Integrated reliable and robust design

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INTEGRATED RELIABLE AND ROBUST

DESIGN

by

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ABSTRACT

The objective of this research is to develop an integrated design methodology for reliability and robustness. Reliability-based design (RBD) and robust design (RD) are important to obtain optimal design characterized by low probability of failure and minimum performance variations respectively. But performing both RBD and RD in a product design may be conflicting and time consuming. An integrated design model is needed to achieve both reliability and robustness simultaneously. The purpose of this thesis is to integrate reliability and robustness. To achieve this objective, we first study the general relationship between reliability and robustness. Then we perform a numerical study on the relationship between reliability and robustness, by combining the reliability based design, robust design, multi objective optimization and Taguchi's quality loss function to formulate an integrated design model. This integrated model gives reliable and robust optimum design values by minimizing the probability of failure and quality loss function of the design simultaneously. Based on the results from the numerical study, we propose a generalized quality loss function that considers both the safe region and the failure region. Taguchi's quality loss function defines quality loss in the safe design region and risk function defines quality loss in the failure region. This integrated model achieves reliability and robustness by minimizing the general quality loss function of the design. Example problems show that this methodology is computationally efficient compared to the other optimization models. Results from the various examples suggest that this method can be efficiently used to minimize the probability of failure and the total quality loss of a design simultaneously.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

1. INTRODUCTION

1.1. BACKGROUND

The objective of this research is to better understand the relationship between reliability and robustness and then to develop a methodology for the integration of reliability-based design and robust design. In today's competitive market, engineers face new challenges due to the creation of complex design models and applications of new technology. With the demand for both reliable and quality products increasing day by day, it has become imperative to create a design model that accounts for both reliability and robustness of new products.

Reliability is defined as the ability of a system or component to perform its required functions under stated conditions for a specified period of time [1]. Another web definition of reliability for mechanical systems is "Mechanical reliability is the probability that a spare, item, or unit will perform its prescribed duty without failure for a given time when operated correctly in a specific environment" [2]. The likelihood of success or failure of a product depends on its reliability. As the number of failures of a product increases, its reliability decreases. The central role in reliability engineering is the concept of failure and efforts need to be put to reduce failure and increase the reliability of a product.

Robustness is defined as the property by which a product performance is insensitive to variation [3]. Numerous methods have been developed to support the design of robust products. The majority of these focus on improving the design so that the variations are reduced. Variations generally occur due to the presence of noise factors. The central role in robust design methodology is the concept of variation and efforts need to be put to control variation.

Design is an important step in the development of a product. The design process has been developed and used for centuries for various different products. Designer's intuition and experience play a major part in the design of systems in the various fields. A design process generally involves analyzing various trial systems before an optimum acceptable deign is obtained [4]. An optimum and acceptable design generally involves reliable, cost effective and durable systems. A design is transformed into objectives and constraints [5-6]. Objectives are the expectations from the design and constraints are the requirements to be met by the design. The region delimited by constraints is known as the feasible region. The designer is faced with the challenge of creating the design that meets the set of constraints. Competitive markets forces the designers to continuously improve the designs. Design improvements generally comply with the same objectives but improve the constraints of the design.

The main goal of an engineer is to come up with a design which is highly reliable and robust. Traditionally, design has been based on engineering judgments and experience. But with the advances in computational methods and new technology, design optimization has become an efficient and easier method to solve design problems. Optimization [7-10] is a design tool that helps designers to identify an optimum design from a number of possible options. Design optimization is increasingly applied in industry since it provides engineers a cheap, easy and flexible means to identify optimal designs. Engineering design focuses on optimizing the performance of the product after meeting all the design requirements. The basic idea in design optimization is to find a set of design variables that optimizes an objective function while satisfying the design requirements. If reliability is involved, the feasibility of the design is formulated probabilistically such that the probability of satisfying the constraints exceeds the desired limit. The main emphasis in these design optimization methods is to achieve high reliability and robustness.

Reliability-based design (RBD) deals with obtaining optimal designs characterized by low probability of failure. The main step in RBD is to characterize the important uncertain variables and the failure modes. Uncertainty is generally characterized using probability theory. The probabilistic distributions of random variables are obtained using statistical models. When designing a product with multiples failure modes, it is important to make the product reliable with respect to each of the failure modes. In a RBD formulation, these failure modes are given as constraints on probabilities of failure corresponding to each of the failure modes. The probability of failure corresponding to each failure mode can be computed by performing probabilistic reliability analysis.

Robust design (RD) optimization deals with obtaining optimal designs characterized by minimum performance variations. In robust design, the performance variations are minimized without eliminating the sources of variation [11]. RD methods are widely used because they can improve the quality of products and processes. Quality is another important factor in any design. High quality products are often desired. But some noise factors lead to unexpected deviations from the function of a product. Robust design has been developed to improve the product quality by making the products insensitive to these unexpected deviations. Robust design optimization is performed by including the robustness concept in the conventional optimization process. In RD, insensitiveness of the objective function is emphasized. Robustness of the objective function is achieved by reducing the change of the objective with respect to the changes in the tolerances of the design variables. Robustness of the constraints means that all the constraints are satisfied within the range of tolerances of the design variables.

A reliable and robust design is important for any system. But, any product design involves several important product characteristics which conflict with each other. For example, robust design requires a trade-off between the target and variability of the quality characteristics. It is also essential for these characteristics to meet the reliability targets. Although existing methods like RD and RBD have proven to be effective, we still need a better approach to address these issues simultaneously at the product design stage. Also performing both reliability-based design and robust design optimization is usually very expensive and time consuming. Therefore, an integrated multi-objective optimization model is needed to capture both reliability-based design and robust design characteristics and to resolve the trade-offs so that a balanced optimization can be carried out to determine optimum values of design with minimum variations and loss.

1.2. LITERATURE REVIEW

1.2.1. Reliability-Based Design (RBD) In engineering design, the traditional deterministic design optimization has been used to improve the design and quality of the products. The design variables are considered to be deterministic and the design is based on the limits of the design constraints. But the deterministic design does not include uncertainties [11] in the design parameters. The uncertainties mainly include variations in the design parameters and need to be taken into consideration in any design optimization problem. Uncertainties are present everywhere and ignorance of uncertainties may lead to a high chance of failure of the design process. So a different optimization model is required which not only improves the quality of the design but also the reliability by taking into consideration the uncertainty. The reliability-based design takes into account these uncertainties and hence provide a more reliable and safe design.

In reliability-based design optimization (RBDO) [12-17], the design parameters are considered as random variables with. The most important step in RBDO is to characterize the design variables with uncertainty and the various failure modes of the design. The design variables and model parameters are described as probability distributions. The probability distributions are generally obtained by statistical models. Variations are represented by the standard deviations of the probability distribution and they are generally considered as constants. The failure modes of the design are translated into constraints on probability of failure in the design optimization problem. The probability of failure is generally calculated using First order Second Moment method (FOSM), Monte Carlo Simulation (MCS) or other reliability analysis.

A typical RBDO model is formulated as follows.

$$\begin{cases} \text{minimize cost}(\mathbf{d}) \\ \text{subject to} \\ \mathrm{P}\{\mathrm{g}_i(\mathbf{d}, \mathbf{X}) > 0\} \geq \mathrm{R_T} \\ h_j(\mathbf{d}) \leq 0,\ j = 1, 2, ..., n \\ d_k{}^l \leq d_k \leq d_k{}^u, k = 1, 2, ..., n \end{cases}$$

where $\mathbf{d} = (d_1, d_2, ..., d_m)$ is the vector for design variables and $\mathbf{X} = (X_1, X_2, ..., X_n)$ is the vector of random variables. The objective of this RBDO model is to minimize the cost which is a function of the design variables. $g_i(\mathbf{d})$ are the performance functions and $h_j(\mathbf{d})$ are the inequality constraints to be considered during the design optimization.

These constraints should be satisfied during the optimization. R_T is the target reliability. $d_k^{\,l}$ and $d_k^{\,u}$ are the lower bound and the upper bound of the design variables d_k. The design variables should be within the bounds.

Some of the commonly used methods to calculate the reliability of a design are presented below:

FOSM: First Order and Second Moment (FOSM) method, also called the moment matching method, is an efficient method to calculate the reliability of a performance function. It involves the first order derivative and the second moment of the function. If the first two moments i.e. the mean and standard deviation of the random variables $\mathbf{X}$ are known, the FOSM method can be used to estimate the mean and standard deviation of the performance function $g(\mathbf{X})$. The mean and standard deviation values can then be used to calculate the probability of failure of the design.

Monte Carlo Simulation: Monte Carlo Simulation (MCS) is a powerful statistical analysis tool and is widely used in engineering applications for sensitivity and probabilistic analysis. It is mainly used for models with high uncertainty and is considered as one of the methods that give accurate results for reliability. MCS is a class of computational algorithms that depend on repeated random sampling and performs large number of experiments to compute the results. MCS performs random sampling of the variables based on the mean value and the standard deviation of the various input variables and performs numerical experiments to satisfy the model objective and performance functions based on the model. From the output variables obtained from the experiments, MCS estimates the statistical characteristics and gives the output based on the objective functions.

1.2.2. Robust Design The main objective of robust design is to minimize the effects of variations in the design parameters. Variability [18-20] is considered as the root cause of the poor product performance. Variations generally occur from manufacturing, material properties. The general practice is to provide tolerances to the design parameters. But variations must be considered to obtain optimum values of the design parameters. Most of the design optimization models are mainly reliability based and do not include uncertainties or variations in the optimization process. Deterministic design optimization models exclude uncertainties in their design process and so, probabilistic design and optimization methods are developed to account for uncertainties in the design. One method is called the robust design optimization (RDO). It is extremely desirable that engineers include robustness in their design so as to reduce the variability and failure costs.

A general method to represent the input variables, the factors affecting them and their response is using a P-diagram as shown in Figure 1.1.

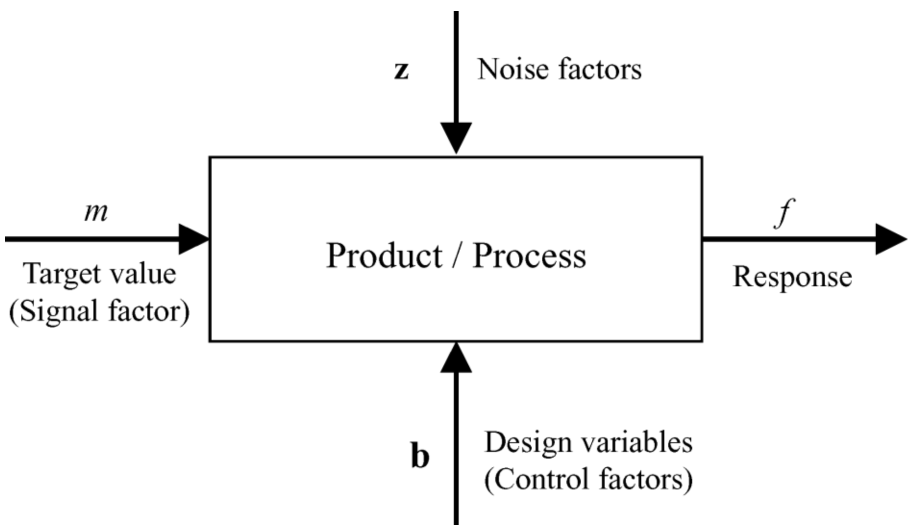

Figure 1.1. P-diagram [22]

The P-diagram [21] shows the functional relationship between the input signal factors (*m*) and the response (*f*). In an ideal condition, the response (*f*) is only a function of the signal factor (*m*). But in reality, it also includes the noise factors (**z**) and the control factors (**b**). Noise factors are the sources of variation and cannot be totally eliminated or controlled even though it causes variation. Some of the noise factors are variations during manufacturing, environmental deterioration etc. To reduce the effects of noise factors, the control factors (**b**) are used. The signal factors in a design are the performance parameters and the control factors are the design variables. Robust design is obtained when the signal to noise ratio is maximized.

The robust design optimization method [22-26] provides an efficient and cost effective method to reduce the variations present in design parameters without eliminating the sources of variation. The main objective is to optimize the mean and minimize the variations by using methods which achieve the performance targets. RDO makes the design parameters insensitive to variations by using the inherent nonlinearity of the relationship between the product parameters and noise factors.
The general form of robust design optimization model is shown below.

$$\begin{cases} \text{minimize } \sigma^2_{f_i(\mathbf{d},\mathbf{X})} \\ \text{subject to} \\ f_i(\mathbf{d},\mathbf{X}) = T_i, i = 1,2,..,n \\ d_k^l \le d_k \le d_k^u, k = 1,2,..,n \\ d_k \ge 0 \end{cases}$$

where $\mathbf{d} = (d_1, d_2, \ldots d_n)$ is the vector for design variables , with d_k^l and d_k^u as its lower and upper limits. $\sigma^2_{f_i(\mathbf{d},\mathbf{X})}$ is the variance of a quality characteristic function $f_i(\mathbf{d},\mathbf{X})$, T_i is the target value for each quality characteristic. The objective is to minimize the variance of the quality characteristic function subject to the constraints.

1.2.3. Integrated Reliable and Robust Design Integrated reliable and robust design combines reliability based design (RBD) and robust design (RD) into a single model to maximize both reliability and robustness simultaneously. RBD is a method to achieve the confidence in product reliability at a given probabilistic level, while RD is a method to improve the product quality by minimizing variability of the output performance function. Since both design methods make use of uncertainties in design variables and other parameters, it is easier for the two different methodologies to be integrated. In this method, both the probability of failure and the variance of the design are minimized. This is done using multi-objective optimization approach to bring both quality and reliability issues simultaneously. Multi-objective optimization is a process of simultaneously optimizing two or more conflicting objectives subject to certain constraints. The two objectives in this model are to minimize the probability of failure and the product quality loss.

The general form of this model is shown below.

$$\begin{cases} \text{minimize } f(\mu_f, \sigma_f^2) \\ \text{subject to} \\ P\{g_i(\mathbf{d}, \mathbf{X}) > 0\} \geq \mathrm{R}_i, i = 1, 2, \ldots, n \\ {d_k}^l \leq d_k \leq {d_k}^u, k = 1, 2, \ldots, n \end{cases}$$

where $f(\mu_f, \sigma_f^2)$ is the objective function, $\mathbf{d} = \mu(\mathbf{X})$ is a design vector, $\mathbf{X}$ is a vector for random variables, g_i is the probabilistic constraint and R_i is the desired reliability. This method minimizes the mean and standard deviation of the design parameters and achieves reliability through the constraint function and hence generally called reliability based robust design optimization [27-31].

Integrated design minimizes the computational effort, time and cost of performing the optimization.

1.3. RESEARCH TASKS

This thesis investigates and develops new methodologies to better understand the relationship between reliability and robustness and then build a model for integrated reliability and robust design. The motivation for our work comes from the fact that an efficient model which integrates both reliability and robustness and minimizes the total quality loss is needed.

The main objective is to completely understand the relationship between reliability and robustness. Once a complete understanding is made, we can create a better design model for integrated reliable and robust design. This better design model can help us make more reliable decisions in terms of reliability and robustness.

The research tasks in this thesis are shown in Figure 1.2.

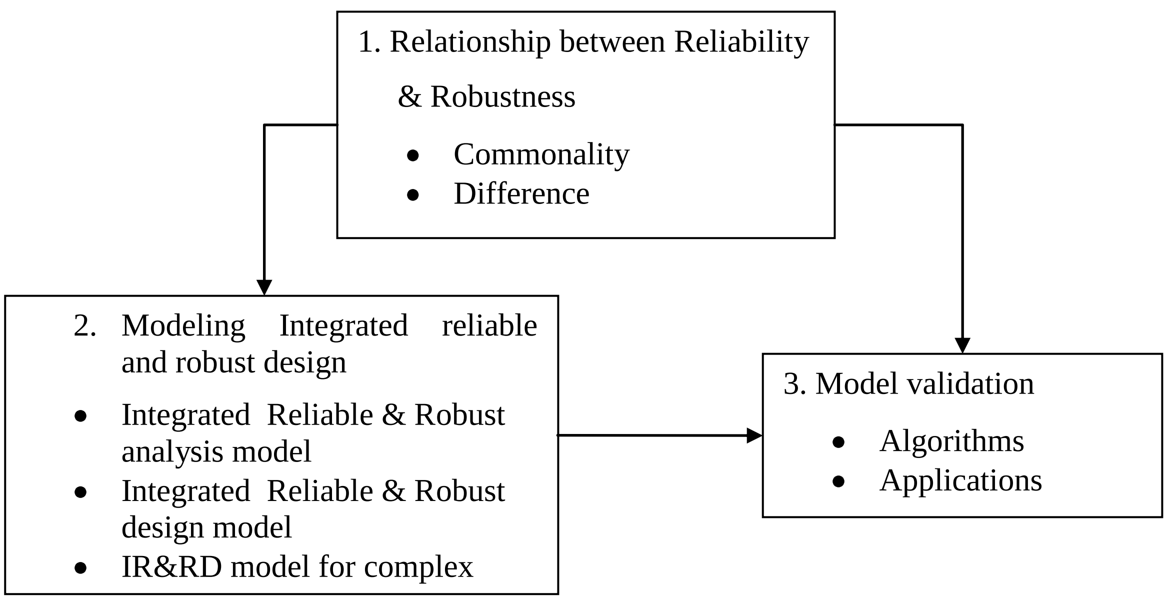

Figure 1.2. Research tasks

Our first task is to perform a study on the relationship between reliability and robustness. This involves studying the various similarities and differences between the two properties. Our second task involves developing an integrated reliable and robust design (IR&RD) model. This IR&RD model is first analyzed based on the requirements and then developed into a design model. IR&RD model is also developed for complex systems with tougher constraints. Our final task involves performing model validation on our integrated reliable and robust design model. The performance of our model is tested on various examples to check the validity.

1.4. ORGANIZATION OF THE THESIS

Chapter 2 presents a study of reliability based design and robust design. It includes the definitions of reliability and robustness and their computations based on limit state function and Taguchi's quality loss functions for different quality characteristics.

Chapter 3 explains the importance of numerical study of the relationship between reliability and robustness and gives a detailed description of the integrated reliable and robust design to study the relationship. Examples to validate the model are also included in this section.

Chapter 4 discusses a general model for integrated design which includes risk present in the design. It deals with a general loss function which includes Taguchi's quality loss function and risk function to measure the total quality loss of the design. Detailed description of the model is presented and an example is used to show the efficiency of the model.

Chapter 5 presents the conclusions which include the summary of research work and the future work.

2. DEFINITIONS OF RELIABILITY AND ROBUSTNESS

2.1. RELIABILITY-BASED DESIGN

2.1.1. Reliability Reliability is one of the most important parameter in the design of any product. The success or failure of a product depends on its reliability. According to IEEE, reliability is defined as “the ability of a system to perform its required functions under stated conditions for a specific period of time [1]”. In other words, reliability is the probability that the random variables $\mathbf{X} = (X_1, X_2, ...X_n)$ is in the safe region defined by $g(\mathbf{X}) > 0$. Higher the reliability better the output obtained from the product. But one factor which reduces the reliability of a product is failure. The probability of failure is defined as the probability that $g(\mathbf{X}) < 0$. In other words, it is the probability that the random variables $\mathbf{X} = (X_1, X_2, ...X_n)$ are in the failure region defined by $g(\mathbf{X}) < 0$. Mathematically, the reliability is computed as shown in equation 1below.

$$R = 1 - pf = P\{g(\mathbf{X}) > 0\} \quad (1)$$

where $pf = P\{g(\mathbf{X}) < 0\}$

The above equation states that the reliability is equal to the probability that the performance function g(**X**) is greater than zero.

2.1.2. Limit state function The reliability of a design is generally determined by knowing the area of the target distribution lying in the safe design space. Safe design space is a region consisting of all the feasible design points. Feasible design represents the design which satisfies all the constraints. If 99% of the target distribution lies in the safe design space, the reliability of the system is 0.99. So knowing the amount of distribution lying in the different design regions is very important. In order to separate the safe design region from the unsafe region, we need a boundary, often called the constraint boundary. The design space is generally defined as a performance function. The performance function $g(\mathbf{X})$ is an important factor in determining the probability of failure of a design. The value of the performance function determines whether a design is in the safe region or not. The constraint boundary defined in terms of the performance function $(g(\mathbf{X}) = 0)$ is generally referred to as a limit state function [32]. The limit state function $(g(\mathbf{X}) = 0)$ separates the safe design space ($g(\mathbf{X}) > 0$) from the failure space ($g(\mathbf{X}) < 0$). Figure 2.1. shows the idea of limit state function for a two dimensional plane X_1-X_2.

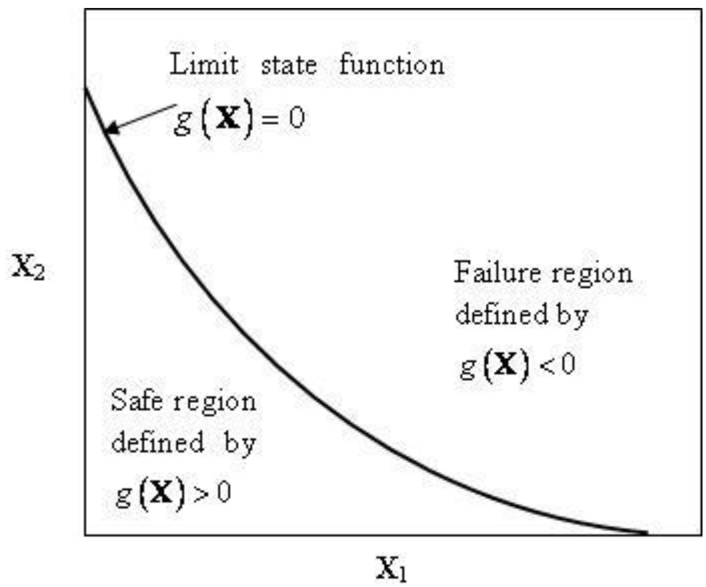

Figure 2.1. Limit state function

$g(\mathbf{X}) = 0$ separates the safe region ($g(\mathbf{X}) > 0$) from the failure region ($g(\mathbf{X}) < 0$). Reliability for this case is computed as the area of the probability density function of the performance function g lying in the safe design region $g(\mathbf{X}) > 0$. Because of the uncertainties present in the random variables defined, the limit state function is a random variable itself. As a result, before the design it is uncertain if g falls into the safe region or the failure region.

2.1.3. Reliability computed with limit state function Limit state function is very important in computing the reliability of a design. The most widely used reliability based design optimization method is the performance level (G-level) method. The G-level method [33] is mainly used in design problems. The key to this method is the concept of limit state function ($g = 0$) which divides the design space into safe region ($g > 0$) and failure region ($g < 0$). But the limit state function need not always be $g(\mathbf{X}) = 0$. Different reliability types may have different limit state functions. The various reliability types and their limit state functions are explained below.

The most common reliability type is the one sided reliability as shown in equation 2.

$$R = P\{g(\mathbf{X}) < C\} \tag{2}$$

where C is a constant.

The above equation states that reliability is equal to the probability that the performance function $g(\mathbf{X})$ is lesser than a constant value C.

This type of reliability is called the one sided reliability as the design space has just one constraint boundary. For this condition, the limit state function is given by $g(\mathbf{X}) = C$. $g(\mathbf{X}) < C$ defines the safe design space and $g(\mathbf{X}) > C$ defines the failure design space. Design parameter with a value lesser than C is desired for this type of design. Smaller the value of the design parameter, better the reliability. Since smaller values of the design parameter are more optimal, this condition is called the smaller-the-better (STB) condition. This is the most common reliability type because most of the design parameters fall under smaller-the-better condition.

The other type of one sided reliability condition is shown in the equation 3 below.

$$R = P\{g(\mathbf{X}) > C\} \tag{3}$$

where C is a constant.

The above equation states that reliability is equal to the probability that the performance function $g(\mathbf{X})$ is greater than a constant value C.

The limit state function is the same for this condition as the previous one and is given by $g(\mathbf{X}) = C$. The difference occurs in the safe and failure regions. The safe region for this condition is defined by $g(\mathbf{X}) > C$ and failure region is defined by $g(\mathbf{X}) < C$. This means that the design parameter with a value greater than C is desired for this type of design. Higher the value of the design parameter, better the reliability. Since large values of the design parameter are desired for this particular design condition, it is called the larger-the-better (LTB) condition.

The reliability can also be double sided, i.e. the design space may have two constraint boundaries. Double sided reliability is shown in the equation below.

$$R = P\{C_1 < g(\mathbf{X}) < C_2\} \tag{4}$$

where C_1 and C_2 are constants.

The above equation states that the reliability is equal to the probability that the performance function $g(\mathbf{X})$ is greater than a constant value C_1 but lesser than a constant value C_2.

This double sided reliability condition has two limit state functions, $g(\mathbf{X}) = C_1$ and $g(\mathbf{X}) = C_2$. The design values falling between the values C_1 and C_2 are safe. $C_1 < g(\mathbf{X}) < C_2$ defines the safe design region and $g(\mathbf{X}) < C_1$, $g(\mathbf{X}) > C_2$ define the failure design region. Since the optimal values are around the nominal value, this condition is called the nominal-the-best (NTB) condition.

2.2. ROBUST DESIGN

2.2.1. Robustness Robustness is a property where a product or a process or any design parameter is insensitive to variation. Robust design is an engineering methodology for improving the productivity during research and development so that high-quality products can be produced quickly and at low cost. Robust design satisfies the functional requirements of a design parameter even though they have large tolerances for ease of manufacturing and assembly. The main aim of robust design is to minimize the product's sensitivity to variation.

2.2.2. Measuring Robustness and Quality Loss Function One of the main ways to improve the robustness of a design is to reduce the variation of the design parameters. Some of the concepts used to describe ways to reduce the variation are robust design methodology, Taguchi methods, quality engineering [34]. According to Taguchi, "quality engineering is not intended to reduce the sources of variation in products directly. Instead, one needs to make the systems of products or production processes less sensitive to sources of uncontrollable noise, or outside influences, through parameter design (off-line quality control) methods." Noise factors are very difficult, expensive or impossible to control as they are so unpredictable. So in order to achieve a robust design, insensitivity to noise factors is a better option than elimination of noise factors. Taguchi came up with a three step procedure based on quality engineering to achieve a robust design [20] – system design, parameter design and tolerance design.

System design is a stage where the different designs are considered involving creativity and innovation. During parameter design [35], the optimum values for the various design parameters are decided. The exact choice of values for the parameters is arrived at based on the noise factors involved with those parameters. This is considered as the major phase to achieve robustness. Finally, during tolerance design, tolerance values are given to each design parameter so as to minimize the effect of variations.
The idea of robust design is to improve the quality of a product by reducing the effects of variation. Higher the quality of a product, better the robustness. Taguchi's methods define a quality loss function (QLF) [36-39] to measure the quality of a product. This method is an off-line quality control method applied at both product and process design stage to improve the product reliability by making the products insensitive to component variations. The quality loss function approximates the financial loss for any particular variation of a product parameter based on the target value of that particular design parameter. QLF states that there is an increasing loss which is a function of the variability

of the design parameter from the target value. The higher the variation from the target value, the higher the loss. Taguchi's expected quality loss function [40] can be expressed in terms of the quadratic relationship

$$\bar{L} = k[(\mu_y - m)^2 + \sigma_y{}^2] \tag{5}$$

where μ_y is the mean value of the design parameter y

m is the target value of the parameter y

σ_y is the standard deviation of the design parameter y

k is a constant defined as

$$k = \frac{A_0}{\Delta_0{}^2} \tag{6}$$

where A_0 is the consumer loss (in dollars)

Δ_0 is the maximum deviation from the target value

This function penalizes the deviation from the target value of a parameter which accounts for the lower performance of a product resulting in loss to the customer. The loss function shown in equation is referred to as the "nominal-the-best" condition as the design parameter has to achieve a nominal value.

The second characteristic is the "smaller-the-better" condition. In this case, the ideal target value is zero. The equation that describes the loss function $\bar{L}$ for this characteristic is

$$\bar{L} = k(\mu_y{}^2 + \sigma_y{}^2) \tag{7}$$

where μ_y is the mean value of the design parameter y

σ_y is the standard deviation of the design parameter y

k is a constant defined as

$$k = \frac{A_0}{{y_0}^2} \tag{8}$$

where A_0 is the consumer loss (in dollars)

y_0 is the maximum tolerated output value of y

The third characteristic is the "larger-the-better" condition. For this characteristic, it is preferred to maximize the result. The ideal target value is infinity. The equation [41] that describes the loss function $\bar{L}$ for this characteristic is

$$\bar{L} = \frac{k}{{\mu_y}^2}\left(1 + \frac{3{\sigma_y}^2}{{\mu_y}^2}\right) \tag{9}$$

where μ_y is the mean value of the design parameter y

σ_y is the standard deviation of the design parameter y

k is a constant defined as

$$k = A_0 {y_0}^2 \tag{10}$$

where A_0 is the consumer loss (in dollars)

y_0 is the minimum output value of y

Using Taguchi's approach, the loss is minimized only by reducing the variation of the design parameters. QLF is mainly used to reduce the variability and move the average of a distribution closer to the target value.

2.3. GENERAL RELATIONSHIP BETWEEN RELIABILITY AND ROBUSTNESS

Reliability is the ability of a product to realize its intended function. If design variables (controllable) and the design parameters (uncontrollable) are denoted by vector **X**, and the safety region is Ω then reliability is defined by $R = P\{\mathbf{X} \in \Omega\}$. For a component with a single failure mode, if its performance $y = g(\mathbf{X}) > 0$ reflects safety, then safe region is $\Omega = \{\mathbf{X} \mid g(\mathbf{X}) > 0\}$, and reliability is $R = P\{\mathbf{X} \in \Omega\} = P\{g(\mathbf{X}) > 0\}$.

On the other hand, robustness is the ability that the performance of a product is not sensitive to uncertainties (or noises). Suppose the performance of the product is $y = g(\mathbf{X})$, the robustness of the product is described by the standard deviation, σ_y of y.

Although it is thought that both reliability and robustness promote each other, they are essentially different. As shown in Figure. 2.2., reliability is targeted to small likelihood events while robustness is suitable for large likelihood events.

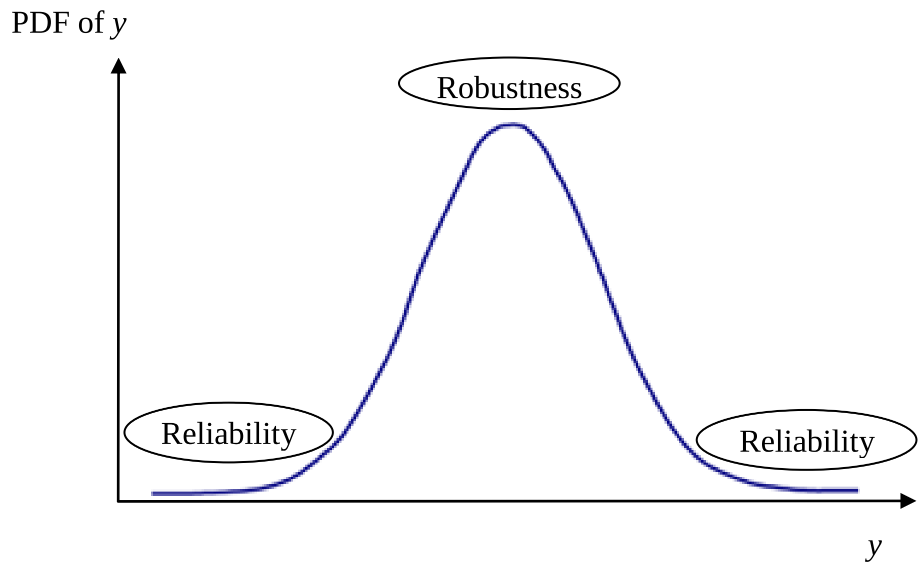

Figure 2.2. Relationship between reliability and robustness

The applications of reliability-based design (RBD) and robust design (RD) are also distinct as illustrated in Figure. 2.3. RBD is primarily used for small likelihood events with but high consequences (zones 1 and 2) while RD is applied to large likelihood (every fluctuation) events with less critical consequences (quality losses) (zone 3). There are no engineering applications where everyday fluctuation leads to critical consequences (zone 4).

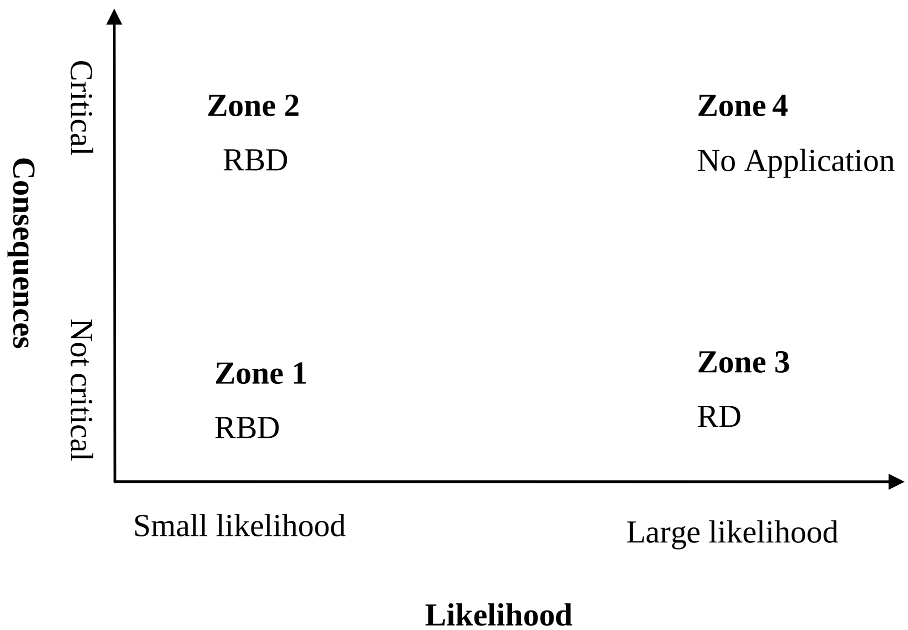

Figure 2.3. Applications of RBD & RD

Reliability and robustness can promote each other, but high reliability does not mean high robustness, and vice versa. This can be explained as follows.
Design 1 and design 2 are two arbitrary distributions [22] as shown in Figure 2.4.

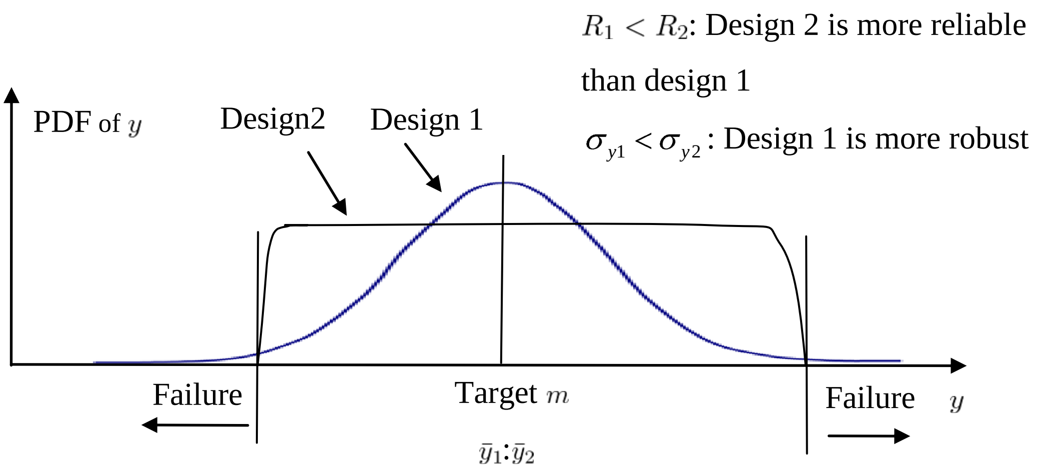

Figure 2.4. The distinction between reliability and robustness

The design range is equal to the allowable range and the means of the two distributions coincide with the target value *m*, i.e. the performance of both design 1 and 2 are on the target *m*. A small part of design 1 is outside the design range whereas none of the design 2 is outside the range. But if any unexpected noise factor becomes active, the distribution of design 2 has a larger probability to be outside the design range than design1.This shows that design 1 is more robust than design 2 as the standard deviation of design 1, σ_{y1} is less than the standard deviation of design 2, σ_{y2} . The probability of failure is generally calculated from the area of the probability density function (PDF) curve in the failure region. From the figure, since the PDF curve in the failure region of design 2 is smaller than that of design 1, the probability of failure of design 2 is lesser than the probability of failure of design 1. So, design 2 is more reliable than design 1. Since neither design 1 nor design 2 is both reliable and robust, reliability and robustness do not mean the same thing.

3. NUMERICAL STUDY ON THE RELATIONSHIP BETWEEN RELIABILITY AND ROBUSTNESS

3.1. INTRODUCTION

The objective of this chapter is to discuss an innovative approach to study the relationship between reliability and robustness and to maximize both simultaneously. A reliable design helps us to reduce the probability of failure of the design. A robust design helps us to reduce the variations of the design parameters. Reliability-based design (RBD) and robust design (RD) are two distinct procedures and do not always promote each other. RBD is mainly used for components where reliability is foremost important and it compromises on the quality of the design. Similarly, RD focuses only on reducing the variations of the parameters and do not give importance to the reliability. Neither RBD nor RD, if used individually, could ensure both reliability and quality simultaneously in a product. Therefore, RBD and RD must be integrated into a single model [28] in order to ensure that a product is robust against the noise factors and reliable over a specified time period.

The objective of our work is to develop an integrated reliable and robust design model which gives us the design with high reliability and robustness. The problems are formulated to minimizing the probability of failure of the design and the failure cost associated with variations. It is not possible to solve this problem accurately and hence only an approximation can be made.

Some approaches [27-31] have been made to integrate both RBD and RD into a single model. But a systematic approach to integrate them into a multi-objective environment is needed.

The robustness of a design is generally increased by reducing the standard deviation of the design parameter. As the standard deviation is reduced, the variation of the parameter from the target value is minimized, thereby increasing the quality. But minimizing the standard deviation may also lead to reducing the probability density function of the design which would reduce the reliability of the design. Therefore, a multi-objective optimization model [28, 30] should be used to combine both reliability and robustness. The reliability is generally measured by the probability that the design will fail to meet the expected values. Robustness can be measured from the standard deviation of the design performance. Since standard deviation and reliability have a positive relationship, measuring robustness from standard deviation may not be the best method. This brings us to the consideration of Taguchi's quality loss function [36] to measure robustness. According to Taguchi, quality is defined as "the losses a product imparts to the society from the time the product is shipped. [20]" These losses are mainly due to the functional variations. Minimizing the variation is the main goal in robust design. The main illustration of this loss in Taguchi's methods is the quality loss function. Taguchi's quality loss function measures these variations as a function of quality loss and provides expressions to measure quality loss for any kind of design with high accuracy. Therefore, Taguchi's quality loss function is used in our design model to maximize the robustness of the design.

This section presents a multi-objective optimization approach to bring both quality and reliability issues simultaneously in a multi-objective environment. The concepts of variability optimization, robust design, reliability based design, multi objective optimization, and Taguchi's quality loss functions are brought together to build the proposed model. The proposed approach ensures reliable, robust, and concurrently cost-effective product design by satisfying all the desired quality characteristics.

3.2. PROCEDURE

The integrated reliable and robust design consists of two basic steps. The first step is to formulate the design problem in terms of reliability and robustness and the second step is to use computational methods to find the relationship between reliability and robustness.

The first step in formulating the design problem is to identify the performance functions. The performance functions define the design problem. They distinguish the safe design from the failure design. The expressions for the design parameters along with their design boundaries are defined. Reliability is calculated as the probability that the performance function lies within the design range.

Any design has a number of characteristics with their design variables falling into a design range. Some characteristics play an important role in the final outcome of the design and are controllable. These characteristics generally have an ideal value with some allowable tolerances. Those characteristics with dimensions within the tolerance range constitute feasible design. They are treated as design variables with lower and upper bounds. These design variables are the essence of the design. Different combinations of design variables constitute different designs. The important step of any design problem is to identify the design variables with their lower and upper bounds as shown in the equation below.

$$d_k^{\,l} \le d_k \le d_k^{\,u}, k = 1, 2, \ldots n \tag{11}$$

where d_k are the design variables with $d_k^{\,l}$ and $d_k^{\,u}$ as their lower and upper bounds.

The next step is to identify the random variables in the performance function. The consideration of design parameters as random variables provides an optimum design in the presence of variability among the design parameters. Most of the random variables used in our examples are normally distributed with the mean value and standard deviation as shown in the equation below.

$$X \sim N(\mu_X, \sigma_X) \tag{12}$$

where μ_X and σ_X are the mean and standard deviation values value of X .

First Order and Second Moment (FOSM) method is used to calculate the probability of failure of the performance function $g(\mathbf{X})$ as shown in the equation below.

$$pf \approx P\{g(\mathbf{X}) < 0\} = \Phi\left(\frac{-\mu_g}{\sigma_g}\right) = \Phi\left(-g(\boldsymbol{\mu}) \middle/ \sqrt{\sum_{i=1}^{n} \left[\left[\frac{\partial g(\mathbf{X})}{\partial X_i}\right]_{\boldsymbol{\mu}}\right]^2 \sigma_i^{\,2}}\right) \tag{13}$$

where μ_g and σ_g are the mean and standard deviations of the function $g(\mathbf{X})$

$\Phi\left(\frac{-\mu_g}{\sigma_g}\right)$ is the cumulative distributive function of $g(\mathbf{X})$

Our objective is to maximize both the reliability and robustness of the design. Reliability is maximized by minimizing the probability of failure (pf) of the performance function i.e. the probability that the performance function falls outside the design space. Robustness in our design is defined by Taguchi's quality loss function based on the quality characteristic. Robustness is maximized by minimizing the expected quality loss function ($\bar{L}$) of the design parameter. To define both reliability and robustness into a single objective function, weights (w_1, w_2) are used. These weights can vary from zero to one and define the relationship between the probability of failure and the loss function. The minimizing function used in our design problem is shown in the equation 14 below.

$$v = \min(w_1 pf + w_2 \bar{L}) \tag{14}$$

Design constraints are added in the optimization model. Constraints are requirements or properties in the design to ensure that the design meets the performance goals. A constraint function can be an inequality constraint $h_i(d_1, d_2, ..., d_n) \leq 0$ or an equality constraint $c_j(d_1, d_2, ..., d_n) = 0$.

Our proposed method studies the relationship between reliability and robustness using a different method. The minimum and maximum values of reliability of the design are calculated first from the above equation using appropriate weights. This reliability region defined between the minimum and maximum values of reliability is divided into a number of equal divisions and the expected quality loss function values are calculated for each corresponding reliability value.

Most of the design models achieve robustness by minimizing the standard deviation of the performance function (σ_g). This method is also used in our design problems so as to compare the results with that from our proposed method. The objective function used for this case is shown in equation 15.

$$v = \min(w_1 pf + w_2 \sigma_g) \tag{15}$$

The general form of our multi-objective optimization model is shown below.

$$\begin{cases} \text{minimize}(w_1 pf + w_2 \bar{L}) \\ \text{subject to} \\ C_1 \le g_i(\mathbf{d}, \mathbf{X}) \le C_2, i = 1, 2, \ldots n \\ h_j(\mathbf{d}, \mathbf{X}) \le 0, \ j = 1, 2, \ldots n \\ d_k{}^l \le d_k \le d_k{}^u, k = 1, 2, \ldots. n \end{cases} \tag{16}$$

where g is the performance function

C_1 and C_2 are the lower and upper design boundaries for the performance function

h is the inequality constraint function

Matlab software is used to perform the optimization. The fmincon function in Matlab is used to minimize the objective function by taking into account the lower and upper bounds of the design variables and the design constraints. The fmincon function finds a constrained minimum of a scalar function of several variables starting at an initial estimate.

To better understand the proposed method, a few engineering problems are taken as examples and are presented in the next section.

Figure 3.1. shows a summary of this procedure in a flowchart.

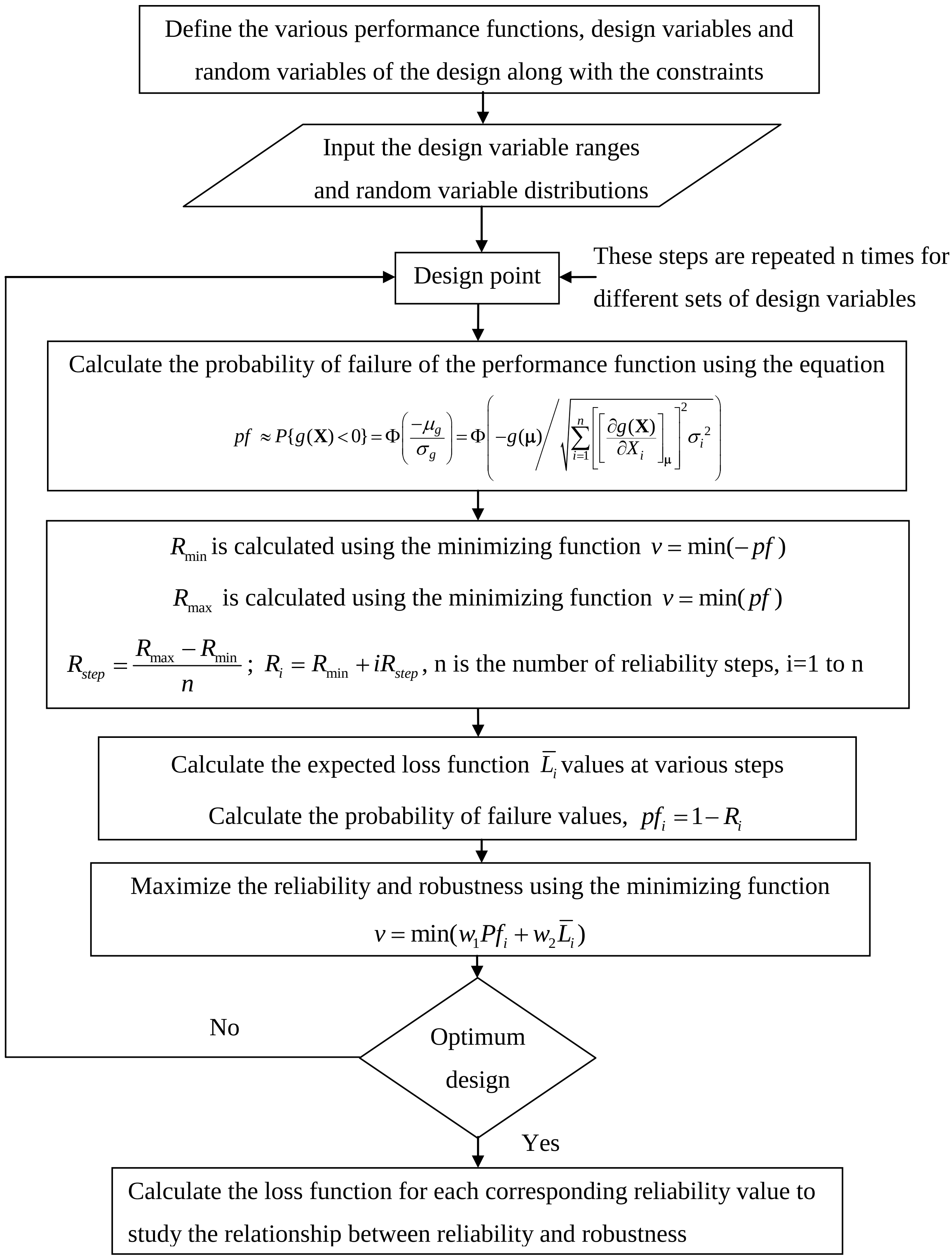

Figure 3.1. Flowchart of the proposed method

3.3. NUMERICAL EXAMPLES

This section presents the study of the proposed design model on design problems with different quality characteristics along with some examples. Design problems would fall under one of the quality characteristics presented below.

3.3.1. Smaller-the-better condition For this case, the reliability increases as the output performance value decreases. The ideal target value is zero. Most of the design problems fall under this category. One sided reliability equation is used for this condition as shown in the equation below.

$$R = P\{g(\mathbf{X}) < C\} \tag{17}$$

where C is the maximum tolerated output value.

3.3.1.1. Quality loss function The main objective is to minimize the probability of failure of the performance function and its loss function. The expected quality loss function used for this case is shown in the equation below.

$$\bar{L} = k({\mu_g}^2 + {\sigma_g}^2) \tag{18}$$

where μ_g is the mean value of the performance function g

σ_g is the standard deviation of the performance function g

k is a constant and is defined as $k = \frac{A_0}{{y_0}^2}$

A_0 is the consumer loss (in dollars)

y_0 is the maximum tolerated output value.

3.3.1.2. Example: Cantilever beam with one design performance A cantilever beam as shown in Figure 3.2. is to be designed.

The objective is to maximize both the reliability and robustness of the design.

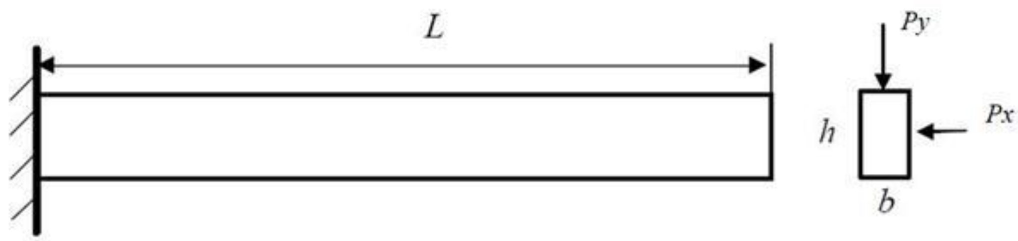

Figure 3.2. Cantilever Beam

L, *b* and *h* are the length, width and height of the cantilever beam respectively. These are our design variables. *Px* and *Py* are the external loads acting on the cantilever beam in the horizontal and vertical directions respectively and they are the random variables.

The performance function used for this example is shown in the equation below.

$$g = Y - S = Y - [(Lb/2Iy)Px + (Lh/2Iy)Py] > 0, \text{ MPa} \tag{19}$$

where g is the performance function for bending stress,

Y is the yield stress of the material and is given by, $Y = 200\text{MPa}$,

S is the stress that occurs due to the loads Px and Py,

$Ix = \frac{bh^3}{12}$ and $Iy = \frac{b^3h}{12}$ are the moments of inertia of the cantilever beam.

The above equation states that the difference between the yield stress of the material and the design stress should be greater than zero. The design stress should not exceed the yield stress of the material.

The various distributions are shown in Table 3.1.

Table 3.1. Distribution of random variables for cantilever beam with one design parameter

Variable	Mean	Standard Deviation	Distribution
Px	2200N	100N	Normal
Py	4400N	220N	Normal

The dimension bounds for the design variables (in mm) are given below.

$$2300 \le L \le 2700$$
$$25 \le b \le 300$$
$$25 \le h \le 300$$

Other values used in this problem are:

Consumer loss (in dollars), A_0=$10

Maximum tolerated output value, y_0=200MPa.

Results:

The values of expected quality loss function are calculated for the different reliability values and the results are plotted as shown in the Figure 3.3.

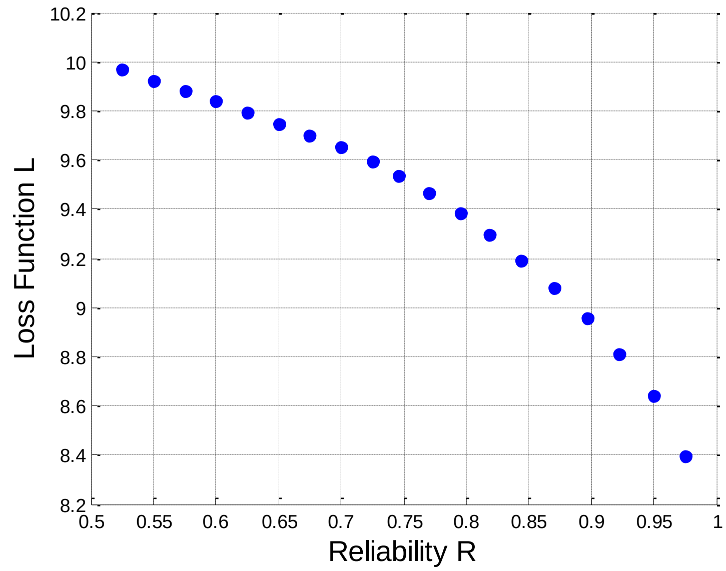

Figure 3.3. Reliability Vs Loss function for cantilever beam with one design parameter

Using standard deviation:

In this case, the robustness is achieved by minimizing the standard deviation of the performance function *g*. The values of standard deviation are calculated for the different reliability values and the results are plotted as shown in Figure 3.4.

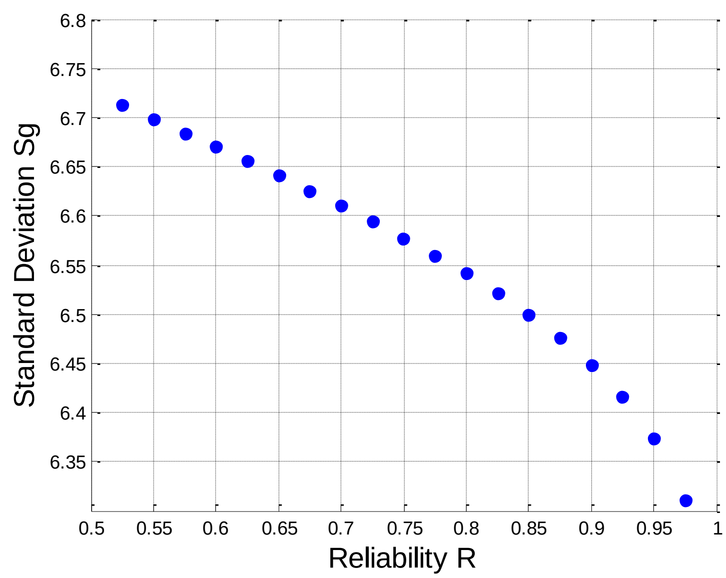

Figure 3.4. Reliability Vs Standard Deviation for cantilever beam with one design parameter

Figure 3.3. shows that as the reliability of the design increases, its quality loss function decreases. Figure 3.4. shows that as reliability of the design increases, its standard deviation decreases. From the above two plots, we find that both the reliability of the bending stress of the design and its robustness increase or decrease simultaneously. Reliability and robustness of the design have a positive relationship. These results clearly demonstrate that the proposed method gives optimum results for Taguchi's smaller the better quality characteristic with one design parameter.

3.3.1.3. Example: Cantilever beam with two design performances

In the previous example, both the reliability and robustness were calculated for the same performance function. In this example, the reliability and robustness are calculated for different design parameters to find the effect of robustness of one parameter on the reliability of the other design parameter.

The two performance functions used for this case are shown below.

$$g_1 = Y - S = Y - \frac{6L}{bh}\left(\frac{Px}{b} + \frac{Py}{h}\right) > 0,\ \text{MPa} \qquad (20)$$

$$g_2 = \frac{4L^3}{E}\sqrt{\left(\frac{Px}{b^3h}\right)^2 + \left(\frac{Py}{bh^3}\right)^2},\ \text{mm} \qquad (21)$$

where g_1 is the performance function for bending stress,

g_2 is the performance function for tip displacement that occurs due to the loading,

E is the Young's modulus of the material and is given as, $E = 200000\text{MPa}$.

The design stress should not exceed the yield stress of the material and the tip displacement during loading should not exceed the allowable displacement.

The performance of the beam is better when the tip displacement of the beam is less. But the maximum tolerated output value of this design is the maximum allowable deflection of the beam given by, y_0=58mm. The consumer loss for this problem is given by, A_0=\$10.

The various distributions are shown in Table 3.2.

Table 3.2. Distribution of random variables for cantilever beam with two design parameters

Variable	Mean	Standard Deviation	Distribution
Px	2200N	100N	Normal
Py	4400N	220N	Normal

The dimension bounds for the design variables (in mm) are given below.

$$2300 \leq L \leq 2700$$
$$25 \leq b \leq 300$$
$$25 \leq h \leq 300$$

Results:

The values of loss function are calculated for the various values of reliability and the results are plotted as shown in Figure 3.5.

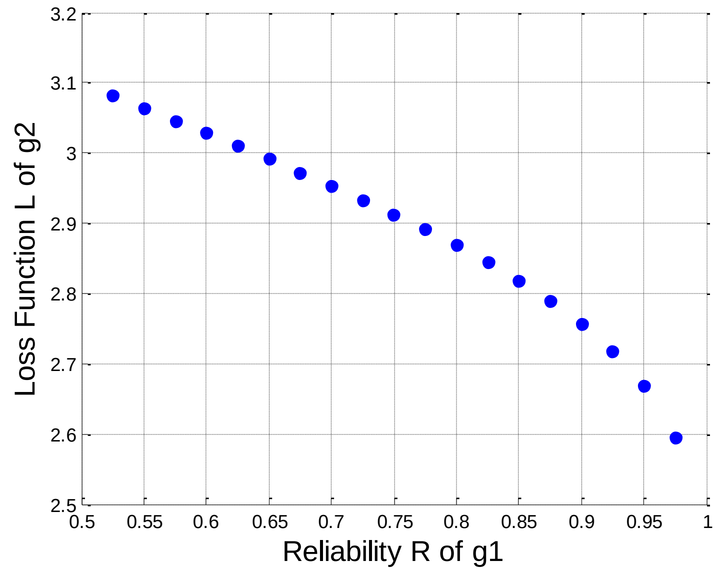

Figure 3.5. Reliability Vs Loss function for cantilever beam with two design parameters

Using standard deviation:

For this case, the robustness of the deflection performance function is maximized by reducing its standard deviation. The values of standard deviation are calculated for the various values of reliability and the results are plotted as shown in Figure 3.6.

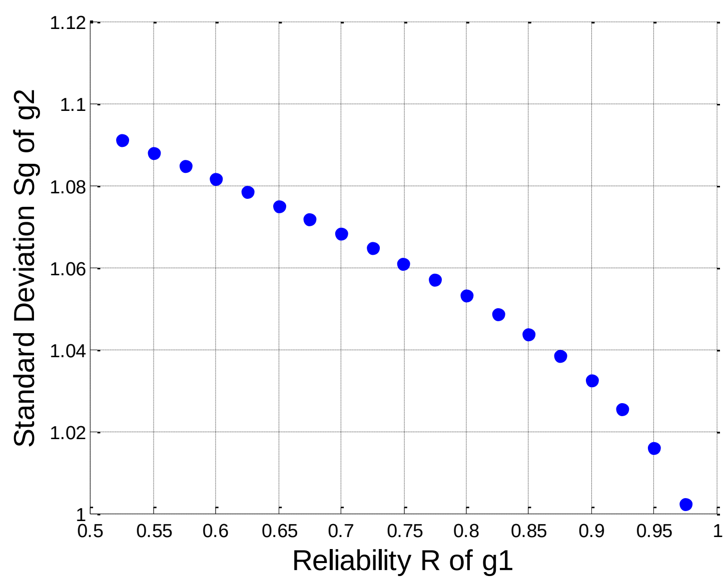

Figure 3.6. Reliability Vs Standard Deviation for cantilever beam with two design parameters

Figure 3.5. shows that as reliability of the design stress increases, the loss function of the deflection decreases. Figure 3.6. shows that as reliability of the design stress increases, the standard deviation of the deflection due the loading decreases. From the above two plots, we find that the reliability of the bending stress of the design and the robustness of the deflection increase or decrease simultaneously. Reliability and robustness of two different performance functions of the design follow a positive relationship. These results clearly demonstrate that the proposed method gives optimum results for Taguchi's smaller the better quality characteristic with two design parameters.

3.3.2. Nominal-the-best condition In some cases, for a characteristic, there is a specified target value. There are also specified upper and lower limits with the target value being the middle point. The optimal value of the design parameter is the target value but any value lying within the limits would be safe. Double sided reliability equation is used for this condition as shown in the equation below.

$$R = P\{C_1 \le g(\mathbf{X}) \le C_2\} \tag{22}$$

where C_1 and C_2 are the lower and upper bounds of the performance function.

3.3.2.1 Quality loss function The expected quality loss function used for this case is shown in the equation below.

$$\bar{L} = k[(\mu_g - m)^2 + {\sigma_g}^2] \tag{23}$$

where m is the target value

k is a constant and is defined as $k = \frac{A_0}{{\Delta_0}^2}$

A_0 is the consumer loss (in dollars)

Δ_0 is the maximum deviation from the target value.

3.3.2.2. Example: Double cantilever beam A double cantilever beam (DCB) is shown in Figure 3.7. [42]. A DCB with an initial crack is used to measure the fracture toughness at the interface, when it is subjected to loads on both sides. The main objective of this problem is to maximize both the reliability and robustness of the design fracture toughness value.

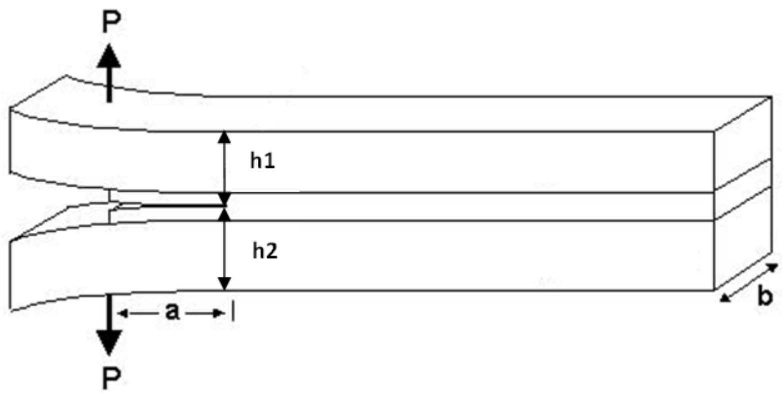

Figure 3.7. Double Cantilever Beam

b is the width of the beam. h_1 and h_2 are the heights of the beam 1 and beam 2, respectively. *a* is the initial crack length present at the interface of the double cantilever beam. *P* is the load acting on the DCB on both sides.

The performance function used for this case is shown in the equation below.

$$225 < g = \frac{P^2 a^2}{2b}\left[\frac{12(1-\upsilon_1^{\,2})}{bE_1 h1^3} + \frac{12(1-\upsilon_2^{\,2})}{bE_2 h2^3}\right] < 265, \text{in-lb/in}^2 \qquad (24)$$

where g is the fracture toughness of the double cantilever beam

E_1 is the Young's modulus of material 1 and is given by, E_1 =30000ksi

E_2 is the Young's modulus of material 2 and is given by, E_2 =10000ksi

υ_1 is the Poisson's ratio of material 1 and is given by $\upsilon_1 = 0.28$

υ_2 is the Poisson's ratio of material 2 and is given by, $\upsilon_2 = 0.30$

The above equation states that the performance function should be greater than 225 but lesser than 265. The ideal value for the performance function is 245.
The various distributions are shown in Table 3.3.

Table 3.3. Distribution of random variables for double cantilever beam

Variable	Mean	Standard Deviation	Distribution
P	230lb	5lb	Normal

The dimension bounds for the design variables (in inch) are given below.

$$0.47 \le b \le 0.7$$
$$0.125 \le h_1 \le 0.15$$
$$0.25 \le h_2 \le 0.35$$

The probability of failure of the design is calculated as the probability that the fracture toughness value falls outside the range. The ideal value of the fracture toughness is 245. But any value falling between 225 and 265 is acceptable. The maximum deviation of the output value is given by, Δ_0 =20. The consumer loss for this problem is given by, A_0=$10.

Results:

The values of loss function are calculated for the various values of reliability and the results are plotted as shown in Figure 3.8.

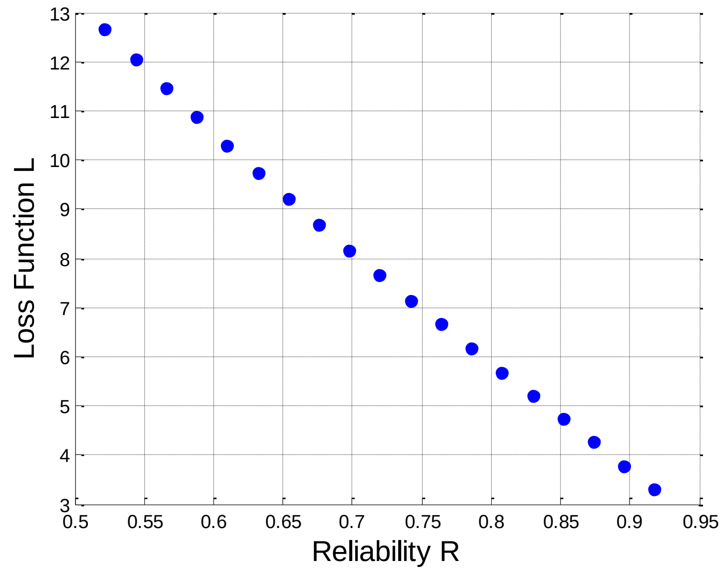

Figure 3.8. Reliability Vs Loss Function for double cantilever beam

Using standard deviation:

In this case, the robustness is maximized by minimizing the standard deviation of the performance function. The values of standard deviation are calculated for the various values of reliability and the results are plotted as shown in Figure 3.9.

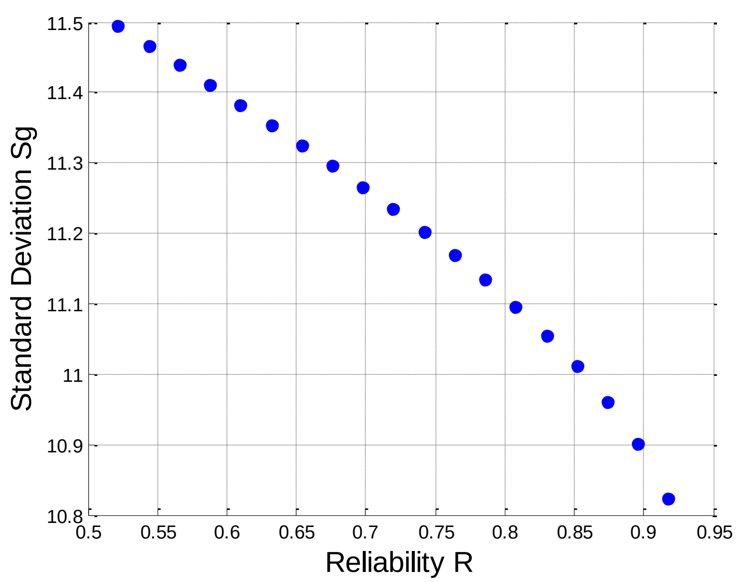

Figure 3.9. Reliability Vs Standard Deviation for double cantilever beam

Figure 3.8. shows that as reliability of the design fracture toughness value of the DCB increases, its quality loss function decreases. Figure 3.9. shows that as reliability of the fracture toughness increases, its standard deviation decreases. From the above two plots, we find that the reliability and robustness of the fracture toughness value of the design increase or decrease simultaneously. Reliability and robustness of the design follow a positive relationship. These results clearly demonstrate that the proposed method gives optimum results for Taguchi's nominal the better quality characteristic.

3.3.3. Larger-the-better condition For this case, the reliability increases as the output performance value increases. The ideal target value is infinity. One sided reliability equation is used for this condition as shown in the equation below.

$$R = P\{g(\mathbf{X}) > C\} \tag{25}$$

where C is the minimum tolerated output value.

3.3.3.1 Quality loss function The main objective is to minimize the probability of failure of the performance function and its loss function. The expected quality loss function used for this case is shown in the equation below.

$$\bar{L} = \frac{k}{{\mu_g}^2}\left(1 + \frac{3{\sigma_g}^2}{{\mu_g}^2}\right) \tag{26}$$

where k is a constant and is defined as $k = A_0 {y_0}^2$

A_0 is the consumer loss (in dollars)

y_0 is the minimum tolerated output value.

3.3.3.2. Example: Engine An engine [43] is shown in the Figure below.

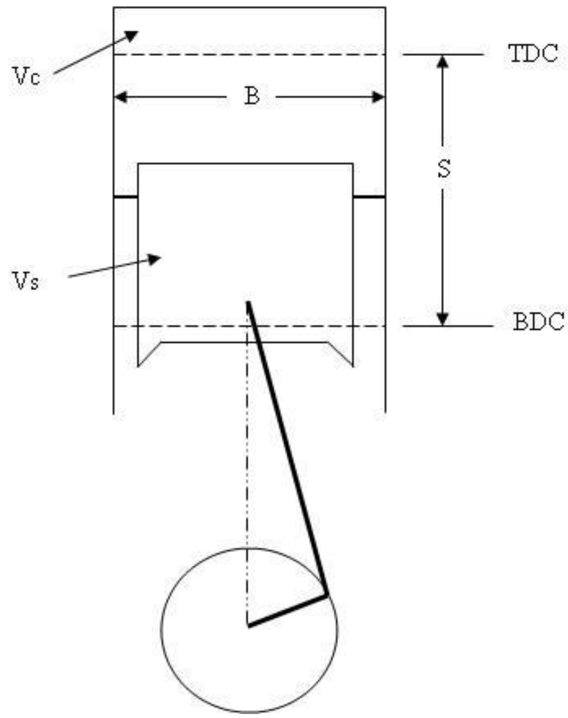

Figure 3.10. Engine

The main objective of this problem is to maximize both the reliability and robustness of the design compression ratio (CR) values of the engine.

The performance function used is shown in the Equation below.

$$g = \frac{Vs + Vc}{Vc} > 9 \tag{27}$$

where g is the compression ratio

Vs and Vc are the swept volume and clearance volume of the engine

The above equation states that the compression ratio value of the design should be greater than 9.

The equations related to this problem are shown below.

$$\text{Swept volume}, Vs = \frac{\pi}{4} B^2 S, \text{cc}$$

$$\text{Gasket volume}, Vg = \frac{\pi}{4} Gb^2 Gt, \text{cc}$$

$$\text{Depression volume}, Vdp = \frac{\pi}{4} B^2 Pd, \text{cc} \tag{28}$$

$$\text{Net piston head volume}, Vph = Vphg - Vdp, \text{cc}$$

$$\text{Clearance volume}, Vc = Vcc + Vg + Vph, \text{cc}$$

where B is the bore diameter of the piston

S is the stroke of the piston

Gb is the bore of the gasket

Gt is the compressed gasket thickness

$Vphg$ is the gross piston head volume

Pd is the piston depression. For this problem Pd =1.27 cm

Vcc is the volume of the combustion chamber in the cylinder head

Vcc and $Vphg$ are the random variables used in this design problem.

The distributions are shown in Table 3.4.

Table 3.4. Distribution of random variables for engine

Variable	Mean	Standard Deviation	Distribution
Vcc	39cc	3cc	Normal
$Vphg$	65.7cc	4cc	Normal

The design variables used in this problem are the bore of the cylinder (B), stroke of the cylinder (S), compressed gasket thickness (Gt) and the bore of the gasket (Gb). The dimension bounds for the design variables (in cm) are given below.

$$4 \leq B \leq 9$$
$$5 \leq S \leq 15$$
$$0.1 \leq Gt \leq 0.4$$
$$5 \leq Gb \leq 12$$

The reliability is maximized by minimizing the probability of failure of the design i.e. the probability that the compression ratio value falls below the least tolerated value. As the value of the compression ratio increases, the performance of the engine gets better. But the least tolerated value of the compression ratio from the design is 9. Any CR value lesser than 9 is not desired for this problem. The consumer loss for this problem is given by, A_0=$40.

Results:

The values of loss functions are calculated for the various values of reliability and the results are plotted as shown in Figure 3.11.

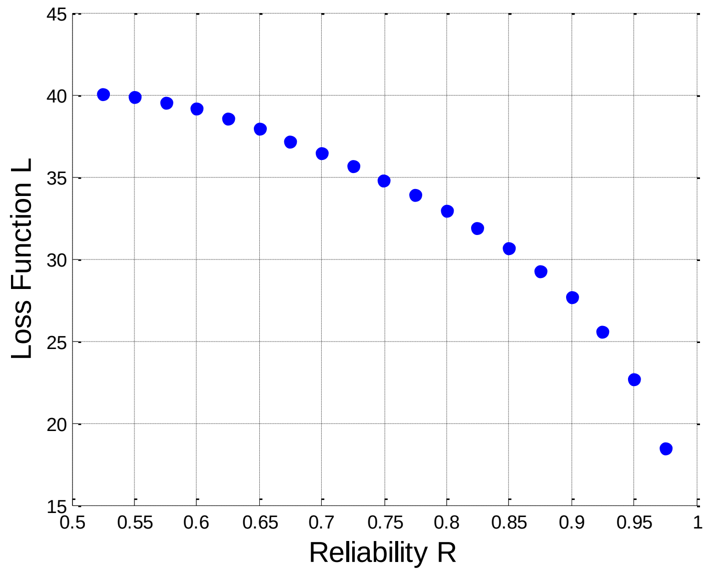

Figure 3.11. Reliability Vs Loss Function for engine

Using standard deviation:

In this case, the robustness is maximized by minimizing the standard deviation of the performance function. The values of standard deviation are calculated for the various reliability values and the results are plotted as shown in Figure 3.12.

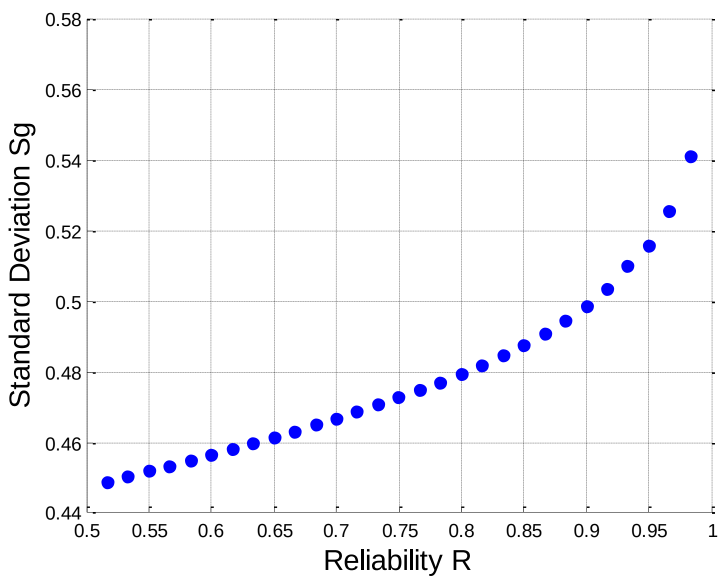

Figure 3.12. Reliability Vs Standard Deviation for engine

Figure 3.11. shows that as reliability of the design compression ratio value increases, its quality loss function decreases. Figure 3.12. shows that as reliability of the performance function increases, its standard deviation also increases. This means that as the reliability increases, robustness decreases. The above two plots give different results. Reliability and robustness follow a positive relationship when we use the proposed method and they follow a negative relationship when we measure robustness from the standard deviation of the design. These results clearly demonstrate that the proposed method gives optimum results for Taguchi's larger the better quality characteristic.

3.4. CONCLUSIONS

This chapter presents a methodology to perform a numerical study on the relationship between reliability and robustness. The various conclusions from the above examples are presented below.

Reliability and robustness may not always change in the same direction. High reliability and robustness are required for every design and a positive relationship between them is often desired. But in some cases, reliability and robustness may not increase simultaneously.

Achieving robustness by minimizing only the standard deviation of the design parameter may not be good for robust design.

Taguchi's quality loss functions provide a better method of measuring robustness compared to standard deviation of the design. The quality loss function involves the failure cost, mean value and standard deviation of the design parameter and hence it provides a better and efficient method to achieve robustness.

The comparison of the results demonstrates that the proposed model provides an efficient and a better method to study the relationship between reliability and robustness of a design.

4. INTEGRATED DESIGN FOR RELIABILITY AND ROBUSTNESS

4.1. INTRODUCTION

The objective of this chapter is to discuss a methodology to formulate a general model of integrated design for reliability and robustness. Most of the current optimization models are only concerned about the safe design space and do not consider the failure region to calculate the loss function. The failure design space is only being used to calculate the reliability of the design. The design values that fall outside the safe region should also be taken into account in the quality loss function. The deviation of one or more results from their expected range is generally considered risk. The objective of our work is to develop a model which minimizes the general loss function of a design. This general loss function includes the losses due to variation of the design parameters from the target value and the losses due to the design parameters falling outside the design range.

Risk is generally defined as the probability that the design values fall outside the design range [44]. Webster's dictionary defines risk as the possibility of loss, injury, disease or death [45]. Another web definition for risk is "Risk is defined as the exposure to the chance of injury or loss." At the most basic level, designers and manufacturers seek to reduce the risk of failure of a product. Since risk is associated with the failure space, risk function is defined as a function of the probability of failure of the design.

The concept of quality loss function (QLF) is important for measuring quality of the design. QLF measures the variation of the design parameters from their target value and calculates the monetary loss associated with the variations. But it does not completely measure the quality of the product. Failure region should also be included when calculating the quality losses as they also contribute to the monetary losses for the design due to loss of quality. This failure region is defined by the risk function which gives the expected value of loss function. Risk provides an appropriate basis for the measurement of the product quality. Risk based quality [46-48] assessment provides a better way of weighing quality expenditures. Therefore, the objective of the product design should also be to minimize the risk associated with the design. The total loss function of a product design should include risk function apart from the quality loss function. This general loss function measures the total cost of quality of the design. Minimizing this loss function maximizes both the reliability and robustness of the design.

The safe design region is defined by Taguchi’s quality loss function and the failure region is defined by the risk function [49-51]. Risk is defined in terms of failure cost i.e. the total cost of rework due to the failure of the product. Taguchi’s quality loss function measures the variation of the output value from the target and is defined in terms of cost i.e. losses due to the variation of target values. Since both the Taguchi’s quality loss function and risk function are measured in terms of cost, it is easier to combine both to define robustness for the entire design space.

Most of the existing multi-objective optimization models allow the use of just one or two performance functions simultaneously to calculate the reliability and robustness of the design. But in reality, any design may have a number of performance functions and all these functions need to be considered when calculating the reliability and robustness of the product. The proposed method can include any number of performance functions as the general loss function of the entire design is calculated as the sum of loss functions from the individual performance functions.

Multi-objective optimization process used in integrated design simultaneously optimizes two conflicting objectives i.e. minimizing the probability of failure and Taguchi’s quality loss function of the design subject to certain constraints. Even though the method is very efficient, some trade-offs need to be made to arrive at an optimal solution. Also, in some cases, there may be more than one optimal solution since the objectives have different units. The efficiency of the method is more when both the objectives are defined in the same units. Since both the objectives in our general loss function are defined in terms of the cost, this proposed method is efficient and gives more accurate solutions.

This section presents a general model for integrated design and the procedure for minimizing the general loss function of a design thereby achieving high reliability and robustness.

4.2. GENERAL MODEL

The general loss function used in the model for Taguchi's nominal-the-best-condition is defined in Equation [29].

$$L_G = \begin{cases} k(y-m)^2 & l \le y \le u \\ k(y-m)^2 + C & \text{otherwise} \end{cases} \tag{29}$$

The general loss function L_G is equal to Taguchi's quality loss function $L = k(y-m)^2$ when the design values are within the design bounds *l* and *u*. When the design values are outside the design range, the general loss function assumes risk which is defined by an additional failure cost C .

The expected general loss function combines all the design values obtained during optimization as shown in the equation below.

$$\bar{L}_G = \int_{-\infty}^{\infty} L_G f_y(y)dy = \int_{-\infty}^{\infty} k(y-m)^2 f_y(y)dy + \int_{-\infty}^{l} Cf_y(y)dy + \int_{u}^{\infty} Cf_y(y)dy \tag{30}$$

$-\infty < y < l$ and $u < y < \infty$ define the failure region and $-\infty < y < \infty$ defines the entire region.

$\int_{-\infty}^{l} Cf_y(y)dy + \int_{u}^{\infty} Cf_y(y)dy$ defines the additional loss function for the failure region and

$\int_{-\infty}^{\infty} k(y-m)^2 f_y(y)dy$ defines the quality loss function for the entire region.

We know that the integration of the quality loss function $k(y-m)^2$ over the entire region $(-\infty, \infty)$ gives the expected value of the loss function as shown in the equation below.

$$\int_{-\infty}^{\infty} k(y-m)^2 f_y(y)dy = k\left[(\mu_y - m)^2 + \sigma_y^2\right] \tag{31}$$

In general, the integration of quality loss function $Lf_y(y)dy$ over the region $(-\infty,\infty)$ for all the quality characteristics gives the expected quality loss function as shown below.

$$\int_{-\infty}^{\infty} Lf_y(y)dy = \bar{L} \tag{32}$$

$[-\infty,l]$ and $[u,\infty]$ define the failure regions and integration of the function $f_y(y)$ over these regions give the probability of failure of the performance function. The general loss function of the failure region is defined as shown in the equation below.

$$\int_{-\infty}^{l} Cf_y(y)dy + \int_{u}^{\infty} Cf_y(y)dy = C[\int_{-\infty}^{l} f_y(y)dy + \int_{u}^{\infty} f_y(y)dy] = CPf \tag{33}$$

The expected general loss function for our model is the combination of the above two equations and is shown in Equation [34].

$$\bar{L}_G = \bar{L} + (1-R)C \tag{34}$$

For multiple performance functions, the general loss function is defined as below.

$$\bar{L}_G = \sum_{i=1}^{p} \bar{L}_{G_i} = \sum_{i=1}^{p} \left[\bar{L}_i + (1 - R_i) C_i \right] \qquad (35)$$

p is the number of performance functions in a design.

Since the general loss function combines both reliability and robustness, our objective is to minimize the general loss function.

The general form of our integrated model is shown below.

$$\begin{cases} \text{minimize}\left(\bar{L}_G = \sum_{i=1}^{p} \bar{L}_{G_i} \right) \\ \text{subject to} \\ C_1 \le g_i(\mathbf{d}, \mu_{\mathbf{X}}) \le C_2, i = 1, 2, \ldots n \\ h_j(\mathbf{d}, \mathbf{X}) \le 0, \ j = 1, 2, \ldots n \\ d_k{}^l \le d_k \le d_k{}^u, k = 1, 2, \ldots. n \\ \{R_i = P(C_1 \le g_i(\mathbf{d}, \mathbf{X}) \le C_2\} \ge R_T \end{cases} \qquad (36)$$

where $\mathbf{d}$ and $\mathbf{X}$ are the vectors for design variables and random variables

C_1 and C_2 are the lower and upper boundaries for the performance function g

h is the inequality constraint function

$d_k{}^l$ and $d_k{}^u$ are the lower and upper bounds of the design variables d_k

R_T is the target reliability for the design

4.3. PROCEDURE

Below is the list of steps involved in our integrated design method.

Step1: The first step is to define the various performance functions in the design, the various design variables and random variables. Design constraints are the conditions that need to be satisfied and they are also defined.

Step 2: The lower and upper bounds of the design variables are defined based on the design requirements and the random variables are defined with the mean and standard deviation values.

Step 3: Initially a starting point of the design is defined so that the optimization process starts from there. The solution obtained from this iteration is used as the design point for the next iteration. This procedure is followed until an optimum design solution is reached.

Within the optimization loop, the following sub-steps are followed.

Step 3-1: The reliability of the various performance functions are calculated using the first order second moment (FOSM) method as shown in the equation below.

$$R = 1 - pf \approx 1 - P\{g(\mathbf{X}) < 0\} = 1 - \Phi\left(\frac{-\mu_g}{\sigma_g}\right) = 1 - \Phi\left(-g(\boldsymbol{\mu}) \Big/ \sqrt{\sum_{i=1}^{n}\left[\left.\frac{\partial g(\mathbf{X})}{\partial X_i}\right|_{\boldsymbol{\mu}}\right]^2 \sigma_i^{\,2}}\right) \qquad (37)$$

Step 3-2: The expected quality loss function $\bar{L}$ is calculated for the various performance functions based on their quality characteristic i.e. smaller the better, nominal the better or larger the better conditions. The various constants used to calculate the loss function are initially defined.

Step 3-3: After calculating the reliability and quality loss function, the general loss function is calculated for each performance function. The cumulative general loss function is calculated as the sum of the general loss functions of each performance function. The equation for the cumulative general loss function of the design is shown below.

$$\bar{L}_G = \sum_{i=1}^{n} \bar{L}_{G_i} = \sum_{i=1}^{n} \left[\bar{L}_i + (1 - R_i) C_i \right] \tag{38}$$

Step 3-4: The objective of this design model is to minimize the general loss function of the design. The minimizing function used in this optimization process is shown below.

$$\text{v} = \min\left(\bar{L}_G = \sum_{i=1}^{n} \bar{L}_{G_i} \right) \tag{39}$$

Matlab software is used to perform the optimization. Fmincon function in matlab is used to minimize the objective function by taking into account the lower and upper bounds of the design variables and the design constraints. Fmincon finds a constrained minimum of a scalar function of several variables starting at an initial estimate.

The results obtained from this method are compared with those obtained from other optimization models like reliability based design optimization and robust design optimization.

Figure 4.1. summarizes this procedure in a flowchart.

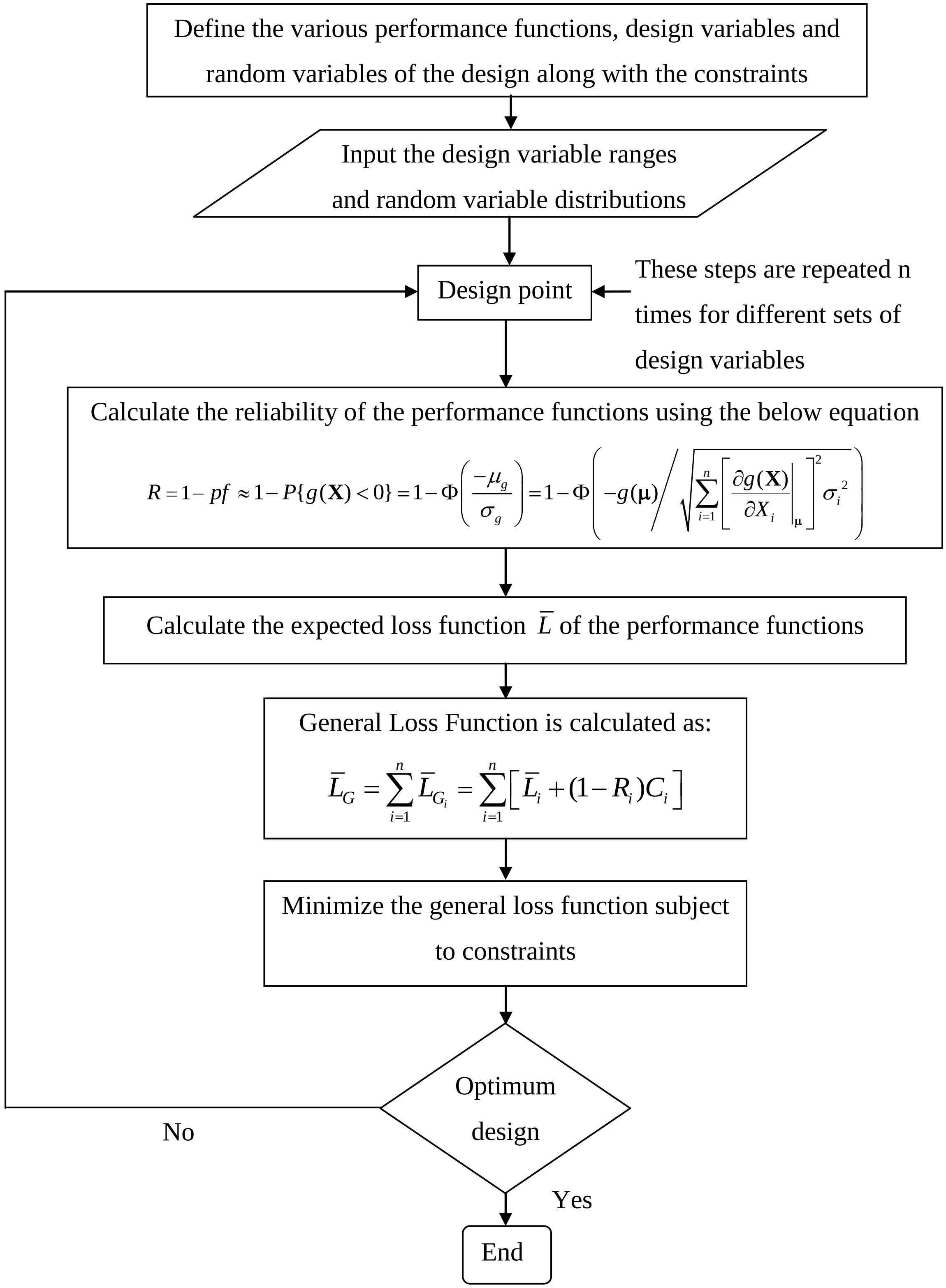

Figure 4.1. Flowchart of the integrated design method

4.4. EXAMPLE: COIL SPRING

A coil spring [10] is shown in Figure 4.2.

The objective is to minimize the general loss function of the coil spring.

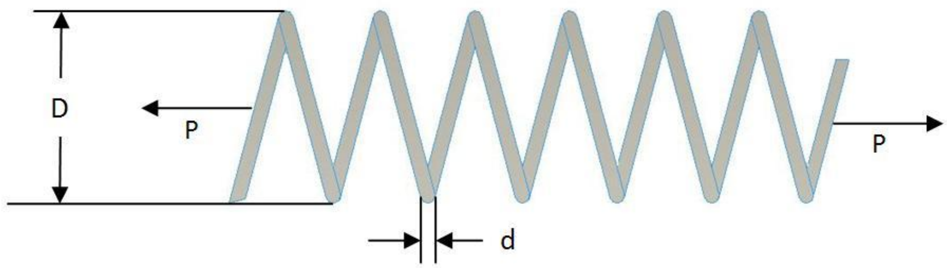

Figure 4.2. Coil Spring

D is the mean coil diameter, *d* is the wire diameter and *N* is the number of active coils and these are our design variables. *P* is the load applied on both sides of the spring and δ is the deflection along the axis of the spring.

There are two performance functions for this spring under load *P* as shown in the equations below.

$$g_1 = \frac{8PD}{\pi d^3}\left[\frac{4D-d}{4D-4d} + \frac{0.615d}{D}\right] < \tau_a,\ \text{lb/in}^2 \tag{40}$$

$$g_2 = \frac{8PD^3N}{d^4G} > \Delta, \text{in} \tag{41}$$

where τ_a is the allowable shear stress of the spring and is given by, $\tau_a = 80000\ \text{lb}/\text{in}^2$,

G is the shear modulus and is given by, $G = 1.15 \text{ x } 10^7 \text{lb}/\text{in}^2$,

Δ is the minimum spring deflection and is given by, $\Delta = 0.3$ in .

The first performance function g_1 states that the shear stress in the wire should not exceed the allowable stress τ_a and the second performance function g_2 states that the deflection of the spring should be greater than the minimum spring deflection Δ.
The various distributions are shown in Table 4.1.

Table 4.1. Distribution of random variables for spring

Variable	Mean	Standard Deviation	Distribution
P	10lb	0.5lb	Normal

The dimension bounds for the design variables (in inch) are given below.

$$0.5 \le D \le 1.3$$
$$0.05 \le d \le 0.2$$
$$2 \le N \le 15$$

The constraint function used in this example is shown in equation 42.

$$\omega_0 - \frac{d}{2\pi D^2 N}\sqrt{\frac{G}{2\rho}} \le 0 \qquad (42)$$

where ω_0 is the lower limit on surge wave frequency and is given by, $\omega_0 = 100$ Hz,

ρ is the mass density of the material, $\rho = 7.38342 \text{ x } 10^{-4} \text{lb} - \text{s}^2 / \text{in}^4$.

The above equation states that the frequency of surge waves should be greater than the lower limit of frequency. The desired reliability of both the performance functions is 0.9999 and they are also given as constraints in the optimization.

The main objective is to minimize the general loss function of the spring for its various failure modes. The first failure mode is that the shear stress exceeds its allowable limit and the second failure mode is that the deflection of the spring falls below its minimum desired value. The expected general loss function for this example is calculated using the equation below.

$$\bar{L}_G = \sum_{i=1}^{2} \left[\bar{L}_i + (1 - R_i) C_i \right] \quad (43)$$

where $\bar{L}_i$ is Taguchi's expected quality loss function

C_i is the failure cost

Taguchi's smaller the better quality loss function is used for the shear stress performance function g_1 and larger the better quality loss function is used for the deflection performance function g_2 as shown in the equation below.

$$\bar{L}_1 = k \left[\mu_{g_1}^2 + \sigma_{g_1}^2 \right] \quad (44)$$

$$\bar{L}_2 = \frac{k}{\mu_{g_2}^2} \left[1 + \frac{3\sigma_{g_2}^2}{\mu_{g_2}^2} \right] \quad (45)$$

The various constants used in this example are shown below.

Shear stress performance function g_1:
$A_0 = \$3$
$y_0 = 80000\text{lb/in}^2$
$C = \$300$

Deflection performance function g_2:
$A_0 = \$2$
$y_0 = 0.3\text{in}$
$C = \$200$

The optimization model used for this example is shown below.

$$\begin{cases} \text{minimize}\left(\bar{L}_G = \sum_{i=1}^{2} \bar{L}_{G_i} \right) \\ \text{subject to} \\ \dfrac{8PD}{\pi d^3}\left[\dfrac{4D-d}{4D-4d} + \dfrac{0.615d}{D} \right] - \tau_a \le 0 \\ \Delta - \dfrac{8PD^3N}{d^4G} \le 0 \\ \omega_0 - \dfrac{d}{2\pi D^2 N}\sqrt{\dfrac{G}{2\rho}} \le 0 \\ \{R_1 = P(g_1 < \tau_a)\} \ge 0.9999 \\ \{R_2 = P(g_2 > \Delta)\} \ge 0.9999 \end{cases} \tag{46}$$

The reliability, standard deviation, loss function and the general loss function are calculated for the two performance functions using first order second moment (FOSM) reliability method, Taguchi's quality loss functions and the general loss function equation and the results are shown in Table 4.2.

Table 4.2. Results for the spring example using integrated reliable and robust design

Property	Shear Stress	Deflection
Reliability	0.9999	1
Standard Deviation	3372.8224MPa	0.11898MPa
General Loss Function	\$2.1683	\$3.9538

The same problem is solved using robust design (RD) optimization and reliability based design (RBD) optimization to compare the results with those in the table above. For the RD, we minimize the standard deviation of both the shear stress and deflection as shown in the equation below.

$$\begin{cases} \text{minimize } (\sigma_{g_1} + \sigma_{g_2}) \\ \text{subject to} \\ \dfrac{8PD}{\pi d^3}\left[\dfrac{4D-d}{4D-4d} + \dfrac{0.615d}{D}\right] - \tau_a \le 0 \\ \Delta - \dfrac{8PD^3N}{d^4G} \le 0 \\ \omega_0 - \dfrac{d}{2\pi D^2 N}\sqrt{\dfrac{G}{2\rho}} \le 0 \\ \{R_1 = P(g_1 < \tau_a)\} \ge 0.999 \\ \{R_2 = P(g_2 > \Delta)\} \ge 0.999 \end{cases} \tag{47}$$

For the RBD, we minimize the cost of the spring calculated from its mass as shown in the equation below.

$$\begin{cases} \text{minimize } \left[\text{Cost} = 55.01754(N+2)\pi^2 D d^2 \rho\right] \\ \text{subject to} \\ \dfrac{8PD}{\pi d^3}\left[\dfrac{4D-d}{4D-4d} + \dfrac{0.615d}{D}\right] - \tau_a \le 0 \\ \Delta - \dfrac{8PD^3 N}{d^4 G} \le 0 \\ \omega_0 - \dfrac{d}{2\pi D^2 N}\sqrt{\dfrac{G}{2\rho}} \le 0 \\ \{R_1 = P(g_1 < \tau_a)\} \ge 0.9999 \\ \{R_2 = P(g_2 > \Delta)\} \ge 0.9999 \end{cases} \tag{48}$$

Results:

The various results obtained are shown in Table 4.3.

Table 4.3. Comparison of results

Property	Integrated reliable and robust design	Robust design	Reliability based design
Probability of failure-shear stress	0.0001	0	0.0001
Probability of failure-deflection	1.042E-68	0.001	0.00010009
Standard deviation-shear stress	3372.8224 MPa	541.775 MPa	3372.821 MPa
Standard deviation-deflection	0.11898 MPa	0.017741 MPa	0.018426 MPa
General Loss Function	\$6.1221	\$178.0853	\$167.0437

Table 4.3. shows the probability of failure, standard deviation and general loss function values calculated from integrated reliable and robust design (IR&RD), robust design (RD) and reliability based design (RBD) models.

The results show the efficiency of the integrated reliable and robust design (IR&RD) model. The general loss function obtained using the IR&RD model is much lesser than that obtained using the other two methods. The required reliability for this example is 0.9999. Our model achieves this reliability value. The inclusion of the reliability target constraint ensures that the model satisfies the specified reliability target while achieving appropriate trade-off among other quality characteristics. Even though the standard deviation values are high compared to robust design model, the robustness is achieved by minimizing the general loss function which gives exceptional results compared to the other two methods. Since high reliability and robustness are achieved, IR&RD proves to be an efficient method for design optimization.

4.5. CONCLUSIONS

This chapter presents a methodology to formulate a general model of integrated design for reliability and robustness. The various conclusions are presented below.

The total quality loss obtained is much lesser when we use the proposed integrated reliable and robust design model than when we use the other optimization models.

High reliability and robustness are achieved by minimizing the general loss function of the design parameters.

The results demonstrate that the integration of the two models achieves a better trade-off among conflicting characteristics and thus provides a better solution.

5. CONCLUSIONS AND FUTURE WORK

5.1. CONCLUSIONS

This thesis presents two methodologies for integrated reliable and robust design. The first work is to perform a numerical study on the relationship between reliability and robustness and the second work is to formulate a general model of integrated design for reliability and robustness. Examples have been shown to show the efficiency of the proposed methods.

The approach for numerical study on relationship between reliability and robustness combines reliability-based design and robust design optimization to formulate an integrated design model which maximizes both reliability and robustness simultaneously in a multi-objective environment. The reliability is measured by the probability of failure of the design and the robustness is measured by Taguchi's quality loss function for different quality characteristics or the standard deviation of the performance function. To achieve both reliability and robustness simultaneously, it is shown in the work that the probability of failure of the performance function and Taguchi's quality loss function are minimized using a multi-objective optimization model. It has been shown that this model gives accurate results for Taguchi's smaller-the-better, nominal-the-best and larger-the-better quality characteristics with less computational effort and time. It is also shown that minimizing loss function is a better method to achieve robustness than minimizing the standard deviation of the performance function.

Another methodology presented in this work is the general model of integrated design for reliability and robustness. Existing methods do not include risk in their optimization models. This general model defines a general loss function which includes both Taguchi's quality loss function and risk defined as a function of cost. It is easier to integrate the above two functions into a general loss function which takes into account both reliability and robustness. Also, large number of performance functions can be used in this model since the general loss function of the design is the sum of the loss functions from the various performance functions. It has been shown that this method described in the thesis gives a quick feasible design compared to other optimization models and satisfies the reliability requirements and minimizes the total failure cost of the system and thereby achieves high reliability and robustness simultaneously.

5.2. FUTURE WORK

The proposed method to study the relationship between reliability and robustness is efficient for design problems with one or two performance functions. This method cannot be used for multiple performance functions. Future work with this method can be to modify the design model so that it can consider multiple performance functions to achieve reliability and robustness. Also, this method can be modified for other reliability methods like Monte Carlo Simulation (MCS), first order and second order reliability methods (FORM and SORM).

The integrated design model used in this research provides an efficient method to calculate the component reliability. But in large systems, there may be large number of components and using this model to calculate the reliability of each component can be computationally expensive and time consuming. Also, reliability is generally time-dependent and it deteriorates with time. So, another future work can be to modify this general model so that it can consider the entire system and can be used for other reliability types like system reliability and time-dependent reliability.

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