A Solution to Finite Escape Time for H∞ Filter based SLAM

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***Abstract*—This paper proposed a solution to the Finite Escape Time problem in H∞ Filter based Simultaneous Localization and Mapping problem. Finite escape time has been one of the obstacle that holding the realization of H∞ Filter in many applications. For this reason, a method of decorrelating some of the updated state covariance of the filter is suggested to avoid the finite escape time from occurred during mobile robot estimations. Two main cases are investigated in this paper to observe the filter performances which are the unstable partially observable and stable partially observable H∞ Filter-SLAM. The simulation results have shown convincing outcomes to the overall estimation, which can prevent the finite escape time in the estimation especially for the stable partially observable H∞ Filter-SLAM case.**

***Index Terms*—H∞ Filter; Kalman Filter; SLAM; Decorrelation.**

I. INTRODUCTION

Nowadays, the development of autonomous robot in various applications can be notably recognized. Especially, in the task of exploration and navigation, the role of autonomous robot is very important in order to explore, observe and plan for its movement. One of the task which attempt to continuously observing landmarks and collecting information while moving through an unknown environment is referred as the SLAM (Simultaneous Localization and Mapping) problem. The problem became famous after a series of influential seminal papers introduced in 1990's such as Smith and Cheeseman et al. [1] introduced the relationship between mobile robot and landmarks. Due to its capability in realizing a truly autonomous robot behavior, the SLAM problem has gained researcher's attention over some past decades. Unfortunately, even though a lot of discussion and development efforts have been continuously conducted, the problem still facing a lot of unsolved issues such as the condition of data association, effects of dynamic environment and uncertainties.

Human limitations to work in hazardous areas is one of the key factor making the SLAM becomes the ultimate way to solve the problem. Hence, the application can be found widely not only in space exploration, but also in underwater navigation, mining operations and military. As the application can be applied in such cases, the system is also well-designed to consider both 2D and 3D configurations [2-5] as shown in Figure 1. Further explanation about SLAM can be found in [6].

With regards to uncertainties, it is a wise decision to model a system that is able to take into account for a worst case of noise or when the noise statistics is unknown. Hence, H∞ Filter could be the best to tolerate with such robust system. H∞ Filter theoretically assumes that the noises are bounded in a level of energy. This approach is recommended to a system where the worst-case estimation is considered. Even though this approach is a family of Kalman Filter approach [7, 8], Kalman Filter do not exhibit such issue that affects the overall estimation.

Throughout this paper, H∞ Filter based SLAM performance in nonlinear SLAM problem under two partially observable SLAM cases is examined; Unstable Partially Observable SLAM and Stable Partially Observable SLAM. Simulation analysis is carried which considers a planar and small environment that consists of some stationary landmarks. Kalman Filter and Extended Kalman filter(EKF) have been studied immensely in the SLAM problem using various approaches [9, 10]. However due to the limitation of incapabilities or incompetency in non-gaussian noise environment, others methods are also welcomed such as the Particle Filter, and the Uncented Kalman Filter. Unfortunately, those two methods suffer in terms of computational cost and for online application. Hence, H∞ Filter is chosen as a solution to SLAM problem.

Despite of current papers that has been published regarding the observability of SLAM using Kalman Filter and H∞ Filter already exists, more analysis still expected. The reason is due to complexity of correlation of the state covariance. The two cases stated above needs proper analysis especially for H∞ where the finite escape time problem exist compared to Kalman Filter, which the problem do not occur. Previous study [11-13] examined the partial observability in SLAM for H∞ Filter about its theoretical analysis. Further analysis in this paper suggested that the finite escape time problem in H∞ Filter can be prevented if proper selection of γ is selected. Furthermore, the analysis is carried longer with more amount of observation noise. To prove this, the simulation result also shows that the finite escape time in H∞ Filter can be avoided.

This paper is organized as follows. In section II, the general SLAM problem and H∞ Filter algorithm are presented with a brief comparison to Kalman Filter, while section III explains about decorrelation strategy applied in this paper. The results are then shown in section IV, which demonstrates the simulation and experimental result of both mentioned cases in H∞ Filter-SLAM problem. Finally section V, concludes the paper.

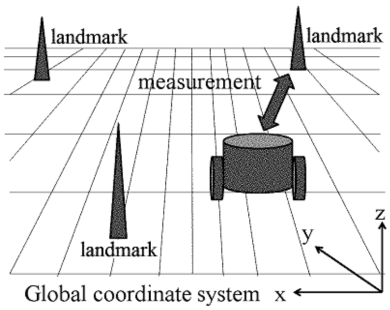

Figure 1: SLAM problem

II. SLAM GENERAL MODEL

SLAM is designed from two base models, which are known as the process model that explains how the robot moves through the environment and the measurement model which calculates and measures the distance between mobile robot and landmarks continuously. For process model, the robot kinematics model should be determined first to understand the robot motion through the environment. The landmarks or features are also important in order to verify the environment. We made an assumption that the landmarks are stationary for convenience. For the SLAM process model, we have the following equations that demonstrate the linear discrete time process.

$$x_{k+1} = F_k x_k + u_k + v_k \tag{1}$$

where F_k is the state transition matrix, x_k is the vehicle and observed landmark states, u_k is the control inputs and v_k is temporally uncorrelated process noise with its associated covariance, Q_k.

In a simpler representation if a linear case is considered, (1) can be represented as follows.

$$x_{k+1} = \begin{bmatrix} F_v & 0 & \cdots & 0 \\ 0 & I_1 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & I_n \end{bmatrix} \begin{bmatrix} x_v \\ l_1 \\ \vdots \\ l_n \end{bmatrix} + \begin{bmatrix} u_v \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} v_k \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{2}$$

For the second stage, the measurement or observation process can be shown as the following equation:

$$z_{k+1} = H_k x_k + w_k \tag{3}$$

where z_{k+1} is the relative angle, and relative distance for any observed landmarks with respect to the mobile robot location. ω_k is the temporally uncorrelated observation error with covariance R_k. H_k on the other hand is the measurement between mobile robot and any measured landmarks based on angle and distance.

H∞ Filter based SLAM

This section includes brief introduction of H∞ Filter-Based SLAM by considering its convergence properties and some comparison to Kalman Filter. Previous works in [7, 9] have presented a satisfactory explanation of the H∞ filtering that will be a good reading for further references. Referring to those, an assumption is made for the noise characteristics.

$$Assumption\ 1: R_k \cong DD^T$$

$$Assumption\ 2: Bounded\ noise\ energy;$$

$$\sum_{t=0}^{N} \|\omega_k\|^2 \geq 0, \sum_{t=0}^{N} \|v_k\|^2 \geq 0$$

where D is the noise variance.

$\Sigma_0 > 0$ is the initial covariance matrix for state x_k, $Q_k > 0$, and $R_k > 0$ are the weighting matrix for process and measurement noises ω_k, and v_k respectively. Above assumption is similar to the standard Kalman Filter assumption where all components of the measurement vector are assumed to be corrupted by noise and bounded at all time. For more detail descriptions of H∞ Filter, refer to [7].

H∞ Filter algorithm is very similar to the Kalman Filter. H∞ Filter concerns about the linear relationship of the system state x_k given by $z_k = l_k x_k$ instead of the state estimation itself as shown in Kalman Filter. The difference between Kalman Filter and H∞ Filter is in its form of gain and covariance characteristics; integration of the prediction and update process. For Kalman Filter, the equation for its gain and covariance are given by;

$$K_k = P_k(I + H_K^T R_K^{-1} H_k P_k)^{-1} \tag{4}$$

$$P_{k+1} = F_k P_k (I + H_K^T R_K^{-1} H_k P_k)^{-1} F_K^T + Q_k \tag{5}$$

The H∞ Filter on the other hand holds the following equations.

$$K_k = P_k(I - \gamma^{-2} P_k + H_K^T R_K^{-1} H_k P_k)^{-1} \tag{6}$$

$$P_{k+1} = F_k P_k (I - \gamma^{-2} P_k + H_K^T R_K^{-1} H_k P_k)^{-1} F_K^T + Q_k \tag{7}$$

As shown above, H∞ Filter relies heavily on the covariance matrix of error signals; Q_k, R_k>0. These two error covariances are designed to achieve the desired performance. Another important parameters to be considered is γ>0. γ must be guaranteed to be positive all the time and selected properly to obtain a good estimation results. If γ becomes bigger, equations (6) and (7) yields the same values as equation (4) and (5) respectively. In other words, the results will be similar to the Kalman Filter estimation.

III. DECORRELATION USING COVARIANCE INFLATION UNIT

The effects of partial observability are being examined in this section. The problem is analyzed in two different categories.

- O(N) but unstable partially observable H∞ Filter-SLAM
- O(N) but stable partially observable H∞ Filter-SLAM

Those two conditions have been investigated in [13], where the analysis focusses on the theoretical development of the Covariance Inflation under defined situations. With respect to the outcomes demonstrated in their results, this paper extends the work to check whether the stable partially observable condition can preserve the best performance. Besides, the study analyzed the results considering a uniform noise characteristic in which H∞ Filter is said robust to. As the decorrelation used Covariance Inflation is a method that adding pseudo-noise to the system, mathematical description for the covariance inflation are included for convenience. This will be the same results to EKF-SLAM [4, 13] as it has almost the same structure.

$$P_{k+1} = F_k P_k F_k^T + \Delta P_k \tag{8}$$

The ΔPk≥0 is design such that it is able to produce a small amount of state covariance, P. Details can be found in [11],[13] regarding the selection of ΔP_k. As stated in [13], the state covariance matrix, which defines the uncertainties on the system is converging to a steady state and almost zero after a period of time. However, it is not clearly demonstrated previously about the conditions of the state covariance during the mobile robot observations. The finite escape time, which is the main problem in H∞ filtering has not been analyzed in different cases of noise. Furthermore, this paper suggests that the proposed approach can avoid the finite escape time problem if the γ is selected appropriately by referring to Theorem 2 proposed in [13].

Assume that the initial covariance $P_0 \geq 0$. The first observation of mobile robot yields a state covariance as follows.

$$P_1 = \begin{bmatrix} P_{0v} & 0 \\ 0 & P_{0m} \end{bmatrix} \tag{9}$$

where P_{0v}, P_{0m} are the initial state covariance for the mobile robot and landmarks respectively.

From the results of [12], the covariance matrix of a stationary robot observing one time of one landmark at point A is given by:

$$\mathrm{P}_1 = \begin{bmatrix} \mathrm{P}_0 & \mathrm{P}_0(\mathrm{A}^{-1}\mathrm{H_A})^{\mathrm{T}} \\ \mathrm{A}^{-1}\mathrm{H_A}\mathrm{P}_0 & \mathrm{A}^{-1}\mathrm{H_A}\mathrm{P}_0(\mathrm{A}^{-1}\mathrm{H_A})^{\mathrm{T}} + \mathrm{A}^{-1}\mathrm{R}^{-1}\mathrm{A}^{-1} \end{bmatrix} \tag{10}$$

where:

$$\mathrm{A} = \begin{bmatrix} \frac{x_m - x_A}{\sqrt{x_m - x_A^2 + y_m - y_A^2}} & \frac{y_m - y_A}{\sqrt{x_m - x_A^2 + y_m - y_A^2}} \\ \frac{y_m - y_A}{\sqrt{x_m - x_A^2 + y_m - y_A^2}} & \frac{x_m - x_A}{\sqrt{x_m - x_A^2 + y_m - y_A^2}} \end{bmatrix} \tag{11}$$

and

$H_A = [e \quad A],\ e = [0 \quad -1]$

Then the Covariance Inflation method adds a pseudo noise $\Delta P^i \geq 0$ to the updated state covariance. Assuming that the process noise covariance is too small and after one step inflation, then the covariance matrix yields:

$$\begin{aligned} P_1 &= F_1 P_0 (I - \gamma^{-2} P_0 + H_1^T R_1^{-1} H_1 P_0)^{-1} F_1^T + \Delta P_0^1 \\ &> F_1 P_0 (I - \gamma^{-2} P_0 + H_1^T R_1^{-1} H_1 P_0)^{-1} F_1^T \end{aligned} \tag{12}$$

where:

$$\Delta P_0^1 = \begin{bmatrix} k\mathrm{P}_0(\mathrm{A}^{-1}\mathrm{H_A})^{\mathrm{T}} & -\mathrm{P}_0(\mathrm{A}^{-1}\mathrm{H_A})^{\mathrm{T}} \\ \mathrm{A}^{-1}\mathrm{H_A}\mathrm{P}_0 & \frac{\mathrm{P}_0(\mathrm{A}^{-1}\mathrm{H_A})^{\mathrm{T}}}{k} \end{bmatrix}$$

From the positive semidefinite matrix properties, any submatrix of a psd is also a psd. Therefore, the map state covariance matrix is also holding the same criteria as above equation. Hence, the following equation of the map state covariance element can be derived.

$$P_{k+1} = F_1 P_0 (I - \gamma^{-2} P_0 + H_1^T R_1^{-1} H_1 P_0)^{-1} F_1^T + \Delta P_{k+1}^1 \geq P_{K+1} \tag{13}$$

For better understanding of above equation, consider that γ posses a very big value such that γ→∞. If this is happening, then the updated state covariance becomes the normal Kalman Filter equation.

Examining further the condition, the map state covariance eventually has become bigger as more observations are being made by the mobile robot. Hence, the process will produce higher amount of noises as time passed by. Such a case normally makes the estimation becomes erroneous as the uncertainties is growing fast per observations. This is supposed not to be happening in SLAM problem.

Consider a case when γ is selected such that it can satisfy the following equation.

$$I + H_K^T R_K^{-1} H_k P_k > \gamma^{-2} P_k > 0 \tag{14}$$

Comparing (5), (12) and (14), if γ>0, then the state covariance is bigger than the previous state covariance and also bigger than the Kalman Filter state covariance.

To understand how a finite escape time occurred during estimation, further examination on (12) can be organized. Assume that γ=1, then:

$$\begin{aligned} P_1 &= F_1 P_0 (I - P_0 + H_1^T R_1^{-1} H_1 P_0)^{-1} F_1^T \\ &> F_1 P_0 (I + H_1^T R_1^{-1} H_1 P_0)^{-1} F_1^T \end{aligned} \tag{15}$$

The left side of equation (15) or the updated state covariance is clearly bigger than the previous equation. The continuation of these properties will yield an increasing value of state covariance or uncertainties and ended with higher value of state covariance. On the other perspective, if the P_0 on the left side equation has the property of $P_0 > 1 + H_1^T R_1^{-1} H_1 P_0$, then the updated state covariance exhibits a negative definite value. Again, this situation is unacceptable in SLAM problem. Several experimental results and analysis have shown this before [13]. Thus, there are two possible values that the updated state covariance shows whether it holds a negative definite matrix of positive definite matrix if γ is not chosen correctly.

To avoid this issue, the addition of pseudo matrix ΔP to the updated state covariance can be a solution. ΔP can be design such that it can tolerate the value of updated state covariance to be positive semidefinite at all time. In other words, $1 - P_0 + H_1^T R_1^{-1} H_1 P_0 + \Delta P \geq 0$ or can be stated as $\Delta P_k > P_k$. These rules must be satisfied in preserving a good estimation in H∞ Filter-SLAM. Furthermore, the addition of ΔP_k has now guarantees that the negative semidefinite matrix is not exist during observations.

One of the important things to be realized is that, ΔP_k can be simply added without control or rules. If ΔP_k is added continuously without depending on the previous state covariance, then the updated state covariance now will be increase unexpectedly. Subsequently, the estimation becomes erroneous again even though finite escape time is not observed. This case explains the first case of partial observability known as unstable partial observability.

A stable partially observable case is a case where the estimation can guarantee a level of good estimation while at the same time can reduce the computational cost of the system. Looking into the H_∞ Filter-SLAM algorithm and Kalman Filter, the measurement matrix, H_k is actually playing an important role that defines whether the state covariance converge at steady state after a successive observation by mobile robot [3]. The measurement matrix, H_k, which describes the relative angle and distance between mobile robot and the landmarks can yield smaller or high value of information. Theoretically if H_k holds small value and $\gamma = 1$, then equation $1 - P_0 + H_k^T R_k^{-1} H_k P_0$ is highly depends on the H_k such that it can leads to negative semidefinite matrix especially if P_o or the previously calculated state covariance is having large uncertainties. Hence, both H_k and γ are related and must be considered to gain better estimation performance in H_∞ Filter-SLAM. Making γ bigger eventually makes H_∞ Filter-SLAM exhibit the similar results to the Kalman Filter. Thus, γ must be defined appropriately to pursue better estimation results according to the condition of H_k.

Rather than adjusting the value of γ, an addition of pseudo matrix ΔP_k can decrease the difficulty of finding the right value of γ. Again, if $\gamma = 1$ then with the addition of $\Delta P_k \geq 0$ the equation becomes:

$$1 - P_0 + H_k^T R_k^{-1} H_k P_0 + \Delta P_k > 0 \tag{16}$$

$\Delta P_k \geq 0$ refrain the possibility of the updated state covariance becomes negative definite matrix or the finite escape time. Even sometimes, H_k contains small value, thanks to ΔP_k, the equation is assured to hold a positive semidefinite matrix. As the ΔP_k is calculated on each of the predicted state covariance, then the updated state covariance will consistently converge. This is how the stable partially observable H_∞ Filter-SLAM is working.

Theoretical explanation is always help to understand how this is happening. The result of [12] is referred to analyze further the condition when a stationary mobile robot is observing two different landmarks at a specified time at point A. The state covariance yields the following equation.

$$P_1 = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} \tag{17}$$

where:

$P_{11} = P_0$
$P_{12} = P_0(A^{-1}H_A)^T = P_{21}^T$
$P_{13} = P_0(\bar{A}^{-1}H_{\bar{A}})^T = P_{31}^T$
$P_{22} = A^{-1}H_A P_0 (A^{-1}H_A)^T + A^{-1}{R_A}^{-1}A$
$P_{23} = A^{-1}H_A P_0 (\bar{A}^{-1}H_{\bar{A}})^T = P_{32}^T$
$P_{33} = \bar{A}^{-1}H_{\bar{A}} P_0 (\bar{A}^{-1}H_{\bar{A}})^T + \bar{A}^{-1}{R_{\bar{A}}}^{-1}\bar{A}$

When the covariance inflation method is applied on the landmarks state covariance of the updated state covariance especially on the diagonal elements of equation (17), the state covariance becomes:

$P_{22} = A^{-1}H_A P_0 (A^{-1}H_A)^T + A^{-1}{R_A}^{-1}A + kP_{23}$
$P_{23} = A^{-1}H_A P_0 (\bar{A}^{-1}H_{\bar{A}})^T = P_{32}^T = 0$
$P_{33} = \bar{A}^{-1}H_{\bar{A}} P_0 (\bar{A}^{-1}H_{\bar{A}})^T + \bar{A}^{-1}{R_{\bar{A}}}^{-1}\bar{A} + P_{23}/k$

where k>0 and other elements are unchanged for all observation times. The addition does not change the properties of the state covariance and can be guaranteed to be converge with a slightly bigger value of uncertainties on the updated landmarks state covariance. However, the computational time can be further reduced as some of the elements do not need to be inverse during calculations.

IV. Simulation Results and Discussion

Simulation for a nonlinear case SLAM for a moving mobile robot observing landmarks is organized to understand the behavior for the two cases of H∞ Filter-SLAM. The mobile robot is assumed to start its task from a global coordinate system of (0, 0). The analysis is mainly covering an indoor environment with a small mobile robot with some randomly placed features. Two cases are examined as stated previously in the preceding section; Unstable partially observable and stable partially observable H∞ Filter-SLAM.

Table 1
Control Parameters for Simulation

Indoor Parameters	Value
Gaussian robot Process Noise, Q_v	0.0000001
Gaussian landmark process noise	0
Gaussian Observation noise, R	$\begin{bmatrix} 0.0001 & 0 \\ 0 & 0.0001 \end{bmatrix}$
Process with random noise	$\begin{bmatrix} Q_{max} = 0.001 \\ Q_{min} = -0.001 \end{bmatrix}$
Observation with random noise	$\begin{bmatrix} R_{\theta max} = 0.16 \\ R_{\theta min} = -0.08 \\ R_{distance(\max)} = 0.5 \\ R_{distance(\min)} = -0.5 \end{bmatrix}$
Initial covariance	$P_v = 0.00001$ $P_m = 10000$
Landmarks location	Defined at certain locations

A. Unstable Partially Observable H_∞ Filter-SLAM

Following Figures 2 and 3 show the results of simulations results based on the unstable partially observable H_∞ Filter-SLAM. It seems that similar results presented on [9] was also achieved. The estimation of both mobile robot and landmarks are far from the expected value and erroneous. This is due to the continuous addition of pseudo-noise to the diagonal elements of covariance matrix. The repetition of pseudo-noise addition eventually has make the covariance matrix bigger and finally effects the overall estimation where mobile robot becomes unsure about its location and landmarks coordinates.

Interestingly, the constructed map illustrates that the unstable H_∞ Filter-SLAM show no finite escape time than the normal H_∞ Filter-SLAM. This subsequently proved that the addition of pseudo psd to H_∞ Filter-SLAM can prevent the finite escape time problem from occurring during observations. Figures 4 and 5 demonstrate that both mobile robot and landmarks updated state covariance is increasing as the mobile robot continues to observe its surroundings.

B. Stable Partially Observable H∞ Filter-SLAM

In this case, the simulation time is increased up to 2000s with the same sampling time as carried by the previous case. Below Figures 6 and 8 illustrate the results of estimations. In contrast with the previous case of unstable partially observable H_∞ Filter-SLAM, now the estimations has becomes better. Both estimations for mobile robot and landmarks agree with the expected results and surpassed the normal performance of H_∞ Filter-SLAM.

It is worth to note that the assumption of each landmark is independent to each other has a significant contribution to the results. It is also interesting to recognize that this is one of the available approached to avoid the finite escape time problem in H_∞ Filter-SLAM. To check the consistency of the proposed technique, the simulation time was extended until 5000s. Even though the time of observation has increased, the proposed technique still yields its best performance and no finite escape time has been observed.

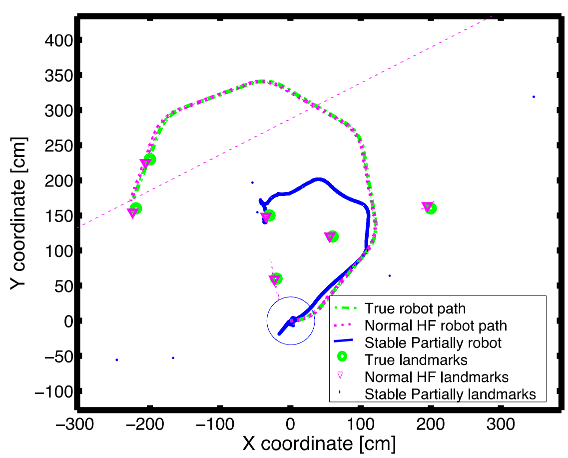

Figure 2: Comparison between normal H_∞ Filter-SLAM and unstable partially observable H_∞ Filter-SLAM

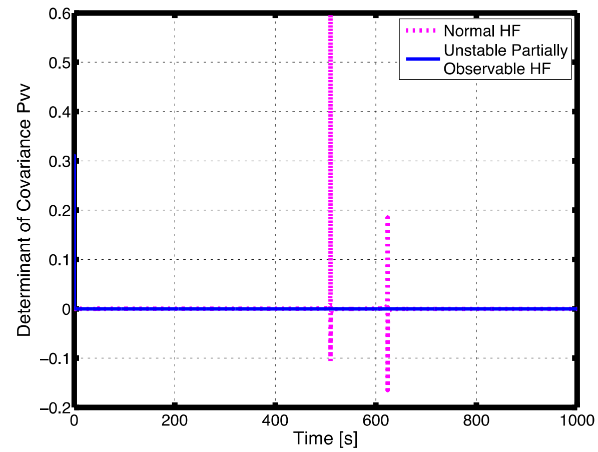

Figure 3: Mobile robot state covariance when observing landmarks

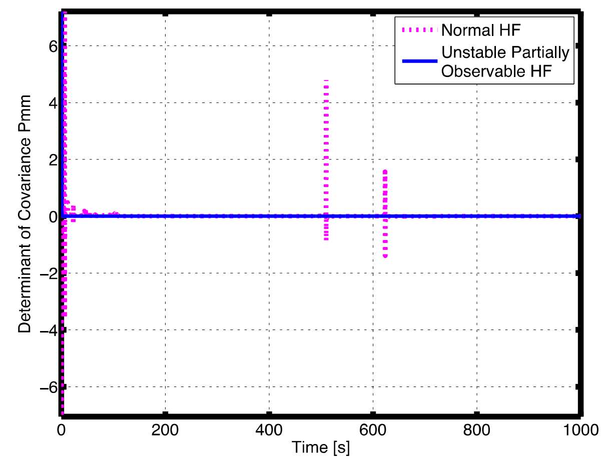

Figure 4: Landmarks state covariance

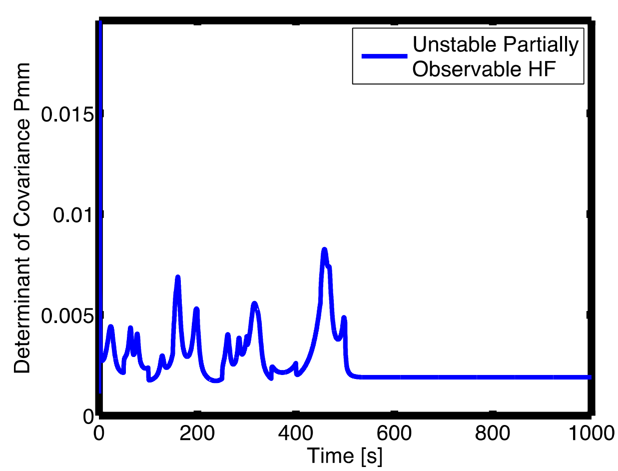

Figure 5: Increasing landmarks state covariance on unstable partially observable H_∞ Filter-SLAM

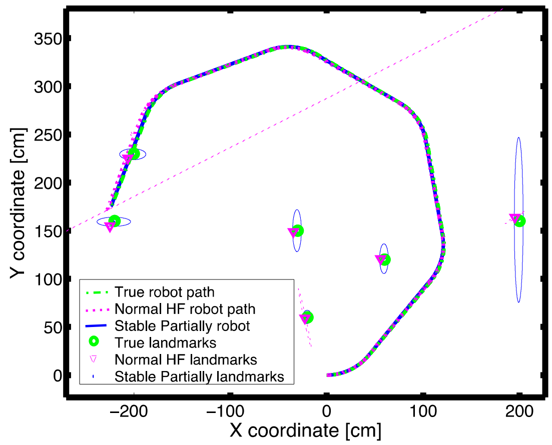

Figure 6: Comparison between normal H_∞ Filter-SLAM and stable partially observable H_∞ Filter-SLAM

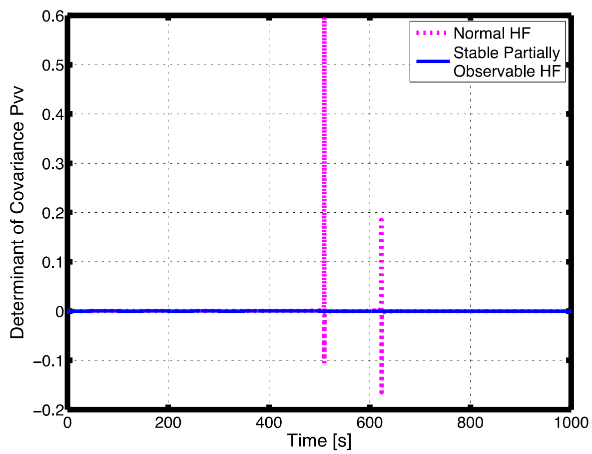

Figure 7: Mobile robot state covariance when observing landmarks

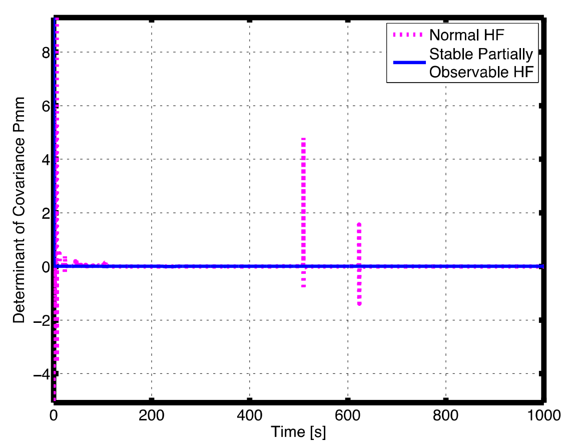

Figure 8: Landmarks state covariance

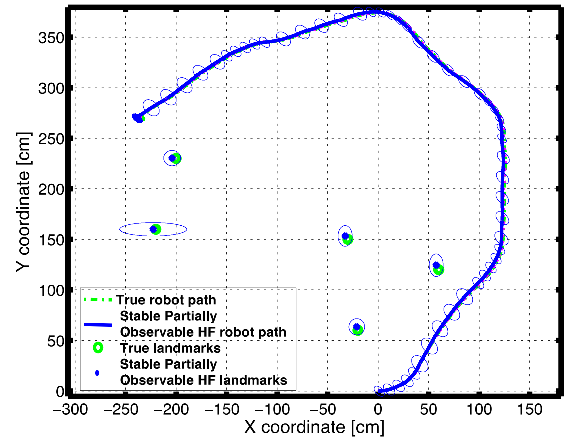

Figure 9: Simulation results for 5000s; no Finite Escape Time is observed

C. *The case of random noise effect to H_∞ Filter-SLAM and Kalman Filter based-SLAM on Stable Partially Observable SLAM*

The result of the stable partially observable H_∞ Filter-SLAM is compared to EKF-SLAM in this section using non-gaussian noise. This is one of the essential requirements when prior information about the environment is unknown. It can be concluded that from Figure 10, H_∞ Filter-SLAM is better than EKF-SLAM although it present slightly bigger covariance than EKF-SLAM about its estimations.

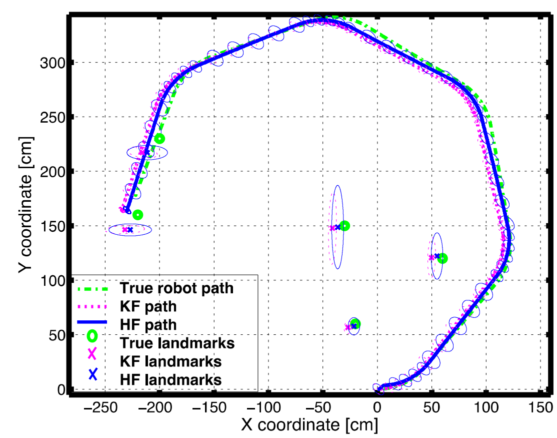

Figure 10: Estimation result with different noise characteristics (random noise) for Kalman Filter based SLAM and H_∞ Filter-SLAM

V. CONCLUSION

This paper proposed H_∞ Filter-SLAM with covariance inflation method to prevent the finite escape time problem. The covariance inflation, which relies on the decorrelation algorithm may sufficiently reduce the cost computation as well as well as the condition where the estimation become erroneous. There is also a possibility that it may result unbounded uncertainties in the estimation as shown by the unstable partially observable SLAM problem. These are the results when a full rank of ΔP is added to the state covariance and without considering the other state covariance elements. If this happen, the estimation becomes erroneous. To overcome such problem, a minor change that relates the mobile robot with the landmark state covariance is taken into account by adding a partial pseudo psd to the state covariance. However, this sequence can also still influence the estimation i.e it could produce high amount of uncertainties if not being design well.

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