft.}$$

The total pictures per flight = $(15 \times 5280)/2800 + 2$
(one extra at each end) = $28 + 2 = 30$.

The distance between flights will depend upon the length of the picture, the 30% overlap between flights, and the scale of the photograph:

$$D_2 = (100\% - 30\%) \times 9 \times 1000 = 6300 \text{ ft.}$$

The total number of flights (one extra at each end) = $(15 \times 5280)/6300 + 2 = 13 + 2 = 15$.

The total number of photographs for the entire area will be the product of the photos per flight and the number of flights, as follows:

$$N = 30 \times 15 = 450 \text{ photos.}$$

5. Sources of Error in Aerial Photos:--The most pronounced errors in aerial photographs are directly the result

of one or more of the following:

- a. Variations in the altitude of the plane, which would cause a corresponding change in the scale of the photographs,
- b. Radial displacements due to ground relief, and
- c. Tilting of the plane.

Good flying will tend to eliminate the first and last of the above, since the essentials of good flying are: constant elevation above the ground; constant overlap, laterally and in the direction of flight; and elimination of tilt.

6. Errors Due to Ground Relief:--One source of error will be discussed herein--that of displacement due to ground relief. This displacement is of prime importance in mapping as it can be used to contour photos, especially if the area be a section of high relief.

In Figure 4, let A-B be the ground relief with respect to the datum plane shown, point A being below the datum plane and point B above the datum plane. The camera lens is at L, and the photographed positions of A and B are at a and b on the negative.

Since maps are orthographic projections, B would be projected on a plane at B', and A at A'. The positions of these points, as photographed, will be displaced. The point B is displaced away from the center C, and point A toward the center C.

The amount of this displacement is given by the following equation: $\Delta D = h \tan \alpha$, or on the plate, $\Delta d = h/S \tan \alpha$.

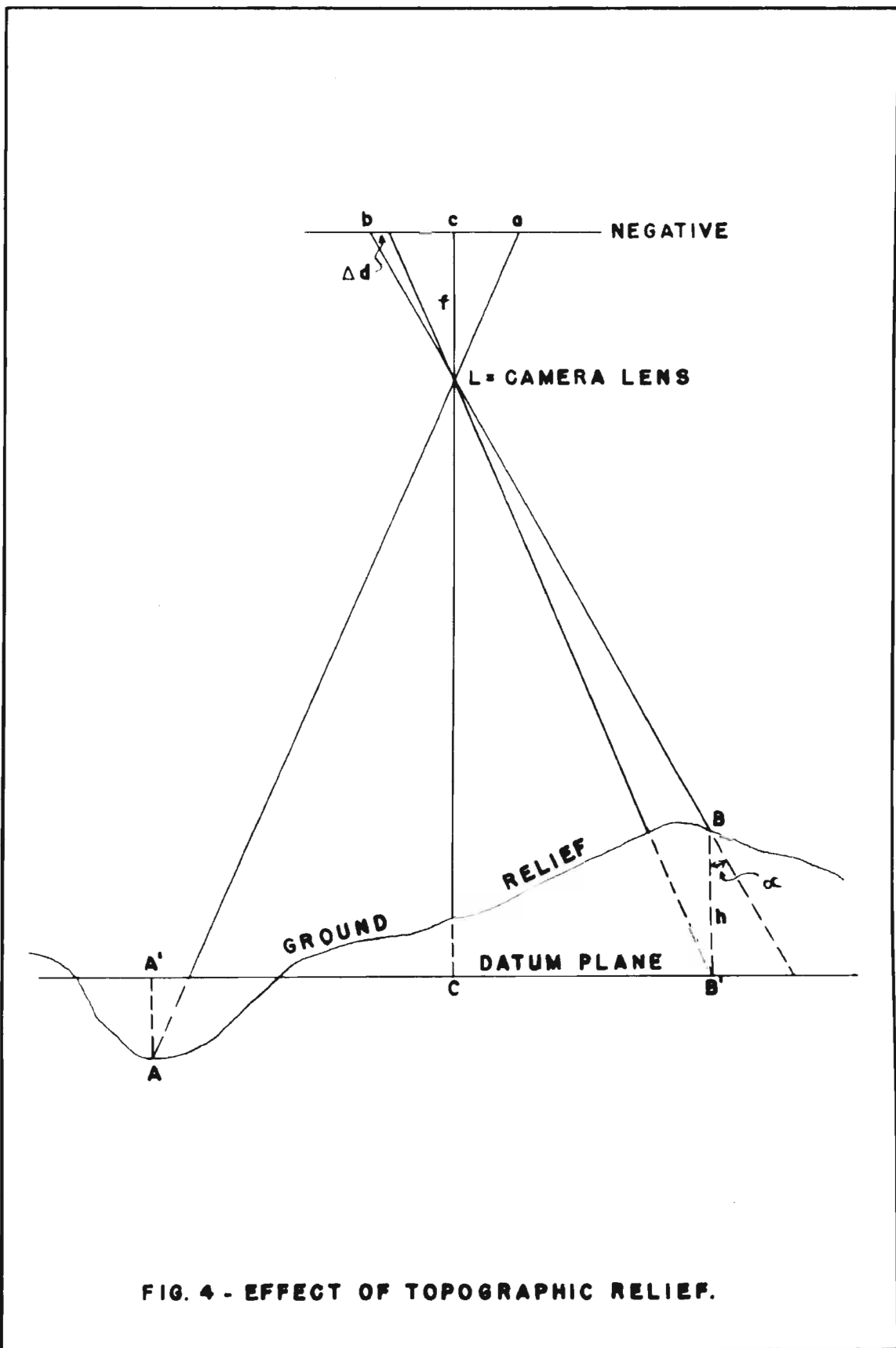


FIG. 4 - EFFECT OF TOPOGRAPHIC RELIEF.

As α varies with the distance from the vertical axis, the displacement varies with this distance and the ground elevation (h). From this, it is seen that the farther away from the center of the photograph, the more the displacement. This would indicate that the central portion of the photo is the least distorted; this is one reason for using overlapped negatives, so that the edges may be discarded.

If we know h , the elevation of B from actual levels, and α , which can be figured from $\tan\alpha = bc/f$, where bc is measured from the photograph, the amount of displacement is given by $\Delta D = h \tan\alpha$.

The following example will show the amount of displacement when the area presents marked relief. Assume that the flight is made at 10,000 ft. altitude; focal length of camera lens = 10 inches; elevation of point A below datum = 520 feet. Referring to Figure 4, measure the distance $a-c$ on the negative, which is found to be 2.5 inches.

The scale = $H/f = 10,000/10 = 1000$ ft. per in.

The horizontal distance on the ground to point A is $1000 \times 2.5 = 2500$ ft.

Also $\tan\alpha = ac/f = 2.5/10 = +0.25$.

Then $\Delta D = h \tan\alpha = 520 \times 0.25 = 130$ ft. (Displacement).

This shows that, in a distance of 2500 feet, the point is displaced 130 feet under the above assumed conditions and is displaced toward the center. If the point were above the datum plane 520 feet, it would have been displaced outward from the center by a like amount.

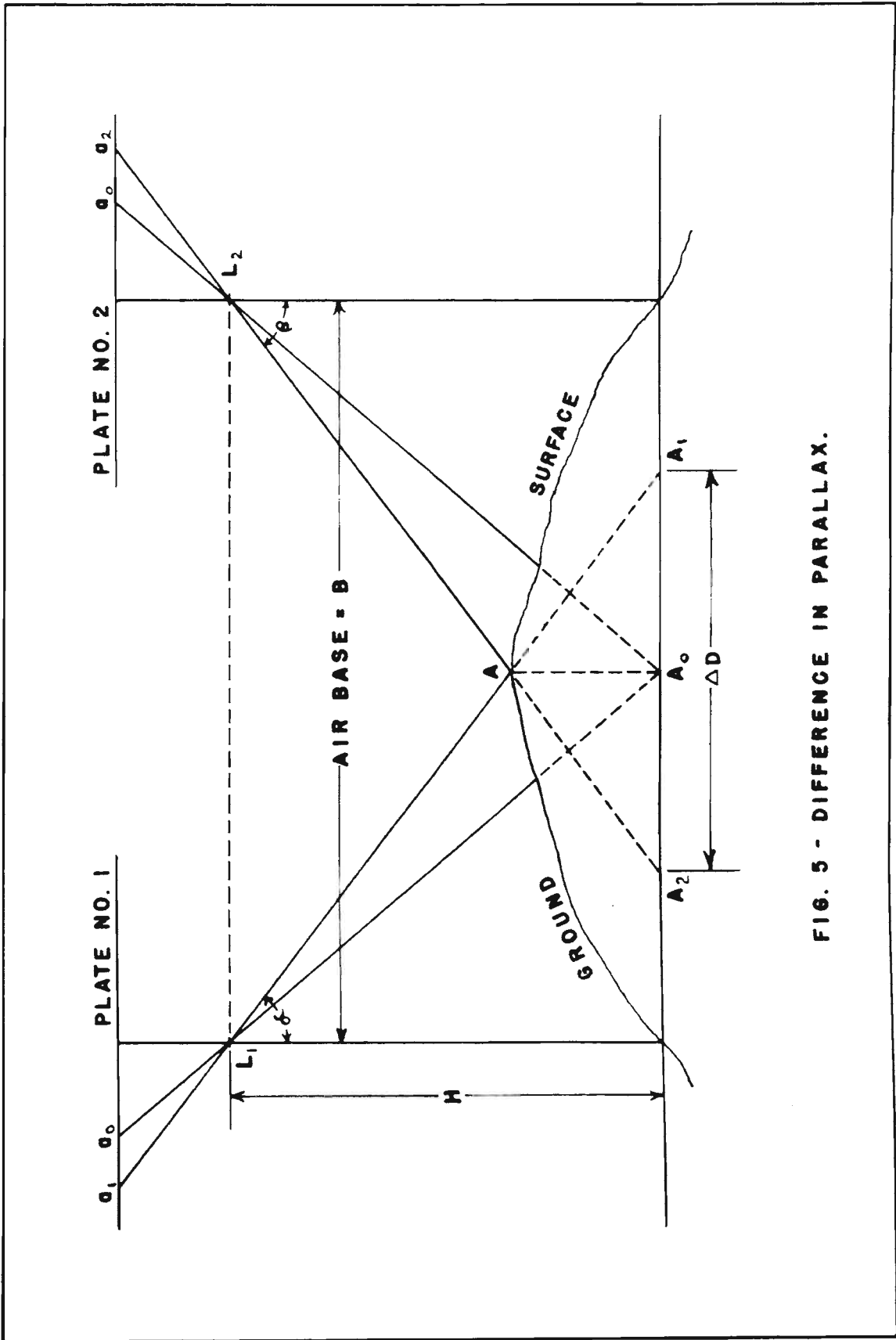


FIG. 5 - DIFFERENCE IN PARALLAX.

7. Parallax Difference:--Assume now that two overlapped photographs are used and placed in the positions shown in Figure 5. Let a_1a_0 in plate 1 = Δd_1 (the displacement in the first photograph) due to elevation (h), and a_2a_0 in plate 2 = Δd_2 (the displacement in the second photograph) due to elevation (h).

From Paragraph 6, $\Delta d_1 = h/s \tan \alpha$ and $\Delta d_2 = h/s \tan \theta$. The total displacement, Δd , on the plate or negative is, therefore, the sum of Δd_1 and Δd_2 , stated as follows:

$$\Delta d = \Delta d_1 + \Delta d_2 = h(\tan \alpha + \tan \theta)/S.$$

Let B = the distance between exposure stations L_1 and L_2 , called the air base. Then, by similar triangles, $B/D = (H - h)/h$.

Let $\Delta D_1 =$ the displacement on the ground A_0A_1 , and $\Delta D_2 =$ the displacement on the ground A_0A_2 . Then $\Delta D = \Delta D_1 + \Delta D_2$.

But $\Delta D_1 = (\text{Scale}) \times \Delta d_1$, and $\Delta D_2 = (\text{Scale}) \times \Delta d_2$.

Let scale $H/f = S$. Therefore $\Delta D = S(\Delta d_1 + \Delta d_2)$, or, since $\Delta d = \Delta d_1 + \Delta d_2$, $\Delta D = S\Delta d$.

Substituting ΔD in the equation $B/D = (H - h)/h$, we have $B/S\Delta d = (H - h)/h$, or $\Delta d = Bh/S(H - h)$.

Substituting $S = H/f$, we have $\Delta d = f/H \times Bh/(H - h)$.

From this equation, the difference in displacement can be computed for a given elevation or, conversely, the elevation can be determined if the displacement is accurately measured on the photograph.

This latter displacement can be measured in the stereoscopic treatment of a pair of overlapped photographs in an

instrument such as the stereocomparator. To illustrate, assume the following problem: Let the airbase (B) = 2500 feet; the altitude (H) = 10,000 feet; the focal length (f) = 10 inches. Therefore scale (S) = $H/f = 10,000/10 = 1000$ ft. per inch.

The parallax displacement measured from the two negatives is 0.147 inch, outward from the center. To determine (h), the height above the datum plane (because the displacement is outward), take the equation just derived, $\Delta d = Bh/S(H - h)$, and solve for (h).

$$\Delta dSH - S\Delta dh = Bh, \text{ and } h(B + \Delta dS) = \Delta dSH$$

$$\text{Then } (h) = \Delta dSH / (B + \Delta dS).$$

Substituting the values of the problem, we have (h) = $(0.147 \times 1000 \times 10,000) / (2500 + 0.147 \times 1000) = 554$ feet (approximately).

CHAPTER III

CONTROL

1. Ground Control:--Ground control is divided into horizontal control and vertical control. A certain amount of ground surveying is required in nearly all the practical methods in use today. Control stations are usually selected after the photographs are taken, because then the points which show up best on the photographs can be chosen. It is best to have these stations in the form of a triangulation system so that angles can be read with an instrument, and distances measured. The coordinates of all ground stations can then be computed. This horizontal control is necessary for tying detail of the photographs into the map. Levels are also run to determine the elevations of all control stations. This is vertical control.

In selecting the control stations, points on the tops of hills do not in general make the best stations, as in terrestrial surveying, but rather points at average elevations of the terrain to be mapped. The reason for not using the tops of hills is evident from the fact that too much change of scale may be introduced because the value of (H) is reduced, thereby decreasing the scale fraction, H/f .

The selection and identification of all control points should be made with the photographs under a stereoscope. All control points should be marked with small pin pricks and circled with colored pencils. It is advisable to use different colors for different types of control, such as ground control and picture control.

Good control points are those which can be definitely spotted on the photographs, such as road intersections, fence corners, railroad and stream crossings, buildings, intersections of walks and paths, and culverts.

2. Picture Control:--Picture control is that control which can be identified easily on the photograph but for which there is no ground control. The purpose of this type of control is to supplement ground control.

These points may be pricked on a stereoscopic pair of photographs under a stereoscope using a small instrument for the purpose called a point selector. It consists of two small bronze frames which carry flat pieces of glass etched on the lower side with symmetrical black crosses. At the intersection of the crosses are small pin holes. One glass is placed with its pin hole over the control point on the one picture; the etched cross on the other glass is brought into fusion with that on the other picture, and the second picture's corresponding control point is pricked.

3. Orientation:--In order to assemble a series of photographs taken in a flight strip so as to make a continuous map without serious error, the photographs must be carefully oriented with respect to each other and with respect to true north. To accomplish the exact orientation involves considerable computation.

Orientation of photographs may be divided into two methods--analytical and graphical.

4. Analytical Orientation:--This method involves the determination of a number of elements which may be sub-

divided into two classes as follows:

- a. Elements of interior orientation are those which affect the negative and camera; they are:
 - (1) The x-coordinate on the plate of the principal point,
 - (2) The y-coordinate on the plate of the principal point, and
 - (3) The principal distance of the camera.
- b. Elements of exterior orientation are those which affect the location of the camera station and photograph, with respect to other camera stations, in a flight strip; they are used in assembling a series of photographs to make one continuous map; they are:
 - (1) Tilt,
 - (2) Swing, and
 - (3) The survey azimuth of the principal plane.

There are several involved mathematical solutions of the above elements; however, these will not be considered here, since the aeroprojector, upon which our experiments will be conducted, has adjustments for each of these elements, and automatically orients by setting up the original camera stations of the photos, with respect to each other, at a miniature scale.

5. Graphical Orientation:--Several graphical methods of orientation of aerial photographs have been used and, for certain purposes, give good results. They may be classified as follows:

- a. Straight-line method,
- b. Section-line method,
- c. Three-point method,
- d. Radial-line method,
- e. Template method,
- f. Slotted-template method.

In all these methods, attempts are made to match photographs by intersections in somewhat the same manner that is used in ordinary plane-table orientation.

As mentioned before, these methods are unnecessary in the orientation of photographs for plotting with the multiplex machine, but must be used where smaller plotting instruments, such as the contour finder or stereocomparagraph, are employed. It is deemed advisable, however, to include herein the various steps in the radial-line method of orientation, as this particular method remains the basis for the later template methods.

6. Radial-Line Method:--In this method, the principal point may be used as the origin of radial directions of displacements of objects shown on photographs tilted less than three degrees without plottable error when the relief is less than ten per cent of the camera altitude. The principal point is the geometric center of the photograph, the point formed by the intersection of the coordinate axes, and will be considered in this method as an "instrument station". Lines radiating from it may be drawn on the photograph through objects which have been, or which are to be, plotted on the map.

Using the principal point of the photograph instead of its nadir point (point on the ground directly beneath the camera lens) causes this method to be only a close approximation. It follows that, in a photograph which has no tilt, the principal point and the nadir point will be identical.

Radial-line plotting includes the following steps:

- a. A certain amount of ground control must be run, and the horizontal and vertical positions determined from precise surveying methods. There should be, preferably, at least three ground-control points visible in each photograph, and these should be at an approximate average elevation of the pictured terrain. It is best to select these after the photograph is taken, because a better choice of points can be made.
- b. Select and mark on the photographs these ground-control points with a small, blue circle.
- c. Determine and mark the principal point of the photograph with a small, red circle. This point can be found at the intersection of lines drawn between the marks registered at the time of exposure at the corners or sides of the photograph (fiducial marks). This principal point may then be used as an instrument station from which radiating lines may be drawn on the photograph to the previously marked ground-control points.
- d. Select a large sheet of low-shrink celluloid and on this plot the ground-control points to the

- scale of the desired map.
- e. Place photograph 1 under the celluloid and cause the radial lines on the photograph to pass through the respective ground-control points on the celluloid. This corresponds to the three-point fix commonly used in plane-table work.
 - f. Select about nine picture-control points, three near the upper margin of print 1, three across the center, and three across the bottom. These should be distributed so that three fall near the right edge of the photograph, three through the center, and three near the left edge. The picture-control points are selected best by studying the adjacent photographs of a flight strip in a stereoscope. They are points such as road intersections, prominent houses, and any other features which can be easily identified in two or more prints. They are marked with small, yellow circles and may be selected and marked at the same time that the ground-control points are marked (See Fig. 6).
 - g. With the photograph under the celluloid, draw on the celluloid radial lines from the principal point through each picture-control point (See Fig. 7).
 - h. Next, photograph 2 is oriented in the same manner, its control points spotted, some of which may occur in photograph 1, and radials drawn on

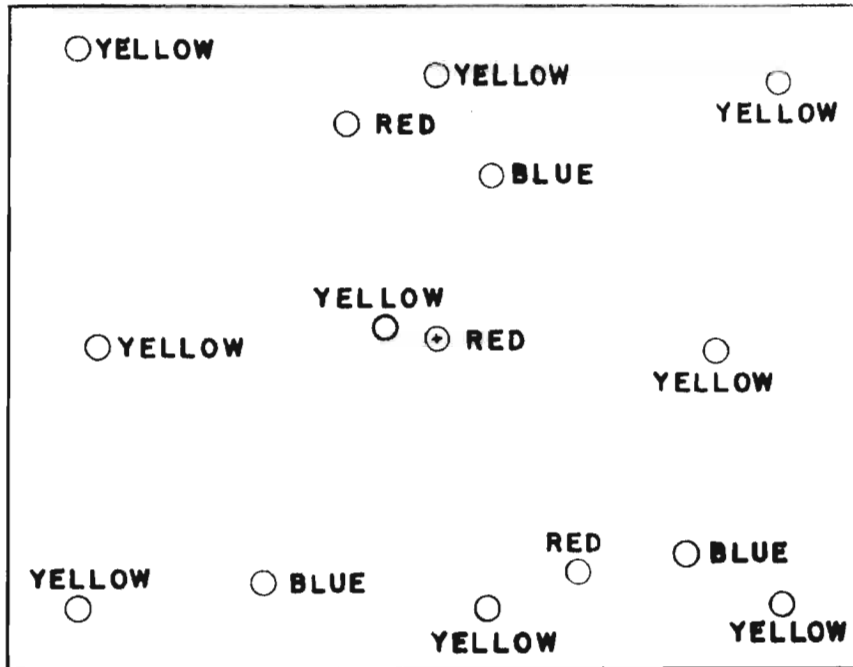


FIG. 6 - CONTROL POINTS.

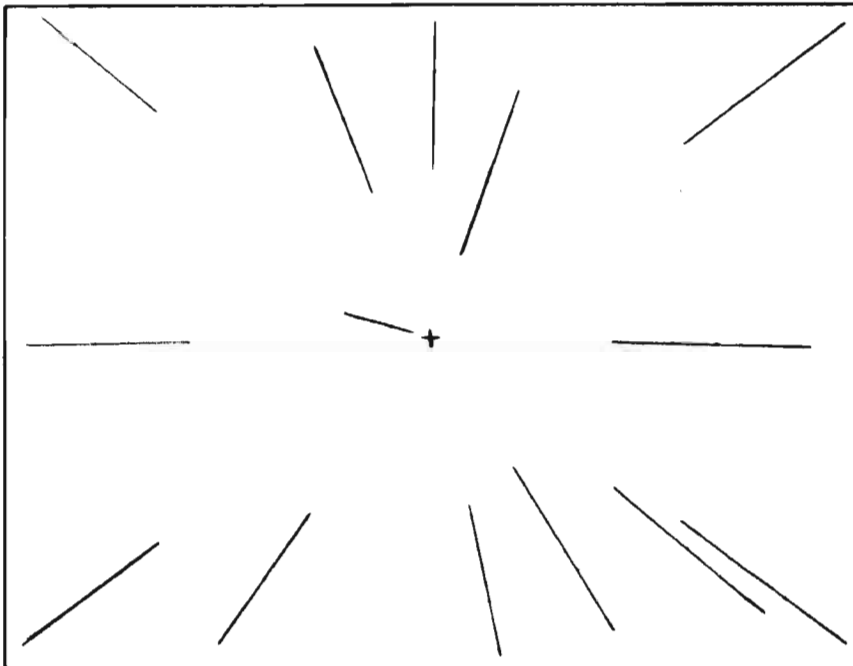


FIG. 7 - DRAWING OF RAYS THROUGH POINTS.

the print through these points from the principal point of photograph 2.

- i. As many of the picture-control points as occur in photograph 1 are marked with yellow circles on print 2, and three new ones selected. The new ones are in the part of photograph 2 which does not overlap photo 1.
- j. Photograph 2 is now placed under the celluloid sheet and radials drawn through each picture-control point. Six of these will intersect radials to corresponding points as drawn from photograph 1, with three new ones. The intersection of these radial lines is the approximate plotted position of the picture-control point.
- k. Next, draw radials in picture 3 in the same manner. From this photograph a third radial will intersect the other two radials in the previously drawn same points, as in Figure 8. Owing to tilt and other causes, this may not happen, and a small triangle of error may result. In such a case, the actual point will be considered as in the center of the triangle.
- l. After passing through a series of photographs in this manner, the final fix of the photographs is made by shifting the photographs under the celluloid until the best orientation is made, using both ground control and picture control. The position of the photograph is then marked and

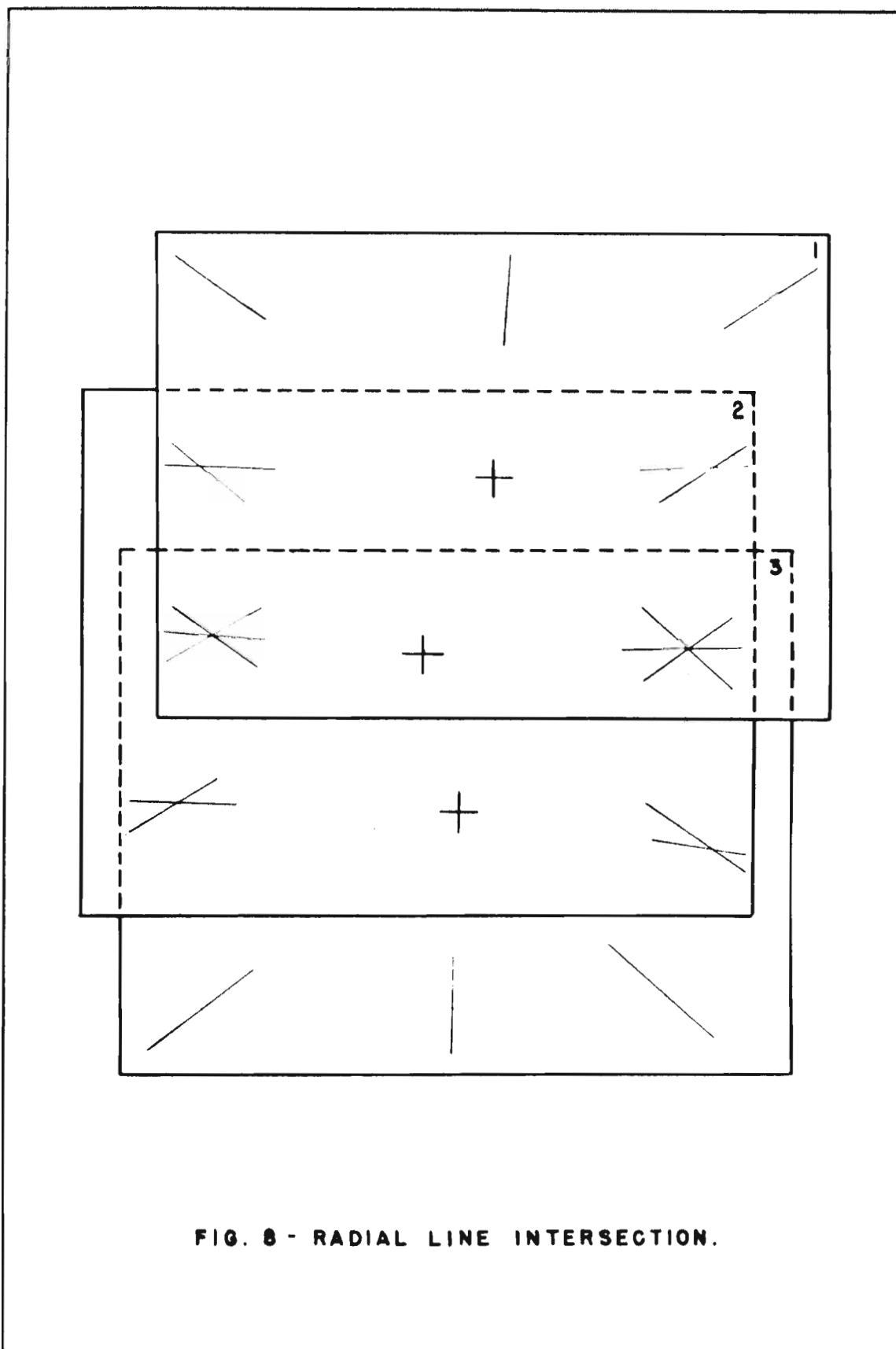


FIG. 8 - RADIAL LINE INTERSECTION.

the detail traced. This is repeated for photograph 2 and so on through the strip. In tracing detail, only the center part of the print is used because of the distortion at the edges.

CHAPTER IV

STEREOSCOPY

1. Definition:--Closely allied to photogrammetry is the field of stereoscopy--the viewing of objects in three dimensions. Its application to photogrammetry is the observation of photographs with optical instruments for the purpose of measuring relative heights of objects thus shown, and also to define the shape and positions of such objects. It is well to here note that all current instruments for the plotting of contours use this principle in their operation.

2. Stereoscopes:--Stereoscopic instruments may be of the mirror (reflecting) type, the prism type, or the lens (refracting) type, or a combination of all. The first recorded optical instrument incorporating the principles of stereoscopy was a mirror-type stereoscope developed by Robert Wheatstone in 1838. The most common present type, the lens stereoscope, consists essentially of two convex lenses separated about $3/8$ inch farther apart than the interpupillary distance of the observer's eyes (See Fig. 9).

3. Principles of Vision:--The principles of stereoscopic perception are better understood with a discussion, from the physiological standpoint, of the workings of the human eye and mind.

The faculty of vision is so natural and customary that we seldom pause to appraise it, or are in the least bit conscious of the intricate processes involved. In the process of vision, three important elements--the eyeball, the optic nerve, and the visual centers of the brain--seem to be



Fig. 9 - Abrams Folding Type Lens Stereoscope.

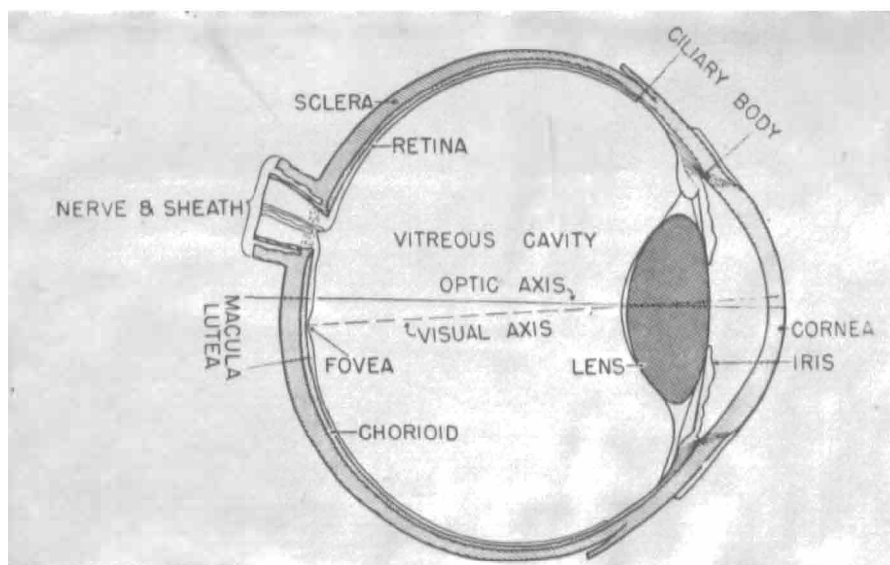


Fig. 10 - Internal Arrangement of the Human Eye.

linked together. The eyeball is globular in form and contains the dioptric apparatus and nervous mechanism which is sensitive to stimulation by luminous radiation from without. The visual impulses thus produced by the impact of light are then transmitted to the brain, where the sensation of vision comes to consciousness (See Fig. 10).

The retina, which constitutes the beginning element of visual perception, is perhaps the most important of all the eye components. A transverse section of the retina would show that it is made up of several million "cones" and "rods" in addition to miscellaneous nerve fibers and cells. As light falls upon the rods, a chemical change occurs within them which, in turn, stimulates the optic nerve and sends a message to the brain. Each rod may be likened to the sensitized coating on a photographic film, with the primary difference being that the rod has the power to regenerate itself. This latter process is continuously being accomplished both in daylight and in darkness at the rate of about five hundred times per minute.

Whenever the eye fixes its attention on an object, the image is sharply focused on a small area of the retina called the macula lutea. The high concentration of cones located at this spot tends to enhance the perception of detail. It is probable that the cones make possible the ability to see objects sharply over a small central field of view, while the rods dominate the viewing of movements and orientation of objects in the remainder of the outer portion of the field of view. Nerve fibers leading from the

retina to the brain carry the numerous stimulations that are thus set up, and "develop" them by a mental process into a composite picture.

Normally, the mobile human eye is capable of covering a horizontal field of view of about 45 degrees inward and 135 degrees outward, and a vertical range of approximately 50 degrees upward and 70 degrees downward.

Although the single human eye (monocular vision) affords a wide range of view in a horizontal and vertical direction, it is much limited in its ability to convey accurate conceptions of depth. Relative directions of objects fixed in space can readily be determined, but the process of being able to determine accurately whether one object is nearer or farther from another is impossible. A perspective view is all that can ordinarily be obtained.

4. Binocular Vision:--Fortunately, man is blessed with two eyes instead of one; his faculty of vision thereby is greatly enlarged and reinforced. Each eye is capable of executing its own movement, but constant training and use in the interest of distinct binocular vision has linked the units together to function as a "double eye". Reactions and movements are invariably made in unison. It will be found that the eyes will work together either as parallel lines of fixation, or as a duplex organ of sight in converging or diverging operations. In the process of convergence, the two eyes tend to work in unison whenever a change is made in the position of fixation. Such unified change in the position of fixation may occur outward or inward along the same line of

vision or as a unified movement to another line of vision.

An optical characteristic which is often encountered in connection with binocular vision is that of a "double image". For example, when the eyes of the observer are focused for a certain distance, any object lying nearer or farther away will be seen as a double image. In normal binocular vision, double images will not ordinarily be noticeable for, as a rule, they are seen only when the viewer's attention is drawn to them by concentration. On the other hand, persons with defective vision may see all objects doubly, although one of the images may be suppressed in consciousness.

Additional factors which affect vision are intensity of light, differences in brightness between adjacent areas of an object, distance of an object from the observer, and sharpness of boundary between adjacent areas. For instance, a brightly colored dot can easily be seen on a black background, but can hardly be seen on a background having a color which differs only slightly from the color of the dot. All objects appear about equal in size at the limit of visibility; vision increases as the angular size of the object increases.

5. Radius of Stereoscopic Perception:--As mentioned before, all that can be determined about the position of an object in monocular vision is its relative direction in the field of view. Binocular vision, on the other hand, affords some estimate of distance and depth perception provided the image is not too far away as compared with the interpupillary distance (average value is about 2.625 inches) of the

observer. Even with normal binocular vision, it has been found that it is impossible to distinguish between objects if the difference of the angle of convergence is less than about 20 seconds of arc (0.0000965 radians).

Thus, for a range of interpupillary distance (d) of 1.97 inches to 2.85 inches (the general range for different persons) and angle (θ) of 20 seconds, the distance from an observer (R) that an object would still appear to have depth would be from 1700 feet to 2450 feet, respectively (See Fig. 11). Beyond that distance the naked eyes, alone, cannot discriminate differences of distances or depths of objects, since beyond that point all objects appear to be projected on the infinite background of space. The distance (R) has often been called the "radius of stereoscopic perception".

When the distance (R) is relatively large, the following relation can be used for approximate results:

$$R = d/\theta = d/0.000097 = 10,315d,$$

where both (R) and (d) are in the same linear units and (θ) is expressed in radians.

In aerial mapping operations involving the use of stereoscopic instruments, it will be found that visual acuity is dependent not only upon the inherent limitations of the instruments used and upon the physical nature of light but also upon the physiological state of the individual. Such factors as stimulants, fatigue, mental depression, distracting noises, unsatisfactory illumination, uncomfortable viewing position, and the improper humidity and ventilation of the workshop, all tend to interfere seriously with results.

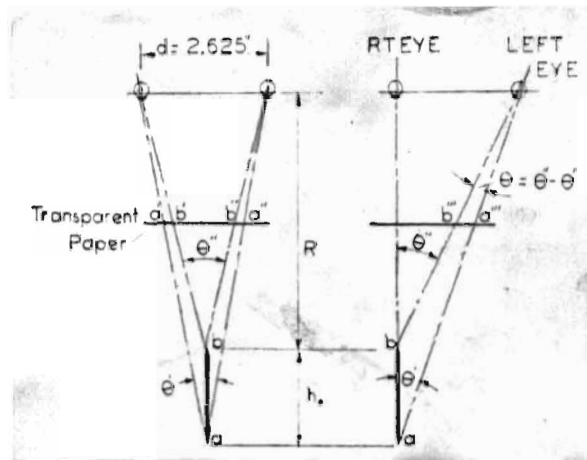


Fig. 11 - Minimum Angle of Depth-Perception.

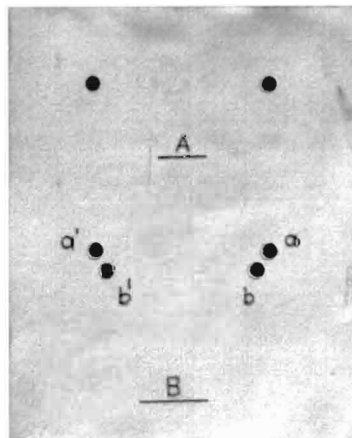


Fig. 12 - Simple Stereogram.

Because of the number of complicated biological factors thus involved, it is very difficult to establish or confirm limits of visual acuity.

The manifestations of stereoscopic vision can best be studied and illustrated by means of pictorial views of the same object as seen from different angles by each eye separately. Figure 12B shows a simple stereogram (spatial model) consisting of two parallel rows of two dots each, with the lower set of dots spaced a little closer together than the upper set. By staring at the two left-hand dots with the left eye and the two right-hand dots with the right eye, it will be found that the dots (a') and (a'') will fuse and the dots (b') and (b'') will also appear to fuse, but above the other fused pair. The phenomenon of the "floating dot" results because the angle of convergence of the upper row of dots is greater than that of the lower row. Hence, it is seen that such distances as (a'b') and (a''b'') may be used as a direct measurement of the relative heights of the objects they represent. The equation of this parallax difference has already been discussed in Chapter II.

6. Limitations of Stereo Perception:--It has been shown that there is a definite limit (20 seconds of arc) to the difference between the two converging angles of fixation upon an object. Such limit would be equivalent to a minimum measurement of differences of elevation of two objects of approximately 0.004 inches, where no magnification is used. At a scale of 1/20,000, differences of elevation of seven feet or more could be determined by the above process, but a differ-

ence of less than seven feet could be determined only by means of some type of magnifying apparatus.

From the formula for determining the radius of stereoscopic perception, it was found that a value of about 2200 feet would result if an average interpupillary distance of 2.55 inches was chosen. It can further be shown that differences in distances can be determined stereoscopically up to the square of the distance to the object divided by the above figure of 2200 feet. Thus, if an object is 1000 feet away from the observer, other objects up to $(1000)^2/2200 = 455$ feet beyond the first object cannot be perceived stereoscopically because, up to that point, the difference in the angles of convergence is 20 seconds or less.

Often, certain geometric figures present themselves as illusions which are purely mental. That such illusions have nothing to do with binocular vision may be proved by the fact that they are more obvious when regarded only with one eye. Figure 13 illustrates an illusion of this type. Monocular conception of depth appears, therefore, to be more of a mental than optical process, as differentiated from binocular vision, which is strictly optical.

Artificial enhancement of the power of stereoscopic vision can be obtained by increasing the virtual base-line (interpupillary distance) or by the introduction of a magnifying optical instrument (which directly tends to lower the effective value of the angle, θ). The "stereo power" of a binocular instrument is found by multiplying the magnifying power of the lens (M) by the ratio of increased optical base

to interpupillary distance (c).

7. Stereoscopic Studies of Aerial Photos:--The object of stereoscopic methods in a study of aerial photos is to obtain a truly three-dimensional picture of the area or object viewed. This requires that the conditions for natural depth perception be artificially reconstructed, and necessitates the following: (a) two slightly different views of the same object or scene from different angles; (b) correct orientation of the two views with respect to one another and to the eyes; and (c) normal binocular vision, allowing the two images to be recombined in the brain so as to give the effect of a single picture having solidity and depth.

The multiplex machine, with which we are primarily concerned, automatically adjusts for the first two of the above conditions. However, it is well for the observer to have a thorough knowledge of the stereoscope, as he will likely use it in picking the prospective control points on the photographs for subsequent reference to the field parties.

In using the stereoscope for the first time, correct procedure and photos of good quality are important. The photos should be fairly well matched for average color tone, should have at least moderate contrast, and should show well-defined features on a topography of at least moderate relief.

In a discussion of correct orientation of the photos with respect to one another and to the instrument, it is well

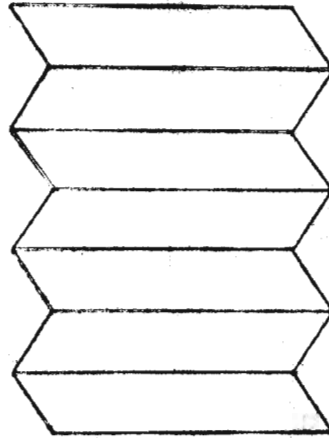


Fig. 13 - Optical Illusion of Depth.



Fig. 14 - Stereograms Showing Correct and "Pseudoscopic" Views.

to note three terms--eye base, stereo base, and photo base. "Eye base" represents an imaginary line between the pupils of the eyes. Its length equals the distance between the pupils when the eyes are focused at infinity (interpupillary distance). "Stereo base" may be defined as the direction and distance between complementary image points on a stereo pair of photos correctly adjusted for comfortable viewing under a given stereoscope. In the lens type of stereoscope, for example, it lies in a plane containing the axes of the two lenses and will vary in distance according to various instruments. "Photo base" is a term which refers to lines on each photo of a stereo pair connecting the center point of one with that point corresponding to the center of the other. In direction, it parallels the line of flight, and in length, it equals the air-line distance between exposure stations reduced to the scale of the photos. Correct orientation of photos for stereo vision requires that eye base, stereo base, and photo base be parallel.

8. Anaglyphs:--Another singular effect which may be produced with stereoscopic pairs of photos is that of the "anaglyph". In this case, two separate pictures of a stereo pair are printed in complementary colors and then superposed upon a single sheet of paper. The image that is to be observed by the right eye is printed in blue-green and that for the left eye in red. Binocular fixation of corresponding points is then obtained by observing the dichromatic overprint with a pair of goggles having a blue-green glass filter in front of the left eye and a red one in front of the

right eye. Thus, the blue-green image will be seen by the right eye alone and the red one by the left eye alone. The resulting effect will be a spatial model in black and white that is formed mentally by observation of the two different optical impressions.

Similar depth-impression effects may be obtained by projecting upon a single screen two stereoscopic pictures that have been illuminated by light of two complementary colors. Spectators are then able to observe the effect of relief by the use of suitable goggles as described above. It is this same principle which is followed in the observance of spatial models in connection with the multiplex plotting machine (See Chapter V).

White light which has been polarized in two planes at right angles to each other may also be used to illuminate two stereo-pictures instead of the complementary colors (red and blue-green). In this method, it is necessary that "polaroid" spectacles be used in the observance of the model which is formed by the illumination of each picture with a different beam of polarized light. Relief visualization is obtained by the right eye receiving one kind of polarized light while the left eye receives the opposite kind. This method will be used extensively with the multiplex machine in experiments to be explained in later chapters. The principles of polarized light will be discussed in Chapter VI.

9. Pseudoscopic Views:--In observing an anaglyph or a stereogram, care must be taken to assure that a reversal of relief is not obtained. Such an effect is known as a

"pseudoscopic illusion". A reversal of relief results if the photo originally intended to be observed by the left eye is placed at the right-hand side of the stereoscope, and if the photo designated to be seen by the right eye is placed at the left-hand of the stereoscope. In the case of complementary colors, as used in the multiplex, this effect is noted if the eyepieces are reversed with respect to colored light coming through the projectors. The same result is obtained by rotating an anaglyph print through 180° so that it will be upside down, and observing it with the spectacles in their original position. Figure 14 shows a simple geometric stereogram in both its correct position and its reverse position.

A pseudoscopic effect may also be noted as an apparent reversal of relief seen on viewing a single photograph with the incorrect orientation. The correct relief impression is obtained when the shadows fall toward the observer.

CHAPTER V

THE MULTIPLEX INSTRUMENT

1. General:--Photogrammetric instruments of the multiplex type have advantages that lend themselves to mapping of regions of moderate to rugged relief. The instrument is simple in principle and construction, and has the great advantage in that it permits adjustment of a long series of overlapping photographs as a single unit. All other types of photogrammetric instruments are confined to the consideration of a single pair of photographs as a unit, and are without provision for adjustment of a series of adjacent stereoscopic models as a group.

The multiplex instrument is so designed that it will project several overlapping photographs in the form of small diapositive plates in such manner that they can be viewed stereoscopically on a small plotting table. The vertical projectors used for this purpose must be oriented in such position as to reproduce exactly the conditions existing at the moment of photography. The relative position of image points in the area common to two adjacent photographs in a series will be defined in space by the intersection of corresponding rays from both projectors when the two are properly oriented. The integration of all such points represents an optical model similar in all respects to the area photographed. The effect of relief is accomplished by projecting the images in complementary colors and observing them through correspondingly colored glasses, as explained in the last chapter. The optical model can then be per-

ceived and measured in three dimensions with a floating mark. Thus, all necessary provision is made for the drawing of either planimetric or topographic maps.

Another advantage of the multiplex instrument is found in the fact that it is of a fixed-focus type and thereby free of all the constructional difficulties that arise in instruments employing the large original aerial negatives without reduction.

One of the superiorities of the multiplex design is its facility of spanning long distances between points of established position. Advantage can be taken of this "bridging" process when maps of great excellence must be dispensed with on account of remoteness of the area and lack of the normal amount of map control. Under this condition mapping can be facilitated by using a horizontal supporting bar with tables 14 or more feet in length on which many vertical projectors can be suspended simultaneously. When maps of good quality are desired, it is customary to use a shorter bar and provide horizontal control at intervals of 8 to 10 miles and elevations in specified positions in every stereoscopic model. With this amount of control, the resulting map should be well within the requirement that the elevation of 90% of the tested points shall not be in error more than one-half the contour interval.

2. Preliminary Planning:--It is important that careful study be given in advance to such matters as map accuracy, map scales and a contour interval suitable to the region to be mapped. Preliminary planning should give consideration

to the type of aerial camera to be used, the determination of its optical characteristics and constants, and other details relating to the actual completion of the aerial photography. The amount of overlap of photographs in the line of flight as well as between parallel flights must be carefully considered, as this has a marked effect on map accuracy. When distances between exposures in the line of flight are too short, accuracy is decreased, and cost is increased when the parallel flight lines are too close together. Cameras of short focal length are generally superior for topographic mapping to those of longer focal length both in respect to the number of stereoscopic models required to cover the given area and in the precision with which elevations can be measured.

When a selection of a contour interval appropriate for the terrain to be mapped is made, it follows that the altitude at which the flights must be made will be determined by the type of camera employed. The effective field of view of the aerial camera, in turn, governs the ratio of reduction to be used in making the diapositive plates in the reduction printer.

The scale at which a map should be plotted on the multiplex table depends on the flight altitude at which the aerial photographs were made.

The control planning operation includes not only a study of the triangulation, traverse and level lines necessary to a good distribution of strong control throughout the area of the map, but also the general location and type of

supplemental control points that may be required in controlling the aerial photographs. The number of supplemental control points needed in any particular case is determined largely by the map accuracy specifications.

The map-sheet preparation consists of constructing the projection lines that define the limiting parallels and meridians of the map, plotting the basic and supplemental horizontal control points thereon, and assembling all the other data that will be needed by the operator.

3. Diapositive Plate Preparation:--The most important element affecting the accuracy of work performed with the multiplex instrument is the quality and size of diapositive plates which are made from the original negatives. These plates, which are used in the vertical projectors, must be made to an exact size and have a quality so excellent that they can be enlarged twelve or more diameters without sufficient loss in sharpness to influence the drawing of the map by the multiplex operator. Error in the size of the diapositive will result in errors of elevation in the map, and the lack of sharpness will be the cause of unsatisfactory stereoscopic models which will prevent the operator from accurately delineating the contour lines.

The diapositive plates are made by means of a special reduction camera (See Fig. 15) which is precisely adjusted to make diapositives of the proper size and of the desired quality from the aerial negatives that are to be used in the mapping operation. Owing to the very small size of the diapositive plates and the great influence of their quality of

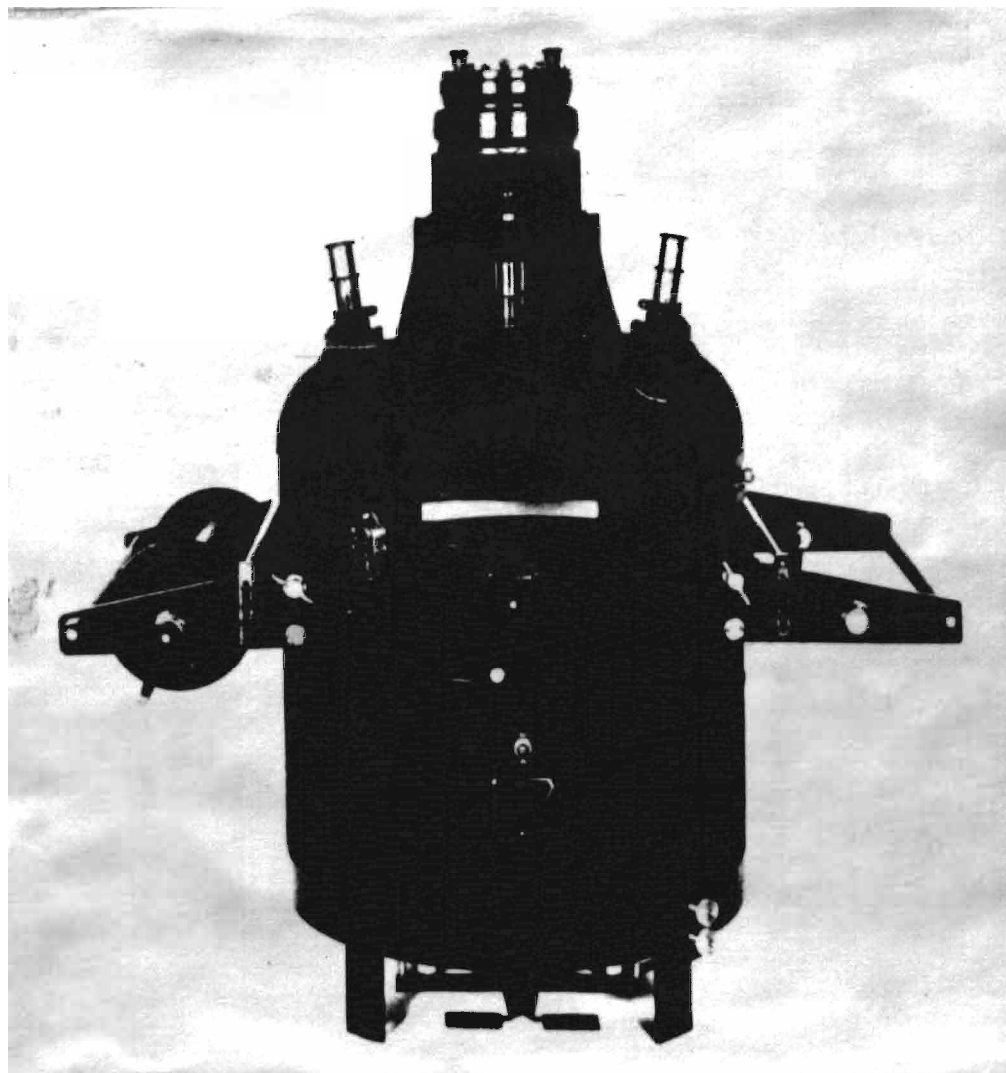


Fig. 15 - Multiplex Reduction
Printer.

image on the accuracy of the resulting map, it is necessary that the settings of the reduction printer be given close attention. It is also apparent that aerial negatives to be utilized in preparing diapositive plates should be used for this purpose soon after the negatives are made and before the negative film has become distorted by age. Any distortion of the negative arising from this cause will be carried over into the stereoscopic model and thus result in maps of a low degree of accuracy.

It is essential that the original negative be reduced in the ratio of the principal distance of the aerial camera to the principal distance of the multiplex projector. Proper reduction ratios must be computed for each aerial camera that is employed, and the reduction printer settings must be altered whenever a change in the film roll so requires.

4. Mapping Operation:--The multiplex instrument (Fig. 16) consists essentially of a horizontal supporting bar on which are suspended a number of small projection cameras. Each projection camera is so constructed that it is adjustable in three directions with respect to the supporting bar and may be tilted or rotated to bring the small diapositive plate into the desired position. It contains a platform on which the diapositive is supported, a light source, a light condensing system, a projection lens, and a light filter to color the projected image as desired. The supporting bar is mounted on a rigid table with a plane surface which is used as the reference plane for the mapping operation. The drawing paper with the projection lines and the horizontal

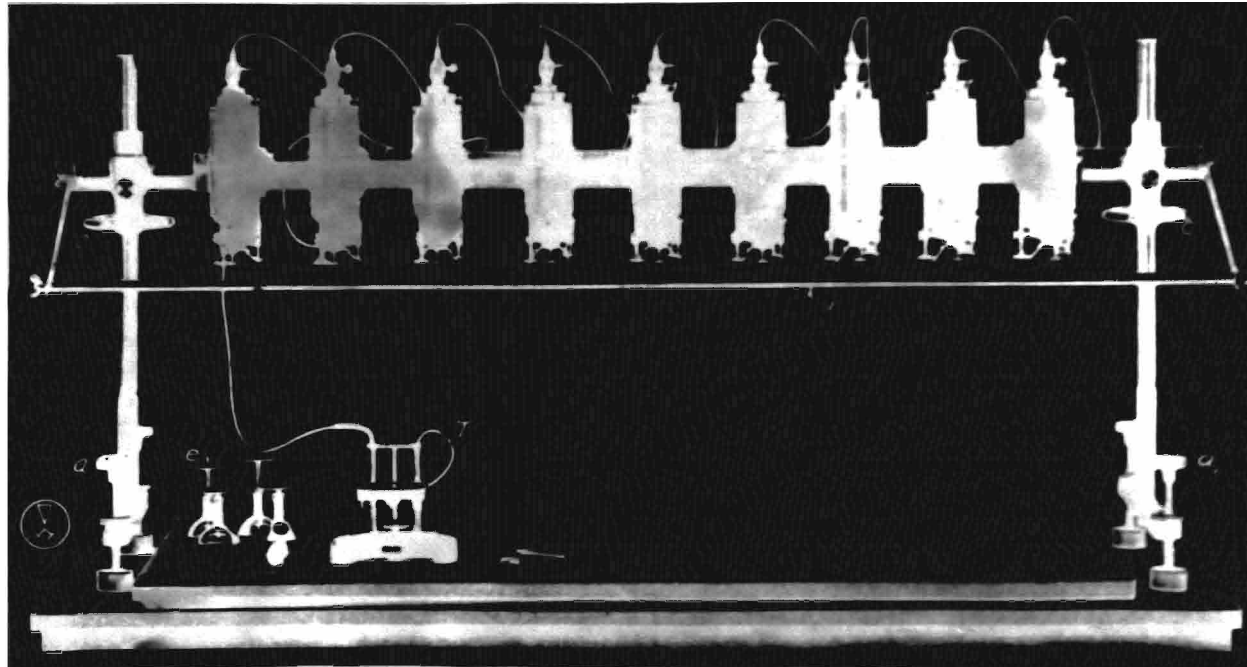


Fig. 16 - Multiplex Plotting Machine.

control drawn thereon is placed on this table top beneath the supporting bar. A light-weight moveable device with a measuring mark on the screen on which the images are cast by the projectors, and a pencil for drawing the map, complete the essential parts of the apparatus. The diapositive plates must first be placed on their supporting platforms in proper sequence, each one being so adjusted as to bring it into coincidence with the principal point of its projector, thus completing what is known as the inner orientation. The stereoscopic model, on which the necessary measurements are made, is the result of proper relative and absolute orientation of the projectors. The relative orientation brings each projector into the same position with respect to neighboring projectors as existed in the air at the moment the photographs were made, and thus makes possible a correct stereoscopic view of the terrain. The absolute orientation has, as its purpose, such adjustment of the model and projectors as a unit with respect to the map plane as will result in the desired scale and orientation with all points on the model having their proper relation to sea level (See Fig. 17).

The drawing of the map is accomplished by moving the drawing device in such manner that it traces out all cultural features, such as roads, railroads, trails, transmission lines and visible property lines, as well as residences and all other buildings which, from their size or importance, merit representation. Following the drawing of cultural features, the operator will take up the representation of

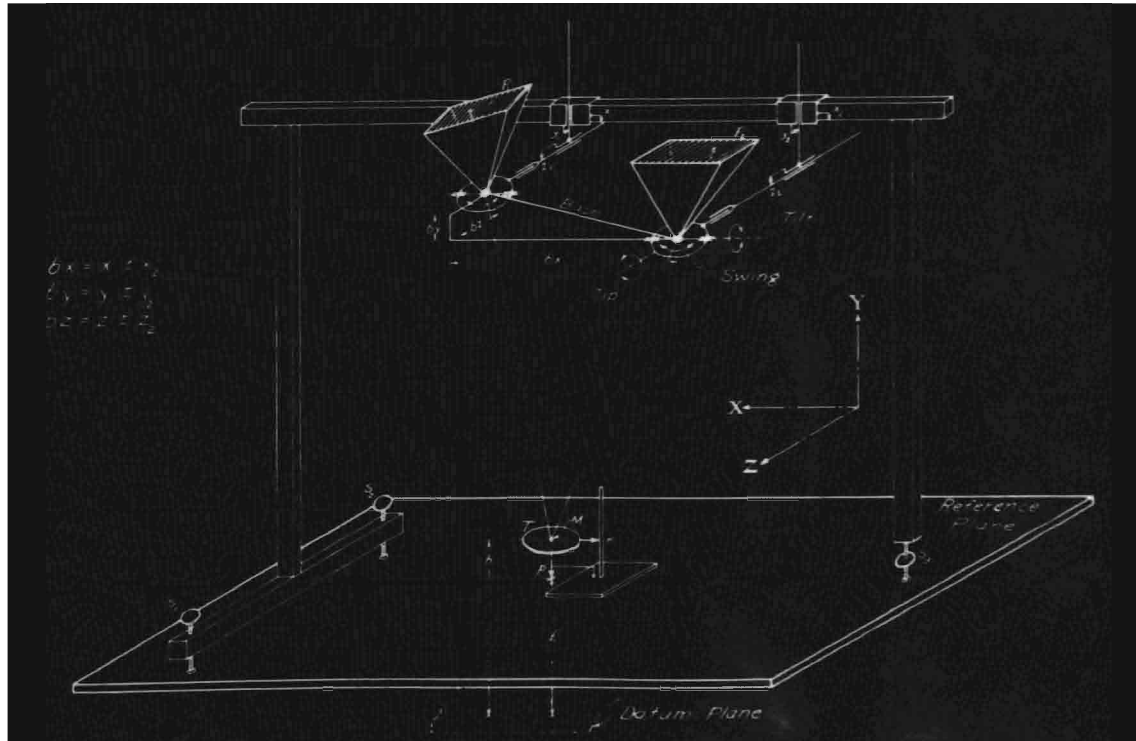


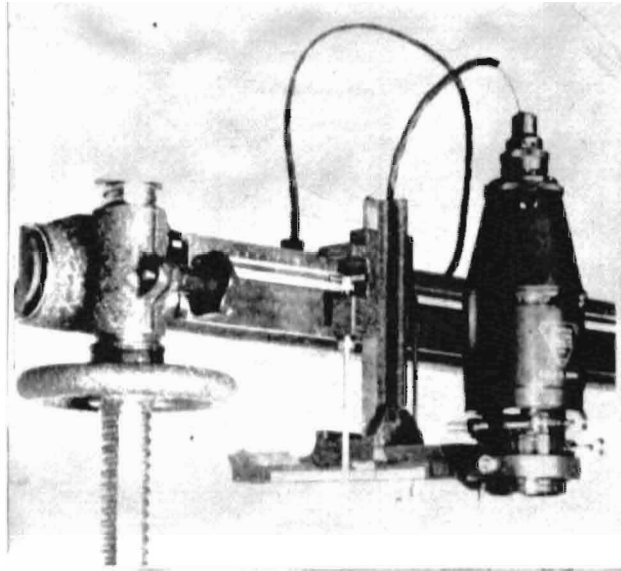
Fig. 17 - Principles of Multiplex Orientation.

drainage features of the map, which includes the outlines of all water bodies and the courses of all rivers and smaller streams. The delineation of wooded regions will follow, and finally the contour lines will be drawn showing the shape and position of all topographic features.

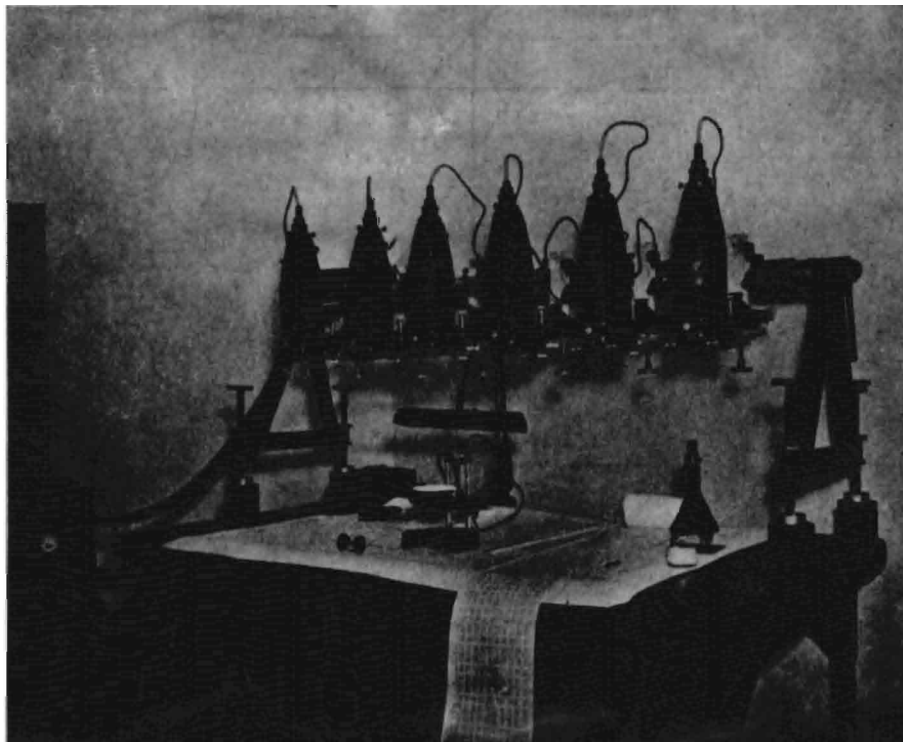
All cultural and drainage features, wooded areas, and contour lines are inked as rapidly as the construction of the map will permit, in order that detail represented in pencil will not smudge and become poor copy for reproduction.

5. The Vertical Projector:--The vertical projector (Fig. 18) consists of a miniature camera of two essential parts, one with a lens, a cone and focal plane, and the other with a lamphouse having the necessary condensing lenses. Provision is made for moving the entire assembly in the X, Y, and Z directions as needed, and for tipping, tilting, or swinging it as may be necessary for orientation. Inasmuch as all rays must be reprojected in directions parallel to the direction of the original rays, it is essential that all critical dimensions of the aerial camera be represented in the camera portion of the projector in the ratio of principal distance of projector to principal distance of aerial camera, as mentioned previously. This relationship is graphically illustrated by Figure 19.

6. The Tracing Table:--The small tracing table (Fig. 20) is the only part of the multiplex equipment having any freedom of motion. It carries the small platen on which the stereoscopic image is observed, the luminous point used in following the contour lines and other detail throughout the



A. Projector on Frame.



B. Series of Projectors.

Fig. 18 - Multiplex Vertical Projectors.

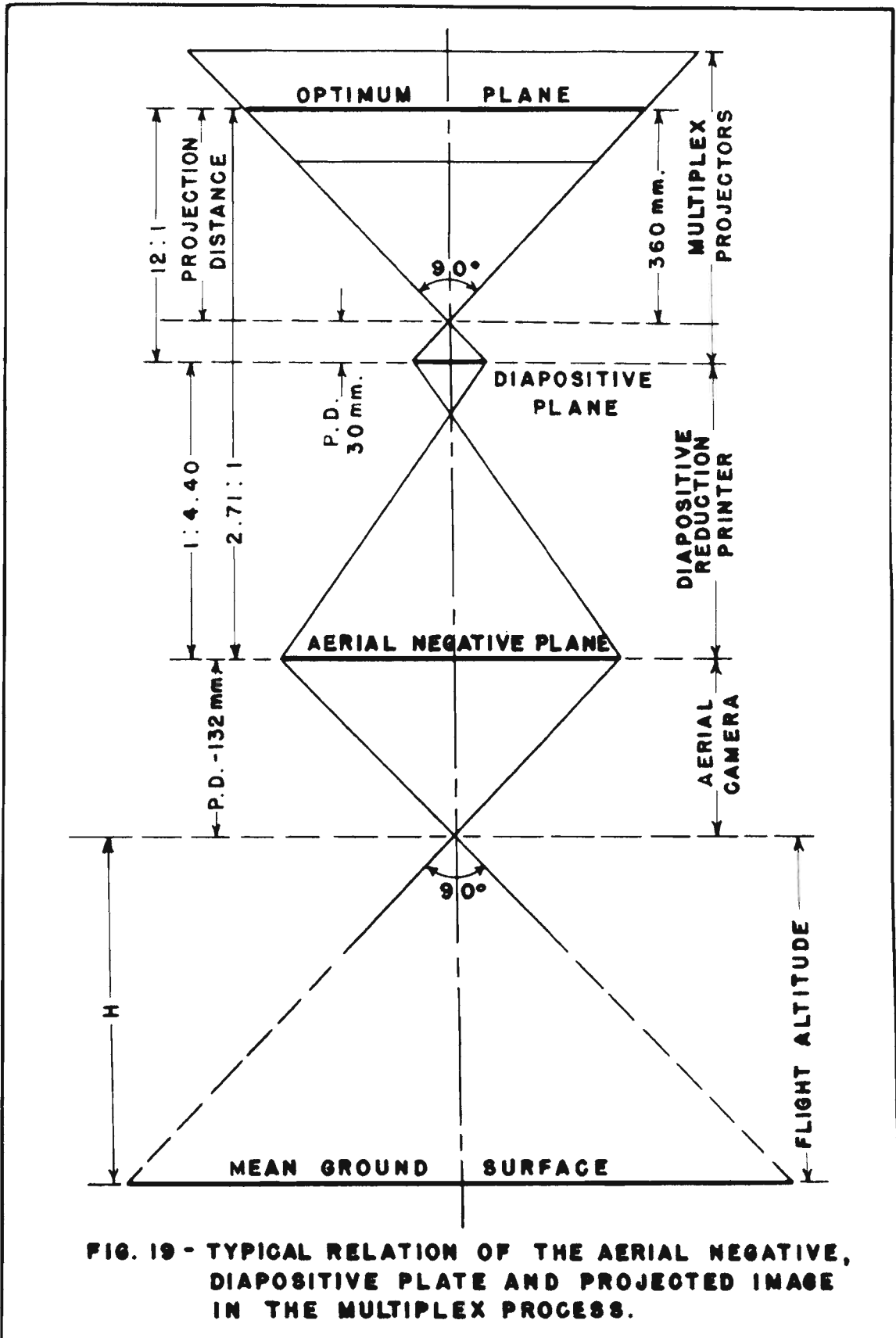


FIG. 19 - TYPICAL RELATION OF THE AERIAL NEGATIVE, DIAPOSITIVE PLATE AND PROJECTED IMAGE IN THE MULTIPLEX PROCESS.

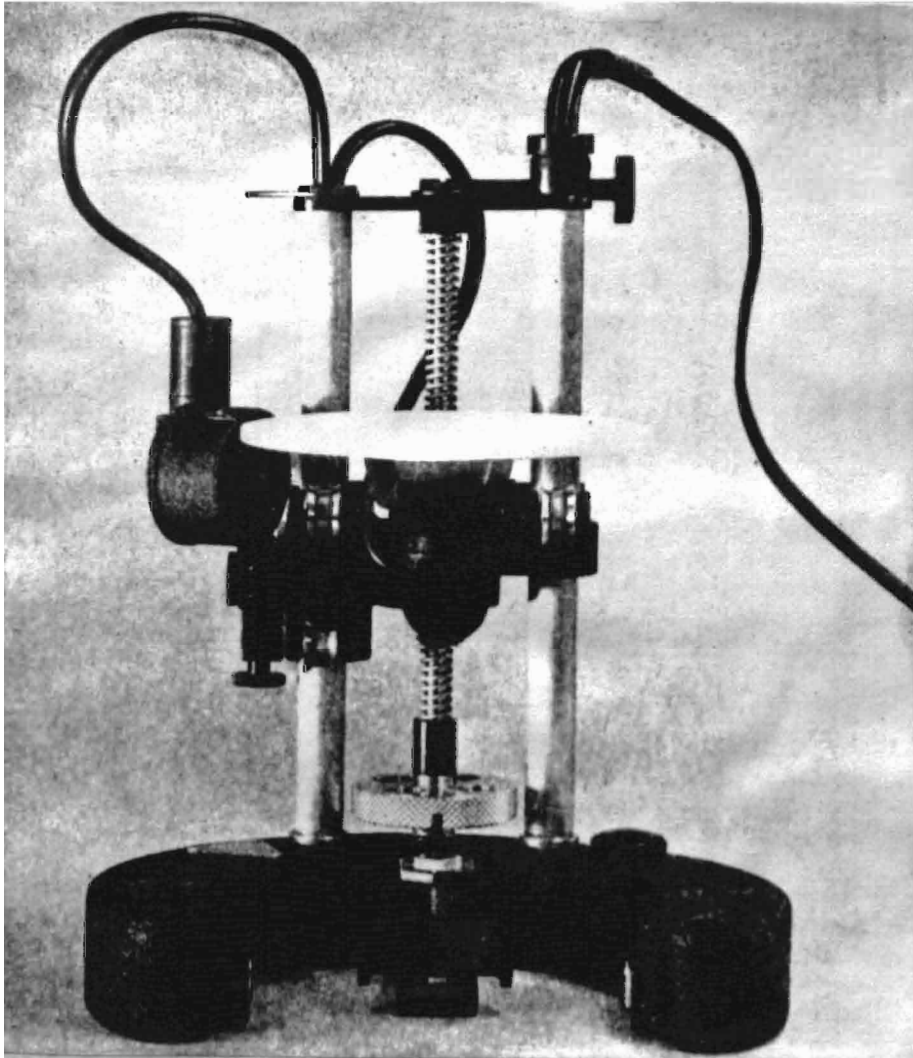


Fig. 20 - Multiplex Tracing Table.

model, and finally the drawing pencil by means of which the path of the tracing table is recorded on the drawing paper.

The illumination of the tracing table is controlled by a small toggle switch which releases light through a minute, pin-point hole in the center of the platen to form the luminous mark. The platen is equipped with an elevating screw which raises or lowers the table as needed for intersecting, by reference to the luminous mark, the rays from the two diapositives of the stereo model. Under the center of the table, directly below the luminous mark, is the drawing pencil, which may be lowered into contact with the table or raised when not in use.

CHAPTER VI

POLARIZATION OF LIGHT

1. Polarization:--The term, "polarization", has been used whenever anything has a property in one direction that it does not have in another. When an electric current flows between two pieces of platinum immersed in acidulated water, a counter electromotive force is produced which acts only in one direction; it opposes the current. This is a case of polarization. A magnet tends to set itself in a north-south direction with the same end always pointing northward. Inside the magnet there must be a polarized condition which causes the magnet always to set itself in the same direction.

2. Polarization of Light by Tourmaline:--The subject of the polarization of light is a large and important branch of the science of light. Several manifestations of light polarization have been discovered to date, including polarization by reflection and polarization by double refraction. However, the method with which we are primarily concerned in this treatise is the polarization of light by absorption into tourmaline. Tourmaline is a crystalline mineral, an aluminum-boron silicate. The plates of tourmaline referred to herein are those which have been cut with their surfaces parallel to what is known as the optic axis of the crystal. Light which passes through a plate of tourmaline crystal becomes polarized. The polarization of the tourmaline plate may be detected by using a second tourmaline. When the axis of the crystals are parallel, light goes through both. As one of the crystals is rotated, the light will grow dimmer and

dimmer until the axes of the crystals are perpendicular, at which time practically no light passes through the crystals, as shown in Figure 21. Either crystal may be rotated in showing this "crossed" effect. The crystal through which the light passes first is called the "polarizer", and the second one is called the "analyzer".

3. Theory of Transverse Nature of Light Waves:--The phenomena of light polarization are considered as a confirmation of the electromagnetic theory of light waves. This theory holds that light waves are of a transverse nature, like electromagnetic waves.

There are two types of wave motion: longitudinal and transverse. In longitudinal waves the direction of vibration is the same as the direction of propagation; in the transverse type, vibrations are at right angles to the direction of propagation.

There is one distinct difference between the two types of waves. Consider the case of longitudinal waves in a long spiral hung from the ceiling. The longitudinal vibrations produce "compressions" and "rarefactions" which travel up and down the spring. These vibrations are completely symmetrical with respect to the direction the waves are traveling, for they will have exactly the same appearance when seen from any side. On the other hand, transverse waves sent along a rope do not have the same appearance when viewed from different positions. If the rope is horizontal and the vibrations are in a vertical plane, the appearance of the rope when seen from above or below is very

different from its appearance when seen from one of the sides. Transverse waves are not completely symmetrical with respect to their direction of propagation. Hence, whenever it is shown that a wave does not have complete symmetry about its direction of propagation, it is assumed that it must be transverse.

A simple illustration will give one test for symmetry. A certain plug fits in a hole. If this plug is rotated, and if it fits for all positions, then there is symmetry; but if the plug fits for one position and not for another, the plug and the hole do not have complete symmetry. Rotation, therefore, is one test of symmetry.

Consider now the experiment of the rotation of a plate of tourmaline held in a beam of plane-polarized light. In one position it transmits the light; in another it does not. A rotation of the crystal is the only change. This experiment shows that the beam of light is not symmetrical. The waves must therefore be transverse.

4. Explanation of Polarization Process:--Acceptance of the theory that waves of light are of a transverse nature carries with it the fact that the waves vibrate at right angles to the direction of their propagation. Ordinary light may vibrate in all directions at right angles to the direction in which it is traveling, but when it is polarized, the direction of the vibrations may be restricted.

A plate of tourmaline absorbs a portion of the incident light and transmits the rest, and that which it does transmit vibrates in a direction parallel to the optic axis of

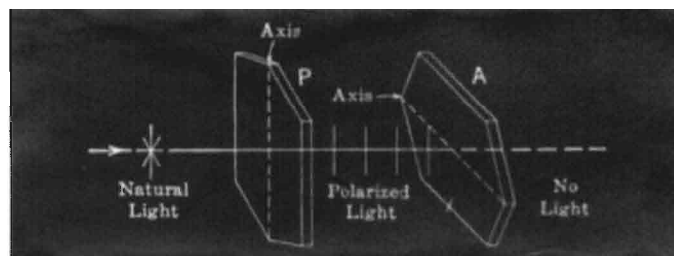


Fig. 21 - Production of Polarized Light.

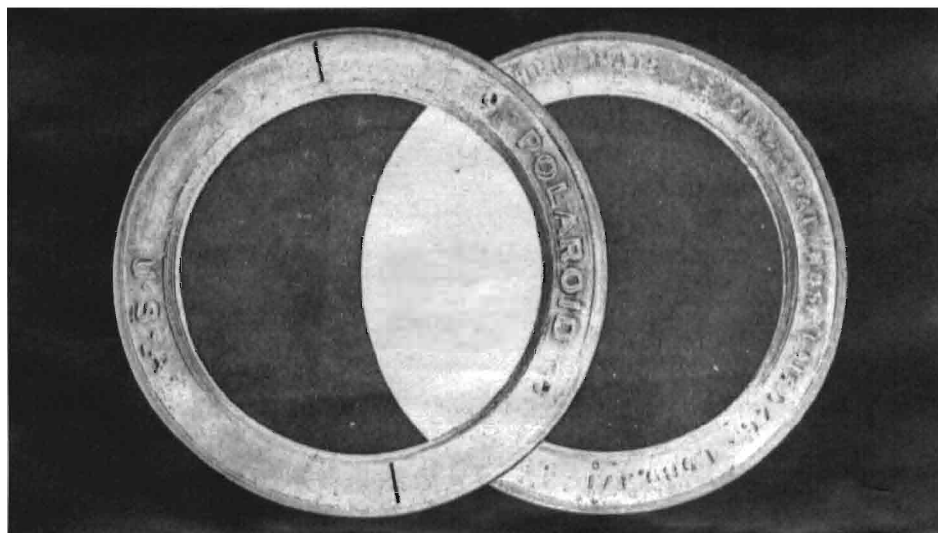


Fig. 22 - Crossed Polaroid Plates.

the crystal. In other words, when ordinary light is incident on tourmaline, the transmitted light is plane-polarized. If the light is already plane-polarized, it may or may not go through a piece of tourmaline, depending on the direction in which the light is vibrating with respect to the plate. When the incident light is vibrating parallel to the optic axis of the crystal, it will go through, but when it is vibrating at right angles to this axis, it will not go through. Hence, as the crystal is rotated, in certain positions the light will go through, while in other positions it will not.

An artificially prepared plate, called a "polaroid" plate, may be used either to polarize light or to analyze it. It consists of a thin film of cellulosic sheeting containing large numbers of submicroscopic crystals. These crystals are turned so that their axes are parallel by stretching the film; the film is then mounted between thin sheets of glass to form a lens-like plate. The crystals, which are of the iodo-quinine group, are similar to tourmaline, for they have the property of absorbing light that is vibrating in one direction and transmitting the light that is vibrating at right angles to this direction. Figure 22 is an illustration of two "crossed" polaroid plates. When they overlap, there is practically no transmitted light.

5. Applications of Polarized Light:--Various optical methods devised for the utilization of polarized light are very useful in different scientific fields.

- a. Optical Rotation--Certain materials, notably quartz and solutions of sugar, have the property of rotating the plane of polarization in transmitting polarized light. This effect may be observed with a "polarimeter", consisting of a polarizer and an analyzer, and means for supporting the optically active substance in the light path between them. Optical rotation is used in determining the percentage of sugar in solutions of unknown concentration. Polarimeters especially designed for testing sugar are known as "saccharimeters", and are arranged to pass the polarized light through a tube filled with the solution under test. If the polarizer and analyzer are crossed initially, giving a dark field, it will be found upon introducing the sugar solution that the analyzer must be turned to a new position to restore darkness. The number of degrees so rotated will indicate the strength of the solution.
- b. Circular Polarization--The induction motor demonstrates that two harmonic vibrations of equal amplitude and frequency, taking place in perpendicular planes, will give circular motion as a resultant when they are combined, provided they are out of phase by one-fourth cycle. When plane-polarized light is passed through a thin, doubly-refracting plate, the components

traverse the plate with different velocities, and in general emerge displaced from each other in phase. If the relative retardation is a quarter-wave length, supposing the components to have equal amplitude, the issuing light is said to be circularly polarized. A doubly-refracting plate designed to produce circular polarization is called a quarter-wave plate. With circularly polarized light, the field remains equally bright for all positions of the analyzer. Applications of the principle of circularly polarized light have led to important deductions concerning the structure of matter; they have also allowed the ratio of the electronic charge and mass to be evaluated.

- c. Photoelasticity--The distribution of internal stresses in structural or machine parts may be observed by passing polarized light through models made to scale from sheet celluloid or Bakelite, which are subjected to external forces simulating those in the actual structure. Though the distribution of stresses lends itself to mathematical calculation in some instances, the photoelastic method solves many problems not amenable to mathematical treatment. Photoelasticity has developed to such a point that it is now a whole branch of engineering in its own right.

6. Projection of Polarized Light:--The projection of white light, which has been polarized in two planes at right angles to each other, as a means of illumination of two stereo pictures to form a three-dimensional image, is one of the more recent developments of light polarization. As mentioned in a previous chapter, it is necessary that polaroid spectacles be used in the observance of the model. In this method of relief visualization, the right eye will receive one kind of polarized light, while the left eye will receive the opposite kind. There are two ways in which polarized light may be projected in order to achieve this three-dimensional effect. They are as follows:

- a. Vectographic Projection--This method employs a single slide or a single strip of film reproducing both images of the stereoscopic pair, superimposed on each other but in opposite directions of polarization. The vectograph is thrown on a screen surfaced with metallic paint and viewed through polarizing picture viewers. Because the system uses existing projection equipment and because of other advantages, it seems reasonable to suppose that it will be used for most three-dimensional projection in the future, particularly when color vectography is made practicable for general use.
- b. Use of Twin Projectors--The double-projector system for showing three-dimensional pictures, like the vectograph system, makes use of polar-

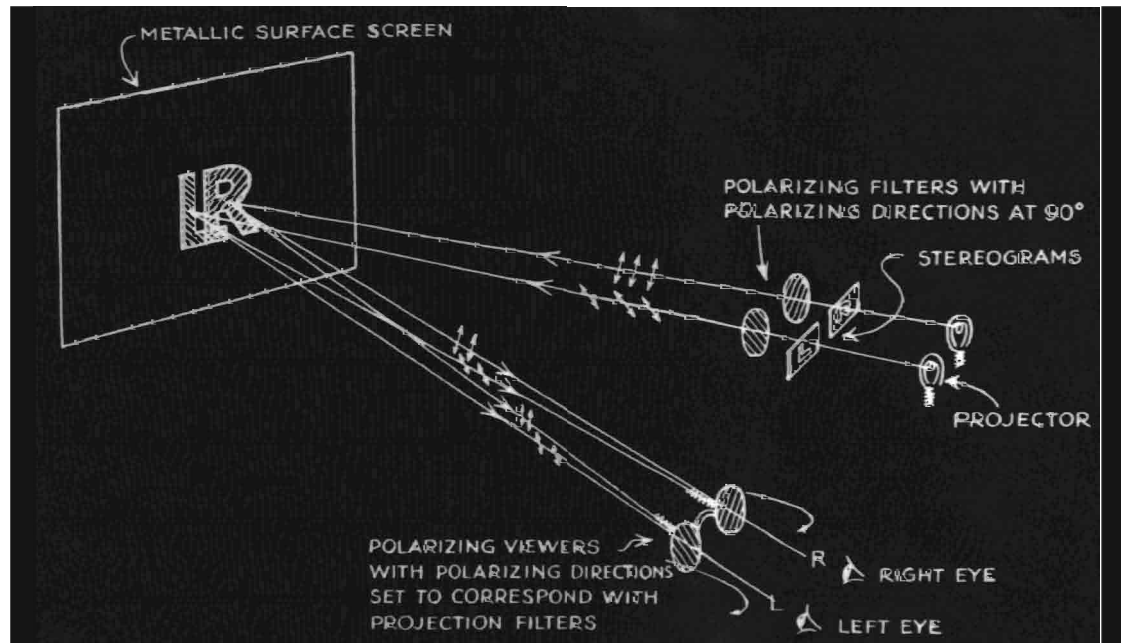


Fig. 23 - Principle of Polarized Light Projection of Three-Dimensional Pictures.

ized light, but in an entirely different way. The two stereograms are projected by a twin projector. Each of the two separate projection systems is fitted with a polarizing filter with its polarizing axis so set that the left-eye picture reaches the screen with all of its light vibrating uniformly in one direction and at 45° left of vertical, while the right-eye picture is made up of light vibrating in a direction at 45° to the right of vertical. The metallic surface of the screen redirects the light of the two images back toward the audience without disturbing the vibration pattern set up by the polarizing filter in the projector. The viewers, worn by each member of the audience, are simple spectacle frames fitted with polarizing eye-pieces similar to those in the projector and set with their axes at corresponding angles. The two images formed on the screen by the twin projectors, a jumbled hodgepodge as seen without viewers, become an easily viewed and convincingly three-dimensional scene. The twin-projector principle, as illustrated in Figure 23, will be used in the multiplex plotting machine to form a polaroid stereoscopic model for the experiments of Chapter VIII.

CHAPTER VII

PREPARATIONS FOR RESEARCH

1. General:--It is not the purpose of this thesis to present a full account of the process of producing maps from aerial photographs. However, those chapters which have preceded our experiments should provide a suitable background for an understanding of the problems involved in the comparisons to be drawn in Chapter VIII.

Before proceeding with the experiments, a short description of the various equipment to be used therein will be given, as well as a brief outline of methods to be employed in conducting the research and presenting the results.

2. Multiplex Machine:--All experiments were performed in the multiplex section of the local branch of the U. S. Geological Survey. The multiplex aeroprinter used for research (See Fig. 24) was manufactured by Bausch and Lomb Optical Company, according to U. S. G. S. specifications. The relation of critical values in the aerial camera, the multiplex reduction printer, and the projectors is demonstrated schematically in Figure 19, on Page 53. Projectors and aerial cameras most commonly employed by the U. S. Geological Survey have principal distances of 30 and 132 mm., respectively; consequently, the ratio of reduction to be employed is $132/30$, or 4.4 diameters. The size of the glass diapositive negatives used in the projectors is 64 by 64 millimeters, with a 3 mm. thickness.

3. Platens:--The small platen on the tracing table was the only piece of equipment, with the exception of the

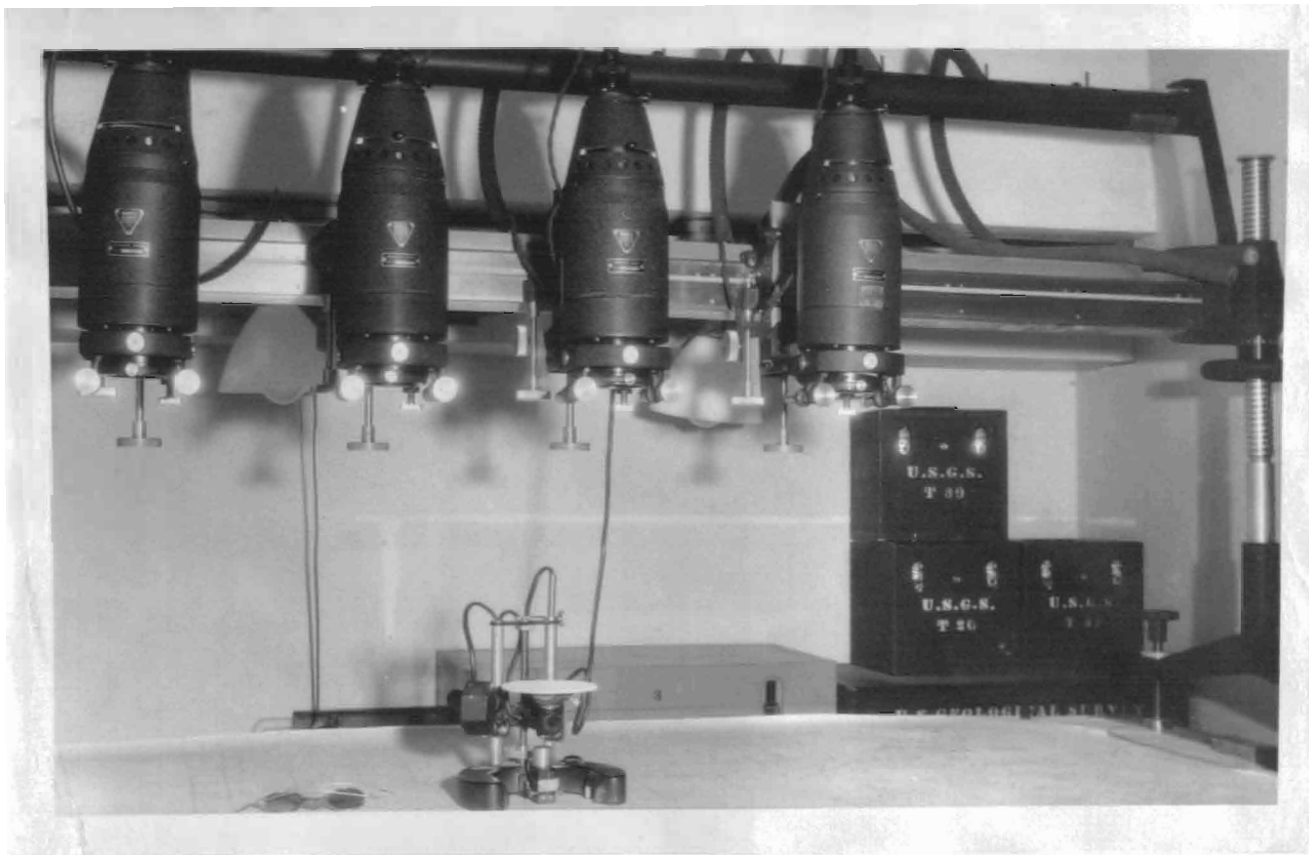


Fig. 24 - Multiplex Machine Used in Research.

filters, that required change as the model was exchanged from anaglyph to polaroid, or vice versa. A platen with the standard white surface was used with the anaglyph model. This surface was obtained by smoking the top of the platen with burning magnesium ribbon.

A separate platen was used for the polaroid model. Its surface was obtained by coating the top of the platen with a fine, aluminum-pigment paint. This was done to meet the requirement of a metallic background for preserving the vibrations of the projected rays of polarized light. The paint used for this purpose was Polaroid Aluminum Pigment No. 689, manufactured by the Polaroid Corporation, Cambridge, Mass. The platens and filters used in the experiments are shown in Figure 25.

4. Filters:--The standard 66-mm. diameter, complementary-color filters were used with the anaglyph model. The red filter was of about 2-mm. thickness, and the blue-green filter was of 4-mm. thickness.

The filters used in the polaroid model were obtained from the Three-Dimension Company, of Chicago. It was impossible to find polaroid lenses of proper size, mounted in a confining rim, ready for direct use in the projectors. The largest size of lens obtainable was 51 millimeters in diameter. It was then necessary to have machined metal adaptor rings of such size as to both mount the lens and fit the projector. The resulting filters are shown in Figure 25.

Each polaroid lens consists of a piece of thin, film-like material mounted between two thin plates of glass. The

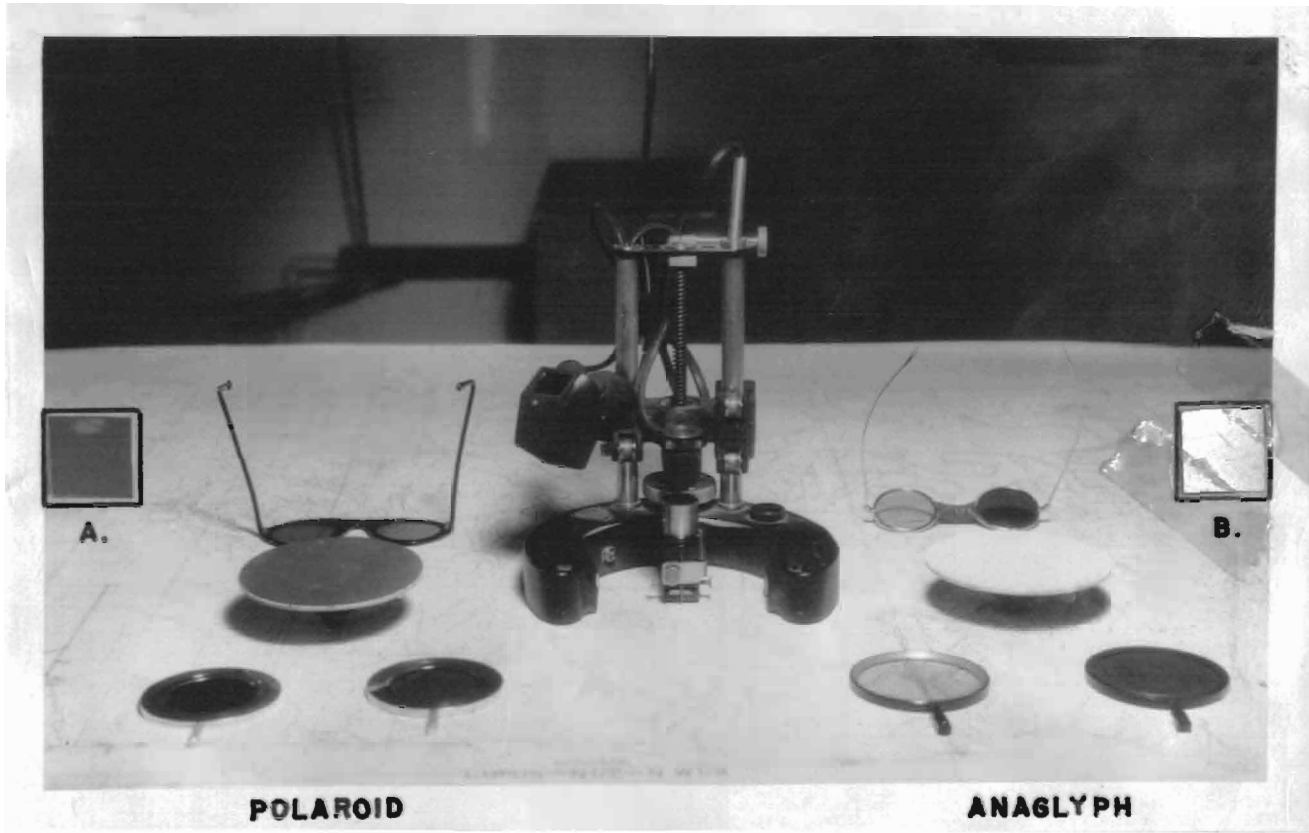


Fig. 25 - Filters and Flatens.

film-like material is a cellulosic sheeting containing sub-microscopic crystals of the iodoquinine group, which has properties analogous to a plate of tourmaline, as explained in Chapter VI. This synthetic light-polarizer is graded into various types. The polaroid filters used in experiments of this thesis contain Type "H" film. The small inset in Fig. 25 holds a sample of Type "J" film. The difference between various types of this film is in the amount of light transmitted and in the color.

5. Method of Presenting Research:--In writing a report of the experiment on each of the points of comparison, as set forth in Chapter VIII, the following system is used, as far as is possible:

- a. The Problem--Definitions; a short discussion of the problem; its effect on the model.
- b. The Experiment--Description of the procedure, step by step, in the experiment; reasons for performing each step.
- c. The Results--A numerical tabulation, or other form, which shows results obtained from the experiment.
- d. The Conclusions--Discussion of different items discovered during the experiment; drawing of comparison between models through results of experiment.

CHAPTER VIII

EXPERIMENTS WITH POLAROID AND COLORED FILTERS

1. Comparison of Light Quantities Coming Through Filters:--It is a well-known fact that the sharpness and clarity of the stereographic model in the multiplex machine is dependent upon the amount of light which comes through the filters and falls on the tracing table. To obtain the maximum quantity of light through the filters, as strong an electric lamp is used in the vertical projectors as is possible without danger of permanently damaging the lenses and filters through the action of the heat generated by the lamp. General practice has settled upon a bulb of 100-watt power. If the projectors are equipped with a blower system (which keeps them appreciably cooler), this value of wattage may be increased somewhat. For this experiment, a bulb of 100-watt power was used, with a setting of 20 volts electro-motive force on the projectors.

- a. Centering of Lamps--Before commencing the measurement of light, the two projectors to be used for the model were tested to see that their lamps were in the center of the lamp housings. This was done to insure that the light from the projectors would be a maximum, with their rays meeting in a focal point directly beneath the projectors.
- b. Light-Measuring Instrument--The instrument used for purposes of measuring light quantities was the Weston Illumination Meter, Model 603 (See

Fig. 26). This instrument indicates the illumination in foot-candles on the surface of the light target, or lumens per square foot evenly distributed. One lumen is defined as the amount of light flux radiating from a uniform 1-candle-power source throughout a solid angle of such size as to surround a unit area at a unit distance from the source. The candle-power is a unit of luminous intensity derived from the international candle; the international candle is expressed as a source which represents the flame of a spermaceti candle that burns 120 grains per hour.

- c. The Experiment--Prior to the actual measurement of the filters, the diapositive plates were removed from the two projectors to be tested. The intensities of light coming from each projector were next equalized by reading the illumination meter under each projector and then adjusting for differences of intensity. After this adjustment, the diapositive plates were replaced, the colored filters were placed in the projectors, and the model was checked for proper orientation. The tracing table was then set at an approximate average elevation of the terrain. The distance from the top of the table to the axis of the light sources was found to be 24 inches. This distance served as a

standard for all subsequent measurements of light intensity. The first measurement taken was the amount of light coming through the diapositive negatives only, with no filters in place. Three readings were taken; one in the center (directly between the two projectors), one under the right projector, and one under the left projector, all with the sensitized element of the illumination meter held at the standard distance of 24 inches from the axis of the light sources. The three points of reading thereby represented measurements along the axis of maximum brightness on the plotting table. The measuring element of the meter, which has two circular apertures, side by side, was held with its longitudinal axis parallel to the axis of maximum brightness. Following these tests, the colored filters were placed in the projectors with the blue-green filter on the right and the red one on the left. Readings of light intensity were taken at the same three points as before. The colored filters were then exchanged, the red filter being placed in the right projector and the blue-green filter in the left one. Another similar set of readings was recorded. The colored filters were switched in order to discover whether or not more light might be coming through one than the

other. The colored filters were removed and were replaced by the polaroid filters. At this point, orientation of the model was checked; it was also noted that the standard distance between the table top and the light source was still constant. Readings of intensity were then recorded for light through the polaroid filters at three points as before.

- d. Results--The following results were obtained from measurements with the illumination meter (Numbers are given in foot-candles):

MODEL	POINT OF MEASUREMENT		
	Left	Center	Right
No Filters	3.2	3.4	3.5
Colored Filters	0.7-R	0.7	0.8-G
	0.7-G	0.7	0.8-R
Polaroid Filters	1.2	1.3	1.4

Table 1 - Light Intensities.

- e. Discussion--For purposes of comparison with intensities of other light sources, the above figures may all be multiplied by 4. This follows from the equation for light intensity, $I = Kc/d^2$, in which it is shown that the illumination on any surface varies inversely as the square of the distance from the source, (d) in the above equation representing the distance. In a comparison of light intensities, all

distances, therefore, are resolved into unity. However, as far as comparing quantities of light through each type of filter is concerned, direct readings of the illumination meter may be used, since these comparisons will be on a percentage basis.

A question that arose prior to experiments with the illumination meter was whether or not it would accurately measure light through the colored filters in terms of the human eye. An instruction sheet on the meter, however, specifically states that it is provided with a filter which changes the color sensitivity of the sensitive plates to match that of the human eye, and can therefore be used to measure illumination of any color.

One of the first things discovered in this experiment was the fact that an enormous amount of light from the projector lamps is eliminated by the diapositive negatives themselves. It follows that negatives with varying shades of darkness in them will transmit corresponding intensities of light through them. A difference between the diapositive negatives in the two adjacent projectors is shown in the measurements of light from directly underneath each. A quantity of 3.5 foot-candles was measured under the right projector, while only 3.2 was

observed under the left projector. This indicates that only $3.2/3.5 = 91.4\%$ as much light came through the left projector as came through the right.

Another point discovered was that the same amount of light came through both the red and the blue-green filters. This is proved by identical readings at each point after the colored filters were switched from one projector to the other (See Table 1).

Using for comparison the readings taken on the tracing table directly between the two projectors, it was found that $1.3/3.4 = 38.2\%$ of the total light came through the projectors with the polaroid filters in place, while $0.7/3.4 = 20.6\%$ of the light was projected with the colored filters in place, both percentages using the full light through the diapositive negatives as a base. With respect to each other, $0.7/1.3 = 53.8\%$ as much light was projected through the colored filters as was projected through the polaroid filters. Hence, it is evident that considerably more light is allowed to come through the polaroid filters onto the tracing table than comes through the colored filters, which represents a considerable advantage for the polaroid filters on this particular point of comparison.



Fig. 26 - Weston Illumination Meter.

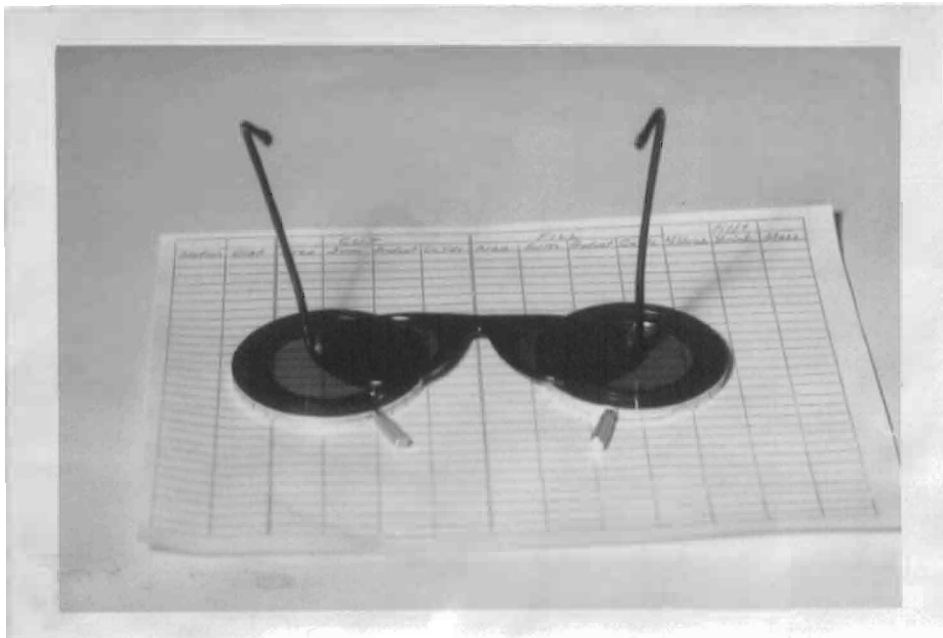


Fig. 27 - Orientation of Polaroid Filters.

2. Placing of Filters in Projectors:--The placing of filters in the projectors has little effect on the accuracy of the map, but should be considered here in the light of its possible inconvenience to the operator. In employing the colored filters, he merely inserts them in the projector with no further thought as to their orientation. In the case of the polaroid filters, however, certain manipulations are necessary to orient them so that light passes through each in such a manner that its particular component is at right angles to the other. This will provide the necessary conditions for a clear stereoscopic model when viewed through the polaroid viewing spectacles. The orientation of the polaroid filters may be divided into two simple steps, which will be called "preliminary" and "final".

- a. Preliminary Step in Orientation of Polaroid Filters--The polaroid viewing spectacles are used as a basis for orientation, since the two eyepieces are already oriented with their optical axes perpendicular to each other. The spectacles are placed flat on the table, over a printed sheet or similar background easy to see, with the ear pieces open and extending upward. One of the polaroid filters is placed under the left lens of the spectacles and rotated until light coming through both the lens and the filter becomes a minimum. This is determined by the obliteration of the background underneath the filter, and indicates

that the optical components of the spectacle lens and the filter are at right angles to each other. The other polaroid filter is placed under the right lens of the spectacles and rotated as before until maximum darkness occurs through that portion of the glass which is in superimposition. (See Fig. 27).

At this point it is apparent that the filters are "crossed" with respect to the lenses of the spectacles. Placing of the left filter in the left projector and the right filter in the right projector would produce a pseudoscopic image on the tracing table. In order to give a correct image, the filter under the left lens of the spectacles is placed in the right projector and the right filter is placed in the left projector. In this process, care should be taken to rotate the filters as little as possible, as they are already in approximate orientation.

b. Final Step in Orientation of Polaroid Filters:--

To achieve final orientation, the light of the left projector is obscured by placing the back of the left hand under its lens, the right eye is closed, and the filter in the right projector is rotated slightly with the right hand until the point is reached at which maximum darkness occurs on the tracing table. Since

the eye looking through the left lens of the spectacles sees a minimum of light coming through the right filter, it is evident that the filter in the right projector is correctly oriented with respect to the right lens of the spectacles.

In like manner, the light of the right projector is blocked off by the back of the right hand, the left eye is closed, and the filter in the left projector is slightly rotated with the left hand until the point of maximum darkness occurs. Complete orientation of the filters has then been reached.

- c. Discussion--From the above description, it may appear that orientation of the polaroid filters is a long, involved process. With a little practice, however, one can easily perform the entire orientation in about one minute's time. The advantage of this particular method of orientation is that it necessitates a minimum rotation of the polaroid filters in the projectors, since most of the rotational movement of the filters is accomplished with the spectacles. Other methods of orientation may even entail rotational movement of the projector's lamp housing assembly. It can be seen that the less movement around the projectors, the less likely will be the necessity for reorientation of the

entire stereoscopic model.

From a comparative standpoint, then, it is apparent that the polaroid filters have disadvantages over the colored filters in that they require, in their orientation, somewhat more time, and may cause a slightly greater disturbance of the projectors.

3. Horizontal Angle of Stereoscopic Perception:--The term, "horizontal angle of stereoscopic perception", as applied to the viewing of a stereoscopic model on the multiplex machine, refers to the horizontal angle through which the observer may swing his head and still perceive the stereoscopic effect of his model. The plane of the angle will be considered as the top of the plotting table; the angle will be centered at the pencil point beneath the center of the tracing table.

Theoretically, the observer's best stereoscopic perception will be attained if the focus of his eyes is on a line perpendicular to the axis of the two projectors and as nearly on a perpendicular above the model as the position of the projectors will allow. However, there is a certain angle through which the observer may swing his head in an arc on each side of the line perpendicular to the axis of the projectors and still be able to see sufficient three-dimensional effect for the plotting of his contours. It will be the purpose of this experiment to measure the maximum angle of stereoscopic perception for both the anaglyph and polaroid methods of light filtration.

- a. The Experiment--In order to obtain a representative comparison of the two methods, angles of stereoscopic perception were measured under each projector and directly between the two, giving three angles for each method. In the interest of greater accuracy, six different measurements were taken of each of the six angles.

The first angle measured was the one with its center on the axis of projection between the two projectors, as seen with the colored filters in place. The axis of projection was defined by moving the tracing table under the left projector with its center on the projected center of the left diapositive and making a dot with the pencil, then moving the tracing table under the right projector and marking the projected center of the right diapositive; a straight line between these two centers represents the axis of projection and is analogous to the line of flight between the two corresponding camera stations. The point halfway between the two centers was marked, and the tracing table was oriented and set directly over it. The point thus selected also was at about an average elevation of the terrain.

In viewing the model for stereoscopic perception, the line of sight was kept at an

angle of around 60° with the horizontal, or about what would be normally used for observation in plotting. The drafting of the angle involved the following: A thin sighting stick was held vertically with its one end on the plotting table and the other end on the line of sight; the line of sight was moved around in an angle to the left of the line perpendicular to the axis of projection, keeping the sighting stick on the line of sight between the eyes and the tracing table, until some point was reached at which the three-dimensional effect was no longer existent; a straightedge was placed on the table from the foot of the sighting stick to the point of the pencil underneath the tracing table, and a line was drawn between these two points; the eyes were then moved in an arc to the right of the perpendicular, the sighting bar was moved at the same time along the line of sight, and a line was drawn between the center and the resulting point of zero stereoscopic perception in the same manner as before. The two lines, so drawn, are the confining limits of an acute angle which represents the maximum horizontal angle through which stereoscopic perception is possible. The above process was performed five more times, and an average of the six angles was computed.

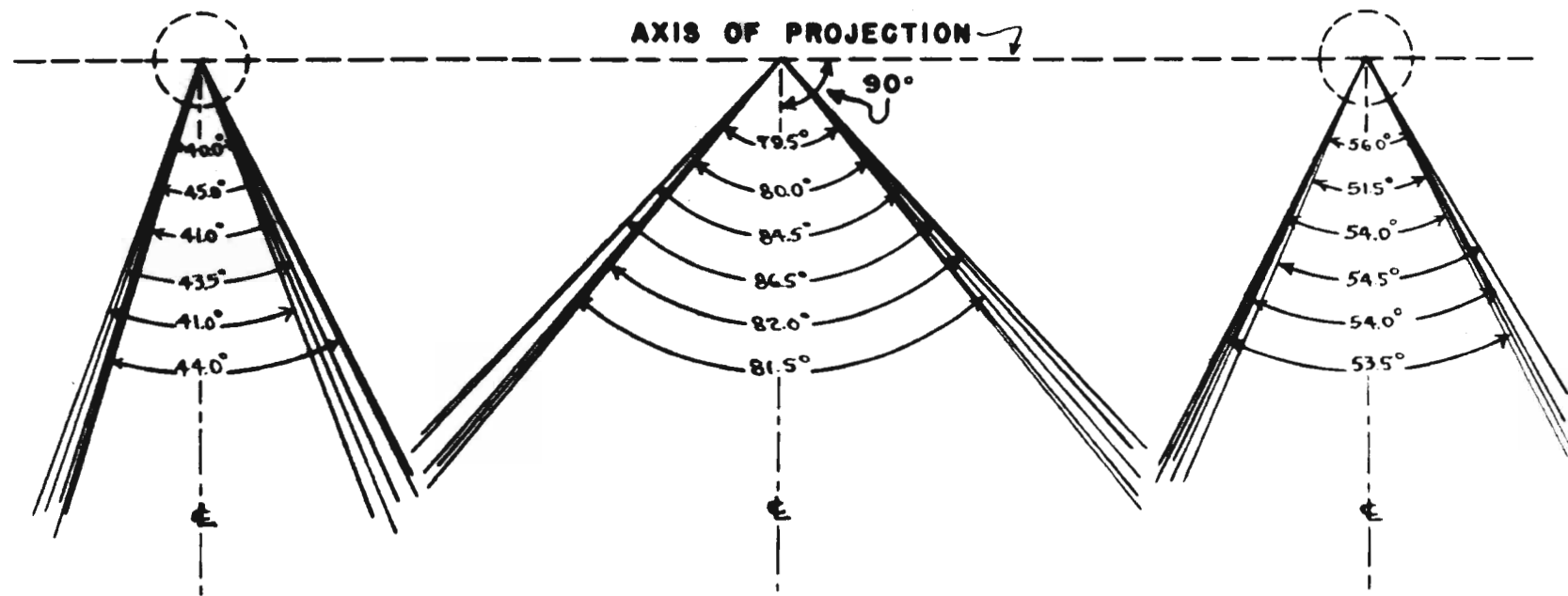
The tracing table was then moved over the point under the left projector, where the angle of stereoscopic perception was measured six times, and an average taken. From there, the table was moved over the projected center of the right projector and another set of angular measurements was made.

For the second half of the experiment, the colored filters were replaced by the polaroid filters. Angular measurements were then made at the three points (under the left projector, at the center, and under the right projector) in the same manner as previously explained. In each case, the average of six different angular measurements was calculated.

- b. Results--Measurements which were recorded in the course of the above experiments are consolidated into Table 2, on Page 85.
- c. Discussion--Attention is called to Figures 28 and 29, wherein are plotted the various angles of stereoscopic perception at the three stations along the axis of projection. Fig. 28 shows those angles as viewed with the colored filters in the projectors, while Fig. 29 shows angles observed under light projected through the polaroid filters. By studying these two diagrams, one may note several pertinent facts concerning the experiments.

STATION	FILTERS	1	2	3	4	5	6	AVE. θ
LEFT	COLORED	40.0°	45.0°	41.0°	43.5°	41.0°	44.0°	42.4°
	POLAROID	43.0°	38.0°	41.0°	44.0°	43.5°	38.5°	41.3°
CENTER	COLORED	79.5°	80.0°	84.5°	86.5°	82.0°	81.5°	82.3°
	POLAROID	41.5°	45.0°	44.5°	43.0°	41.5°	43.0°	43.1°
RIGHT	COLORED	56.0°	51.5°	54.0°	54.5°	54.0°	53.5°	53.9°
	POLAROID	43.0°	48.5°	44.0°	45.0°	48.0°	45.0°	45.6°

TABLE 2 - ANGLES OF STEREOSCOPIC PERCEPTION.

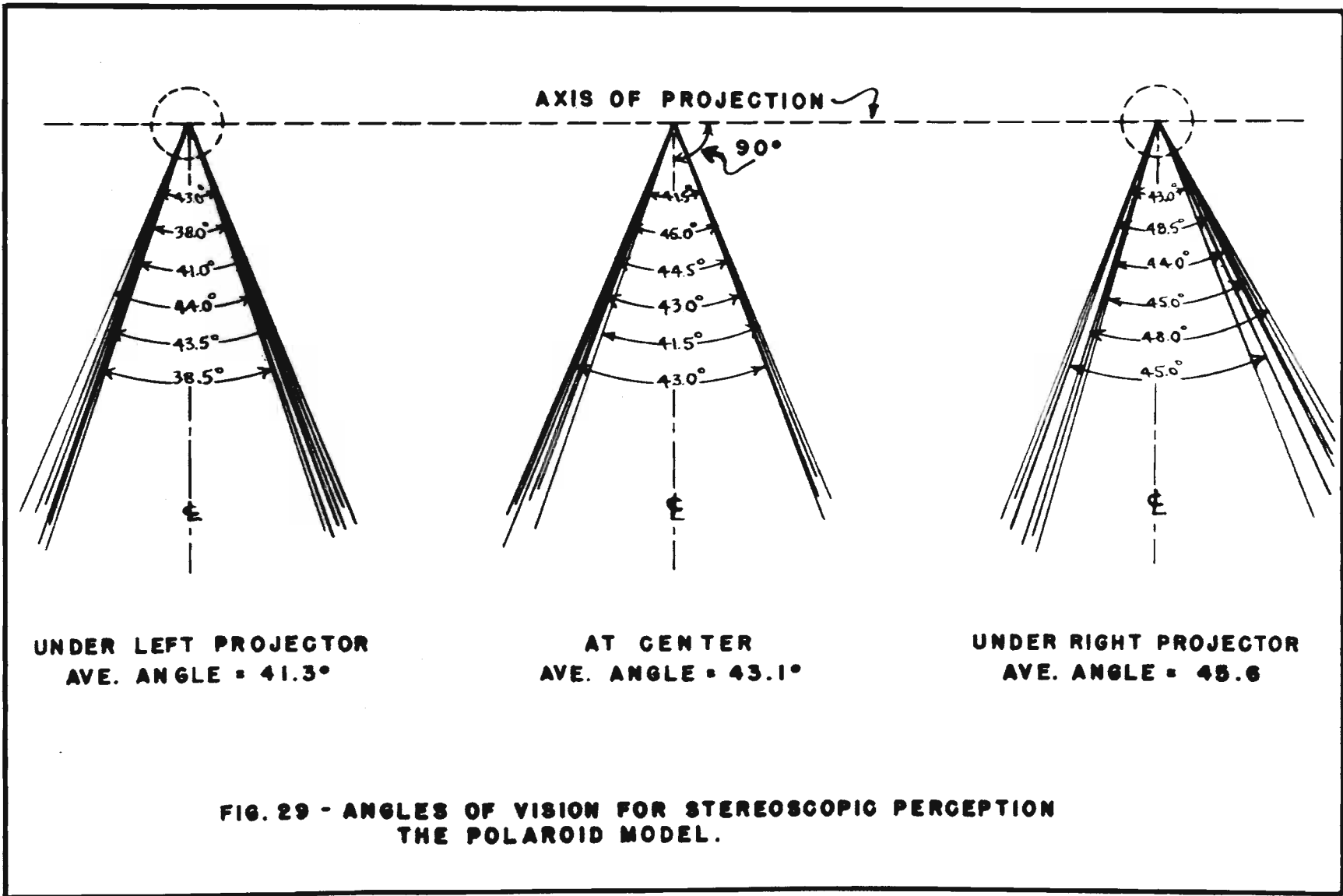


UNDER LEFT PROJECTOR
AVE. ANGLE = 42.4°

AT CENTER
AVE. ANGLE = 82.3°

UNDER RIGHT PROJECTOR
AVE. ANGLE = 53.9°

FIG. 28 - ANGLES OF VISION FOR STEREOSCOPIC PERCEPTION IN THE ANAGLYPH MODEL.



First, when the colored filters are in place, it is evident that there is a greater angle of stereo perception at the point directly between the projectors than under either projector. This means that the operator has greater latitude in moving his head when he is in front of the center of the anaglyph model. Comparing this result with the average angle measured at the center of the model projected through the polaroid filters, we find that the central angle with the colored filters is approximately twice as great. This amount is represented by the percentage, $43.1^{\circ}/82.3^{\circ} = 52.5\%$, which means that the smaller angle is 52.5% as wide as the larger angle.

In comparing the average angle at the center to other angles on the anaglyph model, the angle under the left projector is $42.4/82.3 = 51.5\%$ as large as the angle at the center, while the angle under the right projector is $53.9/82.3 = 65.4\%$ as large as the angle at the center. This indicates that the angle of stereo perception on the anaglyph model appreciably diminishes as the line of sight is moved away from the center of the model.

A study of the average angles on the polaroid model reveals that there is little comparative difference between the three. With

respect to the average central angle of 43.1° , the average angle under the left projector is $41.3/43.1 = 95.8\%$ as large, while the average angle under the right projector is $45.6/43.1 = 105.8\%$ greater. The overall inference to be derived from this series of measurements is that movement of the line of sight to either side of the center makes little relative difference in the horizontal angle of stereoscopic perception.

A comparison of the angles of stereo perception for the two filter systems discloses the following: (a) Under the left projector, the average angle on the polaroid model is $41.3/42.4 = 97.5\%$ as great as the angle on the anaglyph model; (b) at the center, the angle on the polaroid model is 52.5% of that on the anaglyph model, as calculated previously; (c) under the right projector, the average angle on the polaroid model is $45.0/53.5 = 84.2\%$ as large as the average angle on the anaglyph model. It can readily be seen from these figures that a greater angle of stereoscopic perception is present in all parts of the anaglyph model than is present in the polaroid model, and that the value of this angle on the anaglyph model increases as the center of the model is approached by the line of sight. The value of this angle on the polaroid model is compara-

tively constant throughout the entire model.

Figures 28 and 29 show that the average angles of stereo perception under the right projector of both models are considerably greater than the corresponding angles under the left projector. The probable cause of this phenomenon is the fact that the terrain at the right side of the model was much flatter than that area from the center over to the left side of the model. Hence, it can be assumed that this angle on a model of flat terrain will increase through inability of the eyes to perceive the exact point at which the stereoscopic effect dissipates, which in turn is caused by a diminishing of the ratio between the average height of terrain features to the height at which the photographs were taken. Another theory considered was that tilt in the two photographs of the model would tend to throw the nadir point over to the right of center, thereby causing a greater angle of stereoscopic perception on the right side of the model. It is felt, however, that this latter theory is not substantiated because of the negligible value of the tilt which is allowable in aerial photographs used for mapping purposes.

Results of the experiments on horizontal angle of stereoscopic perception definitely

show that the anaglyphic model has the larger angle. The value of this test is not so much as an instrument for proving greater accuracy in map making, but rather shows a possible effect on the inconvenience to, or physical discomfort of, the multiplex operator. It is readily apparent that the strain on a person who is performing tedious or painstaking work is diminished appreciably if that person is able to move around somewhat while he works. Hence, the multiplex operator should be slightly more comfortable in working with the colored filters than with the polaroid filters, as he has a greater angle through which to shift his line of sight while drawing contours on his map.

4. Accuracy in Setting of Light Point on Ground Surface:--The most important single factor in the map-making process is that of accuracy. In this experiment, it is proposed to test the accuracy of plotting with the tracing table on both the anaglyph and polaroid types of stereoscopic model. The delineation of detail, including the horizontal control, in making the map is usually a matter of ordinary drafting ability. The more difficult part of the mapping process involves the plotting of vertical control, which hinges on the ability of the operator to accurately set the light point of his tracing table on the surface of the terrain for any given elevation. Points of known elevation will be used for this test of accuracy; they will

serve as basēs for comparison of average readings on the same points in both the anaglyph and polaroid models. Readings taken on the vernier of the tracing table, when the luminous mark has been set on the surface of the ground at a given point, will indicate the height of the table top in millimeters. The millimeter reading, when converted to feet, gives the elevation of the point above sea level.

- a. The Experiment--A model with average terrain characteristics was selected for the experiment; this particular model came from the Lupus Quadrangle, in Missouri. Six points of varying elevations, which were widely distributed over the model, were selected in order to obtain a representative basis for the test. Two points were located in the central third of the model, two points were near the back edge of the model, and two were along the front edge. Figure 30 illustrates the distribution of the six points over the model. The points, C-1 and C-2, indicate the projected centers of the adjacent diapositives in the model.

In conducting the experiment, the colored filters first were inserted in the machine, and then the tracing table was placed over point 1. The luminous mark was set on the surface of the ground at that point, and a reading in millimeters was made for the corresponding height of the tracing table top. The platen of the

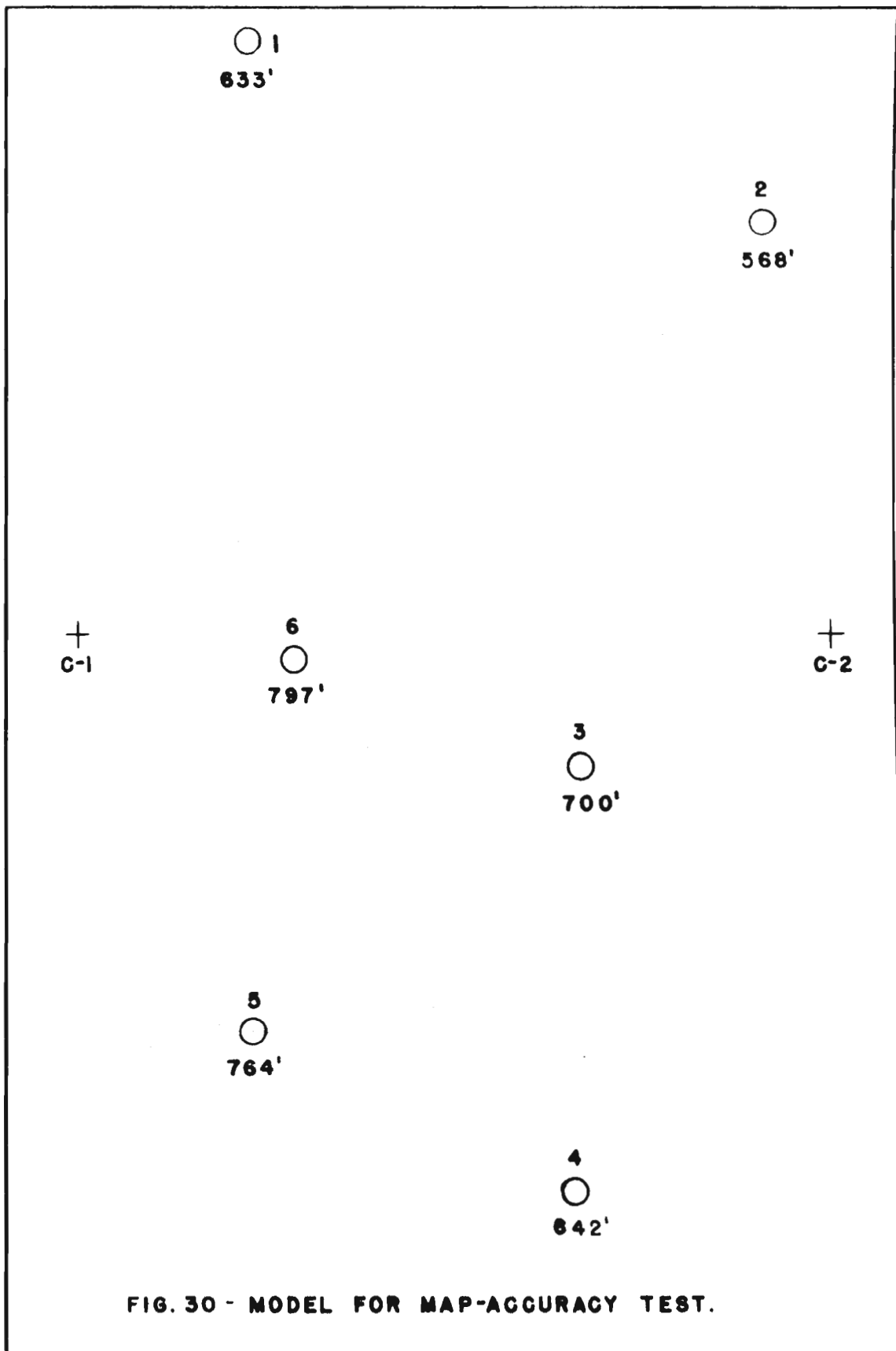


FIG. 30 - MODEL FOR MAP-ACCURACY TEST.

tracing table was lowered, the table was again adjusted over the point, and a second reading of elevation was taken. A total of six readings of millimeters were made for point 1, an average of the six was calculated, and this average was converted into feet of elevation by reference to the multiplex conversion chart shown in Table 3. The tracing table was then moved over the other five points, in turn, and six readings of elevation recorded for each point, as before.

The colored filters were replaced by the polaroid filters. The tracing table was set over the various points, six readings of elevation being recorded at each point in the same manner as with the anaglyph model.

- b. Results--Results of the preceding experiment are condensed in Table 4. Only the average elevation has been converted in each case from the average millimeter reading of the tracing table scale.
- c. Discussion--The veteran multiplex operator should be able to set a point on the map with his tracing table, work elsewhere for a while on the map, and then return to the original point and check it as to elevation. In any event, he should be able to set back on a point within \pm 0.1 millimeter; 0.1 millimeter represents approximately 3.33 feet on a 1:10,000 scale. The

MULTIPLEX CONVERSION CHART

Feet to Millimeters

Interval = 10 ft.		Scale = 1:10,000				Factor = .03048			
Elev.	MM	Elev.	MM	Elev.	MM	Elev.	MM	Elev.	MM
300	9.14	700	21.34	1100	33.53	1500	45.72	1850	56.39
10	9.45	10	21.64	10	33.83	10	46.08	60	56.69
20	9.75	20	21.95	20	34.14	20	46.33	70	57.00
30	10.06	30	22.25	30	34.44	30	46.63	80	57.30
40	10.36	40	22.56	40	34.75	40	46.94	90	57.61
350	10.67	750	22.86	1150	35.05	1550	47.24	1900	57.91
60	10.97	60	23.16	60	35.35	60	47.55	10	58.22
70	11.28	70	23.47	70	35.66	70	47.85	20	58.52
80	11.58	80	23.77	80	35.97	80	48.16	30	58.83
90	11.89	90	24.08	90	36.27	90	48.46	40	59.13
400	12.19	800	24.38	1200	36.58	1600	48.77	1950	59.44
10	12.50	10	24.69	10	36.88	10	49.07	60	59.74
20	12.80	20	24.99	20	37.19	20	49.38	70	60.05
30	13.11	30	25.30	30	37.49	30	49.68	80	60.35
40	13.41	40	25.60	40	37.80	40	49.99	90	60.66
450	13.72	850	25.91	1250	38.10	1650	50.29	2000	60.96
60	14.02	60	26.21	60	38.40	60	50.60	10	61.26
70	14.33	70	26.52	70	38.71	70	50.90	20	61.57
80	14.63	80	26.82	80	39.01	80	51.21	30	61.87
90	14.93	90	27.13	90	39.32	90	51.51	40	62.18
500	15.24	900	27.43	1300	39.62	1700	51.82	2050	62.48
10	15.54	10	27.74	10	39.93	10	52.12	60	62.79
20	15.85	20	28.04	20	40.23	20	52.43	70	63.09
30	16.15	30	28.35	30	40.54	30	52.73	80	63.40
40	16.46	40	28.65	40	40.84	40	53.04	90	63.70
550	16.76	950	28.96	1350	41.15	1750	53.34	2100	64.01
60	17.07	60	29.26	60	41.45	60	53.64	10	64.31
70	17.37	70	29.57	70	41.76	70	53.95	20	64.62
80	17.68	80	29.87	80	42.06	80	54.25	30	64.92
90	17.98	90	30.18	90	42.37	90	54.56	40	65.23
600	18.29	1000	30.48	1400	42.67				
10	18.59	10	30.78	10	42.98	Ft.	MM	Ft.	MM
20	18.90	20	31.09	20	43.28	1	.03	6	.18
30	19.20	30	31.39	30	43.59	2	.06	7	.21
40	19.51	40	31.70	40	43.89	3	.09	8	.24
						4	.12	9	.27
550	19.81	1050	32.00	1450	44.20	5	.15	10	.30
60	20.12	60	32.31	60	44.50	1800	54.88	2150	65.53
70	20.42	70	32.61	70	44.81	10	55.17	60	65.84
80	20.73	80	32.92	80	45.11	20	55.47	70	66.14
90	21.03	90	33.22	90	45.42	30	55.78	80	66.45
						40	56.08	90	66.75

Table 3 - Multiplex Conversion Chart.

		POINT 1		POINT 2		POINT 3		POINT 4		POINT 5		POINT 6	
		MMS.	ELEV.	MMS.	ELEV.	MMS.	ELEV.	MMS.	ELEV.	MMS.	ELEV.	MMS.	ELEV.
ACTUAL		19.29	633	17.31	568	21.34	700	19.57	642	23.28	764	24.29	797
COLORED MODEL	1	19.2		17.3		21.5		19.5		23.3		24.3	
	2	19.0	<i>spread</i>	17.1		21.5		19.4		23.2		24.5	
	3	19.0	<i>.3</i>	17.1		21.4		19.4		23.4		24.4	
	4	19.1		17.1	<i>✓</i>	21.6	<i>✓</i>	19.4	<i>✓</i>	23.4	<i>✓</i>	24.3	<i>✓</i>
	5	19.1		17.3		21.6		19.3		23.2		24.4	
	6	19.3		17.3		21.4		19.5		23.2		24.5	
	Ave.	19.12	627.3	17.20	564.3	21.50	705.3	19.42	637.3	23.28	764.0	24.40	800.7
POLAROID MODEL	1	19.1		17.1		21.3		19.4		23.2		24.3	
	2	19.0		17.2		21.4		19.4		23.4		24.4	
	3	19.2		17.3		21.3		19.5		23.3		24.3	<i>✓</i>
	4	19.1	<i>✓</i>	17.3	<i>✓</i>	21.4	<i>✓</i>	19.4	<i>✓</i>	23.3	<i>✓</i>	24.5	
	5	19.2		17.2		21.3		19.3		23.3		24.4	
	6	19.1		17.3		21.4		19.4		23.2		24.3	
	Ave.	19.12	627.3	17.23	565.3	21.35	700.3	19.40	636.7	23.28	764.0	24.37	799.7

TABLE 4 - RECORDED ELEVATIONS ON MAP-ACCURACY TEST.

writer, however, has digressed from the actual setting by as much as 0.29 millimeter on single readings, and as much as a 0.17-millimeter difference on average readings for any one point. Despite this personal error in manipulation of the machine, it is believed that this test still constitutes a good measure for comparison of one model against the other.

It is interesting to note that the largest personal errors occurred on points 1 and 4, which are the farthest outward from the central axis of the model, as defined by a line through the projected centers of the stereo pair of diapositives. It is also well to here stress the fact that the maximum average personal error of about 5.3 feet is still considerably within the allowable error for plotting 20-ft. contours with the multiplex machine on a Grade A map of 1:10,000 scale.

On point 1, the same average reading, 19.12, was recorded for both models. Using 19.29, the actual reading in millimeters, for a basis of comparison, we find that our error is $(100\% - 19.12/19.29) = 0.9\%$, for both models.

On point 2, the error for the anaglyph model is $(100\% - 17.20/17.31) = 0.7\%$, while the error for the polaroid model is $(100\% - 17.23/17.31) = 0.5\%$. Hence, the polaroid model was found to be slightly more accurate on this particular point.

On point 3, the error for the anaglyph model is $(21.50/21.34 - 100\%) = 0.8\%$; the error for the polaroid model is $(21.35/21.34 - 100\%) = 0.1\%$. This indicates a more sizeable accuracy for the polaroid model.

On point 4, the error for the anaglyph model is $(100\% - 19.42/19.57) = 0.6\%$, and the error for the polaroid model is $(100\% - 19.40/19.57) = 0.8\%$. This shows an advantage in accuracy on this point for the anaglyph model.

On point 5, the reading of both the anaglyph and polaroid models coincided with that of the actual elevation. Hence, there is no error on this point. This point was located at a road intersection along the side of a hill, and was very clearly defined.

On point 6, the error for the anaglyph model is $(24.40/24.29 - 100\%) = 0.4\%$. That for the polaroid model is $(24.37/24.29 - 100\%) = 0.3\%$. This indicates another slightly greater accuracy for the polaroid model.

Summarizing, we have two points on which readings were the same for both models, three points on which the polaroid model proved to be more accurate, and one point on which the anaglyph model was the more accurate. At first glance, it might appear, in view of the above, that the polaroid model would be considerably

more accurate than the anaglyph model. It must be borne in mind, however, that the differential between errors is very small, and that, in working on a larger number of points or a greater number of models, this small initial advantage could easily be equalized. It is important that the polaroid model is at least equal in accuracy to the anaglyph model in the plotting of contours.

Several impressions were noted while observing the two models. It appeared that the polaroid model, in its stereoscopic effect, was much stronger in the central area than at the edges of the model, while the anaglyphic model was more uniform throughout its dimensions.. Also, it was noted that, for good depth perception at the edges of the polaroid model, it helped to hold the eyes as nearly perpendicular to the point as was possible. It has previously been proved that the anaglyphic model has a greater latitude in its horizontal angle of stereoscopic perception. It is likewise evident, from observations of this experiment, that the anaglyphic model also has a greater latitude in the vertical angle of stereoscopic perception.

5. Clearness of Model:--The clearness of a stereoscopic model in the multiplex projector goes hand-in-hand with the accuracy in the drafting of contours. Clearness in a

stereoscopic model implies sharpness of terrain features and other details. This sharpness of detail is dependent largely on the amount of light projected onto the platen of the tracing table. One of the failings of the anaglyph model is encountered where the diapositives have large expanses of forest or other areas of darkness; the resultant model has spots that are correspondingly darkened. This condition hinders to a great extent the proper plotting of the map, particularly in the drawing of the contours. The greater amount of light flowing through the polaroid filters tends to reduce these darkened areas, thereby giving a better model for mapping purposes.

A failing of the polaroid filters is sometimes encountered on a model whose diapositive negatives include large, light-colored areas. The resultant flow of light onto the tracing table produces a reflection, or glare, which is disagreeable to the operator and hinders efficient use of the luminous point. This disadvantage can probably be remedied by using a metallic-pigment paint of a darker shade on the table top; the darker paint would absorb more of the light rays and thus lessen the reflection. A sample of the metallic-painted surface of the tracing table used in experiments of Chapter VIII is contained in Inset B, of Figure 25. Very little light reflection has been encountered on the anaglyph model.

Experiments on clearness of stereoscopic models consisted chiefly of trying various surfaces on the platen of the tracing table. The need of a metallic surface on the

tracing table top for the purpose of collecting vibrations of the rays of polarized light was entirely substantiated. Any other form of surface gave very little or no three-dimensional effect. The white, lusterless finish used for the anaglyph model was also unsatisfactory. Other metallic surfaces were tried, including varying shades of steel, tin, and aluminum. A gilt-painted surface was also used. None of the foregoing surfaces were as satisfactory for the polaroid model as the regulation surface of Polaroid Aluminum Pigment No. 689, manufactured by the Polaroid Corporation.

No mathematical comparison has been derived herein for clearness of the two models. However, from various observations, it is apparent that the average clearness of the polaroid model is more pronounced than that of the anaglyphic model. We may, therefore, consider this point of comparison as favorable to the polaroid model.

6. Removal of Parallax:--The "removal of parallax" in the multiplex machine is a term which is synonymous to the process of exterior orientation of the stereoscopic model. To obtain the true stereoscopic model, it is necessary to make all corresponding rays intersect. On any horizontal plane the two rays of a given point will appear as point images and the line joining them will be parallel to the base line. This distance between the point images is an indication of the height of the actual point in space and is known as horizontal parallax. When this horizontal parallax is reduced to zero by selecting the proper horizontal plane, the actual position of the point in space is determined and

the elevation can be read. However, if corresponding rays do not intersect they cannot lie in the same plane and there is no horizontal plane where the two point images will fuse into one. There will be a component of distance between the two images at right angles to the base line, and hence to the horizontal parallax. This component is called vertical parallax. Relative orientation consists of eliminating this vertical parallax. In multiplex operations, it is commonly referred to simply as "parallax".

At this point, it may be well to review the six elements of exterior orientation--that is, the six movements that can be given to each projector to bring it into its proper position with respect to other projectors. They are as follows:

- a. "X" - movement along the x-axis or along the bar. The mean line of flight should parallel this movement.
- b. "Y" - movement perpendicular to the x-axis and along the y-axis. It is parallel to the datum when the bar is properly leveled.
- c. "Z" - movement in a vertical direction, used to adjust the flight altitude.
- d. Swing - rotation of the projector around its vertical axis.
- e. Tip - rotation of the projector around a horizontal axis parallel to the y-axis, similar to nose up or nose down of the airplane.
- f. Tilt - rotation of the projector around a horizontal axis perpendicular to the tip axis,

similar to a wing up or down in the airplane.

The removal of parallax involves several sequences of the above adjustments, which are continued until the last vestige of a vertical component in the luminous point is eliminated. The easiest method for accomplishing this is to leave one projector entirely stationary and make all movements on the other projector. During the removal process, the presence of parallax is tested at six different points on the model--in each of the four corners and at the projected centers of the diapositives.

When parallax is present in the anaglyph model, the observer sees the luminous point as a pair of dots, one red and one blue-green. The purpose of each individual adjustment is to bring the dots into vertical juxtaposition. Parallax in the polaroid model is seen as a pair of separated dots of approximately equal tone and brightness. The entire removal process requires from five to ten minutes for the average operator.

Experiments on the removal of parallax consisted of performing the adjustments on several models, both anaglyphic and polaroid. The nature of the removal process makes it extremely difficult to arrive at a satisfactory comparison between the two models. Both models were checked as to time required for the removal process and as to completeness of the removal; very little difference between the two models was noted in each case. Hence, as facility in this phase of orientation is concerned, the results indicate that both models compare evenly.

7. Observance of Model from Back of Plotting Table:--

Occasionally, it becomes necessary for the operator to work on certain points along the back edge of his model which are easier observed from the rear side of the plotting table. In order to plot from the back of the anaglyphic model, the operator merely reverses his spectacles by rotating them through an angle of 180° within the plane of the lenses. The spectacles are ordinarily arranged with straight ear pieces and a straight nose bridge which enables them to be worn normally or upside down, depending on the color combination in the projector filters. A correct stereoscopic impression also may be obtained from the rear by switching filters in the projectors and not changing the spectacles. The polaroid spectacles also may be arranged for rotation in the same manner as the anaglyphic spectacles, but it requires some reorientation if change is made in the polaroid filters.

As far as observation from the back of the model is concerned, the anaglyphic model appears to be somewhat easier to see. The position of the vertical projectors on the other side of the bar prohibits observance directly above the model, and thus tends to reduce stereoscopic observance of the polaroid model in a greater degree than the anaglyphic model. It has been mentioned previously that the anaglyphic model has a wider vertical angle of stereoscopic perception than the polaroid model.

It has not been fully determined whether this tendency to lesser sharpness of stereoscopic detail at the edges is inherent in the polaroid model or whether the cause could be

the lesser diameter of the polaroid filters. As far as diameters are concerned, the polaroid filters are only $49/66 = 74\%$ as large as the colored filters. The numbers, 49 and 66, are the diameters of the exposed glass in the polaroid and colored filters, respectively. Polaroid filters of size equal to the colored filters might possibly reduce, or entirely eliminate, this slight haziness toward the edges of the model. In the absence of facilities for further experiment on this problem, however, the colored model will be considered as having an advantage over the polaroid model in clearness of three-dimensional effect as observed from the back of the plotting table.

8. Effect of Heat on Filters:--The effect of heat on filters in the vertical projectors will be considered in this chapter only from a conditional viewpoint. When the multiplex machine is not equipped with a cooling system, heat effects become important because of the close proximity of the filter to the light source. The slot for the filter in the light housing assembly of the projector has a position directly beneath the socket for the electric lamp. Since this lamp is of at least 100-watt power, the resulting heat generated by it is sufficiently great to affect filters or lenses of glass if they are subjected to its effects for a long enough period.

a. Cooling System of Multiplex Machines--Multiplex plotting machines are usually equipped with cooling systems. The multiplex cooling system consists of an electric motor attached to a fan

which blows a continuous stream of air through rubber hoses into, and through, the lamp housing assemblies of the projectors. This system tends to keep both the lens and the filter in the projector sufficiently cool to prevent damage to the glass, while allowing the light to be illuminated for as long a period of operation as is necessary. The blower system is, of course, a recommended item for all mapping operations with the multiplex machine, and most mapping agencies are at present entirely equipped with them. If a multiplex machine is not equipped with the cooling system, it becomes necessary to curtail the periods of its operation so that the intensity of heat from the projector lamps will not crack the filters or (of greater importance) will not damage the projector lenses. The effect of heat on filters, then, will be a consideration only in their use in aeroprojectors not equipped with blower systems.

- b. Heat Effects on Colored Filters--No actual experiment has been conducted to discover how long a colored filter can withstand a certain amount of heat. However, the experiences of a number of veteran multiplex operators in the Multiplex Section of the United States Geological Survey, at Rolla, Missouri, has been collected for the period of several years at this particular

office. All instances of broken filters were accidental, caused probably by one of two reasons, as follows: (a) A flaw in the glass of the filter, and (b) too long a period of illumination of a machine having no cooling system. The latter factor apparently was responsible for much the larger portion of "breaks". Those filters with flaws in the glass were found to break with a much smaller period of illumination. There was no record of the breaking of a red filter. The average length of illumination necessary to crack the blue-green filter was estimated as about four hours. It is evident from the above that the blue-green filters are more sensitive to heat than the red filters. This is explained by the fact that the blue-green filters are about twice as thick as the red filters and hence will crack sooner from the expansion of the glass occasioned by the heat.

- c. Heat Effects on Polaroid Filters--An accidental "break" of a polaroid filter during the course of other experiments makes possible an approximate comparison between the two types of filter as to their heat resistance. Prior to the aforementioned break, it was assumed that the polaroid material in the filter lens was a thin sheet of a crystalline substance, such as tourmaline. Since it was cemented between two thin sheets of

glass, it was expected that the three thin sheets, cemented together, would be more resistant to heat than a glass filter of such thickness as to equal that of the three thin sheets. However, subsequent information from the manufacturer has disclosed that the polaroid material in question is a thin, film-like substance, synthetically made, which, no doubt, accounts for its inability to withstand the effect of the projector lamp for periods of long duration.

An actual instance of the breaking of a polaroid filter from the effect of heat was recorded during research on another comparison problem. The multiplex machine had been operated without the blower system for about 25 minutes, with the colored filters in the projectors. The colored filters were then replaced with the polaroid filters. After about five minutes additional time, one of the filters "exploded" with a rather subdued popping noise. Upon removal, it was discovered that the enclosed film-like material in the center of the lens had curled up and pulled apart in several places. The thin glass sheets of the filter lens were not cracked and were apparently otherwise undamaged. The lens, of course, was of no further value because of the rifts in the polaroid material. It was interesting to note that the dam-

aged lens was the one that was loose in its adaptor ring. It appears that a firm setting of the lens might increase its resistance to heat effects.

- d. Discussion--A true comparison of the heat-resistant qualities of the two types of filter is difficult because it is impossible to test them here under parallel intensities of heat; also, there are so many varied thicknesses of glass, differences in materials, coefficients of expansion, and other factors involved that it would require a special laboratory to properly test them.

As far as our own particular filters are concerned, it is reasonable to assume that the colored filters would withstand the effects of heat much better than the polaroid filters. What little information we have is inadequate to provide an accurate percentage comparison between the two types of filter.

The foregoing material in this section is presented with the realization that the operation of a set of projectors without a cooling system of some sort will soon take its place among the relics of the past. Hence, heating effects on filters and lenses will cease to be an issue for consideration, and all multiplex operators may then plot their maps without fear

of overheating their projectors.

9. Color-Blindness:--A limitation of the colored filters was discovered in their use by people who are color-blind. One requirement of a stereoscopic impression is that each eye see a separate image of the same picture, taken from slightly different angles. In order to fulfil this requirement, the filter must project an image which one eye can see and the other eye cannot see. The anaglyphic model must depend upon the action of complementary colors to accomplish this process. A superimposition of two complementary colors will cause obliteration of practically all light going through them. If a person does not have a complete and normal mental impression of the complementary colors, red and blue-green, his ability to visually interpret the three-dimensional effect of the anaglyphic model will be impaired.

A large percentage of people who are troubled with color-blindness are color-blind in the reds and greens. It is not necessary that a person be color-blind in both of the complementary colors used in the anaglyphic model for non-perception of the stereoscopic effect. If he is color-blind in either the red or the blue-green, he is automatically incapacitated for use as a multiplex operator where the anaglyphic model is used.

Two men who were color-blind tested both models. One man was color-blind only in red, and the other was color-blind both in red and in green. Neither one could see any stereoscopic effect on the anaglyphic model. However, they

could easily observe the three-dimensional effect on the polaroid model. It is probably safe to assume, then, that anyone with ordinary, binocular vision can observe the stereoscopic effect of the polaroid model.

Many people who are color-blind have no other defect as far as their ordinary vision is concerned. The polaroid model opens the field of stereoscopic projection to this group and thereby scores another advantage over the anaglyphic model.

10. Color Projection:--Another advantage of the polaroid model is in the possibility of projecting plane-polarized light in colors. The anaglyphic model depends for its stereoscopic effect on the projection of complementary colors to form a picture in tones of black and white.

We may look forward into the future and find aerial photographs taken and projected in the colors of nature. Such a model would materially aid in photo-interpretation for the plotting of contours and other detail.

CHAPTER IX

CONCLUSION

1. Summary of Comparisons:--The experiments of Chapter VIII are summarized by sections as follows:

- a. Light Quantities Through Filters--Only 53.8% as much light was projected through the colored filters as came through the polaroid filters. Because of the importance of light on the tracing table, this represents a sizeable advantage for the polaroid filters.
- b. Orientation of Filters in Projectors--It was found that the polaroid filters required some orientation in placing them in the projectors, whereas the colored filters merely required insertion in the projectors without regard to orientation. In view of possible disturbance to the projector itself, this was considered as an advantage for the colored filters.
- c. Horizontal Angle of Stereoscopic Perception--In working with the polaroid filters, it was noticed that the three-dimensional effect was lost if the line of sight deviated slightly from a line through the given point perpendicular to the axis of projection. Measurement of this horizontal angle of stereoscopic perception disclosed that the angle at the center of the anaglyphic model was approximately twice as wide as that angle at the center of the polaroid

model. Angles on the axis of projection at the edges of the anaglyphic model also were greater than the corresponding angles on the polaroid model, but to a lesser degree. Since holding to a narrow angle of observation might cause some physical discomfort to the multiplex operator, the anaglyphic model is given the advantage on this point of comparison.

- d. Accuracy in Setting of Light Point on Ground Surface--This important element of mapping was determined by selecting six representative points of known elevation on a typical model, then comparing elevations as measured on each of the anaglyphic and polaroid models. Of the six points, two were ties, on one the anaglyph model was more accurate, and on three the polaroid model was more accurate. The polaroid model, therefore, receives the advantage in this element.
- e. Clearness of Model--The polaroid model was found to be considerably clearer in the center of the model, but progressively lost some of its sharpness as the edges were approached, while the anaglyphic model was found to be more uniformly clear throughout its entire surface. Because of its greater general clearness, however, the polaroid model was adjudged the more advantageous with respect to clarity.

- f. Removal of Parallax--The neutral-colored image points of the polaroid model were found to be equally as easy to follow as the red and blue-green image points of the anaglyphic model in the removal of parallax. No better medium of comparison was found than actual observation during the process of removing parallax. No advantage on this point will be given to either model, and the result will be considered a "draw".
- g. Observance of Model from Rear of Plotting Table--It was found that the stereoscopic effect was somewhat more difficult to observe in the polaroid model, when viewed from the back of the table. The reason for this probably is the smaller vertical angle through which the three-dimensional effect is visible in the polaroid model. The advantage on this point, therefore, goes to the anaglyphic model.
- h. Effect of Heat on Filters--This factor, which is only pertinent to our problem when the multiplex machine is not equipped with a cooling system, rates as an advantage for the anaglyph model. The polaroid filters were found to be much more susceptible to the effects of heat than were the colored filters.
- i. Color-Blindness--A limitation of the anaglyphic model is shown in the inability of people who

are color-blind to perceive any stereoscopic effect in such a model, whereas they may easily observe this effect in the polaroid model. It is felt that a limitation in one model becomes an advantage for the other.

- j. Color Projection--Another advantage of the polaroid model is the possibility of the projection of models in color. These models should not only be more true to nature, but also should be more restful to the eyes.

2. Analysis of Comparisons--Three factors which enter into the mapping process, in the order of their importance, are: (a) Accuracy of the map, (b) time required for its construction, and (c) expense of its preparation. A map is worth little if it does not fully meet the requirements for accuracy. Of the three factors outlined above, this is, by far, the most important. The second and third factors are more or less closely allied. The less time it requires to draft a map, the more maps will be completed in a unit time; as more maps are completed per unit time, the expense per map correspondingly decreases. The time required for constructing the map becomes of prime importance during time of war, when the map perhaps may be required in military operations. The third factor is probably of greater importance to civilian mapping agencies. Mapping is primarily a business to the civilian agency, whereas it is more of a service to the government agency. A knowledge of the above factors will serve as a foundation for evaluating the importance of

the several points upon which our two types of filter have been compared.

The single point of the greatest importance, as far as accuracy is concerned, is the ability of setting the luminous point on the ground surface. A good model which allows the most accurate work with the floating light point insures a map whose contours will most nearly conform to the actual terrain. Although this element has the greater effect on the drafting of vertical control, it also aids, to a lesser degree, in the drafting of horizontal control and other detail.

The point which probably is next in importance is that of amount of light projected onto the tracing table. This item has more of an indirect effect on the accuracy of the map; it is responsible to a large extent for the clearness of the model which, in turn, is conducive to accuracy and facility in the delineation of contours.

The next item which is primarily concerned with map accuracy is that of clearness of the model. Clearness in the model probably is the result of a combination of sharpness of feature in the stereoscopic effect which is inherent in the model itself, plus the proper amount of light falling on the tracing table, plus a receptive surface on the tracing table for preservation of the image.

The removal of parallax affects, to a lesser extent, both the accuracy of the model and the time required for orientation of the model. Its effect on accuracy is noted in the possible incompleteness of removal, while difficulty

in removal probably would constitute a greater consumption of time. In any event, this point will be eliminated in the final analysis, since there appears to be no difference between the two models in this respect.

The horizontal angle of stereoscopic perception apparently has little effect on either accuracy or time, but does have some effect on the physical comfort of the operator. Observance of the model from the rear of the plotting table, which is partially regulated by the vertical angle of stereoscopic perception, will require a greater amount of time for plotting certain points on the polaroid model, but occasions are infrequent when it becomes necessary to plot from the rear of the table.

The orientation of the filters and the effect of heat on filters are both of negligible consequence as comparative factors in our problem. Only a little more time is required for orientation of the polaroid filters than is needed in the mere insertion of the colored filters. Effect of heat was included in Chapter VIII only for the benefit of those few remaining aeroprojectors not equipped with cooling systems.

The limitations of the anaglyphic model, with respect to its use by people who are color-blind and its use for color projection, have no bearing on our factors of accuracy and time, but still are considerations in any comparison between it and the polaroid model.

3. Conclusion:--A decision as to the best stereoscopic model for mapping operations with the aeroprojector is

dependent upon the outcome of a number of comparative tests, such as have been conducted in Chapter VIII. In order to arrive at a logical decision, it is necessary to weigh the advantages of one model against those of the other. The advantages which have been recorded in favor of the anaglyphic model are on the following points: (a) Angles of stereoscopic perception, (b) plotting from rear of model, (c) orientation of filters, and (d) effect of heat on filters. Those points in which an advantage was discovered for the polaroid model were: (a) Accuracy in setting of light point on ground, (b) amount of light coming through filters, (c) clearness of model, (d) application to color projection, and (e) use by color-blind people.

In view of the fact that the elements of accuracy in plotting and amount of projected light are of such great importance in the multiplex process, it is believed that they alone more than balance the advantages discovered in the anaglyphic model. Consequently, it is the opinion of the writer that the polaroid model is fully as effective a basis for good map-production as is the anaglyphic model. It is also believed that further development of the polaroid model, leading to the elimination of the several slight deficiencies noted herein, will result in its eventual displacement of the anaglyphic model in the multiplex process.

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VITA

Robert Greig Livingston was born on May 17, 1914, at De Camp, Mo., the son of Dr. and Mrs. Archibald A. Livingston.

His early education was received in the grade school at Cedar Hill, Mo., and in the high school at Eureka, Mo. He entered the Missouri School of Mines and Metallurgy in September, 1935, and graduated in 1939 with a B. S. Degree in Civil Engineering.

Two of his summers between school years were spent as an engineering aid on the construction of Dam and Locks No. 26, at Alton, Ill. A third summer was spent at the Reserve Officers' Training Camp at Fort Riley, Kans.

Following graduation from Missouri School of Mines in 1939, Livingston was employed by the St. Louis Southwestern Ry. Co. For nearly two years he worked under the Division Engineer at Pine Bluff, Ark.

In early 1941, he accepted active duty in the Army as a Second Lieutenant in the Corps of Engineers, and was assigned to the 30th Engineer Topographical Battalion, at Fort Belvoir, Va., where he remained for two years.

As a Captain, he then activated and took overseas Headquarters Company of the 660th Engr. Topo. Bn. He was engaged in mapping operations with his unit in the European Theater for 2½ years, and was finally separated from the Army after five years of service.

Shortly after his return from overseas, he entered Mo. School of Mines for graduate work in Civil Engineering.