

# The Richardson's Extrapolation Graphical User Interface Excel Spreadsheet Solver

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**Abstract**— In this paper, the fourth version of Richardson's extrapolation Excel spreadsheet calculator for computing differentiations numerically has been upgraded. In this version, a graphical user interface (GUI) form was developed to capture users' inputs so that users will not confuse the input and output in Excel spreadsheet. It makes this version of GUI Excel spreadsheet solver more user friendly, attractive and interactive. A summative evaluation of this Richardson's extrapolation GUI Excel spreadsheet solver has been conducted by involving 20 postgraduate students by using questionnaire. The findings showed that the majority of the students agreed that the Richardson's extrapolation graphical user interface (GUI) spreadsheet solver provides an interesting and interactive learning environment.

**Index Terms**— Richardson's extrapolation; Graphical user interface; Excel; Spreadsheet; Differentiation; Numerical.

## I. INTRODUCTION

In mathematics, differentiation means rate of change and normally is denoted by  $dy/dx$ . If the graph of  $y$  is plotted versus  $x$ , then  $dy/dx$  in this case represents the rate of change of  $y$  with respect to  $x$  which implies the slope of the tangent line to the curve  $y$ . Other real life applications of differentiation, for example, velocity is the rate of change of distance with respect to time, acceleration is the rate of change of velocity with respect to time, current is the rate of change of charge with respect to time and etc.

Numerical differentiation is needed if exact differentiation is impossible to obtain or only numerical data is available. In such situations, an approximation of the definite differentiation is important to provide a practical solution. Richardson's extrapolation [1-2], which is one of the most powerful approaches in numerical differentiation, improves the rate of convergence of a sequence in numerical differentiation.

Implementation of Richardson's extrapolation via mathematical software or any computational tool leads to a desired definite differentiation. According to the works of [3-14], Excel spreadsheet shows the calculation ability for a wide range of numerical methods, such as nonlinear equations [3], system of linear equations [4], interpolation [5-7], numerical differentiation GUI Excel spreadsheet solver by finite-difference approximation, [8], Romberg integral GUI Excel spreadsheet solver [9], eigenvalues problems [10-11], ordinary differentiation equations (ODEs) [12], system of ODEs [13-

14], and Laplace equation [15].

In addition, the application of Excel spreadsheet in implementing Richardson's extrapolation for numerical differentiation has been developed in [16-19] recently. However, the Richardson's extrapolation spreadsheet calculator [16] is limited to level four in calculating numerical differentiation and the initial value  $D(0, 0)$  of approximation to the derivative of a function is required to be keyed in by users using 3-point central difference formula. The second version of Richardson's extrapolation spreadsheet calculator [17] overcomes the previous two limitations, in which users only need to enter  $x$ ,  $f(x)$  and  $h$  instead of the initial value  $D(0, 0)$  and it is able to calculate up to level 10. The third version of Richardson's extrapolation spreadsheet calculator [18] further improves the flexibility of the previous two versions up to any level using VBA programming. The fourth version [19] of this spreadsheet calculator improves the drawback in the third version that users need to enter the function  $f(x)$  by using Excel command rather than programming syntax or mathematical form via one customized written function in VBA programming. However, there is no user input form to separate input and output. Hence, in this Richardson extrapolation GUI Excel spreadsheet solver, a graphical user friendly and interactive user input form is developed to capture users' inputs so that users will not confuse about the input and output in Excel spreadsheet. This GUI Excel spreadsheet solver is targeted to the undergraduate students and lecturers who take and teach numerical methods respectively.

## II. RICHARDSON'S EXTRAPOLATION

There are two ways to improve derivative estimates when employing finite divided differences; decreasing the step size or using a higher-order formula that employs more points. A third approach, based on Richardson's extrapolation which uses two derivatives, estimates a more accurate approximation. [20].

For a given function of  $f$ , approximations  $D$ , to the derivative of  $f$  at a specified value of  $x$ , can be computed for a chosen value of  $h$ , using Richardson's extrapolation method as follows: [21]

$$D(J, K) = \frac{4^K D(J, K-1) - D(J-1, K-1)}{4^K - 1} + O(h^{2K+2}),$$

$$J = 1, 2, \dots, \quad K = 1, 2, \dots \quad (1)$$

$$D(J, 0) = \frac{f(x+h_j) - f(x-h_j)}{2h_j}, \quad J = 0, 1, 2, \dots \quad (2)$$

$$h_0 = h, h_1 = \frac{h}{2}, h_2 = \frac{h}{4}, \dots, h_j = \frac{h}{2^j} \quad (3)$$

The iteration process (1) is repeated until  $|D(J, J) - D(J, J-1)| < \epsilon$  or  $|D(J, J) - D(J-1, J-1)| < \epsilon$  for a specified value of  $\epsilon$ . The value of  $D(J, J)$  then approximates  $f'(x)$  at this level.

Richardson's extrapolation table for  $K$  from 1 to 4 is shown in Table 1.

Table 1  
Richardson's Extrapolation table for level  $K$  values from 1 to 4

J	$h_j$	K				
		0	1	2	3	4
		$D(J,0)$	$D(J,1)$	$D(J,2)$	$D(J,3)$	$D(J,4)$
0	$h_0$	$D(0,0)$				
1	$h_1$	$D(1,0)$	$D(1,1)$			
2	$h_2$	$D(2,0)$	$D(2,1)$	$D(2,2)$		
3	$h_3$	$D(3,0)$	$D(3,1)$	$D(3,2)$	$D(3,3)$	
4	$h_4$	$D(4,0)$	$D(4,1)$	$D(4,2)$	$D(4,3)$	$D(4,4)$

### III. NUMERICAL EXAMPLE

In this section, a numerical example to be solved by Richardson's extrapolation is provided.

#### Question

The charge of a capacitor is given by  $q = e^{-t} \sin t$ , approximate current flows through the capacitor when time  $t = 1$  using Richardson extrapolation with  $h = 0.2$ . Stop if

$$|D_{j,j} - D_{j-1,j-1}| < 0.00005 \quad \text{or} \quad |D_{j,j} - D_{j,j-1}| < 0.00005.$$

Determine the absolute error.

#### Solution

Let  $f(t)$  be the charge of a capacitor function,  $q(t)$ , then

$$f(t) = q(t) = e^{-t} \sin t.$$

Hence, current function is given by  $i(t) = f'(t) = q'(t)$  can be approximated using Richardson's extrapolation at  $t = 1$  with  $h = 0.2$ .

The numerical solution of this example is shown in Figure 2, where  $i(1) = f'(1) = q'(1)$  is approximately -0.11080, while the absolute error at the bottom portion of Figure 2.

### IV. THE RICHARDSON'S EXTRAPOLATION GUI EXCEL SPREADSHEET SOLVER

Figure 1 illustrates the main menu of the Richardson's extrapolation GUI Excel spreadsheet solver. The top portion contains the *Input* button. Once users click the *Input* button, it will open a user input form as given in Figure 2. The bottom portion is the full Richardson's extrapolation table. The user input form let users input the variable used in the numerical differentiation, the function to be differentiated, point of  $x$  to be differentiated and a step size  $h$ . Users can select the desired calculation accuracy of one decimal place, two decimal places, up to nine decimal places calculation from the drop down menu in user input form. For example if the desired calculation accuracy is five decimal places, then the calculation will be stopped if  $|D(J, J) - D(J-1, J-1)| < \epsilon$  or  $|D(J, J) - D(J, J-1)| < \epsilon$ . The value of epsilon is shown corresponding with the desired chosen accuracy. If exact differentiated function is known, users can input it, otherwise just leave it blank. Once the *Compute* button is clicked, the solution of the numerical differentiation via Richardson's extrapolation will be shown at the right top corner of the user input form as given in Figure 1. The full solution will be shown in the bottom portion of the main menu as given in Figure 1. Exact differentiated value and absolute error will be displayed at the left bottom corner of the user input form if the exact differentiated function is inputted. The *Clear* button will clear all the users' input, whereas the *Close* button will close the user input form.

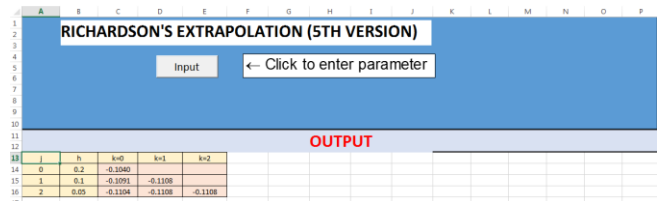


Figure 1: Main menu of the Richardson's extrapolation GUI Excel spreadsheet solver

If the variable defined and the function is not tally, the error message will be popped up as given in Figure 3.

If the value of  $x$  to be differentiated is left to be blank, then another error message will be prompted as given in Figure 4.

Lastly, if the step size  $h$  is not provided, but the *Compute* button is clicked, then the third error message will be displayed as given in Figure 5.

### V. THE EVALUATION OF RICHARDSON'S EXTRAPOLATION GUI EXCEL SPREADSHEET SOLVER

We have tested this Richardson's extrapolation GUI Excel spreadsheet solver on 20 Master of Electrical Engineering students who took the Advanced Engineering Mathematics (AEM) course in Universiti Tun Hussein Onn Malaysia in semester 1 2015/2016. One of the sub topics in the AEM course is Richardson's Extrapolation which is taught in week 5.

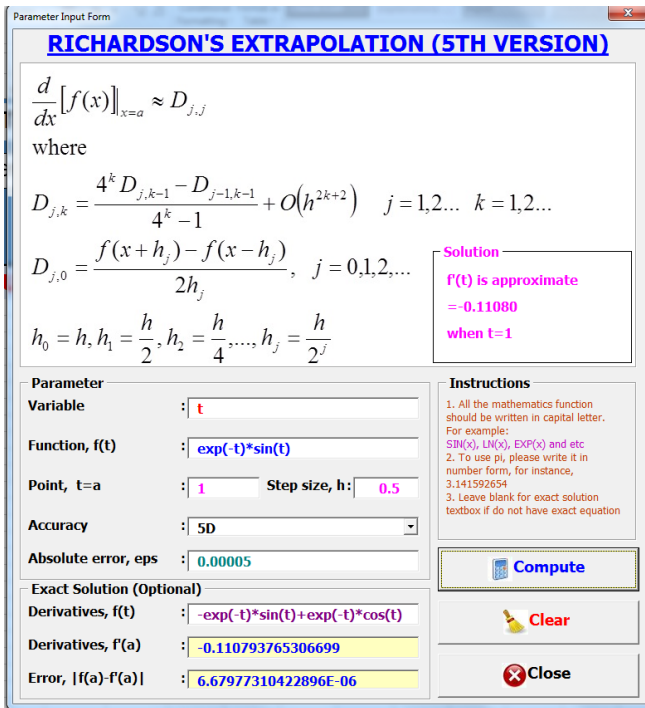


Figure 2: Solution using the Richardson's extrapolation GUI Excel spreadsheet solver

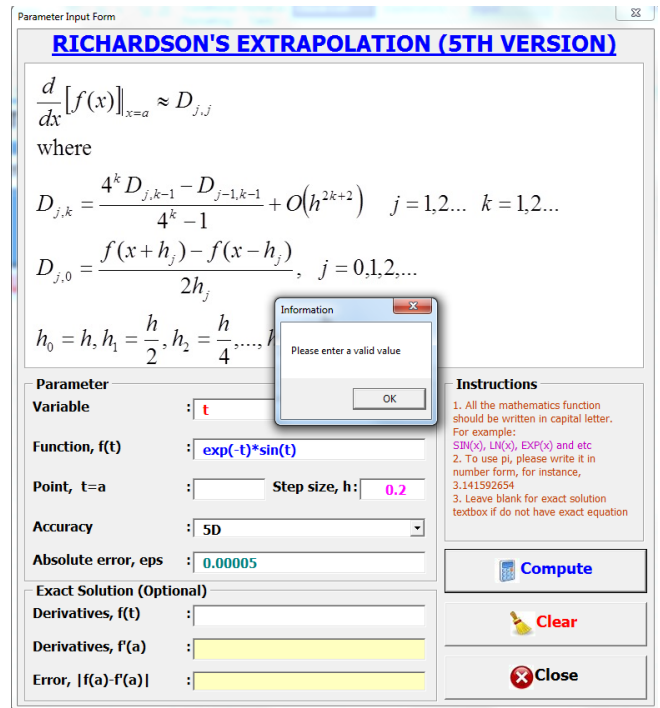


Figure 4: Error if point of x to be differentiated is not given in the Richardson's extrapolation GUI Excel spreadsheet solver

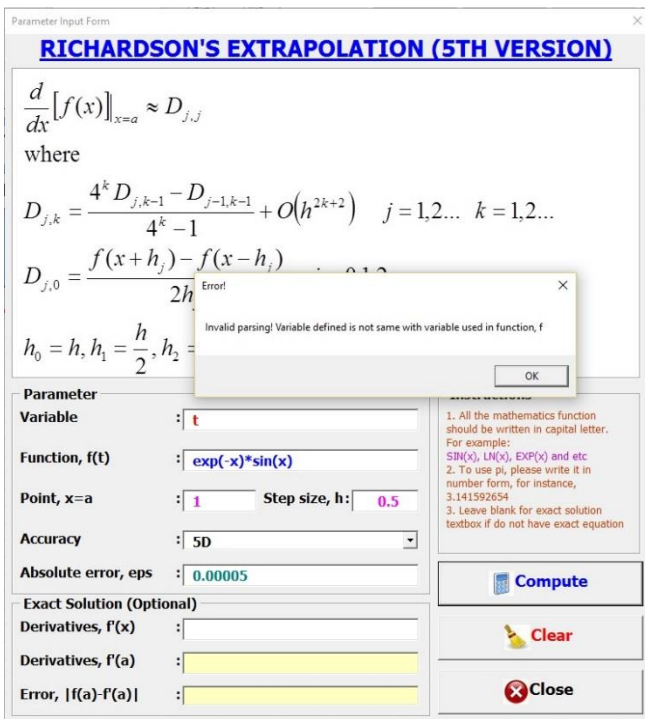


Figure 3: Error if variable and function not tally in the Richardson's extrapolation GUI Excel spreadsheet solver

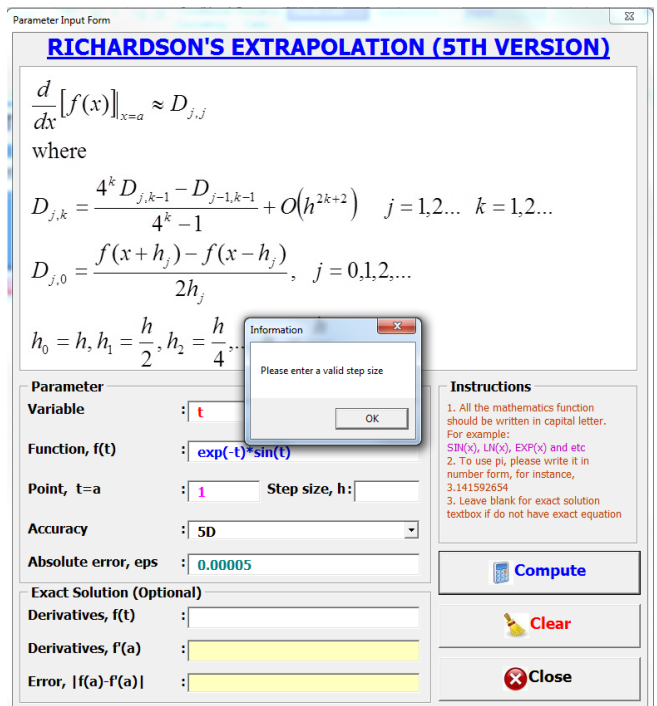


Figure 5: Error if step size h is not provided in the Richardson's extrapolation GUI Excel spreadsheet solver

Initially, the students have been taught on how to approximate numerical differentiation by Richardson's extrapolation traditionally using calculators. After that, they were taught how to solve approximate numerical differentiation by Richardson's extrapolation in standard Excel spreadsheet environment. Lastly, the Richardson's extrapolation GUI Excel spreadsheet solver was given to them

and they were exposed on its usage. Students tested the same differentiation for the third time using this GUI Excel spreadsheet solver. The Richardson's extrapolation GUI Excel spreadsheet solver and questionnaires were later given to them. They were asked to test the GUI Excel spreadsheet solver for other differentiations. The questionnaires were collected in week 14.

There are four aspects of evaluation being conducted for this GUI Excel spreadsheet solver, namely content, design & presentation, teaching strategy design, and technical aspect using a questionnaire that consisted of 24 items. A 5-point Likert scale was used to rate the response of the participants towards the Richardson's extrapolation GUI Excel spreadsheet solver. The participants were asked to answer each question by rating each item using the following scale: 1 = strongly disagree; 2 = disagree; 3 = neutral; 4 = agree and 5 = strongly agree.

The findings of the Richardson's extrapolation GUI Excel spreadsheet solver are shown in Table 2. Findings indicated that the majority of the students agreed that the Richardson's extrapolation GUI Excel spreadsheet solver is useful in terms of content (M=4.60, SD=0.35), design & presentation (M=4.41, SD=0.44), teaching strategy design (M=4.41, SD=0.56) and technical aspect (M=4.34, SD=0.62). According to the results shown in mode, the majority of the participants agreed that this Richardson's extrapolation GUI Excel spreadsheet solver is good when it comes to technical aspect and teaching strategy design, followed by content. This can be proven by the statement given by the participants during the interview. Some of the participants mentioned that "*It allows me to understand Numerical Methods easily*", "*This GUI Excel spreadsheet solver is a very good tool for engineering students to do numerical differentiation in engineering problems and it saves a lot of students' time if to perform manual calculations due to long calculations for some engineering problems*" and "*It is a good tool for students to verify their manual calculation. Besides, it is one of the easiest and interesting tool for students to learn effectively.*" However, they commented "*If the function to be differentiated is complicated, need a proper bracket as in programming syntax in order to get the correct answer because Excel can't display naturally as in mathematics.*"

Table 2  
Descriptive Analyses of Richardson's extrapolation GUI Excel spreadsheet solver Evaluation

Descriptive Statistics	Content	Design & Presentation	Teaching Strategy Design	Technical Aspect
Mean	4.60	4.41	4.41	4.34
Std. Deviation	0.35	0.44	0.56	0.62
Mode	4.67	4.33	5	5

## VI. CONCLUSION

A Richardson's extrapolation GUI Excel spreadsheet solver has been developed to approximate the derivative of the given function at a chosen point with a step size  $h$ . The new feature in this version is a graphical user friendly input form which was developed to capture users input so that users will not confuse about the input and output in Excel spreadsheet. Besides, it is more interactive because users will be prompted by error messages if the variable defined is not tally with the variable in function, the value to be differentiated is not specified or the step size  $h$  is not provided. Once the Compute button is clicked, the VBA programming that calculates Richardson's extrapolation values will be displayed in the

right top corner of the user input form while full solution will be displayed in Excel spreadsheet at bottom part of main menu. The Richardson's extrapolation GUI Excel spreadsheet solver provides an alternative tool for approximating the numerical differentiation by Richardson's extrapolation. It can be used as a marking scheme for educators and students who need the full solutions. Moreover, it reduces calculation time and is hoped to improve students' learning ability.

## ACKNOWLEDGMENT

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