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Practical application of digital computer to distribution systems

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PRACTICAL APPLICATION OF DIGITAL COMPUTER TO DISTRIBUTION SYSTEMS

BY

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ABSTRACT

Given a three phase ac power distribution system, a digital computer program is presented which determines a complete steady state solution for a given condition. The network data is processed by the computer into an admittance matrix, and these parameters are stored in a manner that enables the computer program to operate upon them as the coefficient of a simultaneous set of non-linear equations. This set of equations are solved by the Gauss-Seidel Iterative Process improved by a modification of the relaxation method. Practical experience with the distribution system of the Davenport, Iowa, plant of the Aluminum Company of America (Alcoa), is also described. Results are briefly discussed with relevance to the importance of the inclusion of an improved data for future work.

PREFACE

In the last decade, technological advances and widespread use of digital computers has stimulated researches of numerical methods into the problem of power systems. The development of the large size computer has paralleled the growth of utility systems as a result of interconnected operation, and it has also allowed the power system engineer to utilize more detailed system models for such studies as load flow, short circuit and stability. It is hoped that the present work will serve as a small contribution to such a big effort.

The author is deeply indebted to Dr. J.D. Morgan, Associate Professor of Electrical Engineering, for his technical assistance.

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I. INTRODUCTION

A load flow study is used to determinate the power flow and voltage at various points in an electric network under existing or contemplated condition of normal steady state operation. Load studies are essential in planning future development of a system because satisfactory operation of the system depends on knowing the effects of connection of new loads, new generating stations and new lines before they are installed.

Load flow studies are made on both digital computers and calculating boards. Digital computers have generally replaced a-c calculating boards and have proved to be economical for nearly all load flow studies.

From a mathematical point of view the power flow problem involves the solution of a set of nonlinear simultaneous equations, effectively the digital solution has been approached through formulation of the network equations on either the node or loop basis. In impedance form, using the bus frame of reference, the equation describing the performance of a network is

$$\bar{E}_{BUS} = Z_{BUS} \bar{I}_{BUS} \qquad (I\text{-}1)$$

in admittance form

$$\bar{I}_{BUS} = Y_{BUS} \bar{E}_{BUS} \qquad (I\text{-}2)$$

Using the loop frame of reference, the network performance equation in impedance form is

$$\bar{E}_{LOOP} = Z_{LOOP}\ \bar{I}_{LOOP} \qquad \text{(I-3)}$$

in admittance form

$$\bar{I}_{LOOP} = Y_{LOOP}\ \bar{E}_{LOOP} \qquad \text{(I-4)}$$

Experience has shown that the nodal equations seem to provide the best program. The present work deals with the solution of Equation (I-2); that is, the admittance form of the network equation on a nodal basis.

If P_p and Q_p are the scheduled real and reactive power entering the system at bus p then:

$$P_p - jQ_p = E_p^* I_p \qquad \text{(I-5)}$$

and the current is

$$I_p = \frac{P_p - jQ_p}{E_p^*} \qquad \text{(I-6)}$$

where I_p is positive when flowing into the system.

Now, notice that each element of $\bar{I}_{BUS}$ in Equation (I-2) is equal to

$$I_p = \sum_{q=1}^{n} Y_{pq} E_q \qquad \text{(I-8)}$$

where n is the number of busses and Y_{pq} is an element of the bus admittance matrix Y_{BUS}. Substituting (I-6) in (I-8)

$$\frac{P_p - jQ_p}{E_p^*} = \sum_{q=1}^{n} Y_{pq} E_q \qquad \text{(I-9)}$$

and solving for E_p

$$E_p = \frac{1}{Y_{pp}} \left(\frac{P_p - jQ_p}{E_p^*} - \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} E_q \right) \qquad \text{(I-10)}$$

$$p = 1,2,\ldots,n$$

This is the mathematical approach to the load flow problem. A set of nonlinear simultaneous equations have been developed. Solution techniques are deferred until Chapter III.

Notice that since Equation (I-10) applies only at busses where real and reactive power are specified an additional step is necessary at busses where voltage magnitude is to remain constant.

II. REVIEW OF THE LITERATURE

The load flow problem is not a new one; even more, it is essentially coincident with the development of the electrical power system. Long hand calculation was the only way to solve load flows in the earliest days of system development. These methods were tedious and time consuming for the growing power utility. Since 1929 a-c calculating boards have been relied on by engineers for making load studies. The advent of the large high-speed digital computer provided a second means of making such studies on large systems, although the fifteenth a-c calculating board to be constructed in the United States and Canada was installed in 1959, and by this time new needs were taken care of entirely by digital computers.

The literature describing methods of analysis suitable for use with computers in the power system load flow problem has grown since 1958 until the papers can now be numbered in hundreds. All methods used can be classified according to the divisions commonly considered in linear algebra, namely as direct and iterative methods. The direct methods involving matrix inversion and a final iterative procedure to deal with the restraints at the nodes have advantages for smaller networks. For large networks the completely iterative method is preferable.

One of the first efforts to use a digital method was made by L.A. Dunstan [1], [2]. He described a simple direct method of making load flow studies on punched-card accounting machines. All computing was done in a "master deck" which consisted of a set of cards for each step in the problem. Each card in the master deck were coded and prepunched to control the calculator using an IBM 602A. Applying the method to a 22 busses system required about 4 hours to run from a schedule of load, generation and transformer taps to a printed circuit diagram. J.M. Bennett [3] offered a compact elimination process of triangular solution, utilizing the symmetric properties of the Y matrix.

J.M. Henderson [4] presented in 1955 an iterative method of analysis based on concepts presented by L.A. Dunstan. The distinguishing feature of this paper is the iteration process used to determine successive approximations. Although the method was used successfully to solve very large networks, solution on the card programmed calculator was economic with respect to the network analyzer only in analyzing relatively small systems, those with a maximum of 5 loops and 24 lines.

J.B. Ward and H.W. Hales work [5] is an extremely important paper in the development of load flow computer programs. Solutions as presented here follow four major steps:

1. Description of the network connection and impedances in the form of a list of parameters.

2. Iterative solution for terminal voltages which satisfy the prescribed terminal conditions.

3. Computation of complete terminal information i.e. real and reactive powers, voltage magnitudes and voltage phase angles.

4. Computation of individual line flows.

A later work was that of A.F. Glimm and G.W. Stagg [6]. This work covers much the same areas as does Ward and Hale's work but goes into discussion of developing methods to improve speed of convergence of the iterative solution. The program and the associated routines were developed with the specific requirement that a planning study could be performed in a semiautomatic manner, thus the procedure described in this paper forms the initial steps toward the goal of better economy in the calculation of power system load flows.

J. Brown and W.F. Tinney [7] devoted their paper to a presentation of acceleration methods and discussion of their effectiveness. This paper describes some of the more significant results obtained from testing various schemes for accelerating the nodal iterative method.

Relaxation methods were suggested by R.H. Jordan [8]. Relaxation methods differ from the other iterative methods in that the order in which the unknowns are calculated may be varied in each iteration. A trial solution with rough estimates of voltages will produce a set of residuals [18] and the next step, for example, might be to reduce the largest residual to zero, or by a greater or lesser amount (over or under relaxation), or to change a group of residuals (block relaxation).

Perhaps the next significant work was that of H.P. St. Clair and G.W. Stagg [9]. This paper describes the development of a new load flow program including additional automatic features to increase the speed and usefulness of its application to system planning problems.

Additional automatic features were described by M.S. Dyrkacz and F.J. Maginnis [10].

1. Automatic tap selection to control voltage.
2. Automatic control of power interchange between areas.

A paper entitled, "Digital Computation of Power Flow - Some New Aspects" by H.W. Hale and R.W. Goodrich [11] considers the possibility of reduction of the computation time by reducing the number of iterations through altering the fundamental approach to the problem. A common form of network Equation (I-2) is

$$\begin{bmatrix} I_i \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{ii} & Y_{in} \\ Y_{ni} & Y_{nn} \end{bmatrix} \begin{bmatrix} E_i \\ E_n \end{bmatrix} \qquad \text{(II-1)}$$

Thus, the terminals are separated into two groups, one group consisting of 1 to i and the other of (i+1) to n. Equation (II-1) may now be manipulated into:

$$\begin{bmatrix} I_i \\ E_n \end{bmatrix} = \begin{bmatrix} (Y_{ii}-Y_{in}Y_{nn}^{-1}\,Y_{ni}) & Y_{in}Y_{nn}^{-1} \\ -Y_{nn}^{-1}\,Y_{ni} & Y_{nn}^{-1} \end{bmatrix} \begin{bmatrix} E_i \\ I_n \end{bmatrix} \qquad \text{(II-2)}$$

where the inverse Y_{nn}^{-1} can be found by any of the direct methods of inversion. The matrix relating current and voltage is thus a mixed matrix of impedance elements, admittance elements, and dimensionless elements. All controlled busses are placed in the first i equations, leaving the rest of the equation to be solved with the impedance relationship Y_{nn}^{-1}. The nodal requirement, in terms of P-jQ and voltages, may be substituted for the currents and the equation solved by an iterative method. Hybrid methods reduce the number of iterations, as is shown in this paper, however it is recognized that a reduction in the number of iterations is not the complete answer to the problem of reducing computing time.

J.E. Van Ness [12], [13] made a substantial contribution to the understanding of the characteristic of the iterative solution. He successfully demonstrated that the simultaneous application of correction equations for all busses results in a reduction in the number of iterations required for convergence. These correction equations were evaluated in terms of the independent variable of the load flow problem.

"Elimination Methods for Load Flow Studies" by J.E. Van Ness and J.H. Griffin [14] presents a method for certain types of systems, especially those with negative reactance, which will not converge by the iterative method even though a solution does exist.

"Power System Analysis" published in 1962 [15] covers the basic methods of analysis of power systems and provides a comprehensive introduction to digital solutions of the load flow problem.

A similar work is that of B.M. Weedy [16], where use of the computer is emphasized when dealing with large networks. The book also gives a presentation of essential basics of power system operation.

An outstanding work concerning digital computer techniques was published by Glenn W. Stagg and A.H. El-Abiad [17]. In brief, this book covers network topology, algorithms for the formation of network matrices, short circuit, load flow, and transient stability studies.

Several papers have been published concerning the determination of optimum accelerating factors. For the interested reader reference [18] and [19] in the bibliography will provide much information on this topic.

This discussion would be incomplete if mention was not made of the reference used to explain the requisite background numerical analysis theory. McCalla [20], Lewis and Price [21] and Jenning [22] were consulted for this purpose.

III. COMPONENTS OF THE LOAD FLOW PROBLEM APPLIED TO DISTRIBUTION SYSTEMS

A. Type of busses and data

Three types of busses are represented:

1. swing feeder or bus
2. voltage controlled bus
3. load bus or points

The first is an essential one. Effectively one feeder in the primary circuit or subtransmission system is selected to act as the swing feeder. At this bus the voltage magnitude and phase angle are specified. Since the losses are unknown, power input can not be specified at this terminal, and the feeder input serves to supply whatever output is necessary to balance the other terminals and the losses.

It is practice in the operation of many distribution systems to control the voltage at all terminals having scheduled input. These busses are known as voltage controlled busses where scheduled real power, range of reactive power and voltage magnitude are specified.

The third type of terminal is a common one. The real and reactive powers are specified at a load bus or point. Studies will normally be performed for normal load conditions; however, minimum and maximum load conditions have importance when an attempt is made to schedule the system for optimum economy.

In summary, the following information must be defined to perform a load study of a distribution system:

1. One line diagram of the system
2. Normal operating conditions
3. Impedance or susceptance of all network elements
4. Real and reactive demand at all load points
5. Voltage magnitude, scheduled real power and range of reactive power as well as scheduled reactive power of all feeders, except the swing feeder
6. Voltage magnitude and phase angle at the swing feeder
7. Transformer tap settings

Additional information may be required according to features of the system. Following is the information to be derived in the study:

1. Real and reactive power flow at each end of of all line elements
2. Voltage magnitude and phase angle at all points
3. Real and reactive power loading on the swing feeder

In summary, the information in items 1, 2 and 3 define the network, the independent variables are the quantities mentioned in items 4 through 8, and the dependent variables are the quantities to be founded as results of the study.

B. Modeling the distribution system components

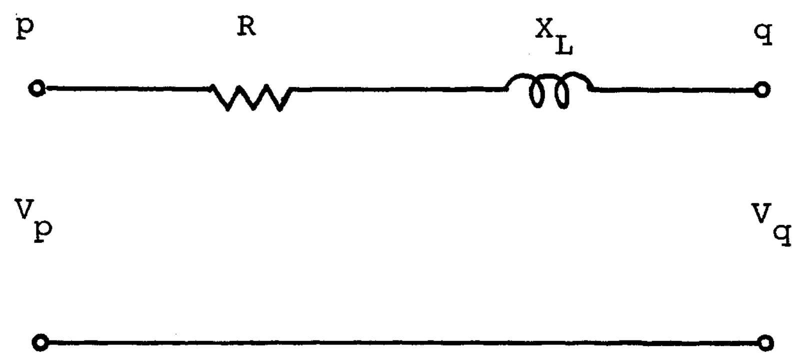
Several approximations are necessary in order to develop a mathematical model to represent the physical system. In a distribution system most of this design involves primary and secondary distribution lines and transformers. Dealing with steady state operation it seems reasonable to model lines and transformers in terms of their equivalent circuits.

1. Representation of Lines

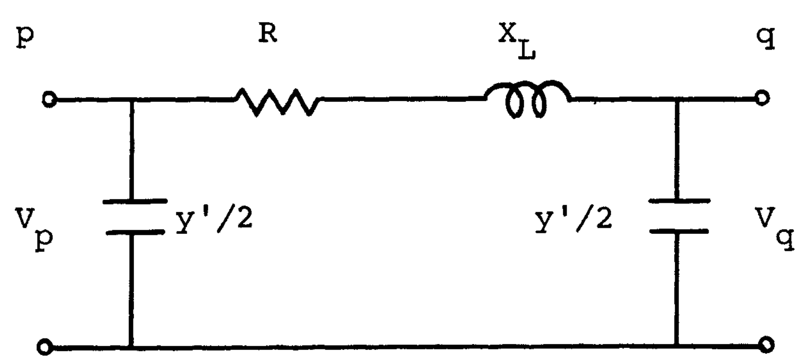
The manner in which lines and cables are represented depend very much on their length and the accuracy required.

For overhead lines in primary circuits a short line representation may be used as shown in Figure (III-1). The effect of capacitance is slight and usually neglected. In longer lines in subtransmission systems of higher voltage capacitance becomes increasingly important. The equivalent circuit is represented with sufficient accuracy by the nominal π equivalent circuit as shown in Figure (III-1b), where y' is the shunt admittance of the line. When this circuit is applied to the load flow program the self admittance elements of each terminal are modified to consider the effect of charging currents, which is the current to ground associated with the capacitance of the line.

The equivalent circuits shown in Figure (III-1) for an overhead line with distributed constants can also be used



(a)



(b)

Figure III-1. Line Equivalent Circuits

a) Short Line Representation

b) Medium Length Representation

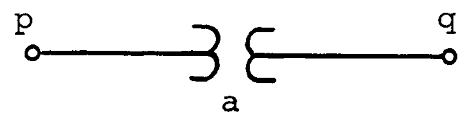
for three-conductor cables. The selection of the correct model depends on its length; however, due to the high capacitance of cables the charging current, especially at high voltages, is an important factor. In a primary and secondary distribution system, the effect of capacitance is usually neglected.

2. Representation of Transformers

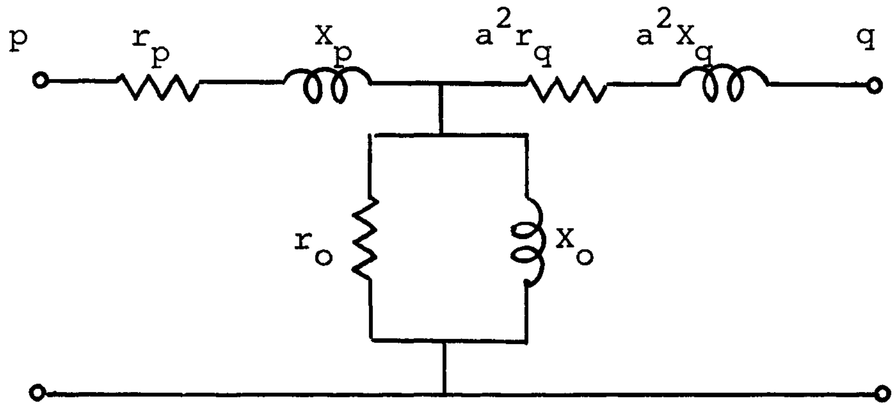
One line representation of a distribution transformer is shown in Figure (III-2a). Resistance, leakage reactance and the path for magnetizing current are shown in the equivalent circuit of Figure (III-2b). Since the magnetizing current of a transformer is usually insignificant compared with the full load current the shunt admittance may be omitted as shown in Figure (III-2c), which is the scheme used in load flow solution for nominal tap ratio transformers.

Notice that auto-transformers may be treated in the same manner as two winding transformers.

In practice the transformation ratios of transformers are frequently changed by the provision of tap-changing equipment. Off nominal turn ratios are indicated in Figure (III-3a). The ratio a can be greater than unity or less than unity. In the following discussion the a to 1 ratio will be uniformly oriented with the 1 terminal connected to the branch representing the associated transformer leakage impedance. To represent an off nominal turns ratio transformer a fictitious autotransformer is connected as



(a)



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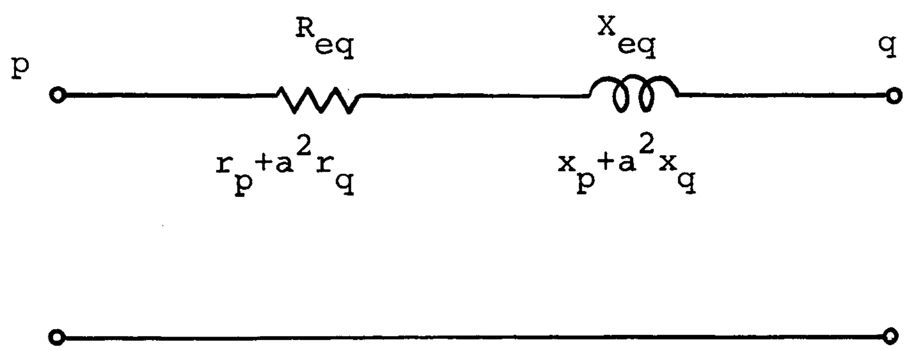

Figure III-2. Transformer Equivalent Circuits

a) One Line Representation

b) Equivalent Circuit in Term of the Primary

c) Equivalent Circuit for Load Flow Calculation

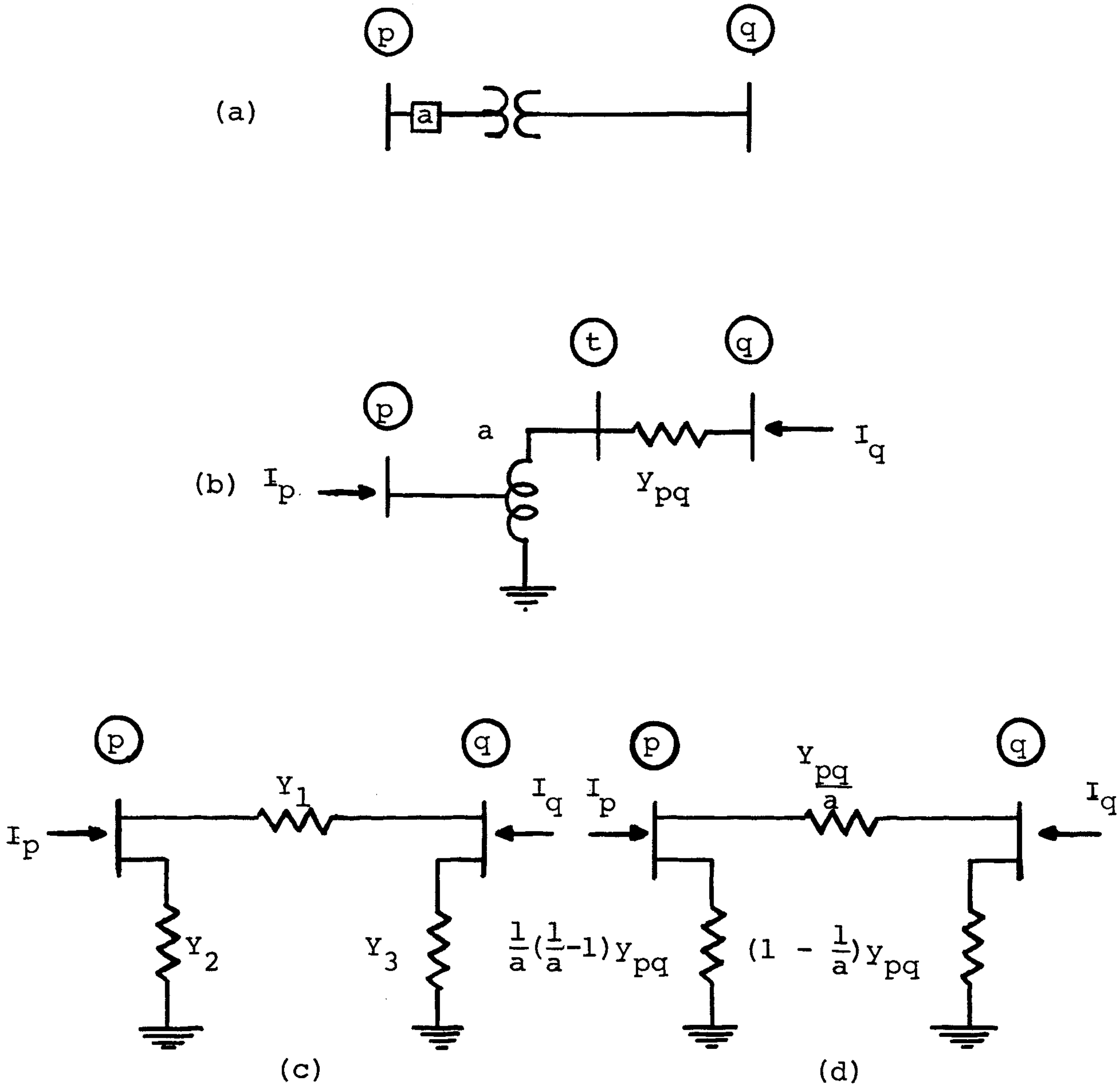

Figure III-3. Off Nominal Transformer Representation

a) One Line Diagram
b) Equivalent Circuit
c) Equivalent π Circuit
d) Equivalent π Circuit with Parameters Expressed in Terms of Admittance and Off-Nominal Turn Ratios.

shown in Figure (III-3b). An equivalent circuit can be obtained to use in the load flow studies, then the elements of the equivalent π circuit can be treated the same as line elements. The parameters of the equivalent π circuit, shown in Figure (III-1d), can be derived by equating the terminal current of the transformer with the corresponding current of the equivalent π circuit. At bus p

$$I_p = \frac{i_{tq}}{a} \tag{III-1}$$

it follows that

$$i_{tq} = (E_t - E_q)\ y_{pq} \tag{III-2}$$

or

$$I_p = (E_t - E_q)\ \frac{y_{pq}}{a} \tag{III-3}$$

Since

$$\frac{E_p}{E_t} = a \tag{III-4}$$

Equation (III-3) becomes

$$I_p = (E_p - a\ E_q)\ \frac{y_{pq}}{a^2} \tag{III-5}$$

Similarly,

$$I_q = (E_q - E_t)\ y_{pq} \tag{III-6}$$

substituting for E_t, Equation (III-6) becomes

$$I_q = (a\,E_q - E_p)\,\frac{y_{pq}}{a} \qquad \text{(III-7)}$$

The terminal currents for the equivalent π circuit shown in Figure (III-3c) are

$$I_p = (E_p - E_q)\,Y_1 + E_pY_2 \qquad \text{(III-8)}$$

$$I_q = (E_q - E_p)\,Y_1 + E_qY_3 \qquad \text{(III-9)}$$

Letting $E_p = 0$ and $E_q = 1$ in Equation (III-5) and (III-8)

$$I_p = -\,\frac{y_{pq}}{a}$$

$$I_p = -\,Y_1$$

then

$$Y_1 = \frac{y_{pq}}{a} \qquad \text{(III-10)}$$

Similarly, letting $E_p = 1$, $E_q = 0$ in both Equation (III-5) and (III-8)

$$I_p = E_p(Y_1 + Y_2)$$

$$I_p = E_p\,\frac{y_{pq}}{a^2}$$

then

$$Y_1 + Y_2 = \frac{y_{pq}}{a^2}$$

and substituting for Y_1 from (III-10),

$$Y_2 = \frac{1}{a}\left(\frac{1}{a} - 1\right)y_{pq} \qquad \text{(III-11)}$$

Similarly, substituting $E_p = 0$ and $E_q = 1$ in both Equation (III-7) and (III-9),

$$I_q = y_{pq}$$

and

$$I_q = Y_1 + Y_3$$

then

$$y_{pq} = Y_1 + Y_3$$

and substituting for Y_1 from Equation (III-10)

$$Y_3 = (1 - \frac{1}{a})y_{pq} \qquad \text{(III-12)}$$

The equivalent π circuit with its parameters expressed in terms of admittance and off-nominal turns ratios are shown in figure (III-1d). The application of Equation (III-10), (III-11) and (III-12) in order to find the Y admittance matrix,is deferred until Chapter IV.

IV. SOLUTION TECHNIQUE

A. The bus admittance matrix

A network matrix equation provides a convenient mathematical model for a digital computer solution. The solution technique to be described may be applied to a transmission or distribution system. The word 'bus' will be used to refer either to a feeder or a load point.

In the nodal frame of reference Equation (I-2)

$$\bar{I}_{BUS} = Y_{BUS}\,\bar{E}_{BUS}$$

describes the performance of the network. The number of busses n in a network of N nodes is (N-1); then N-1 potential differences are all measured with respect to the same reference. The n independent equations are likewise determined by applying Kirchhoff's nodal law to all node other than the reference.

Consider Figure (IV-1). Writing the nodal voltage equation for the system:

$$\begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_p \\ i_q \\ \vdots \\ i_m \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1p} & Y_{1q} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2p} & Y_{2q} & \cdots & Y_{2n} \\ \vdots & & & & & & \\ Y_{p1} & Y_{p2} & \cdots & Y_{pp} & Y_{pq} & \cdots & Y_{pn} \\ Y_{q1} & Y_{q2} & \cdots & Y_{qp} & Y_{qq} & \cdots & Y_{qn} \\ \vdots & & & & & & \\ Y_{n1} & Y_{n2} & \cdots & Y_{np} & Y_{nq} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_p \\ e_q \\ \vdots \\ e_n \end{bmatrix} \quad \text{(IV-1)}$$

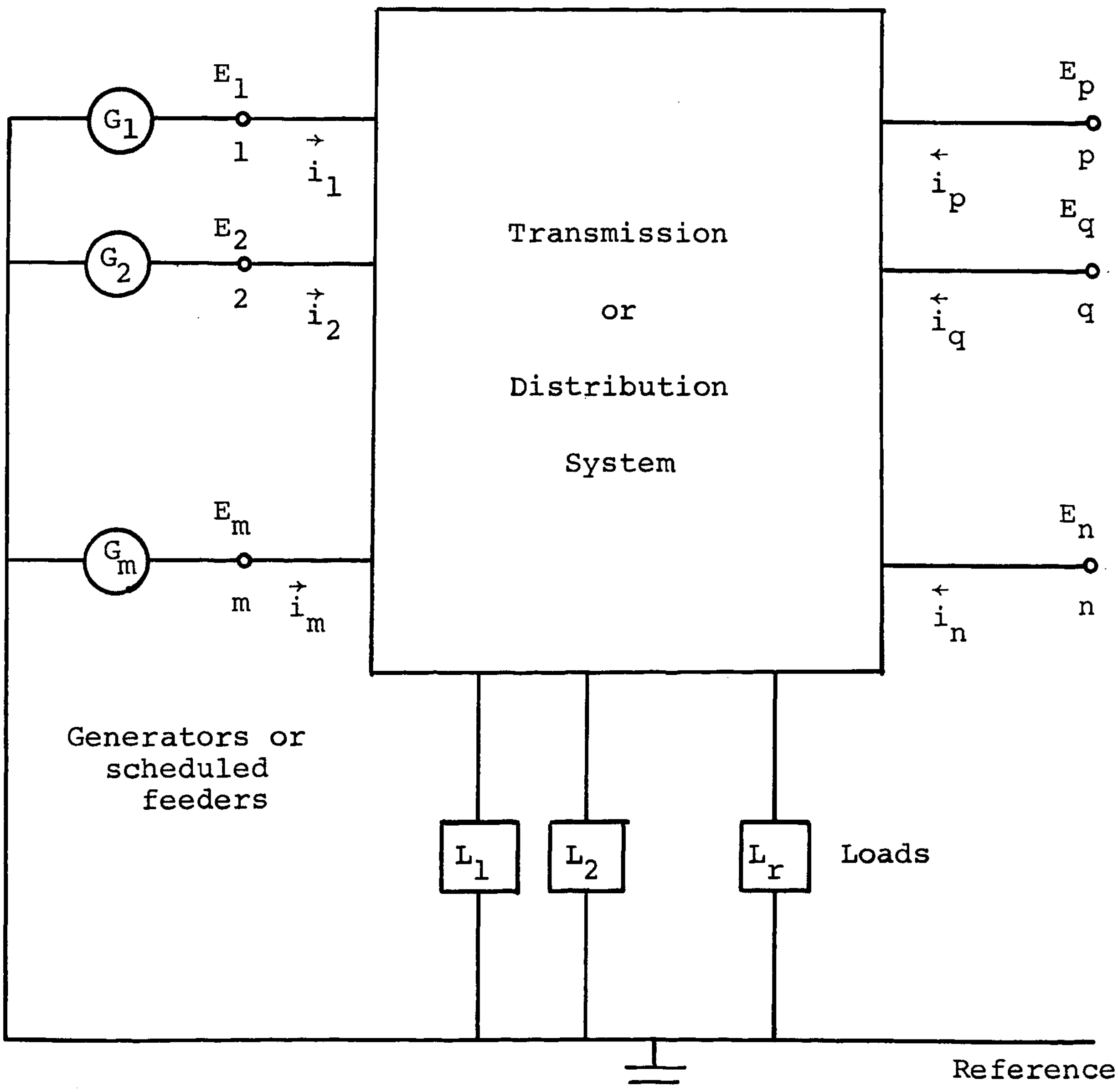

Figure IV-1. The General Power System

i.e. $$\bar{I}_{BUS} = Y_{BUS}\, \bar{E}_{BUS} .$$

Equation (IV-1), defined the bus admittance matrix, practice provides a general rule for writing the admittance matrix directly from a network:

(a) A diagonal element Y_{pp} is the sum of all admittance of all branches which have p as a node. It is always positive. Self admittance elements can be written as:

$$Y_{pp} = Y_{p1} + Y_{p2} + \ldots Y_{pq} \ldots + Y_{pn} \qquad \text{(IV-2)}$$

(b) A non-diagonal element Y_{pq} is minus the sum of the admittances of all branches directly connecting the nodes p and q. Mutual admittance elements can be written as:

$$Y_{pq} = - Y_{pq} \qquad \text{(IV-3)}$$

When line charging is considered Equation (IV-2) is modified.

$$Y_{pp} = Y_{p1} + Y_{p2} + \ldots Y_{pq} + \ldots Y_{pn} + Y'_{pp} \qquad \text{(IV-3)}$$

where y'_{pp} is the sum of the line charging to ground at bus p.

It is possible to find Y_{BUS} by Equations (IV-2) and (IV-3) when mutual inductance is neglected. A more sophisticated approach to the problem is the formation of the metwork matrix by singular transformation and algorithms,

[17] where mutual coupling has been considered. The price of this is larger storage requirements for the incidence and network matrices and more computer time consumed by using algorithm method of matrix formation.

When off-nominal turns ratio is represented at bus p, for a transformer connecting p and q, the equivalent π circuit of Figure (III-1) is used and the self admittance at bus p is

$$Y_{pp} = y_{p1} + \dots \frac{y_{pq}}{a} + \dots y_{pn} + \frac{1}{a}\left(\frac{1}{a} - 1\right) y_{pq}$$
$$Y_{pp} = y_{p1} + y_{p2} + \dots \frac{y_{pq}}{a^2} + \dots y_{pn} \qquad \text{(IV-4)}$$

The mutual admittance from p to q is

$$Y_{pq} = - \frac{y_{pq}}{a} \qquad \text{(IV-5)}$$

The self admittance at bus q is

$$Y_{qq} = y_{q1} + \dots \frac{y_{qp}}{a} + \dots y_{qn} + \left(1 - \frac{1}{a}\right) y_{qp}$$
$$Y_{qq} = y_{q1} + \dots y_{qp} + \dots y_{qn} \qquad \text{(IV-6)}$$

and is unchanged. The mutual admittance from q to p is

$$Y_{qp} = - \frac{y_{qp}}{a}$$

B. Iterative method using Y_{BUS}

This section describes the principle of the iterative procedure for proceeding from a trial set of terminal voltages and converging upon a corrected set of voltages which satisfy the prescribed terminal conditions.

For convenience Equation (I-10) is written here

$$E_p = \frac{1}{Y_{pp}} \left(\frac{P_p - jQ_p}{E_p^*} - \sum_{\substack{q \neq 1 \\ q \neq p}}^{n} Y_{pq} E_q \right)$$

$$p = 1,2,3,\ldots,n$$

where

E_p voltage at bus p

E_q voltage at bus q

Y_{pp} self admittance at bus p

Y_{pq} mutual admittance between bus p and q

P_p net real power at bus p

Q_p net reactive power at bus p

A significant reduction in the computing time for a solution will be obtained by performing as many arithmetic operations as possible before initiating the iterative calculation.

Define the bus parameter KL_p:

$$KL_p = \frac{P_p - jQ_p}{Y_{pp}} \tag{IV-9}$$

and the line parameter YL_{pq}

$$YL_{pq} = \frac{Y_{pq}}{Y_{pp}} \tag{IV-10}$$

and the voltage equation becomes:

$$E_p = \frac{KL_p}{E_p^*} - \sum_{\substack{q \neq 1 \\ q \neq p}}^{n} YL_{pq} E_q \qquad \text{(IV-11)}$$

$$p = 1,2,\ldots,n$$

The procedure for iterative solution involves the following steps:

1. Estimated values are assigned to the voltages. All the voltages may be estimated initially to equal $1.0\underline{/0}$ per unit except the voltages of the swing bus and voltage controlled busses which are fixed.

2. A new set of bus voltages is obtained after substitution of estimated values in Equation (IV-11). As each new corrected voltage is computed, it replaces the previous approximate value in the computation of the next bus voltage.

3. Special treatment is given when a voltage controlled bus is reached. In this case the reactive power Q_p is calculated. Voltage phase angle adjusted, and bus parameter recalculated.

4. Bus voltages are accelerated by means of acceleration factors after each iteration.

5. The process is repeated over and over until all voltage correction appearing in the process become smaller than a determined tolerance.

6. Line flows are then computed as well as reactive power into the network at busses where reactive power is not given.

The method given applies to complex load flows, i.e. real and imaginary components considered and therefore give the full and accurate solution of the network.

1. Voltage controlled bus

A modification of, or deviation from, the normal computational procedures for the solution of the load flow problem is required to take into account voltage controlled busses. Consider the terminal p, where scheduled power P_p and the voltage magnitude $|E_{ps}|$ are specified Equation (I-10) can be written

$$E_p = \frac{P_p - jQ_p}{Y_{pp} E_p^*} - \sum_{\substack{q=1 \\ q \neq p}}^{n} YL_{pq} E_q .$$

Solving for the power:

$$P_p - jQ_p = Y_{pp} E_p E_p^* + E_p^* Y_{pp} \sum_{\substack{q=1 \\ q \neq p}}^{n} YL_{pq} E_q . \qquad \text{(IV-12)}$$

Letting

$$E_p = e_p + jfp \quad \text{(IV-13)}$$

$$E_q = e_q + jfq \quad \text{(IV-14)}$$

$$Y_{pp} = G_{pp} - jB_{pp} \quad \text{(IV-15)}$$

$$YL_{pq} = YL_{pqr} + jYL_{pqi} \quad \text{(IV-16)}$$

and substituting in Equation (IV-12) and separating the real and imaginary parts

$$Q_p = \bar{e}_p{}^2 B_{pp} + \bar{f}_p^2 B_{pp} + (e_p B_{pp} + G_{pp} f_p) \sum_{\substack{q=1 \\ q \neq p}}^{n} (YL_{pqr} e_q - YL_{pi} f_q)$$
$$- (e_p G_{pp} + B_{pp} f_p) \sum_{\substack{q=1 \\ q \neq p}}^{n} (YL_{pqr} f_q - YL_{pqi} e_q) \quad \text{(IV-17)}$$

the values of e_p and f_p must satisfy the relation

$$\bar{e}_p{}^2 + \bar{f}_p{}^2 = \{|E_{ps}|\}^2 \quad \text{(IV-18)}$$

One way to solve Equation (IV-17) and (IV-18) is to adjust the phase angle of the stimated voltage and calculate the reactive bus power required.

In the computer program developed the range of reactive power as well as scheduled reactive power must be specified. If the calculated value of Q_p is greater than the maximum capability $Q_{p\ max}$ of the source, $Q_{p\ max}$ is taken as the reactive power at that bus. If the calculated value of Q_p is less than minimum capability $Q_{p\ min}$, this value is used.

If the calculated value of Q_p is in the range such that

$$Q_{p\ max} > Q_{p\ calculated} > Q_{p\ min}$$

the corrected value of $e_p + jf_p$ must be used for the next iteration. If the calculated value of Q_p is not in the range no change to voltage bus is attempted and it is impossible to obtain a solution for the specified scheduled voltage and range of Q_p.

Notice that any change in Q_p alters the bus parameters, so KL_p must be recalculated for each iteration in this routine.

V. DESCRIPTION OF THE PROGRAM

A. Components

The program developed consists of an integrated set of computer subroutines to perform the load flow calculations and associated data processing. For a detailed description of the user facilities see "Manual for Distribution System Load Flow for the IBM 360". Power Research Center Report PRC 7103MS [23]

The principal components of this program are:

MAIN	To initialize the run and to call other subroutines
LINEIN	Program for line and transformer data assembly
BUSIN	Program for bus data assembly
CONIN	To enter miscellaneous constants
LINECH	To modify line and transformer data for change cases
BUSCH	To modify bus data for change cases
SOLVE	To calculate Y_{BUS}, line and bus parameters and to solve the load flow problem by the iteration process.
OUTPUT	To write output report.

A simplified flow diagram for the program is shown in Figure (V-1).

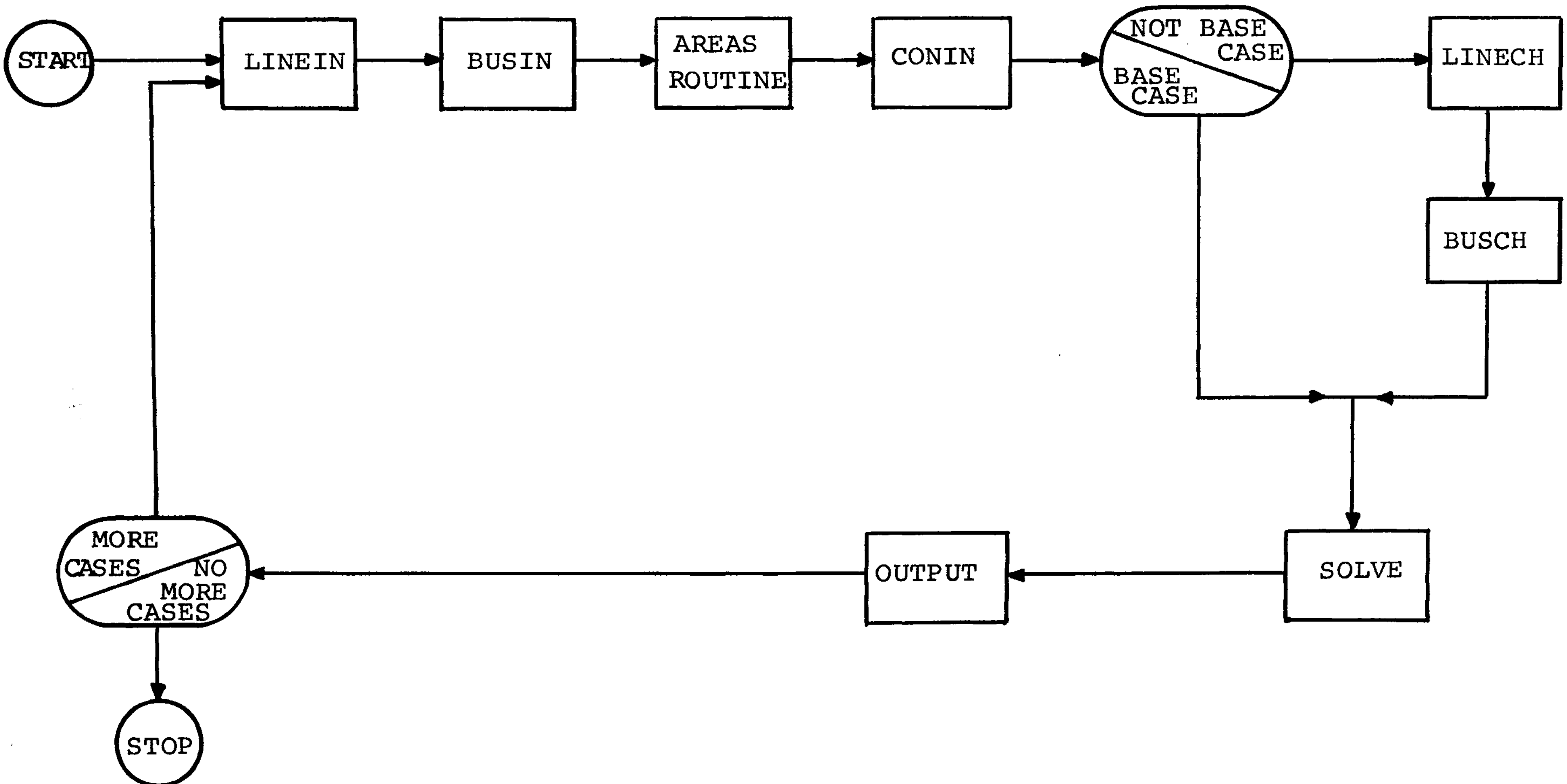

Figure V-1. Simplified Logic Flow Diagram of Load Flow Program

B. Programming Y_{BUS}

Y_{BUS} is classically displayed as a square matrix of order N where n is the number of busses of the system. Neglecting the effects of mutual coupling between lines, Y_{BUS} can be found by inspection following the rules given in Chapter IV. Since every element of Y_{BUS} has two components, representing Y_{BUS} as a square matrix on the computer requires two (n) order matrices. This representation consumes a large amount of computer memory and greatly limits the size of the system that can be studied. For this reason, a more efficient method of representing this matrix is needed.

The following facts are known about a power system:

1. Compared to the size of the system, relatively few lines are connected at any one bus

2. Y_{BUS} is symmetrical about the diagonal.

Using these facts an algorithm is developed that reduces Y_{BUS} to a column matrix that is totally dense and contains only the non-zero off-diagonal terms and the diagonal terms. Thus, the storage requirement are reduced from N^2 to N + 2(NOLIN), where NOLIN is the total number of lines and transformers in the system. Use of this algorithm requires that data be supplied in a definite order. This difficulty has been by-passed by writing a sorting routine (LFSORT program) to prepare the data in the proper sequence. Bus and line parameters are also represented in the same way. A flow chart describing the sequence of steps for the calculation of Y_{BUS}, bus parameters KL_p and line parameters YL_{pq} is shown in Appendix A, Figure A-1. Notice that for simplification a complex representation is given. In the program real and imaginary parts of Y_{BUS}, KL_p and YL_{pq} are calculated separately.

C. Programming the Gauss-Seidel Iterative Method

The bus voltage Equation (IV-11) is then solved by the Gauss-Seidel Iterative Method [6]. In this method the new calculated voltage E_p^{k+1} immediately replaces E_p^k and is used in the calculation of E_{p+1}^{k+1}, E_{p+2}^{k+1}, E_{p+3}^{k+1} .... E_n^{k+1}. Bus voltage equations are applied at each bus, except the swing bus, until

$$\max \Delta E^k \leq \varepsilon \qquad \text{(V-1)}$$

where

$\max \Delta E^k$ = maximum voltage change during kth iteration

ε = specified tolerance

The sequence of steps for the iteration portion of the program, including the Gauss-Seidel method is shown in Appendix A, Figure (A-2).

When bus p is a generator bus a deviation of the normal computational procedure is required. The sequence of steps required to include the effect of voltage controlled busses is shown in Appendix A, Figure A-3.

The Gauss-Seidel Y_{BUS} technique lends itself quite nicely to computer application for two reasons:

1. Y_{BUS} is a sparse matrix and therefore the sparse matrix technique previously discussed can be applied.

2. The algorithm for computing bus voltages requires a relatively small amount of computer memory.

However, the Gauss-Seidel Y_{BUS} techniques has been found to have two major disadvantages when applied to power systems.

1. The convergence of bus voltage is slow
2. The number of iterations required to effect a solution increases rapidly as the number of busses in the system increases.

1. Acceleration of convergence

The convergence rate can be improved by the application of an acceleration or deceleration factor to the real and imaginary components of bus voltage at the end of each iteration. The technique is a simple one. Let

α acceleration factor to real part component of voltage

β acceleration factor to imaginary component of voltage

ξ deceleration factor to real part of voltage

δ deceleration factor to imaginary part of voltage.

If the difference between corrected voltages in two successive iterations changes in one direction, the accelerated value for the real and imaginary part are

$$e_p^{k+1}(\text{accelerated}) = e_p^k + \alpha\ (e_p^{k+1} - e_p^k)$$

$$f_p^{k+1}(\text{accelerated}) = f_p^k + \beta\ (f_p^{k+1} - f_p^k)$$

In the other hand, if this difference is oscillating the deceleration values are

$$e_p^{k+1}(\text{decelerated}) = e_p^k + \xi\ (e_p^{k+1} - e_p^k)$$

$$f_p^{k+1}(\text{decelerated}) = f_p^k + \delta\ (f_p^{k+1} - f_p^k)$$

A general difficulty in the application of this technique is that for any given problem the optimum value of these factors is not usually known prior to solution of the problem. Experience shows that the value

$$\alpha = 1.6$$

$$\beta = 1.7$$

$$\xi = 1.4$$

$$\delta = 1.4$$

are applicable for most of the systems, however it must be pointed out that the actual values depend on the nature of the system under study.

The convergence rate can also be improved by the application of the relaxation method. A variation of this technique is applied in the program that the order in which the corrected voltages are calculated may be varied in such a way to correct first the voltage busses experiencing the largest change.

VI. PRACTICAL APPLICATION TO DAVENPORT DISTRIBUTION SYSTEM

The load flow study presented here is an incidental part of a more general study that includes also short circuit analysis of the Alcoa Davenport plant. The study involves great amounts of data and calculation and it is not feasible to reproduce it here in full. For example, spacing length, wire size, name plate data of transformers, secondary distribution circuits, calculation of percent impedance on the selected system base and final assembly of data input for the load flow program. Nevertheless, it is hoped that this study conveys a clear concept of the method and results.

1. Description of the system

Figure (VII-1) shows the equivalent generating and subtransmission system of Iowa-Illinois Gas and Electric Company, feeding the Alcoa plant.

The system initially consists of 167 busses which are classified as 165 load points and primary feeders, the voltage controlled feeder and the swing feeder. There are 47 distribution transformers, two subtransmission transformers and 182 lines, involving both 13.8 kv and 480 volts cables.

The following notation was used in naming busses:

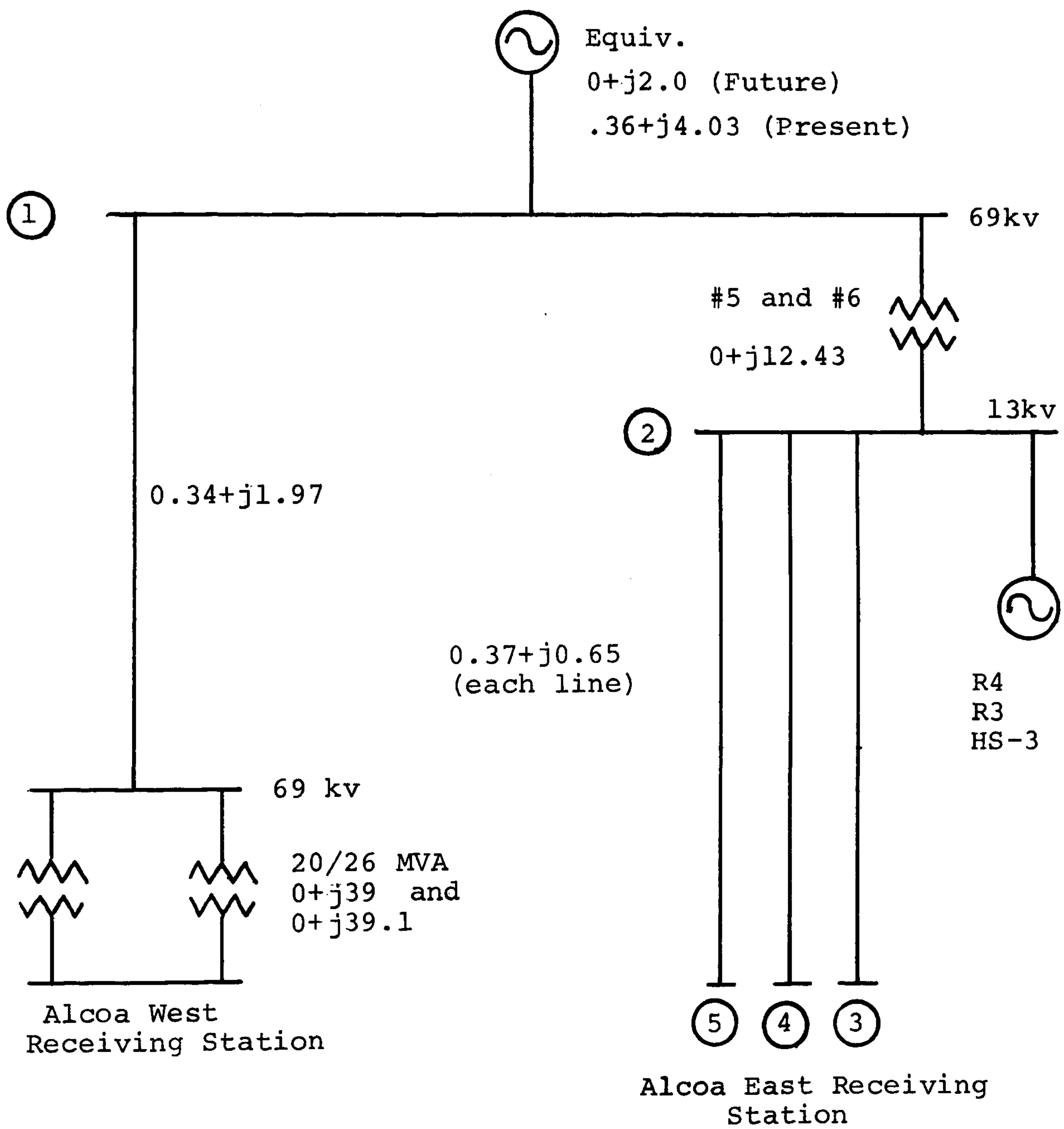

Figure VII-1 Equivalent of I.I.G.E. System
% Z on 100MVA 69kv and 13.8kv

A 13Kv	point at primary of transformers in bank A
Al 480v	point at secondary of transformer Al
Al 4-222	point at the end of secondary main fed by Al through breaker 4-222
SB C84	point on sector bus at column 84

Bus 2 was considered the swing feeder. Voltage was fixed and assumed 0.995 $\underline{/-5}$ on 13.8 kv base. Bus 1 was considered a voltage controlled bus where the utility generated voltage is 1.012$\underline{/0}$.

Normal operation position of breakers and switches were considered to form the base case model of the system. Table VII-1 shows the 13.8 kv primary feeders and the corresponding distribution transformers used in this study.

2. Loading conditions

Loads when this paper was written were known only to a limited extent, for this reason values were assumed, in order to fulfill the requirement of data input.

In representing the secondary distribution system, loads at the end of secondary mains were considered. Loads at radial lines farther than these points were represented as load on the point from which they radiated. This is illustrated in Figure (VII-2), for a typical case at transformers Al, A2, A3. It was noticed that loads are also distributed over the secondary main rather than concentrated at one end. When this was the case

Primary Feeder	Distribution Transformer
13-A-15	P1, P2
	A1, A2, A3
	C1, C2, C3, CR
13-A-12	D1, D2, D3, D4, DR
	E, S
	N3,0
	T1, T2, T3, T4
13-A-13	N1, N2
	FA, FF, FG, FH, FN
	I1, I2, I3, IA
	J2, J3, J4, J5
	LA, LB, LC, LD, LE
	KK, KL, KM, KF

Table VII-1 Primary Feeders and Corresponding Distribution Transformers For Studied Case

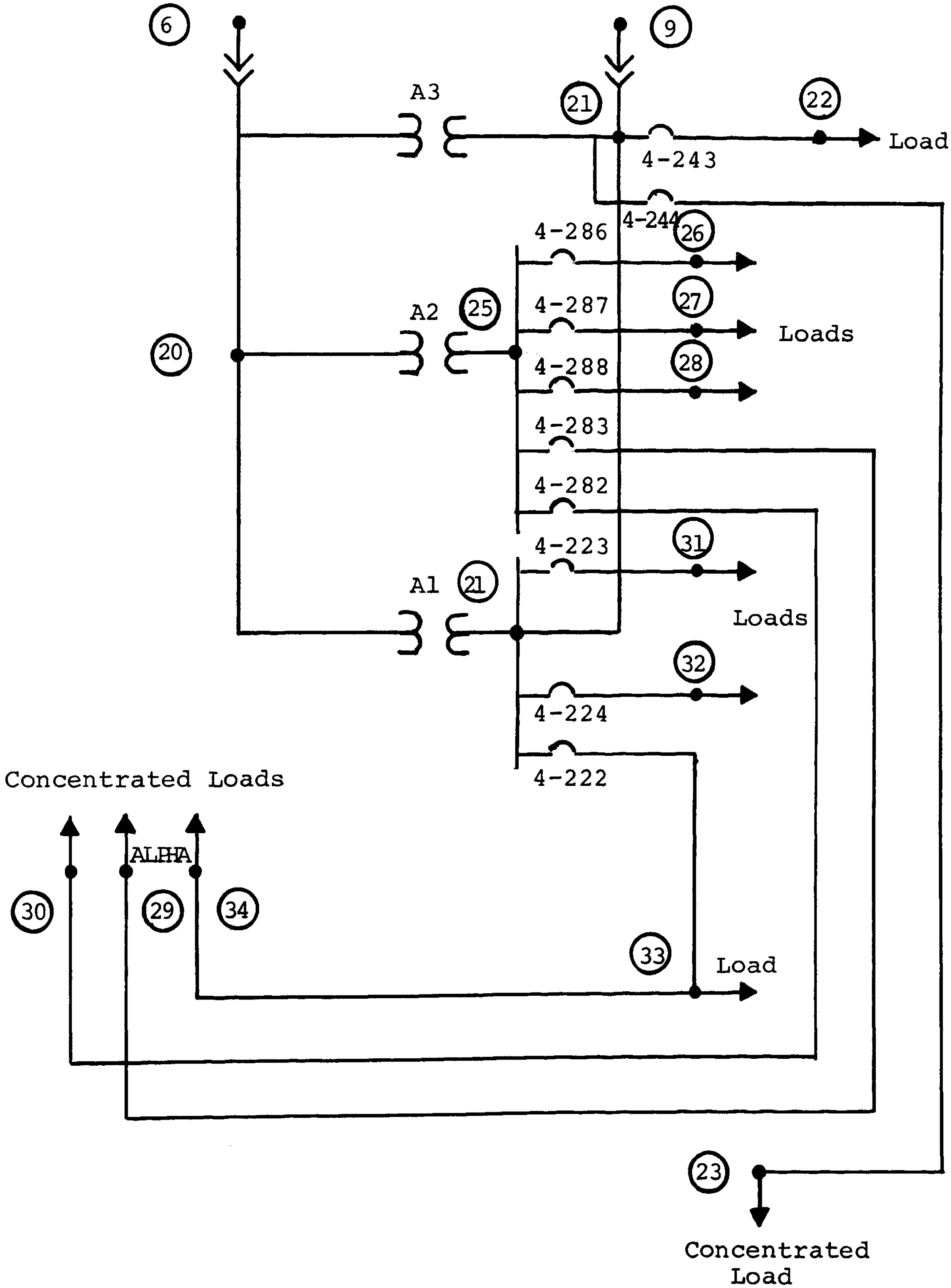

Figure VII-2 Typical Secondary Distribution symplifying assumptions were made. These are shown in Figure (VII-3). For instance, when the load is uniformly distributed over the feeder, the drop to the end of the line was considered the same as if the total load were concentrated at a point half way out on the feeder. Notice that this assumption was not applied for the small number of distributed loads where new points were numbered. Figure (VII-3d) shows assumptions made for uniformly distributed load over part of the line.

3. Computer results

The results for the studied case consists of 38 pages of computer output containing the following reports:

Line and transformer data assembly

Bus data assembly

Record of convergence

Summary of line and bus data, totals and constants

Low voltage summary

High voltage summary

Summary of feeder input data

Summary of mismatch (megawatts and Megavars)

Line overload summary , and

Main report of bus condition and line flows.

It was not possible to reproduce here the program output in full. However, I hope the tables which follow will give an idea of the actual computer output.

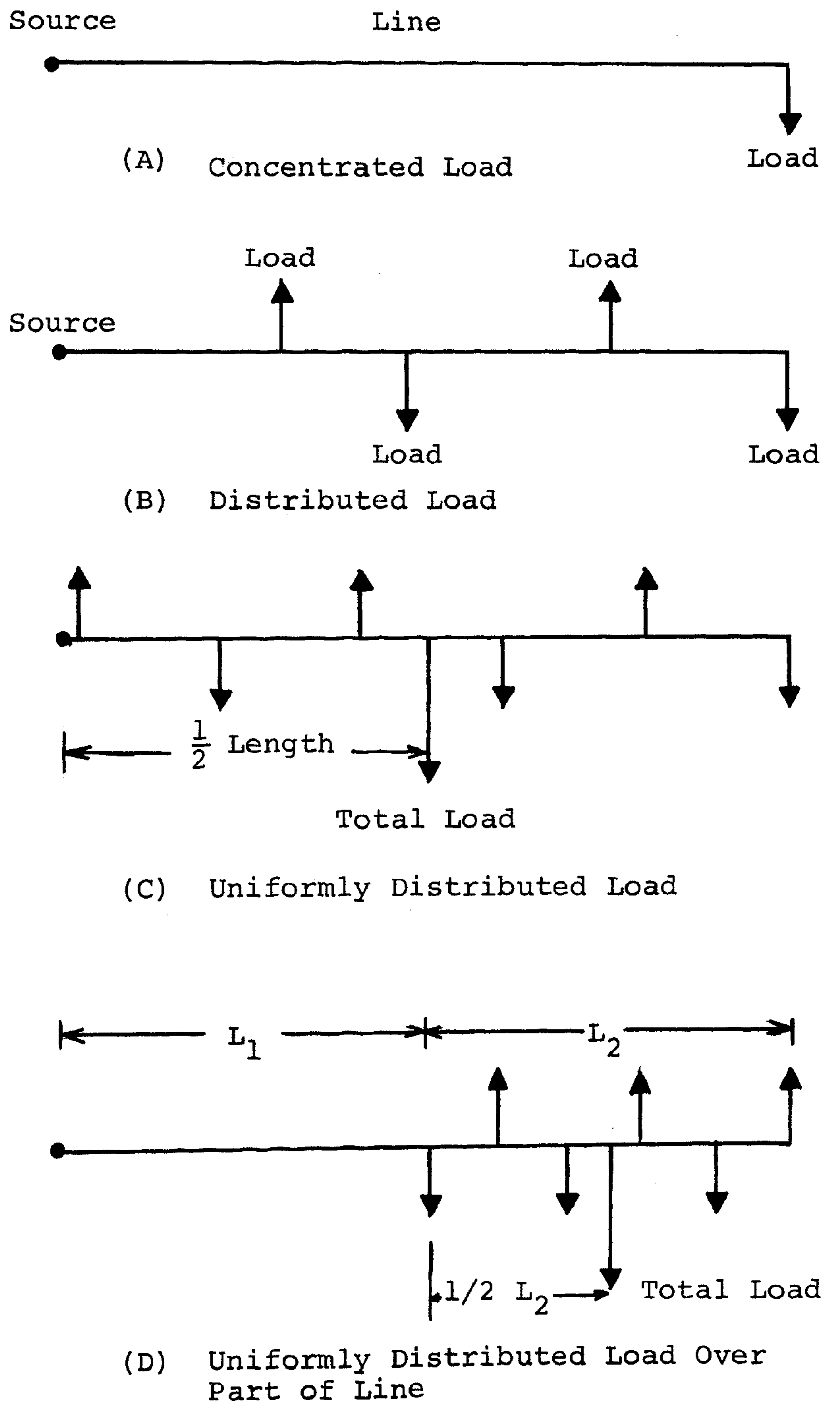

Figure VII-3. Assumptions for Distributed and Concentrated Loads

Table VII-2 shows the record of convergence. Bus voltages magnitude and angle experiencing the greatest changes were printed out every 20 iterations. A voltage tolerance of .0001 per unit for both the real and imaginary components of voltages was used in the test.

On Table VII-3 the summary of line and bus data, totals and constants is shown. Notice that acceleration factors were 1.55 and 1.65 for the real and imaginary components of voltage respectively.

Table VII-4 shows the summary of scheduled generation at Bus 1 and required generation at swing feeder.

Table VII-5 contains the final report of bus voltages condition and line flow for the first busses.

4. Future work

An immediate suggestion for future work is the development of a more accurate load data set, where minimum and maximum load conditions may be considered, and the modification of line parameters where it is necessary, then the study can be performed to investigate the following:

1. Effect of rearranging circuits and incorporation of West receiving station.
2. Effect of temporary loss of generation and subtransmission circuits on system loading.
3. Optimum system running condition and load distribution.

RECORD OF CONVERGENCE

ITERATION COUNT	BUS	EA	MAX DIFF	BUS	EB	MAX DIFF
20	159	0.926045	0.013438	154	-0.064727	-0.019851
40	154	0.982220	-0.006910	159	-0.082594	-0.005428
60	159	0.891798	-0.002365	120	-0.119561	-0.003285
80	154	0.984983	0.001012	159	-0.079350	0.000960
100	159	0.895703	0.000416	154	-0.089331	-0.000416
120	188	0.908316	-0.000315	159	-0.081575	-0.000194
140	188	0.899770	-0.000242	154	-0.089627	0.000073
160	188	0.893278	-0.000182	159	-0.081442	0.000039
180	188	0.888435	-0.000135	154	-0.089597	-0.000011
200	188	0.884871	-0.000098	187	-0.178118	0.000013

Table VII-2. Record of Convergence of Voltages Magnitude and Angle

LINE AND BUS TOTALS	ACTUAL	MAX
DISTRIBUTION LINES	134	900
TRANSFORMERS-FIXED	47	225
-LTC	2	225
TOTAL LINES - - - - - - - -	183	900
ACTIVE BUSSES-NON REG	166	600
-FEEDER	1	225
TOTAL BUSSES - - - - - - -	167	600
CAPACITORS OR REACTORS	0	75

MISCELLANEOUS CONSTANTS	
ACTUAL ITERATIONS	200
MAXIMUM ITERATIONS	500
TOLERANCE - REAL	0.100E-03
-IMAG	0.100E-03
ACC FACT. -REAL	1.55 1.4
-IMAG	1.65 1.4
LTC START	10
SKIP	4
END	400
ACC FACT	1.2

	MW	MVAR
TOTAL LOAD	47.334	22.189
TOTAL LOSSES	0.865	2.691
LINE CHARGING		0.0
FIXED CAP/REAC		0.0
SYSTEM MISMATCH	-0.035	-0.003
TOTAL FEEDER INPUT	48.163	24.877
ITERATIONS BETWEEN BUS-ORDER SORTS		50

Table VII-3. Summary of Line and Bus Data, Totals and Constants

SUMMARY OF FEEDER INPUT DATA
(ASTERISKS INDICATE VOLTAGES NOT HELD)

BUS	FEEDER NAME	MW	MVAR	VAR LIMITS	DESIRED VOLTAGE	ACTUAL VOLTAGE
1	RIVER 69	24	14.2	-5.0 25.0	1.012	1.012
2	RIVER 13	24.16	10.67		0.995	0.995
2	SWING FEEDER					

Table VII-4. Summary of Feeder Input Data

REPORT OF LOAD-FLOW CALCULATION

X BUS	BUS - DATA NAME	VOLTS	ANGLE	LINE - DATA TO BUS	NAME	MW	MVAR	PCT CAP	TAP
1	RIVER 69	1.012	-3.3						
				2	RIVER 13	11.31	6.70		1.000R
				2	RIVER 13	12.69	7.51		1.000R
2	RIVER 13	0.995	-5.0						
				1	RIVER 69	-11.31	-6.25		
				1	RIVER 69	-12.69	-7.01		
				3	EAST A15	15.00	7.59		
				4	EAST A12	15.45	7.65		
				5	EAST A13	17.71	8.70		
3	EAST A15	0.994	-5.0						
				2	RIVER 13	-14.99	-7.57		
				6	P 13KV	7.49	3.78		
				6	P 13KV	7.49	3.78		
4	EAST A12	0.994	-5.0						
				2	RIVER 13	-15.44	-7.63		
				60	DE-3	15.42	7.61		
5	EAST A13	0.994	-5.0						
				2	RIVER 13	-17.70	-8.67		
				100	DE-4	17.69	8.66		
6	P 13KV	0.993	-5.1						
				3	EAST A15	-7.49	-3.77		
				3	EAST A15	-7.49	-3.77		
				7	P2 480V	2.00	1.00	28.0	0.978
				8	P1 480V	0.23	-1.07	43.9	0.978
				20	A 13KV	6.00	4.46		
				48	C 13KV	6.73	3.13		

Table VII-5 Example of Main Report of Bus Conditions and Line Flows
(continued next page)

REPORT OF LOAD-FLOW CALCULATION

BUS	NAME	VOLTS	ANGLE	TO BUS	NAME	MW	MVAR	PCT CAP	TAP
7	P2 480V	1.014	-5.1						
				6	P 13KV	-2.00	-1.00	28.0	
8	P1 480V	1.016	-5.1						
				6	P 13KV	-0.23	1.07	43.9	
				9	SB C206	-0.22	-1.32		
				10	P1 4-027	0.10	0.05		
				11	P1 4-026	0.10	0.05		
9	SB C206	1.032	-5.3						
				8	P1 480V	0.22	1.34		
				21	A1 480V	-1.22	-1.99		
				40	SB C200	1.00	0.64		
10	P1 4-027	1.013	-5.2						
				8	P1 480V	-0.10	-0.05		

Table VII-5 Example of Main Report of Bus Conditions and Line Flows (continued)

4. Optimum losses.
5. Optimum rating and tap ranges of distribution transformers.
6. Recommendation for change of conductor size and system voltage can then be made.

BIBLIOGRAPHY

[1] Dunstan, L.A. "Machine Computation of Power Network Performance", Trans. Amer. Inst. Elect. Engrs., 1947, 66, p 610.

[2] Dunstan, L.A. "Digital Load Flow Studies", ibid., 1954, 73, Pt IIIA, p 825.

[3] Bennett, J.M. "Digital Computers and the Load Flow Problem", Proc. I.E.E., 1956, 103B, Suppl. 1, p 16.

[4] Henderson, J.M. "Digital Computer Solution of Load Flow Studies", Trans. Amer. Inst. Elec. Engrs., 1955, 73, p 1696.

[5] Ward, J.B., and Hale, H.W. "Digital Computer Solution of Power Flow Problems", ibid., 1956, 75, Pt. III, p 398.

[6] Glimm, A.F., and Stagg, G.W. "Automatic Calculation of Load Flow", Trans. Amer. Inst. Elec. Engrs., 1957, 76, Pt. III, p 817.

[7] Brown, R.J., and Tinney, W.F. "Digital Solution for Large Power Network", ibid., 1957, 76, Pt. III, p 347.

[8] Jordan, R.H. "Rapidly Converging Digital Load Flows", ibid., 1957, 76, Pt. III, p 1433.

[9] St. Clair, H.P., and Stagg, G.W. "Experience in Computation of Load Flow Studies Using High Speed Computer", ibid., 1958, 77, Pt. I, p 657.

[10] Dyrkacz, M.S., and MaGinniss, F.J. "A New Automatic Program for Load Flow Studies on the IBM 704", Trans. Amer. Inst. Elect. Engrs., 1959, 78, Pt. III-A, p 52.

[11] Hale, H.W., and Goodrich, R.W. "Digital Computation of Power Flow - Some New Aspects", ibid., 1959, 78, Pt. III, p 919.

[12] Van Ness, J.E. "Iteration Methods for Digital Load Flow Studies", ibid., 1959, 78, Pt. III, p 583.

[13] Van Ness, J.E. "Convergence of Iterative Load Flow Studies", ibid., 1959, 78, Pt. IIIB, p 1590.

[14] Van Ness, J.E., and Griffin, J.H. "Elimination Method for Load Flow Studies", ibid., 1961, 80, Pt. III, p 299.

[15] Stevenson, W.J. Elements of Power System Analysis , McGraw-Hill Book Company, Inc., 1962.

[16] Weedy, B.M. Electric Power Systems, John Wiley and Sons, Inc., 1967.

[17] Stagg, G.W., and El-Abiad, A.H., Computer Methods in Power System Analysis. McGraw-Hill, Inc., 1968.

[18] Brameller, A., and Denmead, J.K. "Some Improved Methods for Digital Network Analysis", Proc. IEE, 1962, 109A, p 109.

[19] Barre', B.A. "The Determination of the Optimum Accelerating Factor for Successive Over-Relaxation", Computer J., 1962, 4, p 73.

[20] McCalla, T.R. Introduction to Numerical Methods, John Wiley and Sons, Inc., 1966.

[21] Lewis, W.E., and Pryce, D.G. The Application of Matrix Theory to Electrical Engineering, E. & F.N. Spon., 1965.

[22] Jennings, W. First Course in Numerical Methods, The MacMillan Company, New York, 1964.

[23] J.V. Saavedra, J.D. Morgan, Manual for Distribution System Load Flow for the IBM 360", UMR Power Research Center Report No. 7103MS.

GLOSSARY OF COMPUTER NOTATION

	Dissertation Symbol	Computer Symbol
Bus parameter real component	KLpr	TKLR(R)
Bus parameter imaginary component	KLpi	TKLI(K)
Line parameter real component	YLpqr	GM(K)
Line parameter imaginary component	YLpqi	BM(K)
Real scheduled feeder input	P_g	PG(K)
Reactive scheduled feeder input	Q_G	QG(K)
Real power load	P_L	PL(K)
Net real power	P_G	PG(K)
Net reactive power	Q_G	QG(K)
Self admittance real component	G_{pp}	TLR(K)
Self admittance imaginary component	B_{pp}	TLI(K)
Mutual admittance real component	G_{pq}	G(L)
Mutual admittance imaginary component	G_{pq}	B(L)
Bus voltage magnitude	$\|E_p\|$	EA(J)
Bus voltage angle	$\underline{/\theta}$	EB(J)
Bus voltage real component during iterations	. . .	EAN
Bus voltage imaginary component during iterations	. . .	EBN
Voltage controlled real component	e_p	DUM7
Voltage controlled imaginary component	f_p	DUM8

GLOSSARY OF COMPUTER NOTATION (continued)

	Dissertation Symbol	Computer Symbol
Maximum reactive power 1	Q_{pmax}	QMAX(M)
Minimum reactive power	Q_{pmin}	QMIN(M)
Acceleration factor for real component of bus voltage	α	ALPHA
Acceleration factor for imaginary component of bus voltage	β	ALPHB
Deceleration factor for real component of bus voltage	ξ	ALPHC
Deceleration factor for imaginary component of bus voltage	σ	ALPHD
Tolerance	ε	EATOL, EBTOL

APPENDIX A

Flow Charts for Mains Routines of the Program

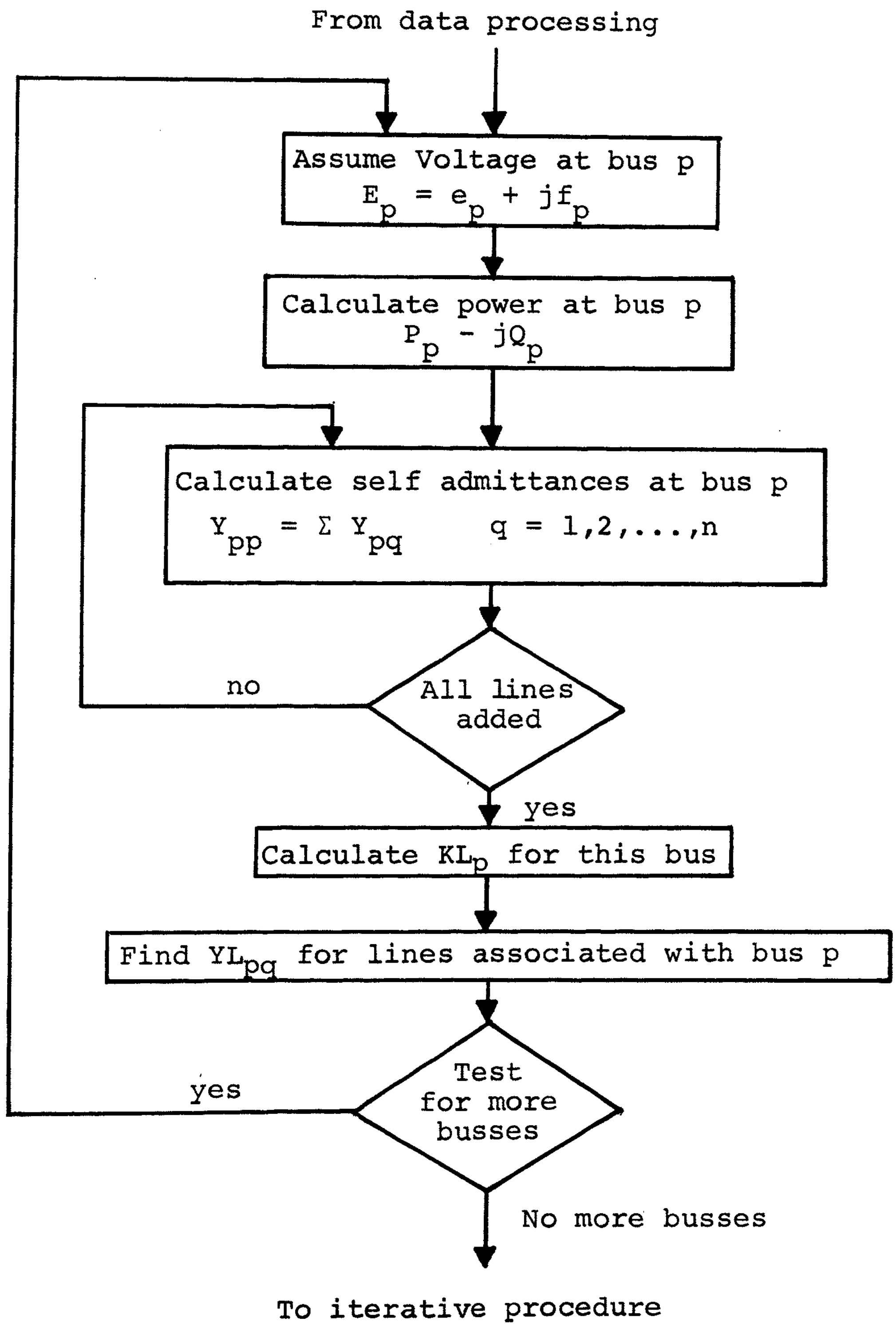

Figure A-1. Flow Chart for the Calculation of Y_{BUS} and Line and Bus Parameters

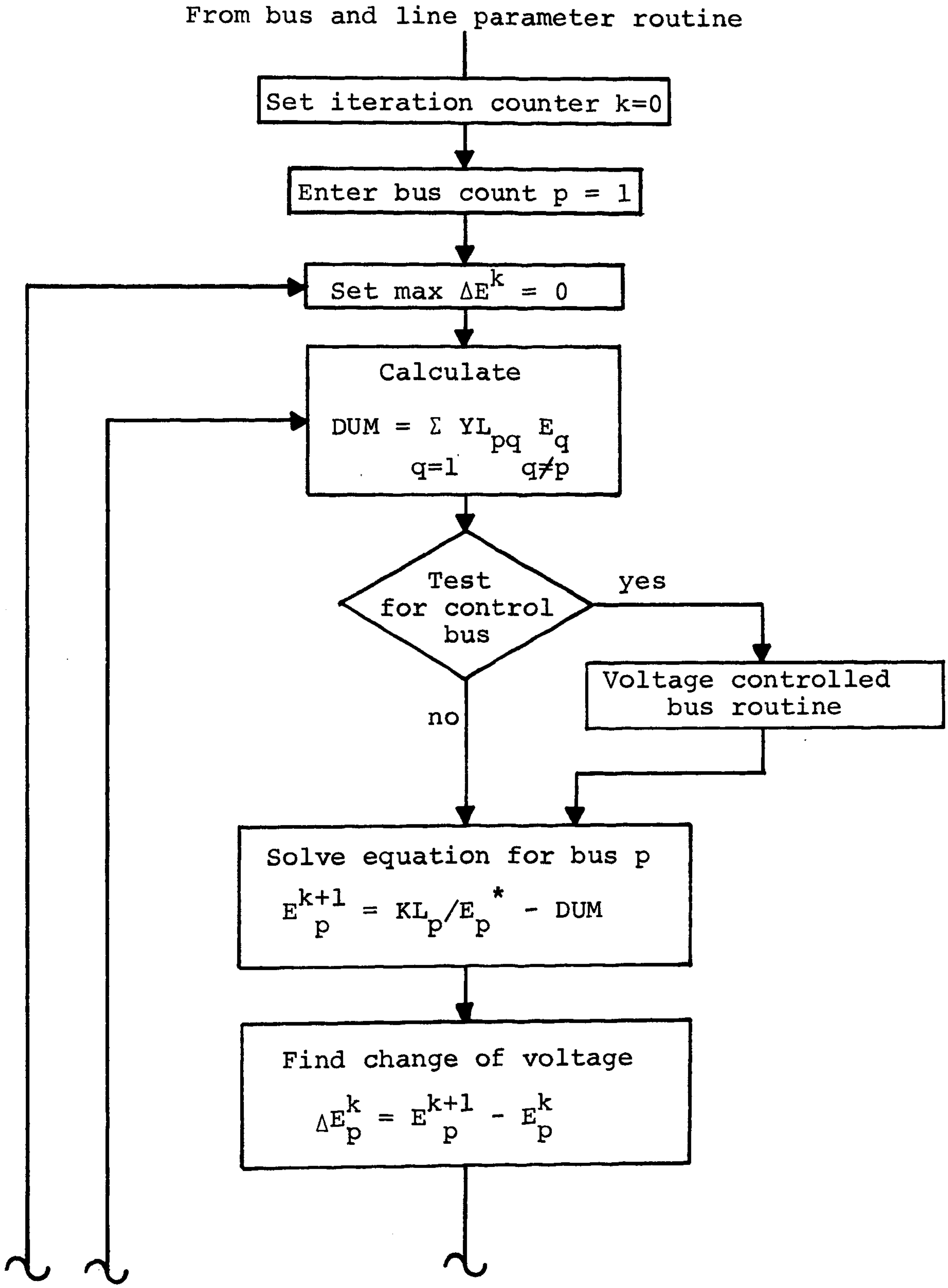

Figure A-2. Flow Chart of Iteration Portion of the Program

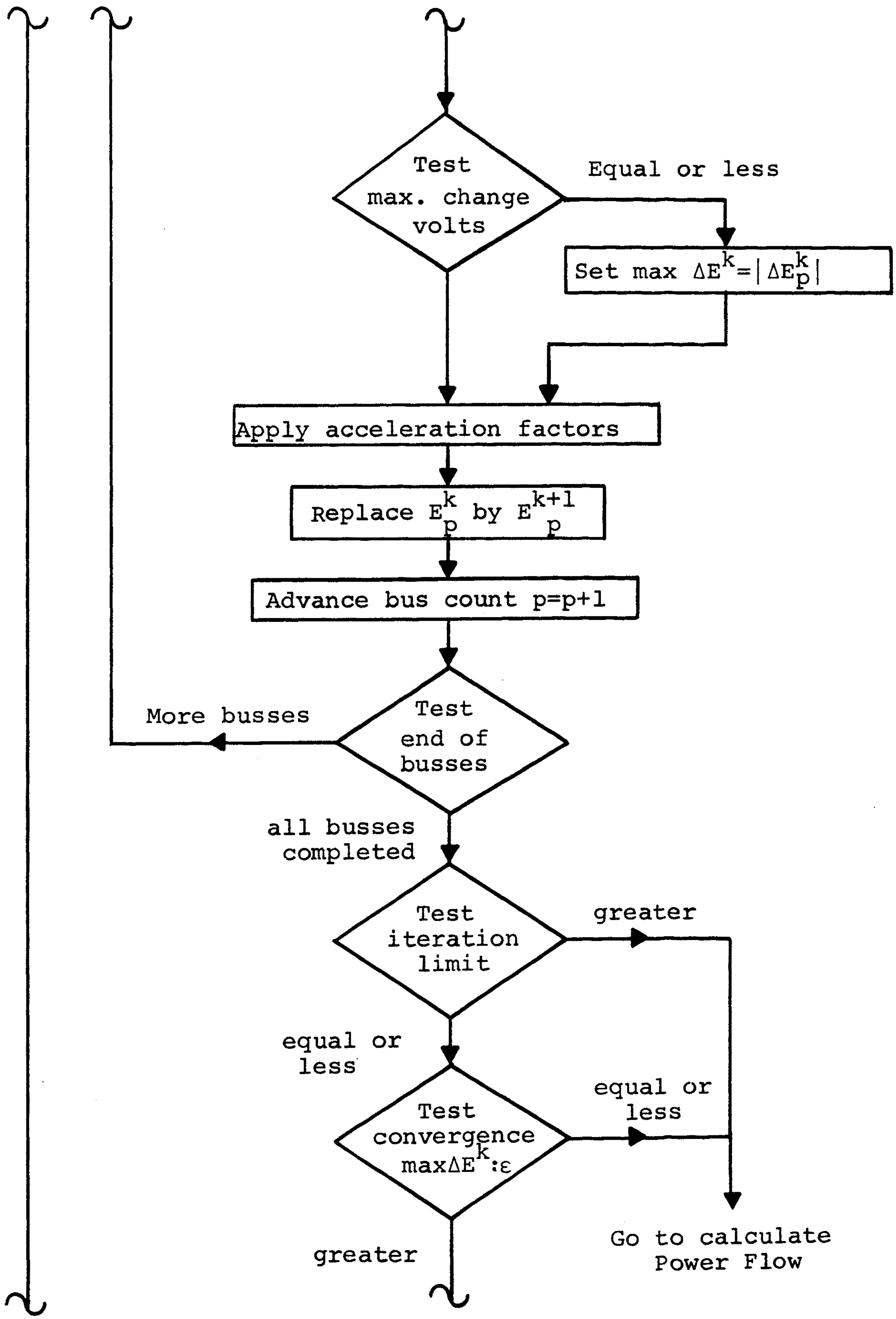

Figure A-2 (continued)

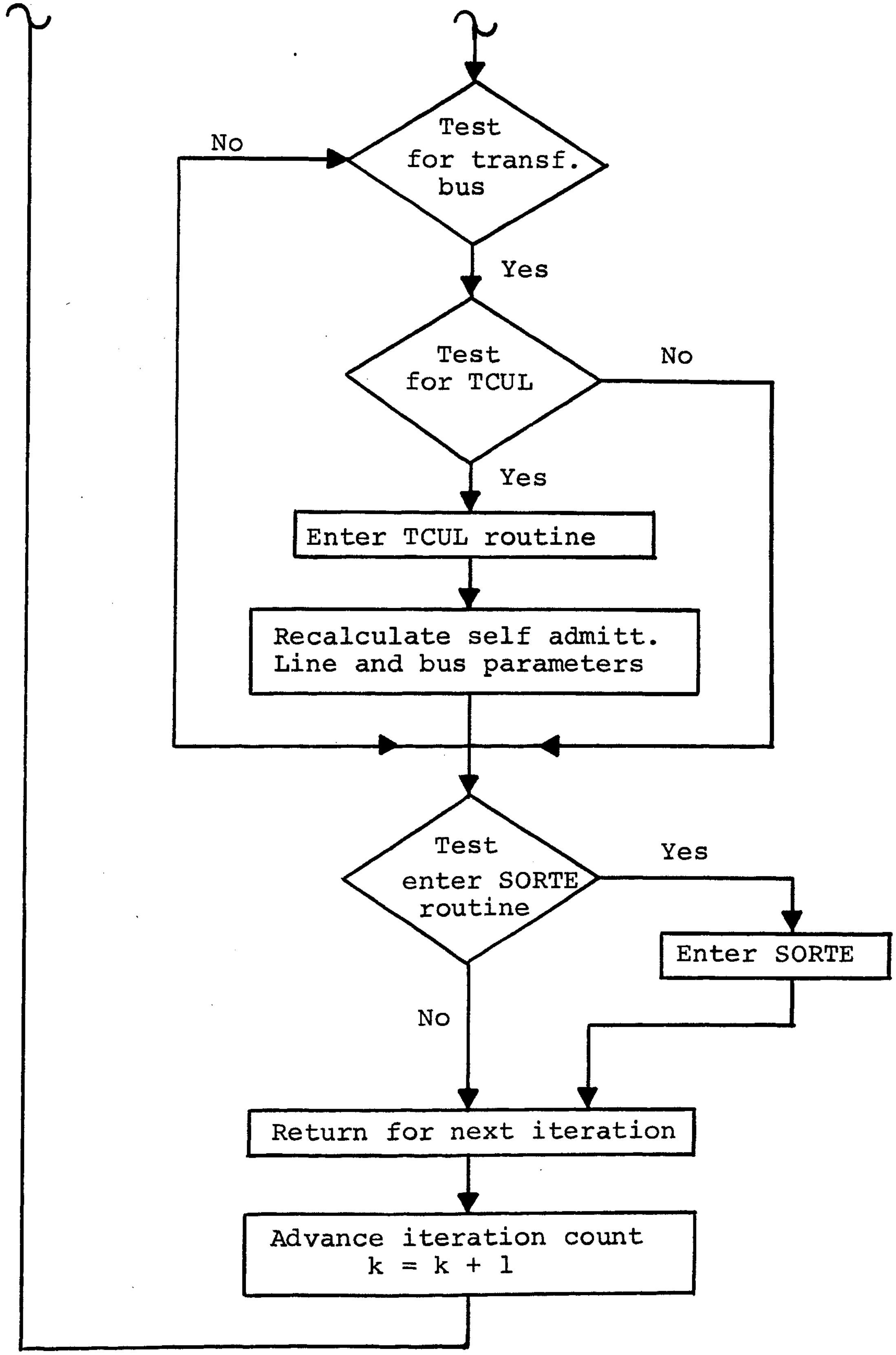

Figure A-2 (concluded)

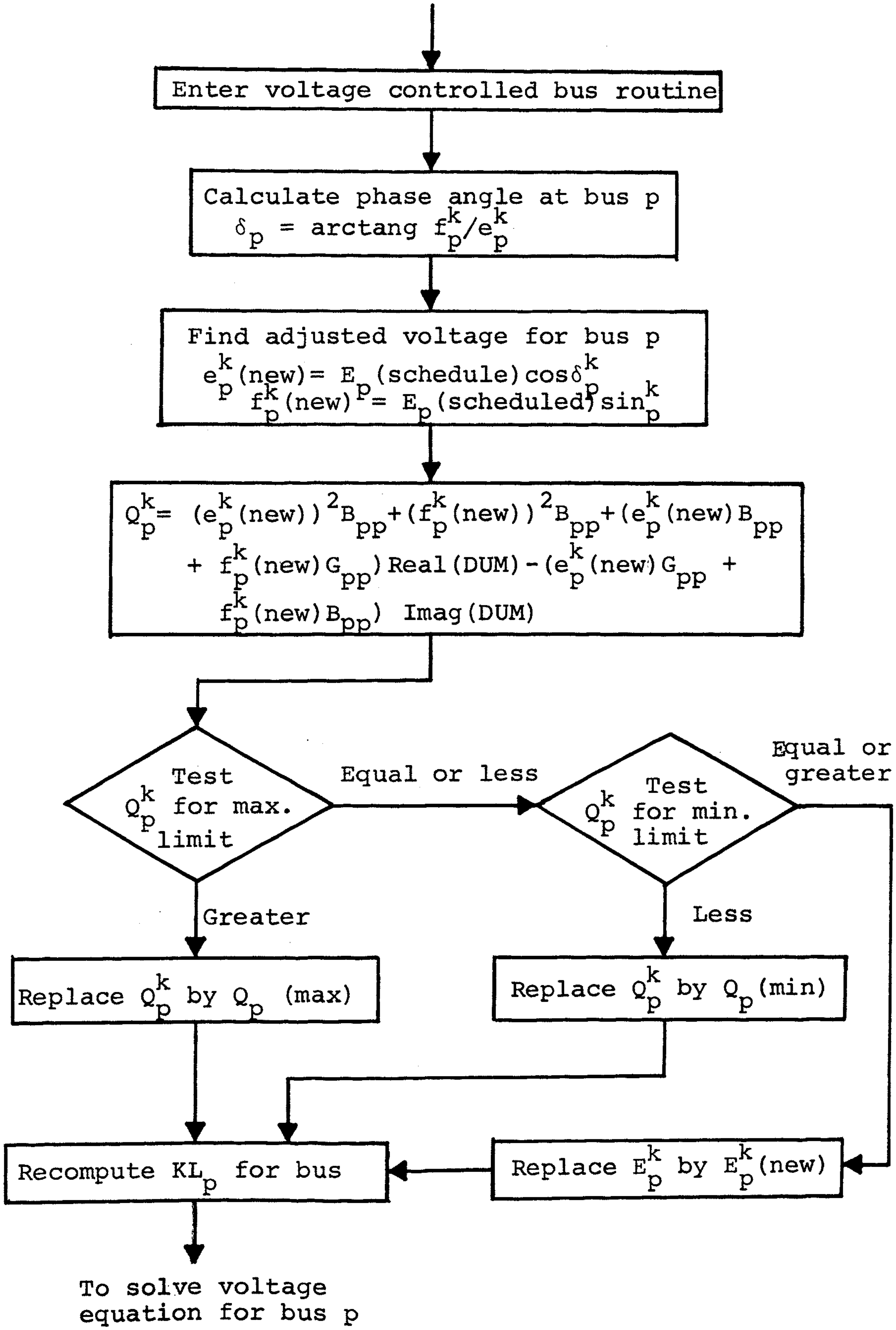

Figure A-3. Flow Chart for Voltage Controlled Bus Routine

APPENDIX B

LINE AND CABLE DATA CALCULATIONS
PERCENTS ON 100 MVA BASE

LINE FROM	TO.NO.	IMPEDANCE R PER 1000 FT $X10^{-3}$	IMPEDANCE X PER 1000 FT $X10^{-3}$	KV BASE	LENGTH FEET	NO. COND.	R%	X%
1	3			13.80			0.3700	0.6500
1	4			13.80			0.3700	0.6500
1	5			13.80			0.3700	0.6500
3	6	19.320	9.370	13.80	2115.00	2	1.0728	1.0406
3	6	19.320	9.370	13.80	2115.00	2	1.0728	1.0406
4	60	19.320	9.370	13.80	1320.00	2	0.6696	0.6495
5	100	19.320	9.370	13.80	2250.00	2	1.1413	1.1070
6	20	39.395	53.575	13.80	320.00	1	0.6620	0.9002
6	48	21.690	17.460	13.80	670.00	3	0.2544	0.6143
8	9	21.690	13.420	0.48	200.00	4	47.0703	116.4930
8	10	21.690	24.900	0.48	250.00	2	117.6758	270.1819
8	11	21.690	24.900	0.48	100.00	2	47.0703	108.0729
9	21	3.950	7.920	0.48	350.00	1	60.0043	120.3125
9	40	3.950	7.920	0.48	120.00	1	20.5729	41.2500

APPENDIX B. (continued)

LINE FROM	TO.NO.	IMPEDANCE R PER 1000 FT $X10^{-3}$	IMPEDANCE X PER 1000 FT $X10^{-3}$	KV BASE	LENGTH FEET	NO. COND.	R%	X%
21	22	21.690	24.900	0.48	100.00	1	94.1406	108.0729
21	23	21.690	24.900	0.48	320.00	2	150.6250	345.8330
21	31	21.690	24.900	0.48	230.00	2	108.2617	248.5674
21	32	21.690	24.900	0.48	100.00	2	47.0703	108.0729
21	33	21.690	24.900	0.48	200.00	2	94.1406	216.1456
25	26	21.690	17.460	0.48	320.00	3	100.4167	242.4999
25	27	21.690	17.460	0.48	300.00	3	94.1406	227.3436
25	28	21.690	17.460	0.48	260.00	3	81.5885	197.0313
25	29	21.690	17.460	0.48	440.00	3	138.0729	333.4375
25	30	21.690	17.460	0.48	830.00	3	260.4556	628.9841
33	34	21.690	24.900	0.48	240.00	2	112.9687	259.3748
40	41	21.690	24.900	0.48	190.00	2	89.4335	205.3384
40	42	3.950	7.920	0.48	120.00	1	20.5729	41.2500
42	43	21.690	24.900	0.48	320.00	2	150.6250	345.8330
43	44	21.690	24.900	0.48	200.00	2	94.1406	216.1456
44	47	21.690	24.900	0.48	380.00	2	178.8671	410.6768
45	48	21.690	17.460	13.80	470.00	3	0.1784	0.4309
46	47	21.690	24.900	0.48	230.00	2	108.2617	248.5674
47	59	21.690	13.420	0.48	650.00	2	305.9568	378.6023

APPENDIX B. (continued)

LINE FROM	To.No.	IMPEDANCE R PER 1000 FT $X10^{-3}$	IMPEDANCE X PER 1000 FT $X10^{-3}$	KV BASE	LENGTH FEET	No. Cond.	R%	X%
50	55	21.690	24.900	2.40	310.00	2	5.8367	13.4010
51	52	21.690	24.900	0.48	80.00	2	37.6562	86.4583
51	53	21.690	24.900	0.48	150.00	2	70.6055	162.1093
51	56	3.950	7.920	0.48	150.00	1	25.7161	51.5625
56	57	21.690	24.900	0.48	230.00	2	108.2617	248.5674
56	58	21.690	24.900	0.48	50.00	2	23.5352	54.0364
56	59	3.950	7.920	0.48	340.00	1	58.2899	116.8750
60	63	19.320	9.370	13.80	660.00	2	0.3348	0.3247
60	90	21.213	93.798	13.80	350.00	1	0.3899	1.7239
60	200	19.320	9.370	13.80	1620.00	2	0.8217	0.7971
63	72	39.395	53.575	13.80	630.00	1	1.3032	1.7723
63	85	212.100	93.798	13.80	570.00	1	6.3483	2.8074
66	70	21.690	13.420	0.48	150.00	4	35.3027	87.3698
67	68	21.690	24.900	0.48	300.00	2	141.2110	324.2183
67	69	21.690	24.900	0.48	250.00	2	117.6758	270.1819
67	70	3.950	7.920	0.48	150.00	1	25.7161	51.5625
70	71	3.950	7.920	0.48	280.00	1	48.0034	96.2500
71	86	21.690	24.900	0.48	300.00	2	141.2110	324.2183
71	87	3.950	7.920	0.48	160.00	1	27.4305	55.0000

APPENDIX B. (continued)

LINE FROM	TO.NO.	IMPEDANCE R PER 1000 FT $X10^{-3}$	X PER 1000 FT $X10^{-3}$	KV BASE	LENGTH FEET	NO. COND.	R%	X%
72	73	232.900	75.397	13.80	960.00	1	11.7404	3.8007
76	77	21.690	24.900	0.48	140.00	2	65.8984	151.3020
76	78	21.690	24.900	0.48	250.00	2	117.6758	270.1819
76	79	21.690	45.040	0.48	340.00	1	320.0779	664.6521
76	80	21.690	24.900	0.48	250.00	2	117.6758	270.1819
76	81	21.690	17.460	0.48	270.00	3	84.7265	204.6093
76	82	21.690	24.900	0.48	40.00	2	18.8281	43.2291
81	86	21.690	17.460	0.48	510.00	3	160.0390	386.4841
82	83	21.690	24.900	0.48	320.00	1	301.2500	345.8330
82	84	21.690	24.900	0.48	340.00	2	160.0390	367.4475
84	86	21.690	24.900	0.48	340.00	2	160.0390	367.4475
86	88	21.690	24.900	0.48	510.50	2	240.0584	551.1711
87	88	3.950	7.920	0.48	260.00	1	44.5746	89.3750
88	89	21.690	24.900	0.48	70.00	2	32.9492	75.6510
91	92	21.690	17.460	0.48	1150.00	3	360.8723	871.4841
100	101	19.320	9.370	13.80	390.00	2	0.1978	0.1919
100	160	19.320	9.370	13.80	1700.00	2	0.8623	0.8364
101	102	14.603	70.058	13.80	1325.00	1	1.0160	4.8743
101	110	184.300	72.748	13.80	970.00	1	9.3873	3.7054

APPENDIX B. (continued)

LINE FROM	TO.NO.	IMPEDANCE R PER 1000 FT $X10^{-3}$	IMPEDANCE X PER 1000 FT $X10^{-3}$	KV BASE	LENGTH FEET	NO. COND.	R%	X%
101	113	184.300	72.748	13.80	600.00	1	5.8066	2.2920
101	123	14.603	70.058	13.80	690.00	1	0.5291	2.5283
111	112	21.690	24.900	0.48	220.00	2	103.5546	237.7602
114	115	21.690	13.420	0.48	190.00	4	44.7168	110.6684
114	117	101.690	13.420	0.48	200.00	4	47.0703	116.4930
115	121	3.950	7.920	0.48	240.00	1	41.1458	82.5000
116	118	184.300	72.748	13.80	1070.00	1	10.3550	4.0874
119	120	21.690	13.420	0.48	200.00	4	47.0703	116.4930
121	122	21.690	24.900	0.48	165.00	2	77.6660	178.3201
121	131	3.950	7.920	0.48	360.00	1	61.7187	123.7500
124	125	21.690	24.900	0.48	345.00	2	162.3926	372.8513
124	126	21.690	24.900	0.48	300.00	2	141.2110	324.2183
124	127	21.690	24.900	0.48	255.00	2	120.0293	275.5854
124	128	21.690	24.900	0.48	200.00	2	94.1406	216.1456
128	129	21.690	24.900	0.48	345.00	2	162.3926	372.8513
128	130	55.873	57.295	0.48	830.00	1	2012.7860	2064.0120
131	132	3.950	7.920	0.48	300.00	1	51.4323	103.1250
131	133	21.690	24.900	0.48	150.00	2	70.6055	162.1093
131	134	21.690	24.900	0.48	150.00	2	70.6055	162.1093

APPENDIX B. (continued)

LINE FROM	TO.NO.	IMPEDANCE R PER 1000 FT $X10^{-3}$	IMPEDANCE X PER 1000 FT $X10^{-3}$	KV BASE	LENGTH FEET	NO. COND.	R%	X%
132	135	21.690	24.900	0.48	150.00	2	70.6055	162.1093
132	136	21.690	24.900	0.48	290.00	2	136.5039	313.4111
132	138	21.690	13.420	0.48	200.00	4	47.0703	116.4930
132	146	21.690	11.046	0.48	400.00	5	75.3125	191.7706
137	145	146.000	70.058	13.80	500.00	1	3.8332	1.8394
137	147	19.320	9.370	13.80	430.00	2	0.2181	0.2116
138	143	21.690	24.900	0.48	130.00	2	61.1914	140.4947
139	141	21.690	24.900	2.40	150.00	1	5.6484	6.4844
140	142	21.690	24.900	2.40	150.00	1	5.6484	6.4844
143	144	21.690	24.900	0.48	300.00	2	141.2110	324.2183
147	148	19.320	9.370	13.80	550.00	2	0.2790	0.2706
147	160	26.327	48.526	13.80	1060.00	1	1.4654	2.7010
148	150	116.000	67.400	13.80	300.00	1	1.8273	1.0618
148	151	116.000	67.400	13.80	500.00	1	3.0456	1.7696
148	152	116.000	67.400	13.80	520.00	1	3.1674	1.8404
148	153	116.000	67.400	13.80	600.00	1	3.6547	2.1235
148	154	293.600	78.076	13.80	740.00	1	11.4085	3.0338
160	161	212.100	93.798	13.80	1140.00	1	12.6966	5.6149

APPENDIX B (continued)

LINE FROM	TO.NO.	IMPEDANCE R PER 1000 FT $X10^{-3}$	IMPEDANCE X PER 1000 FT $X10^{-3}$	KV BASE	LENGTH FEET	NO. COND.	R%	X%
160	170	184.300	72.748	13.80	550.00	1	5.3227	2.1010
160	171	184.300	72.748	13.80	550.00	1	5.3227	2.1010
160	172	184.300	72.748	13.80	550.00	1	5.3227	2.1010
173	174	21.690	24.900	0.48	250.00	2	117.6758	270.1819
173	176	121.690	24.900	0.48	300.00	2	141.2110	324.2183
173	177	21.690	24.900	0.48	450.00	2	211.8164	487.3274
173	178	21.690	24.900	0.48	300.00	2	141.2110	324.2183
173	181	21.690	13.420	0.48	210.00	4	49.4238	122.3177
175	180	21.690	24.900	0.48	100.00	2	47.0703	108.0729
178	179	21.690	24.900	0.48	400.00	2	188.2813	432.2915
181	182	3.950	7.920	0.48	220.00	1	37.7170	75.6250
181	185	21.690	13.420	0.48	120.00	4	28.2422	69.8958
182	183	3.950	7.920	0.48	320.00	1	54.8611	110.0000
182	187	21.690	13.420	0.48	200.00	4	47.0703	116.4930
183	184	21.690	45.040	0.48	540.00	1	508.3589	1055.6240
187	188	21.690	24.900	0.48	400.00	2	188.2813	432.2915
200	201	19.320	9.370	13.80	250.00	2	0.1268	0.1230
202	203	21.690	24.900	2.40	200.00	2	3.7656	8.6458
204	205	21.690	24.900	2.40	140.00	2	2.6359	6.0521

APPENDIX B (continued)

LINE FROM	TO.NO.	IMPEDANCE R PER 1000 FT $X10^{-3}$	IMPEDANCE X PER 1000 FT $X10^{-3}$	KV BASE	LENGTH FEET	NO. COND.	R%	X%
206	207	0.395	0.792	0.48	110.00	1	1.858	3.7812
207	208	21.690	45.040	0.48	295.00	1	277.7146	576.6836
207	209	3.950	7.920	0.48	135.00	1	23.1445	46.4062
208	209	21.690	45.040	0.48	295.00	1	277.7146	576.6836
209	210	26.327	48.430	0.48	470.00	1	537.0522	987.9380
211	212	21.690	9.346	0.48	540.00	6	84.7265	219.0468

APPENDIX C

Transformer Data Calculations Percents on 100 MVA and 13.8 Base

Transformer From	To	NO. TAP	KV BASE OLD	MVA BASE	R%	X%
1	2	0.978	13.80	100.00	0.0000	12.4300
6	7	0.978	13.80	8.00	7.3750	71.6250
6	8	0.978	13.80	2.50	33.6000	213.2000
20	21	0.933	13.11	2.50	30.4323	243.6749
20	21	0.933	13.11	2.50	30.4323	243.6749
20	25	0.975	13.80	5.00	18.6000	117.0000
45	47	0.978	13.80	2.50	33.3200	270.7998
48	49	0.984	13.11	6.50	7.2478	100.2469
48	50	0.958	13.80	5.00	11.0000	115.2000
48	51	0.933	13.11	2.50	30.4323	243.6749
63	64	0.933	13.11	8.00	5.9791	76.8253
63	65	0.933	13.11	8.00	5.9791	76.8253
63	66	0.933	13.11	2.50	30.9016	247.2850
63	67	0.933	13.11	2.50	30.4323	243.6749
72	76	0.978	13.80	3.00	25.3333	187.3333
73	74	1.000	13.80	5.10	12.4118	99.2156
73	75	0.978	13.80	1.00	74.3000	599.3999
85	86	0.978	13.80	2.50	33.2000	216.0000
90	91	1.000	13.80	1.00	93.0000	585.0000
90	91	1.000	13.80	1.00	91.8000	734.5000
102	103	1.010	13.11	1.00	82.8495	662.8860
102	103	1.010	13.11	1.00	82.8495	662.8860
110	111	0.933	13.11	1.00	81.2250	466.5923
113	114	0.885	13.80	2.50	24.0000	275.5999
116	117	0.933	13.11	2.00	26.1725	310.4597
118	119	0.885	13.80	2.50	24.8000	277.2000
123	124	0.933	13.11	2.50	30.9016	247.2850

APPENDIX C (continued)

Transformer From	TO	NO.TAP	KV BASE OLD	IMVA BASE	R%	X%
137	138	0.933	13.11	2.50	30.9016	247.2850
137	139	0.982	13.11	6.50	7.2339	99.9692
137	140	0.980	13.80	5.00	11.8000	107.8000
145	146	0.978	13.80	2.50	28.0000	224.0000
150	155	0.974	13.80	2.50	33.8000	270.0000
151	156	0.974	13.80	2.50	33.8000	270.0000
152	157	0.974	13.80	3.00	27.2333	218.3333
153	158	0.974	13.80	3.00	27.5000	220.0000
154	159	1.100	13.80	1.00	70.2000	561.9998
161	162	0.978	13.80	2.00	42.8000	342.5000
170	173	0.932	13.11	2.50	20.9380	250.8949
171	173	0.932	13.11	2.50	21.2990	244.7579
172	175	0.932	13.11	2.00	27.0750	309.1062
186	187	0.933	13.11	2.50	30.9016	247.2850
186	189	1.000	13.11	2.00	34.4304	390.3311
186	190	1.000	13.11	2.00	34.3401	386.6711
186	191	1.000	13.11	2.00	25.2700	351.0723
201	202	0.978	13.80	7.50	7.2000	74.5333
201	204	0.978	13.80	7.50	7.2000	74.0000
201	206	0.978	13.80	2.50	32.4000	212.8000
201	211	0.974	13.80	2.50	32.3600	223.6000

VITA

Juan Vincente Saavedra was born May 24, 1944 in Ecuador, South America. His elementary and secondary education was received from schools in Guayaquil, Ecuador. He received the degree of B.S.E.E. from the University of Alabama at Tuscaloosa, Alabama in August 1969.

He has been enrolled in the graduate school of the University of Missouri-Rolla since August 1969 and has held an Alcoa Fellowship for the period January 1971 to August 1971. He is a member of Eta Kappa Nu and Institute of Electrical and Electronic Engineers.