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Shop level maintenance of inertial platforms without a surveyed site

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SHOP LEVEL MAINTENANCE

OF INERTIAL PLATFORMS

WITHOUT A SURVEYED SITE

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ABSTRACT

The initial installation of a rotary tilt table for an aircraft inertial navigation system test facility includes accurately aligning the rotary tilt table to local level and true north coordinates. The survey techniques presently employed are an encumbrance during installation and complicate remote deployment. The objective of this thesis is to determine if an inertial platform or platforms can practically be used to replace the role of the survey techniques in the rotary tilt table alignment.

The construction, capability and operation of a rotary tilt table are reviewed. The tests necessary to test and calibrate an inertial platform are outlined, including those requiring an accurately aligned rotary tilt table.

The basic principles of inertial platform self-alignment are stated so a determination of the ideal behavior of the gyrocompass and level servo loops, in the absence of error, can be later used for error models. Typical gaussian error sources, representative of practical inertial components are injected and assessed to determine the standard deviation of the steady state gyrocompass and level servo loop errors for a system. System response times are selected, such that a specific mechanization may be evaluated for specific values of steady state error.

Test equipment and procedures are presented that outline validation measures to be taken to ascertain that system errors are within acceptable limits. The cumulative alignment and readout errors are evaluated to define the rotary tilt table alignment accuracy achievable with one inertial platform The accuracy is improved by utilizing multiple inertial platforms. This rotary tilt table alignment is then compared to the test requirements outlined initially.

The findings are summarized and it is concluded that it is practical to use an inertial system (platform) to perform azimuth alignment of a rotary tilt table and, that leveling is best accomplished using precision spirit levels.

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I.

INTRODUCTION

A rotary tilt table is used as part of a repair and test facility for a self aligning aircraft inertial navigation system because of its ability to accurately tilt and rotate the inertial platform to various positions to evaluate its performance. The rotary tilt table very accurately defines a set of level and azimuth coordinates relative to its mounting surface and, when leveled and aligned in azimuth to true north, provides an east/north/vertical coordinate system.

The initial installation of a repair and test facility includes accurately aligning the rotary tilt table to local level and true north coordinates. This task presently employs survey techniques which require considerable time and involve special equipment and personnel not normally associated with installation of an electronic test facility.

Military users desire a portable test facility, installed in a van or trailer, to facilitate rapid deployment to a new site. One of the goals for this facility is that it be ready for use within a day after arrival. The employment of survey techniques causes the time goal to be exceeded and requires a survey crew which does not normally accompany the deployed personnel.

The installation alignment accuracy required for a particular rotary tilt table is dependent upon the accuracy of the inertial system to be tested. The rotary tilt table should be aligned ten times more accurately than the inertial system test tolerance requirements, if its error contribution is to be neglected. The time consuming survey techniques are imposed because of the high accuracy required for this alignment. As an example, the installation instructions for the AN/ASN-63

Inertial Navigation System Test Facility specify that the rotary tilt table be aligned to within ± 30 arc seconds in both level and azimuth.

Leveling the tilt table is not particularly complicated since spirit levels (such as the Watts TB-19 Angle Level Gage) are available with sufficient accuracy for this requirement. The spirit level may be placed on the adapter plate (see Figure 1) and the leveling screws on the leveling plate adjusted until the spirit level indicates a level condition for all azimuth angles.

True north alignment, however, requires the survey crew, normal survey equipment, and special autocollimation equipment not commercially available. The survey crew can use their normal equipment and techniques to establish a primary true north base line. The equipment and particular procedure they use is selected according to the accuracy specified. This primary true north base line is then transferred to other locations by constructing first, second and third order reference lines, each of which is perpendicular to the last.

Special equipment is required for the survey crew to align the rotary tilt table zero degree azimuth line to the surveyed true north reference line. It is necessary to very accurately fabricate an optical alignment fixture, as shown in Figure 2, and to utilize mirrors, as shown in Figure 3, to effect alignment of the entire assembly of Figure 1 by autocollimation. This involves rotating the adapter plate relative to the zero azimuth of the rotary tilt table until the mirror image of the cross-hairs in the surveyor's theodolite coincides with the actual cross-hairs, as shown in Figure 3. The tilt table is then checked in other attitudes and readjusted until the attitude and azimuth angles trace the same autocollimation line.

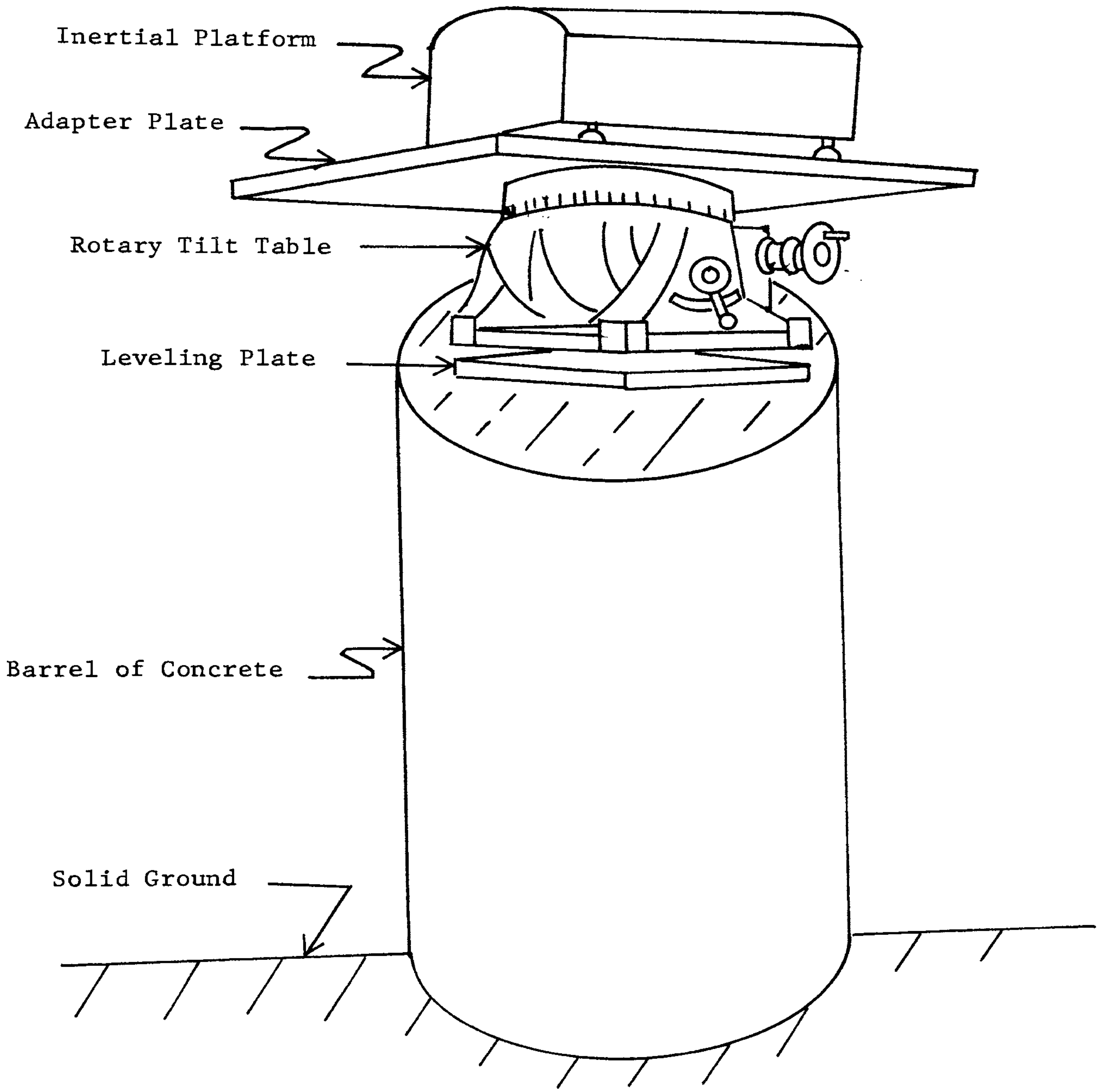


Figure 1 - Rotary Tilt Table Installation

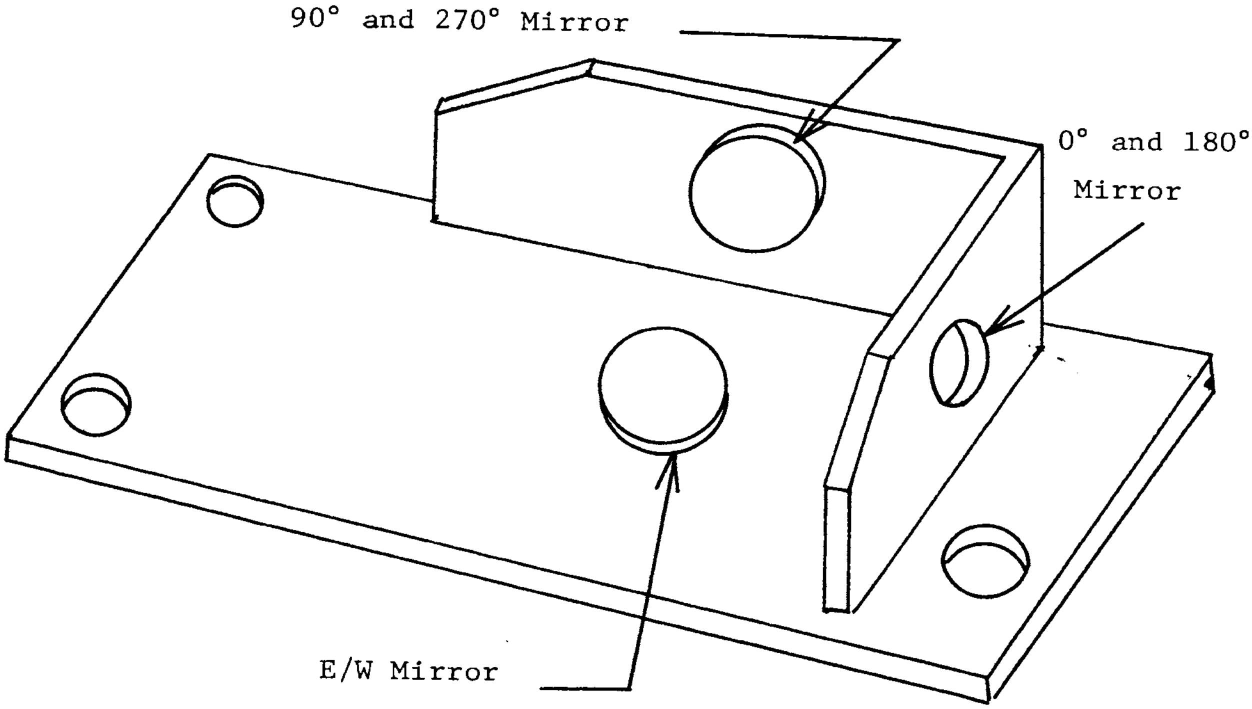


Figure 2- Optical Alignment Fixture

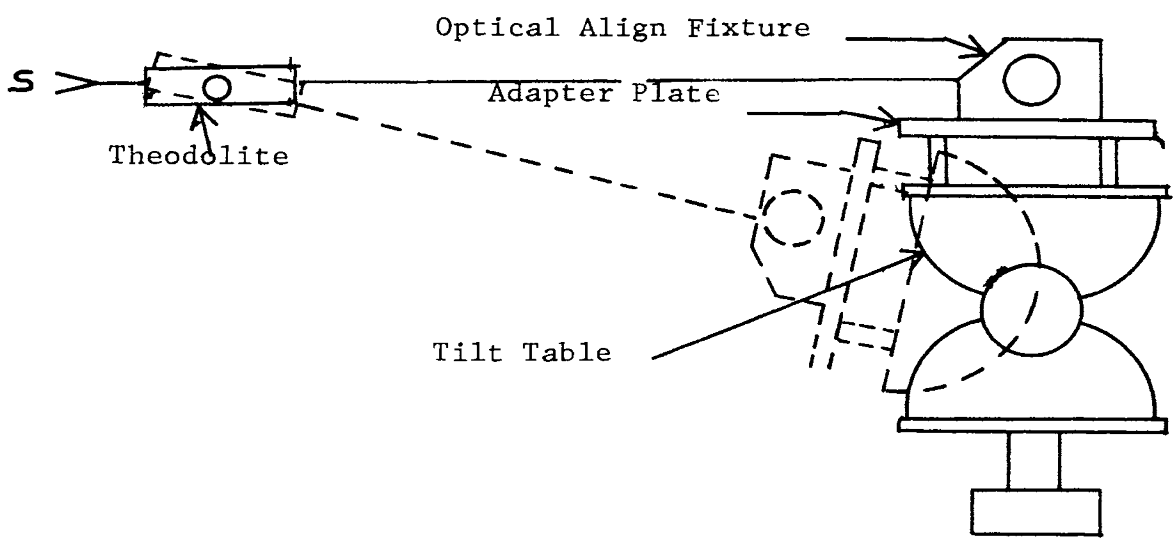


Figure 3 - Autocollimation Alignment

The objective of this thesis is to determine if an inertial platform can practically be used to replace the role of the survey techniques in the rotary tilt table alignment. This is to be accomplished by using the inertial platform's self alignment and leveling capability.

Chapter III defines the rotary tilt table capabilities and outlines the required inertial system tests which require comparison with the tilt table angular position indication.

Chapter IV outlines the basics of the leveling and gyrocompass servo loops used in self alignment to determine their ideal behavior in the absence of error.

Chapter V investigates the level and gyrocompass accuracy of an inertial system whose inertial components are operating within their specification tolerances. Typical error sources are analyzed in the level and gyrocompass loops to determine the system's resultant steady state level and gyrocompass accuracy.

The rotary tilt table alignment method is outlined in Chapter VI. This chapter first discusses the procedures and equipment that may be used without an accurate established coordinate system to ascertain whether the major inertial platform errors are within specified tolerances, thus guaranteeing accuracy of a particular inertial system. Second, the alignment method utilizing an inertial platform and the resulting alignment accuracy is discussed. Third, the use of multiple inertial platforms to improve the tilt table alignment accuracy is investigated.

Chapter VII investigates the utility of the rotary tilt table alignment method. First the rotary tilt table alignment accuracy is compared with the accuracy requirements for inertial platform tests.

Then it is evaluated on the basis of how well it identifies inertial platforms which are in or out of specification.

Chapter VIII summarizes the comparison of the resulting accuracy of the rotary tilt table alignment to the inertial platform testing requirements. Conclusions are drawn as to whether or not a coordinate frame defined by an inertial system or systems is a practical replacement for the present spirit level and survey techniques.

II.

REVIEW OF RELATED LITERATURE

There is considerable published literature available in the area of inertial platform design, mechanization and analysis, which is closely related to, and essential to the analysis of this thesis topic. The open literature surveyed concentrates primarily on the design and analysis of inertial guidance systems, which establishes the basis for test facility accuracy requirements. Some common alignment and test requirements are briefly described by bibliographic reference item (4), but no reference is made to related test facility requirements.

Bibliographic reference items (1) through (5) and (7) contain extensive discussion on the mechanization of leveling and gyrocompass servo loops. The effects of errors on leveling and gyrocompassing, and methods for analysis of errors are described in reference items (1), (3), (4) and (5).

The basic installation criteria and procedures used for rotary tilt table installation using survey techniques, is described in unclassified government installation procedures. Specific test tolerances and test procedures involved in testing an inertial platform are also described in unclassified government test procedures.

There was no existing criteria in the literature reviewed to indicate previous work toward defining methods for, or evaluation of using an inertial platform to accurately align a rotary tilt table for its own test facility. The government documents give an insight to the accuracy requirements but use survey techniques to effect the required alignment accuracy.

III.

ROTARY TILT TABLE: CAPABILITIES, OPERATION AND APPLICATION

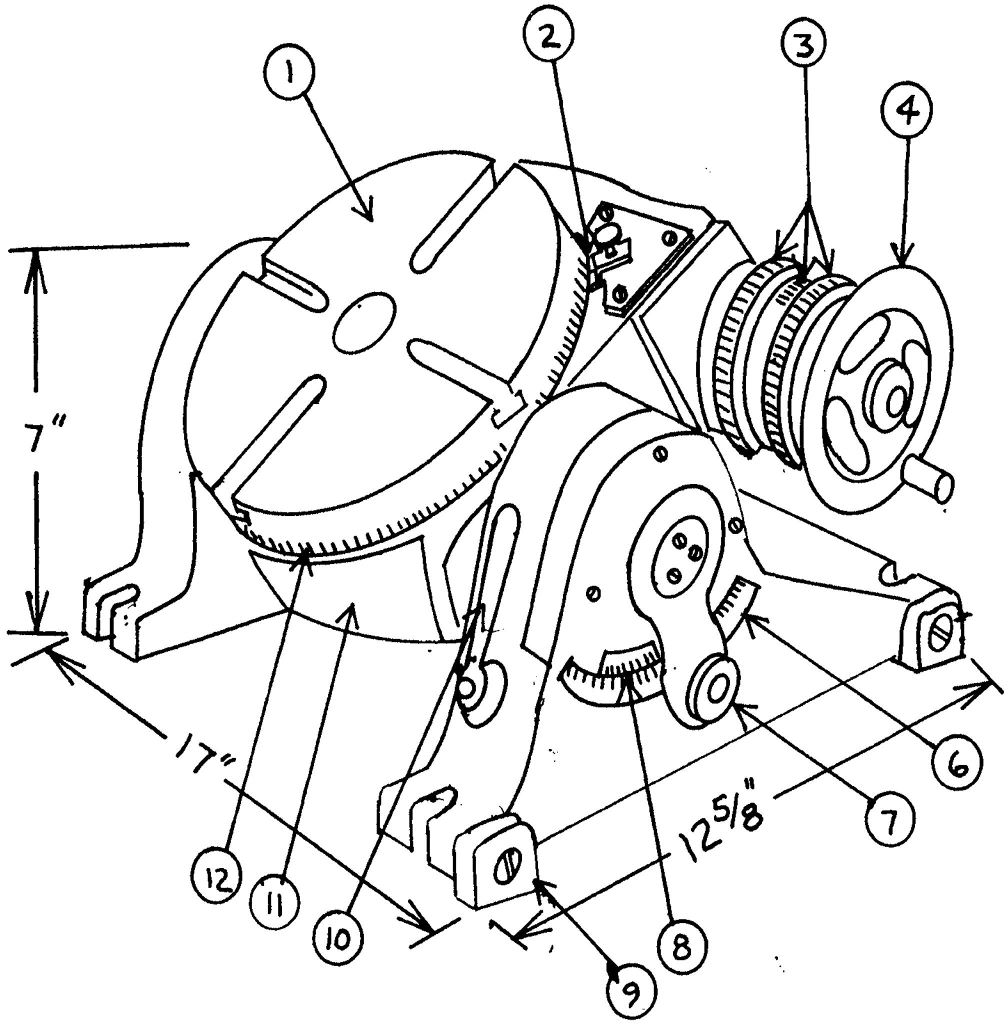
A rotary tilt table is used with the inertial platform test facility because of its ability to accurately position the gyro stabilized platform while performing the variety of tests necessary to verify directional accuracy of an inertial navigation system. The capabilities, operation, and application of the rotary tilt table and those tests which utilize it and involve comparisons with its angular position indication are discussed below to establish criteria for judging the tilt table accuracy requirements.

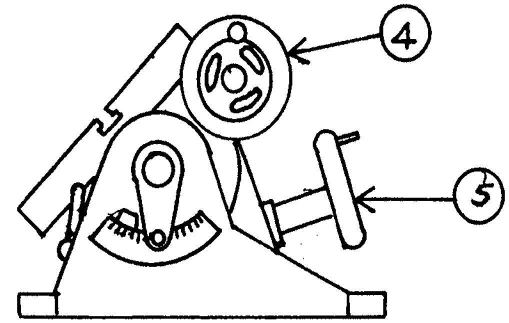
A. Capabilities and Operation

The rotary tilt table is an accurately machined device consisting basically of a flat table surface, called a platen, coupled through two axes to a rigid base. The platen is adjustable through ± 360 degrees about a vertical axis and +90 degrees from level about one horizontal axis by means of gear drives operated by handwheels.

The Swiss-made Society Genevoise, type PI-2 rotary tilt table, shown in Figure 4, will be used for the following brief description of construction and operation. This assembly has envelope dimensions of 17 inches wide by 12 5/8 inches deep and 7 inches high when level. It has a 7 7/8 inch platen, and weighs approximately 115 pounds. The platen can be rotated continuously in either direction through 360 degrees and can be tilted from 0 to 90° in one direction only. It is designed to handle loads up to 55 pounds on its platen.

The platen is rotated by means of the rotation handwheel. It is attached to a worm gear that mates with gear teeth on the bottom





1. Platen
2. Rotation pointer
3. Rotation vernier indicator and controls
4. Rotation handwheel
5. Tilt handwheel
6. Tilt angle indicator
7. Magnifying glass-tilt indicator
8. Tilt angle vernier scale
9. Aligning block
10. Cradle locking lever
11. Cradle
12. Rotation indicator

Figure 4 - Rotary Tilt Table

of the platen. One revolution of the rotation handwheel corresponds to an angular rotation of three degrees. The rotation graduations on the platen are in one degree increments but the platen is adjustable to an accuracy of one minute by a vernier adjustment. Its adjustment is further improved by a second vernier whose graduations are in five second intervals.

The platen supporting cradle is tilted by means of the tilt handwheel. This is also attached to a worm gear engaging gear teeth on the cradle. The tilt angle graduations are in half degrees. Accuracy is improved to one minute of angle by reading the tilt angle vernier scale through the magnifying glass and refined to 15 seconds by interpolation. Two reference stops define the 0 degree and 90 degree positions.

B. Application

The accuracy required of the rotary tilt table and its installation alignment is determined by the accuracy specified for the particular inertial platform tests to be performed. This topic briefly describes the inertial platform tests to be performed and identifies error sources involved in performing the tests. In particular those tests affected by the tilt table alignment accuracy are identified.

1. Gyrocompass Test

The gyrocompass test is performed to ascertain that the inertial platform stable element levels and gyrocompasses to specified accuracies. This consists of comparing the inertial platform's azimuth and level indications with the rotary tilt table. The alignment accuracy of the rotary tilt table limits the accuracy to which this test can be performed. Potential system errors that contribute to this test result are gyro drift, accelerometer bias, latitude setting, gimbal synchros, the external indicator, and the rotary tilt table alignment.

2. Accelerometer Bias

The accelerometer bias test is performed to determine that the accelerometer bias potentiometers are adjusted for no output when there is no acceleration input. This is performed by comparing the accelerometer's open loop output when subjected to earth's gravity, first in one direction, then in the other. The sum of the two measurements is equal to twice the accelerometer bias. The rotary tilt table is only used as a convenient fixture for position adjustment since the definition of vertical during this test is zero output from the two accelerometers not being tested. Therefore, this test does not involve comparison of inertial platform angle to rotary tilt table angle, but the tilt table ability for fine adjustment is needed here.

3. Gyro Drift Test

This test is performed to ascertain whether the gyro bias potentiometers are adjusted to eliminate inherent gyro drift. The level axis gyros are tested by opening the stabilization loop and allowing the gyro drift rate to drive the stable element off level. The resulting rate of change of the accelerometer output is then representative of the gyro drift rate. The azimuth gyro drift is identified by opening the loop and measuring the rate of change of azimuth. Potential system errors that may contribute to the results of this test are latitude setting, gimbal synchros, and external measurement equipment. The heading of the inertial platform must be compared to the rotary tilt table to assure that a component of earth rate is not identified as gyro drift, thus rotary tilt table alignment accuracy affects the results.

4. Synchro Null Tests

This test is performed to ascertain that the inertial platform synchro electrical nulls are within specified limits. With the rotary tilt table set to zero azimuth and level, each of the gimbal synchros angles is measured and must be within specified tolerance. The only errors involved are the gimbal synchros, the external measurement equipment, and the rotary tilt table alignment.

5. Synchro Linearity Tests

This test involves rotating and tilting the inertial platform and comparing its angular indications to those of the rotary tilt table. The gimbal synchros and external measurement equipment are the error sources involved in the results of this test.

6. Accelerometer Scale Factor Test

This test is performed to determine if the accelerometer produces the correct output for a known acceleration input in the -1g to +1g range. The known acceleration inputs are the component of gravity sensed as the stable element, caged to the case, is tilted from 0° in successively greater angles to 90°. The sources of error applicable to this test are the accelerometer, the external measurement equipment, and the rotary tilt table level alignment.

IV.

INERTIAL PLATFORM SELF ALIGNMENT

The basic theory of the level and gyrocompass servo loops for an inertial platform must be discussed in order to establish mathematical models of the leveling and gyrocompassing operations. These models can then be used to establish the ideal behavior of the level and gyrocompass operations in the absence of error, which will show that self alignment is possible.

The heart of the inertial system is the gimbal suspended stable element which is within the inertial platform and is used to define a three-axis coordinate system. The stable element contains either two or three orthogonally mounted gyros to give a stable attitude in the three axes. It contains a set of three orthogonally mounted accelerometers to sense acceleration in any direction of the three dimensional coordinate frame.

Initial platform self alignment uses the system's inertial instruments (gyros and accelerometers) to sense deviation from the desired coordinate frame. At least two noncollinear vectors are required to define a three-axis orthogonal coordinate system. For self alignment of an earth referenced inertial system, the mass attraction vector is used for leveling. An angular rate vector, such as the earth's rotational vector, is used for azimuth alignment.[1] The three earth-referenced coordinates are then defined as north, east and vertical by the sensitive axes of the three accelerometers. The north and east accelerometers sense acceleration in the ground plane and the third accelerometer senses vertical acceleration including the mass attraction vector.

In the following discussion, it is assumed that the platform initially has been roughly aligned to within a few degrees of the desired orientation so that small angle approximations are valid.[1] This rough alignment can be achieved by slaving the platform pitch and roll gimbals to the attitude of the test surface, which is approximately level and by slaving the azimuth gimbal to some external heading reference such as a magnetic compass corrected for local variation.

A. Leveling

Platform level may be defined as the attitude where the level (north and east) accelerometers do not sense a component of gravity. Simplified forms of self leveling loops are shown in Figures 5 and 6.

The east/west axis level loop is shown in Figure 5 where θ_E is the platform misalignment angle about the east/west axis. A component of gravity, $g \sin\theta_E$, will be sensed by the north accelerometer when the platform is tilted about the east/west axis. This gravity component output from the north accelerometer, approximated by $g\theta_E$ due to the small angle, causes torquing of the east gyro which, in turn, drives the gimbal servo to rotate the platform about the east/west axis until the north accelerometer output is zero. The additional input, as illustrated by Figure 7, is the component of earth rate sensed by the east gyro when the inertial platform is not oriented in azimuth. Since the gyro input axis is not perpendicular to earth rate, it senses the component of earth rate, $-\Omega \cos\phi \sin\psi$, which becomes an input to the loop driving it off level. For consideration of error free leveling, this input can be disregarded for systems which are aligned with north.[2]

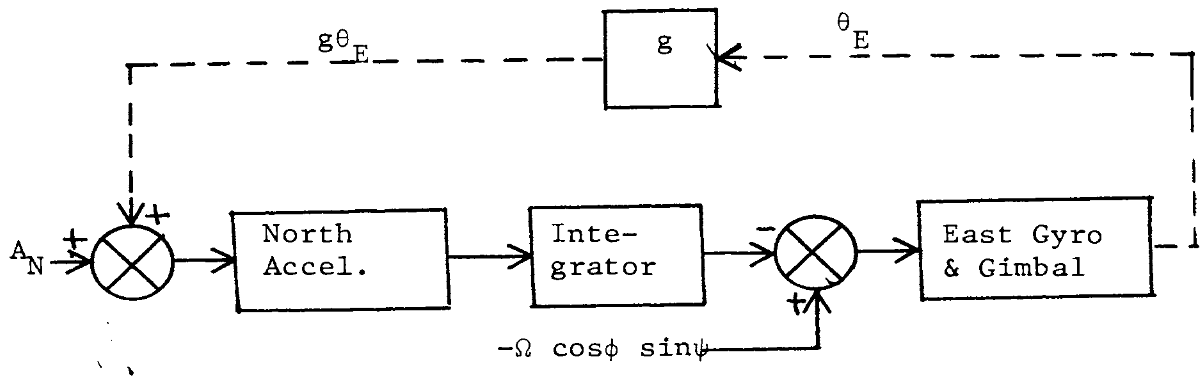


Figure 5 - East/West Axis Self Level Loop

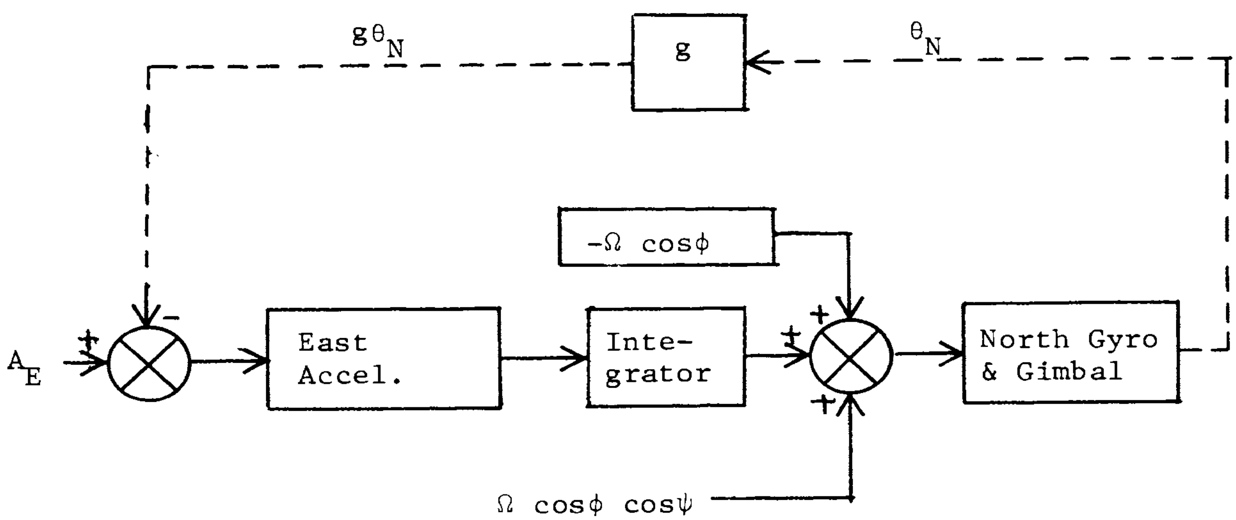
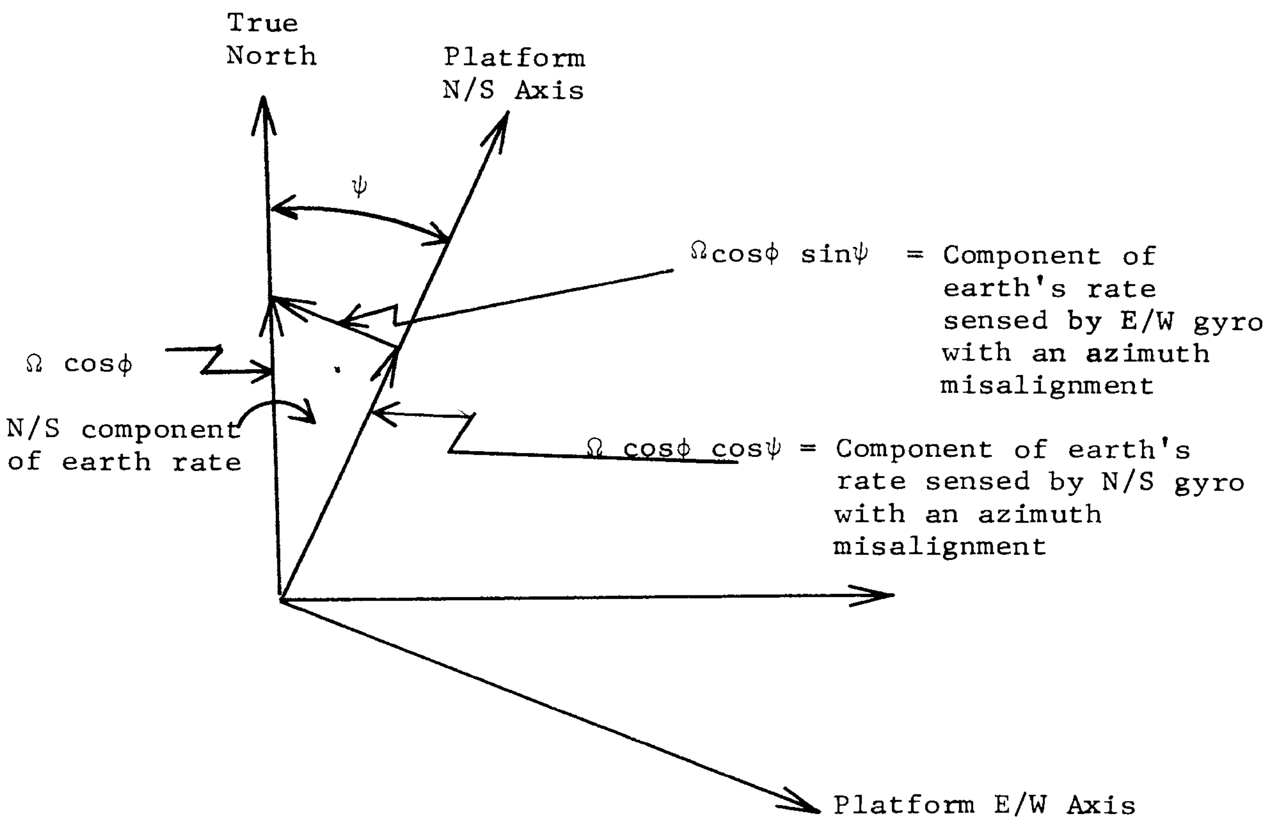


Figure 6 - North/South Axis Self Level Loop



ϕ = Latitude

ψ = Azimuth misalignment

Figure 7 - Components of Earth Rate Vector

Sensed by the Level Axis Gyro

The north/south axis level loop is shown in Figure 6 where θ_N is the platform misalignment angle about the north/south axis. A component of gravity, $-g \sin\theta_N$, will be sensed by the east accelerometer when the platform is tilted about the north/south axis. This gravity component is approximated by $g\theta_N$, due to the small angle. The east accelerometer output signal is applied to torque the north/south gyro, which in turn, drives the gimbal servo to rotate the platform until the east accelerometer output is zero. Referring again to Figure 7, the earth rate component sensed by the north gyro can be assumed to be the full $\Omega\cos\phi$ vector when the system is aligned with north and level. This loop input would drive the loop off level about the north/south axis except that the gyro torquing input, $-\Omega\cos\phi$, is applied to the gyro to offset the earth rate input. Again for consideration of error free leveling, this cancels out earth rate for systems aligned to north. This makes the north/south level loop closely resemble the east/west level loop. After alignment, during navigation this input is used to keep the platform level as the earth rotates in space.

Since the north/south and the east/west leveling loops are essentially the same, only a general mechanization of the leveling loop will be developed here. Consider the leveling loop to be mechanized as the typical level axis stabilization loop in the normal navigation mode of operation, as shown in Figure 8. Since the time constant of the gimbal servo is usually small, the gyro and gimbal dynamics may be represented as an integrator[3], thus the gyro may be represented as shown in Figure 8. In this loop, any platform tilt angle, ϕ, from the vertical will be sensed by the accelerom-

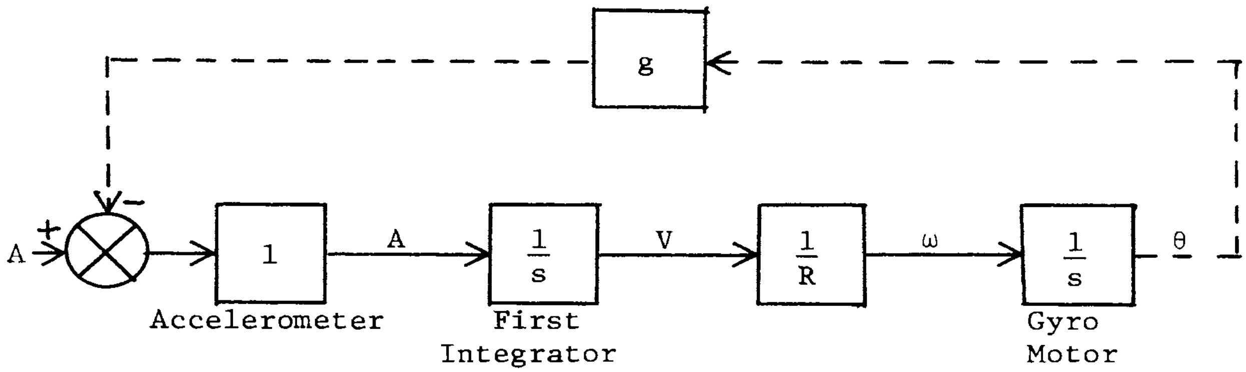


Figure 8 - Simplified Level Axis Stabilization Loop

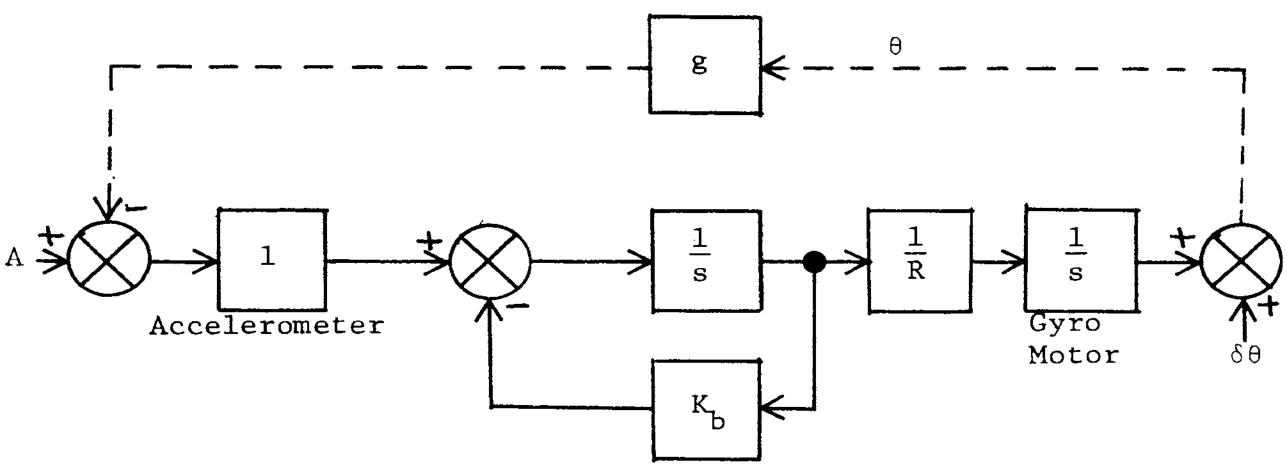


Figure 9 - Simplified Level Axis Damped Stabilization Loop

eter and will result in an apparent velocity, V, which is divided by the earth's radius, R, to give an angular rate, ω. This angular rate is applied to the gyro in the correct sense to drive the platform to null θ. Using the small angle approximation so that $\sin\theta = \theta$, the equation of motion for the leveling loop can be written as:

$$\ddot{\theta} + \frac{g}{R}\theta = 0 \tag{4.1}$$

If the platform is started from rest with an angular error θ_0, then the vertical error at any time is given by:

$$\theta = \theta_0 \cos\omega_0 t \tag{4.2}$$

where $\omega_0 = \sqrt{\frac{g}{R}} = 1.204 \times 10^{-3}$ rad/sec is the undamped natural frequency. The stabilization loop of Figure 8 is, therefore, an undamped low frequency servo system with a maximum bounded error of θ_0 and a period of 84.4 minutes. This loop is known as the Schuler Loop, and the period is known as the Schuler Period.

During alignment, damping is added around the first integrator as shown in Figure 9 to allow the platform to attain a steady state level condition in the presence of an initial tilt error θ_0. The transfer function for this damped loop is:

$$\frac{\theta}{\delta\theta_0}(s) = \frac{s(s + K_b)}{s^2 + K_b s + \frac{g}{R}} \tag{4.3}$$

If $\delta\theta$ is the step error $\frac{\theta_0}{s}$ (which corresponds to an initial tilt error θ_0), then applying the final value theorem, shows that the tilt angle θ damps to zero. This equation still has a natural frequency of $\omega_0 = 1.204 \times 10^{-3}$ rad/sec but the initial error damps out in time.

Faster inertial platform leveling is achieved by increasing the forward gain by K_R, as shown in the level alignment loop in Figure 10, thus increasing the natural frequency. The transfer function for this loop is as follows:

$$\frac{\theta}{\delta\theta}(s) = \frac{s(s + K_b)}{s^2 + K_b s + \frac{g}{R} K_R} \tag{4.4}$$

Applying the final value theorem with $\delta\theta = \frac{\theta_0}{s}$ shows that the initial error still damps to zero, but the natural frequency has been increased by the factor $\sqrt{K_R}$ and thus the time constant has been decreased by $1/\sqrt{K_R}$.

B. Gyrocompassing

The term "gyrocompassing" is used to indicate the appropriate alignment about the local vertical (that is, azimuth alignment). Correct azimuth orientation may be defined as alignment of the east/west axis perpendicular to the earth's rotation rate vector as indicated in Figure 7. This process is accomplished by causing rotation of the previously leveled platform until the input axis of the east gyro no longer senses a component of the earth rate vector (i.e. the input axis is perpendicular to the earth's rotation vector)[4]. Thus, the north gyro will then have its input axis in the north direction and the azimuth angle of the inertial platform will be zero.

Figure 11 illustrates how a gyrocompass loop is mechanized as an extension of the east/west leveling loop. The east/west axis leveling loop, as mentioned during its discussion, will be driven off level by an input of earth rate to the east gyro. Re-

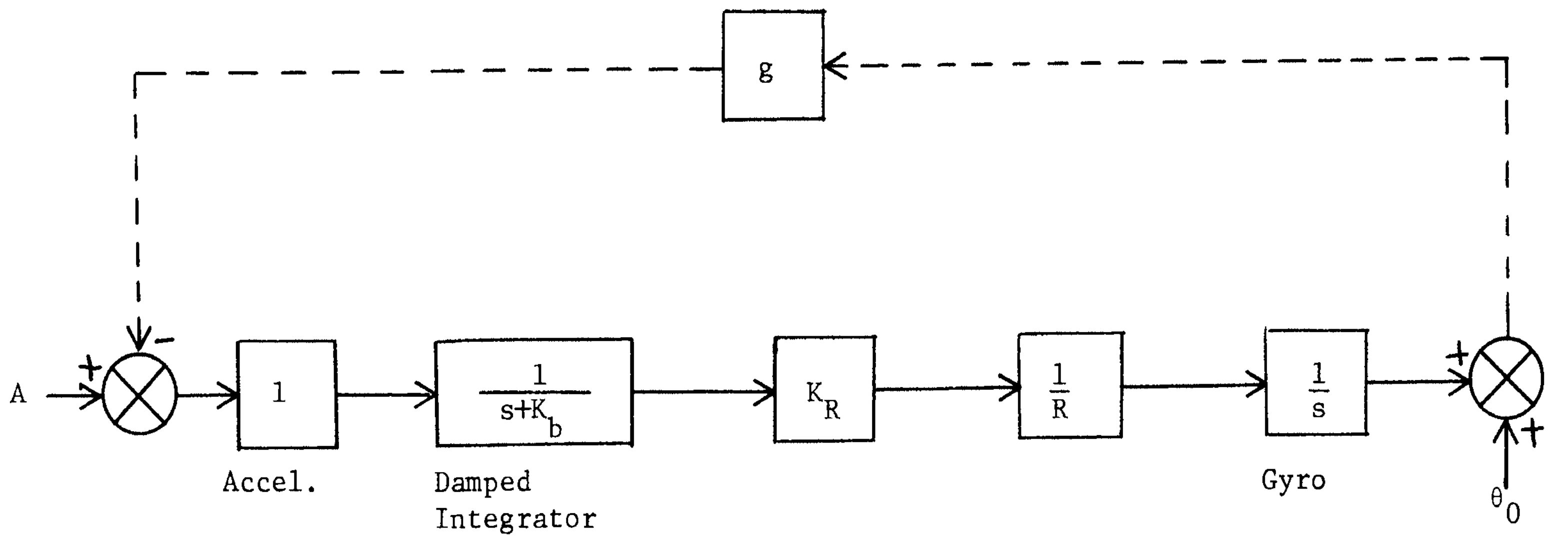


Figure 10 - Simplified Single Axis Level Alignment Loop

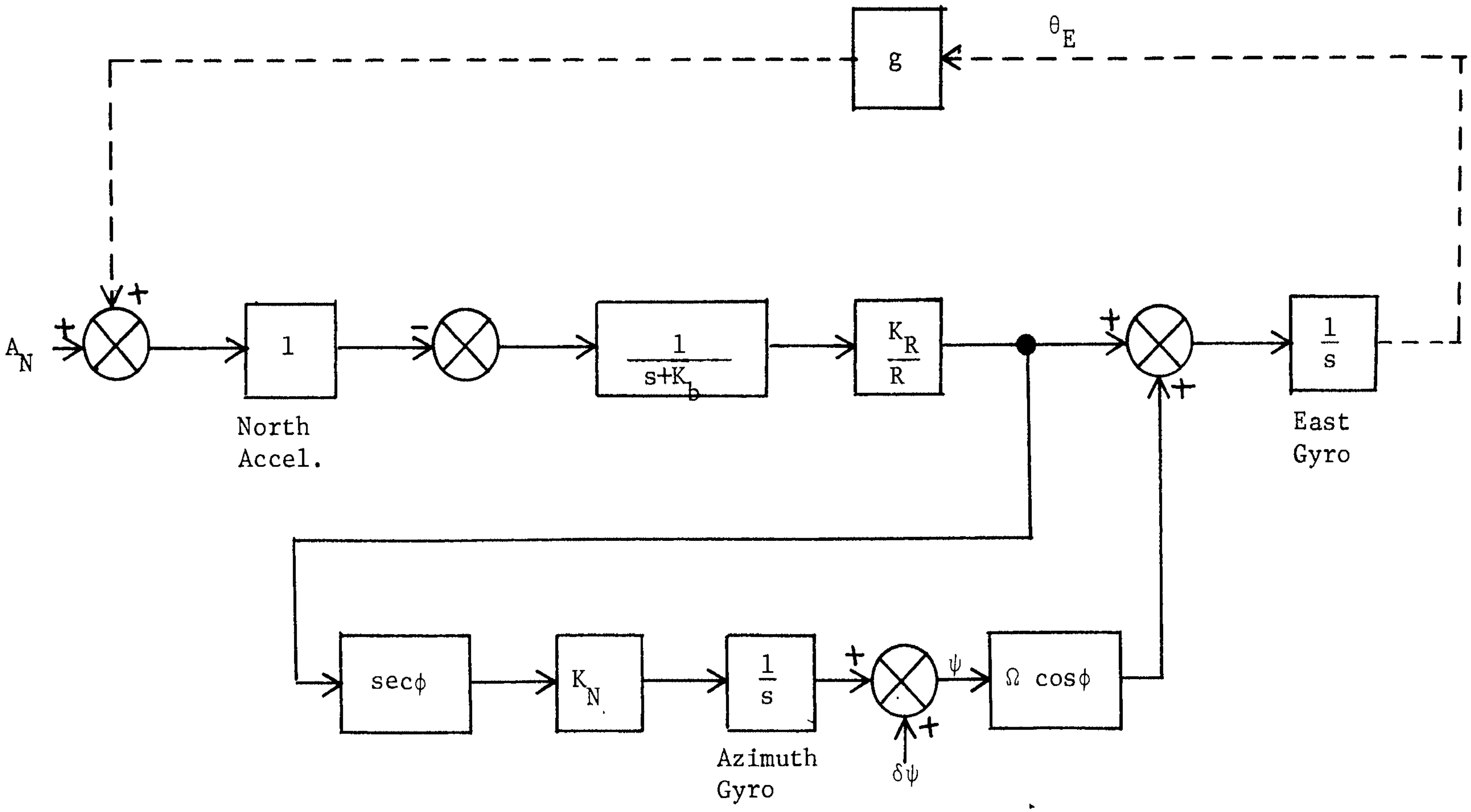


Figure 11 - Gyrocompass Loop

ferring to the implementation of the gyrocompass loop in Figure 11, if the platform is at an angle ψ from true north, the east gyro will sense the component of earth rate $-\Omega\cos\phi\ \sin\psi$ (see Figure 7). It is assumed that the azimuth angle is small due to rough alignment so $\sin\psi \approx \psi$. This earth rate component causes torquing of the east gyro, driving the platform off level, causing the north accelerometer to sense a component of gravity. The north accelerometer output re-levels the platform and torques the azimuth gyro until the azimuth angle, ψ, is nulled out.

The transfer function for the azimuth misalignment angle ψ, as a function of the angular input $\delta\psi$, for the gyrocompass is:

$$\frac{\psi}{\delta\psi}(s) = \frac{s[s^2 + K_b s + (\frac{g}{R})K_R]}{s^3 + K_b s^2 + (\frac{g}{R})K_R s + (\frac{g}{R})K_R \Omega K_N}$$

The gains may be chosen such that the characteristic equation has three equal roots[2]. This is typical design practice. The azimuth angular error $\delta\psi$ is used to incorporate the initial azimuth misalignment, ψ_0, by making it a step function of magnitude ψ_0 whereupon equation 4.5 becomes:

$$\psi(s) = \frac{\psi_0(s^2 + 3\alpha s + 3\alpha^2)}{(s + \alpha)^3}$$

Where $\alpha = \sqrt[3]{(\frac{g}{R})K_R \Omega K_N}$

Applying the final value theorem to equation 4.6 shows that gyrocompassing can damp out initial azimuth misalignment errors and thus can perform azimuth alignment in error free systems.

V.

SELF ALIGNMENT ERRORS

The accuracy of an inertial platform's self alignment is limited by erroneous initial conditions and mechanical imperfections which are introduced as error inputs to the level and gyrocompass servo loops[5]. These error inputs are nulled by the servo loops causing the inertial platform to orient to false level and azimuth coordinates.

A. Error Sources Considered

There are many possible sources of error and their importance is dependent upon the particular mechanization and its application. The error sources which are considered here to be typical for an aircraft inertial navigation system are; initial level and azimuth misalignment angles, latitude error, gyro drift, accelerometer bias and interaxis coupling errors. The level and azimuth misalignment angles, θ_0 and ψ_0 respectively, refer to the initial orientation of the platform that the servo loops are mechanized to correct. Latitude is required to establish correct earth rate torquing; therefore any latitude error, $\delta\phi$, results in an erroneous earth rate input. Gyro drift, ε, is an output from the gyro pickoff synchro which was not caused by the rotation of the earth or by movement of the vehicle over the earth's surface. Accelerometer bias, A_B, is non-zero output from an accelerometer when it is sensing no acceleration. Interaxis coupling, λ, will occur if the principal axes of the stable element do not coincide with the input axes of the gyros resulting in erroneous earth rate inputs.

These error sources are introduced into the level and gyrocompass loops as constant error forcing functions, as shown in Figures 12 and 13. The azimuth misalignment error, ψ_0, the latitude error, $\delta\phi$, and the interaxis coupling error, λ, require further explanation since they enter the loops as a function of an earth rate component coupled into the gyros.

As shown in Figure 7 the component of earth rate sensed by the east gyro is $\omega_E = \Omega\cos\phi\ \sin\psi$. With a small change in azimuth angle, $\delta\psi$, it becomes:

$$\omega_E' = \Omega\cos\phi\ \sin(\psi + \delta\psi)$$

$$= \Omega\cos\phi\ \sin\psi\ \cos\delta\psi + \Omega\cos\phi\ \cos\psi\ \sin\delta\psi$$

and for small $\delta\psi$

$$\omega_E' = \Omega\cos\phi\ \sin\psi + (\delta\psi)\Omega\cos\phi\ \cos\psi$$

$$= \omega_E + (\delta\psi)\ \cos\phi\ \cos\psi.$$

Therefore, if the nominal azimuth angle is zero and is in error by $\delta\psi$, then the earth rate correction input is in error by:

$$\delta\omega_E = (\delta\psi)\Omega\cos\phi$$

and since $\delta\psi$ is equal to ψ_0 this becomes

$$\delta\omega_E = \psi_0\Omega\cos\phi \tag{5.1}$$

as shown in Figure 13. Similarly, the component of earth rate sensed by the north gyro is $\omega_N = \Omega\cos\phi\ \cos\psi$. With a small change in azimuth angle, $\delta\psi$, it becomes:

$$\omega_N' = \Omega\cos\phi\ \cos(\psi + \delta\psi)$$

$$= \Omega\cos\phi\ \cos\psi\ \cos\delta\psi - \Omega\cos\phi\ \sin\psi\ \sin\delta\psi$$

and for small $\delta\psi$

$$\omega_N' = \Omega\cos\phi\ \cos\psi\ [1 - \frac{(\delta\psi)^2}{2}] - (\delta\psi)\Omega\cos\phi\ \sin\psi$$

$$= \omega_N[1 - \frac{(\delta\psi)^2}{2}] - (\delta\psi)\Omega\cos\phi\ \sin\psi$$

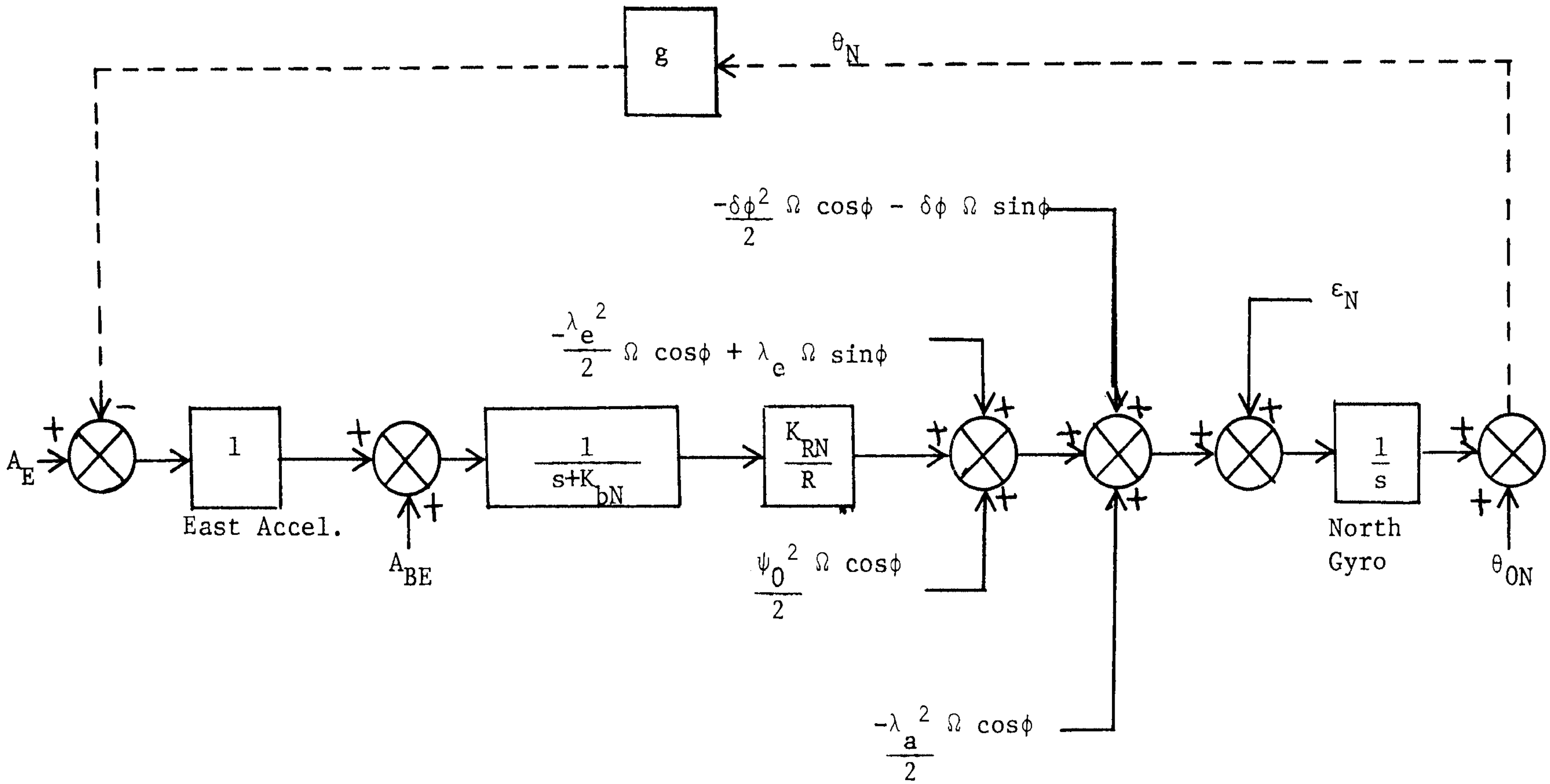


Figure 12 - North/South Leveling Loop Error Diagram

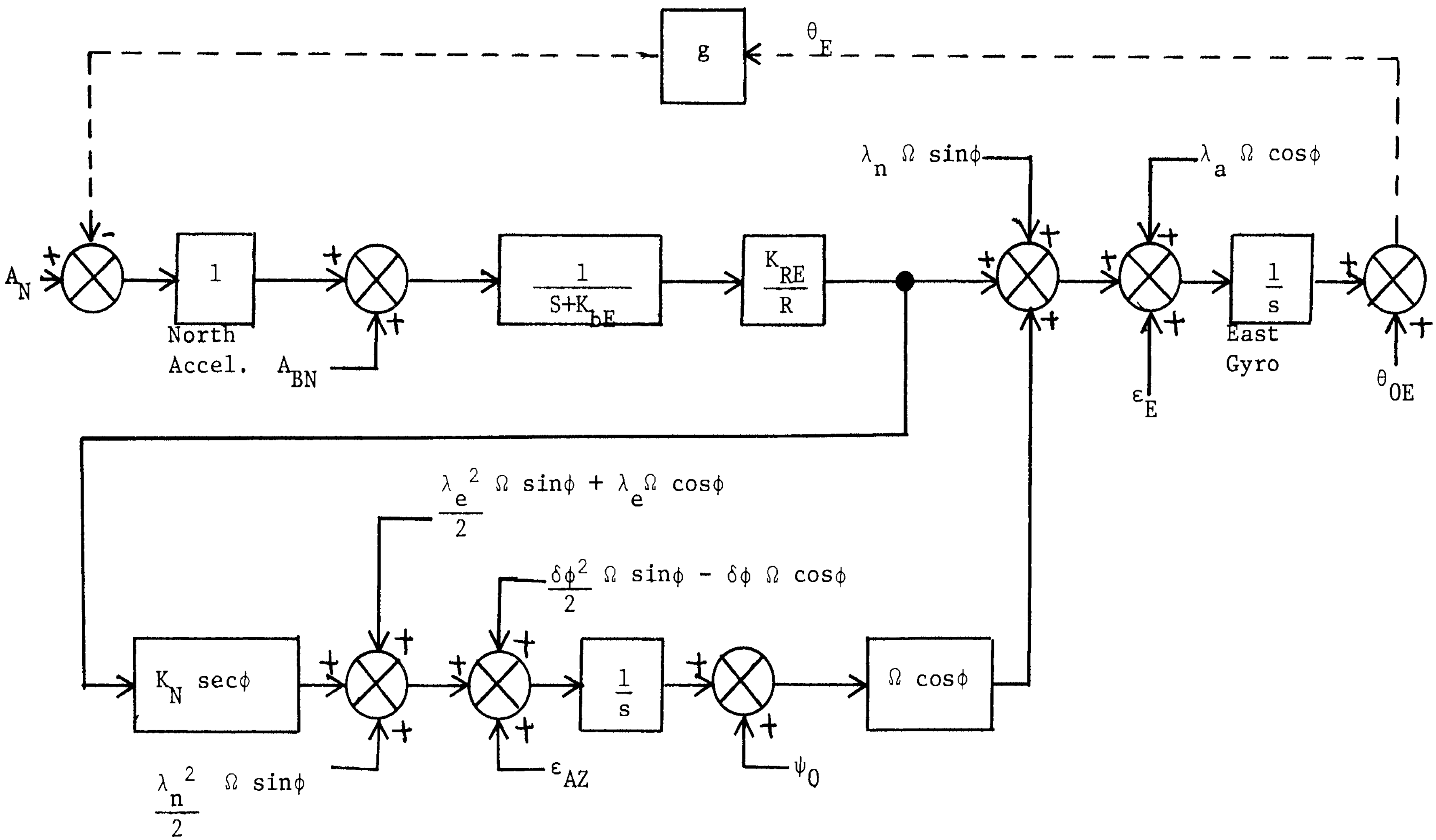


Figure 13 - East/West and Gyrocompass Loop Error Diagram

Therefore, if the nominal azimuth angle is zero and is in error by $\delta\psi$, then the earth rate correction input is in error by

$$\delta\omega_N = -\frac{(\delta\psi)^2}{2}\Omega\cos\phi$$

and since $\delta\psi$ is equal to ψ_0 this becomes:

$$\delta\omega_N = \frac{-\psi_0{}^2}{2}\ \Omega\cos\phi \qquad (5.2)$$

as shown in Figure 12.

Referring again to the earth rate components shown in Figure 7, the earth rate sensed by the east gyro is $\omega_E = \Omega\cos\phi\ \sin\psi$. With a small change in latitude, $\delta\phi$, it becomes

$$\omega_E{}' = \Omega\cos(\phi + \delta\phi)\ \sin\psi$$

$$= \Omega\sin\psi\ \cos\phi\ \cos\delta\phi - \Omega\sin\psi\ \sin\phi\ \sin\delta\phi$$

and for small $\delta\phi$

$$\omega_E{}' = \Omega\cos\phi\ \sin\psi - (\delta\phi)\Omega\sin\psi\ \sin\phi.$$

Therefore, if the nominal azimuth angle is zero, $\omega_E{}'$ is equal to zero, indicating that latitude error has no affect on the east gyro, as shown in Figure 13. Similarly the component of earth rate sensed by the north gyro is $\omega_N = \Omega\cos\phi\ \cos\psi$. With a small change in latitude, $\delta\phi$, it becomes:

$$\omega_N{}' = \Omega\cos(\phi + \delta\phi)\ \cos\psi$$

$$= \Omega\cos\psi\ \cos\phi\ \cos\delta\phi - \Omega\cos\psi\ \sin\phi\ \sin\delta\phi$$

and for small $\delta\phi$

$$\omega_N{}' = \Omega\cos\phi\ \cos\psi\ [1 - \frac{(\delta\phi)^2}{2}] - (\delta\phi)\Omega\cos\psi\ \sin\phi$$

$$= \omega_N - \frac{(\delta\phi)^2}{2}\ \Omega\cos\phi\ \cos\psi - (\delta\phi)\Omega\cos\psi\ \sin\phi.$$

Therefore if the azimuth angle is zero and the latitude error is $\delta\phi$, the latitude earth rate correction to the north gyro is in error by:

$$\delta\omega_N = -\frac{(\delta\phi)^2}{2}\ \Omega\cos\phi - (\delta\phi)\Omega\sin\phi \tag{5.3}$$

as shown in Figure 12. The component of earth rate sensed by the azimuth gyro is $\omega_{AZ} = -\Omega\sin\phi$. With a small change in latitude $\delta\phi$ it becomes:

$$\omega_{AZ}' = -\Omega\sin(\phi + \delta\phi)$$

$$= -\Omega\sin\phi\ (1 - \frac{\delta\phi^2}{2}) - \Omega(\delta\phi)\ \cos\phi$$

so the latitude correction for the azimuth gyro is in error by:

$$\delta\omega_{AZ} = \frac{(\delta\phi)^2}{2}\ \Omega\sin\phi - (\delta\phi)\ \Omega\cos\phi \tag{5.4}$$

as shown in Figure 13.

Interaxis coupling errors, λ, result from misalignment of the gyros' input axes relative to the orthogonal coordinates defined by the accelerometers. The east gyro input axis can be misaligned in either azimuth or level angle directions. The component of earth rate sensed by the east gyro due to azimuth misalignment is $\omega_{E1} = \Omega\cos\phi\ \sin\lambda_a$. With a small variation, λ_a, from a zero nominal azimuth it behaves like an azimuth error, which results in an earth rate error of:

$$\delta\omega_{E1} = \lambda_a\Omega\cos\phi \tag{5.5}$$

as shown in Figure 13.

The component of earth rate sensed by the east gyro due to level misalignment, λ_n, is $\omega_{E2} = -\Omega\sin\phi\ \sin\lambda_n$. Since λ_n is a small angle the earth rate error is:

$$\delta\omega_{E2} = -\lambda_n\Omega\sin\phi \tag{5.6}$$

as shown in Figure 13. The component of earth rate sensed by the north gyro due to azimuth misalignment is $\omega_{N1} = \Omega\cos\phi\ \cos\psi$. With a small variation in the azimuth direction, λ_a, from a nominal zero azimuth it appears like an azimuth error and therefore can be written as:

$$\delta\omega_{N1} = -\frac{\lambda_a^2}{2}\ \Omega\cos\phi \tag{5.7}$$

as shown in Figure 12. The component of earth rate sensed by the north gyro due to level misalignment appears exactly like a negative latitude error, thus the earth rate error may be written:

$$\delta\omega_{N2} = -\frac{\lambda_e^2}{2}\ \Omega\cos\phi + \lambda_e\Omega\sin\phi \tag{5.8}$$

as shown in Figure 12. The component of earth rate sensed by the azimuth gyro is $\omega_{AZ1} = -\Omega\sin\phi$ and with a small level misalignment about the east/west axis it appears as a negative latitude error and the earth rate error may be written as:

$$\delta\omega_{AZ1} = \frac{\lambda_e^2}{2}\ \Omega\sin\phi + \lambda_e\Omega\cos\phi \tag{5.9}$$

as shown in Figure 13. For a small misalignment about the north/south axis the earth rate sensed by the azimuth gyro is $\omega_{AZ2} = -\Omega\sin\phi\ \cos\lambda_n$ and for small λ_n the resultant earth rate error is:

$$\delta\omega_{AZ2} = \frac{\lambda_n^2}{2}\ \Omega\sin\phi \tag{5.10}$$

as shown in Figure 13.

B. Error Effects on Leveling

The effects of the error sources which have been defined on leveling will be investigated by introducing the error forcing functions into the servo loops as they occur in the physical system. The leveling loops, with error inputs, are shown in Figures 12 and 13.

The east/west level loop is tightly coupled in the gyrocompass loops, but the north/south level loop is only loosely coupled to azimuth for a north aligned platform, therefore the north/south level loop can be analyzed separately and will be discussed first. The transfer functions relating the platform north/south axis level misalignment angle, θ_N, to the error inputs as obtained from Figure 12 are as follows:

$$\frac{\theta_{N1}}{A_{BE}}(s) = \frac{K_{RN}\left(\frac{\omega_0^2}{g}\right)}{(s + \omega_0\sqrt{K_{RN}})^2} \tag{5.11}$$

$$\frac{\theta_{N2}}{\varepsilon_N}(s) = \frac{(s + 2\omega_0\sqrt{K_{RN}})}{(s + \omega_0\sqrt{K_{RN}})^2} \tag{5.12}$$

$$\frac{\theta_{N3}}{(\psi_0)^2}(s) = \frac{\Omega\cos\phi(s + 2\omega_0\sqrt{K_{RN}})}{2(s + \omega_0\sqrt{K_{RN}})^2} \tag{5.13}$$

$$\frac{\theta_{N4}}{\theta_{0N}}(s) = \frac{s(s + 2\omega_0\sqrt{K_{RN}})}{(s + \omega_0\sqrt{K_{RN}})^2} \tag{5.14}$$

$$\frac{\theta_{N5}}{-\frac{(\delta\phi)^2}{2}\cos\phi - (\delta\phi)\sin\phi}(s) = \frac{\Omega(s + 2\omega_0\sqrt{K_{RN}}}{(s + \omega_0\sqrt{K_{RN}})^2} \tag{5.15}$$

$$\frac{\theta_{N6}}{-\frac{\lambda_e^{2}}{2}\cos\phi + \lambda_e \sin\phi}(s) = \frac{\Omega(s + 2\omega_0\sqrt{K_{RN}})}{(s + \omega_0\sqrt{K_{RN}})^2} \tag{5.16}$$

$$\frac{\theta_{N7}}{-\frac{\lambda_a^{2}}{2}\cos\phi}(s) = \frac{\Omega(s + 2\omega_0\sqrt{K_{RN}})}{(s + \omega_0\sqrt{K_{RN}})^2} \tag{5.17}$$

The steady state north/south axis level error caused by the constant error inputs may be obtained by application of the final value theorem to equations 5.11 thru 5.17 which gives:

$$\theta_{N1ss} = \frac{A_{BE}}{g} \tag{5.18}$$

$$\theta_{N2ss} = \frac{2\varepsilon_N}{\omega_0\sqrt{K_{RN}}} \tag{5.19}$$

$$\theta_{N3ss} = \frac{\psi_0{}^2\Omega \cos\phi}{\omega_0\sqrt{K_{RN}}} \tag{5.20}$$

$$\theta_{N4ss} = 0 \tag{5.21}$$

$$\theta_{N5ss} = -\frac{2\Omega[\frac{(\delta\phi)^2}{2}\cos\phi + (\delta\phi)\sin\phi]}{\omega_0\sqrt{K_{RN}}} \tag{5.22}$$

$$\theta_{N6ss} = \frac{2\Omega[-\frac{\lambda_e^{2}}{2}\cos\phi + \lambda_e \sin\phi]}{\omega_0\sqrt{K_{RN}}} \tag{5.23}$$

$$\theta_{N7ss} = -\frac{\Omega\lambda_a^{2}\cos\phi}{\omega_0\sqrt{K_{RN}}} \tag{5.24}$$

It can be seen from equations 5.18 through 5.24 that initial tilt of the platform has no affect on the steady state level condition about the north/south axis since the level loop is designed to eliminate initial tilt. The remainder of the errors do cause a steady state off level condition at the end of alignment.

The transfer functions relating the east/west axis level misalignment angle, θ_E, to the error inputs are as follows (where $\rho = s^3 + K_{bE}s^2 + \omega_0{}^2K_{RE}s + \omega_0{}^2K_{RE}K_N\Omega$):

$$\frac{\theta_{E1}}{A_{BN}}(s) = \frac{\left(\frac{K_{RE}}{g}\right)(\omega_0{}^2)(s + K_N\Omega)}{\rho} \tag{5.25}$$

$$\frac{\theta_{E2}}{\varepsilon_E}(s) = \frac{s(s + K_{bE})}{\rho} \tag{5.26}$$

$$\frac{\theta_{E3}}{\theta_{0E}}(s) = \frac{s^2(s + K_{bE})}{\rho} \tag{5.27}$$

$$\frac{\theta_{E4}}{\lambda_a\Omega\cos\phi}(s) = \frac{s(s + K_{bE})}{\rho} \tag{5.28}$$

$$\frac{\theta_{E5}}{\lambda_n\Omega\sin\phi}(s) = \frac{s(s + K_{bE})}{\rho} \tag{5.29}$$

$$\frac{\theta_{E6}}{\psi_0}(s) = \frac{s(s + K_{bE})(\Omega\cos\phi)}{\rho} \tag{5.30}$$

$$\frac{\theta_{E7}}{\varepsilon_{AZ}}(s) = \frac{(\Omega\cos\phi)(s + K_{bE})}{\rho} \tag{5.31}$$

$$\frac{\theta_{E8}}{\frac{\lambda_n{}^2}{2}\Omega\cos\phi}(s) = \frac{(\Omega\cos\phi)(s + K_{bE})}{\rho} \tag{5.32}$$

$$\frac{\theta_{E9}}{\frac{\delta\phi^2}{2}\Omega\sin\phi - \delta\phi\Omega\cos\phi}(s) = \frac{(\Omega\cos\phi)(s + K_{bE})}{\rho} \tag{5.33}$$

$$\frac{\theta_{E10}}{\frac{\lambda_e^2}{2}\Omega\sin\phi + \lambda_e\Omega\cos\phi}(s) = \frac{(\Omega\cos\phi)(s + K_{bE})}{\rho} \tag{5.34}$$

The steady state east/west axis level error caused by the constant error inputs may be obtained by application of the final value theorem to equations 5.25 through 5.34 which gives:

$$\theta_{E1ss} = \frac{A_{BN}}{g} \tag{5.35}$$

$$\theta_{E2ss} = 0 \tag{5.36}$$

$$\theta_{E3ss} = 0 \tag{5.37}$$

$$\theta_{E4ss} = 0 \tag{5.38}$$

$$\theta_{E5ss} = 0 \tag{5.39}$$

$$\theta_{E6ss} = 0 \tag{5.40}$$

$$\theta_{E7ss} = \frac{\varepsilon_{AZ}K_{bE}\cos\phi}{\omega_0{}^2K_{RE}K_N} \tag{5.41}$$

$$\theta_{E8ss} = \frac{-(\frac{\lambda_n{}^2}{2})\Omega(\cos\phi)^2K_{bE}}{\omega_0{}^2K_{RE}\ K_N} \tag{5.42}$$

$$\theta_{E9ss} = \frac{[(\frac{\delta\phi^2}{2})\sin\phi - (\delta\phi)\cos\phi]\Omega K_{bE}\cos\phi}{\omega_0{}^2K_{RE}K_N} \tag{5.43}$$

$$\theta_{E10ss} = \frac{[\ \frac{\lambda_e{}^2}{2}\sin\phi + \lambda_e\cos\phi\ ]\Omega K_{bE}\cos\phi}{\omega_0{}^2K_{RE}K_N} \tag{5.44}$$

It can be seen from equations 5.35 through 5.44 that east gyro drift, initial tilt or azimuth error, and east gyro misalignment cause no steady state error in the east/west axis level condition. The remainder of the errors cause a steady state off level condition at the end of alignment.

C. Error Effects on Gyro Compassing

The effects of the error sources on gyrocompassing are investigated by introducing the error forcing functions into the gyrocompass loop as shown in Figure 13. The transfer functions relating the azimuth misalignment angle, ψ, to the error inputs are as follows (where $\rho = s^3 + K_{bE}s^2 + \omega_0^{\,2}K_{RE}s + \omega_0^{\,2}K_{RE}K_N\Omega$):

$$\frac{\psi_1}{\psi_0}(s) = \frac{s(s^2 + K_{bE}s + \omega_0^{\,2}K_{RE})}{\rho} \tag{5.45}$$

$$\frac{\psi_2}{\theta_{OE}}(s) = \frac{-\,s\omega_0^{\,2}K_{RE}K_N\sec\phi}{\rho} \tag{5.46}$$

$$\frac{\psi_3}{\varepsilon_E}(s) = \frac{-\,\omega_0^{\,2}K_{RE}K_N\sec\phi}{\rho} \tag{5.47}$$

$$\frac{\psi_4}{\varepsilon_{AZ}}(s) = \frac{s^2 + K_{bE}s + \omega_0^{\,2}K_{RE}}{\rho} \tag{5.48}$$

$$\frac{\psi_5}{A_{BN}}(s) = \frac{s\left(\frac{K_{RE}}{R}\right)K_N\sec\phi}{\rho} \tag{5.49}$$

$$\frac{\psi_6}{\frac{\lambda_n^{\,2}}{2}\Omega\sin\phi}(s) = \frac{s^2 + K_{bE}s + \omega_0^{\,2}K_{RE}}{\rho} \tag{5.50}$$

$$\frac{\psi_7}{\frac{\delta\phi^2}{2}\,\Omega\sin\phi - (\delta\phi)\Omega\cos\phi}(s) = \frac{s^2 + K_{bE}s + \omega_0{}^2 K_{RE}}{\rho} \tag{5.51}$$

$$\frac{\psi_8}{\lambda_n \Omega\sin\phi}(s) = \frac{-\,\omega_0{}^2 K_{RE} K_N \sec\phi}{\rho} \tag{5.52}$$

$$\frac{\psi_9}{\lambda_a \Omega\cos\phi}(s) = \frac{-\,\omega_0{}^2 K_{RE} K_N \sec\phi}{\rho} \tag{5.53}$$

$$\frac{\psi_{10}}{\frac{\lambda_e^2}{2}\,\Omega\sin\phi + \lambda_e \Omega\cos\phi}(s) = \frac{s^2 + K_{bE}s + \omega_0{}^2 K_{RE}}{\rho} \tag{5.54}$$

The steady state azimuth error caused by the constant error inputs are obtained by application of the final value theorem to equations 5.45 through 5.54 which gives:

$$\psi_{1ss} = 0 \tag{5.55}$$

$$\psi_{2ss} = 0 \tag{5.56}$$

$$\psi_{3ss} = \frac{-\,\varepsilon_E \sec\phi}{\Omega} \tag{5.57}$$

$$\psi_{4ss} = \frac{\varepsilon_{AZ}}{K_N \Omega} \tag{5.58}$$

$$\psi_{5ss} = 0 \tag{5.59}$$

$$\psi_{6ss} = \frac{\frac{\lambda_n{}^2}{2}\sin\phi}{K_N} \tag{5.60}$$

$$\psi_{7ss} = \frac{\frac{\delta\phi^2}{2}\sin\phi - (\delta\phi)\cos\phi}{K_N} \tag{5.61}$$

$$\psi_{8ss} = \lambda_n \operatorname{Tan}\phi \tag{5.62}$$

$$\psi_{9ss} = \lambda_a \tag{5.63}$$

$$\psi_{10ss} = \frac{\frac{\lambda_e^2}{2}\sin\phi + \lambda_e \cos\phi}{K_N} \tag{5.64}$$

It can be seen from equations 5.55 through 5.64 that gyro drift, interaxis coupling, and latitude error will cause a steady state azimuth misalignment at the end of gyrocompassing.

D. Relative Importance of Errors

Some typical system components and characteristic time constants for level and gyrocompass response are selected in order to evaluate the relative importance of the errors. This permits evaluation of the performance of an inertial platform in aligning a tilt table.

Typical response time for the leveling loops is ten seconds. Selection of damping ratio is a design consideration for a particular mechanization involving a trade-off between alignment time and accuracy. Critical damping will be assumed for this example, whereby equation 4.4 may be written:

$$\frac{\theta}{\theta_0}(s) = \frac{s(s + K_{bN})}{(s + \frac{1}{\tau})^2} \tag{5.65}$$

where $(\frac{1}{\tau})^2 = (\frac{g}{R})K_{RN} = \omega_0^2 K_{RN}$, $K_{bN} = 2\omega_0\sqrt{K_{RN}}$ and $\tau = 10$ seconds.

Then

$$K_{RN} = \frac{(1/\tau)^2}{\omega_0^2} = \frac{10^{-2}}{(1.204 \times 10^{-3})^2} = 6900 \tag{5.66}$$

$$K_{bN} = 2\omega_0\sqrt{K_{RN}} = (2)(1.204 \times 10^{-3})\sqrt{6900} = 0.20 \tag{5.67}$$

The gyrocompass response time is chosen to be 15 minutes (900 seconds) and the design is such that the characteristic equation of equation 4.5 has three equal roots. The last term $[\frac{g}{R} K_{RE}\Omega K_N]$

is equal to α^3 where $\alpha = \frac{1}{\tau}$, and it follows that $K_{bE} = 3\alpha$ and $\omega_0{}^2K_{RE} = 3\alpha^2$. Then solving for the constants where $\tau = 900$ seconds gives:

$$K_{bE} = 3\alpha = 0.00333 \tag{5.68}$$

$$K_{RE} = \frac{3\alpha^2}{\omega_0{}^2} = 2.56 \tag{5.69}$$

$$K_N = \frac{\alpha^3}{\omega_0{}^2K_{RE}\Omega} = 5.07 \tag{5.70}$$

For this performance analysis the component errors used will be those of Kearfott's C70 2401 005 accelerometer as shown in Table I and Kearfott's Alpha series floated rate integrating gyro as shown in Table II. From Table I the accelerometer bias error is 0.00001 g and from Table II the gyro drift is 0.02 deg./hr. It is assumed that these errors are normally distributed and the values are 1σ values.

It is assumed that local latitude is known within one mile which represents an error of approximately one arc minute. In order to evaluate interaxis coupling errors, it will be assumed that the input axes of the gyros may deviate as much as ten arc seconds from the principal axes of the stable element.

Assuming the inertial platform has been roughly aligned to within one degree of north and assuming a forty degree latitude, the resultant north/south level axis errors are derived from equations 5.18 thru 5.24 as follows:

$$\theta_{N1ss} = \frac{A_{BE}}{g} = \frac{10^{-5}g}{g} \text{ rad} = 2.06 \text{ arc sec.} \tag{5.71}$$

	C70 2401 005
Scale Factor (output)	4.9475 ma/g of applied acceleration
Operating Temperature	150°F ± 10°F
Linearity	5×10^{-6} g/g^2
Threshold	2×10^{-7} g
Zero Stability	0.00001 g
Vibration	Up to 20 g peak to 2000 cps
Storage Temp.	-65°F to +200°F
Scale Factor Variation	0.03% per year
Excitation	6 volts, 3860 cps
Natural Frequency	220 cps
Frequency Response	Flat to 250 cps
Shock	60 g's
Weight	4 ounces

Table I

Characteristics of Kearfott C70 2401 005 Accelerometer

	C70 2516 010
Size	1.837 dia. x 2.765
Weight (lbs.)	0.70
Angular Momentum (gm cm^2/sec)	100,000
Gyro Gain (output/input)	5.9
Transfer Function (v/° input)	.5
Characteristic Time (msec.)	7
Gimbal Freedom (degrees)	$\pm$ 3
Operating Temperature (°F)	180
*Drift (short term) (°/hr) vertical	0.02
*Drift (short term) (°/hr) azimuth	0.03
Mass Unbalance (maximum untrimmed along each axis) (°/hr/g)	1.0
Fixed Torque (maximum untrimmed at null) (°/hr)	1.0
Mass Unbalance Shift Max. Spread (°/hr/g)	0.5
Fixed Torque Shift Max. Spread (°/hr)	0.5
Elastic Restraint (°/hr/°)	3.0
Anisoelastic Error (Max. under vibratory acceleration) (°/hr/g^2)	0.02
Torquer Scale Factor	2°/hr./ma^2 dc
Maximum Torquing Rate (°/hr)	20,000
Torquer Linearity (% of Max. at Null) (proportional)	0.25
Signal Output (mv/°)	87
Motor Excitation (44 cps 3 ϕ)	26v
Heater Type	cycling
Stabilization Time (maximum minutes)	4
Vibration (0-2000 cycles) (g)	$\pm$25
Schock (g)	$\pm$50
Operating Life (hr)	2,000
Maximum Altitude	no limit

*Drift (short term) is based on the standard deviation of 5 consecutive 1° of arc drift readings during a total elapsed time of approximately 1 hour.

Table II

Characteristics of Kearfott C70 2516 010

Miniature Floated Rate Integrating Gyro

θ_{N2ss} = 0.4 arc sec. (5.72)

θ_{N3ss} = 0.032 arc sec. (5.73)

θ_{N5ss} = 0.0562 arc sec. (5.74)

θ_{N6ss} = 0.00935 arc sec. (5.75)

$\theta_{N7ss} = 2.7 \times 10^{-8}$ arc sec. (5.76)

The resultant east/west level axis steady state errors from equations 5.33 thru 5.42 are as follows:

θ_{E1ss} = 2.06 arc sec. (5.77)

θ_{E7ss} = 2.68 arc sec. (5.78)

$\theta_{E8ss} = 3.62 \times 10^{-6}$ arc sec. (5.79)

θ_{E9ss} = 0.45 arc sec. (5.80)

θ_{E10ss} = 0.0076 arc sec. (5.81)

The azimuth steady state errors present after gyrocompassing is completed are derived from equations 5.55 thru 5.64 as follows:

ψ_{3ss} = 6.0 arc min. (5.82)

ψ_{4ss} = 0.9 arc min. (5.83)

$\psi_{6ss} = 5.12 \times 10^{-9}$ arc min. (5.84)

ψ_{7ss} = 0.15 arc min. (5.85)

ψ_{8ss} = 0.0139 arc min. (5.86)

ψ_{9ss} = 0.0166 arc min. (5.87)

ψ_{10ss} = 0.0026 arc min. (5.88)

It is concluded from the investigation of relative importance of errors, that gyro drift and accelerometer bias are the major sources of error[7]. Since the errors due to latitude and initial azimuth error are small with respect to accelerometer bias and gyro drift, they will be neglected from further consideration.

Interaxis coupling errors due to gyro input axis deviation may be neglected, but the cross coupling of azimuth gyro drift into the east/west level loop is significant. Therefore, the standard deviation of leveling and gyrcompoassing errors are:

$$\theta_{Nss} = \sqrt{\Sigma(\theta_{Niss})^2} = 2.09 \text{ arc sec.} \quad (5.89)$$

$$\theta_{Ess} = \sqrt{\Sigma(\theta_{Eiss})^2} = 3.32 \text{ arc sec.} \quad (5.90)$$

$$\psi_{ss} = \sqrt{\Sigma(\psi_{iss})^2} = 6.07 \text{ arc min.} \quad (5.91)$$

when an in specification inertial platform is used.

VI.

ROTARY TILT TABLE ALIGNMENT METHOD AND ACCURACY

The inertial platform alignment accuracy as analyzed in Chapter V is contingent upon inertial components performing within specified error limits. Therefore, the procedure for using an inertial platform to align the rotary tilt table first validates inertial platform accuracy by ascertaining that these inertial components are performing within their accuracy requirements. This is done without using an aligned tilt table. The validated inertial platform is then utilized to align the tilt table. As additional good inertial platforms are available, they are used to obtain data on the misalignment of the tilt table so its alignment can be improved by platform alignment error averaging.

A. Inertial Platform Validation

The major causes of inertial platform level and gyrocompass error have been identified as accelerometer bias and gyro drift. In order to obtain the accuracies quoted it is necessary to appropriately bias the accelerometers and gyros. This can be done with the aid of the tilt table (only rough tilt table alignment is required) and must be done prior to utilizing the inertial platform to align the tilt table. The equipment and procedures that are utilized to ascertain whether the accelerometer bias and gyro drift errors are within the desired good platform limits are outlined.

1. Accelerometer Bias

Accelerometer bias is corrected by adjustment of a potentiometer to null the accelerometer output for zero acceleration conditions.

The magnitude of accelerometer bias may be determined by comparing the accelerometer output caused by plus one g of acceleration to the output caused by minus one g of acceleration. In order to do this, it is necessary to manually manipulate the platform gimbals, so the gyros are disabled. The leveling and gyrocompass loops are opened at each accelerometer output and a digital voltmeter is connected as shown in Figure 14.

The platform gimbals are manually manipulated until the outputs of any pair of accelerometers are zero. If these two accelerometers are properly biased, the third accelerometer will be vertical and, as such will sense the full one g gravity vector. Figure 15, however, illustrates that if one or both level axis has a bias error, the component of gravity sensed is $g \cos\beta$ and the indicated acceleration would be $g \cos\beta + A_B$.

The platform gimbals are manually manipulated to reverse the sense of gravity on the vertically oriented accelerometer. The level accelerometer biases are still of the same magnitude so the vertical accelerometer still maintains an angle of β with respect to true vertical. Thus, the vertical accelerometer senses $-g \cos\beta + A_B$.

If no accelerometer bias is present, the sum of the two voltages (i.e. the difference in magnitude) represents $2A_B$, which is equal to twice the amount of correction required for the accelerometer bias potentiometer. Correction involves adjustment of the potentiometer in a direction to reduce the

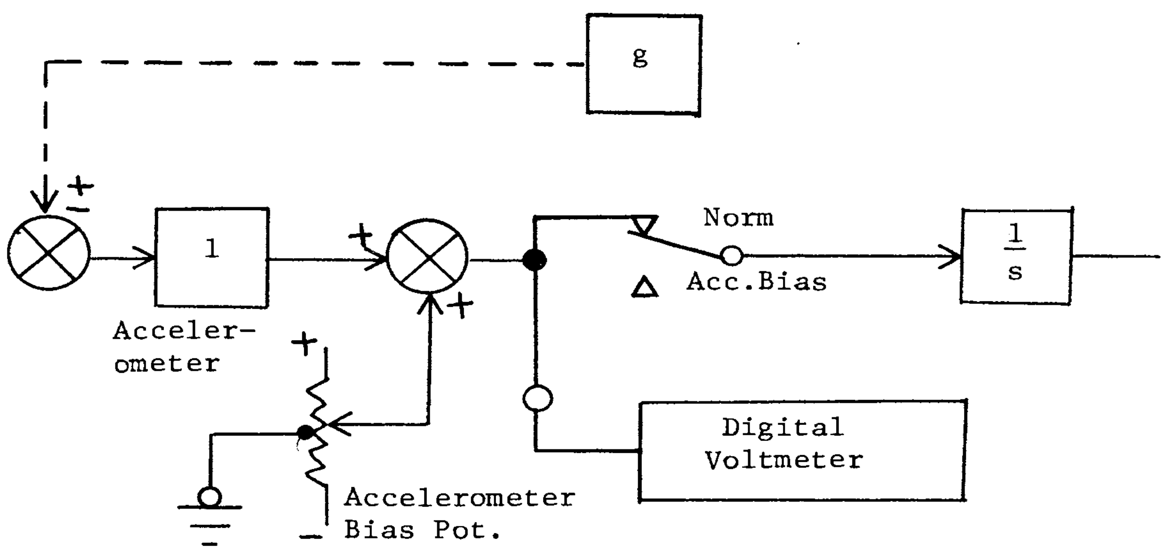
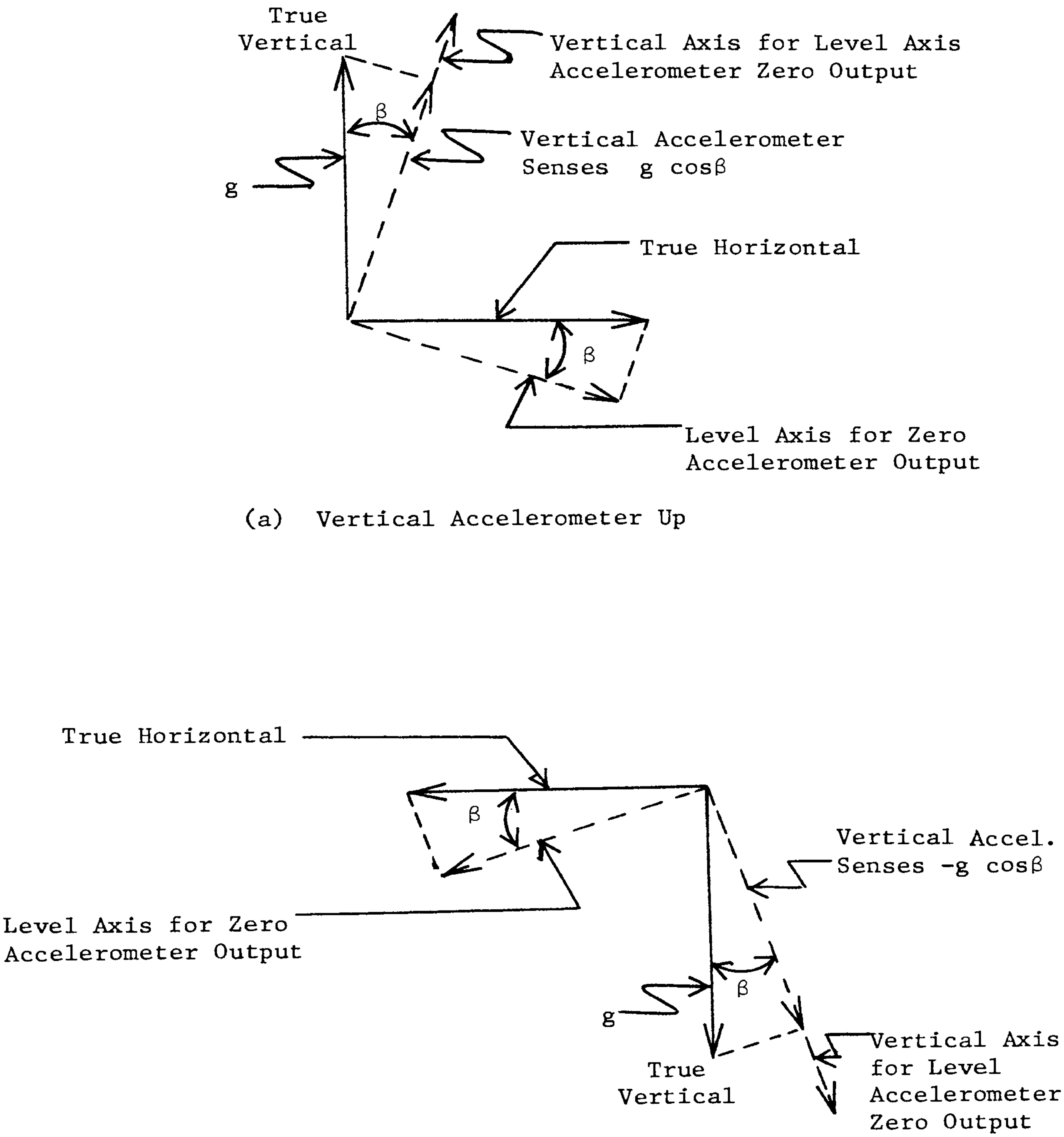


Figure 14 - Accelerometer Bias Measurement



(a) Vertical Accelerometer Up

(b) Vertical Accelerometer Down

Figure 15

Vertical Misalignment Angle Due to Accelerometer Bias difference in magnitude of the two voltages to zero.

The other two accelerometers may be tested for bias by manual manipulation of the gimbals in the same manner. It can be seen that the tilt table alignment is not required for this accelerometer biasing. The tilt table is only used as a convenient method of rotating the platform.

2. Gyro Drift

Gyro drift is both the most common and the most significant source of error in inertial platforms. Most systems make provision for both measurement and correction of gyro drift with the inertial platform installed in the using vehicle.

Gyro drift is compensated by applying an electrical torque to the gyro equal in magnitude, but opposite in direction, to the drift producing torques. For example, if the gyro were drifting in a clockwise direction at 0.02 deg/hr. due to a drift producing torque, then an electrical torque equivalent to the 0.02 deg/hr. torquing rate would be applied to the gyro in the counter clockwise direction giving a net drift of zero deg/hr. which causes the gyro to appear perfect (i.e. a gyro with zero drift rate).

a. North/South Gyro

A typical north/south level axis loop mechanized to monitor gyro drift with external equipment is shown in Figure 16. The amount of gyro drift is measured and corrections made according to the following procedure:

(1) A strip chart recorder is connected to the test point at T.P.

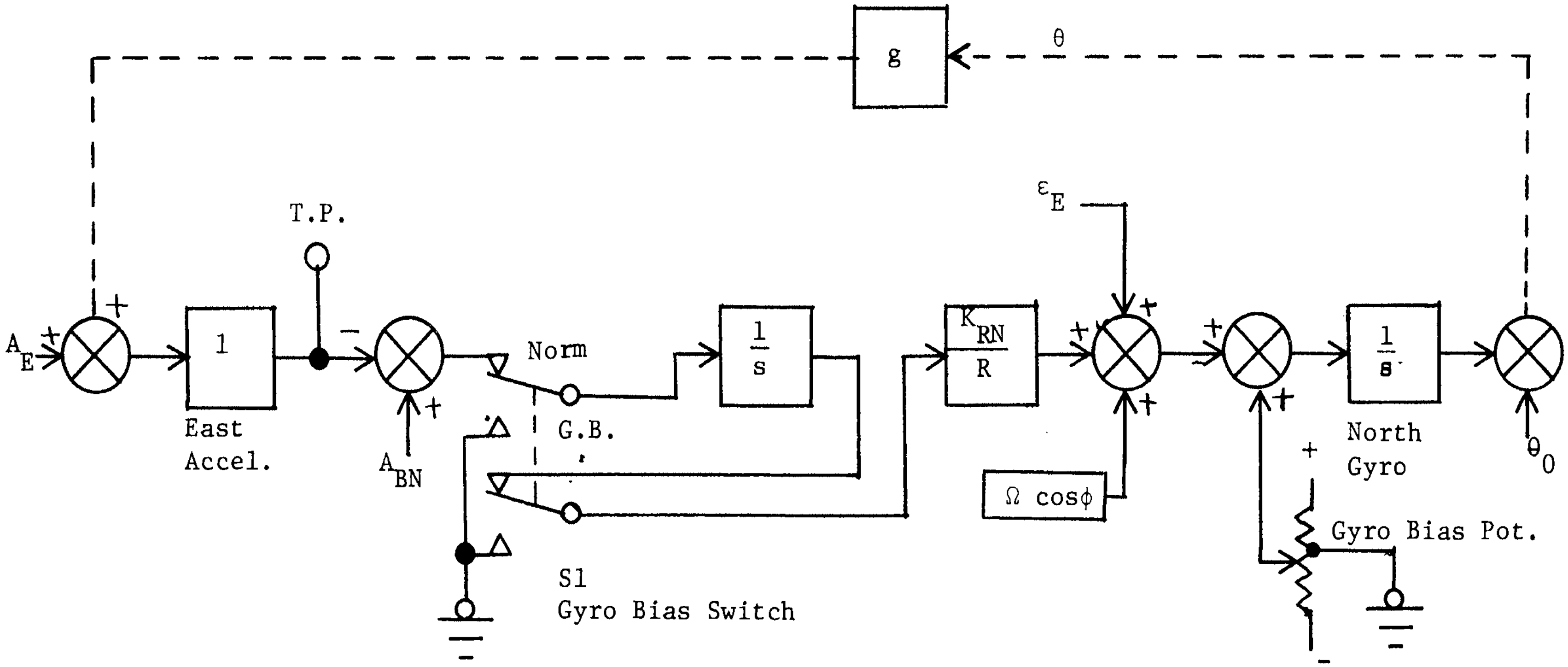


Figure 16 - Typical North/South Level Axis Loop Mechanized for Gyro Drift Measurement

(2) At the completion of alignment the system is put into the Navigate mode. The switch, S1, is switched from normal to the gyro bias position.

(3) As the gyro drifts at a given rate, ε, the level misalignment angle, θ, increases which, in turn, increases the gravity acceleration sensed by the accelerometer, causing the voltage at T.P. to increase.

(4) The voltage change being monitored at T.P. by a strip chart recorder, as shown in Figure 17, is converted to drift rate in degrees per hour by using system scale factors.

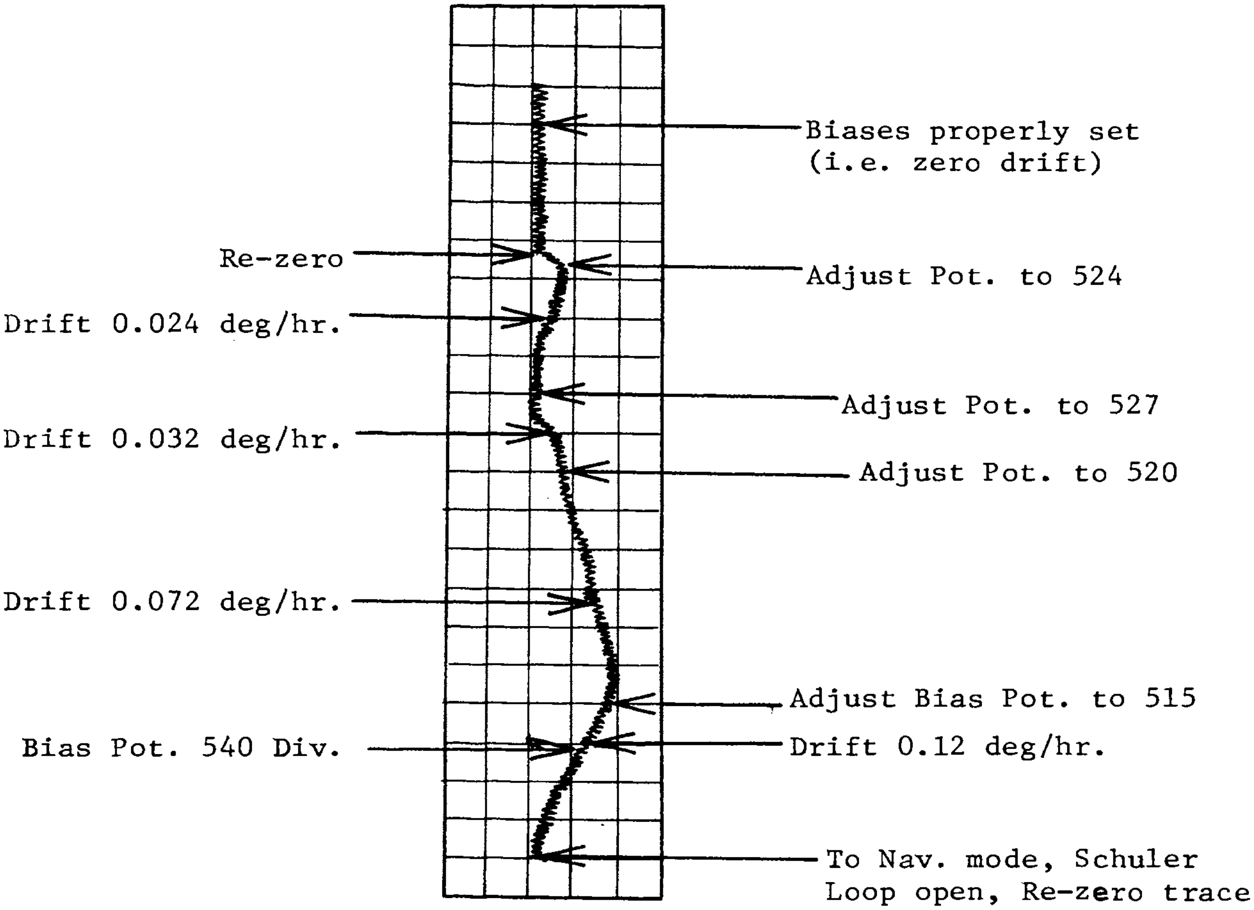
(5) The gyro bias potentiometer is then adjusted in the appropriate direction and amount until the rate of change of voltage at T.P. is zero.

This measurement is actually the result of all of the rate inputs to the north gyro including the effects of latitude error, azimuth misalignment angle and interaxis coupling which were shown to be at least an order of magnitude less than the attainable gyro drift rate accuracy in Chapter IV (i.e. affect on measuring drift is small).

A sample strip chart recording obtained during biasing of an inertial platform is shown in Figure 17.

b. East/West Gyro

Measurement of the east/west gyro drift is complicated due to its association with the gyrocompass loop. Any east/west gyro drift will cause an azimuth misalignment which will cause a component of earth rate to appear as an



North Gyro Drift Run
Litton LN-12 INS Platform
Serial No. 12-3714
Scale: 0.2 mv./cm.
Speed: 0.12 mm./sec.

Recorder: Sanborn 350 with 350-1500 Low Level Preamp

Figure 17 - Strip Chart Recording of a Gyro Drift Measurement

east/west gyro drift rate. The amount of earth rate error introduced into the north gyro due to an azimuth misalignment angle of one degree, according to equation 5.2 is:

$$\delta\omega_N = -\frac{\psi_0^2}{2}\,\Omega\cos\phi = 0.00225 \text{ deg/hr.} \tag{6.1}$$

The earth rate error introduced into the east gyro due to this same one degree azimuth misalignment, according to equation 5.1 is:

$$\delta\omega_E = \psi_0\Omega\cos\phi = 0.261 \text{ deg/hr.} \tag{6.2}$$

Thus, the same azimuth error gives an error in measured drift rate which is two orders of magnitude larger than for the north gyro and precludes correct biasing of the east gyro directly.

The east gyro is drift tested as if it were a north/south gyro by slewing the platform 90 degrees in azimuth and removing the azimuth gyro (gyrocompass loop) input. Even if the azimuth accuracy is poor, the earth rate error is reduced to acceptable levels. An azimuth misalignment of one degree would inject an earth rate error of only 0.00225 deg/hr. into the east gyro in this position, which is well within acceptable tolerance.

c. Azimuth Gyro

The adjustment of the azimuth gyro is similar to that of the level axis gyro except that the change in the platform azimuth angle due to gyro drift is measured instead of an accelerometer output, since there is not an accelerometer whose output can be correlated with azimuth gyro drift. A typical circuit for monitoring and correcting azimuth gyro drift is shown in Figure 18. In this circuit, the control transformer (CT) is adjusted for zero output at the end of alignment prior to going into the gyro bias mode. After going into the gyro bias mode, any azimuth gyro drift results in an output from CT which is recorded on the strip chart recorder. Again, the earth rate error contribution is much smaller than the drift rate error and thus can be neglected.

B. Rotary Tilt Table Alignment

The rotary tilt table alignment is performed by using an aligned validated inertial platform to indicate when the rotary tilt table is leveled and aligned to true north. The inertial platform is tested and calibrated in accordance with the preceding procedure. It is then shut down and re-aligned to minimize the errors which build up as a function of time. After the inertial platform is aligned it is switched to the navigate mode, after which it is ready for use as a coordinate reference possessing the accuracies described in Chapter V.

The entire rotary tilt table assembly is first rotated in azimuth (by rotating the barrel) until the external readout device on the inertial platform indicates that the rotary tilt table's zero degree azimuth is oriented to north. Finer azimuth adjustment is easily accomplished by tapping the edge of the barrel in the appropriate direction with a lead hammer.

The leveling is accomplished by adjustment of the leveling

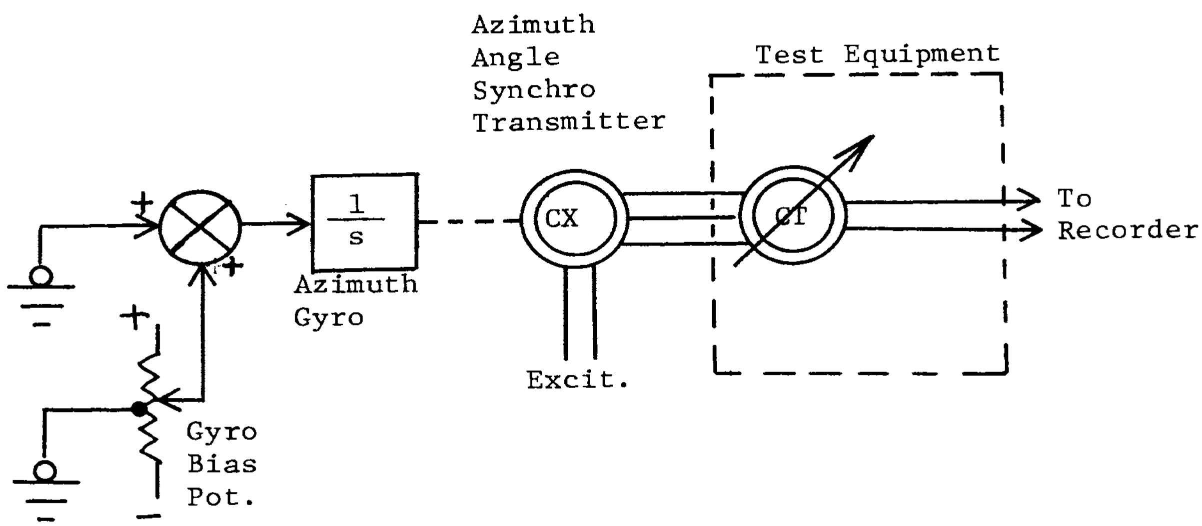


Figure 18

Simplified Circuit for Azimuth Gyro Drift Measurement screws on the leveling plate, using the north/south and east/west axis gyro pickoff synchro readouts to define level. The gyro pickoff synchro outputs should remain at null as the inertial platform is rotated through a complete circle. Both azimuth and level may require readjustment several times before the rotary tilt table alignment is complete.

C. Tilt Table Alignment Accuracy

After using one platform for alignment of the tilt table, the total tilt table alignment error will consist of the inertial platform alignment errors plus the data readout errors. The data readout errors occur in both the platform associated readout equipment and the external readout devices.

The inertial platform may have the ability to level and gyrocompass to the accuracies computed in Chapter V, however, it does not have the ability to transmit the intelligence this accurately to an external readout device. The gyro pickoff synchros which are used to transmit the tilt and azimuth angles of the stable element to the electrical interface of the inertial platform are only accurate to approximately 6 arc minutes[8].

Error is also possible due to the test instrument used to display the synchro output. A very convenient and widely used readout device is a precise angle indicator such as the Clifton Precision Products model 394 which has an accuracy of 6 arc minutes[8]. Better readout accuracy may be achieved if this device is replaced by a synchro bridge such as the Theta Instrument Corporation model SB-11, which is accurate to 10 arc seconds[9].

Assuming the readout error values are 1σ values, then the total level and azimuth tilt table alignment accuracies are determined by combining these readout errors with the inertial platform alignment errors. The resulting standard deviation of the tilt table alignment errors are:

$$\text{(level)} \quad \theta_{ss} = 6 \text{ arc minutes} \tag{6.3}$$

$$\text{(azimuth)} \quad \psi_{ss} = \sqrt{(6)^2 + (6)^2} = 8.5 \text{ arc minutes} \tag{6.4}$$

Note that the 6 arc minute error of the gyro pickoffs becomes the only significant level axis error source.

D. Increasing the Accuracy of the Tilt Table Alignment

The accuracy of the tilt table alignment determined by an inertial platform has been predicated thus far upon the accuracy of alignment of a single platform and the accuracy to which this alignment can be identified externally. The accuracy of this alignment can now be improved by using the data from alignments with several inertial platforms. If it is assumed that the tilt table is aligned with several different platforms and that all error sources are independent, the number of alignments with different inertial platforms required to achieve a given probability, or confidence level, γ, that the true tilt table misalignment, μ, is within an interval k, about the mean value determined by repeated alignment is:[10]

$$n = \left[\frac{c\sigma}{k}\right]^2 \tag{6.5}$$

Where the standard deviation of the alignment error for each alignment is σ and c is the number of standard deviation intervals required to give the required confidence level. This, then gives the number of inertial platform alignments required to obtain a given tilt table alignment accuracy. As an example, the number of inertial platforms required to provide a confidence level of 95% that the tilt table confidence interval will be ± 8.5 arc minutes in azimuth is as follows:

$$n = \left[\frac{(1.96)(8.5)}{(8.5)}\right]^2 = 3.84 \approx 4 \text{ inertial platforms} \qquad (6.6)$$

Figure 19 illustrates how the tilt table azimuth alignment confidence interval for a given confidence level can be reduced by using more inertial platforms. It illustrates that only one inertial platform is required for a 90% confidence level that the tilt table misalignment is known within 15 arc minutes, whereas 780 inertial platforms would be required for a 99.9% confidence level of a 1 arc minute confidence interval.

Figure 20 is a similar illustration showing how the tilt table level alignment confidence interval for a given confidence level can be reduced by using more inertial platforms. It illustrates that only one inertial platform is required for 90% confidence that the tilt table misalignment is within 10 arc minutes whereas 375 inertial platforms are required for 99.9% confidence level of a 1 arc minute confidence interval.

E. Additional Alignment Error Considerations

The use of several inertial platforms makes it necessary to consider the accuracy of repeatability in mounting the inertial platform to the adapter plate, and to reconsider classification of latitude error and readout error as random errors.

The adapter plate is a very accurately fabricated device, as

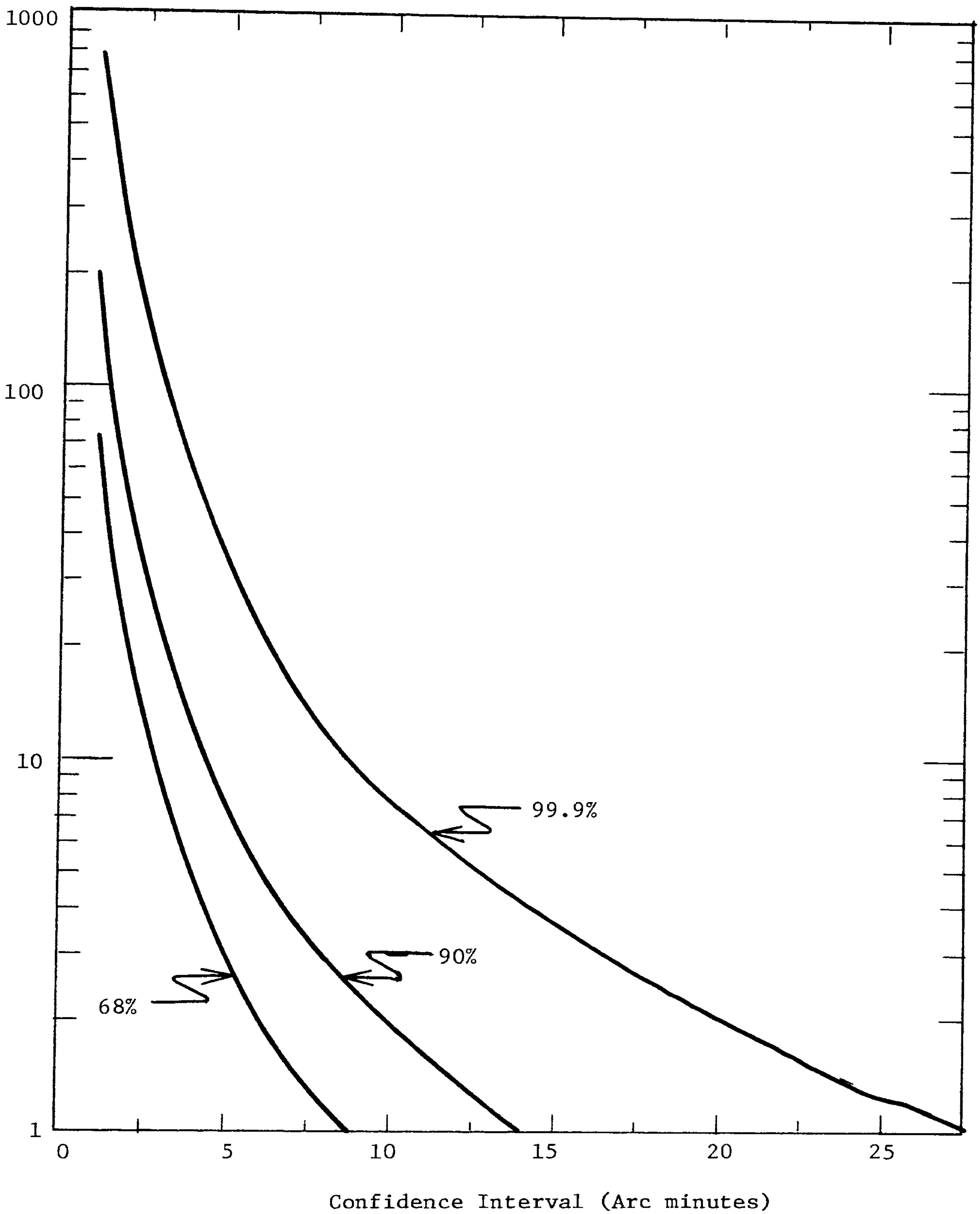


Figure 19 - Azimuth Alignment Conficence Interval

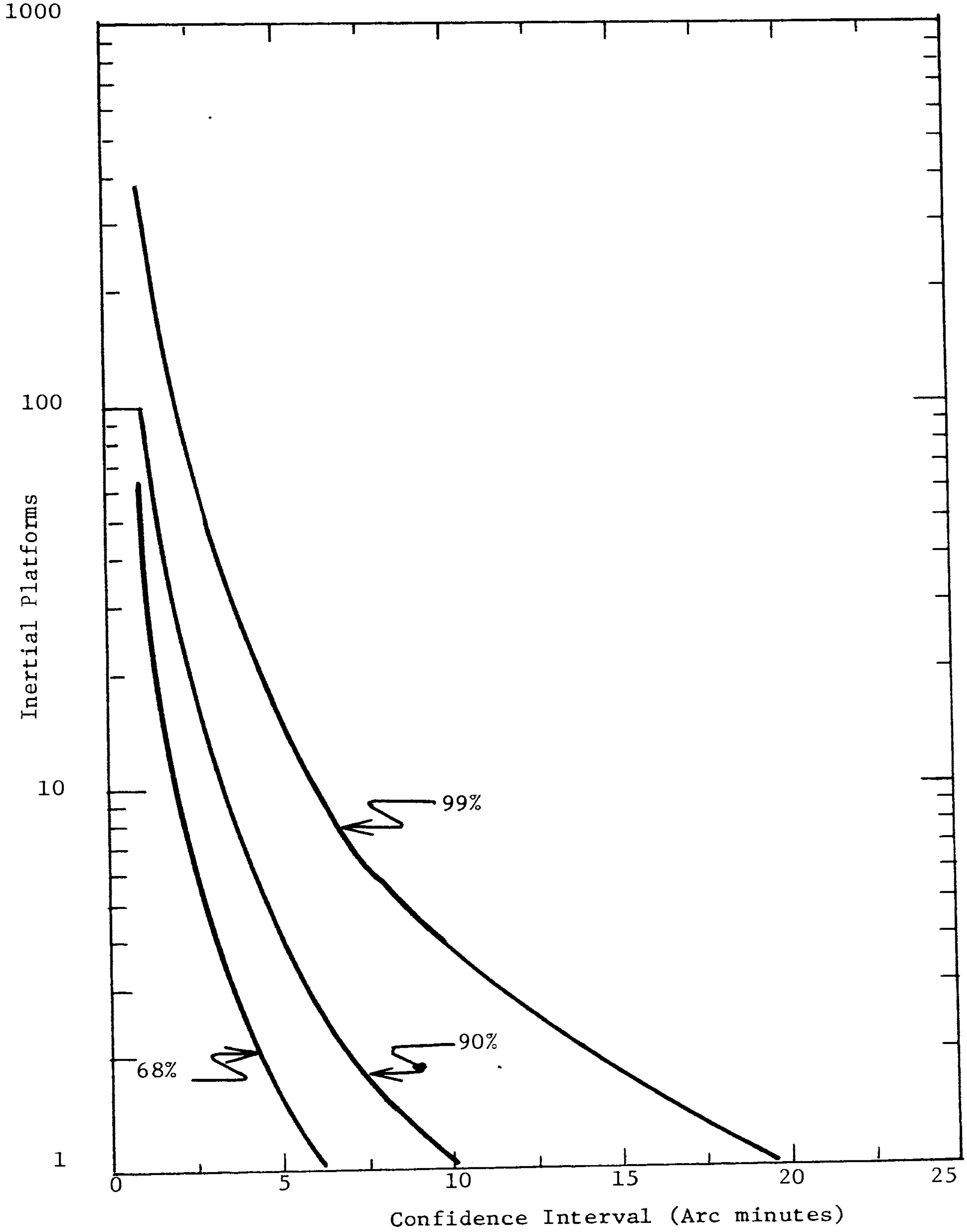


Figure 20 - Level Alignment Confidence Interval

is the mounting surface of the inertial platform. A three-hole mounting pattern is located on the mounting surface of the inertial platform to an accuracy of ± 0.0001 inches, by use of a template. The adapter plate is fabricated with threaded moveable plates, which are located by using the same template. The inertial platforms are mounted using very accurately machined bolts, resulting in a repeatability between platforms of ± 0.0001 inches. Assuming a 10 inch spacing between mounting holes, this represents less than a 20 arc second error.

The latitude error and the readout error are fixed errors for a particular test site and test equipment, therefore they can no longer be considered random errors. They had no significant affect on the error computed for the inertial platform alignment, but it should be remembered that the tilt table alignment accuracy cannot be made better than these errors.

VII.

ALIGNMENT METHOD UTILITY

The utility of the tilt table alignment method proposed in the previous chapter must be assessed. This is now done by considering the various inertial platform test requirements and then evaluating the performance of the aligned tilt table when performing these tests.

A. Inertial Platform Test Requirements

The requirements of particular tests are evaluated here, to establish acceptance criteria for both azimuth and level alignment.

1. Gyrocompass Test

The purpose of the gyrocompass test is to ascertain whether the inertial platform levels and gyrocompasses to acceptable accuracies. The 1σ accuracy, as identified at the electrical interface, to which an in-specification inertial platform should align per equations 6.3 and 6.4 is ± 6 minutes in level and ± 8.5 minutes in azimuth.

2. Accelerometer Scale Factor Test

The accelerometer scale factor test is performed to ascertain whether the accelerometers produce correct outputs for known gravity component inputs. The accelerometers are mounted on the inertial platform's stable element and would not normally tilt with the outer case, but course align is accomplished by caging the stable element in level to the outer case and holding the inertial platform in this mode. Both the level and azimuth positions are displayed externally, subject to the accumulated readout errors.

The rotary tilt table is initially set to the zero tilt and zero azimuth position, and the inertial platform is initially aligned and then caged to the outer case in both level and azimuth. The output of the vertical accelerometer at this point should be 1.0 $\pm$ 0.00001 g which is the zero stability (see Table I). The component of gravity sensed is g $\cos\theta$, and θ may vary as much as 11 arc minutes around zero and stay within this tolerance.

The rotary tilt table is then tilted to 30° 00' which puts the north accelerometer in a 30° pitch down position and the vertical accelerometer 30° off vertical. The resultant gravity components sensed should be $-g \sin 30° = -0.5g$ for the north accelerometer and $-g \sin 60° = -0.866g$ for the vertical accelerometer. The accelerometer scale factor variation per Table I is 0.03% per year. Since the maximum acceleration to be measured in this test is 1g, the level angle must be known well enough to provide 0.0003g sensitivity. The resultant acceptable range of acceleration tolerances are 0.5 $\pm$ 0.0003g for the north accelerometer and 0.866 $\pm$ 0.0003g for the vertical accelerometer. The level angle error which would cause this value of error is that which causes the sine of the angle to deviate 0.0003 from the value for 30° and 60° respectively. Accordingly arc sin (0.500 + 0.0003) is 30°1.2' and the arc sin (0.866 + 0.0003) is 60°2'. This illustrates that a 6 arc minute error in the level angle of the rotary tilt table is greater than the full tolerance margin for accelerometer scale factor linearity.

The east accelerometer is tested by rotating the rotary tilt table in azimuth by 90° such that the east accelerometer is pitched down 30°. Then rotation of the rotary tilt table to 180° and 270° permits testing both accelerometers for opposite sense acceleration inputs. The tilt table can then be tilted to 60° and the rotary tilt table rotation repeated for another set of measurements. Throughout, the results and accuracy requirements accordingly will be consistent with the result derived above.

3. Gyro Drift Test

The gyro drift test is performed to identify gyro drift rates. The heading of the inertial platform must be compared to the rotary tilt table to assure that a component of earth rate is not identified as gyro drift. The acceptance criteria for the gyrocompass test is sufficiently accurate for the gyro drift test, as has been previously shown when considering platform validation.

4. Synchro Null Test

The synchro null test is performed to ascertain that the inertial platform synchro electrical nulls are within specified limits. The rotary tilt table is adjusted to zero azimuth and level, and each of the gyro pickoff synchro angles is measured. The acceptance criteria is the same as for the gyrocompass test.

5. Synchro Linearity Test

The synchro linearity test determines that the gyro pickoff synchros accurately indicate the correct angular position. The rotary tilt table is adjusted to various angles and the external readout of tilt table and inertial platform should agree. The acceptance criteria is the same as for the gyrocompass test.

6. Test Requirement Summary

The inertial platform azimuth requirement is determined by the requirements of the gyrocompass test. The 1σ azimuth accuracy established by this test is 8.5 arc minutes. The level axis requirement is established by the accelerometer scale factor test and the 1σ accuracy requirement is 1.2 arc minutes.

B. Alignment Method Evaluation

The inertial platform test requirements as specified in the previous section can now be utilized to evaluate the utility of the proposed alignment method. The accelerometer scale factor test requires a significantly higher level accuracy than the inertial platform's gyro pickoffs can provide and spirit levels with the required accuracy are available and easily utilized. Therefore, the rotary tilt table level alignment will be accomplished by means of the spirit level. The remainder of this utility evaluation will address itself to the azimuth alignment of the rotary tilt table.

The acceptability of the rotary tilt table alignment accuracy obtained by using multiple inertial platforms for alignment is evaluated on the basis of how well it identifies good (in specification) or bad (out of specification) inertial platforms as measured by the gyrocompass test azimuth requirements. In order to perform this evaluation, the following probabilities are defined:

$P_a(G)$ = Probability that a platform tests good with a perfect tilt table and an acceptance interval of $\pm a$.

$P_a(\overline{G})$ = Probability that a platform tests bad with a perfect tilt table and an acceptance interval of $\pm a$.

$P_a(A)$ = Probability of accepting a platform when tested with an erroneous tilt table and an acceptance interval of $\pm a$.

$P_a(\overline{A})$ = Probability of rejecting a platform when tested with an erroneous tilt table and an acceptance interval of $\pm a$.

The probability of accepting an inertial platform given that it is good, $P_a(A|G)$, and the probability of accepting a platform given that it is bad, $P_a(A|\overline{G})$, will be determined to evaluate the performance of the tilt table alignment method. $P_a(A|G)$ is defined[11]:

$$P_a(A|G) = \frac{P_a(A,G)}{P_a(G)} \tag{7.1}$$

where $P(A,G)$ is the joint probability of A and G. $P(A|\overline{G})$ is defined as follows:

$$P_a(A|\overline{G}) = \frac{P_a(\overline{G}|A)\ P_a(A)}{P_a(\overline{G})} \tag{7.2}$$

where:

$$P_a(\overline{G}) = 1 - P_a(G)$$

and:

$$P_a(\overline{G}|A) = 1 - \frac{P_a(A,G)}{P_a(A)}$$

Thus from equation 7.2

$$P_a(A|\overline{G}) = \frac{P_a(A) - P_a(A,G)}{1 - P_a(G)} \qquad (7.3)$$

Therefore, $P_a(G)$, $P_a(A)$ and $P_a(A,G)$ need to be computed.

Assuming the inertial platform errors have a normal distribution with zero mean, the following probability density function applies:

$$f_X(x) = \frac{1}{\sigma_x \sqrt{2\pi}} e^{-\frac{x^2}{2\sigma_x^2}}$$

where x is the value of the inertial platform error, and σ_x^2 is the variance of the inertial platform error. If the platform is defined as good when the magnitude of the error is less than or equal to some acceptance criteria, a, then:

$$P_a(G) = \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-a}^{a} e^{-\frac{x^2}{2\sigma_x^2}} dx$$

$$= 2[\, N(\frac{a}{\sigma_x}) - N(0) \,] \qquad (7.4)$$

where: $$N(c) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{c} e^{-\frac{\kappa^2}{2}} d\kappa$$

The tilt table error is assumed to be normally distributed with zero mean and thus has the following normal probability density function:

$$f_Y(y) = \frac{1}{\sigma_y \sqrt{2\pi}} e^{-\frac{y^2}{2\sigma_y^2}}$$

where y is the value of the tilt table error and σ_y^2 is the variance of the tilt table error. The total error during a test is the sum of the platform and tilt table errors:

$$Z = X + Y \tag{7.5}$$

therefore, since the errors are independent:

$$f_Z(z) = \frac{1}{\sigma_z \sqrt{2\pi}} e^{-\frac{z^2}{2\sigma_z^2}} \tag{7.6}$$

where $$\sigma_z^2 = \sigma_x^2 + \sigma_y^2 \tag{7.7}$$

Assuming the inertial platform acceptance limit once again is $\pm a$, then

$$P_a(A) = \frac{1}{\sigma_z \sqrt{2\pi}} \int_{-a}^{a} e^{-\frac{z^2}{2\sigma_z^2}} dz$$

$$= 2 \left[N\left(\frac{z}{\sigma_z}\right) - N(0)\right] \tag{7.8}$$

The joint probability, $P_a(A,G)$ needs to be evaluated to complete the solution.

$$P_a(A,G) = P[(-a \leq X \leq a) \text{ and } (-a \leq Z \leq a)]$$

since $Z = X + Y$

$$P_a(A,G) = P \; [(-a \leq X \leq a) \text{ and } (-a - X) \leq Y \leq (a - X)]$$

$$= \int_{-a}^{a} \int_{-a-x}^{a-x} f_{X,Y}(x,y) \, dy \, dx \tag{7.9}$$

Since X and Y are independent and normal, their joint probability density function is:

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_x\sigma_y} e^{-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2}\right)} \tag{7.10}$$

therefore

$$P_a(A,G) = \frac{1}{\sqrt{2\pi}} \int_{-\frac{a}{\sigma_x}}^{\frac{a}{\sigma_x}} e^{-\frac{\kappa^2}{2}} \left[N\left(\frac{a-x}{\sigma_y}\right) - N\left(\frac{-a-x}{\sigma_y}\right) \right] d\kappa \tag{7.11}$$

Equation 7.11 cannot be integrated directly since N is not available in closed form, therefore it can only be evaluate numerically by replacing the integral with the sum:

$$P_a(A,G) = \sum_{i=1}^{n} \left\{ \left[\frac{1}{2\pi} e^{-\frac{\kappa^2}{2}}\right] \left[N\left(\frac{a-x}{\sigma_y}\right) - N\left(\frac{-a-x}{\sigma_y}\right) \right] \Delta\kappa \right\} \tag{7.12}$$

where: $x_i = -a + (i - \frac{1}{2})\,\Delta x$

$n = \frac{2a}{\Delta x}$

$\kappa_i = \frac{x_i}{\sigma_x}$

and $\Delta\kappa = \frac{\Delta x}{\sigma_x}$

This numerical integration can be performed by looking up the values of the components in a normal distribution function table for each value of i and summing the results.

The probabilities $P_a(A|G)$ and $P_a(A|\bar{G})$ are computed for rotary tilt table alignments using 1, 4. 16 and 64 inertial platforms to perform the alignment and for inertial platform acceptance intervals of ±17 arc minutes (2σ) and ±8.5 arc minutes (1σ). The standard deviations for the rotary tilt table alignment using 1, 4, 16 and 64 inertial platforms are as follows:

$$\sigma_{y1} = \frac{\sigma_x}{\sqrt{n}} = \frac{(8.5)}{1} = 8.5 \text{ arc min.} \quad (7.13)$$

$$\sigma_{y4} = \frac{(8.5)}{\sqrt{4}} = 4.25 \text{ arc min.} \quad (7.14)$$

$$\sigma_{y16} = \frac{(8.5)}{\sqrt{16}} = 2.125 \text{ arc min.} \quad (7.15)$$

$$\sigma_{y64} = \frac{(8.5)}{\sqrt{64}} = 1.0625 \text{ arc min.} \quad (7.16)$$

Using the relation defined by equation 7.7, the values of σ_Z are:

$$\sigma_{Z1} = \sqrt{72.25 + 72.25} = 12.1 \quad (7.17)$$

$$\sigma_{Z4} = \sqrt{72.25 + 18} = 9.49 \quad (7.18)$$

$$\sigma_{Z16} = \sqrt{72.25 + 4.5} = 8.77 \quad (7.19)$$

$$\sigma_{Z64} = \sqrt{72.25 + 1.13} + 8.56 \quad (7.20)$$

The probability that an inertial platform is good when tested with a 17 arc minute (2σ) acceptance interval is computed from equation 7.4 and normal distribution function tables as follows:

$$P_2(G) = 2\ [N(2) - N\ (0)\ ] = 0.9544 \quad (7.21)$$

Now using equation 7.8 and the normal distribution function table, the related value of $P_2(A_1)$ (i.e. a 2σ acceptance interval and a 1 platform alignment) is:

$$P_2(A_1) = 2\ [\ N(1.41) - N\ (0)\ ] = 0.8414 \quad (7.22)$$

The probabilities of acceptance using 4, 16 and 64 platform alignments are computed in a similar manner and presented in Table III.

The joint probabilities for the platform being good and accepted for rotary tilt table alignments using 1, 4, 16 and 64 inertial platforms are also tabulated in Table III.

Platforms Used for Alignment	$P_2(G)$	$P_2(A)$	$P_2(A,G)$
1	0.9544	0.8414	0.8272
4	0.9544	0.9266	0.9134
16	0.9544	0.9476	0.9390
64	0.9544	0.9534	0.9492

Table III

Summary of probabilities for 2σ Acceptance Interval

Assume that the inertial platform acceptance criteria is reduced to 8.5 arc minutes (1σ). Then from equation 7.4, $P_1(G)$ is:

$$P_1(G) = 2(0.8413 - 0.5000) = 0.6826 \tag{7.23}$$

The values of σ_y and σ_z remain as computed in equations 7.17 through 7.20, but the new integral limits for P(A) as shown in equation 7.8 yields the following for a 1 platform alignment:

$$\begin{aligned} P_1(A_1) &= 2\ [\ N(0.71) - N(0)\ ] \\ &= 2\ (0.7611 - 0.5000) = 0.5222 \end{aligned} \tag{7.24}$$

The probabilities of acceptance using 4, 16 and 64 platform alignments, which are computed in the same manner, are presented in Table IV.

The joint probabilities for the platform being good and accepted for rotary tilt table alignments using 1, 4, 16 and 64 inertial platforms are also tabulated in Table IV.

The resulting probabilities for accepting an inertial platform, given that it is good, $P_a(A|G)$, and accepting it, given that it is bad, $P_a(A|\overline{G})$, is computed for a 2σ acceptance interval from the data presented in Table III. The computations for a 2σ acceptance interval using a 1 platform alignment are as follows:

$$P_2(A_1|G) = \frac{P(A_1,G)}{P(G)} = \frac{0.8272}{0.9544} = 0.8667 \tag{7.25}$$

$$P_2(A_1|\overline{G}) = \frac{P(A_1) - P(A_1,G)}{1 - P(G)} = \frac{0.8414 - 0.8272}{1 - 0.9544} = 0.3114 \tag{7.26}$$

The probabilities of acceptance using 4, 16 and 64 platform alignments, which are computed in the same manner, are presented in Table V.

Platforms Used for Alignment	$P_1(G)$	$P_1(A)$	$P_1(A,G)$
1	0.6826	0.5222	0.4292
4	0.6826	0.6266	0.5600
16	0.6826	0.6680	0.6290
64	0.6826	0.6778	0.6996

Table IV

Summary of Probabilities for 1σ Acceptance Interval

Number of Platforms Used for Alignment	2σ Acceptance Interval		1σ Acceptance Interval	
	$P_2(A\|G)$	$P_2(A\|\overline{G})$	$P_1(A\|G)$	$P_1(A\|\overline{G})$
1	0.8667	0.3114	0.6280	0.2932
4	0.9570	0.2894	0.8203	0.2100
16	0.9838	0.1885	0.9214	0.1220
64	0.9945	0.0921	0.9963	0.0564

$P_a(A|G)$ = Probability of acceptance given a good platform

$P_a(A|\overline{G})$ = Probability of acceptance given a bad platform

Table V

Evaluation of the Tilt Table Alignment Method

The resulting probabilities for accepting an inertial platform, given a good platform, $P_a(A|G)$, and given a bad platform, $P_a(A|\overline{G})$, may now be computed for the 1σ acceptance interval from the data presented in Table IV. The computations for the 1 platform alignment are as follows:

$$P_1(A_1|G) = \frac{P_1(A_1,G)}{P_1(G)} = \frac{0.4246}{0.6826} = 0.6280 \tag{7.27}$$

$$P_1(A_1|\overline{G}) = \frac{P_1(A_1) - P_1(A_1,G)}{1 - P_1(G)}$$

$$= \frac{0.5222 - 0.4292}{1 - 0.6826} = 0.2932 \tag{7.28}$$

The probabilities of acceptance using 4, 16 and 64 platform alignments, which are computed in the same manner, are presented in Table V for comparison with the data for the 2σ acceptance interval.

Table V compares the probabilities of accepting good and bad platforms as the number of platforms used for alignment of the tilt table is increased. It also illustrates the affect of varying the acceptance interval. As the number of platforms used for the alignment increases from 1 to 4, the performance of the facility improves very rapidly. The rate of improvement is much slower as the number increases to 16 and then to 64. The tighter 1σ acceptance interval essentially shifts 27% of the platforms from acceptable to not acceptable status. This results in an overall reduction in accepting both good and bad platforms. As expected, greater tilt table accuracy is required in order to successfully utilize the tighter tolerance.

VIII.

CONCLUSIONS

It has been determined that an inertial platform will self align, and thus define a coordinate system, to predictable accuracies if its inertial components are operating within specification tolerances and if local latitude is known. This self aligned inertial platform can then be used to align a rotary tilt table. The probable alignment errors have been analyzed and specific errors determined for a particular system mechanization. The alignment accuracies were improved through use of several inertial platforms. Actual testing requirements were evaluated to establish realistic inertial platform acceptance interval tolerances and it was determined at this point that the inertial platform should not be used for level alignment of the rotary tilt table. The acceptability of the rotary tilt table azimuth alignment was then further evaluated relative to probabilities of accepting good (in specification) inertial platforms and rejecting bad (out of specification) ones, when performing the gyrocompass test since its accuracy requirement was the limiting azimuth accuracy requirement. This comparison showed that the rotary tilt table azimuth alignment using four inertial platforms is marginally acceptable.

The unacceptability of the level alignment of a rotary tilt table using an inertial platform is of little consequence since leveling the tilt table is not particularly complicated. The Watts model TB-19 Angle Level Gage is a spirit level with a bubble sensitivity of 5 arc seconds per division. This device is of sufficient accuracy to level the rotary tilt table to the desired accuracy of ten times better than 1.2 arc minutes, such that rotary tilt table level errors may be neglected in establishing acceptable tolerances for inertial platform tests.

The results of the azimuth alignment using four inertial platforms shows 96% probability of accepting a good inertial platform with a 2σ acceptance interval, with a 29% probability of accepting a bad platform. When the inertial platform acceptance interval is reduced to σ, the percentages are lower, 82% and 21%. As expected, the tighter test tolerance requires a more accurate tilt table alignment with the greatest effect on the acceptance of good platforms but the conditions follow the same trend of improvement as more inertial platforms are used for the alignment. It is concluded that a rotary tilt table aligned using four verified inertial platforms is acceptable as an interim alignment until the azimuth accuracy can be improved.

The azimuth accuracy of the rotary tilt table can be improved as part of its normal use. This may be accomplished by keeping a record of the gyrocompass test results and periodically improving the azimuth alignment. After the initial four inertial platform alignment, it may be used normally and then after 16 more inertial platforms have been tested the rotary tilt table could be readjusted according to the average azimuth misalignment as determined by the additional gyrocompass alignments which are performed after repair and calibration.

The procedure for correcting the rotary tilt table azimuth would be as described in Chapter VI, using the readout from an installed inertial platform to indicate the "difference angle" during rotary tilt table azimuth correction. The rotary tilt table can then be used with the confidence of a 16 inertial platform alignment as shown in Table V.

As the rotary tilt table is used for additional maintenance, a running record of gyrocompass test results will eliminate the need for periodic re-alignment and after sufficient data becomes available, further improvement of the alignment can be accomplished.

It is, therefore, the conclusion of this thesis research that it is practical to use inertial platforms, of the type to be tested, to align a rotary tilt table in azimuth, but the level alignment should be accomplished by use of a spirit level.

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