1957

Servomechanism compensation with an active phase-lead network

Robert T. DeWoody

SERVOMECHANISM COMPENSATION WITH AN ACTIVE PHASE-LEAD NETWORK

BY

ROBERT T. DeWOODY

A

THESIS

submitted to the faculty of the

SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI

in partial fulfillment of the work required for the

Degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

Rolla, Missouri

1957

Approved by - Roger E. Nolte
Professor of Electrical Engineering

ACKNOWLEDGMENTS

The author wishes to thank Dr. R. E. Nolte for his suggestions and assistance; R. Glover for drawing the graphs; and Miss Virginia Libera for the typing.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

I. INTRODUCTION

The problem is to develop a phase-lead compensation network for a servomechanism which will not reduce the low-frequency gain and thus decrease the velocity-lag error. The principal part of this thesis deals with the design of such an active phase-lead network. The active network does not have d-c coupling; this eliminates drift problems.

To illustrate an active phase-lead network a simple second order servomechanism is used as a basis for comparing three methods of compensation.

The first method is compensation with a conventional phase-lead network with a gain decrease. The second method is compensation with a conventional phase-lead network with a gain increase. The third method is compensation with an active phase-lead network with no gain change.

The three compensated systems were selected to have about the same peak overshoot but not necessarily the same rise time.

The basic servomechanism was simulated on the MSM Analog Computer and used to design the active phase-lead network.

Photographic recordings were made to illustrate the effectiveness of the design.

II. REVIEW OF LITERATURE

Although there are many published articles on servomechanism compensation with phase-lead networks, there is very little written on linear feedback compensation networks.

An article by George J. Schwartz(1) deals with the design of a high performance servo system which is compensated by a passive phase-lead network and output velocity feedback.

Herbert Harris(2) compares the results of compensating with a passive phase-lead network and output velocity feedback. Harris concludes that the two methods give essentially the same results.

A review of the use of analog computers in linear servo design is presented by Robert A. Bruns(3). Bruns gives attention to systems that do not lend themselves to direct mathematical methods.

---

(1) Schwartz, George J., The Application of Lead Networks and Sinusoidal Analysis to Automatic Control Systems. Transactions of the Institute of Electrical Engineers. Vol. 66, pp. 66-77 (1947)

(2) Harris, Herbert, A Comparison of Two Basic Servomechanism Types. Transactions of the Institute of Electrical Engineers. Vol. 66, pp. 83-93 (1947)

(3) Bruns, Robert A., Analogue Computers for Feedback Control Systems. Transactions of the Institute of Electrical Engineers. Vol. 71, Part 11, pp. 250-254 (1952)

A nonlinear feedback compensation method, using tachometer generators, is presented by J. B. Lewis[4].

(4) Lewis, J. B., The Use of Nonlinear Feedback to Improve the Transient Response of a Servomechanism. Transactions of the Institute of Electrical Engineers. Vo. 71, Part 11, pp. 449-453 (1952)

This method varies the damping ratio as a function of the error signal. The damping ratio is increased as the error approaches zero.

George J. Biernson[5] has written a paper on an

(5) Biernson, George J., A General Technique for Approximating Transient Response from Frequency Response Asymptotes. Transactions of the Institute of Electrical Engineers. Vol. 75, Part 11, pp. 253-273 (1956)

approximate method of determining the transient response, of a servomechanism, from the frequency response. Biernson uses the open and closed loop frequency response curves to determine the individual transient response terms. The results of the approximate method compare very well with the exact transient response.

A basic coverage of servomechanism synthesis on analog computers is presented in a book by C. L. Johnson[6].

(6) Johnson, Clarence L., Analog Computer Techniques. N. Y. McGraw-Hill, pp. 45-64 (1956)

The book shows methods of generating many useful transfer functions.

III. GENERAL THEORY

The general theory of feedback control systems is well covered in periodicals and books. A brief review of the transient and frequency response analysis is presented here to define symbols and correlate the two methods of analysis. The following symbols are used.

KG - Open-loop transfer function

K_v - Velocity constant

T_m - Time constant of motor and load

θ_i - Input angle (radians)

θ_o - Output angle (radians)

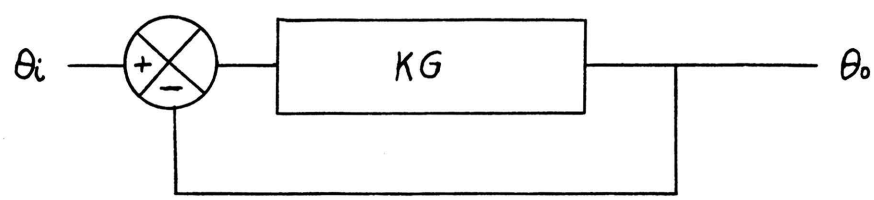

Figure 1. Block Diagram of Servomechanism

The transfer function of a simple servomechanism has the following form[7]

---

(7) Thaler, George J., Elements of Servomechanism Theory. N. Y., McGraw-Hill, 1955. pp. 67-74.

$$KG = \frac{K_v}{s(T_m s + 1)} \qquad \text{------------------------------------} \quad (1)$$

Let $K = K_v/T_m$

$\alpha = 1/2T_m$

$\beta = \sqrt{(K_v/T_m) - (1/4T_m^2)}$

then $$\frac{\theta_o}{\theta_i} = \frac{K}{(s+\alpha)^2+\beta^2} \qquad \text{(2)}$$

For a step input[8]

(8) Thomson, William T., Laplace Transformation. N. Y., Prentice-Hall, 1950. pp. 71-75.

$$\frac{\theta_o}{\theta_i} = 1 - \frac{\sqrt{\alpha^2+\beta^2}}{\beta} e^{-\alpha t} \sin\left(\beta t + \tan^{-1}\frac{\beta}{\alpha}\right) \qquad \text{(3)}$$

Equation 3 is the equation for the transient response of a servomechanism with a step displacement input.

The transient characteristics of a servomechanism are evaluated in terms of the time required for the output to settle to within a certain percentage (usually 2 to 5%) of the final position, when a step displacement is applied to the input[9]. The transient characteristics may also be evaluated in terms of the rise time and peak overshoot of the transient response. The rise time is the time, from the application of a step displacement input, until the output is first equal to the input. The peak overshoot is the ratio of the maximum value of the output to the input.

(9) Thaler, op.cit., pp. 13-19.

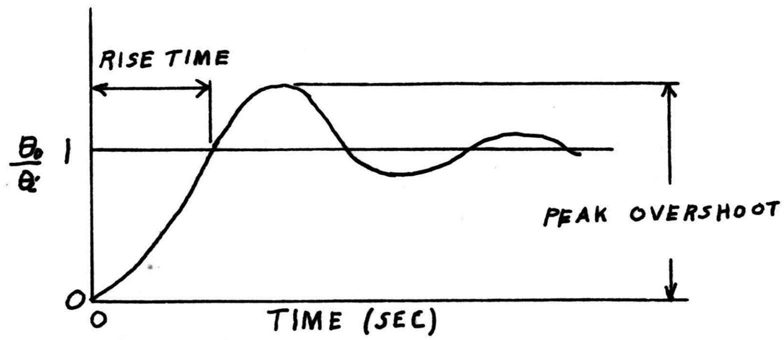

Figure 2. Transient Response Curve

The rise time is determined by finding the time at which equation 3 is equal to one or when $\sin(\beta t + \tan^{-1}\frac{\beta}{\alpha})$ is equal to zero.

Let $\beta t + \tan^{-1}\frac{\beta}{\alpha} = 2\pi$

then $t = (2\pi - \tan^{-1}\frac{\beta}{\alpha})/\beta$ -------------------------- (4)

Equation 4 shows that an increase in β reduces the rise time. Since $\beta = \sqrt{(K_v T_m - (1/4T_m^2))}$, β may be increased by increasing K_v/T_m.

The peak overshoot occurs when $\sin(\beta t + \tan^{-1}\frac{\beta}{\alpha}) = -1$ and is determined by the value of $(\sqrt{\alpha^2 + \beta^2}/\beta)e^{-\alpha t}$.

$$\text{Peak overshoot} = 1 + \frac{\sqrt{\alpha^2+\beta^2}}{\beta} e^{-\alpha t} \qquad \text{---------------- (5)}$$

The value of equation 5 at any particular time is determined by α. If α is a large positive number equation 5 will approach one very quickly, but if α is a small positive number then equation 5 would approach one at a slow rate. Therefore if β is held constant the peak overshoot will be large for small values of α and small for large values of α .

Transient analysis of servomechanisms becomes more difficult as networks are added to the system and the effects of these additional components are not readily evaluated. In order to circumvent this problem frequency response analysis is used.

The open loop frequency response is the steady state sinusoidal value of the system transfer function, equation 1. The frequency response is calculated by substituting $j\omega$ for s in equation 1, and determining the magnitude and phase for several values of frequency. The magnitude function is plotted in decibels versus $\log_{10}\omega$ and the phase angle is plotted in degrees versus $\log_{10}\omega$.

Figure 3 shows the phase curve and asymptotic approximation of the magnitude curve for equation 1.

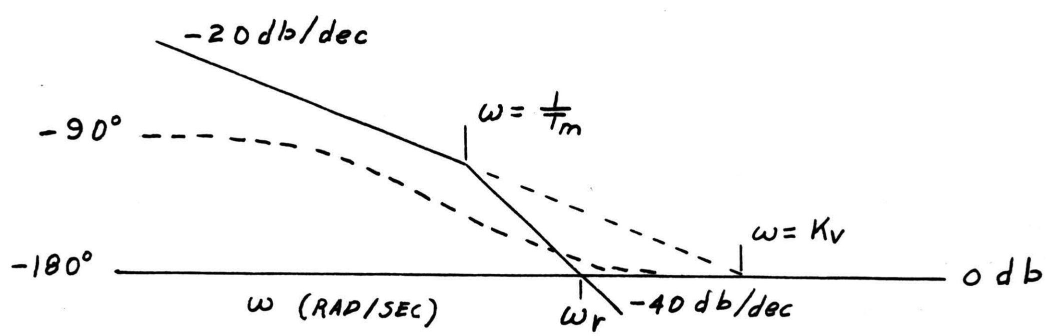

Figure 3. Frequency Response Curve

From Figure 3, $\omega_r = \sqrt{K_v/T_m}$, where ω_r is the frequency at which the magnitude curve crosses the zero decibel axis, and will be referred to as the crossover frequency. This value of ω_r is compared to $\beta = \sqrt{(K_v/T_m) - (1/4T_m^2)}$ and it is observed that an increase in ω_r results in an increase in β.

It is concluded at this point that an increase in the crossover frequency results in a decrease in the rise time of the transient response.

A negative approach is taken to correlate α with the frequency response plot. If the phase margin is defined as 180° minus the phase angle at ω_r, the system has sustained oscillations if the phase margin is zero. The condition for equation 3 to have sustained oscillations is $\alpha = 0$. Therefore a small phase margin indicates a small value of α and a large transient overshoot while a large phase margin indicates a large value of α and a small overshoot.

The phase margin may be changed by using a phase shift network or by changing the crossover frequency without shifting the phase. The latter may be accomplished by changing the gain or by using a low pass filter. The low pass filter can reduce the crossover frequency but cannot increase it.

The transient equations in this section apply only to systems that are underdamped and no other condition of damping is considered in this paper.

IV. BASIC UNCOMPENSATED SERVO SYSTEM

The transfer function of the basic uncompensated system is

$$KG = \frac{50}{s(0.1s + 1)} \qquad (6)$$

Figure 4 is the frequency response plot of equation 6. The phase margin is 25 degrees and the crossover frequency is 21 radians per second.

The equation for the transient response of the basic system is

$$\frac{\theta_o}{\theta_i} = 1 - 1.025e^{-5t}\sin(21.8t - 77.1^\circ) \qquad (7)$$

Curve 1 of Figure 15 is a plot of equation 7. The rise time is 0.083 seconds and the peak overshoot is 1.48.

It is apparent from the transient response plot that the basic system requires too much time to settle to the final value and that the peak overshoot is too high.

Two things may be done to improve the transient response. The phase margin may be increased and the crossover frequency may be increased. Increasing the phase margin decreases the peak overshoot and increasing the crossover frequency decreases the rise time. The system will then have a shorter response time.

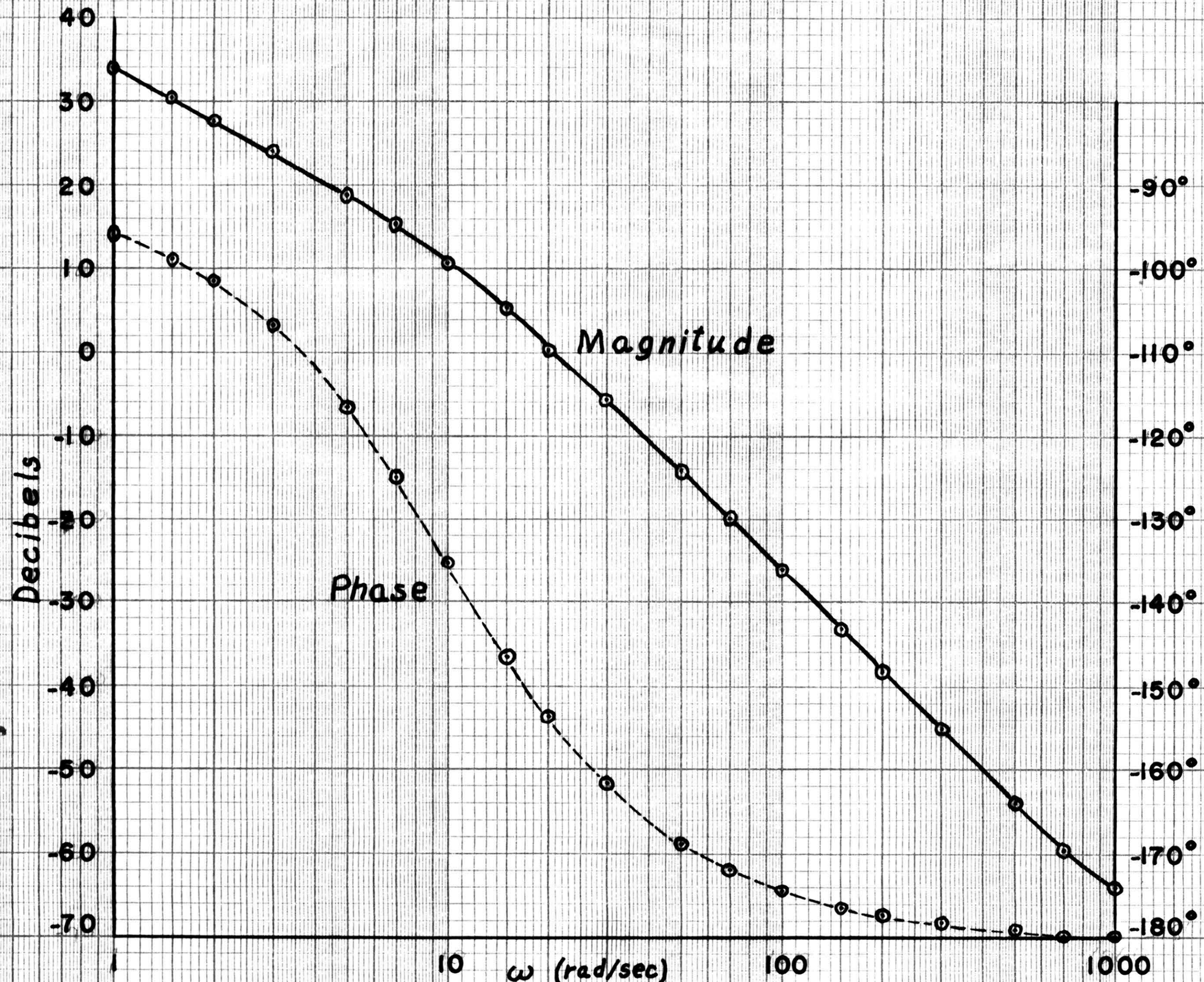

Figure 4. Frequency Response of Basic System

V. CONVENTIONAL PHASE-LEAD COMPENSATION WITH GAIN DECREASE

The conventional phase-lead compensation network is shown in Figure 5[10].

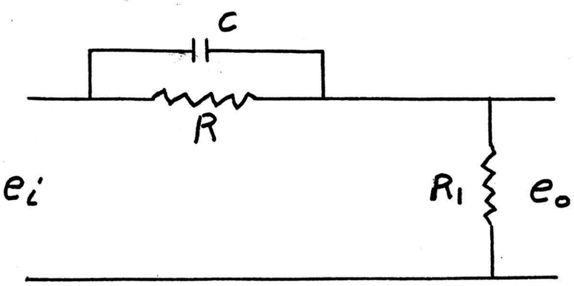

Figure 5. Phase-Lead Network

The transfer function of this network is

$$\frac{E_o}{E_1} = \left(\frac{R_1}{R+R_1}\right)\left(\frac{RCs+1}{\left(\frac{R_1}{R+R_1}\right)RCs+1}\right) \qquad (8)$$

---

(10) Brown, Gordon S., and Campbell, Donald P., Principles of Servomechanisms. N. Y. John Wiley Sons, 1948, pp. 219-222

The values of the components were selected so that

$$\frac{E_o}{E_1} = \frac{1}{2}\left(\frac{0.1s+1}{0.05s+1}\right) \qquad (9)$$

The gain of the servomechanism was reduced to 20 to make the peak overshoot less than 1.1. This reduction in gain could be obtained with a low-pass filter which would give the desired high frequency gain without reducing the low frequency gain. The low-pass filter is not considered in the analysis of this system.

The equation for the servo and network is

$$KG = \left(\frac{20}{s(0.1s+1)}\right)\frac{1}{2}\left(\frac{0.1s+1}{0.05s+1}\right) \qquad (10)$$

$$KG = \frac{10}{s(0.05s+1)} \qquad (11)$$

Figure 6 is the frequency response plot of equation 11. The phase margin is 66 degrees and the crossover frequency is 9 radians per second.

The transient equation is

$$\frac{\theta_o}{\theta_i} = 1 - 1.414e^{-10t}\sin(10t - 45°) \qquad (12)$$

Curve 2 of Figure 15 is a plot of equation 12. The peak overshoot is 1.03 and the rise time is 0.24 seconds.

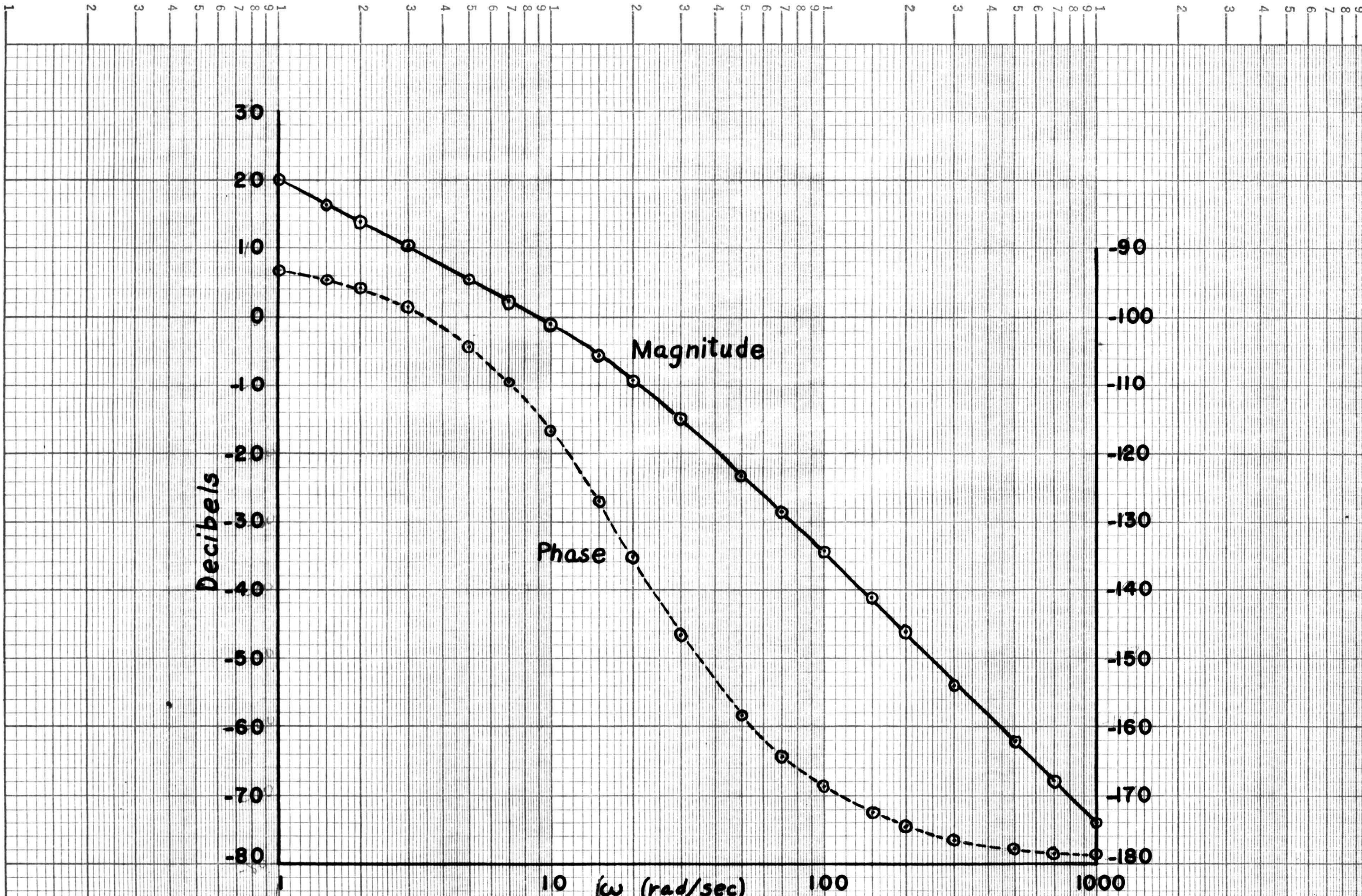

Figure 6. Frequency Response of Compensated System with Gain Decrease

VI. CONVENTIONAL PHASE-LEAD COMPENSATION WITH GAIN INCREASE

The network is of the same form as Figure 5. The network transfer function is

$$\frac{E_o}{E_i} = \frac{1}{10}\left(\frac{0.1s+1}{0.01s+1}\right) \qquad (13)$$

The gain of the servomechanism was increased to 500 to prevent the compensated system from being overdamped and for the peak overshoot to be less than 1.1.

The transfer function of the compensated system is

$$KG = \left(\frac{500}{s(0.1s+1)}\right)\frac{1}{10}\left(\frac{0.1s+1}{0.01s+1}\right) \qquad (14)$$

$$KG = \frac{50}{s(0.01s+1)} \qquad (15)$$

Figure 7 is the frequency response plot of equation 15. The phase margin is 65 degrees and the crossover frequency is 45 radians per second.

The transient equation is

$$\frac{\theta_o}{\theta_i} = 1 - 1.414e^{-50t}\sin(50t - 45°) \qquad (16)$$

Curve 3 Figure 15 is a plot of equation 16. It is noted that the peak overshoot is about 1.05 and the rise time 0.048 seconds.

This system has a low peak overshoot and a fast rise time which results in a very satisfactory response time. The disadvantage of this system is that the gain was increased.

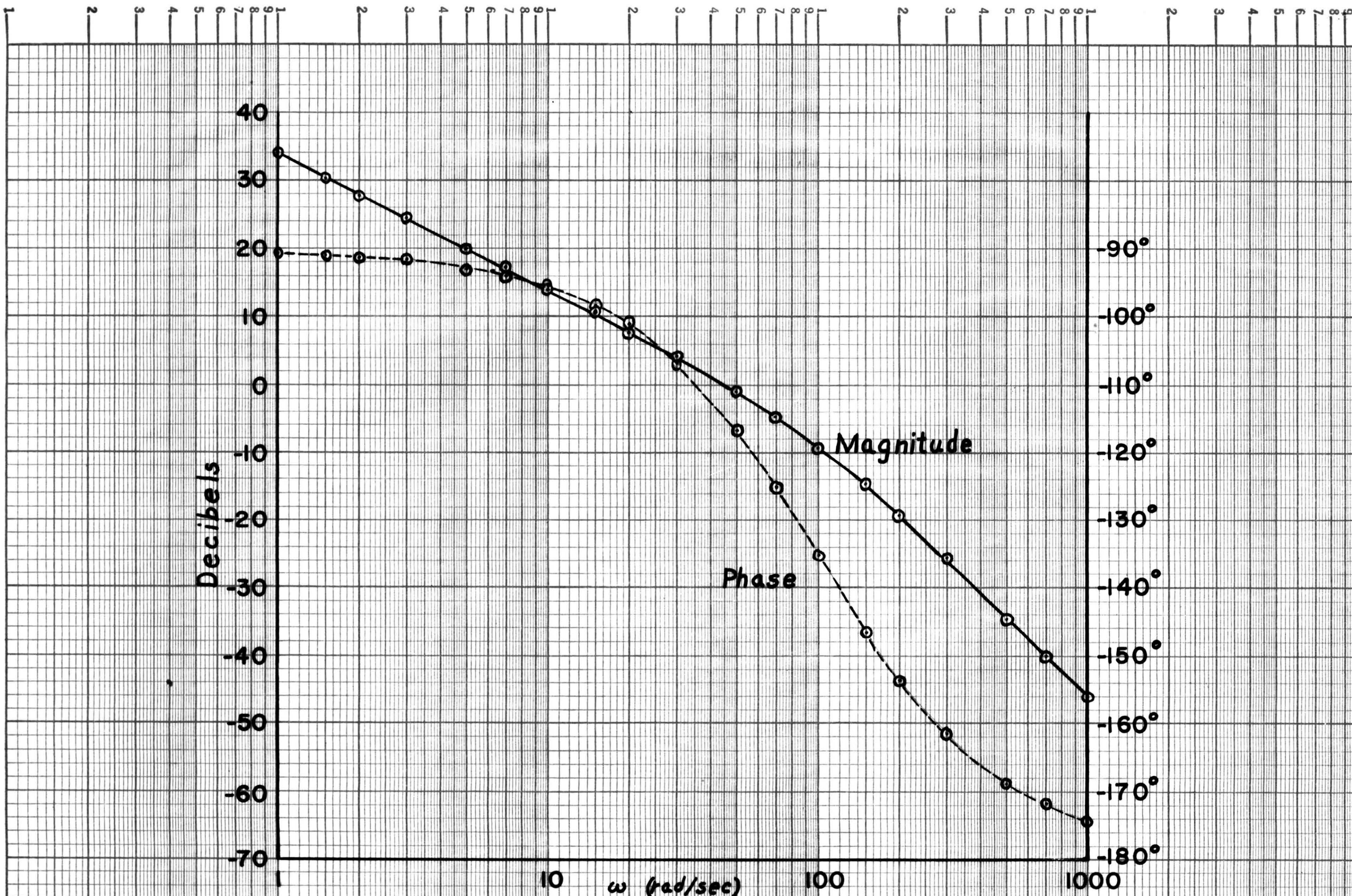

Figure 7. Frequency Response of Compensated System with Gain Increase

VII. ACTIVE PHASE-LEAD COMPENSATION NETWORK

It is desired that this compensation network provide a phase lead and a gain increase in the region of 30 radians per second without adversly affecting the low frequency gain of the system.

It is proposed that the passive part of the network be of the form shown in Figure 8.

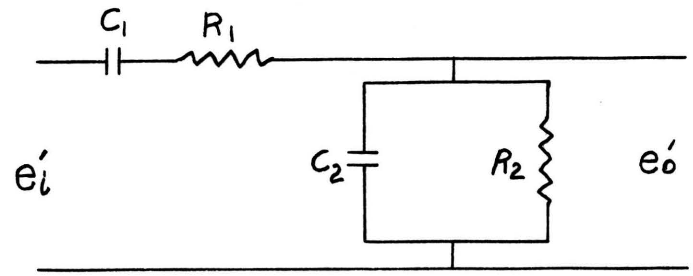

Figure 8. Feedback Network

The transfer function for the circuit of Figure 8 is

$$\frac{E_o'}{E_i'} = \frac{\dfrac{R_2}{R_2 C_2 S + 1}}{\dfrac{R_2}{R_2 C_2 S + 1} + \dfrac{R_1 C_1 S + 1}{C_1 S}} \quad \text{-----------------} \quad (17)$$

$$\frac{E_o'}{E_i} = \frac{R_2 C_1 S}{R_1 C_1 R_2 C_2 S^2 + (R_1 C_1 + R_2 C_2 + R_2 C_1) S + 1} \quad \text{--------------} \quad (18)$$

Let $R_1C_1 = T_1$

$R_2C_2 = T_2$

$R_2C_1 = T_3$

then

$$\frac{E_o'}{E_i'} = \frac{T_3 s}{T_1 T_2 s^2 + (T_1 + T_2 + T_3)s + 1} = G_N \qquad (19)$$

The network of Figure 8 will be placed in the feedback path of an amplifier as shown in Figure 9.

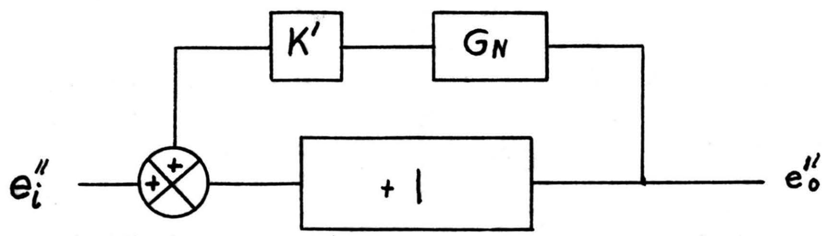

Figure 9. Active Feedback Compensation Network

The transfer function for the circuit of Figure 9 is

$$\frac{E_o''}{E_i''} = \frac{1}{1 - K' G_N} \qquad (20)$$

Substituting equation 19 into equation 20

$$\frac{E_o''}{E_i''} = \frac{1}{1 - \dfrac{K' T_3 s}{T_1 T_2 s^2 + (T_1 + T_2 + T_3)s + 1}} \qquad (21)$$

or

$$\frac{E_o''}{E_i''} = \frac{T_1 T_2 s^2 + (T_1 + T_2 + T_3) s + 1}{T_1 T_2 s^2 + (T_1 + T_2 + T_3 - K' T_3) s + 1} \qquad \text{-------------- (22)}$$

Substitute $j\omega$ into equation 22

$$\frac{E_o''}{E_i''} = \frac{(1 - \omega^2 T_1 T_2) + j\omega (T_1 + T_2 + T_3)}{(1 - \omega^2 T_1 T_2) + j\omega (T_1 + T_2 + T_3 - K' T_3)} \qquad \text{-------------- (23)}$$

It can be seen, from equation 23, that when K' is greater than zero the network will yield a phase lead and a gain increase. It is only left to determine the component values that will give the best transient response.

VIII. DESIGN OF ACTIVE COMPENSATION NETWORK

Due to the large number of components in the network and the long process necessary to determine the effect each component has on the transient response, the network was designed on the MSM Analog Computer.

In order to design the network on the analog computer the basic servo system and active network were simulated on the computer. The network components were General Radio decade resistors and condensers. The computer circuit is shown in Figure 10.

The design procedure consisted of observing the transient response on the cathode-ray oscilloscope, with the computer on repetitive operation, and adjusting R_1, C_1, R_2, and C_2 for the best transient response.

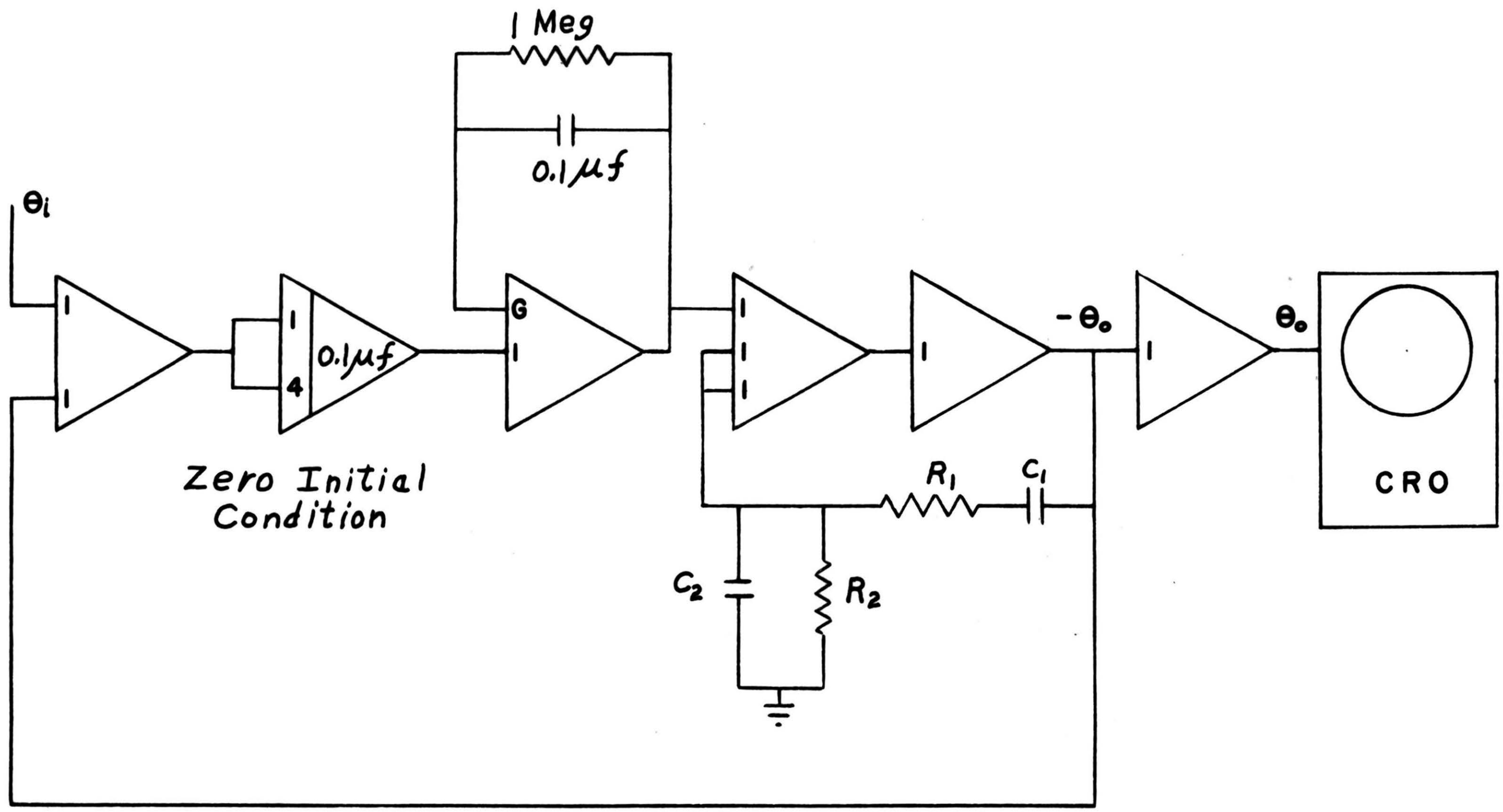

Figure 10. Analog Computer Simulation of Servo System and Active Compensation Network

IX. RESULTS FROM ANALOG COMPUTER

The active network components as determined from the analog computer are

R_1 = 4000 ohms	T_1 = 0.0016 sec.
C_1 = 0.4 micro farads	T_2 = 0.04 sec.
R_2 = 80,000 ohms	T_3 = 0.032 sec.
C_2 = 0.5 micro farads	K' = 2

The network transfer function is

$$\frac{E_o}{E_i} = \frac{T_1T_2s^2 + (T_1 + T_2 + T_3)s + 1}{T_1T_2s^2 + (T_1 + T_2 + T_3 - K'T_3)s + 1} \quad \text{----------- (22)}$$

$$\frac{E_o}{E_i} = \frac{0.64\text{x}10^{-4}s^2 + 7.36\text{x}10^{-2}s + 1}{0.64\text{x}10^{-4}s^2 + 0.96\text{x}10^{-2}s + 1} \quad \text{---------------- (23)}$$

Figure 11 is a frequency response plot of equation 24. This plot shows a phase lead and a gain increase in the region of 10 to 100 radians per second. This is the type of response desired.

Photographs of the computer results as presented on the cathode-ray oscilloscope are shown on page 24. Figure 12 is a photograph of the basic uncompensated system and Figure 13 is a photograph of the feedback compensated system. While there was no accurate time scale available on the oscilloscope, the two photographs were taken at the same sweep frequency and therefore may be compared with each other.

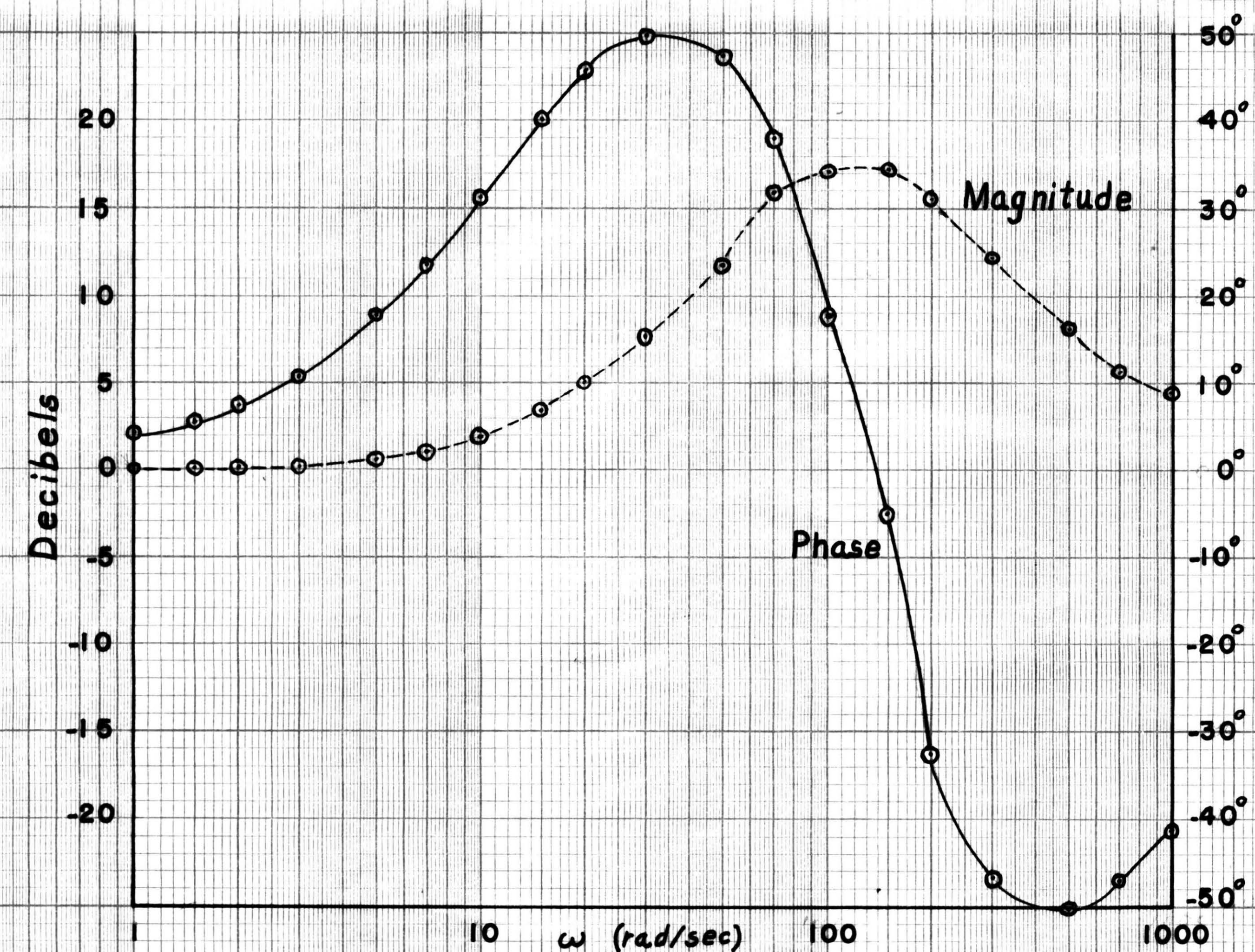

Figure II. Frequency Response of Active Compensation Network

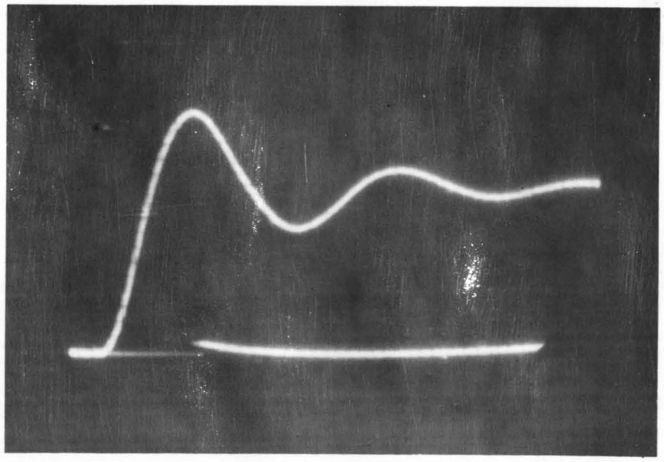

Figure 12. Transient Response of Basic System

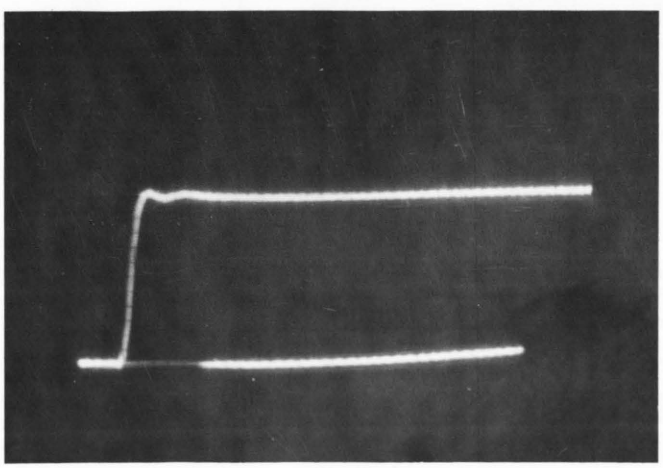

Figure 13. Transient Response of Compensated System

X. MATHEMATICAL ANALYSIS OF ACTIVE PHASE-LEAD NETWORK

The transient response of the feedback compensated system is analyzed here for a better comparison with the other methods of compensation.

The open loop compensated transfer function is

$$KG = \frac{50}{s(0.1s + 1)} \; \frac{0.64\times10^{-4}s^2 + 7.36\times10^{-2}s + 1}{0.64\times10^{-4}s^2 + 0.96\times10^{-2}s + 1} \quad \text{----- (25)}$$

The closed loop compensated transfer function is

$$\frac{\theta_o}{\theta_i} = \frac{500(s^2 + 1150s + 15{,}600)}{s^4 + 160s^3 + 17{,}600s^2 + 731{,}000s + 7.8\times10^6} \quad \text{-----(26)}$$

Factoring equation 26 gives

$$\frac{\theta_o}{\theta_i} = \frac{500(s+1136)(s+14)}{(s+49.6+j91)(s+49.6-j91)(s+44.7)(s+16.1)} \quad \text{----- (27)}$$

For a step displacement input

$$\frac{\theta_o}{\theta_i} = \frac{K_1}{S} + \frac{K_2}{S+49.6+j91} + \frac{K_3}{S+49.6-j91} + \frac{K_4}{S+44.7} + \frac{K_5}{S+16.1} \quad \text{---- (28)}$$

Taking the inverse laplacian

$$\frac{\theta_o}{\theta_i} = K_1 - 2|K_2|e^{-49.6t}\sin\left[91t - (90^\circ + \angle K_2)\right] + K_4e^{-44.7t} + K_5e^{-16.1t} \quad \text{-- (29)}$$

K_1, K_2, K_3, K_4, and K_5 are determined by partial fractions expansion.

$$\frac{\theta_o}{\theta_i} = 1 - 0.64 e^{-49.6t} \sin(91t - 25.7^\circ) - 1.56 e^{-44.7t} + 0.269 e^{-16.1t} \qquad \text{-----} \quad (30)$$

The term of equation 30, which will be effective for the longest time is the last term. This term has the smallest exponent and causes the overshoot to last for a time longer than would be anticipated from the frequency of the periodic term.

Figure 14 is a plot of equation 25, the open loop transfer function of the compensated system. The phase margin is 66 degrees and the crossover frequency is 37 radians per second.

Curve 4 Figure 15 is a plot of equation 30, the transient response of the compensated system. The rise time is 0.056 seconds and the peak overshoot is 1.03.

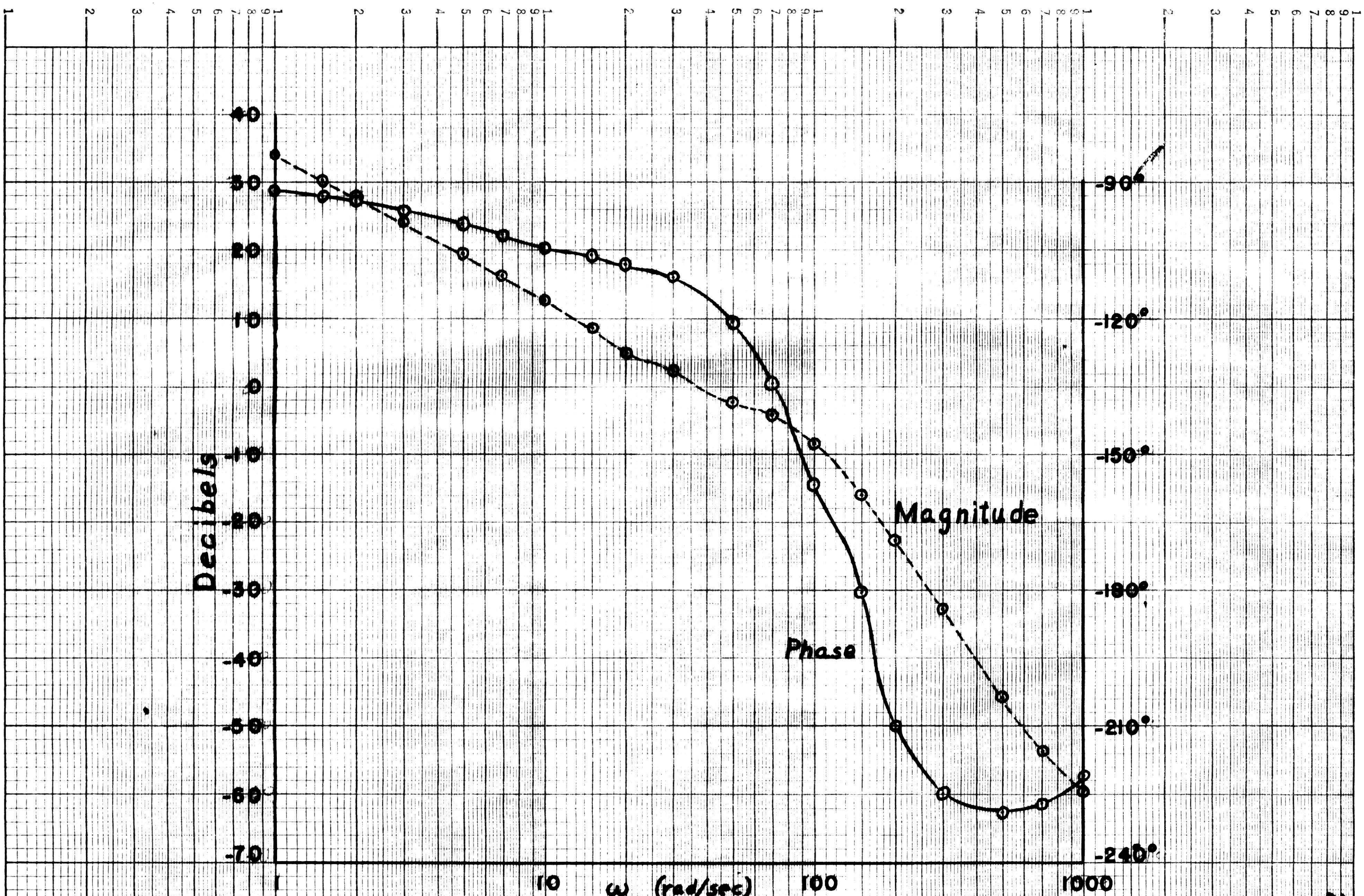

Figure 14. Frequency Response of Active Compensated System

XI. COMPARISON OF COMPENSATED SYSTEMS

The transient response of four servo systems are presented on page 29. The four systems are:

1. The basic uncompensated system.
2. The basic system compensated by a passive phase-lead network with a gain decrease.
3. The basic system compensated by a passive phase-lead network with a gain increase.
4. The basic system compensated by an active phase-lead network with no gain change.

From the transient response curves it is seen that systems 3 and 4 have similar transient characteristics. System 2 has about the same peak overshoot as systems 3 and 4 but the rise time is much longer.

While systems 3 and 4 have similar transient responses, system 3 must have a higher value of electronic gain. This is because the passive phase-lead network attenuates the low-frequencies and it is necessary to increase the electronic gain in order for the crossover frequency to remain high enough to give a fast rise time. The electronic gain was not increased on system 2 and while the phase margin of systems 2, 3, and 4 are about the same the crossover frequency of system 2 is lower than the other systems. This is why system 2 has the longest rise time.

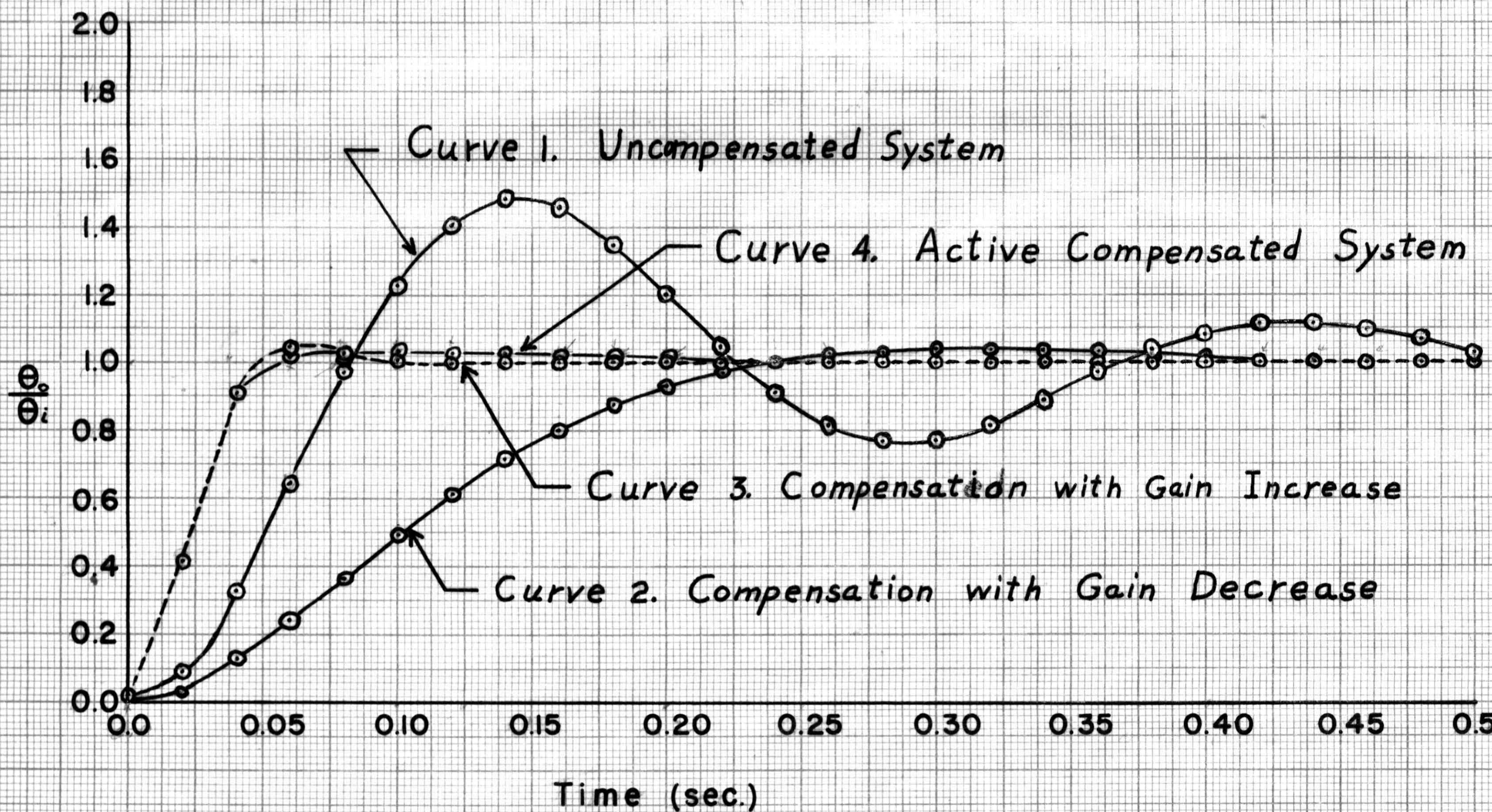

Figure 15. Transient Responses of Uncompensated and Compensated Systems

XII. METHOD OF INSERTING THE ACTIVE PHASE-LEAD NETWORK INTO A SERVOMECHANISM

High performance servomechanisms normally use carrier amplifiers for the electronic gain and a d-c motor for the output device. Since it is necessary to drive the d-c motor with a d-c amplifier, the carrier signal must be changed to a d-c type of signal by the use of a circuit such as a phase detector.

Figure 16 shows the proposed method of inserting the active network in a servo system. The network is placed between the power amplifier and the phase detector and uses the filter circuit of the phase detector as part of the compensation network. The great advantage of this method is that it is possible to modify an existing equipment without major circuit changes.

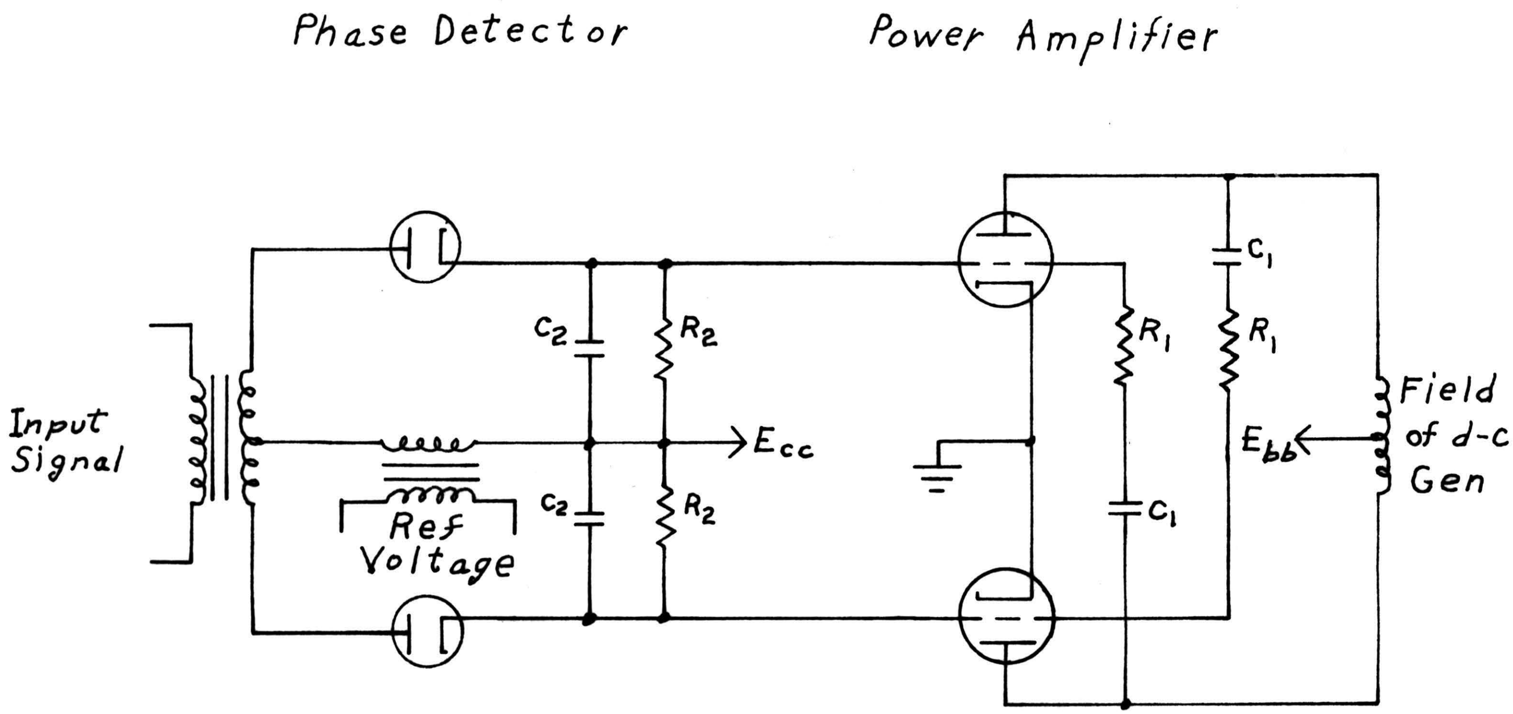

Figure 16. Active Network Inserted into a Servomechanism

XIII. CONCLUSIONS AND SUMMARY

The importance of a phase-lead compensation network which does not affect the low-frequency gain of the system lies in the velocity-lag error. The velocity-lag error is determined by the zero frequency gain, the higher the gain the lower the velocity-lag error. The electronic gain of high performance servo systems is set as high as possible without exceeding signal to noise ratio requirements. If it is possible to increase the gain and use a passive phase-lead network then it is possible to use the active phase-lead network at the increased gain and have the same transient response with a lower velocity-lag error.

Compensation with conventional passive phase-lead networks results in a compromise between steady state and transient characteristics, for high performance servomechanisms. This compromise occurs because the passive phase-lead network attenuates low-frequencies.

The active phase-lead network, presented in this thesis, has no effect on low-frequencies. Therefore the steady state and transient characteristics may be determined independently within the limitations of the basic system components.

If the steady state and transient requirements are not high, the active-phase lead network has no advantage over the passive network.

BIBLIOGRAPHY

Biernson, George J., A General Technique for Approximating Transient Response from Frequency Response Asymptotes. Transactions of the Institute of Electrical Engineers. Vol. 75, Part 11, pp. 253-273 (1956)

Brown, Gordon S., and Campbell, Donald F., Principles of Servomechanisms. N. Y., John Wiley Sons, 1948, pp. 219-222.

Bruns, Robert A., Analogue Computers for Feedback Control Systems. Transactions of the Institute of Electrical Engineers. Vol. 71, Part 11, pp. 250-254 (1952)

Harris, Herbert, A Comparison of Two Basic Servomechanism Types. Transactions of the Institute of Electrical Engineers. Vol. 66, pp. 83-93 (1947)

Johnson, Clarence L., Analog Computer Techniques. N. Y., McGraw-Hill, 1956, pp. 45-64.

Lewis, J. B., The Use of Nonlinear Feedback to Improve the Transient Response of a Servomechanism. Transactions of the Institute of Electrical Engineers. Vol. 71, Part 11, pp. 449-453 (1952)

Schwartz, George J., The Application of Lead Networks and Sinusoidal Analysis to Automatic Control Systems. Transactions of the Institute of Electrical Engineers. Vol. 66, pp. 66-77 (1947)

Thaler, George J., Elements of Servomechanisms Theory. N. Y., McGraw-Hill, 1955, pp. 67-74.

Thomson, William T., Laplace Transformation. N. Y., Prentice-Hall, 1950, pp.71-75.

VITA

The author was born on December 29, 1928, in Summersville, Missouri.

He attended public school at Summersville, Missouri between the years 1934 and 1945. In September, 1945, he enrolled in the Missouri School of Mines and Metallurgy.

On July 28, 1948, the author entered the United States Air Force as a private and served as an instructor in the radar school at Keesler Field, Mississippi. He was discharged from the Air Force on July 28, 1952.

In August, 1952, the author entered the employ of the Radio Corporation of America Service Company as an instructor of radar and radio and remained in this position until February, 1954.

In February, 1954, the author enrolled in the Missouri School of Mines and Metallurgy and received the degree of Bachelor of Science in Electrical Engineering in May, 1955. In September, 1955, he accepted a position as Instructor of Electrical Engineering at the Missouri School of Mines and Metallurgy while completing the requirements for the Master of Science Degree in Electrical Engineering.