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Clipper amplitude modulator

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CLIPPER AMPLITUDE MODULATOR

BY

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A

THESIS

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INTRODUCTION

An amplitude modulated wave is defined as a wave form in which the amplitude of a sinusoidal signal (carrier) is varied in accordance with a modulating signal whose frequency is much lower than the frequency of the carrier signal. Numerous devices have been found for generating an amplitude modulated wave. In some modulators the amplitude of the carrier is actually varied in accordance with the modulating signal. In other types of modulators the carrier and the modulating signal are simultaneously applied to a non-linear element or circuit. The output of this non-linear element is a distorted wave containing many frequencies. The amplitude modulated wave is then obtained by selecting those particular frequency components needed to constitute an amplitude modulated wave.

In this thesis, the possibility of obtaining an amplitude modulated wave by a different method has been investigated. Namely, the carrier signal is clipped with a modulating signal by employing the clipping action of a full-wave diode clipper. The resulting clipped wave form closely resembles an amplitude modulated wave. The close agreement between the mathematical analysis of the wave form in question and the experimental results substantiates the validity of this method.

SECTION I
REVIEW OF LITERATURE

1. MODULATION:

Modulation, as used in the technical sense, means to vary some properties of a wave in some desired manner in order to convey intelligence. The wave whose properties are to be changed is called the carrier wave, and the wave which causes this variation is termed the modulating signal. The carrier is usually in the form of a single frequency sine wave. The sources of the modulating signal are numerous, depending on the particular application.

2. PRINCIPLES OF AMPLITUDE MODULATION:

Suppose that a wave form is given by the expression

$$e = A \sin(wt + \theta) \qquad (1)$$

where (A) is the amplitude, $w = 2\pi f$ is angular frequency, t = time, and θ is the phase angle. According to the definition for modulation stated above, the properties of the wave expressed with equation (1) can be changed by varying any one of the three quantities A, w, or θ in accordance with the modulating signal. If θ varies with the modulating signal, the wave of the equation (1) is said to be phase modulated. Variation of (w) with the modulating signal results in frequency modulation, and variation of A with the modulating signal is termed amplitude modulation. Amplitude modulation is the most common and is the type considered in this thesis.

Since expression (1) is a sine wave, it is apparent that if (A) is varied in accordance with a sinusoidal modulating signal, the amplitude, or the envelope, of expression (1) should also vary sinusoidally resulting in figure (1). In the usual case w is much larger than the angular frequency of the modulating signal.

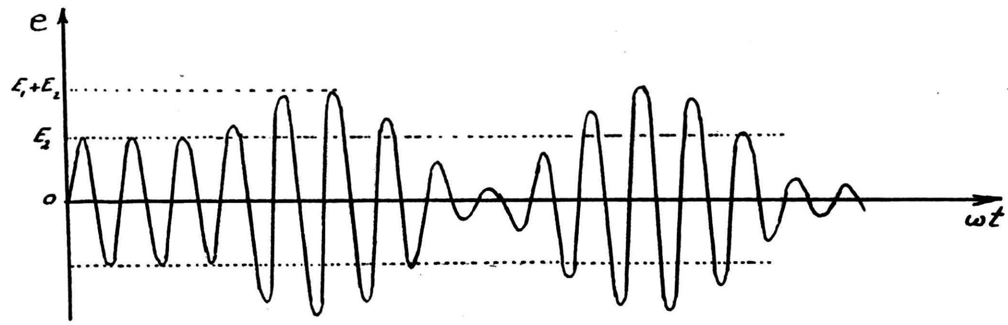

Figure (1). Amplitude modulated wave

Let the single frequency carrier wave be given by the expression

$$e_2 = E_2 \operatorname{Sin} w_2 t \qquad (2)$$

and the modulating signal be given as

$$e_1 = E_1 \sin w_1 t \qquad (3)$$

where

E_2 = Amplitude of carrier voltage

E_1 = Amplitude of modulating voltage

$w_2 = 2\pi f_2$, f_2 being the frequency of the carrier

$w_1 = 2\pi f_1$, f_1 being the frequency of the modulating signal

t = time

$m = \frac{E_1}{E_2}$ modulation index.

From figure (1) it is seen that the expression for the amplitude modulated wave will be,

$$e = (E_2 + E_1 \operatorname{Sin} w_1 t) \operatorname{Sin} w_2 t \qquad (4)$$

in which $(E_2 + E_1 \operatorname{Sin} w_1 t)$ is the sinusoidally varying amplitude, or envelope of the quantity $\operatorname{Sin} w_2 t$, assuming $w_2 \gg w_1$. Rearranging equation (4)

$$e = E_2 \left(1 + \frac{E_1}{E_2} \operatorname{Sin} w_1 t\right) \operatorname{Sin} w_2 t$$

$$e = E_2 \left(1 + m \operatorname{Sin} w_1 t\right) \operatorname{Sin} w_2 t$$

$$e = E_2 \operatorname{Sin} w_2 t + \frac{mE_2}{2} \operatorname{Cos.} (w_2 - w_1) t + \frac{mE_2}{2} \operatorname{Cos} (\omega_2 + \omega_1) t \qquad (5)$$

Equation (5) consists of three terms: the unchanged carrier, and the lower and upper side band terms respectively. In most modulating devices the non-linearity present developes other frequency components whose magnitudes are reduced to insignificance by tuned circuits.

In an amplitude modulated wave the intelligence or program is entirely in the side bands, and no intelligence is conveyed by the carrier. From equation (5) it is seen that the sole function of the carrier is to translate low frequency intelligence into high frequences. In radio communication, for instance, if the voice frequencies were impressed on the transmitting antenna, a physically very large antenna would be required since the dimensions of an antenna must be of the same order of magnitudes as the wavelength of the frequences to be radiated. For example, a quarter-wave antenna for 1000 cycles would be about 46.5 miles long. Suppose that a transmitter could effectively radiate the voice frequences; without modulation, all the transmitters would have to operate over the same frequency band, and it would not be possible to select the desired station.

In wire communication the cost of transmission lines is very high. By modulating different frequency carriers with separate conversation many calls can be made simultaneously over the same pair of wires.

(3) POWER DURING AMPLITUDE MODULATION

The power contained in a modulated wave is the sum of the powers of the separate frequency components. Thus, if there is no over modulation, the total power in the modulated wave is greater than the power contained in the carrier because of the presence of the side bands. If the total power of an amplitude modulated wave is maintained in a resistance R, the average power due to the carrier component is $\frac{E_2^2}{2R}$, and that due to each of the side-band components is $\frac{m^2E_2^2}{8R}$. Therefore the total average power in the ideal amplitude modulated wave is

$$P = \frac{E_2^2}{2R} + \frac{m^2E_2^2}{8R} + \frac{m^2E_2^2}{8R}$$

$$P = \frac{E_2^2}{2R} \left(1 + \frac{m^2}{2}\right)$$

$$P = P_2 \left(1 + \frac{m^2}{2}\right)$$

where

$P_2 = \frac{E_2^2}{2}$ is the average power due to the carrier component, or the power without modulation.

4. METHODS OF AMPLITUDE MODULATION

"Amplitude modulation may be produced by many methods and devices. Those which have been used commonly are*

1. Absorbtion processes
   a. Microphone in antenna system
   b. White system
   c. Magnetic amplifier
2. Modulated class C amplifier
   a. Plate-modulated
   b. grid-modulated
   c. Cathode-modulated

3. Amplitude modulated oscillator.
4. Square-law processes:
   a. Nonlinear circuit element
   b. Triode square-law modulator
   c. Balanced square-law modulator."

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*Electronics Training Staff of the Cruft Laboratory, Electronic Circuits and Tubes, pp. 648, 1947.

At the present time, most amplitude modulation is accomplished by a modulated class C amplifier, an amplitude-modulated oscillator, or a square-law processes. As an example, a grid-modulated Class C amplifier will be analysed.

5. GRID-MODULATED CLASS C AMPLIFIER:

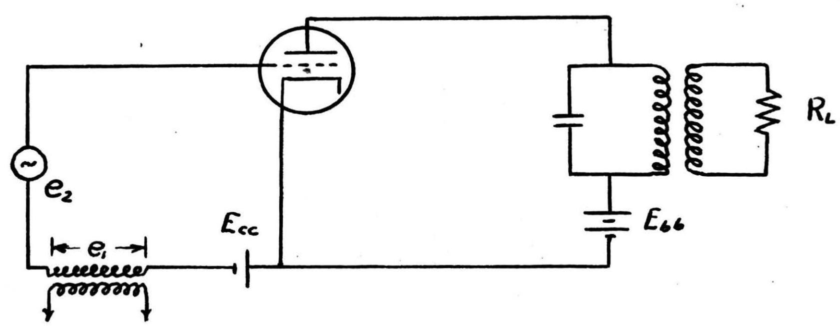


Figure (2). Basic circuit of a grid-modulated Class C amplifier

$e_2 = E_2 \text{ Cos } w_2 t$, carrier signal
$e_1 = E_1 \text{ Cos } w_1 t$ modulating signal.

Figure (2) shows the basic circuit of a grid-modulated Class C amplifier. The output is controlled by varying the grid-bias voltage in accordance with the modulating signal, thereby obtaining amplitude modulation. The relative values of the voltages are such that at the crest of the modulation cycle (i.e. when the grid is the least negative), the tube operates under typical Class C amplifier conditions, with a grid excitation that is not quite sufficient to give saturation conditions resulting from positive grid potentials.

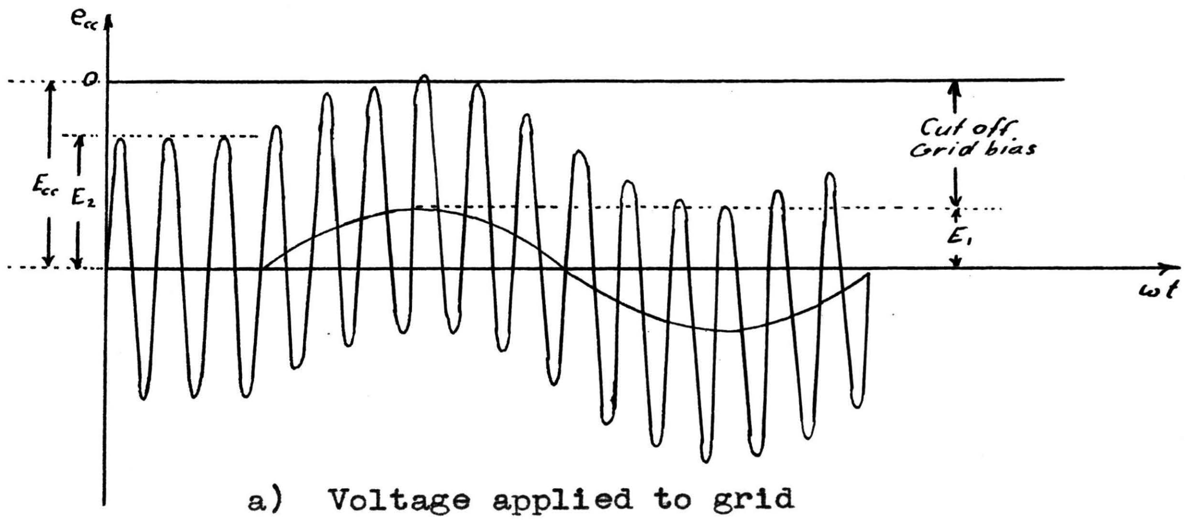
Reference to figure (2) shows that the total voltage in the grid circuit is

$$e_{cc} = E_{cc} + E_1 \text{ Cos } w_1 t + E_2 \text{ Cos } w_2 t$$

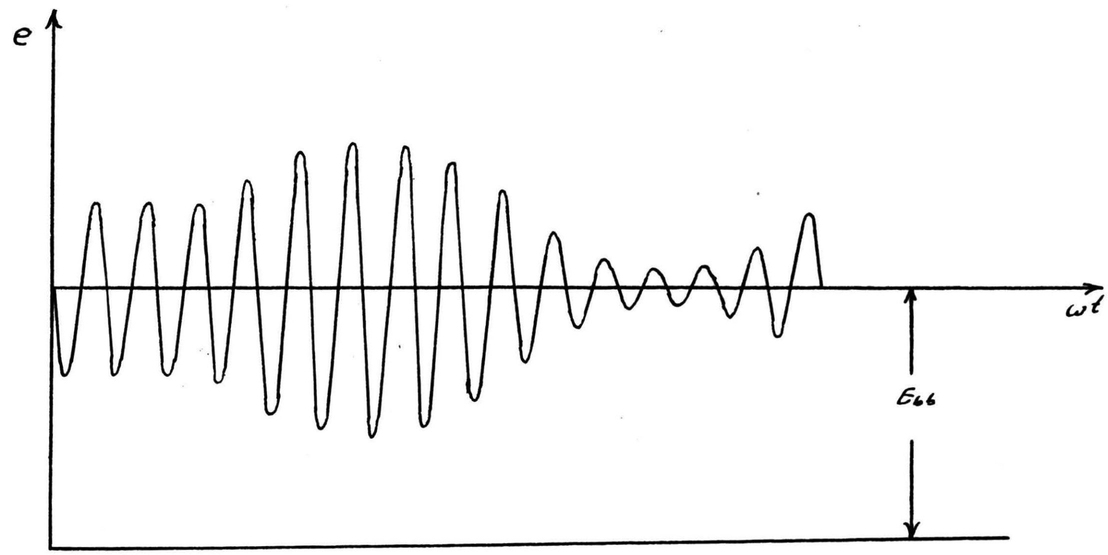
Figure (3a) shows the total voltage applied to the grid circuit, and figure (3b) is the corresponding plate-cathode voltage.

To obtain reasonably linear modulation the grid should not be driven positive during the peaks of the modulating signal. Driving the grid positive causes the grid to draw current, which in turn, causes distortion due to impedance drop in the driving source. On the other hand, driving the grid slightly positive during the peaks of the modulation cycle gives greater output power. Driving voltages in the grid circuit should also have good voltage regulation for linear modulation, Increasing the load resistance improves the linearity of the dynamic characteristic curve of the amplifier, thereby improving the linearity of the operation. But increasing the load resistance decreases the power output.

The efficiency of the grid-modulated Class C amplifier varies during the modulation cycle. At the portion of the negative half cycle of the modulating signal when the effective grid-bias (i.e. d-c grid bias plus modulating signal) drives the grid to cut off, the tube operates under typical Class C conditions with good efficiency. During the other portions of the modulating signal, the instantaneous plate voltage is quite high during the time of plate-current flow,



a) Voltage applied to grid



b) Plate-cathode voltage

Figure (3). Wave forms in a grid-modulated Class C amplifier.

resulting in increased plate loss. The over all efficiency during the complete modulation cycle tends to be in the neighborhood of 50 per cent. Hence, the output of a grid-modulated Class C amplifier is less than the output of an unmodulated Class C amplifier.

The main advantage of a grid-modulated Class C amplifier is that it requires a small driving power.

6. DISTORTION:

If the equation for the envelope of an amplitude modulated wave does not contain the various frequency components present in the original intelligence in their correct relative magnitudes and phases the modulated wave is said to be distorted.

If the envelope equation contains frequences that are not present in the original intelligence, this distortion is known as nonlinear or amplitude distortion. The amplitude distortion caused by the nonlinear characteristics of the tube is not serious, since all the harmonics generated, other than the desired ones, can be minimized by appropriate filter circuits.

Frequency distortion exists when the ratio of the output to input voltage varies as the frequency of the input signal varies. This is caused by the circuit impedances which vary with frequency.

Phase shift is said to be introduced when the relative phases of the various frequency components of the input are different at the output. It must be emphasized that if the phases of various frequency components are shifted by the same amount at the output no phase distortion is introduced. In the vacuum tube amplifiers the time delay is introduced by the reactances at low and high frequencies. Also, at high frequencies, additional time delay is caused by the transit time of the electrons in the tube.

SECTION 11
THEORETICAL STUDY OF THE CLIPPER AMPLITUDE MODULATOR

1. INTRODUCTION:

In the preceeding section the principles of amplitude modulation were discussed briefly. In this section the possibility of obtaining an amplitude modulated wave by the clipping action of diodes will be discussed.

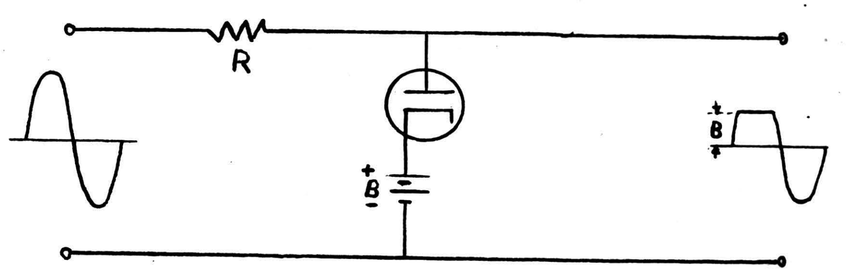


Figure (4). Diode clipper circuit, for clipping positive peaks.

Figure (4) shows one form of a diode clipper circuit. (B) is the d-c bias voltage on the diode, placed as shown. The resistor (R) has a value higher than the low internal resistance presented by the diode while conducting. The load impedance across the output terminals is high in comparison with (R) so that the load does not draw appreciable current when the tube is conducting. If the input is a sinusoidal voltage having an amplitude greater than (B), the diode of figure (4) can not conduct until the potential on the plate exceeds (B). For all negative values of input voltage, and for all positive values of input smaller than (B), the plate will be at a lower potential than the cathode. Under this condition, the diode presents an open circuit and hence the output potential accurally reproduces all the variations of input potential, until the input voltage rises above the bias voltage. At this point, the plate is at a higher potential than the cathode and therefore the tube starts conducting. During the conduction period the diode has a very low internal impedance. This arrangement produces the voltage output wave as shown in figure (4). The positive half cycle is clipped at a voltage approximately equal to (B).

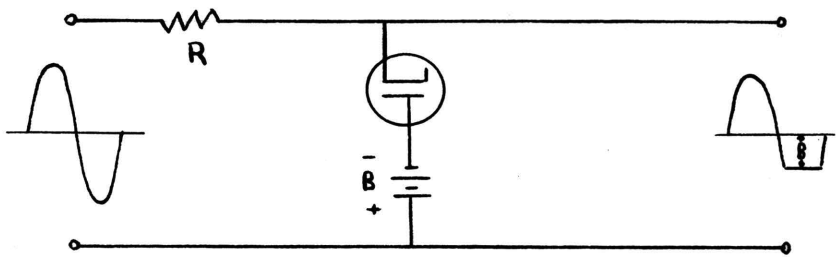


Figure (5). Diode clipper circuit , for clipping negative peaks.

If the connection of figure (4) is slightly changed as in figure (5), the tube will conduct only during the period when the cathode is more negative than the plate, giving the output wave form that has its negative half cycles clipped at the negative level (B), as indicated in figure (5).

If figures (4) and (5) are combined as in figure (6), the input sine wave will have both its positive and negative peaks clipped at the output.

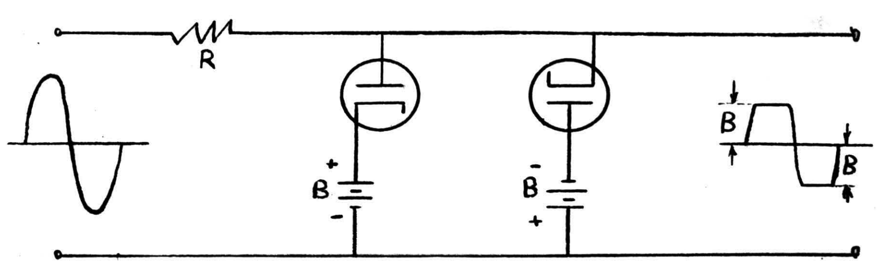


Figure (6). Diode clipper circuit, for clipping both negative and positive peaks.

In figure (6) the tubes are biased with equal magnitude supplies, so that positive and negative clipping levels of the output wave are equal.

2. CLIPPER AMPLITUDE MODULATOR:

In connection with the diode clipper circuits it has been mentioned that the clipping level of a wave is solely dependent on and is almost equal to the magnitude of the bias voltage (B). This suggests that if the bias voltage is varied with time, the clipping level will also vary accordingly. Therefore, if a sinusoidal voltage is superimposed on the d-c bias of figure (6), the clipping level of the output wave form will also vary sinusoidally about (B).

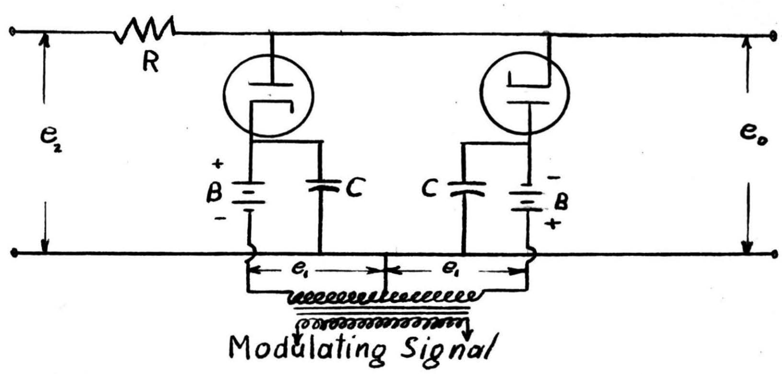


Figure (7). Clipper amplitude modulator.

$e_2 = E_2 \text{ Sin} w_2 t$, input carrier voltage.

$e_1 = E_1 \text{ Sin} w_1 t$, time - varying bias or modulating signal, as appears on the secodary of the audio transformer.

B = d-c bias voltage.

C = By-pass condensers for the carrier signal.

The circuit of figure (7) explains the way of obtaining a time-varying bias and it is the circuit used for the clipper amplitude modulator — the subject of this thesis. The modulating signal is placed in series with the d-c bias voltages through a center-tapped audio transformer, making the total bias on the first tube $+(B + e_1)$, and, $-(B + e_1)$ on the second. The capacitance C has a value such as to provide a low impedance path to carrier frequency, thereby isolating the modulation source from the carrier voltage. The high frequency sinusoidal carrier wave is applied to the input terminals as in figure (7). Because of the sinusoidally varying bias voltages on the diodes, the carrier will be clipped with a sinusoidal envelope. The output wave form will be as shown in figure (8).

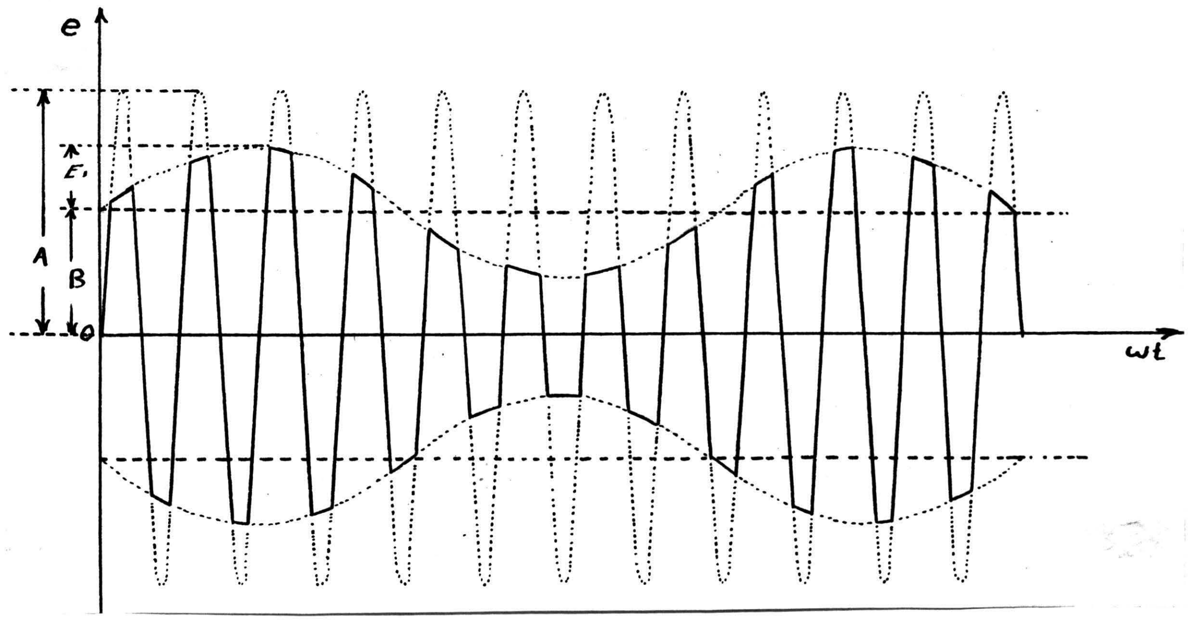


Figure (8). Sinusoidally clipped carrier signal.

Where

(A) = amplitude of the carrier signal as measured between the resistance R and the first diode in figure (7) with the diodes removed.

Figure (8) somewhat resembles an amplitude modulated wave. However, an equation for this wave must be found in order to see if it contains the frequency components that constitute an amplitude modulated wave.

3. MATHEMATICAL ANALYSIS OF A SINUSOIDALLY CLIPPED SINE WAVE:

The main task is to analyse the wave form of figure (8) mathematically and show that it is an amplitude modulated wave. In other words, the frequency components that costitute the wave form of figure (8) are to be found. The method of attack to analyse this wave will be as follows: First, suppose that the high frequency sine wave (carrier) is clipped at the constant level (B) on both sides of (wt)-axis. This gives a periodic function as in figure (9). By expressing the curve of figure (9) mathematically and replacing (B) by ($B+E_1 \sin w_1 t$) in the expression of figure (9) an equation for figure (8) is obtained.

Let $w_1 t = x_1$ and $w_2 t = x_2$. Also assume that $x_2 \gg x_1$ and $|B+E_1| \leq A$.

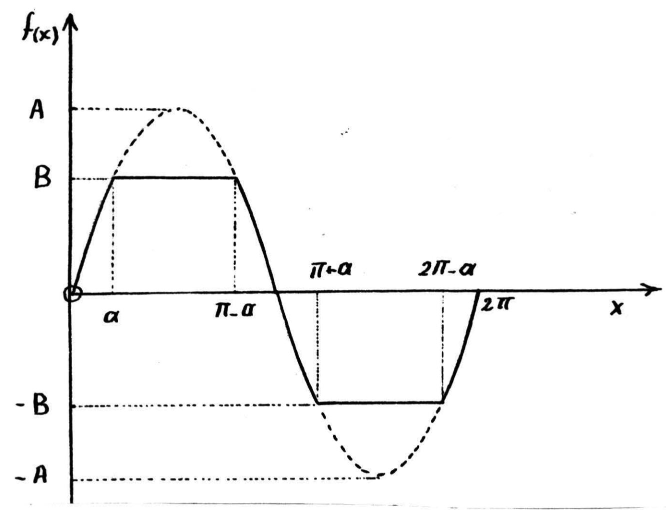


Figure (9). Sinusoidal carrier clipped at the constant level (B).

In general, the Fourier Series is given by the expression

$$f_{(x)} = \frac{a}{2} + \sum_{n=1}^{\infty} (a_n \operatorname{Cos} nx + b_n \operatorname{Sin} nx)$$

where

$$a_o = \frac{I}{\pi}\int_{-\pi}^{\pi} f_{(x)}\, dx$$

$$a_n = \frac{I}{\pi}\int_{-\pi}^{\pi} f_{(x)} \text{ Cosnx } dx$$

$$b_n = \frac{I}{\pi}\int_{-\pi}^{\pi} f_{(x)} \text{ Sinnx } dx$$

n = Any positive integer

However if the origin is chosen as in figure (9), according to the principles of Fourier Series, the curve of figure (9) is said to be an odd function. For this particular case the Fourier expression reduces to the form

$$f_{(x)} = \sum_{n=1}^{\infty} b_n \text{ Sinnx } dx \qquad (7)$$

where, in equation (7)

$$b_n = \frac{2}{\pi}\int_{0}^{\pi} f_{(x)} \text{ Sinnx } dx \qquad (8)$$

n = Any positive odd integer

From figure (9)

$f_{(x)}$ = A Sinx for $0 < x < a$

$f_{(x)}$ = B for $a < x < (\pi - a)$

$f_{(x)}$ = A Sinx for $(\pi - a) < x < \pi$

When x=a , A Sinx = B

or

$$x = a = \text{Arc Sin}\left(\frac{B}{A}\right)$$

When n = I , from equation (8)

$$b_1 = \frac{2}{\pi}\int_{0}^{a}(A \text{ Sinx}) \text{ Sinx } dx + \frac{2}{\pi}\int_{a}^{\pi-a} B \text{ Sinx } dx + \frac{2}{\pi}\int_{\pi-a}^{\pi}(A \text{ Sinx}) \text{ Sinx } dx$$

$$b_1 = \frac{2A}{\pi}\left[\frac{x}{2} - \frac{\mathrm{Sin}2x}{4}\right]_0^{a} - \frac{2B}{\pi}\left[\mathrm{Cos}x\right]_a^{\pi-a} + \frac{2A}{\pi}\left[\frac{x}{2} - \frac{\mathrm{Sin}2x}{4}\right]_{\pi-a}^{\pi}$$

$$b_1 = \frac{2A}{\pi}\left(\frac{a}{2} - \frac{\mathrm{Sin}2a}{4}\right) - \frac{2B}{\pi}\left[\mathrm{Cos}(\pi-a) - \mathrm{Cos}a\right] + \frac{2A}{\pi}\left[\frac{\pi}{2} - \frac{\pi-a}{2} + \frac{\mathrm{Sin}2(\pi-a)}{4}\right]$$

$$b_1 = \frac{2}{\pi}\left(Aa - \frac{A}{2}\mathrm{Sin}2a + 2B\,\mathrm{Cos}a\right) \qquad (9)$$

Substituting the values of (a), Sin2a , and Cosa from appendix A,

$$b_1 = \frac{2}{\pi}\left(A\,\mathrm{ArcSin}\frac{B}{A} + B\,\frac{\sqrt{A^2-B^2}}{A}\right) \qquad (\mathrm{IO})$$

When $n > I$, from equation (8)

$$b_n = \frac{2}{\pi}\int_0^a A\,\mathrm{Sin}x\,\mathrm{Sin}nx\,dx + \int_a^{\pi-a} B\,\mathrm{Sin}nx\,dx + \int_{\pi-a}^{\pi} A\,\mathrm{Sin}x\,\mathrm{Sin}nx\,dx$$

$$b_n = \frac{2}{\pi}\left\{\frac{A}{2}\left[\frac{I}{I-n}\mathrm{Sin}x(I-n) - \frac{I}{I+n}\mathrm{Sin}x(I+n)\right]_0^a - \frac{B}{n}\left[\mathrm{Cos}nx\right]_a^{\pi-a} + \frac{A}{2}\left[\frac{I}{I-n}\mathrm{Sin}x(I-n) - \frac{I}{I+n}\mathrm{Sin}x(I+n)\right]_{\pi-a}^{\pi}\right\}$$

$$b_n = \frac{2}{\pi}\left[\frac{A}{2(I-n)}\mathrm{Sin}a(I-n) - \frac{A}{2(I+n)}\mathrm{Sin}a(I+n) - \frac{B}{n}\mathrm{Cos}n(\pi-a) + \frac{B}{n}\mathrm{Cos}na + \left(-\frac{A}{2(I-n)}\right)\mathrm{Sin}(\pi-a)(I-n) + \frac{A}{2(I+n)}\mathrm{Sin}(I+n)(\pi-a)\right]$$

Since (n) is any positive odd integer,

$$b_n = \frac{2}{\pi}\left[\frac{A}{2(I-n)}\mathrm{Sin}a(I-n) - \frac{A}{2(I+n)}\mathrm{Sin}a(I+n) + \frac{2B}{n}\mathrm{Cos}a + \frac{A}{2(n-I)}\mathrm{Sin}(\pi-a)(I-n) + \frac{A}{2(I+n)}\mathrm{Sin}(\pi-a)(I+n)\right] \qquad (\mathrm{II})$$

For $n = 3$

$$b_3 = \frac{2}{\pi}\left(\frac{A}{4}\,\text{Sin}2a - \frac{A}{8}\,\text{Sin}4a + \frac{2B}{3}\,\text{Cos}3a + \frac{A}{4}\,\text{Sin}2a - \frac{A}{8}\text{Sin}4a\right)$$

$$b_3 = \frac{2}{\pi}\left(\frac{A}{2}\,\text{Sin}2a - \frac{A}{4}\,\text{Sin}4a + \frac{2B}{3}\,\text{Cos}3a\right)$$

Substituting the values of Sin2a, Sin4a, and Cos3a from appendix A and simplifying,

$$b_3 = \left(\frac{4B}{3\pi} - \frac{4B^3}{3A^2\pi}\right)\frac{\sqrt{A^2-B^2}}{A} \qquad (I2)$$

For $n = 5$, from equation (II)

$$b_5 = \frac{2}{\pi}\left[\frac{A}{8}\,\text{Sin}4a - \frac{A}{I2}\,\text{Sin}6a + \frac{2B}{5}\,\text{Cos}5a - \frac{A}{8}\,\text{Sin}(4\pi-4a) + \frac{A}{I2}\,\text{Sin}(6\pi-6a)\right]$$

$$b_5 = \frac{2}{\pi}\left(\frac{A}{4}\,\text{Sin}4a - \frac{A}{6}\,\text{Sin}6a + \frac{2B}{5}\,\text{Cos}5a\right)$$

Substituting the values of Sin4a, Sin6a, and Cos5a from appendix A and simplifying,

$$b_5 = \frac{2}{\pi}\left(\frac{2B}{5} - \frac{22B^3}{I5A^2} + \frac{I6B^5}{I5A^4}\right)\frac{\sqrt{A^2-B^2}}{A} \qquad (I3)$$

Therefore, from equations (7), (8), (I0), (I2), and (I3)

$$\begin{aligned} f_{(x)} = & \left(\frac{2A}{\pi}\,\text{Arc Sin}\frac{B}{A} + \frac{2B}{\pi}\frac{\sqrt{A^2-B^2}}{A}\right)\text{Sin}x \\ & + \left(\frac{4B}{3\pi} - \frac{4B^3}{3A^2\pi}\right)\frac{\sqrt{A^2-B^2}}{A}\,\text{Sin}3x \\ & + \left(\frac{4B}{5\pi} - \frac{44B^3}{I5A^2\pi} + \frac{32B^5}{I5A\pi}\right)\frac{\sqrt{A^2-B^2}}{A}\,\text{Sin}5x \\ & + \cdots\cdots \end{aligned} \qquad (I4)$$

It is desirable to develop some neans of checking for possible mistakes made in deriving equation (I4). If the quantity (A) approaches infinity, figure (9) reduces to the form of a square wave clipped at the level (B) on both sides of x-axis. Taking the limit of $f_{(x)}$ as (A) approaches infinity equation (I4) reduces to the form identical to that obtained from a square wave by similar analysis.

First, express equation (I4) in the following fashion and replace Arc $\sin\frac{B}{A}$ by its equivalent series form.

$$f_{(x)} = \frac{2A}{\pi}\left(\frac{B}{A} + \frac{B^3}{6A^3} + \frac{3B^5}{40A^5} + \ldots\right) + \frac{2B}{\pi}\sqrt{I - \frac{B^2}{A^2}}\ \sin x$$
$$+ \left(\frac{4B}{3\pi} - \frac{4B^3}{3A^2\pi}\right)\sqrt{I - \frac{B^2}{A^2}}\ \sin 3x$$
$$+ \left(\frac{4B}{5\pi} - \frac{44B^3}{15A^2\pi} + \frac{32B^5}{15A\pi}\right)\sqrt{I - \frac{B^2}{A^2}}\ \sin 5x + \ldots\ldots$$

$$\lim_{A\to\infty}\left[f_{(x)}\right] = \lim_{A\to\infty}\left[\left(\frac{2B}{\pi} + \frac{2B^3}{6A^2\pi} + \frac{6B^5}{40A\pi} + \ldots\right) + \frac{2B}{\pi}\sqrt{I - \frac{B^2}{A^2}}\,\sin x\right.$$
$$+ \left(\frac{4B}{3\pi} - \frac{4B^3}{3A^2\pi}\right)\sqrt{I - \frac{B^2}{A^2}}\ \sin 3x$$
$$\left. + \left(\frac{4B}{5\pi} - \frac{44B^3}{15A^2\pi} + \frac{32B^5}{15A\pi}\right)\sqrt{I - \frac{B^2}{A^2}}\ \sin 5x + \ldots\right]$$

Which reduces to

$$\lim_{A\to\infty}\left[f_{(x)}\right] = \frac{4B}{\pi}\sin x + \frac{4B}{3\pi}\sin 3x + \frac{4B}{5\pi}\sin 5x + \ldots.$$

This is seen to be identical with the Fourier expansion of a square wave. (See appendix E)

To get an expression for figure (8), every quantity (B) appearing in equation (I4) must be replaced by ($B + E_1 \sin x_1$) as suggested on page (14).

The first term of equation (I4) becomes

$$b_1 = \frac{2A}{\pi}\text{Arc } \sin\frac{B}{A} + \frac{2B}{\pi}\sqrt{I - \frac{B^2}{A^2}}$$

Substituting the series forms for Arc $\sin\frac{B}{A}$ and $\sqrt{I - \frac{B^2}{A^2}}$ as given **in appendix - B.**

$$b_1 = \frac{2}{\pi}\left[A\left(\frac{B}{A} + \frac{B^3}{6A^3} + \frac{3B^5}{40A^5} + \frac{5B^7}{112A^7} + \frac{35B^9}{1152A^9} + \frac{63B^{11}}{2816A^{11}} + \frac{231B^{13}}{13312A^{13}} + \frac{143B^{15}}{10240A^{15}} + \ldots\right)\right.$$
$$\left. + B\left(I - \frac{B^2}{2A^2} - \frac{B^4}{8A^4} - \frac{B^6}{16A^6} - \frac{5B^8}{128A^8} - \frac{7B^{10}}{2^9A^{10}} - \frac{21B^{12}}{2^{10}A^{12}} - \frac{33B^{14}}{2^{11}A^{14}} - \ldots\right)\right]$$

$$b_1 = \frac{2}{\pi}\left(B + \frac{B^3}{6A^2} + \frac{3B^5}{40A^4} + \frac{5B^7}{112A^6} + \frac{35B^9}{1152A^8} + \frac{63B^{11}}{2816A^{10}} \right.$$

$$+ \frac{231B^{13}}{13312A^{12}} + \frac{143B^{15}}{10240A^{14}} + \cdots\cdots\cdots\cdots$$

$$+ B - \frac{B^3}{2A^2} - \frac{B^5}{8A^4} - \frac{B^7}{16A^6} - \frac{5B^9}{128A^8} - \frac{7B^{11}}{2^8\ A^{10}}$$

$$\left. - \frac{21B^{13}}{2^{10}\ A^{12}} - \frac{33B^{15}}{2^{11}\ A^{14}} - \cdots\cdots\cdots \right)$$

$$b_1 = \frac{2}{\pi}\left(2B - \frac{B^3}{3A^2} - \frac{B^5}{20A^4} - \frac{B^7}{56A^6} - \frac{5B^9}{576A^8} - \frac{7B^{11}}{1408A^{10}} \right.$$

$$\left. - \frac{21B^{13}}{6656A^{12}} - \frac{11B^{15}}{5120A^{14}} - \cdots\cdots\cdots \right) \qquad (15)$$

In equation (15) replace (B) by ($B + E_1 \sin x_1$) where ($E_1 \sin x_1$) is the modulating signal. The expanded form of the quantity ($B + E_1 \sin x_1$)^n is given in appendix C.

From equation (15) and appendix C,

$$b' = \frac{2}{\pi}\Big[2\,(B+E_1\sin x_1) - \frac{1}{3A^2}\,(B^3 + 3B^2E_1\sin x_1 + 3BE_1^2\sin^2 x_1 + E_1^3\sin^3 x_1)$$

$$- \frac{1}{20A^4}\,(B^5 + 5B^4E_1\sin x_1 + 10B^3E_1^2\sin^2 x_1 + 10B^2E_1^3\sin^3 x_1 + 5BE_1^4\sin^4 x_1 + E_1^5\sin^5 x_1)$$

$$- \frac{1}{56A^6}\,(B^7 + 7B^6E_1\sin x_1 + 21B^5E_1^2\sin^2 x_1 + 35B^4E_1^3\sin^3 x_1 + 35B^3E_1^4\sin^4 x_1 + 21B^2E_1^5\sin^5 x_1 + 7BE_1^6\sin^6 x_1 + E_1^7\sin^7 x_1)$$

$$- \frac{5}{576A^8}\,(B^9 + 9B^8E_1\sin x_1 + 36B^7E_1^2\sin^2 x_1\ \ 84B^6E_1^3\sin^3 x_1 + 126B^5E_1^4\sin^4 x_1 + 126B^4E_1^5\sin^5 x_1 + 84B^3E_1^6\sin^6 x_1\ \ 36B^2E_1^7\sin^7 x_1 + 9BE_1^8\sin^8 x_1 + E_1^9\sin^9 x_1)$$

$$- \frac{7}{1408A^{10}} (B^{11} + 11B^{10}E_1 \sin x_1 + 55B^9E_1^2 \sin^2 x_1$$
$$+ 165B^8E_1^3 \sin^3 x_1 + 330B^7E_1^4 \sin^4 x_1$$
$$+ 462B^6E_1^5 \sin^5 x_1 + 462B^5E_1^6 \sin^6 x_1$$
$$+ 330B^4E_1^7 \sin^7 x_1 + 165B^3E_1^8 \sin^8 x_1$$
$$+ 55B^2E_1^9 \sin^9 x_1 + 11BE_1^{10} \sin^{10} x_1$$
$$+ E_1^{11} \sin^{11} x_1)$$

$$- \frac{21}{6656A^{12}} (B^{13} + 13B^{12}E_1 \sin x_1 + 78B^{11}E_1^2 \sin^2 x_1$$
$$+ 286B^{10}E_1^3 \sin^3 x_1 + 715B^9E_1^4 \sin^4 x_1$$
$$+ 1287B^8E_1^5 \sin^5 x_1 + 1716B^7E_1^6 \sin^6 x_1$$
$$+ 1716B^6E_1^7 \sin^7 x_1 + 1287B^5E_1^8 \sin^8 x_1$$
$$+ 715B^4E_1^9 \sin^9 x_1 + 286B^3E_1^{10} \sin^{10} x_1$$
$$+ 78B^2E_1^{11} \sin^{11} x_1 + 13BE_1^{12} \sin^{12} x_1$$
$$+ E_1^{13} \sin^{13} x_1)$$

$$- \cdots\cdots\cdots\cdots \Big]$$

$$b_1' = \frac{2}{\pi} \Big[\Big(2B - \frac{B^3}{3A^2} - \frac{B^5}{20A^4} - \frac{B^7}{56A^6} - \frac{5B^9}{576A^8} - \frac{7B^{11}}{1408A^{10}}$$
$$- \frac{21B^{13}}{6656A^{12}} \cdots\cdots \Big)$$
$$+ \Big(2 - \frac{B^2}{A^2} - \frac{B^4}{4A^4} - \frac{B^6}{8A^6} - \frac{5B^8}{64A^8} - \frac{7B^{10}}{128A^{10}}$$
$$- \frac{21B^{12}}{512A^{12}} \cdots \Big) E_1 \sin x_1$$

$$- \Big(\frac{B}{A^2} + \frac{B^3}{2A^4} + \frac{3B^5}{8A^6} + \frac{5B^7}{16A^8} + \frac{35B^9}{128A^{10}}$$
$$+ \frac{63B^{11}}{256A^{12}} \cdots \Big) E_1^2 \sin^2 x_1$$

$$-\left(\frac{1}{3A^2} + \frac{B^2}{2A^4} + \frac{5B^4}{8A^6} + \frac{35B^6}{48A^8} + \frac{105B^8}{128A^{10}} + \frac{231B^{10}}{256A^{12}} \dots\right)E_1^3 \sin^3 x_1$$

$$-\left(\frac{B}{4A^4} + \frac{5B^3}{8A^6} + \frac{35B^5}{32A^8} + \frac{105B^7}{64A^{10}} + \frac{1155B^9}{512A^{12}} + \dots\right)E_1^4 \sin^4 x_1$$

$$-\left(\frac{1}{20A^4} + \frac{3B^2}{8A^6} + \frac{35B^4}{32A^8} + \frac{147B^6}{64A^{10}} + \frac{2079B^8}{512A^{12}} + \dots\right)E_1^5 \sin^5 x_1$$

$$-\left(\frac{B}{8A^6} + \frac{35B^3}{48A^8} + \frac{147B^5}{64A^{10}} + \frac{693B^7}{128A^{12}} + \dots\dots\right)E_1^6 \sin^6 x_1$$

$$-\left(\frac{1}{56A^6} + \frac{5B^2}{16A^8} + \frac{105B^4}{64A^{10}} + \frac{693B^6}{128A^{12}} + \dots\right)E_1^7 \sin^7 x_1$$

$$-\left(\frac{5B}{64A^8} + \frac{105B^3}{128A^{10}} + \frac{2079B^5}{512A^{12}} + \dots\right)E_1^8 \sin^8 x_1$$

$$-\left(\frac{5}{576A^8} + \frac{35B^2}{128A^{10}} + \frac{1155B^4}{512A^{12}} + \dots\dots\right)E_1^9 \sin^9 x_1$$

$$-\left(\frac{7B}{128A^{10}} + \frac{231B^3}{256A^{12}} + \dots\dots\right)E_1^{10} \sin^{10} x_1$$

$$-\left(\frac{7}{1408A^{10}} + \frac{63B^2}{256A^{12}} + \dots\dots\right)E_1^{11} \sin^{11} x_1$$

$$-\left(\frac{21B}{512A^{12}} + \dots\dots\dots\dots\right)E_1^{12} \sin^{12} x_1$$

$$- \dots\dots\dots\dots\dots\dots\dots\dots \Big] \qquad (16)$$

Let a_0 , a_1 , a_2 , etc... be the corresponding constant terms of equation (16). Therefore,

$$
\begin{aligned}
b_1' = a_0 &+ a_1 \operatorname{Sin}x_1 + a_2 \operatorname{Sin}^2x_1 + a_3 \operatorname{Sin}^3x_1 + a_4 \operatorname{Sin}^4x_1 \\
&+ a_5 \operatorname{Sin}^5x_1 + a_6 \operatorname{Sin}^6x_1 + a_7 \operatorname{Sin}^7x_1 + a_8 \operatorname{Sin}^8x_1 \\
&+ a_9 \operatorname{Sin}^9x_1 + a_{IO} \operatorname{Sin}^{IO}x_1 + a_{II} \operatorname{Sin}^{II}x_1 \\
&+ a_{I2} \operatorname{Sin}^{I2}x_1 + \ldots\ldots\ldots\ldots\ldots \qquad \text{(I7)}
\end{aligned}
$$

Substituting the trigonometric identities for the sine terms from appendix D,

$$
\begin{aligned}
b_1' = a_0 &+ a_1 \operatorname{Sin}x_1 + a_2 \left(\tfrac{I}{2} - \tfrac{I}{2} \operatorname{Cos}2x_1 \right) \\
&+ a_3 \left(\tfrac{3}{4} \operatorname{Sin}x_1 - \tfrac{I}{4} \operatorname{Sin}3x_1 \right) \\
&+ a_4 \left(\tfrac{3}{8} - \tfrac{I}{2} \operatorname{Cos}2x_1 + \tfrac{I}{8} \operatorname{Cos}4x_1 \right) \\
&+ a_5 \left(\tfrac{5}{8} \operatorname{Sin}x_1 - \tfrac{5}{I6} \operatorname{Sin}3x_1 + \tfrac{I}{I6} \operatorname{Sin}5x_1 \right) \\
&+ a_6 \left(\tfrac{5}{I6} - \tfrac{I5}{32} \operatorname{Cos}2x_1 + \tfrac{3}{I6} \operatorname{Cos}4x_1 - \tfrac{I}{32} \operatorname{Cos}6x_1 \right) \\
&+ a_7 \left(\tfrac{35}{64} \operatorname{Sin}x_1 - \tfrac{2I}{64} \operatorname{Sin}3x_1 + \tfrac{7}{64} \operatorname{Sin}5x_1 \right. \\
&\qquad \left. - \tfrac{I}{64} \operatorname{Sin}7x_1 \right) \\
&+ a_8 \left(\tfrac{35}{I28} - \tfrac{56}{I28} \operatorname{Cos}2x_1 + \tfrac{28}{I28} \operatorname{Cos}4x_1 \right. \\
&\qquad \left. - \tfrac{8}{I28} \operatorname{Cos}6x_1 + \tfrac{I}{I28} \operatorname{Cos}8x_1 \right) \\
&+ \tfrac{a_9}{I28} \left(63 \operatorname{Sin}x_1 - 42 \operatorname{Sin}3x_1 + I8 \operatorname{Sin}5x_1 \right. \\
&\qquad \left. - \tfrac{9}{2} \operatorname{Sin}7x_1 + \tfrac{I}{2} \operatorname{Sin}9x_1 \right) \\
&+ \tfrac{a_{IO}}{256} \left(63 - I05 \operatorname{Cos}2x_1 + 60 \operatorname{Cos}4x_1 - \tfrac{45}{2} \operatorname{Cos}6x_1 \right. \\
&\qquad \left. + 5 \operatorname{Cos}8x_1 - \tfrac{1}{2} \operatorname{Cos}IOx_1 \right) \\
&+ \tfrac{a_{11}}{1024} \left(462 \operatorname{Sin}x_1 - 330 \operatorname{Sin}3x_1 + 165 \operatorname{Sin}5x_1 \right. \\
&\qquad \left. - 55 \operatorname{Sin}7x_1 + 11 \operatorname{Sin}9x_1 - \operatorname{Sin}11x_1 \right) \\
&+ \tfrac{a_{12}}{2048} \left(462 - 792 \operatorname{Cos}2x_1 + 495 \operatorname{Cos}4x_1 \right. \\
&\qquad - 220\operatorname{Cos}6x_1 + 66\operatorname{Cos}8x_1 - 12\operatorname{Cos}10x_1 + \operatorname{Cos}12x_1 \\
&+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}
$$

$$b_1' = \left(a_0 + \frac{a_2}{2} + \frac{3a_4}{8} + \frac{5a_6}{16} + \frac{35a_8}{128} + \frac{63a_{10}}{256} + \frac{462a_{12}}{2048} + \dots \right)$$

$$+\left(a_1 + \frac{3a_3}{4} + \frac{5a_5}{8} + \frac{35a_7}{64} + \frac{63a_9}{128} + \frac{462a_{10}}{1024} + \dots \right).\mathrm{Sin}x_1$$

$$-\left(\frac{a_2}{2} + \frac{a_4}{2} + \frac{15a_6}{32} + \frac{56a_8}{128} + \frac{105a_{10}}{256} + \frac{792a_{12}}{2048} + \dots \right)\mathrm{Cos}2x_1$$

$$-\left(\frac{a_3}{4} + \frac{5a_5}{16} + \frac{21a_7}{64} + \frac{42a_9}{128} + \frac{330a_{11}}{1024} + \dots\dots \right)\mathrm{Sin}3x_1$$

$$+\left(\frac{a_4}{8} + \frac{3a_6}{16} + \frac{28a_8}{128} + \frac{60a_{10}}{256} + \frac{495a_{12}}{2048} + \dots\dots \right)\mathrm{Cos}4x_1$$

$$+\left(\frac{a_5}{16} + \frac{7a_7}{64} + \frac{18a_9}{128} + \frac{165a_{11}}{1024} + \dots\dots\dots\dots \right)\mathrm{Sin}5x_1$$

$$-\left(\frac{a_6}{32} + \frac{8a_8}{128} + \frac{45a_{10}}{512} + \frac{220a_{12}}{2048} + \dots\dots\dots\dots \right)\mathrm{Cos}6x_1$$

$$-\left(\frac{a_7}{64} + \frac{9a_9}{256} + \frac{55a_{11}}{1024} + \dots\dots\dots\dots\dots\dots \right)\mathrm{Sin}7x_1$$

$$+\left(\frac{a_8}{128} + \frac{5a_{10}}{256} + \frac{66a_{12}}{2048} + \dots\dots\dots\dots\dots\dots \right)\mathrm{Cos}8x_1$$

$$+\left(\frac{a_9}{256} + \frac{11a_{11}}{1024} + \dots\dots\dots\dots\dots\dots\dots\dots \right)\mathrm{Sin}9x_1$$

$$-\left(\frac{a_{10}}{512} + \frac{12a_{12}}{2048} + \dots\dots\dots\dots\dots\dots\dots\dots \right)\mathrm{Cos}10x_1$$

$$-\left(\frac{a_{11}}{1024} + \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \right)\mathrm{Sin}11x_1$$

$$+ \dots\dots\dots\dots\dots\dots \qquad (18)$$

Let C_0 , C_1 , C_2, etc... be the respective constant terms of equation (18). Then,

$$b_1' = C_0 + C_1\mathrm{Sin}x_1 + C_2\,\mathrm{Cos}2x_1 + C_3\,\mathrm{Sin}3x_1 + C_4\,\mathrm{Cos}4x_1$$
$$+ C_5\,\mathrm{Sin}5x_1 + C_6\,\mathrm{Cos}6x_1 + C_7\,\mathrm{Sin}7x_1 + C_8\,\mathrm{Cos}8x_1$$
$$+ C_9\,\mathrm{Sin}9x_1 + C_{10}\,\mathrm{Cos}10x_1 + C_{11}\,\mathrm{Sin}11x_1 + \dots \qquad (19)$$

The first term, e_1', for a sinusoidally clipped sine wave becomes, from equations (14) and (19),

$$e_1' = b_1' \operatorname{Sin} x_2$$

$$\begin{aligned} e_1' = C_0 \operatorname{Sin} x_2 &+ C_1 \operatorname{Sin} x_1 \operatorname{Sin} x_2 + C_2 \operatorname{Sin} x_2 \operatorname{Cos} 2x_1 \\ &+ C_3 \operatorname{Sin} 3x_1 \operatorname{Sin} x_2 + C_4 \operatorname{Cos} 4x_1 \operatorname{Sin} x_2 \\ &+ C_5 \operatorname{Sin} 5x_1 \operatorname{Sin} x_2 + C_6 \operatorname{Cos} 6x_1 \operatorname{Sin} x_2 \\ &+ C_7 \operatorname{Sin} 7x_1 \operatorname{Sin} x_2 + C_8 \operatorname{Cos} 8x_1 \operatorname{Sin} x_2 \\ &+ C_9 \operatorname{Sin} 9x_1 \operatorname{Sin} x_2 + C_{10} \operatorname{Cos} 10x_1 \operatorname{Sin} x_2 \\ &+ \cdots\cdots \end{aligned} \tag{20}$$

or

$$\begin{aligned} e_1' = C_0 \operatorname{Sin} x_2 &+ \frac{C_1}{2} \operatorname{Cos}(x_2 - x_1) - \operatorname{Cos}(x_2 + x_1) \\ &+ \frac{C_2}{2} \operatorname{Sin}(x_2 - 2x_1) + \operatorname{Sin}(x_2 + 2x_1) \\ &+ \frac{C_3}{2} \operatorname{Cos}(x_2 - 3x_1) - \operatorname{Cos}(x_2 + 3x_1) \\ &+ \frac{C_4}{2} \operatorname{Sin}(x_2 - 4x_1) + \operatorname{Sin}(x_2 + 4x_1) \\ &+ \cdots\cdots \end{aligned} \tag{21}$$

where

$x_1 = w_1 t = 2\pi f_1 t$, f_1 being the frequency of the modulating signal

$x_2 = w_2 t = 2\pi f_2 t$, f_2 being the frequency of the carrier signal

The constants C_0, C_1, C_2, etc... have the dimensions in volts as seen from equations (16) and (18). This point will be more clear when C_0, C_1, C_2, etc... are expressed in terms of the known quantities (A), (B), and (E_1) in the following pages. Also, expressions (20) or (21) clearly shows that (e_1') is an amplitude modulated wave, ($C_0 \sin x_2$) being the carrier signal of this modulated wave.

The constants C_0 , C_1 , C_2 , etc... can be expressed in terms of the known quantities (A), (B), and (E_1). From equation (18),

$$C_0 = a_0 + \frac{a_2}{2} + \frac{3a_4}{8} + \frac{5a_6}{16} + \frac{35a_8}{128} + \frac{63a_{10}}{256} + \frac{462a_{12}}{2048} + \cdots\cdots$$

Substituting for a_0 , a_1 , a_2 , etc.., from equation (16)

$$\begin{aligned} C_0 = \frac{2}{\pi} \Big[& \left(2B - \frac{B^3}{3A^2} - \frac{B^5}{20A^4} - \frac{B^7}{56A^6} - \frac{5B^9}{576A^8} - \frac{7B^{11}}{1408A^{10}} - \frac{21B^{13}}{6656A^{12}} \cdots \right) \\ & - \frac{1}{2} \left(\frac{B}{A^2} + \frac{B^3}{2A^4} + \frac{3B^5}{8A^6} + \frac{5B^7}{16A^8} + \frac{35B^9}{128A^{10}} + \frac{63B^{11}}{256A^{12}} + \cdots \right) E_1^2 \\ & - \frac{3}{8} \left(\frac{B}{4A^4} + \frac{5B^3}{8A^6} + \frac{35B^5}{32A^8} + \frac{105B^7}{64A^{10}} + \frac{1155B^9}{512A^{12}} + \cdots\cdots \right) E_1^4 \\ & - \frac{5}{16} \left(\frac{B}{8A^6} + \frac{35B^3}{48A^8} + \frac{147B^5}{64A^{10}} + \frac{693B^7}{128A^{12}} + \cdots \right) E_1^6 \\ & - \frac{35}{128} \left(\frac{5B}{64A^8} + \frac{105B^3}{128A^{10}} + \frac{2079B^5}{512A^{12}} + \cdots\cdots \right) E_1^8 \\ & - \frac{63}{256} \left(\frac{7B}{128A^{10}} + \frac{231B^3}{256A^{12}} + \cdots\cdots\cdots\cdots \right) E_1^{10} \\ & - \frac{462}{2048} \left(\frac{21B}{512A^{12}} + \cdots\cdots\cdots\cdots\cdots\cdots \right) E_1^{12} \\ & - \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \Big] \end{aligned}$$

Rearranging,

$$C_o = \frac{4B}{\pi}\left(1 - \frac{B^2}{6A^2} - \frac{B^4}{40A^4} - \frac{B^6}{112A^6} - \frac{5B^8}{1152A^8} - \frac{7B^{10}}{3416A^{10}} - \frac{21B^{12}}{13312A^{12}} + \ldots\right)$$

$$- \frac{E_1^2 B}{\pi A^2}\left(1 + \frac{B^2}{2A^2} + \frac{3B^4}{8A^4} + \frac{5B^6}{16A^6} + \frac{35B^8}{128A^8} + \frac{63B^{10}}{256A^{10}} + \ldots\right)$$

$$- \frac{3E_1^4 B}{16\pi A^4}\left(1 + \frac{5B^2}{2A^2} + \frac{35B^4}{8A^4} + \frac{105B^6}{16A^6} + \frac{1155B^8}{128A^8} + \ldots\right)$$

$$- \frac{5E_1^6 B}{64\pi A^6}\left(1 + \frac{35B^2}{6A^2} + \frac{147B^4}{8A^4} + \frac{693B^6}{16A^6} + \ldots\right)$$

$$- \frac{175E_1^8 B}{4096\pi A^8}\left(1 + \frac{105B^2}{10A^2} + \frac{2079B^4}{40A^4} + \ldots\right)$$

$$- \frac{441E_1^{10} B}{16384\pi A^{10}}\left(1 + \frac{33B^2}{9A^2} + \ldots\right)$$

$$- \ldots \qquad (22)$$

Similarly

$$C_1 = a_1 + \frac{3a_3}{4} + \frac{5a_5}{8} + \frac{35a_7}{64} + \frac{63a_9}{128} + \frac{462a_{11}}{1024} + \ldots$$

$$C_1 = \frac{4E_1}{\pi}\left(1 - \frac{B^2}{2A^2} - \frac{B^4}{4A^4} - \frac{B^6}{16A^6} - \frac{5B^8}{128A^8} - \frac{7B^{10}}{256A^{10}} - \frac{21B^{12}}{1024} - \ldots\right)$$

$$- \frac{E_1^3}{2\pi A^2}\left(1 + \frac{3B^2}{2A^2} + \frac{15B^4}{8A^4} + \frac{35B^6}{16A^6} + \frac{315B^8}{128A^8} + \frac{693B^{10}}{256A^{10}} + \ldots\right)$$

$$- \frac{E_1^5}{16\pi A^4}\left(1 + \frac{15B^2}{2A^2} + \frac{175B^4}{8A^4} + \frac{735B^6}{16A^6} + \frac{6395B^8}{128A^8} + \ldots\right)$$

$$- \frac{E_1^7}{256\pi A^6}\left(1 + \frac{35B^2}{2A^2} + \frac{735B^4}{8A^4} + \frac{4851B^6}{16A^6} + \ldots\ldots\ldots\right)$$

$$- \frac{35E_1^9}{4096\pi A^8}\left(1 + \frac{63B^2}{2A^2} + \frac{2079B^4}{8A^4} + \ldots\ldots\ldots\ldots\ldots\right)$$

$$- \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (23)$$

$$C_2 = -\left(\frac{a_2}{2} + \frac{a_4}{2} + \frac{15a_6}{32} + \frac{56a_8}{128} + \frac{105a_{10}}{256} + \frac{792a_{12}}{2048} + \ldots\right)$$

$$C_2 = \frac{E_1^2 B}{\pi A^2}\left(1 + \frac{B^2}{2A^2} + \frac{3B^4}{8A^4} + \frac{5B^6}{16A^6} + \frac{35B^8}{128A^8} + \frac{63B^{10}}{256A^{10}} + \ldots\right)$$

$$+ \frac{E_1^4 B}{4\pi A^4}\left(1 + \frac{5B^2}{2A^2} + \frac{35B^4}{8A^4} + \frac{105B^6}{16A^6} + \frac{1155B^8}{128A^8} + \ldots\ldots\right)$$

$$+ \frac{15E_1^6 B}{128\pi A^6}\left(1 + \frac{35B^2}{6A^2} + \frac{147B^4}{8A^4} + \frac{693B^6}{16A^6} + \ldots\ldots\ldots\ldots\right)$$

$$+ \frac{35E_1^8 B}{512\pi A^8}\left(1 + \frac{105B^2}{10A^2} + \frac{2079B^4}{40A^4} + \ldots\ldots\ldots\ldots\ldots\right)$$

$$+ \frac{735E_1^{10} B}{16384\pi A^{10}}\left(1 + \frac{33B^2}{9A^2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\right.;$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (24)$$

$$C_3 = -\left(\frac{a_3}{4} + \frac{5a_5}{16} + \frac{21a_7}{64} + \frac{42a_9}{128} + \frac{330a_{11}}{1024} + \ldots\ldots\ldots\right)$$

$$C_3 = \frac{E_1^3}{6\pi A^2}\left(1 + \frac{3B^2}{2A^2} + \frac{15B^4}{8A^4} + \frac{35B^6}{16A^6} + \frac{315B^8}{128A^8} + \frac{693B^{10}}{256A^{10}} + \ldots\right)$$

$$+ \frac{E_1^5}{32\pi A^4}\left(1 + \frac{15B^2}{2A^2} + \frac{175B^4}{8A^4} + \frac{735B^6}{16A^6} + \frac{6395B^8}{128A^8} + \ldots\ldots\right)$$

$$+ \frac{3E_1^7}{1280\pi A^6}\left(1 + \frac{35B^2}{2A^2} + \frac{735B^4}{8A^4} + \frac{4851B^6}{16A^6} + \ldots\ldots\ldots\right)$$

$$+ \frac{35E_1^9}{6144\pi A^8}\left(1 + \frac{63B^2}{2A^2} + \frac{2079B^4}{8A^4} + \ldots\ldots\ldots\ldots\ldots\ldots\right)$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (25)$$

$$c_4 = \left(\frac{a_4}{8} + \frac{3a_6}{16} + \frac{28a_8}{128} + \frac{60a_{10}}{256} + \frac{495a_{12}}{2048} + \ldots\ldots\ldots\right)$$

$$c_4 = -\frac{E_1^4 B}{16\pi A^4}\left(1 + \frac{5B^2}{2A^2} + \frac{35B^4}{8A^4} + \frac{105B^6}{16A^6} + \frac{1155B^8}{128A^8} + \ldots\ldots\right)$$

$$-\frac{3E_1^6 B}{64\pi A^6}\left(1 + \frac{35B^2}{6A^2} + \frac{147B^4}{8A^4} + \frac{693B^6}{16A^6} + \ldots\ldots\ldots\ldots\right)$$

$$-\frac{35E_1^8 B}{1024\pi A^8}\left(1 + \frac{105B^2}{10A^2} + \frac{2079B^4}{40A^4} + \ldots\ldots\ldots\ldots\ldots\ldots\right)$$

$$-\frac{1155E_1^{10} B}{16384\pi A^{10}}\left(1 + \frac{33B^2}{9A^2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\right)$$

$$- \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (26)$$

$$c_5 = \left(\frac{a_5}{16} + \frac{7a_7}{64} + \frac{18a_9}{128} + \frac{165a_{11}}{1024} + \ldots\ldots\ldots\ldots\ldots\ldots\right)$$

$$c_5 = -\frac{E_1^5}{160\pi A^4}\left(1 + \frac{15B^2}{2A^2} + \frac{175B^4}{8A^4} + \frac{735B^6}{16A^6} + \frac{6395B^8}{128A^8} + \ldots\right)$$

$$-\frac{E_1^7}{1280\pi A^6}\left(1 + \frac{35B^2}{2A^2} + \frac{735B^4}{8A^4} + \frac{4851B^6}{16A^6} + \ldots\ldots\ldots\right)$$

$$-\frac{5E_1^9}{2048\pi A^8}\left(1 + \frac{63B^2}{2A^2} + \frac{2079B^4}{8A4} + \ldots\ldots\ldots\ldots\ldots\ldots\right)$$

$$- \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (27)$$

$$c_6 = -\left(\frac{a_6}{32} + \frac{8a_8}{128} + \frac{45a_{10}}{512} + \frac{220a_{12}}{2048} + \cdots\cdots\cdots \right)$$

$$c_6 = \frac{E_1^6 B}{128\pi A^6}\left(1 + \frac{35B^2}{6A^2} + \frac{147B^4}{8A^4} + \frac{693B^6}{16A^6} + \cdots\cdots\cdots\right)$$
$$+ \frac{5E_1^8 B}{512\pi A^8}\left(1 + \frac{105B^2}{10A^2} + \frac{2079B^4}{40A^4} + \cdots\cdots\cdots\right)$$
$$+ \frac{245E_1^{10} B}{32768\pi A^{10}}\left(1 + \frac{33B^2}{9A^2} + \cdots\cdots\cdots\right)$$
$$+ \cdots\cdots\cdots \qquad (28)$$

$$c_7 = -\left(\frac{a_7}{64} + \frac{9a_9}{256} + \frac{55a_{11}}{1024} + \cdots\cdots\cdots \right)$$

$$c_7 = \frac{E_1^7}{8960\pi A^6}\left(1 + \frac{35B^2}{2A^2} + \frac{735B^4}{8A^4} + \frac{4851B^6}{16A^6} + \cdots\cdots\cdots\right)$$
$$+ \frac{5E_1^9}{8192\ \pi A^8}\left(1 + \frac{63B^2}{2A^2} + \frac{2079B^4}{8A^4} + \cdots\cdots\cdots\right)$$
$$+ \cdots\cdots\cdots \qquad (29)$$

$$c_8 = \left(\frac{a_8}{128} + \frac{5a_{10}}{256} + \frac{66a_{12}}{2048} + \cdots\cdots\cdots \right)$$

$$c_8 = -\frac{5E_1^8 B}{4096\pi A^8}\left(1 + \frac{105B^2}{10A^2} + \frac{2079B^4}{40A^4} + \cdots\cdots\cdots\right)$$
$$- \frac{35E_1^{10} B}{16384\pi A^{10}}\left(1 + \frac{33B^2}{9A^2} + \cdots\cdots\cdots\right)$$
$$- \cdots\cdots\cdots \qquad (30)$$

$$c_9 = \left(\frac{a_9}{256} + \frac{11a_{11}}{1024} + \cdots\cdots\cdots \right)$$

$$c_9 = -\frac{5E_1^9}{73828\pi A^8}\left(1 + \frac{63B^2}{2A^2} + \frac{2079B^4}{8A^4} + \cdots\cdots\cdots\right)$$
$$- \cdots\cdots\cdots \qquad (31)$$

Comparing equation (23) with (31), it is seen that $C_9 \ll C_1$. Therefore the evaluation of further constants is not necessary.

Inspection of the expressions (22) through (31) shows that C_0, C_1, C_2, etc... have the dimensions in volts, as expected.

If $E_1 = 0$, there is no modulation. For this case, from equations (22) through (31),

$$C_1 = C_2 = C_3 = \ldots\ldots\ldots\ldots C_n = 0$$

$$C_0 = \frac{2}{\pi}\left(2B - \frac{B^3}{3A^2} - \frac{B^5}{20A^4} - \frac{B^7}{56A^6} - \frac{5B^9}{576A^8} - \frac{7B^{11}}{1408A^{10}} - \frac{21B^{13}}{6656A^{12}} \ldots\ldots\right)$$

which is identical with equation (15), for the unmodulated case.

If (A) approaches infinity, the high frequency wave, or the carrier signal, becomes a square wave.

Letting (A) approach infinity,

$$C_0 = \frac{4B}{\pi}$$

$$C_1 = \frac{4E_1}{\pi}$$

$$C_2 = C_3 = C_4 = \ldots\ldots\ldots\ldots C_n = 0$$

and the formula (20), the first term of the expression for figure (9), becomes

$$e_1' = \frac{4B}{\pi}\,\mathrm{Sin}x_2 + \frac{4E_1}{\pi}\,\mathrm{Sin}x_1\,\mathrm{Sin}x_2$$

This is the same expression obtained if the carrier signal were assumed to be a square wave. (See appendix E)

For the case $E_1 = 0$ and $B = A$ the carrier signal is not changed.

When $E_1 = 0$

$$C_1 = C_2 = C_3 = \ldots\ldots\ldots\ldots\ldots C_n = 0$$

$$C_0 = \frac{4B}{\pi}\left(1 - \frac{B^2}{6A^2} - \frac{B^4}{40A^4} - \frac{B^6}{112A^6} - \frac{5B^8}{1152} - \frac{7B^{10}}{3416A^{10}} - \frac{21B^{12}}{13312A^{12}} \ldots\ldots\right)$$

Letting $B = A$

$$C_0 = \frac{4A}{\pi}\left(1 - \frac{1}{6} - \frac{1}{40} - \frac{1}{112} - \frac{5}{1152} - \frac{7}{3416} - \frac{21}{13312} \ldots\ldots\right)$$

$$C_0 = \frac{4A}{\pi}\left(1 - .1668 - .025 - .00898 - .00433 - .00205 - .001578 - \ldots\ldots\right)$$

$$C_0 = \frac{4A}{\pi}\left(1 - .208738\right)$$

$$C_0 = \frac{4A}{\pi}\left(.791\right)$$

$$C_0 = A$$

Therefore equation (20) reduces to

$e_1' = A \operatorname{Sin} x_2$, as expected.

The second term of equation (14) can be expanded as was the first term. From equation (14),

$$b_3 = \left(\frac{4B}{3\pi} - \frac{4B^3}{3A^2\pi} \right) \frac{\sqrt{A^2 - B^2}}{A}$$

$$b_3 = \frac{4B}{3\pi} \left(1 - \frac{B^2}{A^2} \right) \sqrt{1 - \frac{B^2}{A^2}}$$

$$b_3 = \frac{4B}{3\pi} \left(1 - \frac{B^2}{A^2} \right)^{3/2}$$

Substituting the series form of $\left(1 - \frac{B^2}{A^2} \right)^{3/2}$ from appendix B

$$b_3 = \frac{4B}{3\pi} \left(1 - \frac{3B^2}{2A^2} + \frac{3B^4}{2^3A^4} + \frac{B^6}{2^4A^6} + \frac{3B^8}{2^7A^8} + \frac{3B^{10}}{2^8A^{10}} + \frac{7B^{12}}{2^{10}A^{12}} + \frac{9B^{14}}{2^{11}A^{14}} + \dots\dots\dots \right)$$

$$b_3 = \frac{4}{3\pi} \left(B - \frac{3B^3}{2A^2} + \frac{3B^5}{2^3\,A^4} + \frac{B^7}{2^4A^6} + \frac{3B^9}{2^7A^8} + \frac{3B^{11}}{2^8A^{10}} + \frac{7B^{13}}{2^{10}A^{12}} + \frac{9B^{15}}{2^{11}A^{14}} + \dots\dots\dots \right) \qquad (32)$$

Replace (B) by $(B + E_1 \text{ Sin}x_1)$ in order to introduce the sine variation of the envelope as explained on page (14). The expanded form of $(B + E_1 \text{ Sin}x_1)^n$ is listed in appendix C for different values of (n).

$$b_3' = \frac{4}{3\pi}\Big[(B + E_1 \operatorname{Sin} x_1)$$

$$- \frac{3}{2A^2} (B^3 + 3B^2E_1 \operatorname{Sin} x_1 + 3BE_1^2 \operatorname{Sin}^2 x_1 + E_1^3 \operatorname{Sin}^3 x_1)$$

$$+ \frac{3}{2^3A^4} (B^5 + 5B^4E_1 \operatorname{Sin} x_1 + 10B^3E_1^2 \operatorname{Sin}^2 x_1$$

$$+ 10B^2E_1^3 \operatorname{Sin}^3 x_1 + 5BE_1^4 \operatorname{Sin}^4 x_1 + E_1^5 \operatorname{Sin}^5 x_1)$$

$$+ \frac{1}{2^4A^6} (B^7 + 7B^6E_1 \operatorname{Sin} x_1 + 21B^5E^2 \operatorname{Sin}^2 x_1$$

$$+ 35B^4E_1^3 \operatorname{Sin}^3 x_1 + 35B^3E_1^4 \operatorname{Sin}^4 x_1 + 21B^2E_1^5 \operatorname{Sin}^5 x_1$$

$$+ 7BE_1^6 \operatorname{Sin}^6 x_1 + E_1^7 \operatorname{Sin}^7 x_1)$$

$$+ \frac{3}{2^7A^8} (B^9 + 9B^8E_1 \operatorname{Sin} x_1 + 36B^7E_1^2 \operatorname{Sin}^2 x_1$$

$$+ 84B^6E_1^3 \operatorname{Sin}^2 x_1 + 126B^5E_1^4 \operatorname{Sin}^4 x_1$$

$$+ 126B^4E_1^5 \operatorname{Sin}^5 x_1 + 84B^3E_1^6 \operatorname{Sin}^6 x_1$$

$$+ 36B^2E_1^7 \operatorname{Sin}^7 x_1 + 9BE_1^8 \operatorname{Sin}^8 x_1 + E_1^9 \operatorname{Sin}^9 x_1)$$

$$+ \frac{3}{2^8A^{10}} (B^{11} + 11B^{10}E_1 \operatorname{Sin} x_1 + 55B^9E_1^2 \operatorname{Sin}^2 x_1$$

$$+ 165B^8E_1^3 \operatorname{Sin}^3 x_1 + 330B^7E_1^4 \operatorname{Sin}^4 x_1$$

$$+ 462B^6E_1^5 \operatorname{Sin}^5 x_1 + 462B^5E_1^6 \operatorname{Sin}^6 x_1$$

$$+ 330B^4E_1^7 \operatorname{Sin}^7 x_1 + 165B^3E_1^8 \operatorname{Sin}^8 x_1$$

$$+ 55B^2E_1^9 \operatorname{Sin}^9 x_1 + 11BE_1^{10} \operatorname{Sin}^{10} x_1$$

$$+ E_1^{11} \operatorname{Sin}^{11} x_1)$$

$$+ \frac{7}{2^{10}A^{12}} (B^{13} + 13B^{12}E_1 \sin x_1 + 78B^{11}E_1^2 \sin^2 x_1$$

$$+ 286B^{10}E_1^3 \sin^3 x_1 + 715B^9E_1^4 \sin^4 x_1$$

$$+ 1287B^8E_1^5 \sin^5 x_1 + 1716B^7E_1^6 \sin^6 x_1$$

$$+ 1716B^6E_1^7 \sin^7 x_1 + 1287B^5E_1^8 \sin^8 x_1$$

$$+ 715B^4E_1^9 \sin^9 x_1 + 286B^3E_1^{10} \sin^{10} x_1$$

$$+ 78B^2E_1^{11} \sin^{11} x_1 + 13BE_1^{12} \sin^{12} x_1$$

$$+ E_1^{13} \sin^{13} x_1)$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots \Big]$$

$$b'_3 = \frac{4}{3\pi}\Big[\Big(B - \frac{3B^3}{2A^2} + \frac{3B^5}{2^3A^4} + \frac{B^7}{2^4A^6} + \frac{3B^9}{2^7A^8} + \frac{3B^{11}}{2^8A^{10}}$$

$$+ \frac{7B^{13}}{2^{10}A^{12}} + \ldots\ldots\ldots\Big)$$

$$+ \Big(1 - \frac{9B^2}{2A^2} + \frac{15B^4}{2^3A^4} + \frac{7B^6}{2^4A^6} + \frac{27B^8}{2^7A^8} + \frac{33B^{10}}{2^8A^{10}}$$

$$+ \frac{91B^{12}}{2^{10}A^{12}} + \ldots\Big)E_1 \sin x_1$$

$$+ \Big(- \frac{9B}{2A^2} + \frac{30B^3}{2^3A^4} + \frac{21B^5}{2^4A^6} + \frac{108B^7}{2^7A^8} + \frac{165B^9}{2^8A^{10}}$$

$$+ \frac{546B^{11}}{2^{10}A^{12}} + \ldots\Big)E_1^2 \sin^2 x_1$$

$$+ \Big(- \frac{3}{2A^2} + \frac{30B^2}{2^3A^4} + \frac{35B^4}{2^4A^6} + \frac{252B^6}{2^7A^8} + \frac{495B^8}{2^8A^{10}}$$

$$+ \frac{2002B^{10}}{2^{10}A^{12}} + \ldots\Big)E_1^3 \sin^3 x_1$$

$$+ \left(\frac{15B}{2^3A^4} + \frac{35B^3}{2^4A^6} + \frac{3\cdot 126B^5}{2^7A^8} + \frac{3\cdot 330B^7}{2^8A^{10}} + \frac{7\cdot 715B^9}{2^{10}A^{12}} \right.$$

$$\left. + \ldots\ldots \right) E_1^4 \sin^4 x_1$$

$$+ \left(\frac{3}{2^3A^4} + \frac{21B^2}{2^4A^6} + \frac{3\cdot 126B^4}{2^7A^8} + \frac{3\cdot 462B^6}{2^8A^{10}} + \frac{7\cdot 1287B^8}{2^{10}A^{12}} \right.$$

$$\left. + \ldots\ldots \right) E_1^5 \sin^5 x_1$$

$$+ \left(\frac{7B}{2^4A^6} + \frac{3\cdot 84B^3}{2^7A^8} + \frac{3\cdot 462B^5}{2^8A^{10}} + \frac{7\cdot 1716B^7}{2^{10}A^{12}} \right.$$

$$\left. + \ldots\ldots \right) E_1^6 \sin^6 x_1$$

$$+ \left(\frac{1}{2^4A^6} + \frac{3\cdot 36B^2}{2^7A^8} + \frac{3\cdot 330B^4}{2^8A^{10}} + \frac{7\cdot 1716B^6}{2^{10}A^{12}} \right.$$

$$\left. + \ldots\ldots \right) E_1^7 \sin^7 x_1$$

$$+ \left(\frac{27B}{2^7A^8} + \frac{3\cdot 165B^3}{2^8A^{10}} + \frac{7\cdot 1287B^5}{2^{10}A^{12}} + \ldots\ldots \right) E_1^8 \sin^8 x_1$$

$$+ \left(\frac{3}{2^7A^8} + \frac{3\cdot 55B^2}{2^8A^{10}} + \frac{7\cdot 715B^4}{2^{10}A^{12}} + \ldots\ldots \right) E_1^9 \sin^9 x_1$$

$$+ \left(\frac{33B}{2^8A^{10}} + \frac{7\cdot 28B^3}{2^{10}A^{12}} + \ldots\ldots\ldots\ldots\ldots \right) E_1^{10} \sin^{10} x_1$$

$$+ \left(\frac{3}{2^8A^{10}} + \frac{7\cdot 78B^2}{2^{10}A^{12}} + \ldots\ldots\ldots\ldots\ldots \right) E_1^{11} \sin^{11} x_1$$

$$+ \left(\frac{7\cdot 13B}{2^{10}A^{12}} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \right) E_1^{12} \sin^{12} x_1$$

$$\left. + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \right]$$

Rearranging,

$$b_3' = + \frac{4B}{3\pi}\left(1 - \frac{3B^2}{2A^2} + \frac{3B^4}{8A^4} + \frac{B^6}{16A^6} + \frac{3B^8}{2^7A^8} + \frac{3B^{10}}{2^8A^{10}} + \frac{7B^{12}}{2^{10}A^{12}} + \ldots\right)$$

$$+ \frac{4E_1}{3\pi}\left(1 - \frac{9B^2}{2A^2} + \frac{15B^4}{8A^4} + \frac{7B^6}{16A^6} + \frac{27B^8}{2^7A^8} + \frac{33B^{10}}{2^8A^{10}} + \frac{91B^{12}}{2^{10}A^{12}} + \ldots\right)\sin x_1$$

$$- \frac{6BE_1^2}{\pi A^2}\left(1 - \frac{5B^2}{6A^2} - \frac{7B^4}{24A^4} - \frac{3B^6}{16A^6} - \frac{55B^8}{3\cdot 2^7A^8} - \frac{91B^{10}}{3\cdot 2^8A^{10}} - \ldots\right)\sin^2 x_1$$

$$- \frac{2E_1^3}{\pi A^2}\left(1 - \frac{5B^2}{2A^2} - \frac{35B^4}{24A^4} - \frac{21B^6}{16A^6} - \frac{165B^8}{2^7A^8} - \frac{1001B^{10}}{3\cdot 2^8A^{10}} - \ldots\right)\sin^3 x_1$$

$$+ \frac{5BE_1^4}{2\pi A^4}\left(1 + \frac{7B^2}{6A^2} + \frac{63B^4}{40A^4} + \frac{33B^6}{16A^6} + \frac{1001B^8}{3\cdot 2^7A^8} + \ldots\ldots\ldots\ldots\right)\sin^4 x_1$$

$$+ \frac{E_1^5}{2\pi A^4}\left(1 + \frac{7B^2}{2A^2} + \frac{63B^4}{8A^4} + \frac{231B^6}{16A^6} + \frac{3003B^8}{2^7A^8} + \ldots\ldots\ldots\ldots\right)\sin^5 x_1$$

$$+ \frac{7BE_1^6}{12\pi A^6}\left(1 + \frac{9B^2}{2A^2} + \frac{99B^4}{8A^4} + \frac{429B^6}{16A^6} + \ldots\right)\sin^6 x_1$$

$$+ \frac{E_1^7}{12\pi A^6}\left(1 + \frac{27B^2}{2A^2} + \frac{495B^4}{8A^4} + \frac{3003B^6}{16A^6} + \ldots\right)\operatorname{Sin}^7 x_1$$

$$+ \frac{9BE_1^8}{32\pi A^8}\left(1 + \frac{55B^2}{6A^2} + \frac{1001B^4}{24A^4} + \ldots\ldots\ldots\ldots\right)\operatorname{Sin}^8 x_1$$

$$+ \frac{E_1^9}{32\pi A^8}\left(1 + \frac{55B^2}{2A^2} + \frac{5005B^4}{24A^4} + \ldots\ldots\ldots\ldots\right)\operatorname{Sin}^9 x_1$$

$$+ \frac{11BE_1^{10}}{64\pi A^{10}}\left(1 + \frac{49B^2}{33A^2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\right)\operatorname{Sin}^{10} x_1$$

$$+ \frac{E_1^{11}}{64\pi A^{10}}\left(1 + \frac{91B^2}{2A^2} + \ldots\ldots\ldots\ldots\ldots\ldots\right)\operatorname{Sin}^{11} x_1$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (33)$$

Let d_0, d_1, d_2, etc... be the respective constant terms of equation (33). Therefore,

$$b_3' = d_0 + d_1 \operatorname{Sin}x_1 + d_2 \operatorname{Sin}^2 x_1 + d_3 \operatorname{Sin}^3 x_1 + d_4 \operatorname{Sin}^4 x_1 + d_5 \operatorname{Sin}^5 x_1 + d_6 \operatorname{Sin}^6 x_1 + \ldots \qquad (34)$$

Replacing $\operatorname{Sin}^2 x_1$, $\operatorname{Sin}^3 x_1$, $\operatorname{Sin}^4 x_1$, etc... by their trigonometric identities from appendix D,

$$\begin{aligned} b_3' = d_0 &+ d_1 \operatorname{Sin}x_1 + d_2\left(\tfrac{1}{2} - \tfrac{1}{2}\operatorname{Cos}2x_1\right) \\ &+ d_3\left(\tfrac{3}{4}\operatorname{Sin}x_1 - \tfrac{1}{4}\operatorname{Sin}3x_1\right) \\ &+ d_4\left(\tfrac{3}{8} - \tfrac{1}{2}\operatorname{Cos}2x_1 + \tfrac{1}{8}\operatorname{Cos}4x_1\right) \\ &+ d_5\left(\tfrac{5}{8}\operatorname{Sin}x_1 - \tfrac{5}{16}\operatorname{Sin}3x_1 + \tfrac{1}{16}\operatorname{Sin}5x_1\right) \\ &+ d_6\left(\tfrac{5}{16} - \tfrac{15}{32}\operatorname{Cos}2x_1 + \tfrac{3}{16}\operatorname{Cos}4x_1 - \tfrac{1}{32}\operatorname{Cos}6x_1\right) \\ &+ d_7\left(\tfrac{35}{64}\operatorname{Sin}x_1 - \tfrac{21}{64}\operatorname{Sin}3x_1 + \tfrac{7}{64}\operatorname{Sin}5x_1 - \tfrac{1}{64}\operatorname{Sin}7x_1\right) \end{aligned}$$

$$+ d_8\left(\frac{35}{128} - \frac{56}{128} \operatorname{Cos}2x_1 + \frac{28}{128} \operatorname{Cos}4x_1 - \frac{8}{128} \operatorname{Cos}6x_1 + \frac{1}{128} \operatorname{Cos}8x_1 \right)$$

$$+ \frac{d_9}{128}\left(63 \operatorname{Sin}x_1 - 42 \operatorname{Sin}3x_1 + 18 \operatorname{Sin}5x_1 - \frac{9}{2} \operatorname{Sin}7x_1 + \frac{1}{2} \operatorname{Sin}9x_1 \right)$$

$$+ \frac{d_{10}}{256}\left(63 - 105 \operatorname{Cos}2x_1 + 60 \operatorname{Cos}4x_1 - \frac{45}{2} \operatorname{Cos}6x_1 + 5 \operatorname{Cos}8x_1 - \frac{1}{2} \operatorname{Cos}10x_1 \right)$$

$$+ \frac{d_{11}}{1024}\left(462 \operatorname{Sin}x_1 - 330 \operatorname{Sin}3x_1 + 165 \operatorname{Sin}5x_1 - 55 \operatorname{Sin}7x_1 + 11 \operatorname{Sin}9x_1 - \operatorname{Sin}11x_1 \right)$$

$$+ \frac{d_{12}}{2048}\left(462 - 792 \operatorname{Cos}2x_1 + 495 \operatorname{Cos}4x_1 - 220 \operatorname{Cos}6x_1 + 66 \operatorname{Cos}8x_1 - 12 \operatorname{Cos}10x_1 + \operatorname{Cos}12x_1 \right)$$

$$+ \frac{d_{13}}{4096}\left(1716 \operatorname{Sin}x_1 - 1287 \operatorname{Sin}3x_1 + 715 \operatorname{Sin}5x_1 - 286 \operatorname{Sin}7x_1 + 78 \operatorname{Sin}9x_1 - 13 \operatorname{Sin}11x_1 + \operatorname{Sin}13x_1 \right)$$

$$+ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

rearranging,

$$b'_3 = + \left(d_0 + \frac{d_2}{2} + \frac{3d_4}{8} + \frac{5d_6}{16} + \frac{35d_8}{128} + \frac{63d_{10}}{256} + \frac{462d_{12}}{2048} + \cdots\cdots \right)$$

$$+ \left(d_1 + \frac{3d_3}{4} + \frac{5d_5}{8} + \frac{35d_7}{64} + \frac{63d_9}{128} + \frac{462d_{11}}{1024} + \frac{1716d_{13}}{4096} + \cdots \right) \operatorname{Sin}x_1$$

$$- \left(\frac{d_2}{2} + \frac{d_4}{2} + \frac{35d_6}{32} + \frac{56d_8}{128} + \frac{105d_{10}}{256} + \frac{792d_{12}}{2048} + \ldots\ldots\ldots \right) \operatorname{Cos} 2x_1$$

$$- \left(\frac{d_3}{4} + \frac{5d_5}{16} + \frac{21d_7}{64} + \frac{42d_9}{128} + \frac{330d_{11}}{1024} + \frac{1287d_{13}}{4096} + \ldots\ldots\ldots\ldots \right) \operatorname{Sin} 3x_1$$

$$+ \left(\frac{d_4}{8} + \frac{3d_6}{16} + \frac{28d_8}{128} + \frac{60d_{10}}{256} + \frac{495d_{12}}{2048} + \ldots \right) \operatorname{Cos} 4x_1$$

$$+ \left(\frac{d_5}{16} + \frac{7d_7}{64} + \frac{18d_9}{128} + \frac{165d_{11}}{1024} + \frac{715d_{13}}{4096} + \ldots \right) \operatorname{Sin} 5x_1$$

$$- \left(\frac{d_6}{32} + \frac{8d_8}{128} + \frac{45d_{10}}{512} + \frac{220d_{12}}{2048} + \ldots\ldots\ldots\ldots \right) \operatorname{Cos} 6x_1$$

$$- \left(\frac{d_7}{64} + \frac{9d_9}{256} + \frac{55d_{11}}{1024} + \frac{286d_{13}}{4096} + \ldots\ldots\ldots\ldots \right) \operatorname{Sin} 7x_1$$

$$+ \left(\frac{d_8}{128} + \frac{5d_{10}}{256} + \frac{66d_{12}}{2048} + \ldots\ldots\ldots\ldots\ldots\ldots \right) \operatorname{Cos} 8x_1$$

$$+ \left(\frac{d_9}{256} + \frac{11d_{11}}{1024} + \frac{78d_{13}}{4096} + \ldots\ldots\ldots\ldots\ldots\ldots \right) \operatorname{Sin} 9x_1$$

$$- \left(\frac{d_{10}}{512} + \frac{12d_{12}}{2048} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \right) \operatorname{Cos} 10x_1$$

$$- \left(\frac{d_{11}}{1024} + \frac{13d_{13}}{4096} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \right) \operatorname{Sin} 11x_1$$

$$+ \left(\frac{d_{12}}{2048} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \right) \operatorname{Cos} 12x_1$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (35)$$

Let k_0 , k_1 , k_2 , etc... be the respective constant terms of equation (35). Then,

$$b'_3 = k_0 + k_1 \operatorname{Sin}x_1 + k_2 \operatorname{Cos}2x_1 + k_3 \operatorname{Sin}3x_1 + k_4 \operatorname{Cos}4x_1 + k_5 \operatorname{Sin}5x_1 + k_6 \operatorname{Cos}6x_1 + \ldots \qquad (36)$$

The second term of equation (14) for a sinusoidally clipped wave becomes,

$$e_3 = b'_3 \operatorname{Sin}3x_2$$

$$e_3 = k_0 \operatorname{Sin}3x_2 + k_1 \operatorname{Sin}x_1 \operatorname{Sin}3x_2 + k_2 \operatorname{Cos}2x_1 \operatorname{Sin}3x_2 + k_3 \operatorname{Sin}3x_1 \operatorname{Sin}3x_2 + k_4 \operatorname{Cos}4x_1 \operatorname{Sin}3x_2 + k_5 \operatorname{Sin}5x_1 \operatorname{Sin}3x_2 + k_6 \operatorname{Cos}6x_1 \operatorname{Sin}3x_2 + \ldots$$

$$\begin{aligned} e_3 = {} & k_0 \operatorname{Sin}3x_2 \\ & + \frac{k_1}{2} \operatorname{Cos}(3x_2 - x_1) - \operatorname{Cos}(3x_2 + x_1) \\ & + \frac{k_2}{2} \operatorname{Sin}(3x_2 - 2x_1) + \operatorname{Sin}(3x_2 + 2x_1) \\ & + \frac{k_3}{2} \operatorname{Cos}(3x_2 - 3x_1) - \operatorname{Cos}(3x_2 + 3x_1) \\ & + \frac{k_4}{2} \operatorname{Sin}(3x_2 - 4x_1) + \operatorname{Sin}(3x_2 + 4x_1) \\ & + \frac{k_5}{2} \operatorname{Cos}(3x_2 - 5x_1) - \operatorname{Cos}(3x_2 + 5x_1) \\ & + \ldots \end{aligned} \qquad (37)$$

The constants k_0 , k_1 , k_2 , etc... have the dimensions of volts as expected from equations (33) and (35). This point will be more clear when k_0 , k_1 , k_2 , etc...

are expressed in terms of the known quantities (A), (B), and (E_1) in the following pages. Also expression (37) clearly shows that e_3 is an amplitude modulated wave, ($k_0 \sin 3x_2$) being the carrier signal.

From equations (33) and (35),

$$k_0 = d_0 + \frac{d_2}{2} + \frac{3d_4}{8} + \frac{5d_6}{16} + \frac{35d_8}{128} + \frac{63d_{10}}{256} + \frac{462d_{12}}{2048} + \dots$$

$$k_0 = + \frac{4B}{3\pi} \left(1 - \frac{3B^2}{2A^2} + \frac{3B^4}{8A^4} + \frac{B^6}{16A^6} + \frac{3B^8}{2^7A^8} + \frac{3B^{10}}{2^8A^{10}} + \frac{7B^{12}}{2^{10}A^{12}} + \dots \right)$$

$$- \frac{3BE_1^2}{\pi A^2} \left(1 - \frac{5B^2}{6A^2} - \frac{7B^4}{24A^4} - \frac{3B^6}{16A^6} - \frac{55B^8}{3 \cdot 2^7A^8} - \frac{91B^{10}}{3 \cdot 2^8A^{10}} \dots \right)$$

$$+ \frac{15BE_1^4}{16\pi A^4} \left(1 + \frac{7B^2}{6A^2} + \frac{63B^4}{40A^4} + \frac{33B^6}{16A^6} + \frac{1001B^8}{3 \cdot 2^7A^8} + \dots \right)$$

$$+ \frac{35BE_1^6}{192A^6\pi} \left(1 + \frac{9B^2}{2A^2} + \frac{99B^4}{8A^4} + \frac{429B^6}{64A^6} + \dots \right)$$

$$+ \frac{315BE_1^8}{2^{12}A^8\pi} \left(1 + \frac{55B^2}{6A^2} + \frac{1001B^4}{24A^4} + \dots \right)$$

$$+ \frac{693BE_1^{10}}{2^{14}A^{10}\pi} \left(1 + \frac{49B^2}{33A^2} + \dots \right)$$

$$+ \dots$$

(38)

$$k_1 = d_1 + \frac{3d_3}{4} + \frac{5d_5}{8} + \frac{35d_7}{64} + \frac{63d_9}{128} + \frac{462d_{11}}{1024} + \cdots\cdots$$

$$k_1 = + \frac{4E_i}{3\pi}\left(1 - \frac{9B^2}{2A^2} + \frac{15B^4}{8A^4} + \frac{7B^6}{16A^6} + \frac{27B^8}{2^7A^8} + \frac{33B^{10}}{2^8A^{10}} + \frac{91B^{12}}{2^{10}A^{12}} + \cdots\cdots\right)$$

$$- \frac{3E_i^3}{2\pi A^2}\left(1 - \frac{5B^2}{2A^2} - \frac{35B^4}{24A^4} - \frac{21B^6}{16A^6} - \frac{165B^8}{2^7A^8} - \frac{1001B^{10}}{3\cdot 2^8A^{10}} - \cdots\cdots\right)$$

$$+ \frac{5E_i^5}{16\pi A^4}\left(1 + \frac{7B^2}{2A^2} + \frac{63B^4}{8A^4} + \frac{231B^6}{16A^6} + \frac{3003B^8}{2^7A^8} + \cdots\cdots\right)$$

$$+ \frac{35E_i^7}{768\pi A^6}\left(1 + \frac{27B^2}{2A^2} + \frac{495B^4}{8A^4} + \frac{3003B^6}{16A^6} + \cdots\cdots\right)$$

$$+ \frac{63E_i^9}{2^{12}\pi A^8}\left(1 + \frac{55B^2}{2A^2} + \frac{5005B^4}{24A^4} + \cdots\cdots\right)$$

$$+ \frac{231E_i^{11}}{2^{15}\pi A^{10}}\left(1 + \frac{91B^2}{2A^2} + \cdots\cdots\right)$$

$$+ \cdots\cdots \qquad (39)$$

$$k_2 = -\left(\frac{d_2}{2} + \frac{d_4}{2} + \frac{35d_6}{32} + \frac{56d_8}{128} + \frac{105d_{10}}{256} + \frac{792d_{12}}{2048} + \cdots\right)$$

$$k_2 = + \frac{3BE_i^2}{\pi A^2}\left(1 - \frac{5B^2}{6A^2} - \frac{7B^4}{24A^4} - \frac{3B^6}{16A^6} - \frac{55B^8}{3\cdot 2^7A^8} - \frac{91B^{10}}{3\cdot 2^8A^{10}} - \cdots\cdots\right)$$

$$- \frac{5BE_1^4}{4\pi A^4} \left(1 + \frac{7B^2}{6A^2} + \frac{63B^4}{40A^4} + \frac{33B^6}{16A^6} + \frac{1001B^8}{3 \cdot 2^7 A^8} + \ldots\ldots \right)$$

$$- \frac{245BE_1^6}{384\pi A^6} \left(1 + \frac{9B^2}{2A^2} + \frac{99B^4}{8A^4} + \frac{429B^6}{16A^6} + \ldots\ldots \right)$$

$$- \frac{63BE_1^8}{512\pi A^8} \left(1 + \frac{55B^2}{6A^2} + \frac{1001B^4}{24A^4} + \ldots\ldots \right)$$

$$- \frac{1155BE_1^{10}}{2^{14}\pi A^{10}} \left(1 + \frac{49B^2}{33A^2} + \ldots\ldots \right)$$

$$- \ldots\ldots \qquad (40)$$

$$k_3 = - \left(\frac{d_3}{4} + \frac{5d_5}{16} + \frac{21d_7}{64} + \frac{42d_9}{128} + \frac{330d_{11}}{1024} + \ldots\ldots \right)$$

$$k_3 = + \frac{E_1^3}{2\pi A^2} \left(1 - \frac{5B^2}{2A^2} - \frac{35B^4}{24A^4} - \frac{21B^6}{16A^6} - \frac{165B^8}{2^7 A^8} - \frac{1001B^{10}}{3 \cdot 2^8 A^{10}} \ldots \right)$$

$$- \frac{5E_1^5}{32\pi A^4} \left(1 + \frac{7B^2}{2A^2} + \frac{63B^4}{8A^4} + \frac{231B^6}{16A^6} + \frac{3003B^8}{2^7 A^8} + \ldots\ldots \right)$$

$$- \frac{7E_1^7}{192\pi A^6} \left(1 + \frac{27B^2}{2A^2} + \frac{495B^4}{8A^4} + \frac{3003B^6}{16A^6} + \ldots\ldots \right)$$

$$- \frac{21E_1^9}{2^{11}\pi A^8} \left(1 + \frac{55B^2}{2A^2} + \frac{5005B^4}{24A^4} + \ldots\ldots \right)$$

$$- \frac{165E_1^{11}}{2^{15}\pi A^{10}} \left(1 + \frac{91B^2}{2A^2} + \ldots\ldots \right)$$

$$- \ldots\ldots \qquad (41)$$

$$k_4 = + \left(\frac{d_4}{8} + \frac{3d_6}{16} + \frac{28d_8}{128} + \frac{60d_{10}}{256} + \frac{495d_{12}}{2048} + \ldots\ldots\ldots\ldots \right)$$

$$k_4 = + \frac{5BE_i^4}{16\pi A^4} \left(1 + \frac{7B^2}{6A^2} + \frac{63B^4}{40A^4} + \frac{33B^6}{16A^6} + \frac{1001B^8}{3\cdot 2^7 A^8} + \ldots\ldots \right)$$

$$+ \frac{7BE_i^6}{64\pi A^6} \left(1 + \frac{9B^2}{2A^2} + \frac{99B^4}{8A^4} + \frac{429B^6}{16A^6} + \ldots\ldots\ldots\ldots \right)$$

$$+ \frac{63BE_i^8}{2^{10}\pi A^8} \left(1 + \frac{55B^2}{6A^2} + \frac{1001B^4}{24A^4} + \ldots\ldots\ldots\ldots \right)$$

$$+ \frac{165BE_i^{10}}{2^{12}\pi A^{10}} \left(1 + \frac{49B^2}{33A^2} + \ldots\ldots\ldots\ldots \right)$$

$$+ \ldots\ldots\ldots\ldots \qquad (42)$$

$$k_5 = + \left(\frac{d_5}{16} + \frac{7d_7}{64} + \frac{18d_9}{128} + \frac{165d_{11}}{1024} + \ldots\ldots\ldots\ldots \right)$$

$$k_5 = + \frac{E_i^5}{32\pi A^4} \left(1 + \frac{7B^2}{2A^2} + \frac{63B^4}{8A^4} + \frac{231B^6}{16A^6} + \frac{3003B^8}{2^7 A^8} + \ldots\ldots \right)$$

$$+ \frac{7E_i^7}{768\pi A^6} \left(1 + \frac{27B^2}{2A^2} + \frac{495B^4}{8A^4} + \frac{3003B^6}{16A^6} + \ldots\ldots\ldots\ldots \right)$$

$$+ \frac{9E_i^9}{2^{11}\pi A^8} \left(1 + \frac{55B^2}{2A^2} + \frac{5005B^4}{24A4} + \ldots\ldots\ldots\ldots \right)$$

$$+ \frac{165E_i^{11}}{2^{16}\pi A^{10}} \left(1 + \frac{91B^2}{2A^2} + \ldots\ldots\ldots\ldots \right)$$

$$+ \ldots\ldots\ldots\ldots \qquad (43)$$

$$k_6 = - \left(\frac{d_6}{32} + \frac{8d_8}{128} + \frac{45d_{10}}{512} + \frac{220d_{12}}{2048} + \ldots\ldots\ldots\ldots \right)$$

$$k_6 = -\frac{7BE_1^6}{384\pi A^6}\left(1 + \frac{9B^2}{2A^2} + \frac{99B^4}{8A^4} + \frac{429B^6}{16A^6} + \ldots\ldots\ldots\ldots\right)$$

$$-\frac{9BE_1^8}{2^9\pi A^8}\left(1 + \frac{55B^2}{6A^2} + \frac{1001B^4}{24A^4} + \ldots\ldots\ldots\ldots\right)$$

$$-\frac{495BE_1^{10}}{2^{15}\pi A^{10}}\left(1 + \frac{49B^2}{33A^2} + \ldots\ldots\ldots\ldots\right)$$

$$-\ \ldots\ldots\ldots\ldots\ldots\ldots \qquad (44)$$

$$k_7 = -\left(\frac{d_7}{64} + \frac{9d_9}{256} + \frac{55d_{11}}{1024} + \ldots\ldots\ldots\ldots\right)$$

$$k_7 = -\frac{E_1^7}{768\pi A^6}\left(1 + \frac{27B^2}{2A^2} + \frac{495B^4}{8A^4} + \frac{3003B^6}{16A^6} + \ldots\ldots\ldots\right)$$

$$-\frac{9E_1^9}{2^{13}\pi A^8}\left(1 + \frac{55B^2}{2A^2} + \frac{5005B^4}{24A^4} + \ldots\ldots\ldots\ldots\right)$$

$$-\frac{55E_1^{11}}{2^{16}\pi A^{10}}\left(1 + \frac{91B^2}{2A^2} + \ldots\ldots\ldots\ldots\right)$$

$$-\ \ldots\ldots\ldots\ldots\ldots\ldots \qquad (45)$$

$$k_8 = +\left(\frac{d_8}{128} + \frac{5d_{10}}{256} + \frac{66d_{12}}{2048} + \ldots\ldots\ldots\ldots\right)$$

$$k_8 = +\frac{9BE_1^8}{2^{12}\pi A^8}\left(1 + \frac{55B^2}{6A^2} + \frac{1001B^4}{24A^4} + \ldots\ldots\ldots\ldots\right)$$

$$+\frac{11BE_1^{10}}{2^{14}\pi A^{10}}\left(1 + \frac{49B^2}{33A^2} + \ldots\ldots\ldots\ldots\right)$$

$$+\ \ldots\ldots\ldots\ldots\ldots\ldots \qquad (46)$$

$$k_9 = + \left(\frac{d_9}{256} + \frac{11d_{11}}{1024} + \frac{78d_{13}}{4096} + \dots\dots\dots\dots \right)$$

$$k_9 = + \frac{E_1^9}{2^{13}\pi A^8} \left(1 + \frac{55B^2}{2A^2} + \frac{5005B^4}{24A^4} + \dots\dots\dots \right)$$

$$+ \frac{11E_1^{11}}{2^{16}\pi A^{10}} \left(1 + \frac{91B^2}{2A^2} + \dots\dots\dots\dots \right)$$

$$+ \dots\dots\dots\dots \qquad (47)$$

$$k_{10} = - \left(\frac{d_{10}}{512} + \frac{12d_{12}}{2048} + \dots\dots\dots\dots \right)$$

$$k_{10} = - \frac{11BE_1^{10}}{2^{15}\pi A^{10}} \left(1 + \frac{49B^2}{33A^2} + \dots\dots\dots\dots \right)$$

$$- \dots\dots\dots\dots \qquad (48)$$

$$k_{11} = - \left(\frac{d_{11}}{1024} + \dots\dots\dots\dots \right)$$

$$k_{11} = - \frac{E_1^{11}}{2^{16}\pi A^{10}} \left(1 + \frac{91B^2}{2A^2} + \dots\dots\dots\dots \right)$$

$$\dots\dots\dots\dots \qquad (49)$$

Since k_{11} is seen to be very small compared to k_1, it is not necessary to evaluate further constants. k_0 , k_1 , k_2 , etc... are the constant terms of equation (37) , and they have the dimensions of volts as can be seen from expressions (38) through (49).

If $E_1 = 0$, there is no modulation. For this case, from equations (38) through (49),

$$k_1 = k_2 = k_3 = \dots\dots\dots\dots = k_n = 0$$

and

$$k_o = \frac{4B}{3\pi}\left(1 - \frac{3B^2}{2A^2} + \frac{3B^4}{8A^4} + \frac{B^6}{16A^6} + \frac{3B^8}{2^7A^8} + \frac{3B^{10}}{2^8A^{10}} + \frac{7B^{12}}{2^{10}A^{12}} + \ldots\ldots\right)$$

which is identical with equation (32) for the unmodulated case.

If (A) approaches infinity, the high frequency wave , or the carrier signal, becomes a square wave. Therefore, letting (A) approach infinity, from equation (38) through (49) ,

$$k_o = \frac{4B}{3\pi}$$

$$k_1 = \frac{4E_1}{3\pi}$$

$$k_2 = k_3 = k_4 = \ldots\ldots\ldots = k_n = 0$$

and the formula (37), the second part of the expression for figure (8), reduces to

$$e_3 = \frac{4B}{3\pi}\operatorname{Sin}3x_2 + \frac{4E_1}{3\pi}\operatorname{Sin}x_1 \operatorname{Sin}3x_2$$

This is the same expression obtained when the carrier signal is considered to be a square wave . (See appendix E).

The final expression for figure (8)

$$e = e_1' + e_3 + e_5 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\begin{aligned} e = {} & G_o \operatorname{Sin}w_2t + G_1 \operatorname{Sin}w_1t \operatorname{Sin}w_2t + G_2\operatorname{Cos}2w_1t \operatorname{Sin}w_2t \\ & + G_3 \operatorname{Sin}3w_1t \operatorname{Sin}w_2t + G_4\operatorname{Cos}4w_1t \operatorname{Sin}w_2t \\ & + \ldots\ldots\ldots\ldots\ldots\ldots \\ & + k_o\operatorname{Sin}3w_2t + k_1\operatorname{Sin}w_1t \operatorname{Sin}3w_2t + k_2\operatorname{Cos}2w_1t \operatorname{Sin}3w_2t \\ & + k_3\operatorname{Sin}3w_1t \operatorname{Sin}3w_2t + k_4\operatorname{Cos}4w_1t \operatorname{Sin}3w_2t \\ & + \ldots\ldots\ldots\ldots\ldots \end{aligned} \qquad (50)$$

where the constants C_0 , C_1 , C_2 , etc... have the dimensions of volts, and they are defined with the expressions (22) through (31). The constants k_0, k_1 , k_2 , etc... also have dimensions of volts, and they are given with the relations (38) through (49).

Equation (50) is clearly seen to be an amplitude modulated wave. This equation contains infinite number of frequency components. But with a tank circuit the most desired frequency components may be selected. For instance, if the tank circuit is tuned to the frequency (f_2), equation (50) is approximated by

$$e = C_0 \operatorname{Sin} w_2 t + C_1 \operatorname{Sin} w_1 t \operatorname{Sin} w_2 t$$

or

$$e = C_0 \operatorname{Sin} w_2 t + \frac{C_1}{2} \operatorname{Cos}(w_2 - w_1)t + \frac{C_1}{2} \operatorname{Cos}(w_2 + w_1)t \tag{51}$$

which is the equation for an ideal amplitude modulated wave.

If the tank circuit is tuned to $(3f_2)$, equation (50) is approximated by

$$e = k_0 \operatorname{Sin} 3w_2 t + k_1 \operatorname{Sin} w_1 t \operatorname{Sin} 3w_2 t$$

or

$$e = k_0 \operatorname{Sin} 3w_2 t + \frac{k_1}{2} \operatorname{Cos}(3w_2 - w_1)t + \frac{k_1}{2} \operatorname{Cos}(3w_2 + w_1)t \tag{52}$$

Equation (52) is also an amplitude modulated wave having a carrier frequency of $(3f_2)$.

SECTION 111

EXPERIMENTAL STUDY

Figure (10) is the wiring diagram of the clipper amplitude modulator used in the experiment. It is similar to figure (7) except that a twin diode is employed instead of two single diodes, and the low-impedance paths across the d - c bias batteries, and the secondary of the audio transformer are arranged as shown.

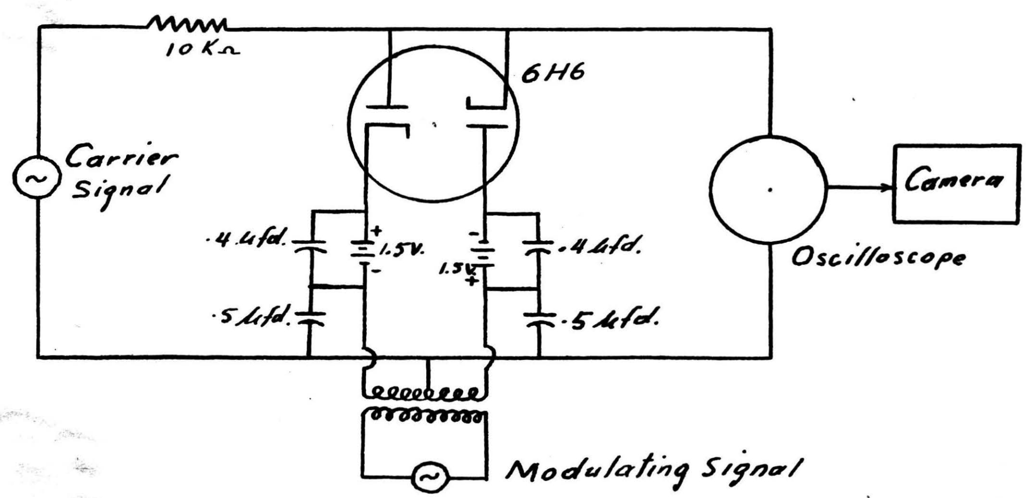


Figure (10). Clipper amplitude modulator, experimental circuit.

Some data on 6H6 twin diodes are given in table 1.

Table 1

For 6H6 twin diode

Plate No. 1 to cathode No.1 Capacitane	3.0 μμ farad
Plate No. 2 to cathode No. 2 Capacitane	3.4 μμ farad
Plate No. 1 to plate No.2 Capacitane	0.1 μμ farad
Forward plate resistance	720 ohms
Backward plate resistance	∞ ohms

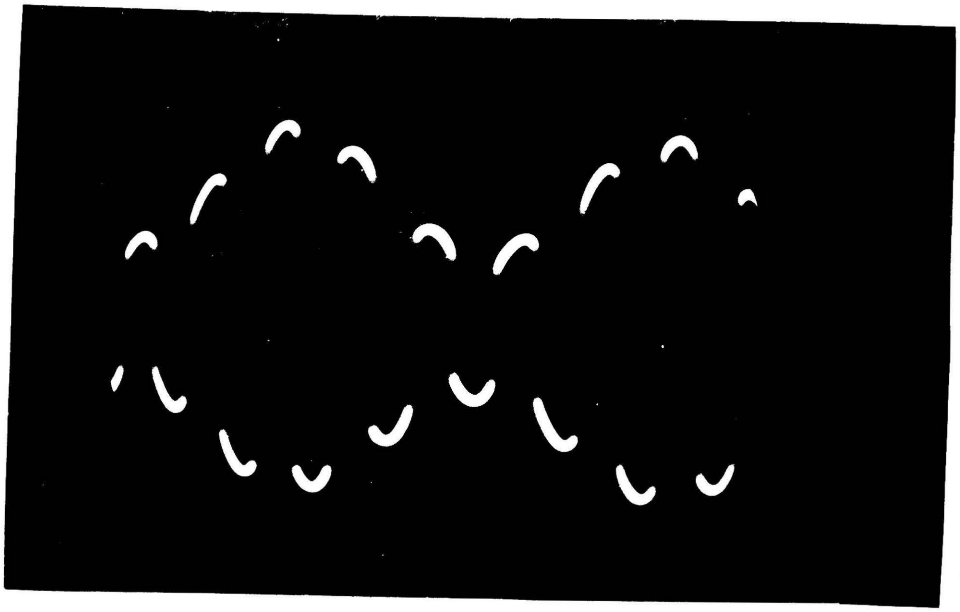


Figure (11). The photograph of a sinusoidally clipped carrier for f_2 = 450 cycles per second and f_1 = 150 cycles per second.

Figure (11) is the photograph of a sinusoidally clipped carrier taken at low frequencies to demonstrate individual radio frequency cycles and the general pattern of clipping level with modulating signal. In this figure, it is seen that the envelope closely approximates a sinusoidal pattern; but the top portion of a radio frequency cycle somewhat deviates from being an exact segment of the modulating signal. This is because the diode has a finite plate resistance while conducting, and the voltage drop across this resistance appears at the output and therefore alters the clipping level slightly.

Figure (12) is the photograph of the envelope of an overmodulated wave.

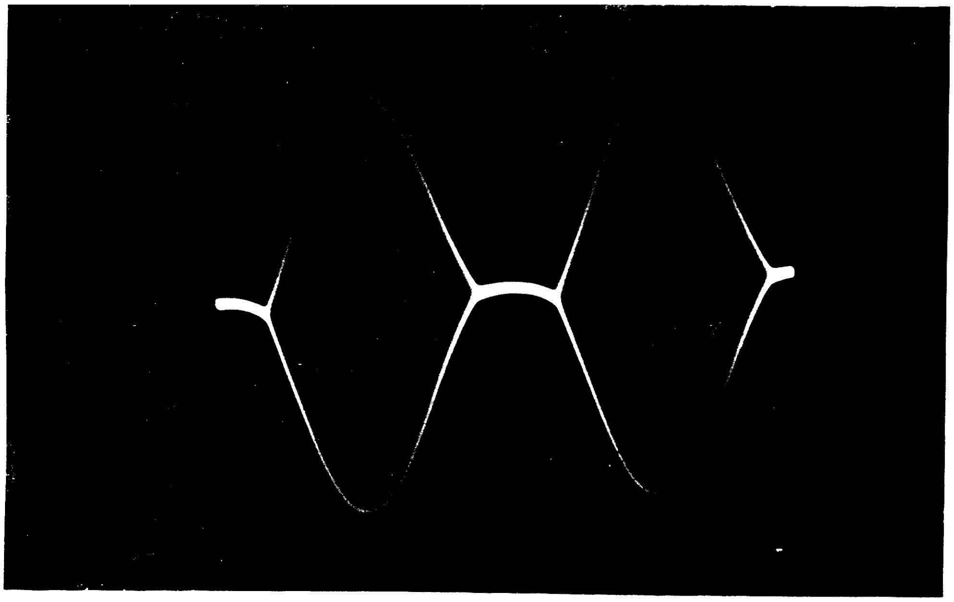


Figure (12). The photograph of the envelope of an overmodulated wave for f_2 = 85 Kc, f_1 = 280 cycles per second.

Figure (13) is also an overmodulated wave for f_2 = 85 Kc and f_1 = 280 cycles per second showing the carrier clipped with a modulating signal. Over modulation exists when $E_1 > B$, where (E_1) is the amplitude of the modulating signal and (B) is the magnitude of the d-c bias. This fact is self explanatory from figure (8).

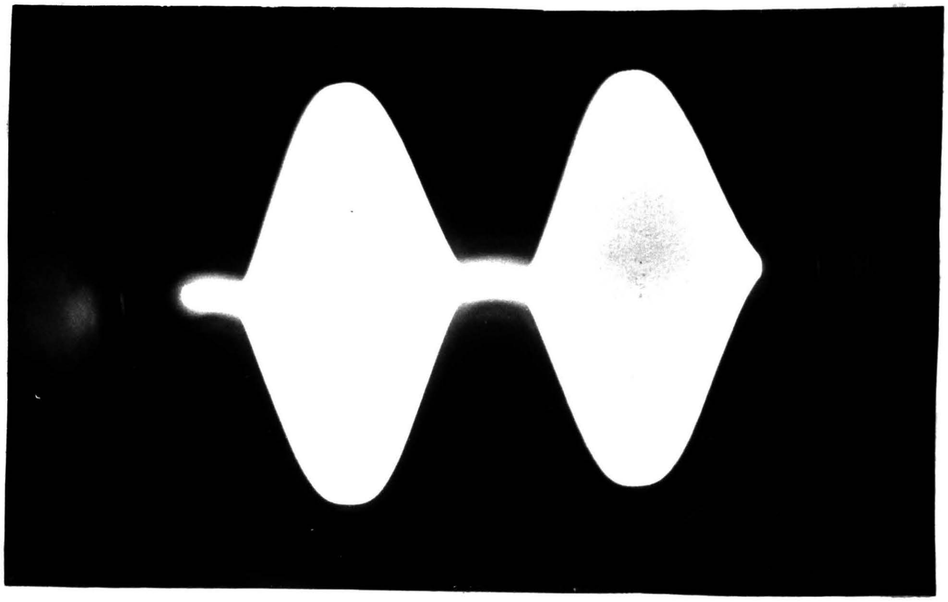


Figure (13). The photograph of an overmodulated wave for f_2 = 85 Kc and f_1 = 280 cycles per second.

Figure (14) shows 100 percent modulation. This occurs when the modulating signal swings to the (wt)-axis of figure (8) on alternate half cycles.

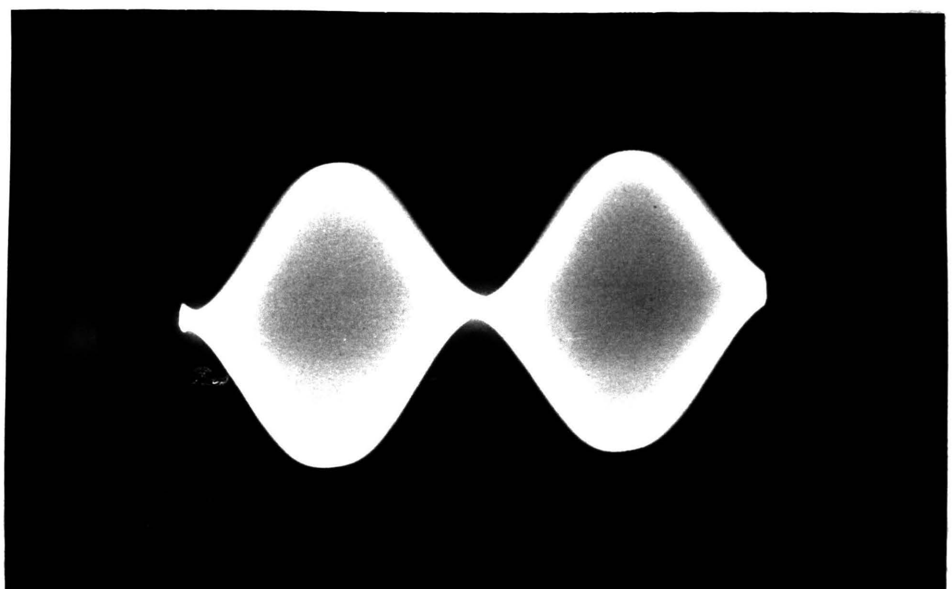


Figure (14). 100 percent modulation, f_2 = 120 Kc and f_1 = 400 cycles per second.

In figure (14) there is a phase shift between the upper and the lower envelopes. This is caused by the fact that the time-varying bias voltages (modulating signal) on the secondary of the center-tapped audio transformer are not 180 degrees out of phase. It has been stated earlier that the total bias on the first diode must be equal to the total bias on the second diode with 180 degrees phase difference to obtain a symmetrical clipping.

Figure (15) is another photograph of the sinusoidally clipped carrier for f_2 = 200 Kc and f_1 = 1000 cycles per second.

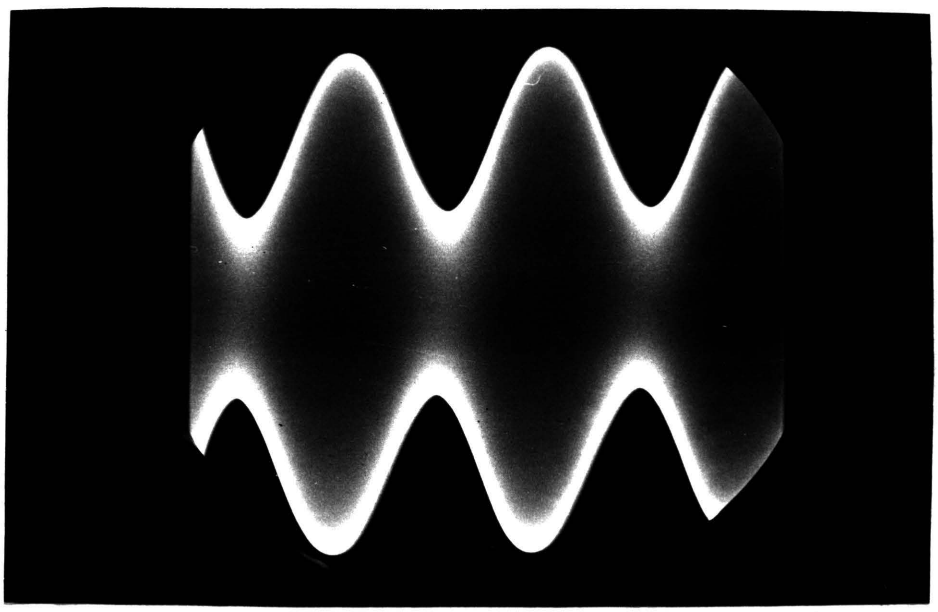


Figure (15). The photograph of a sinusoidally clipped carrier for f_2 = 200 Kc and f_1 = 1000 cycles per second.

In the actual operation of a clipper amplitude modulater a tuned circuit will be present at the output terminals to offer high impedance to the carrier component and less to the unwanted harmonics. This reduces the magnitudes of the unwanted harmonics at the output.

At high freqencies the interelectrode capacitances of a 6H6 twin diode becomes a low impedance path to the carrier frequencies, thereby reducing the output. Consequently, a 6H6 twin diode or any other ordinary diode will not operate satisfactorily at high radio frequency because of capacity effect. Therefore, crystal rectifiers must be substituted for vacuum tube diodes at these high frequencies; for instance, General Electric, type G-7 crystal diode is designed to operate up to 500 Mc.

CONCLUSIONS

It is possible to obtain an amplitude modulated wave by the clipping action of diodes, provided the clipping level, or the bias of figure (7), varies with the modulating signal. The resulting wave form contains an infinite number of frequency components from which the desired ones can be selected by a tuned circuit.

Some characteristics of a clipper amplitude modulator can be compared with the conventional vacuum tube amplitude modulators as follows:

a) The conventional vacuum tubes such as triodes and pentodes have constant amplification characteristics only over a certain range of frequencies, called the mid-frequency range. Those frequency components of the input signal falling on either side of the mid-frequency range are reduced in amplitude and shifted in phase, introducing frequency and phase distortion. The clipper amplitude modulator can be made free from frequency and phase distortion within the frequency range of operation of the diodes.

b) If crystal rectifiers are employed instead of ordinary diodes, amplitude modulation can be accomplished at much higher frequencies than when conventional vacuum tube modulators are used. For instance, General Electric, type G - 7 crystal diode is designed for use up to 500 Mc.

c) The output of a clipper amplitude modulator contains an infinite number of frequency components, thereby introducing amplitude distortion. But this is not serious since the undesired frequency components can be reduced to insignificance by a tuned circuit.

d) Clipper amplitude modulator has an inexpensive and simple circuit. It is also small, compact, and light.

SUMMARY

Amplitude modulation is obtained by varying the amplitude of a high frequency signal (carrier) in accordance with an intelligence. There are numerous systems producing an amplitude modulated wave. In this thesis, another system and method is introduced. Namely, an amplitude modulated wave is obtained by clipping the carrier with a modulating signal by employing the clipping action of a full-wave diode clipper. Consequently, the system may be called a "clipper amplitude modulator". The mathematical analysis and the experimental results have shown that the output wave form of a clipper amplitude modulator is an amplitude modulated wave.

Ordinary vacuum tube diodes do not operate satisfacorily at frequencies above 800 Kc. Therefore at high frequencies crystal rectifiers must be used.

The clipper amplitude modulator has some advantages over the modulators using conventional triodes or pentodes. Distortion can be made very small in the clipper amplitude modulator. It is small in physical size, and it has a simple circuit.

APPENDIX- A

Evaluation of Sin(na) and Cos(na) for figure (9):

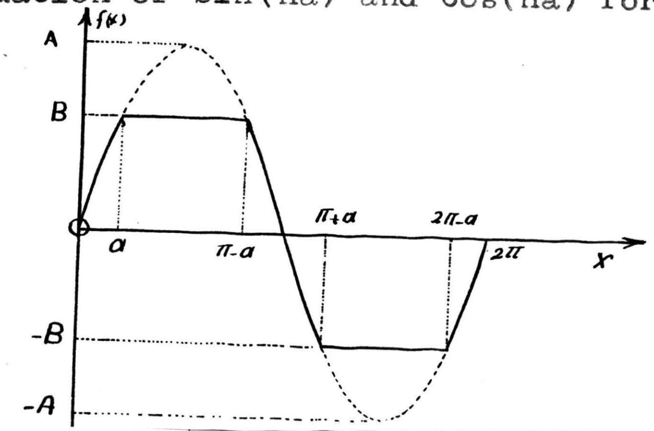


Figure (9). Sinusoidal carrier clipped at the constant level (B).

At the point where x= a , B = A Sina

or

$$\text{Sina} = \frac{B}{A}$$

$$\text{Cosa} = \frac{\sqrt{A^2-B^2}}{A}$$

Therefore,

$$\text{Sin2a} = 2\ \text{Sina Cosa}$$

$$\text{Sin2a} = \frac{2B}{A}\cdot\frac{\sqrt{A^2-B^2}}{A}$$

$$\text{Sin4a} = 2\text{Sin3a Cosa} - \text{Sin2a}$$

$$\text{Sin4a} = 2(3\ \text{Sina} - 4\text{Sin}^3\text{a})\text{Cosa} - \text{Sin2a}$$

$$\text{Sin4a} = 2\left(3\frac{B}{A} - 4\frac{B^3}{A^3}\right)\frac{\sqrt{A^2-B^2}}{A} - \frac{2B}{A}\frac{\sqrt{A^2-B^2}}{A}$$

$$\text{Sin4a} = \left(\frac{4B}{A} - \frac{8B^3}{A^3}\right)\frac{\sqrt{A^2-B^2}}{A}$$

$$\text{Cos3a} = 4\text{Cos}^3\text{a} - 3\text{Cosa}$$

$$\text{Cos3a} = 4\left(\frac{\sqrt{A^2-B^2}}{A}\right)^3 - \frac{3\sqrt{A^2-B^2}}{A}$$

$$\text{Cos3a} = \left(1 - \frac{4B^2}{A^2}\right)\frac{\sqrt{A^2-B^2}}{A}$$

$$\mathrm{Cos}5a = 2\mathrm{Cos}4a\ \mathrm{Cos}a - \mathrm{Cos}3a$$

$$\mathrm{Cos}5a = 2(2\mathrm{Cos}3a\ \mathrm{Cos}a - \mathrm{Cos}2a)\mathrm{Cos}a - \mathrm{Cos}3a$$

$$\mathrm{Cos}5a = 4\mathrm{Cos}3a\ \mathrm{Cos}^2a - 2(1 - 2\mathrm{Sin}^2a)\mathrm{Cos}a - \mathrm{Cos}3a$$

$$\mathrm{Cos}5a = 4\mathrm{Cos}3a\ \mathrm{Cos}^2a - 2\mathrm{Cos}a + 4\mathrm{Sin}^2a\ \mathrm{Cos}a - \mathrm{Cos}3a$$

$$\mathrm{Cos}5a = 4\left(1 - \frac{4B^2}{A^2}\right)\frac{\sqrt{A^2-B^2}}{A}\left(\frac{\sqrt{A^2-B^2}}{A}\right)^2 - \frac{2\sqrt{A^2-B^2}}{A} + \frac{4B^2}{A^2}\frac{\sqrt{A^2-B^2}}{A}$$

$$-\left(1 - \frac{4B^2}{A^2}\right)\frac{\sqrt{A^2-B^2}}{A}$$

$$\mathrm{Cos}5a = \left(1 - \frac{12B^2}{A^2} + \frac{16B^4}{A^4}\right)\frac{\sqrt{A^2-B^2}}{A}$$

$$\mathrm{Sin}6a = 2\ \mathrm{Sin}5a\ \mathrm{Cos}a - \mathrm{Sin}4a$$

$$\mathrm{Sin}6a = 2(2\mathrm{Sin}4a\ \mathrm{Cos}a - \mathrm{Sin}3a)\mathrm{Cos}a - \mathrm{Sin}4a$$

$$\mathrm{Sin}6a = 4\ \mathrm{Sin}4a\ \mathrm{Cos}^2a - 2\ \mathrm{Sin}3a\ \mathrm{Cos}a - \mathrm{Sin}4a$$

$$\mathrm{Sin}6a = 4\ \mathrm{Sin}4a\ \mathrm{Cos}^2a - 2(3\mathrm{Sin}a - 4\mathrm{Sin}^3a)\mathrm{Cos}a - \mathrm{Sin}4a$$

$$\mathrm{Sin}6a = 4\ \mathrm{Sin}4a\ \mathrm{Cos}^2a - 6\ \mathrm{Sin}a\ \mathrm{Cos}a + 8\mathrm{Sin}^3a\ \mathrm{Cos}a - \mathrm{Sin}4a$$

$$\mathrm{Sin}6a = 4\left(\frac{4B}{A} - \frac{8B^3}{A^3}\right)\frac{\sqrt{A^2-B^2}}{A}\left(\frac{\sqrt{A^2-B^2}}{A}\right)^2 - \frac{6B}{A}\frac{\sqrt{A^2-B^2}}{A}$$

$$+\frac{8B^3}{A^3}\frac{\sqrt{A^2-B^2}}{A} - \left(\frac{4B}{A} - \frac{8B^3}{A^3}\right)\frac{\sqrt{A^2-B^2}}{A}$$

$$\mathrm{Sin}6a = \left(\frac{6B}{A} - \frac{32B^3}{A^3} + \frac{32B^5}{A^5}\right)\frac{\sqrt{A^2-B^2}}{A}$$

APPENDIX - B

The following relations are to be used in expanding equation (8).

$$\text{ArcSin}\,x = x + \frac{x^3}{6} + \frac{3x^5}{40} + \frac{5x^7}{112} + \frac{35x^9}{1152} + \frac{63x^{11}}{2816} + \frac{231x^{13}}{13312} + \frac{143x^{15}}{10240} + \ldots\ldots \qquad x^2 < 1$$

$$\text{ArcSin}\left(\frac{B}{A}\right) = \frac{B}{A} + \frac{B^3}{6A^3} + \frac{3B^5}{40A^5} + \frac{5B^7}{112A^7} + \frac{35B^9}{1152A^9} + \frac{63B^{11}}{2816A^{11}} + \frac{231B^{13}}{13312A^{13}} + \frac{143B^{15}}{10240A^{15}} + \ldots\ldots\ldots \qquad \left(\frac{B}{A}\right)^2 < 1$$

$$(1+X)^m = 1 + mx + \frac{m(m-1)}{2}x^2 + \frac{m(m-1)(m-2)}{3}x^3 + \ldots\ldots\ldots\ldots + \frac{m(m-1)(m-2)\ldots\ldots(m-n+1)}{n}x^n \qquad x < 1$$

Letting $x = \left(-\frac{B^2}{A^2}\right)$ *and* $m = \frac{1}{2}$, *and* $m = \frac{3}{2}$

$$\left(1 - \frac{B^2}{A^2}\right)^{\frac{1}{2}} = 1 - \frac{B^2}{2A^2} - \frac{B^4}{8A^4} - \frac{B^6}{16A^6} - \frac{5B^8}{128A^8} - \frac{7B^{10}}{2^8A^{10}} - \frac{21B^{12}}{2^{10}A^{12}} - \frac{33B^{14}}{2^{11}A^{14}} \ldots\ldots$$

$$\left(1 - \frac{B^2}{A^2}\right)^{\frac{3}{2}} = 1 - \frac{3B^2}{2A^2} + \frac{3B^4}{2^3A^4} + \frac{B^6}{2^4A^6} + \frac{3B^8}{2^7A^8} + \frac{3B^{10}}{2^8A^{10}} + \frac{7B^{12}}{2^{10}A^{12}} + \frac{9B^{14}}{2^{11}A^{14}} \ldots\ldots$$

APPENDIX - C

EXPANSION OF THE TERM $(B + E\ SINX)^n$

Bionomial Theorem;

$$(a+b)^n = a^n + na^{n-I}b + \frac{n(n-I)}{2}a^{n-2}b^2 + \frac{n(n-I)(n-2)}{3}a^{n-3}b^3 + \ldots\ldots + nab^{n-I} + b^n$$

where (n) is any positive integer.

$$(B+E\ Sinx)^2 = B^2 + 2BE\ Sinx + E^2\ Sin^2x$$

$$(B+E\ Sinx)^3 = B^3 + 3B^2E\ Sinx + 3BE^2\ Sin^2x + E^3\ Sin^3x$$

$$(B+E\ Sinx)^4 = B^4 + 4B^3E\ Sinx + 6B^2E^2\ Sin^2x + 4BE^3\ Sin^3x + E^4\ Sin^4x$$

$$(B+E\ Sinx)^5 = B^5 + 5B^4E\ Sinx + IOB^3E^2\ Sin^2x + IOB^2E^3\ Sin^3x + 5BE^4\ Sin^4x + E^5\ Sin^5x$$

$$(B+E\ Sinx)^6 = B^6 + 6B^5E\ Sinx + I5B^4E^2\ Sin^2x + 20B^3E^3\ Sin^3x + I5B^2E^4\ Sin^4x + 6BE^5\ Sin^5x + E^6Sin^6x$$

$$(B+E\ Sinx)^7 = B^7 + 7B^6E\ Sinx + 2IB^5E^2\ Sin^2x + 35B^4E^3\ Sin^3x\ 35B^3E^4\ Sin^4x + 2IB^2E^5\ Sin^5x + 7BE^6\ Sin^6x + E^7\ Sin^7x$$

$$(B+E\ Sinx)^8 = B^8 + 8B^7E\ Sinx + 28B^6E^2\ Sin^2x + 56B^5E^3\ Sin^3x + 70B^4E^4\ Sin^4x + 56B^3E^5\ Sin^5x + 28B^2E^6\ Sin^6x + 8BE^7\ Sin^7 + E^8\ Sin^8x$$

$$(B+E\ Sinx)^9 = B^9 + 9B^8E\ Sinx + 36B^7E^2\ Sin^2x + 84B^6E^3\ Sin^3x + I26B^5E^4\ Sin^4x + I26B^4E^5\ Sin^5x + + 84B^3E^6\ Sin^6x + 36B^2E^7\ Sin^7x + 9BE^8\ Sin^8x + E^9\ Sin^9x$$

$$(B+E\ \sin x)^{10} = B^{10} + 10B^{9}E\ \sin x + 45B^{8}E^{2}\ \sin^{2}x$$
$$+ 120B^{7}E^{3}\ \sin^{3}x + 210B^{6}E^{4}\ \sin^{4}x$$
$$+ 252B^{5}E^{5}\ \sin^{5}x + 210B^{4}E^{6}\ \sin^{6}x$$
$$+ 120B^{3}E^{7}\ \sin^{7}x + 45B^{2}E^{8}\ \sin^{8}x$$
$$+ 10BE^{9}\ \sin^{9}x + E^{10}\ \sin^{10}x$$

$$(B+E\ \sin x)^{11} = B^{11} + 11B^{10}E\ \sin x + 55B^{9}E^{2}\ \sin^{2}x$$
$$+ 165B^{8}E^{3}\ \sin^{3}x + 330B^{7}E^{4}\ \sin^{4}x$$
$$+ 462B^{6}E^{5}\ \sin^{5}x + 462B^{5}E^{6}\ \sin^{6}x$$
$$+ 330B^{4}E^{7}\ \sin^{7}x + 165B^{3}E^{8}\ \sin^{8}x$$
$$+ 55B^{2}E^{9}\ \sin^{9}x + 11BE^{10}\ \sin^{10}x$$
$$+ E^{11}\ \sin^{11}x$$

$$(B+E\ \sin x)^{12} = B^{12} + 12B^{11}E\ \sin x + 66B^{10}E^{2}\ \sin^{2}x$$
$$+ 220B^{9}E^{3}\ \sin^{3}x + 495B^{8}E^{4}\ \sin^{4}x$$
$$+ 792B^{7}E^{5}\ \sin^{5}x + 924B^{6}E^{6}\ \sin^{6}x$$
$$+ 792B^{5}E^{7}\ \sin^{7}x + 495B^{4}E^{8}\ \sin^{8}x$$
$$+ 220B^{3}E^{9}\ \sin^{9}x + 66B^{2}E^{10}\ \sin^{10}x$$
$$+ 12BE^{11}\ \sin^{11}x + E^{12}\ \sin^{12}x$$

$$(B+E\ \sin x)^{13} = B^{13} + 13B^{12}E\ \sin x + 78B^{11}E^{2}\ \sin^{2}x$$
$$+ 286B^{10}E^{3}\ \sin^{3}x + 715B^{9}E^{4}\ \sin^{4}x$$
$$+ 1278B^{8}E^{5}\ \sin^{5}x + 1716B^{7}E^{6}\ \sin^{6}x$$
$$+ 1716B^{6}E^{7}\ \sin^{7}x + 1287B^{5}E^{8}\ \sin^{8}x$$
$$+ 715B^{4}E^{9}\ \sin^{9}x + 286B^{3}E^{10}\sin^{10}x$$
$$+ 78B^{2}E^{11}\ \sin^{11}x + 13BE^{12}\ \sin^{12}x$$
$$+ E^{13}\ \sin^{13}x$$

$$
\begin{aligned}
(B+E\ \mathrm{Sin}x)^{14} = B^{14} &+ 14B^{13}E\ \mathrm{Sin}x + 91B^{12}E^{2}\ \mathrm{Sin}^{2}x \\
&+ 364B^{11}E^{3}\ \mathrm{Sin}^{3}x + 1001B^{10}E^{4}\ \mathrm{Sin}^{4}x \\
&+ 2002B^{9}E^{5}\ \mathrm{Sin}^{5}x + 3003B^{8}E^{6}\ \mathrm{Sin}^{6}x \\
&+ 3432B^{7}E^{7}\ \mathrm{Sin}^{7}x + 3003B^{6}E^{8}\ \mathrm{Sin}^{8}x \\
&+ 2002B^{5}E^{9}\ \mathrm{Sin}^{9}x + 1001B^{4}E^{10}\ \mathrm{Sin}^{10}x \\
&+ 364B^{3}E^{11}\ \mathrm{Sin}^{11}x + 91B^{2}E^{12}\ \mathrm{Sin}^{12}x \\
&+ 14BE^{13}\ \mathrm{Sin}^{13}x + E^{14}\ \mathrm{Sin}^{14}x
\end{aligned}
$$

APPENDIX - D

TRIGONOMETRIC IDENTITIES OF $(\mathrm{Sin}x)^n$

$$\mathrm{Sin}^2x = \frac{1}{2} - \frac{1}{2}\mathrm{Cos}2x$$

$$\mathrm{Sin}^3x = \frac{3}{4}\mathrm{Sin}x - \frac{1}{4}\mathrm{Sin}3x$$

$$\mathrm{Sin}^4x = \frac{3}{8} - \frac{1}{2}\mathrm{Cos}2x + \frac{1}{8}\mathrm{Cos}4x$$

$$\mathrm{Sin}^5x = \mathrm{Sin}^2x\ \mathrm{Sin}^3x$$

$$\mathrm{Sin}^5x = \frac{3}{8}\mathrm{Sin}x - \frac{1}{2}\mathrm{Sin}x\ \mathrm{Cos}2x + \frac{1}{8}\mathrm{Sin}x\ \mathrm{Cos}4x$$

$$\mathrm{Sin}^5x = \frac{3}{8}\mathrm{Sin}x - \frac{1}{2}\left(\frac{1}{2}\mathrm{Sin}3x + \frac{1}{2}\mathrm{Sin}(-x)\right) + \frac{1}{8}\left(\frac{1}{2}\mathrm{Sin}5x - \frac{1}{2}\mathrm{Sin}3x\right)$$

$$\mathrm{Sin}^5x = \frac{5}{8}\mathrm{Sin}x - \frac{5}{16}\mathrm{Sin}3x + \frac{1}{16}\mathrm{Sin}5x$$

$$\mathrm{Sin}^6x = \mathrm{Sin}^5x\ \mathrm{Sin}x$$

$$\mathrm{Sin}^6x = \frac{5}{8}\mathrm{Sin}^2x - \frac{5}{16}\mathrm{Sin}3x\ \mathrm{Sin}x + \frac{1}{16}\mathrm{Sin}5x\ \mathrm{Sin}x$$

$$\mathrm{Sin}^6x = \frac{5}{8}\left(\frac{1}{2} - \frac{1}{2}\mathrm{Cos}2x\right) - \frac{5}{16}\left(\frac{1}{2}\mathrm{Cos}2x - \frac{1}{2}\mathrm{Cos}4x\right) + \frac{1}{16}\left(\frac{1}{2}\mathrm{Cos}4x - \frac{1}{2}\mathrm{Cos}6x\right)$$

$$\mathrm{Sin}^6x = \frac{5}{16} - \frac{15}{32}\mathrm{Cos}2x + \frac{3}{16}\mathrm{Cos}4x - \frac{1}{32}\mathrm{Cos}6x$$

$$\mathrm{Sin}^7x = \mathrm{Sin}^6x\ \mathrm{Sin}x$$

$$\mathrm{Sin}^7x = \frac{5}{16}\mathrm{Sin}x - \frac{15}{32}\mathrm{Sin}x\ \mathrm{Cos}2x + \frac{3}{16}\mathrm{Sin}x\ \mathrm{Cos}4x - \frac{1}{32}\mathrm{Sin}x\mathrm{Cos}6x$$

$$\mathrm{Sin}^7x = \frac{35}{64}\mathrm{Sin}x - \frac{21}{64}\mathrm{Sin}3x + \frac{7}{64}\mathrm{Sin}5x - \frac{1}{64}\mathrm{Sin}7x$$

$$\mathrm{Sin}^8x = \mathrm{Sin}^7x\ \mathrm{Sin}x$$

$$\mathrm{Sin}^8x = \frac{35}{64}\mathrm{Sin}^2x - \frac{21}{64}\mathrm{Sin}3x\ \mathrm{Sin}x + \frac{7}{64}\mathrm{Sin}5x\ \mathrm{Sin}x - \frac{1}{64}\mathrm{Sin}7x\mathrm{Sin}x$$

$$\mathrm{Sin}^8x = \frac{35}{128} - \frac{56}{128}\mathrm{Cos}2x + \frac{28}{128}\mathrm{Cos}4x - \frac{8}{128}\mathrm{Cos}6x + \frac{1}{128}\mathrm{Cos}8x$$

$$\sin^9 x = \sin^8 x \cdot \sin x$$

$$\sin^9 x = \frac{35}{128}\sin x - \frac{56}{128}\sin x\cos 2x + \frac{28}{128}\sin x\cos 4x$$
$$- \frac{8}{128}\sin x\cos 6x + \frac{1}{128}\sin x\cos 8x$$

$$\sin^9 x = \frac{1}{128}\left(63\sin x - 42\sin 3x + 18\sin 5x - \frac{9}{2}\sin 7x + \frac{1}{2}\sin 9x\right)$$

$$\sin^{10} x = \frac{1}{128}\left(63\sin x - 42\sin 3x + 18\sin 5x - \frac{9}{2}\sin 7x + \frac{1}{2}\sin 9x\right)\sin x$$

$$\sin^{10} x = \frac{1}{256}\left(63 - 105\cos 2x + 60\cos 4x - \frac{45}{2}\cos 6x + 5\cos 8x\right.$$
$$\left. - \frac{1}{2}\cos 10x\right)$$

$$\sin^{11} x = \frac{1}{256}\left(63 - 105\cos 2x + 60\cos 4x - \frac{45}{2}\cos 6x + 5\cos 8x\right.$$
$$\left. - \frac{1}{2}\cos 10x\right)\sin x$$

$$\sin^{11} x = \frac{1}{1024}\left(462\sin x - 330\sin 3x + 165\sin 5x - 55\sin 7x\right.$$
$$\left. + 11\sin 9x - \sin 11x\right)$$

$$\sin^{12} x = \frac{1}{1024}\left(462\sin x - 330\sin 3x + 165\sin 5x - 55\sin 7x\right.$$
$$\left. + 11\sin 9x - \sin 11x\right)\sin x$$

$$\sin^{12} x = \frac{1}{2048}\left(462 - 792\cos 2x + 495\cos 4x - 220\cos 6x\right.$$
$$\left. + 66\cos 8x - 12\cos 10x + \cos 12x\right)$$

$$\sin^{13} x = \frac{1}{2048}\left(462 - 792\cos 2x + 495\cos 4x - 220\cos 6x\right.$$
$$\left. + 66\cos 8x - 12\cos 10x + \cos 12x\right)\sin x$$

$$\sin^{13} x = \frac{1}{4096}\left(1716\sin x - 1287\sin 3x + 715\sin 5x - 286\sin 7x\right.$$
$$\left. + 78\sin 9x - 13\sin 11x + \sin 13x\right)$$

Appendix E

MATHEMATICAL ANALYSIS OF A SQUARE-WAVE CARRIER CLIPPED WITH A SINUSOIDAL MODULATING SIGNAL:

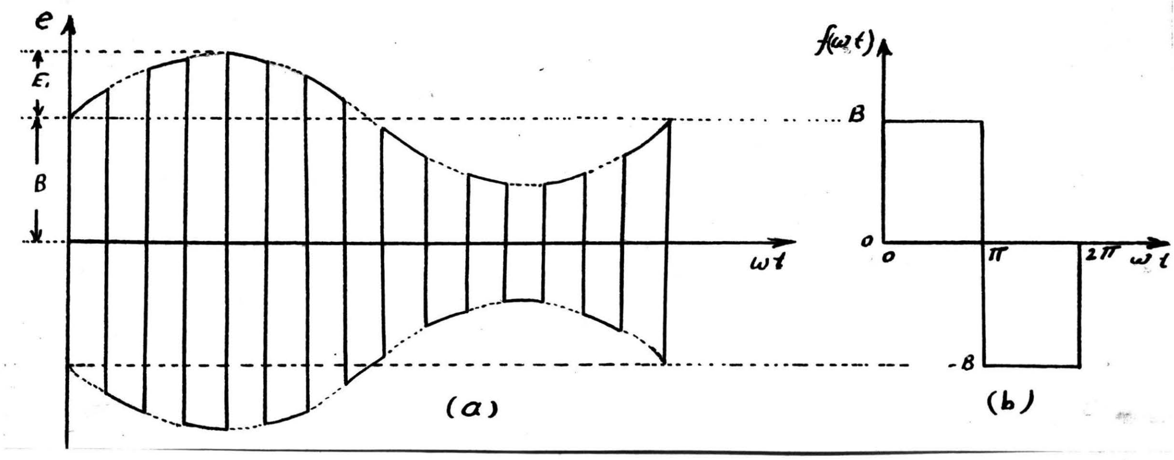


Figure (16). a) Sinusoidally clipped square- wave carrier, b) square wave-carrier clipped at the constant level (B)

$e_1 = E_1 \sin w_1 t$, modulating signal as expressed with respect to the constant level $e = B$

$w_2 t$ = angular frequency of the square-wave carrier.

Analysis of figure (b) by Fourier methods gives

$$f_{(w_2 t)} = \frac{4B}{\pi} \sin w_2 t + \frac{4B}{3\pi} \sin 3w_2 t + \frac{4B}{5\pi} \sin 5w_2 t + \ldots\ldots$$

Therefore the coefficients or the amplitudes are

$$b_1 = \frac{4B}{\pi} \quad , \quad b_2 = \frac{4B}{3\pi} \quad , \quad b_3 = \frac{4B}{5\pi} \quad , \quad \ldots\ldots$$

Introducing sine variation in the amplitudes

$$e_1' = \frac{4}{\pi} (B + E_1 \sin w_1 t) \sin w_2 t$$

$$e_1' = \frac{4B}{\pi} \sin w_2 t + \frac{4E_1}{\pi} \sin w_1 t \sin w_2 t$$

$$e_3 = \frac{4}{3\pi} (B + E_1 \ \mathrm{Sin} w_1 t) \ \mathrm{Sin} 3w_2 t$$

$$e_3 = \frac{4B}{3\pi} \mathrm{Sin} 3w_2 t + \frac{4E'}{3\pi} \mathrm{Sin} w_1 t \ \mathrm{Sin} 3w_2 t$$

$$e_5 = \frac{4}{5\pi} (B + E_1 \ \mathrm{Sin} w_1 t)$$

$$e_5 = \frac{4B}{5\pi} \mathrm{Sin} 5w_2 t + \frac{4E'}{5\pi} \mathrm{Sin} w_1 t \ \mathrm{Sin} 5w_2 t$$

Finally,

$$e = e_1' + e_3 + e_5 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$e = \frac{4B}{\pi} \mathrm{Sin} w_2 t + \frac{4E'}{\pi} \mathrm{Sin} w_1 t \ \mathrm{Sin} w_2 t$$

$$+ \frac{4B}{3\pi} \mathrm{Sin} 3w_2 t + \frac{4E'}{3\pi} \mathrm{Sin} w_1 t \ \mathrm{Sin} 3w_2 t$$

$$+ \frac{4B}{5\pi} \mathrm{Sin} 5w_2 t + \frac{4E'}{5\pi} \mathrm{Sin} 5w_2 t \ \mathrm{Sin} w_1 t$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

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VITA

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