1969

A study of different types of connections to structural-steel square tubings

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A STUDY OF

DIFFERENT TYPES OF CONNECTIONS

TO STRUCTURAL-STEEL SQUARE TUBINGS

BY 440

CHAO-SHENG CHEN, 1930

A

THESIS

submitted to the faculty of

THE UNIVERSITY OF MISSOURI-ROLLA

in partial fulfillment of the requirement for the

Degree of

MASTER OF SCIENCE IN CIVIL ENGINEERING

Rolla, Missouri

1969

Approved by

Oktay Ural (advisor) Emmett [illegible]

[illegible]

ABSTRACT

Although square tubular element has long been recognized as an ideal structural member, its acceptance was limited by the lack of detailed knowledge on suitable connections related to this geometric shape.

It is structurally inefficient to use a strong column poorly connected to a strong beam. So it is necessary to understand, in advance of adoption, the effect and behavior of different types of connections and to develop an efficient structural system.

In this study, the ordinary column-to-beam framing with different types of connections has been investigated. The three types of connections are single-plate connection, double-angle connection, and multi-angle connection. Six specimens were prepared, instrumented and tested during this study. The research covers only the important factors affecting the design of a connection, such as the rotation and deflection of the beam end, the moment-rotation and deformation of the connection, and stress distribution and stress concentration on the column wall.

ACKNOWLEDGEMENT

The author wishes to take this opportunity to express his deep appreciation to Dr. Oktay Ural, his advisor, for his advice and guidance throughout this study.

The author is especially grateful to Professor John W. Hubler and Dr. Ernest M. Spokes for their generous help and guidance.

The author is also indebted to Mr. Chenchin Liu, Mr. Wei-Jiun Huang and Mr. Richard K. H. Chen, for their valuable suggestions and unforgettable cooperation whenever required.

Thanks are also due to Mrs. Diane Jones for correcting the English of the manuscript.

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LIST OF SYMBOLS

The letter symbols adopted for use in this thesis are defined where they first appear and listed here in alphabetical order.

a = clear distance or plasticity reduction factor

c = distance from neutral axis to the point at which stress is desired

C_c = upper limit of slenderness ratio

d = depth of beam

D = outside diameter of a circular tube or the diagonal dimension of a rectangular tube

D_i = inside diameter of the tube

D_o = outside diameter of the tube

e = eccentricity

E = modulus of elasticity of material

E_t = tangent modulus of elasticity

f_b = allowable bending stress

f_{cr} = critical buckling stress

f'_{cr} = critical buckling stress calculated on the basis of a linear stress-strain relationship

I = moment of inertia

J_w = polar moment of inertia of weld

k = factor of stress concentration

kL = effective unbraced length of the column

L = unsupported length of member

L_h = length of the angle

L_v = angle's leg length

M = moment

M_p = plastic moment

N = factor of safety

P = load

P_u = ultimate load

P_y = theoretical yield load

Q = statical moment of section lying outside the line on which the shear stress is desired, taken about the neutral axis

r = least radius of gyration of section

R = radius of the tube or reaction

S = section modulus

t = thickness of the tube wall

t_w = thickness of the beam web

V = vertical shear on beam

w = leg size of fillet weld

y = distance of center of gravity of area to neutral axis

σ_b = bending stress, may be tension or compression

σ_y = yield stress

τ = horizontal shear stress at any point

Δ = deflection or deformation

ϕ = end rotation in radian

I INTRODUCTION

In the early days of the 18th century, malleable iron was produced in small quantities by fusing pig iron with coke or charcoal. The invention of the puddling process in 1874 made it possible to produce malleable iron in large quantities.

Steel, during this period, was made from puddled iron bars. Pure iron bars were packed in a box with charcoal, bone, or some carboniferous material free of oxygen. This box was tightly closed and kept in a furnace for several days. The carbon from the packing made its way into the hot iron bars and changed them into steel.

Engineers discovered that steel combined strength, workability and low cost to a degree unparalleled in any other material for construction. The need of steel in construction then increased rapidly stimulating the work to improve the quality of steel. Some people believe that the standard of civilization in a nation might be measured by the quantity of steel produced and consumed.

Although steel has been widely used in structural elements, such as beams, columns, frames, and most of all the reinforcement of concrete structures, the square tubular shapes were only recently introduced. The square tubular type of section has many advantages. It has good resistance to bending and high section moduli in both directions. It is easy to weld square tubular shapes to other sections because of its flat sides. These sections offer good torsional resistance and this in turn provides greater lateral stability under compression. However, its acceptance has been restricted by a lack of detailed information on suitable connections, particularly for ordinary beam and column framing.

Obviously, it is wasteful to use a strong column poorly connected to a strong wide-flange beam. The whole frame would be structurally insufficient if the connection could not resist the force and moment caused by the allowable design loads of the column and the beam.

The purpose of this research is to study the efficiencies of different types of connections between tubular columns and beams. When a structural elements is subjected to external loads, its behavior depends not only upon the magnitude of the loads and the strength of the material, but also upon the shape of the element itself. So the types of connections are important factors to be determined.

In this study, three different types of connections are introduced: single-plate connection, double-angle connection, and multi-angle connection. The single-plate connection is designed to apply the loads on the center of the column wall. The double-angle connection distributes the applied loads over a rectangular area on the center of the column wall. And the multi-angle connection distributes the loads to the perpendicular column wall.

The requirements for adequate connections are as follows:

1) The connection should have adequate strength to safely carry the imposed beam reactions.

2) The connection should not induce stresses or distortions in the column that would cause an appreciable reduction in column strength.

3) The connection should not be so stiff as to restrict the transmission of bending moment to the connected column.

4) The connection must have satisfactory moment-rotation characteristics, although it is not designed to carry bending moments. The moment developed by the load will tend to deform the connection and unless some part of the connection can freely deform, the connection itself may

become overstressed to the point of possible failure.

5) The connection should not require complex erection procedures.

The general assumptions of elastic theory applied in this study are:

1) The material is homogeneous and isotropic.

2) A plane section before bending remains plane after bending.

3) The values of yield stress in tension and compression are the same.

Generally speaking, an experimental engineering study is both an art and a science. As an art it requires human thoughts, spirits and repetitious practices for its successful completion, and as a science it requires the use of mathematical analysis and intelligent interpretation of results. The accuracy of results depends upon the following factors:

1) The accuracy and sensitivity of the testing machine and the instruments used.

2) The care used in the preparation of the specimens.

3) The environmental control.

4) The knowledge, experience and care technique of the operator.

Perhaps the accuracy of the results depends more upon the last factor than anything else. Even though the best machines and instruments are available and the specimens have been properly prepared and the environment is under control, we still can not predict the success of the tests. Unless the researcher is experienced and careful and has a good knowledge of the technique of the test, the results obtained may be far from satisfactory. Experience may be gained only through repeated testing and knowledge may be gained from both experience and analytic studies, but a careful technique must be developed by the person himself.

II REVIEW OF LITERATURE

In 1964, Jack G. Bouwkamp* wrote a paper about his studies on the concept of tubular-joint design. He stated that although the circular tube has long been recognized as an ideal structural element to carry concentric compression loads, the use of tubular columns for truss-construction developed only after welding became an accepted method of joining structural elements. The reason for the use of tubular columns is that local buckling is of no concern. Only in extreme cases of thin-walled large diameter tubular member does the D/t ratio become a factor to be considered.

1) AISI Light Gage Cold-formed Steel Design Manual* specifies:

$$D/t \leqslant 33{,}000{,}000/f_y \qquad (1)$$

2) British Standard* specifies:

$$t \geqslant 0.1\sqrt[3]{D} \qquad (2)$$

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* Bouwkamp, J. G., " Concept of Tubular-Joint Design ", Proceedings of ASCE, Structural Division, April, 1964.

* American Iron and Steel Institute Light Gage Cold-Formed Steel Design Manual Commentary on the 1962 Edition. P.27

* " Use of Tubular Steel in Buildings ", Addendum No. 1 to British Standard 449 (" the Use of Structural Steel in Building "), November, 1953.

$$t_{min} = 0.128 \text{ in.} \qquad (3)$$

(for tube not exposed to weather)

$$t_{min} = 0.160 \text{ in.} \qquad (4)$$

(for tube exposed to weather)

where D = the outside diameter of a circular tube or the diagonal dimension of a rectangular tube

t = the thickness of the tube wall

f_y = yield point stress, psi

The purpose of limiting the D/t ratio is to prevent buckling. The over-all stability of the member can be secured by using the specified allowable compressive stress for different grades of steel.

3) German Buckling Specifications* restrict L/r to:

$$20 < L/r < 115 \qquad (5)$$

(for St 37-Steel, $f_y = 32,600$ psi)

$$20 < L/r < 90 \qquad (6)$$

(for St 52-Steel, $f_y = 48,200$ psi)

* " Stahlleichtbau und Stahlrohrbau im Hochbau; Richtlinien fur die Zulassung, Ausfuhrung, Bemessung, " Deutchen Normenausschuss, Beuth-Vertrieb, GmbH, Koln, Germany, 1950.

where $L =$ unsupported length of member

$r =$ least radius of gyration of the section

$$r_{min} = \sqrt{\frac{I_{min}}{A}} \qquad (7)$$

and

$$I_{\substack{max \\ min}} = \frac{I_x + I_y}{2} \pm \sqrt{\frac{(I_x - I_y)^2}{2} + I_{xy}^{\,2}} \qquad (8)$$

where $I =$ principal moment of inertia

Considering connection design of circular column, Bouwkamp noted that in the early days it was essential to weld a column member to a gusset plate because of the difficulty in cutting the tubular column to fit flat as well as cylindrical surfaces (Fig. 1).

The invention of a fully automatic oxyacetylene tube-cutting machine made it possible to cut any desired shape. The gusset plate is no longer necessary and the directly connected tubular joint reduces the cost as well (Fig. 2).

Also Bouwkamp pointed out that in the design of a connection that is only subjected to moderate static load, the flexibility normally does not need to be considered. However, for dynamically loaded connections, special attention should be given to the flexibility of the wall in order to limit the development of stress concentrations as much as possible. This procedure is also advisable for

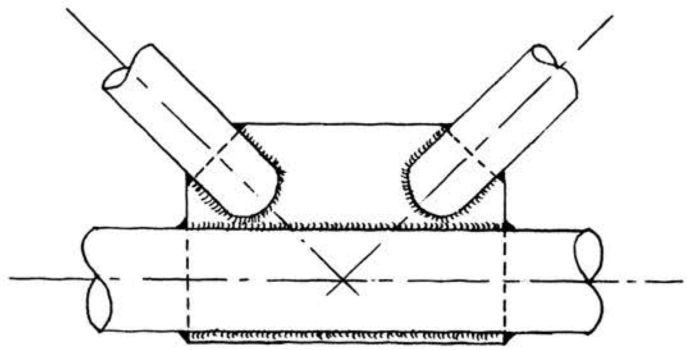

Fig. 1 Welding with Gusset Plate

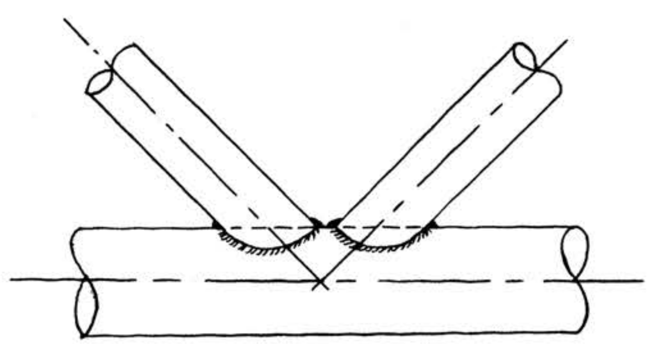

Fig. 2 Directly Connected Joints

connection subjected to high static loads.

Richard N. White and Pen Jeng Fang* presented a paper concerning framing connection for square structural tubing to the ASCE Structural Engineering Conference in New York on October 19, 1964. In that paper they introduced Batho's beam line concept* which, in conjunction with the moment-rotation ($M-\phi$) plot for the connection, is assumed to be the most convenient method for checking the efficiency of connection. This will be discussed to detail in chapter III.

They also introduced several factors influencing the behavior of connections:

1) Ratio of the width of the tube wall to the tube thickness--- As this ratio increases, any connection fastened directly to the tube wall rather than at the tube, will tend to become flexible.

2) Ratio of the connection length to the tube size --- This could also be expressed as the ratio of depth of connected beam to tube size, because web connection depths

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* White, R. N. and Fang, P. J., " Framing Connection for Square Structural Tubing ", Proceedings of ASCE, April, 1966.

* Batho's Beam Line Concept, from the Second Report, Steel Structures Research Committee, Department of Scientific and Industrial Research of Great Britain, London, England, 1934.

are ordinarily proportional to beam depths.

3) Shape of the tube--- The behavior of rectangular tubes is nearly the same as that of square tubes.

4) Type of the fastener--- The total rotation of the beam end at a connection is a function of all possible sources of rotation including distortions of the tube, deformations of the section used in the connection, deformation of the connecting devices, and deformation of the beam web, etc.

White and Fang believed that the distortion in the loaded face of the tubular column which sometimes occurs at early stages of loading may be the governing factor in designing the joint connection.

In conclusion, they stated that the shear strength is more than adequate in all connections tested, and nearly all connections have sufficient strength to resist the moment induced by beams of ordinary L/d ratio, where d is the depth of the beam.

Charles J. Schilling* published a paper in 1965 concerning the buckling strength of tubular columns. He introduced the Engesser formula or the tangent modulus

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* Schilling, C. J., " Buckling Strength of Circular Tubes", Proceedings of ASCE, October, 1965.

equation of a perfect axially loaded column:

$$f_{cr} = \frac{\pi^2 E_t}{(kL/r)^2} \qquad (9)$$

where f_{cr} = critical buckling stress of the column

E_t = tangent modulus of elasticity

kL = effective unbraced length of the column

for a circular tube

$$r = \frac{\sqrt{D_o^2 + D_i^2}}{4} \cong \frac{R}{\sqrt{2}} \qquad (10)$$

where D_o = the outside diameter of the tube

D_i = the inside diameter of the tube

R = radius of the tube

when f_{cr} is below the proportional limit of the material, that is, $E_t = E$

then,

$$f'_{cr} = \frac{\pi^2 E}{(kL/r)^2} \qquad (11)$$

where f'_{cr} = buckling stress calculated on the basis of a linear stress-strain relationship

thus,

$$f_{cr} = a\, f'_{cr} \qquad (12)$$

and

$$a = \frac{E_t}{E} \qquad (13)$$

where a = plasticity reduction factor

Since the actual columns contain residual stresses and geometric imperfections specially when loaded eccentrically, they fail at stresses below the theoretical buckling stress calculated from Equation 9. Hence, the preceding equations need some correction. Both American Institute of Steel Construction (AISC) and American Association of State Highway Officials (AASHO) use the following empirical formula:

$$f_a = \frac{f_y}{N}\left[1 - \frac{f_y}{4\pi^2 E}\,(kL/r)^2\right] \qquad (14)$$

where f_a = allowable axial stress

N = factor of safety

Eq. 14 has a restriction in L/r slenderness ratio

$$C_c = \sqrt{\frac{2\pi^2 E}{f_y}} \qquad (15)$$

where C_c = upper limit of slenderness ratio

AISC divides the Euler formula, Eq. 11, by a factor of safety N. In Eq. 14, AISC uses a factor of safety that varies from 1.67 to 1.92 and is defined as:

$$N = \frac{5}{3} + \frac{3(kL/r)}{8C_c} - \frac{(kL/r)^3}{8C_c^3} \qquad (16)$$

A constant factor of 1.92 is used in Eq. 11. AASHO uses a constant factor of safety of 2.25 in Eq. 14. The above AISC and AASHO formulae are for structural steel columns independent of their cross-section.

Regarding electric-resistance-welded tubular shapes, D. S. Wolford and M. J. Rebhotz*presented their formula derived from tests of carbon steel tubes with yield strength of 45,000 psi and 55,000 psi and a proportional limit of 50% of the yield strength:

$$f_a = \frac{f_y}{N}\left[1 - 0.385\sqrt{\frac{f_y}{\pi^2 E}}\ (kL/r)\right] \qquad (17)$$

$$C_c = 1.73\sqrt{\frac{\pi^2 E}{f_y}} \qquad (18)$$

A factor of safety of 2.16 should be used to divide the Euler formula, Eq. 11, when the slenderness ratio exceeds the limiting value.

From a practical view point, James F. Lincoln Arc Foundation* suggests the basic criteria for connection

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* Wolford, D. S. and Rebholz, M. J., " Beam and Column Tests of Welded Steel Tubing with Design Recommendations", Bulletin No. 233 American Society of Testing Materials, Philadelphia, Pennsylvania, October, 1958.

* James F. Lincoln Arc Foundation, " The Design of Welded Structures ", Cleveland, Ohio, 1968.

design as follows:

1) The bending forces from the end moment lie entirely within the flange of the beam. The most direct and effective method to transfer these forces is the flange weld. A plate welded to the top flange of a beam and to the column wall may transfer the tensile forces back into the column and a plate welded to the bottom flange of the beam and to the column wall may transfer the compressive forces back into the column (Fig. 3).

2) The shear forces lie almost entirely within the web of the beam and must be transferred directly out to the supporting column by a connection on the web. The length of these welds is determined by the shear reaction to be transferred (Fig. 4). A vertically stiffened seat has sufficient welding to transfer the shear reaction back into the column as suggested by the Foundation (Fig. 5). The seat also serves as a support for the beam during erection.

3) Rigid, continuous connections are used to form a delicate structure. This reduces the beam weight and usually reduces the overall weight of the complete structure.

4) Plastic design will reduce steel weight and also the design time.

5) The connection should be done in the shop and in

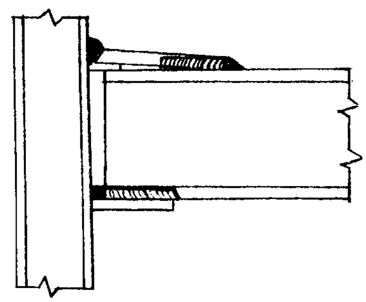

Fig. 3 Connection with Top and Bottom Flanges

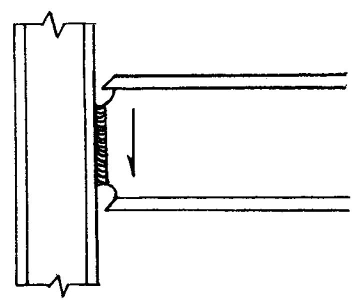

Fig. 4 Connection with Shear Plate on Web

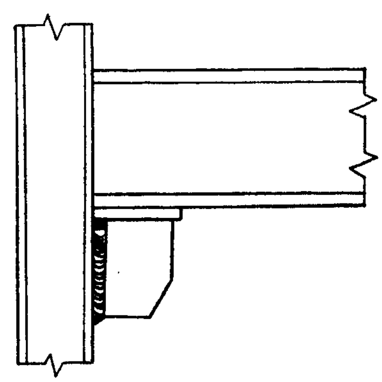

Fig. 5 Connection with Vertical Seat

flat position if possible.

6) The connection must offer proper accessibility for welding, whether done in shop or field.

A further assumption made by T. R. Higgins* is useful for checking the safety of the column wall.

1) A distance of six times the thickness of the wall above and below the application would be the action range of line forces.

2) The upper and lower boundaries of this portion are fixed.

3) The rectangular portion fails at ultimate load P_u.

4) The tensile line forces applied to this area are uniformly distributed.

Thus, the internal work done by this resisting portion is the summation of the plastic moment M_p multiplied by the angle ϕ along the edges (Fig. 6). The external work done is the ultimate load P_u multiplied by the virtual displacement Δ . In fig. 6 at ultimate loading, the plastic moment M_p will build up along the dashed line to form plastic hinges. The internal work done will be the plastic moment M_p multiplied by the corresponding angle change along these lengths:

* Higgins, T. R., Director of Engineering and Research of AISC, made the plastic analysis of connection.

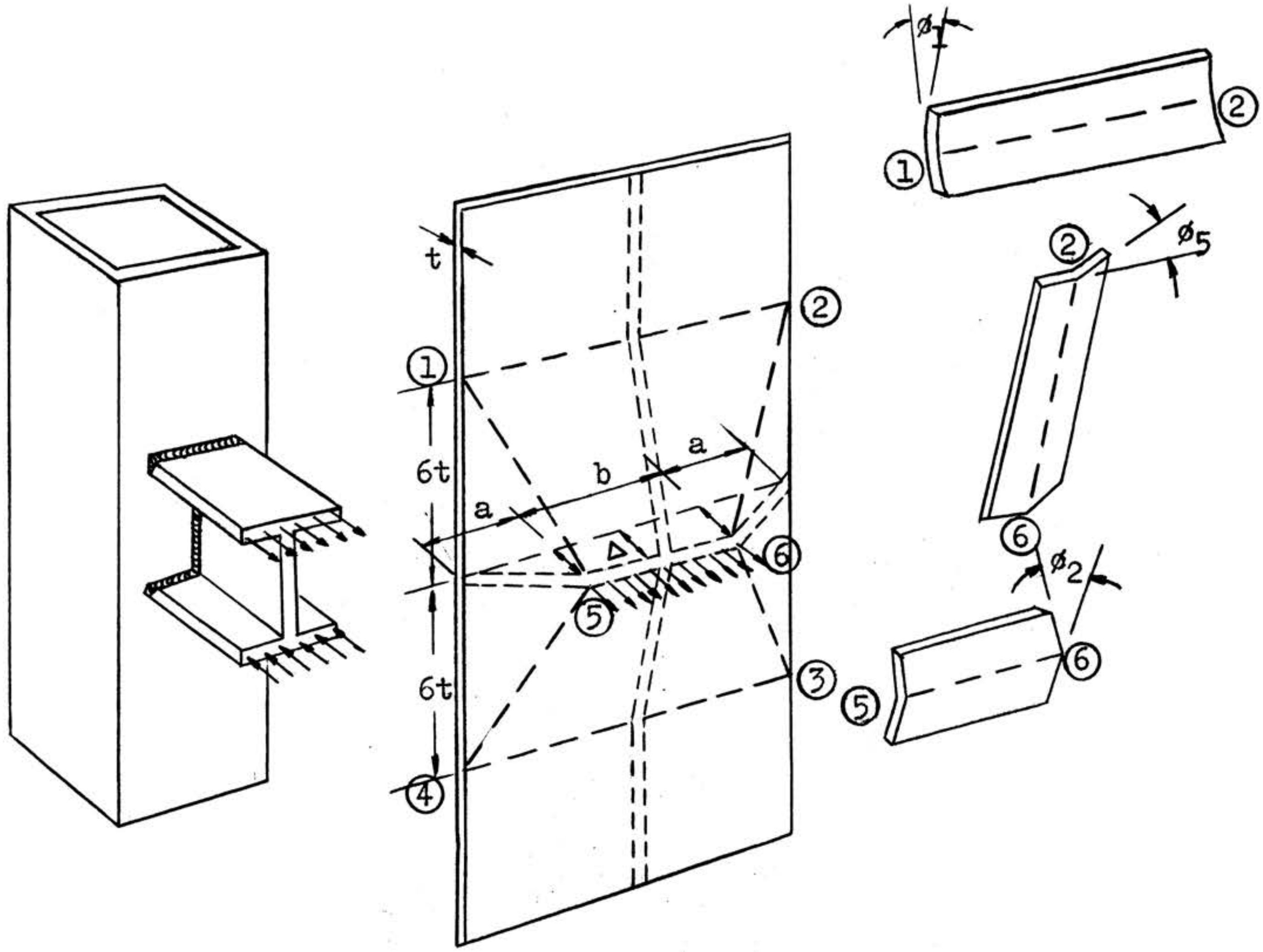

Fig. 6 **Analysis of Column** Wall Deformation I

(grangerized from " Design of Welded Structures ",

Lincoln Arc Welding Foundation, 1968)

angle ϕ_1 along ①~② & ③~④

angle ϕ_2 along ⑤~⑥

angle ϕ_5 along ①~⑤, ②~⑥, ③~⑥ & ④~⑤

With the help of Fig. 7, we can determine:

distance $②\sim⑥ = \sqrt{a^2 + 36t^2}$

$$\frac{⑥\sim Ⓨ}{\sqrt{a^2+36t^2}} = \frac{6\ t}{a}$$

distance $⑥\sim Ⓨ = \frac{a}{6\ t}\sqrt{a^2+36t^2}$

and $\tan \phi_6 = \frac{a}{6\ t}$

the angle changes ϕ along the hinges are:

$$\phi_1 = \Delta/6t$$

$$\phi_2 = 2\phi_1 = \Delta/3t$$

$$\phi_3 = \frac{\Delta\, a}{6t\sqrt{a^2+36t^2}}$$

$$\phi_4 = \frac{6\Delta t}{a\sqrt{a^2+36t^2}}$$

and

$$\phi_5 = \phi_3 + \phi_4 = \frac{\Delta}{6at}\sqrt{a^2+36t^2}$$

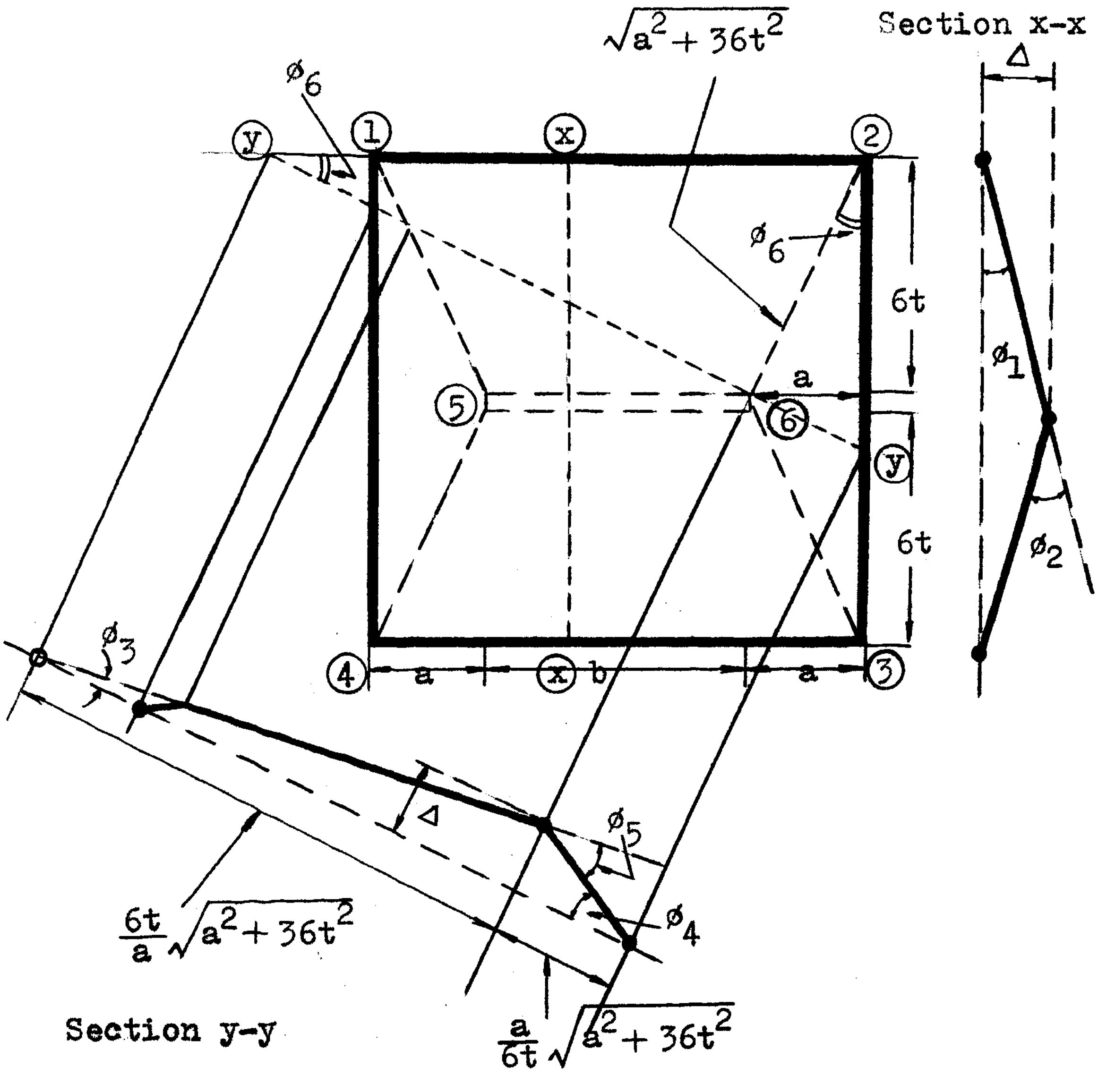

Fig. 7 Analysis of Column Wall Deformation II

(grangerized from " Design of Welded Structures ", Lincoln Arc Welding Foundation, 1968) then, the internal work

$$= M_p \left[\phi_1 \, 2(2a+b) + \phi_2 \, b + \phi_5 \, 4\sqrt{a^2+36t^2} \right]$$

$$= M_p \left[\frac{\Delta}{6t} 2(2a+b) + \frac{\Delta b}{3t} + \frac{\Delta}{6at} \left(\sqrt{a^2+36t^2}\right)\left(4\sqrt{a^2+36t^2}\right) \right]$$

$$= M_p \Delta \frac{2(a+b)}{3t} + 4\left(\frac{a^2+36t^2}{6at}\right)$$

$$= \frac{2M_p \Delta}{3t}(2a+b+36t^2/a) \qquad (19)$$

the plastic moment M_p, in in.-lbs/linear inch is (from Fig. 8)

$$M_p = (2\,\sigma_y \times \frac{t}{2} \times 1'' \times \frac{t}{4}) = \frac{\sigma_y t^2}{4} \qquad (20)$$

$$\text{the external work} = P_u \Delta \qquad (21)$$

since internal work = external work

then
$$P_u \Delta = \left(\frac{2\Delta}{3t}\right)\left(\frac{\sigma_y t^2}{4}\right)(2a+b+36t^2/a)$$

$$= \frac{\Delta \sigma_y t}{6}(2a+b+36t^2/a) \qquad (22)$$

If we apply a load factor of 2, and use the yield strength, the allowable force P, which may be applied to the plate

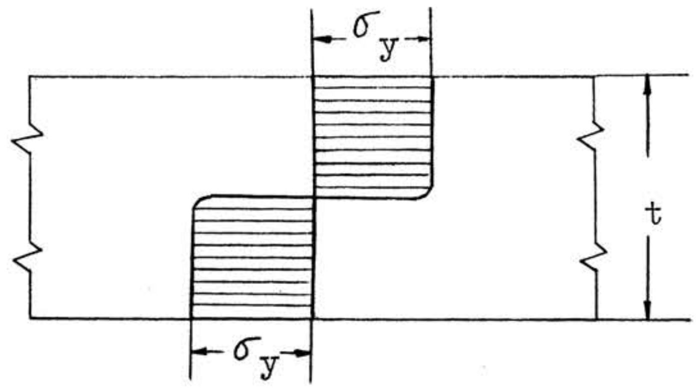

Fig. 8 Plastic Moment M_p

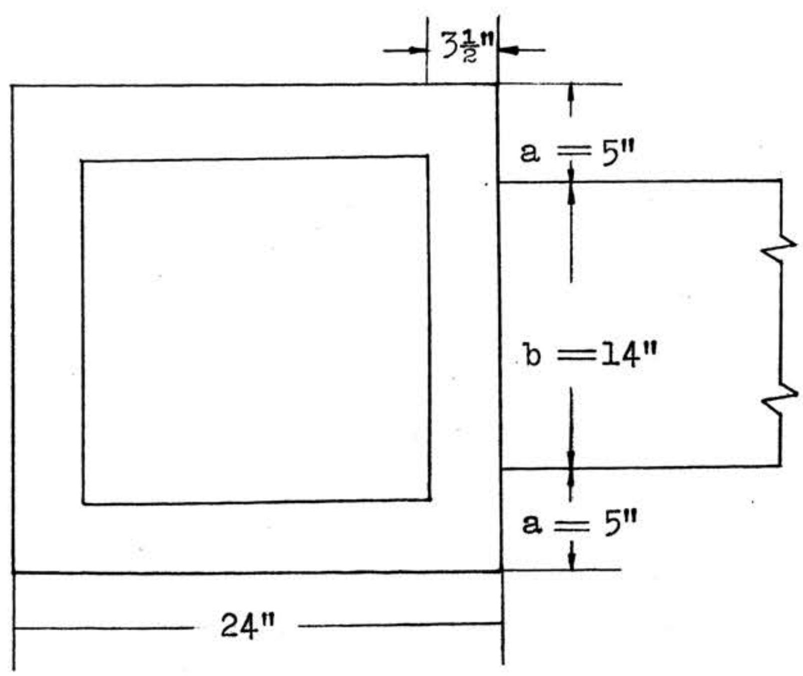

Fig. 9 Example for Allowable Force

would be

$$P = \frac{t\,\sigma_y}{12}(2a + b + 36t^2/a) \qquad (23)$$

For example (Fig. 9)

here: $t = 3\frac{1}{2}"$ $\quad b = 14"$

$a = 5"$ $\quad \sigma = 22,000$ psi

the calculated tensile force on beam flange is 368 kips,

the allowable force

$$P = \frac{3\frac{1}{2}"(36\text{ksi})}{12}\left(2 \times 5" + 14" + 36(3\tfrac{1}{2}")^2/5"\right)$$

$$= 1178 \text{ kips} > 368 \text{ kips} \qquad \underline{\text{OK}}$$

III THEORY

Because of the rigidity of the welded connection, the specimen tested in this study was considered as two cantilever beams rigidly framed into the tubular column.

Studying just one end of this two-ended cantilever beam as shown in Fig. 10, the bending stresses are zero at the neutral axis and are assumed to increase linearly to a maximum at the outer fiber of the section. The fibers stressed in tension elongate while the fibers stressed in compression contract. This causes each section stressed in this way to rotate. The resulting effect of this movement is an overall deflection (or bending) of the beam.

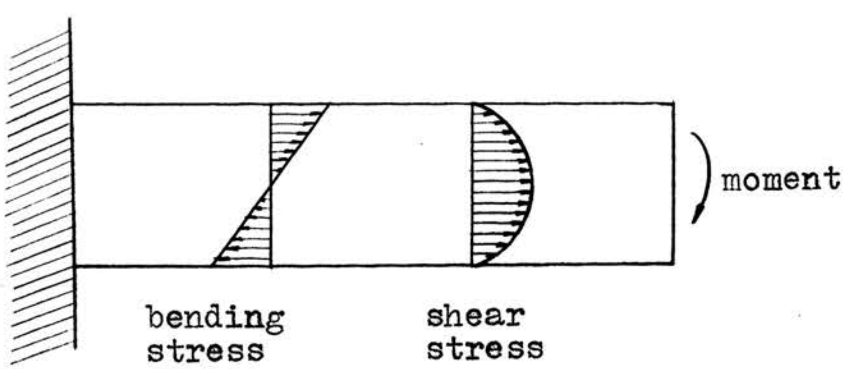

Fig. 10 Stress diagrams of a cantilever beam

In addition to pure bending stresses, horizontal shear stress is often present in beams. It depends on vertical shear and only occurs if the bending moment varies along the beam. The horizontal shear has a maximum value at the neutral axis and is zero at the outer fibers.

for bending
$$\sigma_b = \frac{M\ c}{I} \qquad (24)$$

for shear
$$\tau = \frac{V\ A\ y}{I\ t} = \frac{V\ Q}{I\ t} \qquad (25)$$

where σ_b = bending stress, may be tension or compression, psi

M = bending moment at the section, in.-lbs

I = moment of inertia of the section, $in.^4$

c = distance from neutral axis to the point at which stress is desired, in.

τ = horizontal shear stress at any point, psi

V = external vertical shear on beams, lbs.

A = area of section beyond the plane where stress is desired, $in.^2$

y = distance of center of gravity of area to neutral axis, in.

t = thickness of section at plane where stress is desired, in.

Q = statical moment of section lying outside the line on which the shear stress is desired, taken about the neutral axis

Formulae for a cantilever beam subjected to a concentrated load are as follows: (Fig. 11)

$$R = V = P \tag{26}$$

at support $$M_{max} = P\,b \tag{27}$$

when $x > a$ $$M_x = P(\,x - a\,) \tag{28}$$

at free end $$\Delta_{max} = \frac{P\,b^2}{6EI}(\,3L - b\,) \tag{29}$$

at load $$\Delta = \frac{P\,b^3}{3EI} \tag{30}$$

when $x < a$ $$\Delta_x = \frac{P\,b^2\,(\,3L - 3x - b\,)}{6\,E\,I} \tag{31}$$

when $x > a$ $$\Delta_x = \frac{P\,(L - x)^2}{6EI}(3b - L + x) \tag{32}$$

The end rotation of a cantilever beam can be derived from the conjugate beam method* :

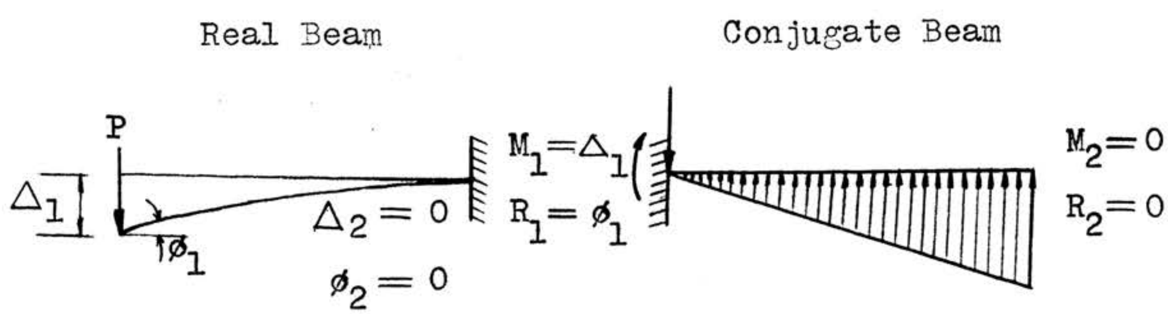

Fig. 12 End rotation by conjugate beam

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* The theory of conjugate beam is primarily due to H.F.B. Mueller-Breslau, Beitrag zur Theorie des Fachwerks, Z. Architekt. u. Ing. Hannover. 1885.

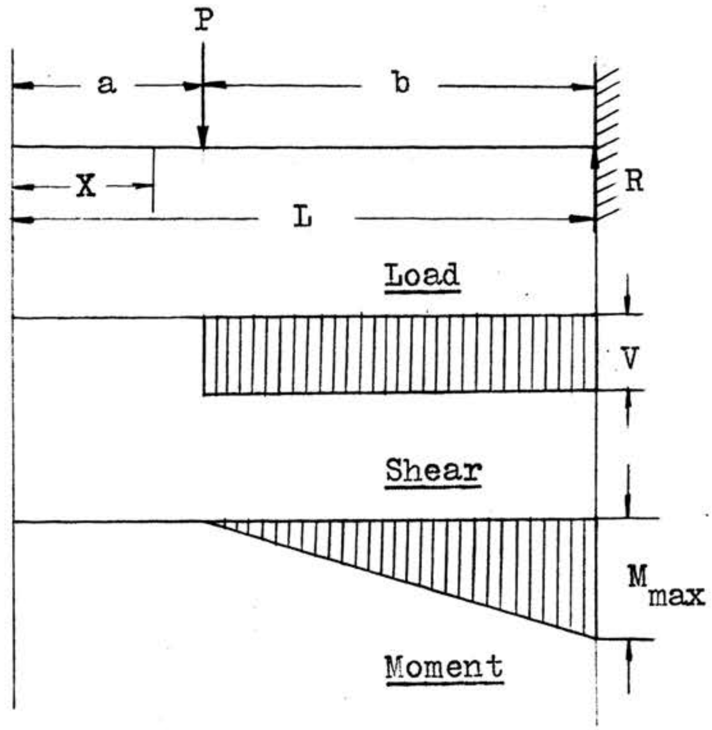


Fig. 11 The Shear and Moment Diagrams of a Cantilever Beam Subjected to a Concentrated Load at Any Point

$$\phi_1 = R = \frac{M\ L}{2EI} \qquad (33)$$

where ϕ = end rotation in radians

In checking the moment-rotation characteristics of the connection, the beam-line concept is the most convenient method. Suppose a prismatic beam is subjected to a uniformly distributed load and to equal end moments as shown in Fig. 13.

$$\phi_1 = \frac{1}{8EI}\ wL^2 \times \frac{2}{3} \cdot \frac{L}{2} = \frac{wL^3}{24EI}$$

$$\phi_2 = \frac{1}{2EI}(\ M + M\ ) \times \frac{L}{2} = \frac{M\ L}{2EI}$$

$$\phi = \phi_1 - \phi_2 = \frac{wL^3}{24EI} - \frac{M\ L}{2EI} \qquad (34)$$

Obviously, from Eq. 34 the end rotation ϕ is a linear function of the end moment M.

When the connection is completely restrained or $\phi = 0$, in other words a fixed-end beam,

then

$$M_o = \frac{wL^2}{12} \qquad (35)$$

When the connection has no restraint or $M = 0$, in other words a simply supported beam,

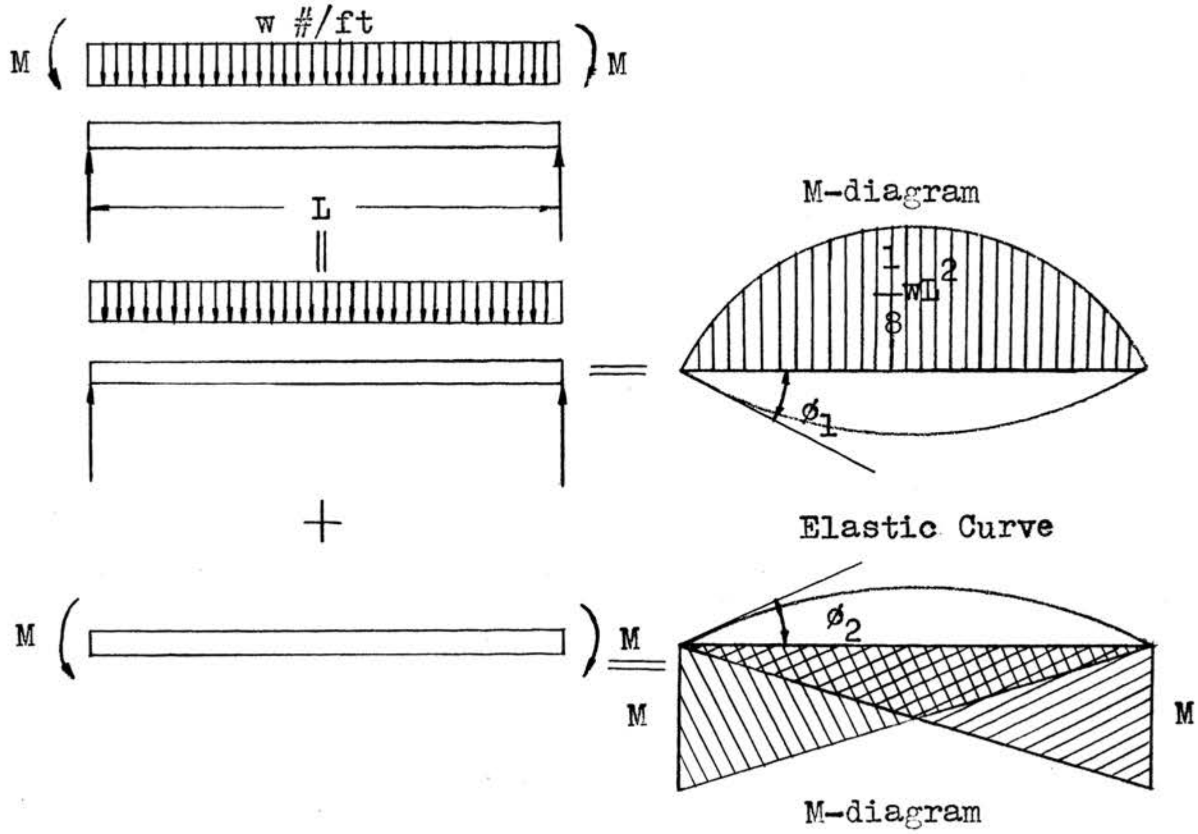


Fig. 13 Beam-line Concept I

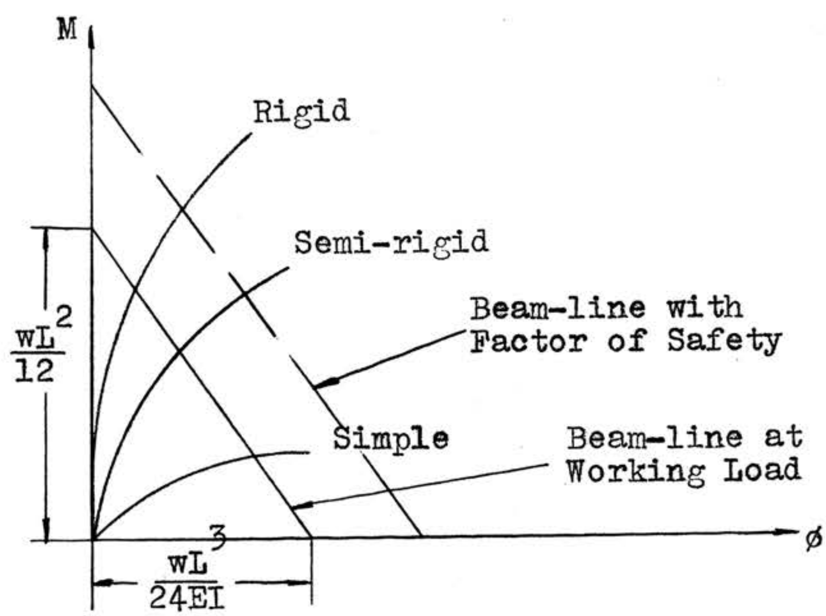


Fig. 14 Beam-line Concept II

then
$$\phi_o = \frac{wL^3}{24EI} \qquad (36)$$

from Eq. 36
$$\phi_o = \frac{M\ L}{3EI} = \frac{fIL}{3EIc} = \frac{2}{3}\left(\frac{f_b}{E}\right)\left(\frac{L}{d}\right) \qquad (37)$$

where d = depth of the beam

We plot the M-ϕ curve as shown in Fig. 14. Thus, for a given beam stress f, the ϕ_o is directly proportional to the L/d ratio of the connected beam.

The intersection of the M-ϕ curve of a connection with the beam line of the connected beam defines completely the end condition. If the curve reaches the safe beam line (beam line adjusted by a factor of safety) it is adequate. For example, if the simple connection shown in Fig. 14 is not satisfactory because it failed before the M-ϕ curve had reached the safe beam line then the rigid and semi-rigid connections are adequate. It is to be noted that a connection might be satisfactory for short spans (low L/d ratio) but unsatisfactory for long spans (high L/d ratio).

Since we assume one half of a specimen is a cantilever beam and since we assume a rigid connection is provided, then when a load is applied on the beam end the induced bending moment is transferred to the column wall-face by the flange of the beam. Thus, in analyzing the stress distribution on the wall-face it is convenient and reasonable to assume the wall-face is subjected to a couple M and a vertical load P. The couple M can then be represented by a pair of uniformly distributed parallel line forces acting horizontally toward and away from the wall face as shown in Fig. 15 a. The maximum intensity of these parallel line forces is located at the position of the two flanges and goes to zero at the center.

We can consider a unit strip for study as shown in Fig. 15 b, after checking the real condition, such as a beam with one end fixed and other hinged and subjected to a double triangular load as shown in Fig. 15 c.

Refering to Fig. 15 d:

$$M = \frac{4\ f}{2} \times \frac{8 \times 2}{3} = \frac{32}{3} f \text{ in.-lb}$$

$$R_A = -\frac{3}{2} \times \frac{32\ f}{3\ \ 30} \left(\frac{30^2 - 12^2}{30^2} \right)$$

$$= -0.448 \text{ f lbs.}$$

$$R_B = +0.448 \text{ f lbs.}$$

$$M_A = 0$$

$$M_B = \frac{1}{2} \times \frac{32}{3} f \left(1 - 3\ \frac{12^2}{30^2}\right)$$

$$= 2.773 \text{ f in.-lb}$$

$$M_C = -\ 0.448\ f \times 8 = -\ 3.584 \text{ f in.-lb}$$

$$M_D = +\ 0.448\ f \times 14 - 2.773\ f = 3.500 \text{ f in.-lb}$$

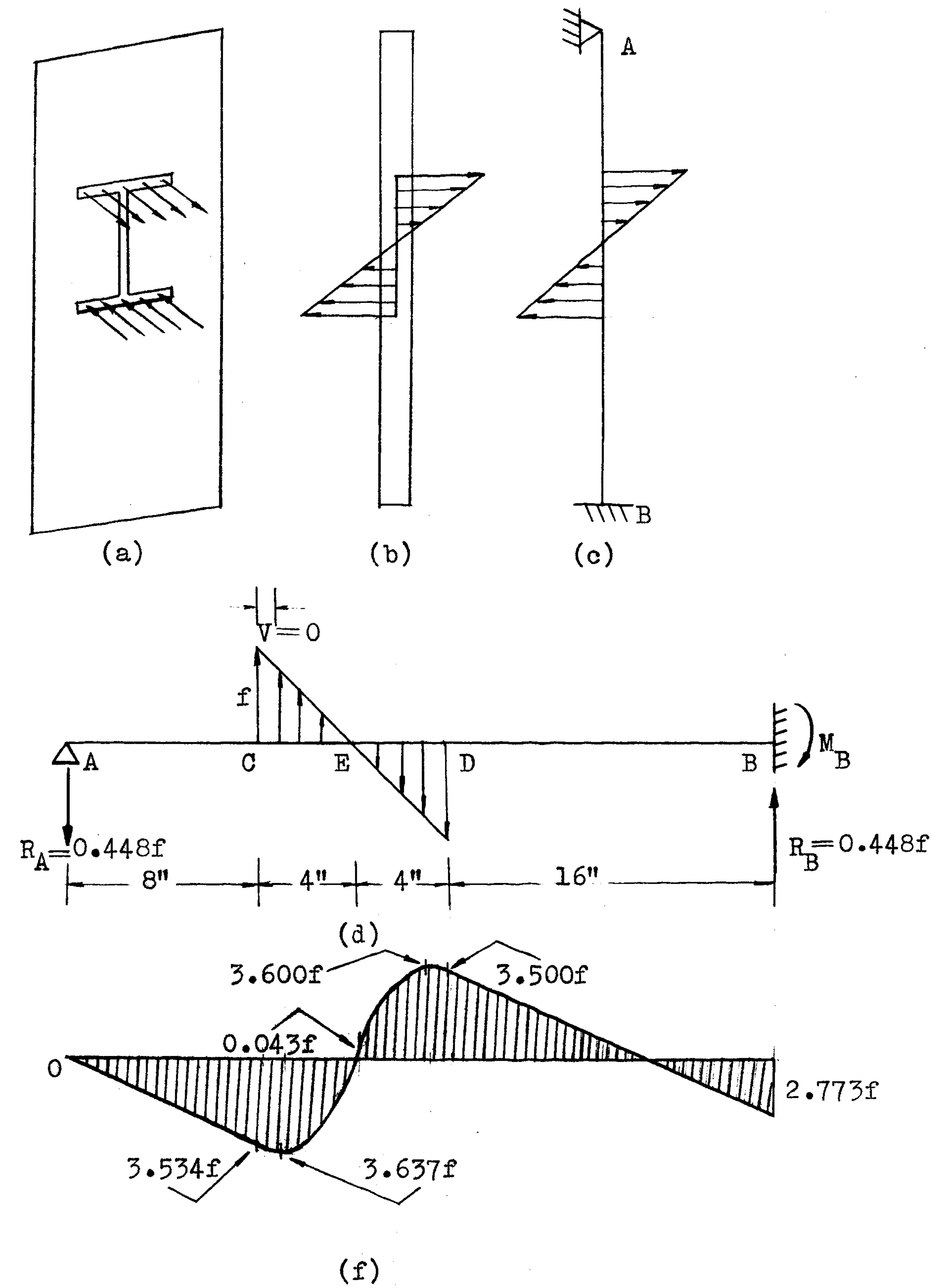


Fig. 15 Analysis of Stresses on Column Wall

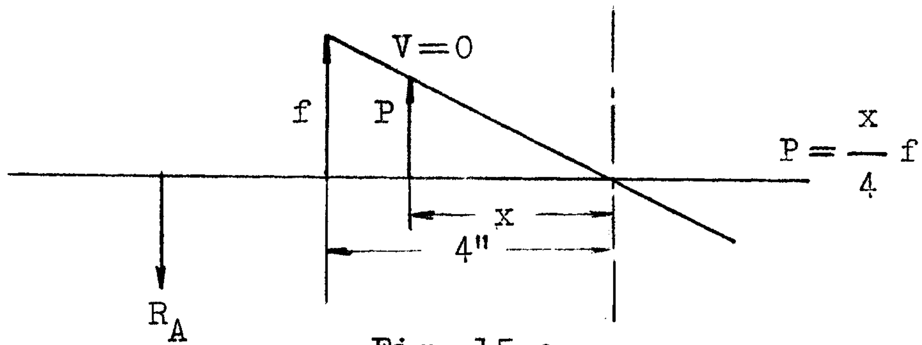


Fig. 15 e

$\sum V = 0,$ $$R_A = \frac{4}{2} f - P\frac{x}{2}$$

$$x = 3.469 \text{ in.}$$

then Maximum moment between A and E

$$M_{max} = -0.448\ f \times (8+0.531) + \frac{f+P}{2} 0.531 \times 0.27$$

$$= -3.637\ f \text{ in.-lb}$$

Maximum moment between E and B

$$M_{max} = -2.773\ f + 0.448\ f \times 14.531$$

$$- \frac{f+P}{2} 0.531 \times 0.27$$

$$= +3.600\ f \text{ in.-lb}$$

$$M_E = -0.448\ f \times 12 - 2f \times 4 \times 2/3$$

$$= +0.043\ f \text{ in.-lb}$$

or $$M_E = -2.773\ f + 0.448\ f \times 18 - 2f \times 4 \times 2/3$$

$$= +0.043\ f \text{ in.-lb} \qquad \underline{\text{OK}}$$

The moment diagram is shown in Fig. 15 f. To find stresses, use the moment diagram and the interaction formula

$$f = \frac{P}{A} \pm \frac{M\,c}{I} \qquad (38)$$

where for any section above C, $P = 0$.

Theoretical analysis of connection strength is as follows:

1) Single-plate connection:

The single-plate connection, in general, is no better than the directly welded web connection shown in Fig. 16 and Fig. 17. Unless the plate is as thick as the beam web the resulting connecting fillet welds will be smaller and will reduce the strength of the connection. The strength of a single-plate connection can be calculated as a directly welded web connection.

a) for parallel load (Fig. 16)

using A36 Steel with E70 Electrode*

$$\tau = 15,800 \text{ psi}$$

$$f = 11,200 \text{ w}$$

then $$(11200 \text{ w})\, L = t_w\, 15800\, L$$

$$w = 1.4\, t_w \qquad (39)$$

where w = leg size of fillet weld, in.

---

* E70 Electrode having a minimum yield strength of 60,000 psi and a minimum tensile strength of 72,000 psi.

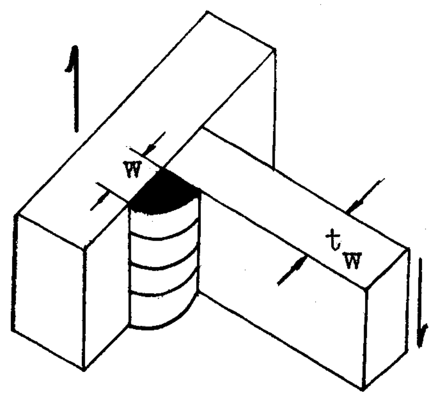


Fig. 16 Directly Welded Web Connection with Parallel Load

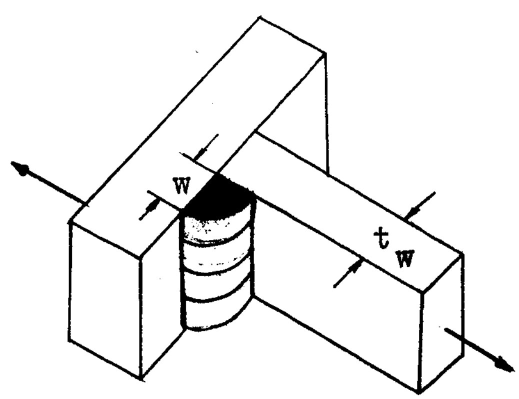


Fig. 17 Directly Welded Web Connection with Transverse Load

t_w = thickness of the beam web, in.

$f = 11,200\ w$ = allowable stress for welds

That is, to develop the full shear stress, the weld leg should be 1.4 times the thickness of the web.

b) For transverse load (Fig. 17)

using A36 Steel with E70 Electrode

$$\sigma_t = 21,600 \text{ psi}$$

then
$$(11200\ w)\ L = t_w\ 21600\ L$$

$$w = 1.9\ t_w \qquad (40)$$

That is, to develop the full tensile stress the weld leg should be 1.9 times the thickness of the web.

2) Double-angle connection:

The analysis of this type of connection is divided into two parts:

a) Analysis of the weld of the angle to the column wall. It is assumed the two angles bear against each other for a vertical distance equal to 1/6 of their length: The remaining 5/6 of the length is resisted by the connecting weld (Fig. 18). It is also assumed these forces increase linearly to the edge of the weld.

for horizontal force on weld

since applied moment = resisting moment

then
$$\frac{R}{2} L_h = \frac{2}{3} P\ L_v$$

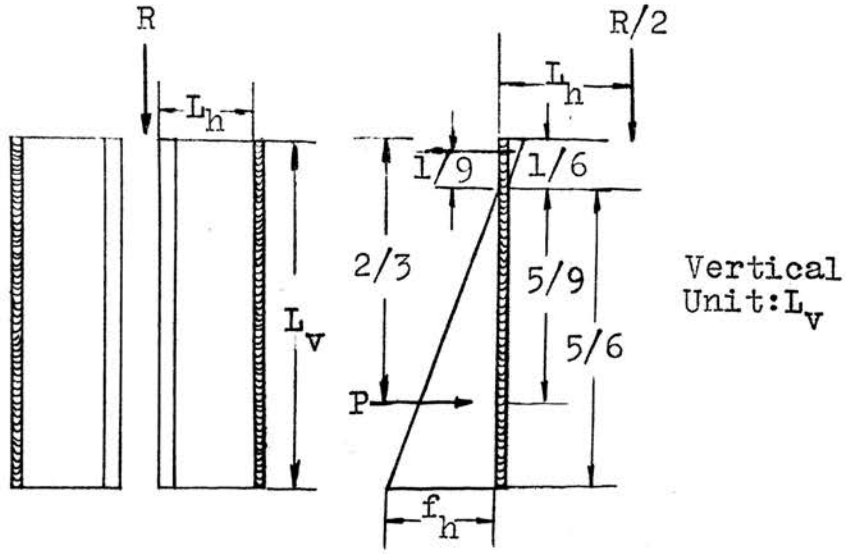


Fig. 18 Analysis of Angle to Column

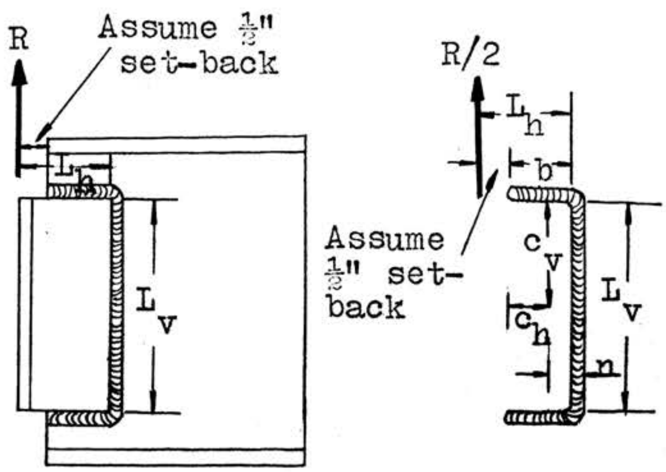


Fig. 19 Analysis of Angle to Beam

or $$P = 0.75\,R\,L_h/L_v \tag{41}$$

and from force triangle

$$P = \tfrac{1}{2}(\,f_h\,)(\,\frac{5}{6}L_v\,) \tag{42}$$

from Eqs. 41 and 42

$$f_h = \frac{9\,R\,L_h}{5\,L_v^2} \tag{43}$$

where L_v = length of angle, in.

L_h = angle's leg length, in.

for vertical force on weld

$$f_v = R/2L_v \tag{44}$$

so, resultant force on weld

$$f_r = \sqrt{f_h^2 + f_v^2} = \sqrt{\left(\frac{9\,R\,L_h}{5\,L_v}\right)^2 + \left(\frac{R}{2L_v}\right)^2} \tag{45}$$

or $$f_r = \frac{R}{2L_v^2}\sqrt{L_v^2 + 12.96\,L_h^2} \tag{46}$$

It is noted that the top and bottom welds of the angle to the column help the carrying capacity of the connection.

b) Analysis of the weld of the angle to the beam web:

from Fig. 19 $$n = \frac{b^2}{2b + L_v} \tag{47}$$

$$c_h = L_h - n - \tfrac{1}{2}''$$

$$c_v = L_v/2$$

$$b = L_h - \tfrac{1}{2}''$$

$$J_w = \frac{(2b + L_v)^3}{12} - \frac{b^2(b + L_v)^2}{2b + L_v} \qquad (48)$$

twisting (horizontal)

$$f_h = \frac{T\ c_v}{J_w} = \frac{R(L_h - n)\ c_v}{2\ J_w} \qquad (49)$$

twisting (vertical)

$$f_{v1} = \frac{T\ c_h}{J_w} = \frac{R(L_h - n)\ c_h}{2\ J_w} \qquad (50)$$

shear (vertical)

$$f_{v2} = \frac{R/2}{2b + L_v} \qquad (51)$$

resultant

$$f_r = \sqrt{f_h^{\ 2} + (f_{v1} + f_{v2})^2} \qquad (52)$$

where J_w = polar moment of inertia of weld, in.^{4}

3) Multi-angle connection:

Since the multi-angle connection is the combination of a double-angle connection with a thin top angle and a thin bottom angle, we need only to study the top and bottom angles.

a) Bottom angles: The top leg of the bottom angle is subjected to bending stress and will deflect downward. If the angle is too thin, the top of the connection weld tends to tear because only this portion of the weld resists the bending. AISC specifies that the compressive stress on the fillet of a beam at the web toe shall not exceed

$$\sigma = 0.75\ \sigma_y \text{ psi}$$

and is located at a distance K up from the flange face (Fig. 20). That is,

$$\frac{R}{t_w\ (N + K)} = \text{not over } 0.75\ \sigma_y \text{ psi} \qquad (53)$$

for A36 Steel $\quad 0.75\,\sigma_y = 27{,}000$ psi

and then

$$N = \frac{R}{t_w\ (27000)} - K \qquad (54)$$

$$e_f = a + N/2 \qquad (55)$$

$$e = e_f - t - 3/8'' \qquad (56)$$

then $\quad M = R\ e = \sigma S$

and $\quad S = b\ t^2/6$

so

$$R\ e = M = \sigma S = \frac{\sigma\ b\ t^2}{6} \qquad (57)$$

where $\quad a =$ clear distance, in.

$e =$ eccentricity, in.

$S =$ section modulus, in.^{3}

The minimum horizontal dimension of the angle for easy

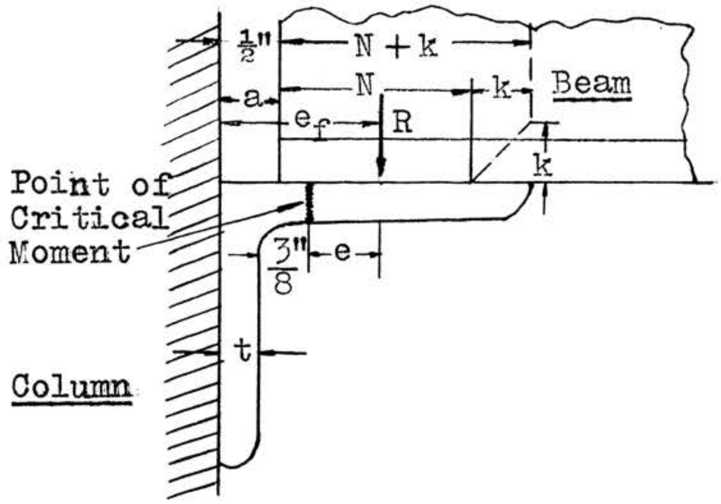


Fig. 20 Analysis of Bottom Angle

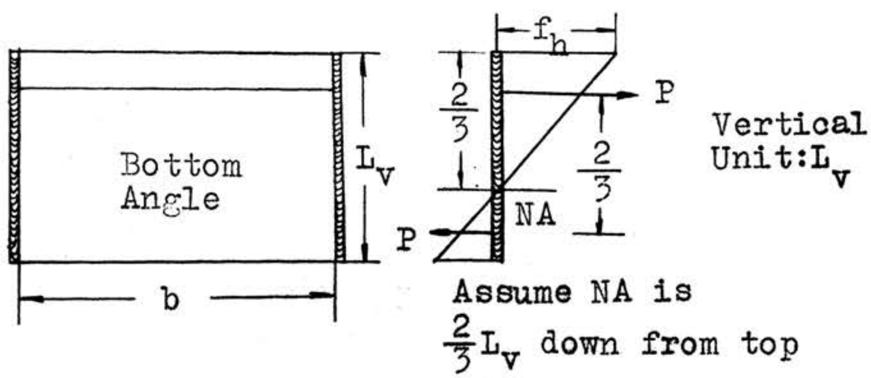


Fig. 21 Analysis of Vertical Weld on Bottom Angle

erection is :

$$L_h = a + N \qquad (58)$$

Assume the vertical length of the connecting fillet weld equals the vertical leg of the angle, then from Fig. 21, the horizontal force on weld will be :

$$\text{moment (each weld)} = \frac{R}{2}(e_f) = P\left(\frac{2}{3} L_v\right) \qquad (59)$$

also

$$P = \frac{1}{2} (f_h)\left(\frac{2}{3} L_v\right) \qquad (60)$$

then

$$f_h = \frac{2.25\ R\ e_f}{L_v^2} \qquad (61)$$

the vertical force on weld will be :

$$f_v = \frac{R}{2\ L_v} \qquad (62)$$

and the resultant force on the weld in question will be :

$$f_r = \sqrt{f_v^2 + f_h^2}$$

$$f_r = \sqrt{\left(\frac{R}{2\ L_v}\right)^2 + \left(\frac{2.25\ R\ e_f}{L_v^2}\right)^2} \qquad (63)$$

or

$$f_r = \frac{R}{2L_v^2}\sqrt{L_v^2 + 20.25\ e_f^2} \qquad (64)$$

b) Top angle: The top angle is used to give horizontal stability to the beam. The greatest movement or rotation occurs in the fillet weld connecting the upper leg of the angle to the column. Thus it is necessary to weld the upper leg completely.

IV EXPERIMENTAL PROCEDURE

The six specimens used for the experimental test are referred to as Specimen A, B, C, D, E and F respectively. Each specimen consists of a square structural tubing of 30 in. in length and a cross section of 10 in. by 10 in. On each of the opposite sides of the column a 24 in. long 8 WF 17 beam is connected to form a cantilever arrangement. The dimensions and properties of tubular column and beam are described in Table I. All material used was A36 Steel.

The variables in the test specimens were the wall thicknesses of the square structural tubing and the types of connections. Because of the closed cross section of the tubing, welding was the only way of connecting. This made the connection very rigid.

Specimens A and B were the same except that the wall thicknesses of the tubing were $\frac{1}{4}$" and $\frac{1}{2}$" respectively. The connection consisted of a 6" by $3\frac{1}{2}$" by 3/16" plate welded to the web of the WF beam and then welded directly on both sides to the tubing wall at the center of the column face. This is called a single-plate connection (Fig. 22).

Specimens C and D were the same except that the wall thicknesses of the tubing were $\frac{1}{4}$" and $\frac{1}{2}$" respectively. The

TABLE I

Dimension and Property of Sections

Member		Beam	Square Tubing $\frac{1}{2}$"		Square Tubing $\frac{1}{4}$"	
Section		8 WF 17	10"	10"	10"	10"
A	in.^{2}	5.00	17.93		9.48	
I	in.^{4}	XX 56.40 YY 6.72	259.80		147.90	
S	in.^{3}	XX 14.10 YY 2.60	51.96		29.58	
r	in.	XX 3.36 YY 1.16	3.81		3.95	
wt/ft	#	17.00	60.95		32.23	
d	in.	8.00				
t_f	in.	5/16				
b	in.	$5\frac{1}{4}$				
t_w	in.	$\frac{1}{4}$				

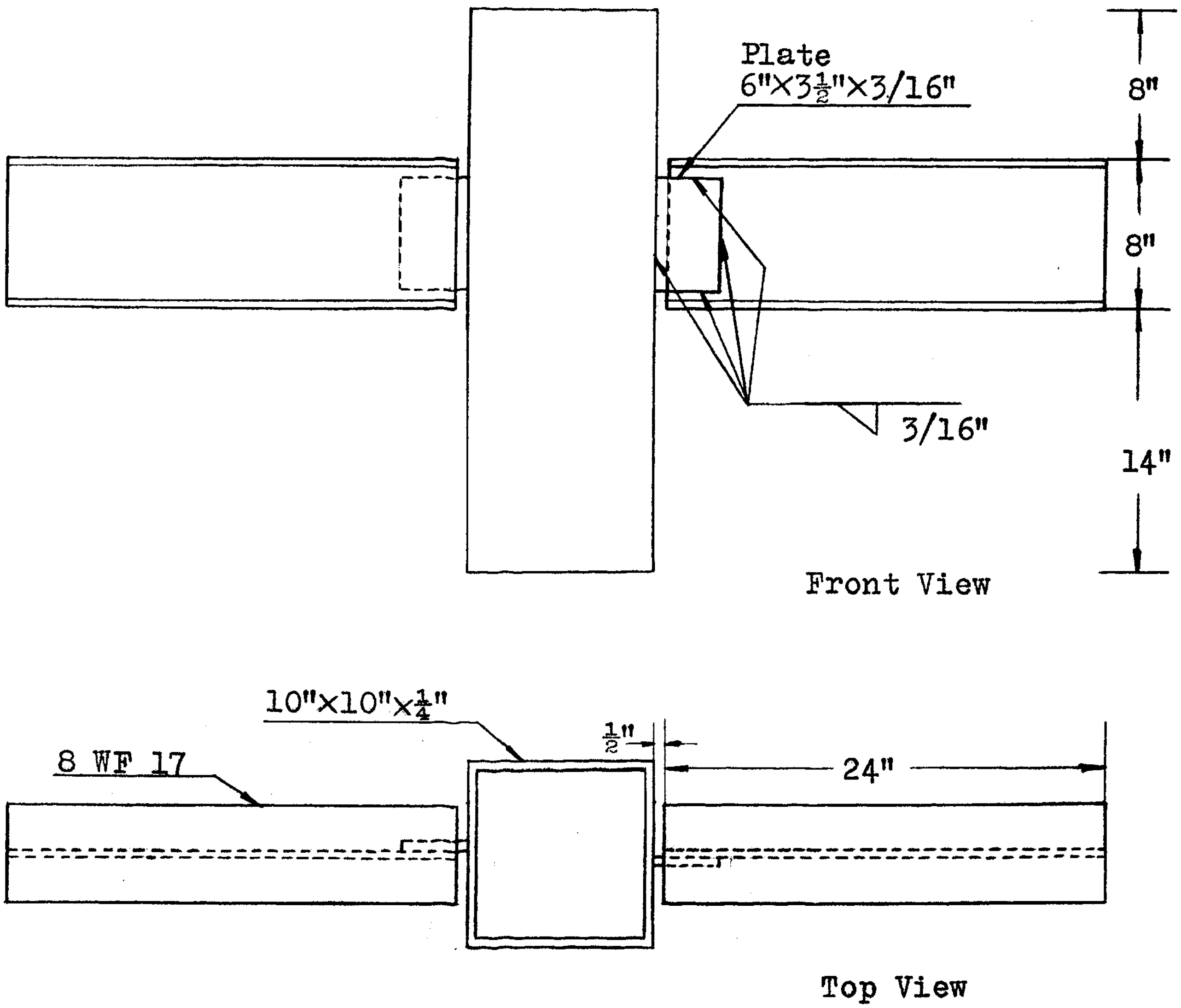


Fig. 22 The Single-plate Connection

connection consisted of two 6-in. 3" by 2" by 3/16" angles welded to each side of the WF beam web and then welded to the center of the column wall. This is called a double-angle connection and is designed to distribute the load over a rectangular area in the center of the column face as shown in Fig. 23.

Specimens E and F were the same except that the wall thicknesses of the tubing were $\frac{1}{4}$" and $\frac{1}{8}$" respectively. The connection was the same as the double-angle connection used in Specimens C and D except it had two additional angles, $1\frac{1}{2}$" by $1\frac{1}{2}$" by 1/8", each 10 in. in length, welded across the bottom and top flange of the WF beam and extending to the edge of the tubing. This is called a multi-angle connection and is designed to distribute the load to the edges of the column and to allow the perpendicular walls having a greater shear capacity to help support the load as shown in Fig. 24.

The general data of these specimens are described in Table II. The load carrying capacities of the members and the connections are explained in the Appendix.

The machine and instruments used in this study are as follows:

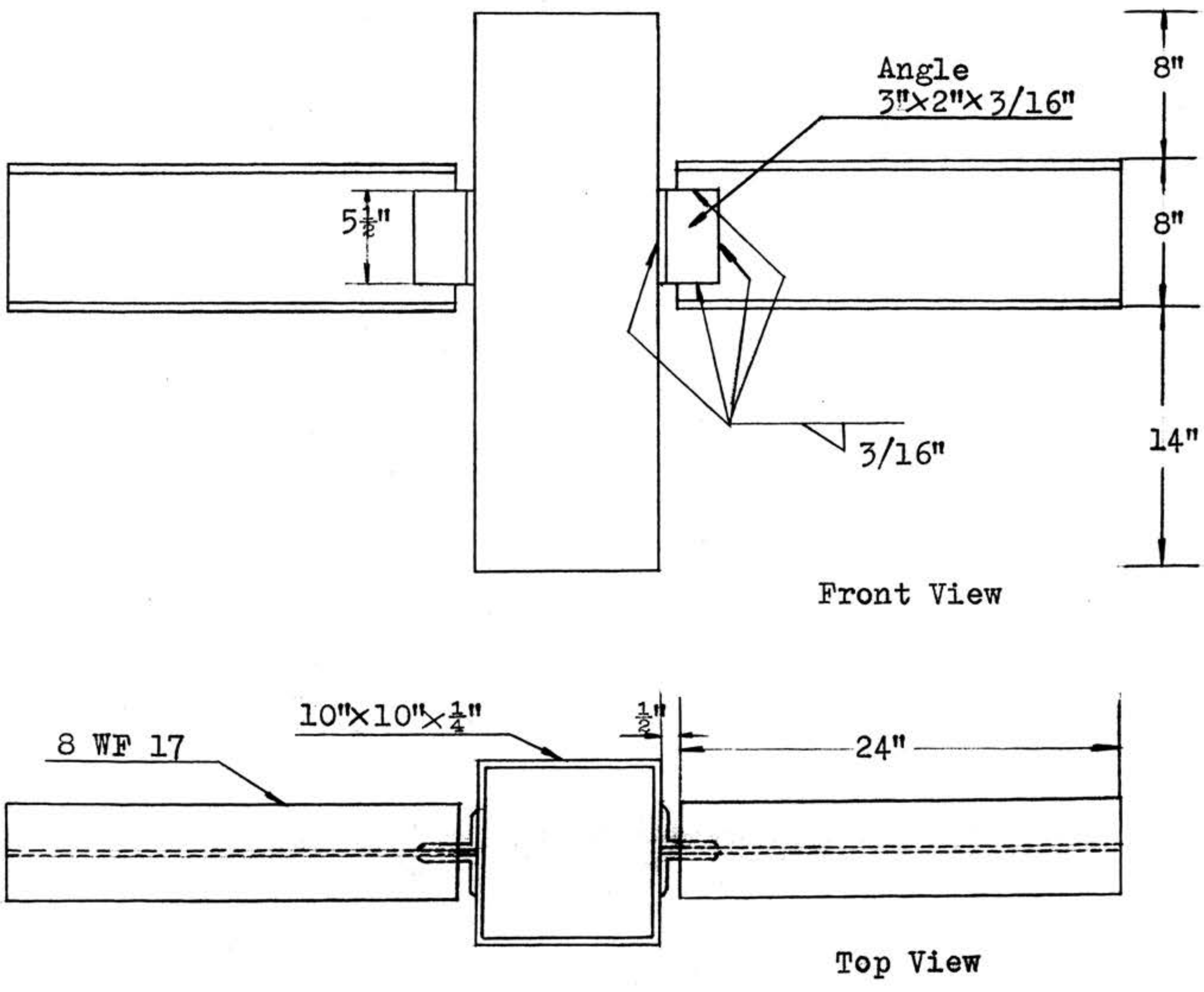


Fig. 23 The Double-angle Connection

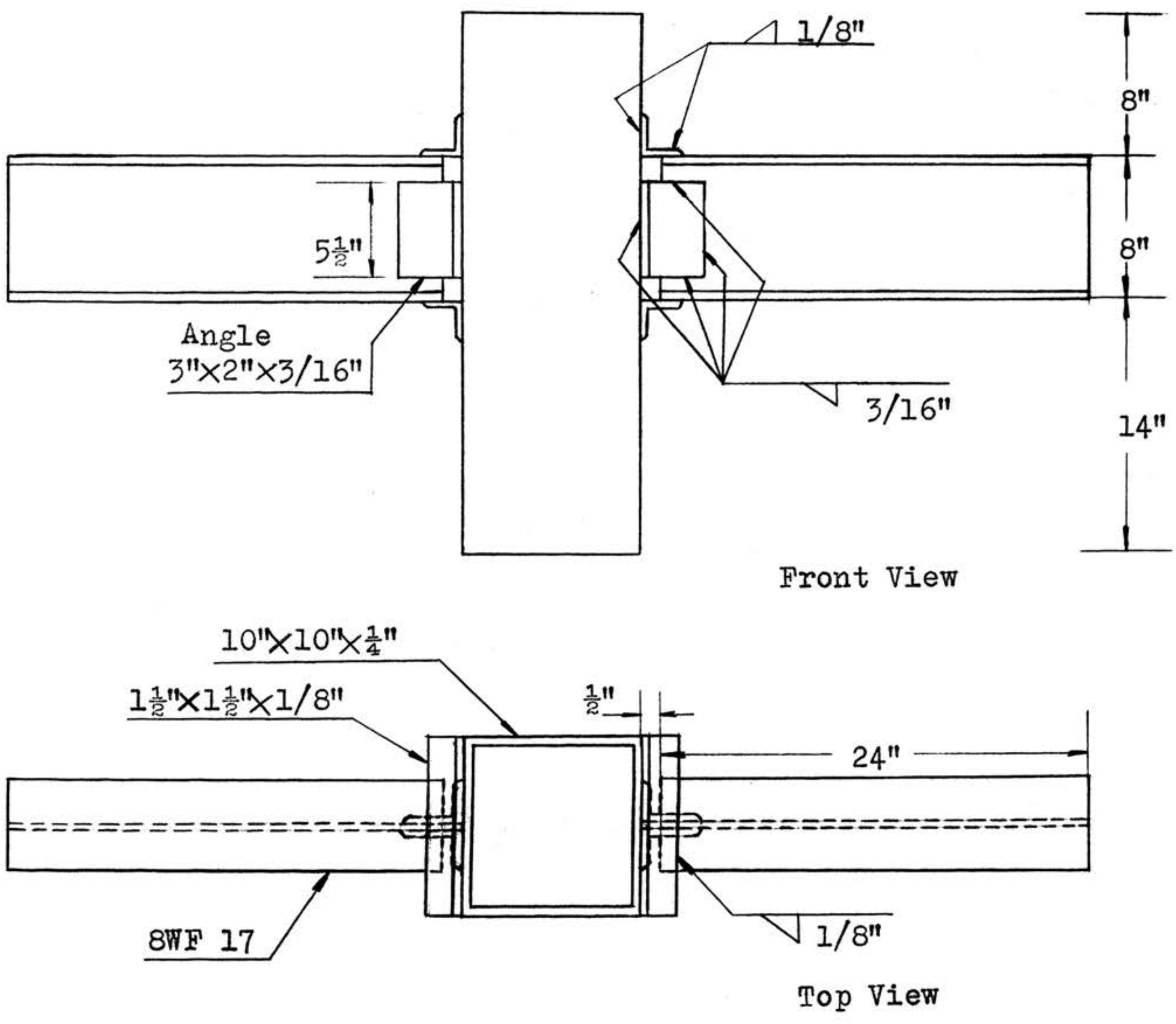


Fig. 24 The Multi-angle Connection

TABLE II

General Data of Specimens

Specimen	A	B	C	D	E	F
Column $10 \times 10 \times 30$	Wall $\frac{1}{4}$"	Wall $\frac{1}{2}$"	Wall $\frac{1}{4}$"	Wall $\frac{1}{2}$"	Wall $\frac{1}{4}$"	Wall $\frac{1}{2}$"
Beam 2×24"	8 WF 17					
Connection	Single-plate $1 \times 6 \times 3\frac{1}{2} \times 3/16$		Double-angle $2 \times 3 \times 2 \times 3/16$		Multi-angle $2 \times 3 \times 2 \times 3/16$ $2 \times 1\frac{1}{2} \times 1\frac{1}{2} \times 1/8$	
Welded Length	19"	19"	22"	22"	22" 40"	22" 40"
Total Wt. lbs	149.673	221.473	151.645	223.445	152.875	224.675
Number of Gages	24	24	16	16	20	20

1) Testing Machine : Tinius Olsen products, having a maximum loading capacity of 200,000 lbs., dividing into 4 loading scales of 2,000 lbs., 10,000 lbs., 50,000 lbs., and 200,000 lbs. as shown in Fig. 25.

2) Balancing Unit : Arthur R. Anderson products, Model 301, having 24 terminals.

3) Strain Indicator : Baldwin products, SR4 Type MB, ranging from 0 to 20,000 microinches per inch.

4) Deflection Gage : Lufkin products, Model J48D-3, ranging from 0.001 in. to 3 in. as shown in Fig. 26.

The strain gages used in the tests were SR-4 A-7 gages, provided by the Baldwin, Lima, Hamilton (BLH) Electronics, Inc.. The A-7 gage has a resistance of 119.5 ± 0.3 ohms and a gage factor of $1.96 \pm 2\%$.

When installing the strain gages on the surface of the specimens the following steps were observed very carefully:

1) Rust removal with electric sand disc and sand paper.

2) The surface cleaned with trichlorethylene or acetone.

3) Duco cement applied to area and strain gages affixed to the proper locations.

4) After drying, a waterproof coat was sprayed on.

Strain gages were located on the location where high

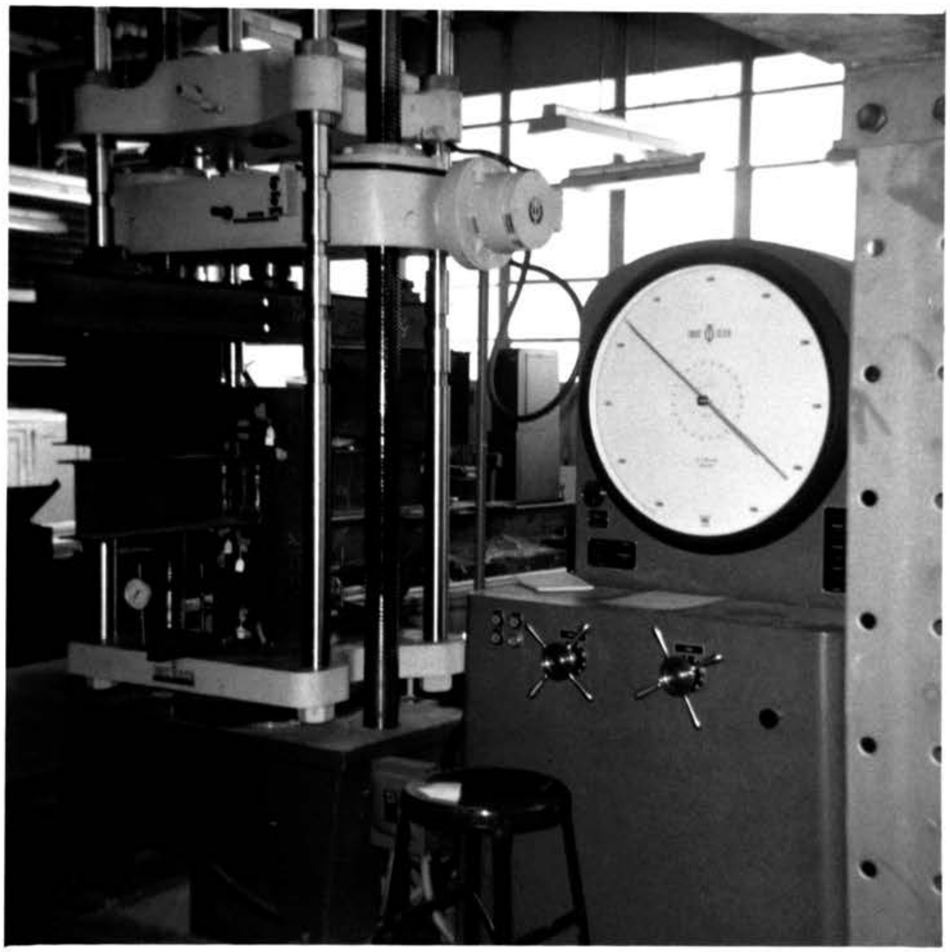


Fig. 25 Tinius Olsen Testing Machine

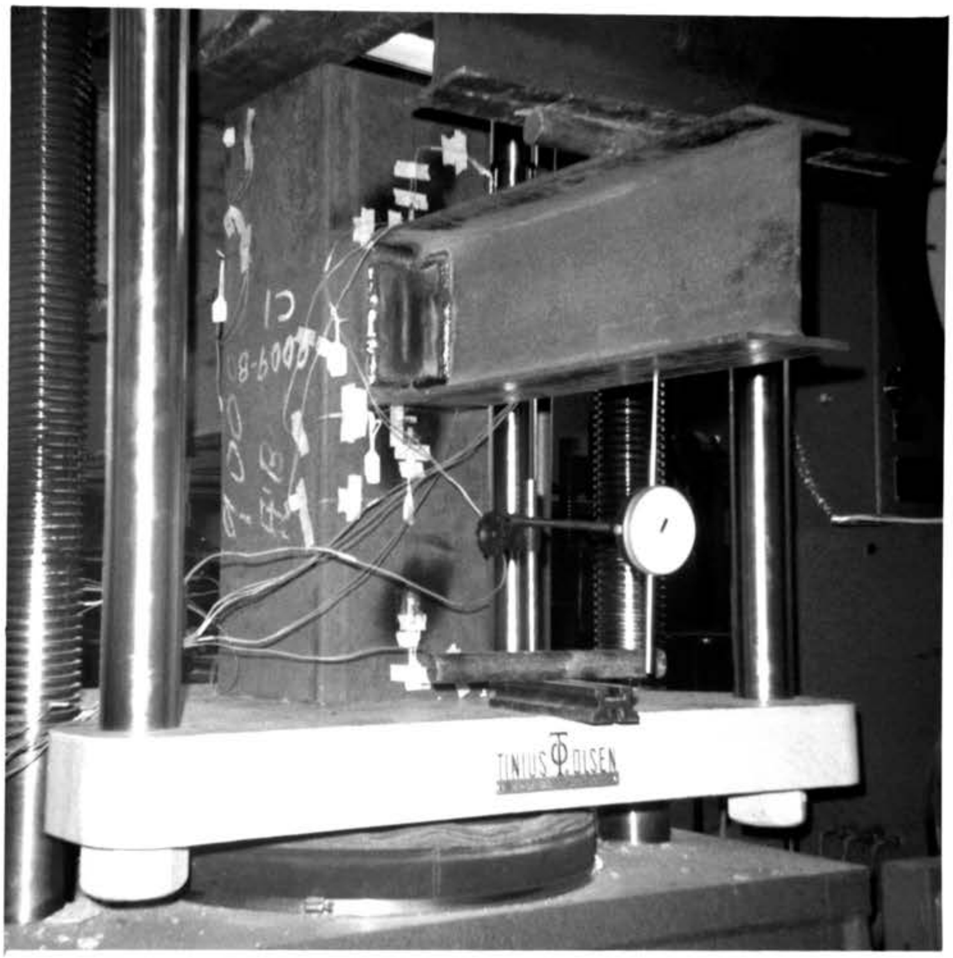


Fig. 26 Lufkin Deflection Gage

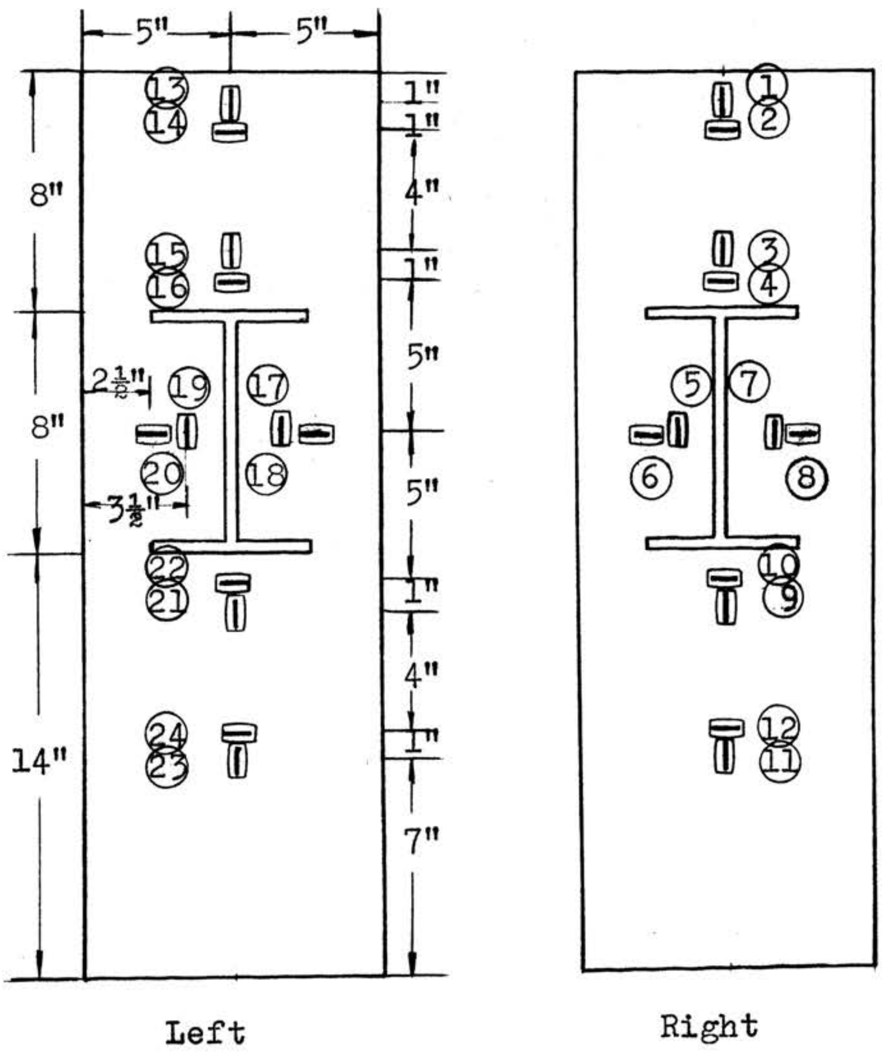
stresses were expected. Locations were determined both theoretically and experimentally. In this study, Specimens A and B, C and D, and E and F had the same gage locations as shown in Fig. 27, Fig. 28, and Fig. 29 respectively.

Both moment-rotation tests and shear-capacity tests were conducted. In the moment-rotation test, the specimens were loaded at two points each 15 inches from the tubular column wall as shown in Fig. 30 and Fig. 31. The deflection of the cantilever beam was measured at a point directly beneath the point of load application.

The setup for shear-capacity test was the same as for the moment-rotation test with the exception that the load was applied at a distance 5 inches from the tubular column wall. This is the usual loading configuration used in the tests which were performed.

It is noted here that to insure the proper concentric loading condition a roller was placed below each short load column. Although this increased the difficulty of setting up the test it helped obtain more accurate data.

A 66 inches long 6 WF 20 beam weighing 110 lbs. was chosen as the load beam. Two 12 B 14 beams, each 24 inches in length and 28 lbs. in weight, were used as short load

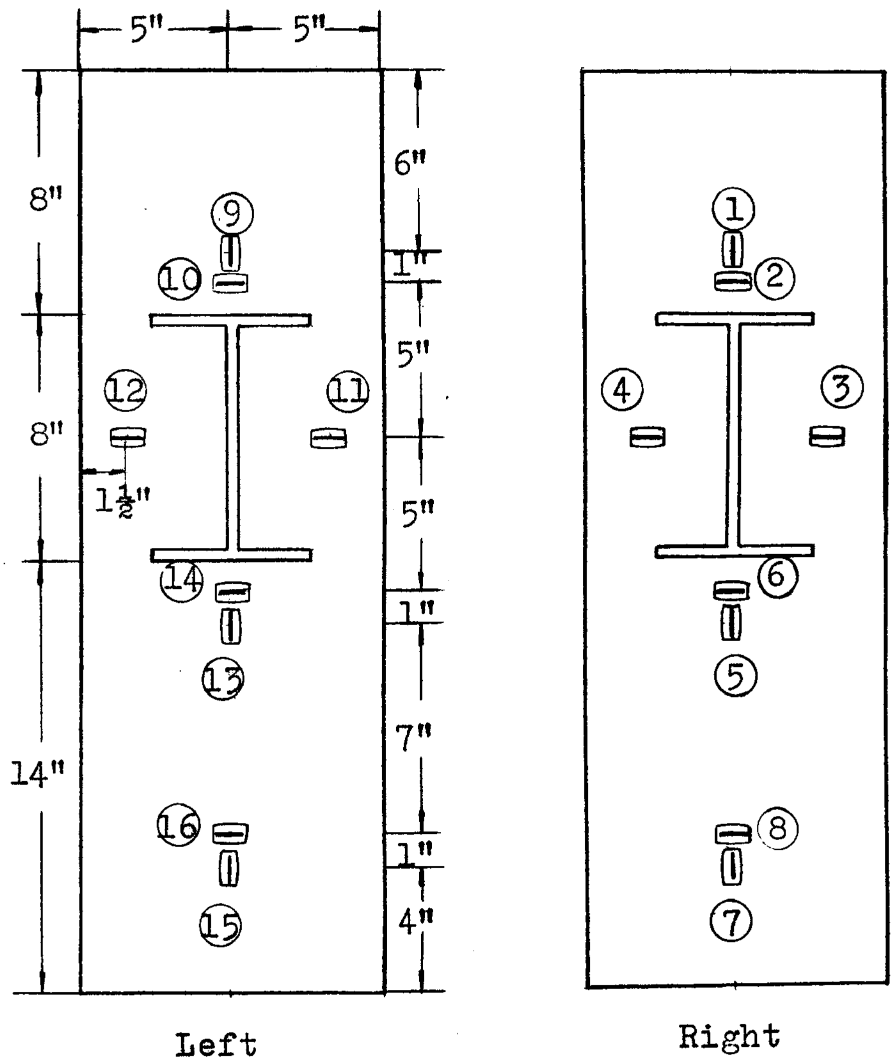


▯ --- Gages

◯ --- Gage Identifications

Fig. 27 Gage Locations of

Specimens A and B



▯ ---- Gages

◯ ---- Gage Identifications

Fig. 28 Gage Locations of

Specimen C and D

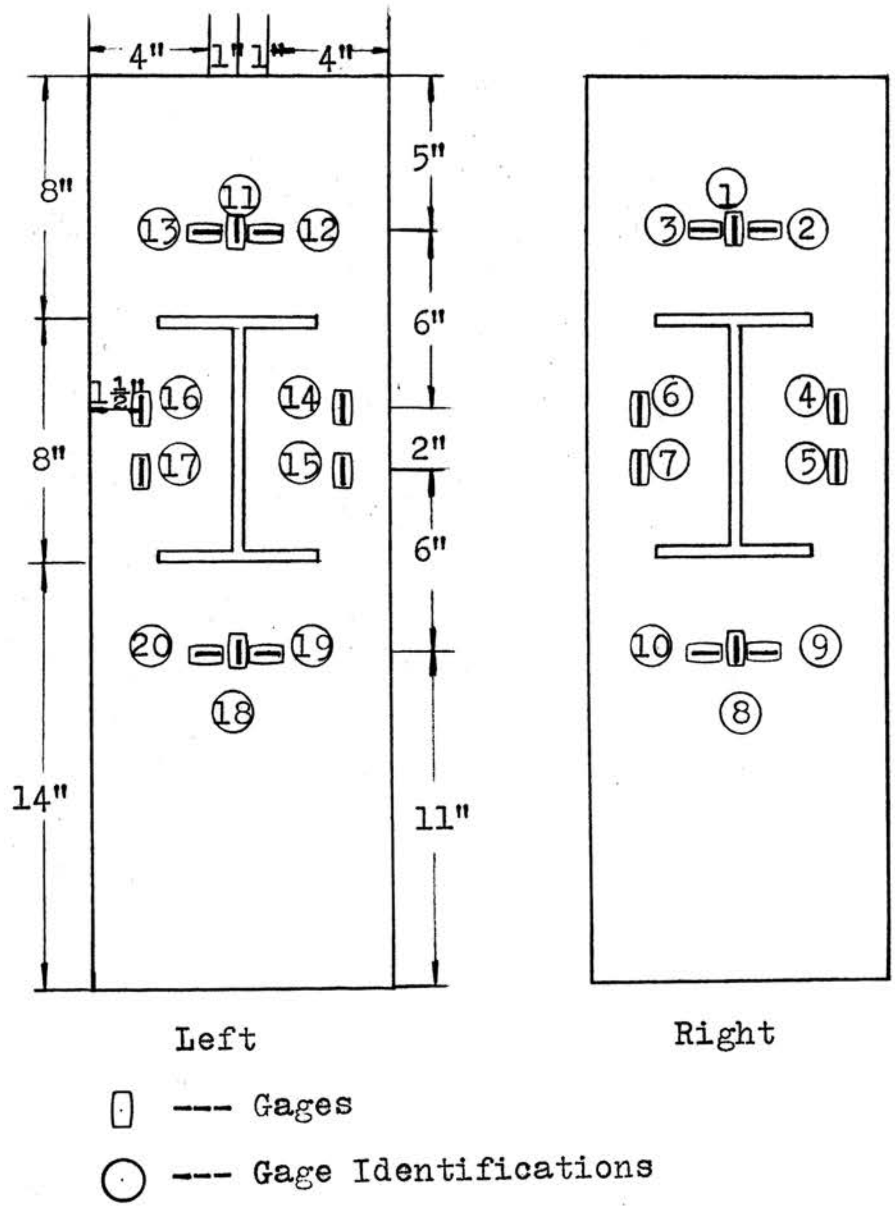


Fig. 29 Gage Locations of Specimens E and F

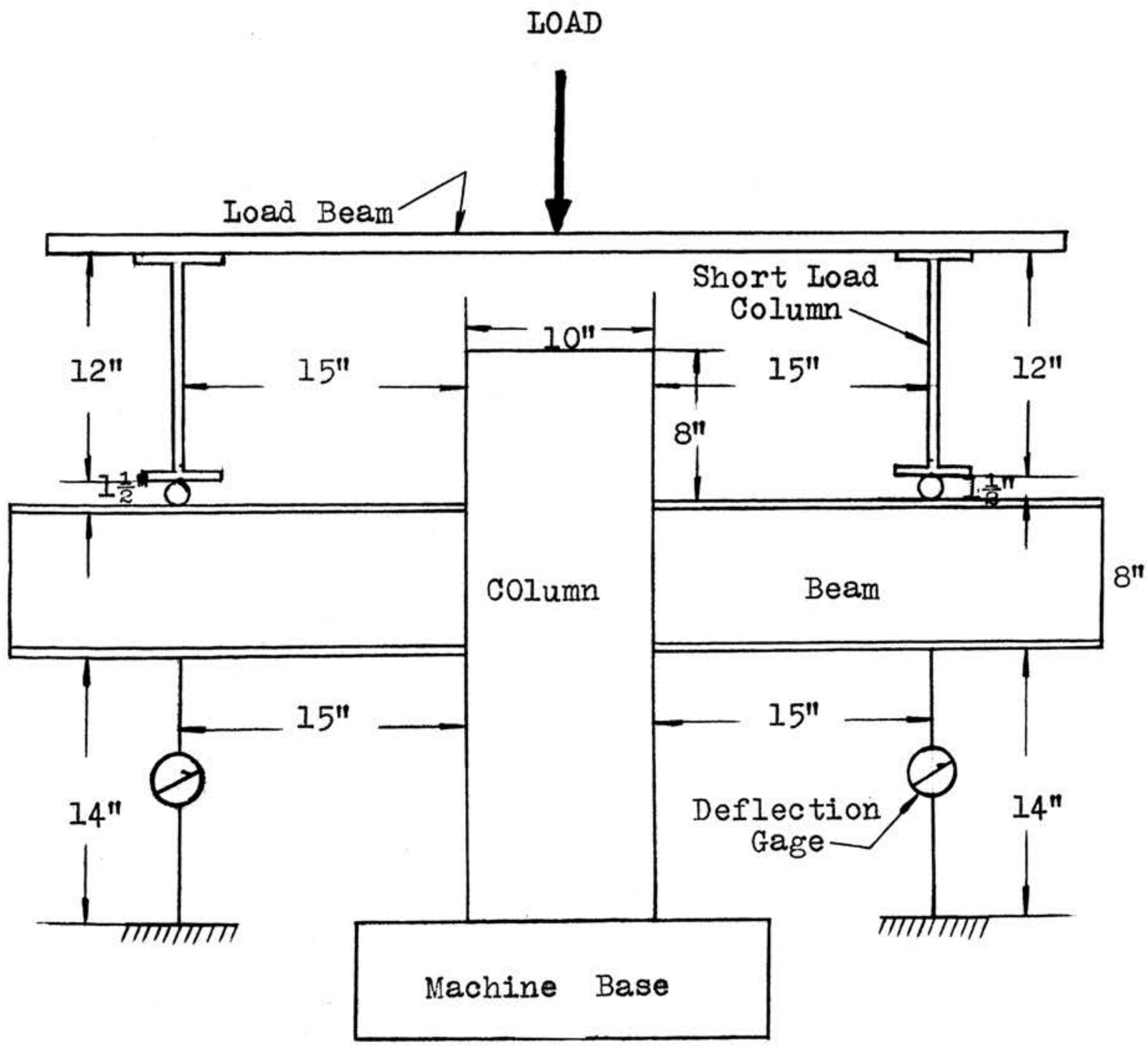


Fig. 30 Moment-rotation Test Setup I

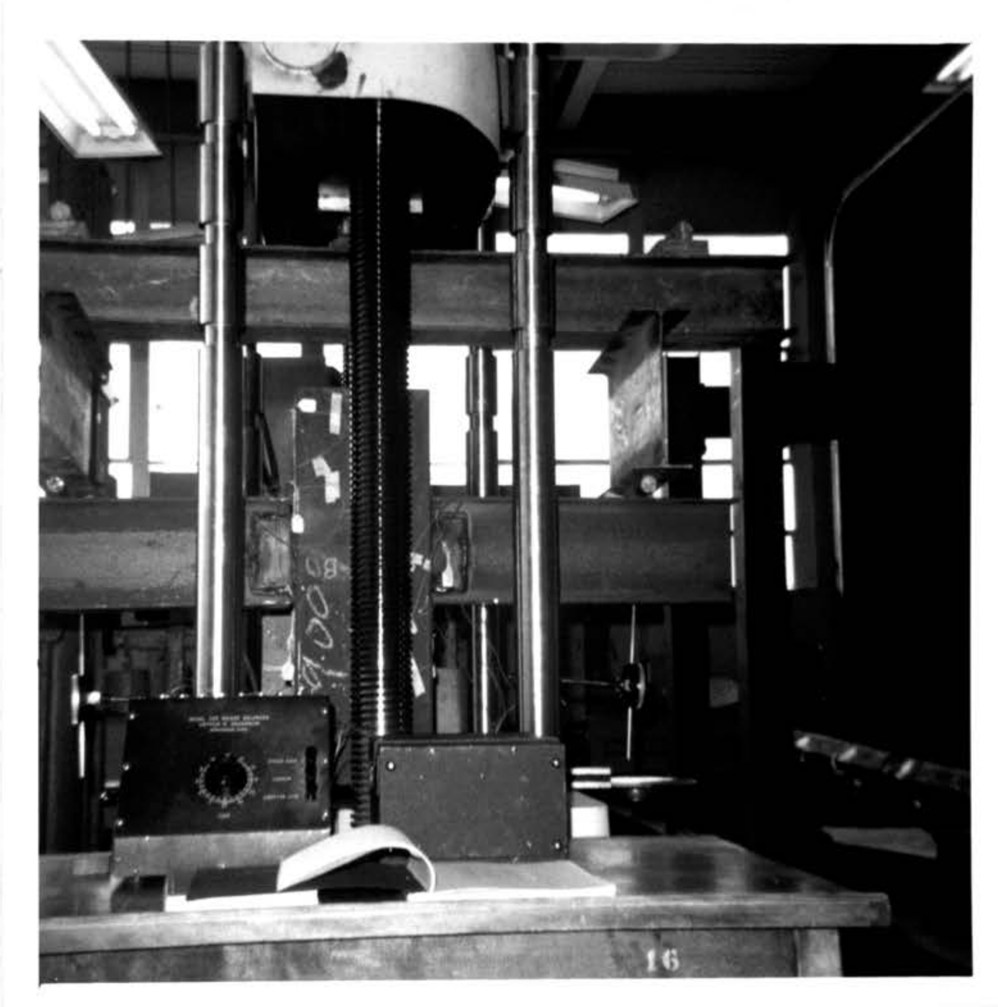


Fig. 31 Moment-rotation Test Setup II

columns. Thus, a total dead load was 166 lbs. This load was neglected in the tests due its negligible magnitude with respect to the high loading conditions.

Both the moment-rotation test and the shear-capacity test were started with zero loading. Increments of 300~500 lbs. for the moment-rotation tests and of 1000~2000 lbs. for the shear-capacity tests were used. Before adding each increment the strain reading of each gage and the deflection reading were carefully recorded. After each test, the specimen was examined and checked. Pictures were taken to show the results (Fig. 32, 33, and 34).

Fig. 32 Specimen was examined after test

Fig. 33 Connection Failure

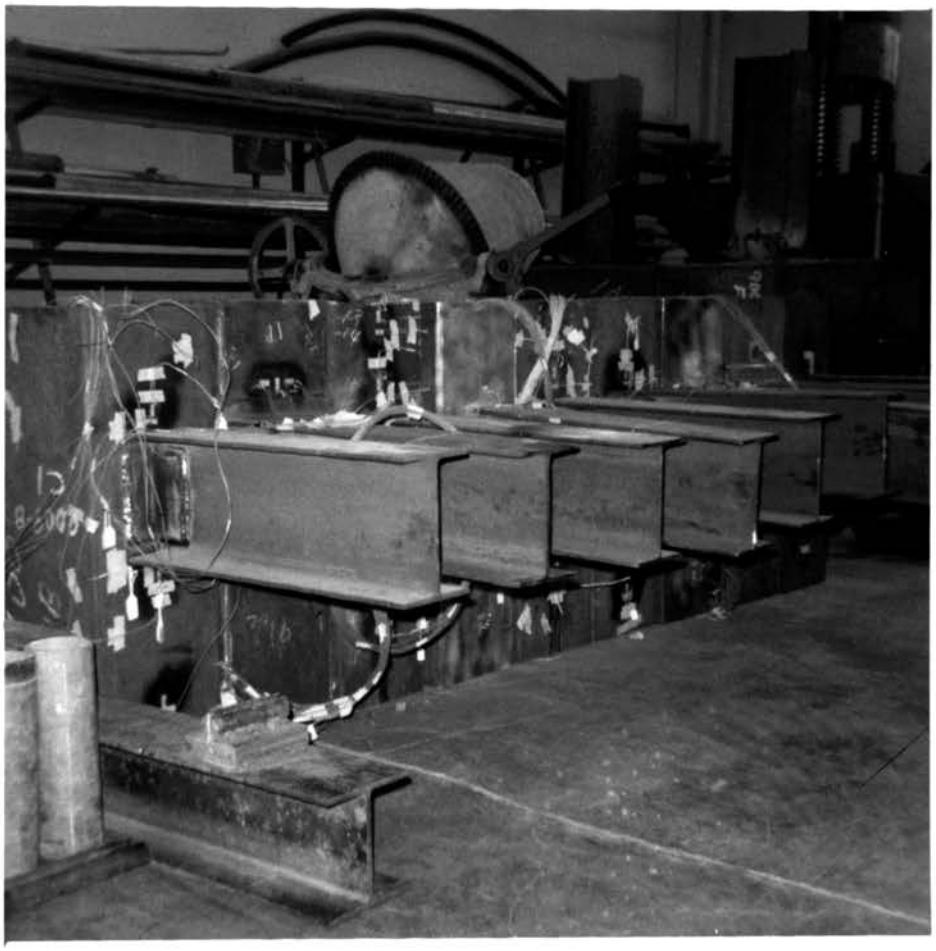


Fig. 34 Specimens after testing

V DISCUSSION OF RESULTS

Ten tests were conducted with six specimens. Seven of the ten were moment-rotation tests and the remaining three were shear-capacity tests.

Without considering the effects of temperature changes the general results can be summarized as follows:

1) Although the whole column wall was stressed during the load application, the maximum stresses were distributed over the area near the connection. The stress varies linearly with distance.

2) As was anticipated, the top strain gages were in tension and the bottom gages were in compression.

3) The connection yielded before the beam did.

4) The single-plate connection failed at both ends of the connecting plate, the double-angle connection failed at only the beam end of the connecting angle, and the multi-angle connection failed at neither end except the buckling of the column wall.

5) Approximately no strain induced at a distance of about twice the length of the connecting plate or angle from the connection. It was assumed then, that the action was terminated at this distance and a point of inflection existed nearby. This phenomenon is close to the assumption

made in the preceding chapter.

6) Shear capacity of the connections was great enough to prevent shearing failure.

Table III was constructed from the recorded deflection data and the theoretical data computed from the formula

$$\Delta = \frac{1\ P\ L^3}{3\ E\ I} \qquad (\ 65\ )$$

and in order to make comparison easy several load versus deflection curves were drawn as shown in Fig. 35. The discussion of the Table and Curves lead to some statements concerning the behavior of beam end deflections:

1) It should be noted that the deflection measured at the beam end is the summation of the results of all possible factors affecting the deflection. These factors might be the deflection of the beam end due to applied moment, the deformation of the connection, and the local buckling of the column wall caused by the concentration of stress.

2) Since applied load was always less than the theoretical yield load, P_y, the beam in each test can be considered as being elastic during the entire test. Thus, the difference in deflection between specimens was due to the difference of connection and column wall thickness only.

TABLE III

Values of Deflections

Load lbs.	Deflections unit: 0.001 in.						
	Theoretical	Specimen					
		A	C	E	B	D	F
250	0.1715	50	45	2	13	12	
500	0.3430	100	91	6	26	24	
750	0.5145	148	140	10	39	35	
1000	0.6860	195	189	14	52	47	
1250	0.8575	245	247	17	65	57	
1500	1.0290	295	305	21	79	73	
1750	1.2005	394	383	25	92	87	
2000	1.3720	510	461	29	106	102	14
2250	1.5435	636	563	33	121	118	
2500	1.7150	781	665	37	137	137	
2750	1.8865		792	41	154	157	
3000	2.0580		938	45	171	181	
3500	2.4010			55	210	239	
4000	2.7440			64	266	276	27
4500	3.0870			69	361	323	
5000	3.4300			87			
6000	4.1160						40
8000	5.4880						57
10000	6.8600						75
12000	8.2320						92
14000	9.6040						121
16000	10.9760						139
18000	12.3480						167
20000	13.7200						193
22000	15.0920						240
24000	16.4640						306

** Theoretical $\Delta = \dfrac{P\,L^3}{3EI}$

Where $L = 15$ in.

$E = 29{,}000{,}000$ psi

$I = 56.40$ in.^{4}

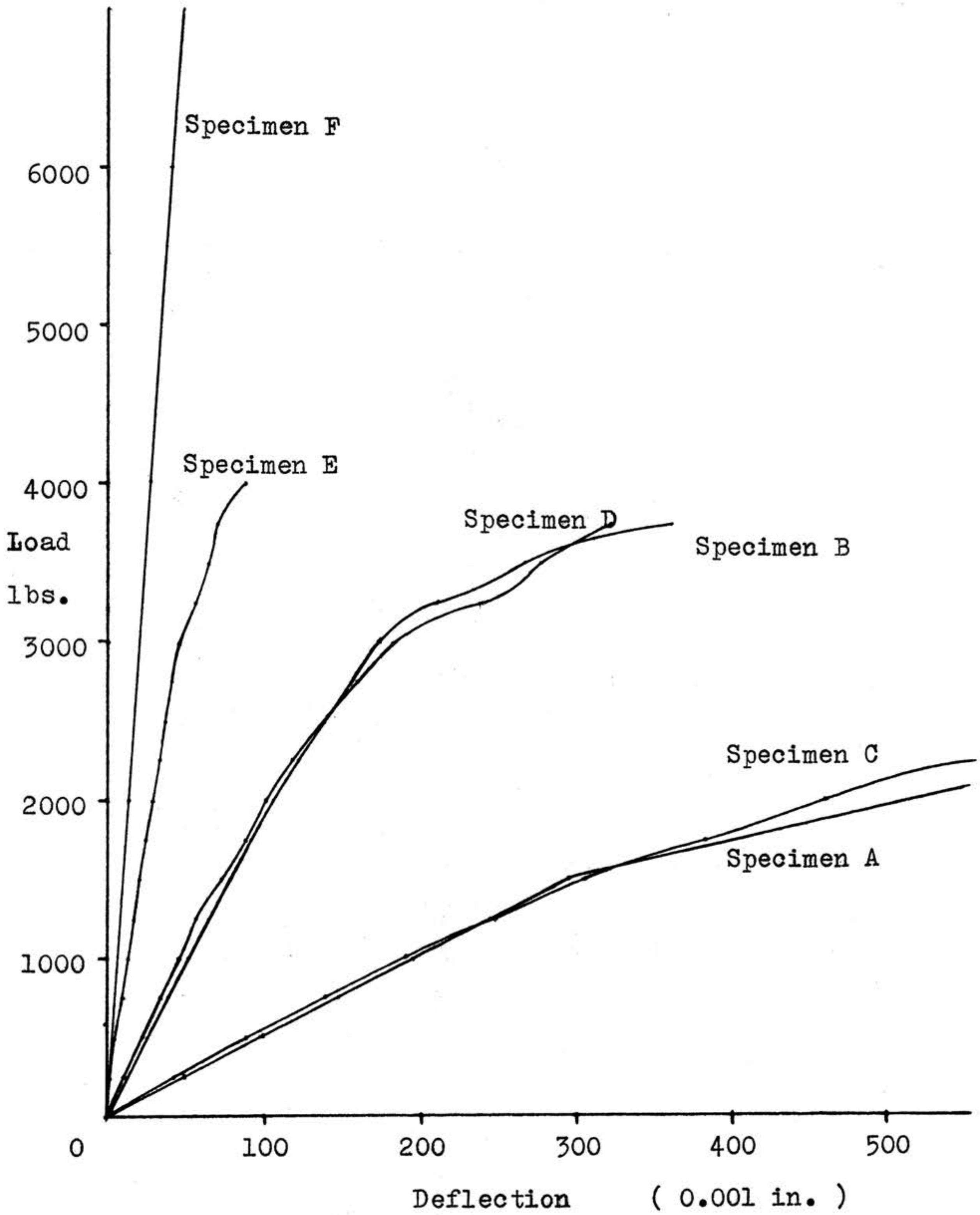


Fig. 35 Load versus Deflection

where $$P_y = \frac{36000\ I}{L \times c}\ \text{lbs.}$$

3) Comparing the deflections of Specimen A with C, and B with D, we discover that the single-plate connection induces almost the same amount of deflection as the double-angle connection. This is because vertical welds on beam webs are weaker in resisting horizontally distributed line forces. Hence, even though the double-angle connection maintains twice the length of vertical weld on web, there will be almost an equal capability in resisting the horizontal line forces. It should be noted that the single-plate connections fail at both ends of the connecting plate (i,e., the beam end and column end) and the double-angle connection fails at the beam end only. This is because the double-angle connection possesses a horizontal weld of six inches on the top and bottom edges of the angles.

4) Comparing the deflections of Specimen E with A and C, and F with B and D, we discover the multi-angle connections have strong capabilities in resisting the horizontal line forces. This is because the top and bottom angles take most of the tensile and compressive line forces.

5) Comparing the deflections of Specimen B with A, D with C, and F with E, it is evident that the column wall thickness plays a dominant role in affecting the deflections. Under the same conditions the thinner wall column induces

much larger buckling and results in rotation of the local wall. This rotation adds a large amount of deflection to the free end where

$$\Delta = \frac{2}{3} L \cdot \phi \qquad (66)$$

The other important beam end behavior is the end rotation. Since the end rotation cannot be measured during the test, the only way to obtain the experimental value of end rotation is from the recorded deflection.

For a cantilever beam

$$\Delta = \frac{1}{3}\frac{P L^3}{E I} = \frac{1}{3}\frac{M L^2}{E I}$$

and

$$\phi = \frac{M L}{2EI}$$

then

$$\phi = \frac{3 \Delta}{2 L} \qquad (67)$$

Using the above relationship together with Table III, a moment versus end rotation table can be constructed. Because of the stress concentration yielding occurred in the early stages of loading in Specimens A and C, and this greatly influenced the end rotation. Hence, we should waive Specimens A and C for the sake of obtaining a more accurate relationship for the end rotation exerted on each type of connection. Table IV lists only Specimens B, D, E and F. Since calculated end rotation is a linear function

TABLE IV

Values of End Rotations

Moment in.-#	End Rotation unit: radian Specimen B	Specimen D	Specimen E	Specimen F
3750	0.0013	0.0012	0.0002	
7500	0.0026	0.0024	0.0006	
11250	0.0039	0.0035	0.0010	
15000	0.0052	0.0047	0.0014	
18750	0.0065	0.0057	0.0017	
22500	0.0079	0.0073	0.0021	
26250	0.0092	0.0087	0.0025	
30000	0.0106	0.0102	0.0029	0.0014
33750	0.0121	0.0118	0.0033	
37500	0.0137	0.0137	0.0037	
41250	0.0154	0.0157	0.0041	
45000	0.0171	0.0181	0.0045	
52500	0.0210	0.0239	0.0055	
60000	0.0266	0.0276	0.0064	0.0027
67500	0.0361	0.0323	0.0069	
75000			0.0087	
90000				0.0040
120000				0.0057
150000				0.0075
180000				0.0092
210000				0.0121
240000				0.0139
270000				0.0167

of end deflection it presents almost the same relation and effect with each type of connection as the end deflection did as described in the preceding paragraph.

In checking the moment-rotation characteristics of the connection, a beam-line chart, which is based on the beam-line concept derived by Batho in 1934 should be used. Refer to Equations 35 and 37, for an allowable stress of 23,760 psi, the uniformly distributed working load would be 11,800 lbs./in.

then
$$M_o = \frac{w\ L^2}{12} \qquad (35)$$

$$= \frac{11800 \times 15 \times 15}{12}$$

$$= 221,000 \text{ in.-lb.}$$

and
$$\phi_o = \frac{2\ f_b\ L}{3\ E\ d} \qquad (37)$$

$$= \frac{2}{3} \times \frac{23760}{29 \times 10^6} \times \frac{15}{8}$$

$$= 0.00103 \text{ radian.}$$

Applying a safety factor of 1.65 to the beam-line of working load, a safe beam-line can be drawn which serves the purpose of checking moment-rotation characteristics of the connections as shown in Fig. 36. The intersection of each M-ϕ curve to the safe beam-line shows every connection

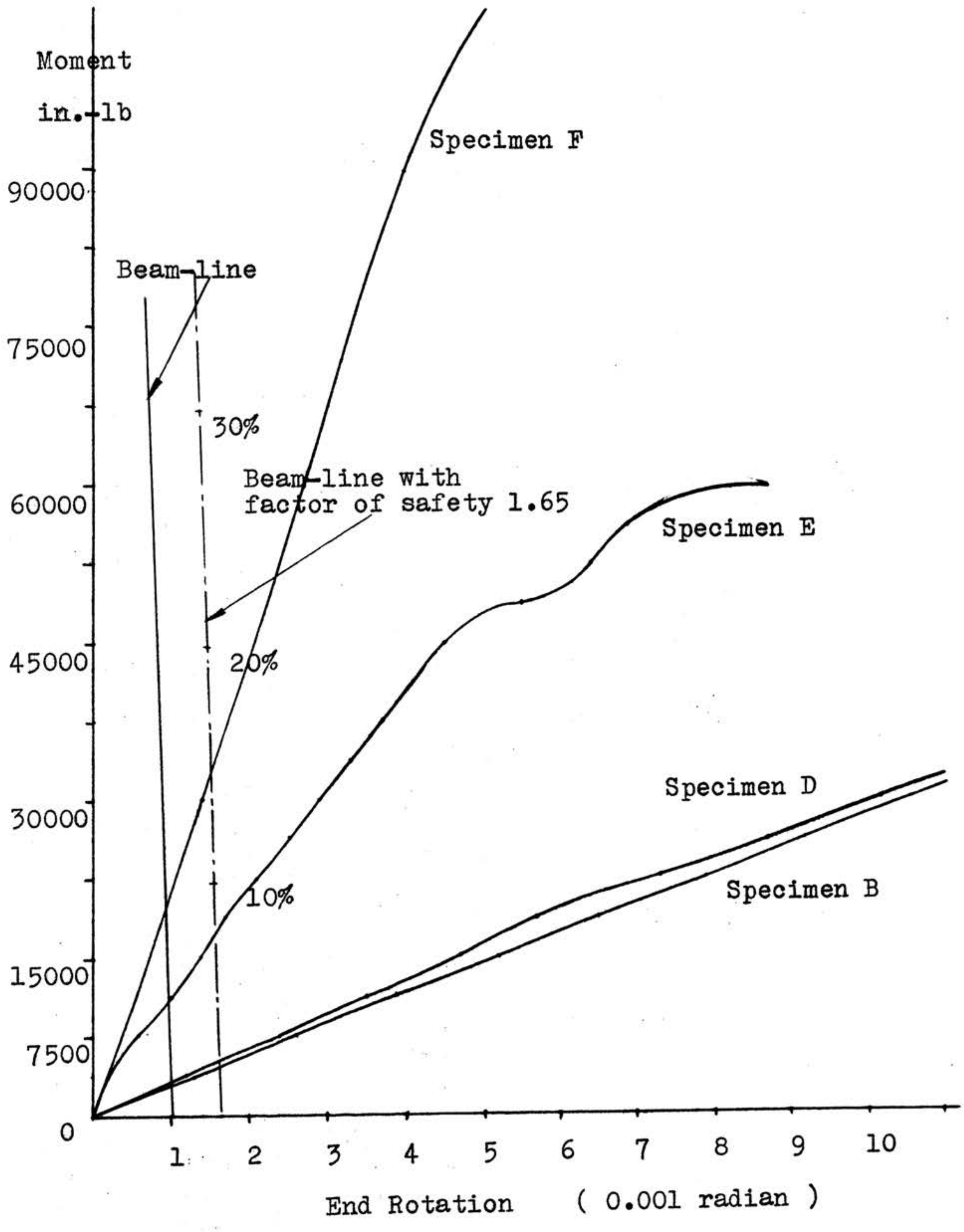


Fig. 36 M-ø for Beam-line Concept

is suitable if the moment-rotation characteristics are only examined. However, the stiffness of each connection differs to a large extent.

Generally, a square tubular column with a low width-to-thickness ratio exhibites moderate stiffness, whereas connections on the thinner tubings were quite flexible. The stiffness of different types of connections varies from 5% to 20% and is independent of beam size. For design, stiffness is also an important factor in evaluating a connection. The acceptable stiffness depends on the usage of the structure. Usually 20% to 30% can be considered as quite stiff.

As expected, the maximum stress intensity occurs at the portion just above and below the connection on the column wall. Column wall buckling is present at this area and tension goes to the top and compression goes to the bottom. The yielding load of the column wall differs with different types of connections. They are 1600 lbs. to 1800 lbs. for Specimens A and C, 4500 lbs. for Specimens B and D, 16000 lbs. for Specimen E, and 26000 lbs. for Specimen F.

The major cause of the wide range of differences of yielding load of the column wall can be explained by the stress concentrations as follows (the explanation is under the assumption of a rigid structure):

In single-plate connection the moment produced by an applied load is carried by the beam, and then the connecting plate. Fiber stress on the beam can be calculated by the formula

$$f_b = \frac{M\ c_b}{I_b}\ .$$

The nominal stress on the connecting plate can also be calculated by the ordinary formula

$$f_p = \frac{M\ c_p}{I_p}\ .$$

But the sudden change of dimension and shape of the cross-section causes a stress concentration at both edges of the plate. A factor of stress concentration, k, should be applied to the nominal fiber stress of the plate, f_p, for obtaining a reasonable result.

$$k = \frac{f'}{f} = \frac{\text{actual stress}}{\text{nominal stress}} \qquad (68)$$

The ratio of actual stress to nominal stress is the factor of stress concentration, k, for this particular case, where the actual stresses are obtained from experiments.

In our case, if the applied load is 1500 lbs., then $\qquad M = 1500 \times 15 = 22500$ in.-lb and $c_b = 4''$ $\qquad I_b = 56.4 \text{ in.}^4$

$c_p = 3''$ $\qquad I_p = 3.375 \text{ in.}^4$

then $f_b = 22500 \times 4/56.4 = 1600$ psi

$$f_p = 22500 \times 3/3.375 = 20000 \text{ psi}$$

$$\frac{f_p}{f_b} = \frac{20000}{1600} = 12.5$$

and here $f_p' = 34670$ psi

$$k = \frac{f_p'}{f_p} = \frac{34670}{20000} = 1.734$$

This k value is for Specimen A only. When Specimen B was subjected to the same moment, the stress measured at the same location was only 9976 psi, which made the factor of stress concentration, k, less than one. This shows the importance of the properties of the column wall on the results of the tests.

Tables V and VI, and Figures 37 and 38 show the relationship between loads and stresses both in tension and in compression. They are very important in checking the characteristics of the connections. Even though it

TABLE V

Values of Tensile Stresses

Load lbs.	Tensile Stresses unit: psi					
	Specimen					
	A	B	C	D	E	F
250	5655	1102	3799	1421	232	
500	11296	2668	6438	2755	493	
750	19343	4234	10005	4205	667	
1000	24302	5800	14007	5365	841	
1250	29145	7279	17806	6902	1073	
1500	34307	8758	22127	8410	1189	
1750		10440		9802	1392	
2000		12238		11339	1624	1044
2250		14094		12721	1769	
2500		15979		14125	1943	
2750		17864		15631	2175	
3000		19749		17255	2465	
3500		24070		20880	3016	
4000		28652		24070	3683	2204
4500		33959			4524	
5000					5278	
6000						3364
8000						4843
10000						7303
12000						9135
14000						11977
16000						16646
18000						19778
20000						23954
22000						27840
24000						34133

TABLE VI

Values of Compressive Stresses

Load lbs.	Compressive Stresses unit: psi					
	Specimen					
	A	B	C	D	E	F
250	5510	1479	4321	1276	261	
500	10962	3045	7888	2697	551	
750	16269	4553	11165	3973	696	
1000	22330	6119	15689	5249	870	
1250	28594	7569	19256	6699	1160	
1500	34670	9077	22939	8149	1450	
1750		10469		9570	1740	
2000		12093		11049	2030	1015
2250		13601		12557	2291	
2500		15283		14065	2552	
2750		16733		15631	2784	
3000		18299		17255	3045	
3500		21808		20648	3654	
4000		25984		22968	4350	1943
4500		32741			5133	
5000					5945	
6000						2668
8000						4205
10000						6061
12000						7395
14000						9831
16000						12165
18000						15718
20000						17893
22000						24215
24000						29986

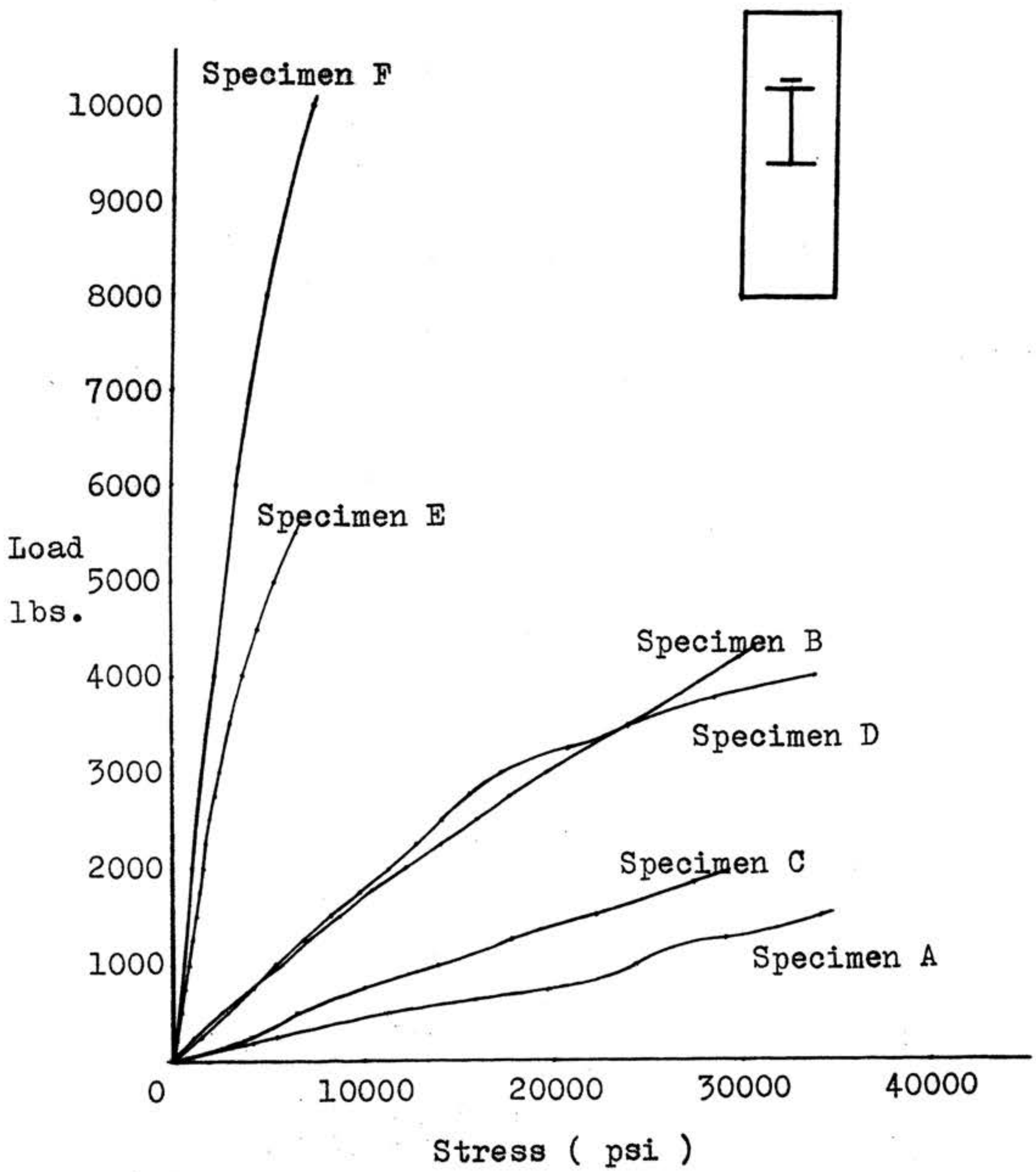


Fig. 37 Load versus Tensile Stress

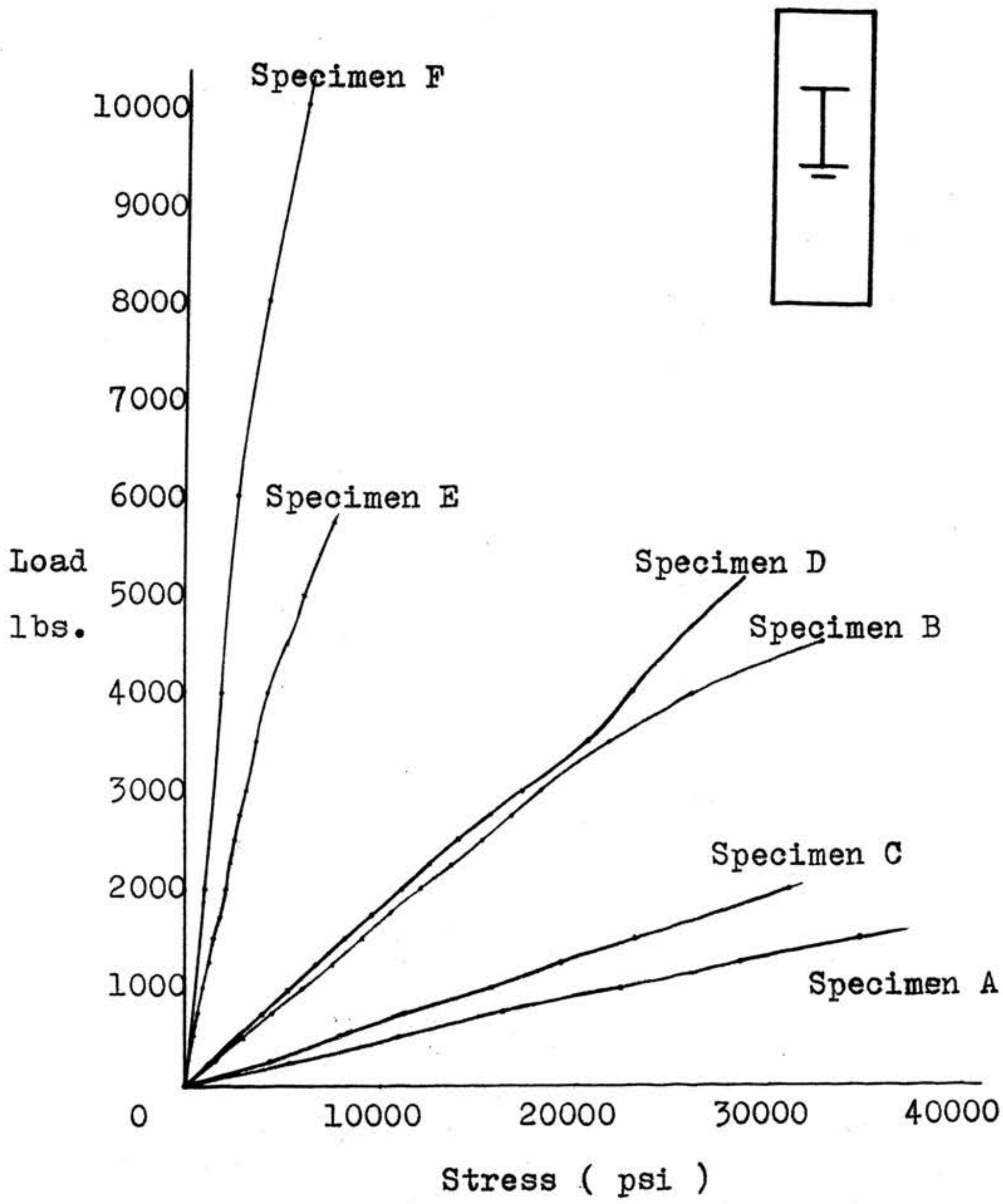


Fig. 38 Load versus Compressive Stress

is hard to plot a stress pattern or curve because of the great effect of stress concentration, it is shown in the experiment that the stress distribution closely follows the preceding assumption. Stress changes from compression to tension near the extreme end of the column showing the existance of an inflection point nearby.

VI CONCLUSIONS

The following statements can be concluded:

1) All connections have sufficient shear capacity. This can be proved by the fact that, in shear capacity tests, failure was only caused by flange crippling or web deforming of the connected beam.

2) The vertical welds can hardly resist the horizontal line forces which are induced by the bending moment. Unfortunately, the bending moment is the governing factor of failure.

3) The whole column wall is stressed during loading. Maximum stress occurs at the portion near the connection. The stress decreases linearly with distance.

4) Stress concentration is the major cause of the local buckling of the column wall.

5) The thickness of the column wall plays an important role in its behavior especially due to local buckling.

An obvious disadvantage of the single-plate connection is that its vertical weld can hardly resist the horizontal line forces. Furthermore, the sharp change of cross section between beam and connection **causes** the stress concentration to occur. The stress concentration causes the connection to yield at low load. Also, the stress concentration develops the local buckling, which, in turn, weakens the load carrying capacity of the column. The local buckling of the column wall would not be a serious problem should the column wall be thick enough to resist the intensive stress as Specimen B did. However, it is not economical to use thick-wall column just for this single purpose.

Since the single-plate connection is the simplest connection and is least expensive to fabricate, it is suggested that this type of connection could be used on condition that the load is moderate or the full column strength is not needed, such as in the upper part of a two or three-story building.

The double-angle connection appears almost the same behavior and effect to the column and to the beam as the single-plate connection does. This is because the horizontal line forces govern most of the behavior. Hence, it would be wasteful to adopt this type of connection except where the shear capacity would be the major factor of failure.

However, the bilateral symmetry of the two angles with respect to the web presents a better appearance. Besides, the double vertical welds make the double-angle connection more stable than that of the single-plate connection.

The top and bottom angles of the multi-angle connection distribute the stress on a wider section preventing the occurrence of the stress concentration. In other words, these two angles bear effectively the horizontal line forces. This protects the column wall from local buckling caused by stress concentration as in other types of connections. So it is suggested that the multi-angle connection be used in important joints.

In order to assure a maximum area of action or a minimum of force transfer into the flexible wall face, it is necessary to extend the bottom angle to its maximum length. This means that all welds have to be along the corners of the column. In case a relatively narrow tubular column is to be connected, the bottom angle should exceed the width of the column. This might be architectually undesirable and is expensive to fabricate.

APPENDIX

Capacity of Members

1) Load Beam 6 WF 20

$$S_{xx} = 13.4 \text{ in.}^3$$

$$d = 6.25 \text{ in.}$$

$$w_t = 0.25 \text{ in.}$$

for A36 Steel

$$F_b = 23760 \text{ psi}$$

$$F_v = 14400 \text{ psi}$$

from $M = F_b \times S_{xx} = \frac{1}{4}PL$

$$= 23760 \times 13.4$$

$$= 318,000 \text{ in.-lb}$$

for moment-rotation test $L = 40$ in.

$$P = \frac{4}{40} \times 318000$$

$$= 31,800 \text{ lbs.}$$

for shear capacity test $L = 20$ in.

$$P = \frac{4}{20} \times 318000$$

$$= 63,600 \text{ lbs.}$$

and $V = F_v \times w_t \times d$

$$= 14400 \times 0.25 \times 6.25$$

$$= 22,500 \text{ lbs.}$$

allowable total load $= 22,500$ lbs.

2) Load Short Beam 12 B 14

$$d = 11.875 \text{ in.}$$

$$w_t = 0.1875 \text{ in.}$$

then $$V = 14400 \times 0.1875 \times 11.875$$

$$= 32,000 \text{ lbs.}$$

allowable total load $= 32000 \times 2$

$= 64,000$ lbs.

3) Structural Beam 8 WF 17

$$S_{xx} = 14.1 \text{ in.}^3$$

$$d = 8.00 \text{ in.}$$

$$w_t = 0.25 \text{ in.}$$

then $$P = \frac{23760 \times 14.1}{15}$$

$$= 22,300 \text{ lbs.}$$

$$V = 14400 \times 0.25 \times 8.00$$

$$= 28,800 \text{ lbs.}$$

allowable total load $= 22300 \times 2$

$= 44,600$ lbs.

4) Single-plate Connection

$$F_v = 13600 \text{ psi}$$

$$w = 0.1875 \text{ in.}$$

$$l = 12 \text{ in.}$$

then $$V = 13600 \times 0.1875 \times 12 \times 0.707$$

$$= 21,600 \text{ lbs.}$$

allowable total load $= 21600 \times 2$

$= 43,200$ lbs.

5) Double-angle Connection

$$w = 0.1875 \text{ in.}$$

$$l = 17 \text{ in.}$$

then $$V = 13600 \times 0.1875 \times 17 \times 0.707$$

$$= 30,600 \text{ lbs.}$$

allowable total load $= 30600 \times 2$

$$= 61,200 \text{ lbs.}$$

6) Multi-angle Connection

$$w_i = 0.125 \text{ in.}$$

$$l_i = 40 \text{ in.}$$

then $$V = 30600 + 13600 \times 0.125 \times 40 \times 0.707$$

$$= 78,600 \text{ lbs.}$$

allowable total load $= 78600 \times 2$

$$= 157,200 \text{ lbs.}$$

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