1965

A study of radiation heat transfer in rectangular ducts and enclosures

Jenn-Wuu Ou

A STUDY OF RADIATION HEAT TRANSFER IN RECTANGULAR DUCTS AND ENCLOSURES

BY

JENN-WUU OU

A

THESIS

submitted to the faculty of the

UNIVERSITY OF MISSOURI AT ROLLA

in partial fulfillment of the work required for the

Degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

Rolla, Missouri

1965

Approved by

John E. Francis (Advisor) H. J. Sauer

Elton L. Park Jr. R. A. Schaefer

ABSTRACT

The purpose of this study is to determine the radiant heat transfer in rectangular configurations and construct a general computer program. Two specific cases are studied in this work.

The first case is a rectangular duct with openings at each end. The duct is separated into two sections which are called the source and the sink respectively. A linear temperature profile is imposed on the source section. The energy loss of both the source and the sink is investigated.

The second case is a complete rectangular enclosure. The two end plates are called the source and the sink respectively. A linear temperature profile is imposed on the duct like section between the two plates. The energy loss of both the source and the sink is investigated.

The method of analysis is Gebhart's unified method. The computer programs are as generalized as possible. Each program contains two main parts: (A) The evaluation of the configuration factors between any two surfaces in the enclosure. (B) The evaluation of the radiant energy loss of the source and the sink.

TABLE OF CONTENTS

LIST OF FIGURES

Figure Page

LIST OF TABLES

LIST OF PLATES

ACKNOWLEDGEMENT

The author wishes to express special acknowledgement to his professor, Dr. John E. Francis, for his guidance and suggestions which made this investigation possible. The author also wishes to thank the members of the Computer Science Center for their assistance.

INTRODUCTION AND REVIEW OF LITERATURE

Much work has been done in the field of thermal radiation. Hamilton and Morgan (1)* first developed the configuration factors for many cases and stated the configuration factor algebra. The analysis of radiant heat transfer has been presented using approaches, such as: (A) Hottel (2) introduced a method, by which the equivalent shape factor can be solved and the radiant heat transfer then determined. (B) The method originally proposed by Poljak and later refined by Oppenheim (3) is called "Radiation Analysis by Network". This method makes use of the analogy between radiation interchange and electrical circuits. (C) Ishimoto and Bevans (4) presented a method using the "Script F" in their paper. This method states that the net exchange between two surfaces in an enclosure must be of the form $\sigma(T_1^4-T_2^4)$ multiplied by a factor $\mathcal{F}_{12}$ (called script F) which is solved by a matrix solution. (D) The method developed by Gebhart (5) makes use of determinants and introduces the so termed absorption factors and uses certain relations to reduce the amount of labor required in obtaining a numerical solution for the rate of heat transfer to or from a given surface.

This study is concerned with the radiant heat transfer in both a rectangular duct and a rectangular enclosure. The

---

* Numbers in parentheses refer to Bibliography.

open rectangular duct is separated into two sections, the source and the sink. The temperature of the source is changing from T_1 to T_2 in a linear fashion and the temperature of the sink is a uniform value given by T_3. The complete rectangular enclosure or box has temperatures of T_1 and T_2 at either end. The duct like section between these ends has a linear temperature profile varying from the end temperatures.

Since the radiation properties are dependent upon the temperature distribution and the temperature along a wall may not be uniform, the method of numerical analysis must be used to approximate the radiant heat transfer. That is, the wall with varying temperature must be divided into several sections, the solution is then based on Gebhart's unified method for radiation-exchange calculations. In solving the simultaneous equations, the Gauss-Jordan reduction method is used.

The configuration factors between any two surfaces are evaluated by means of "configuration factor algebra". The two special classes of configuration factors used are: (A) The configuration factors for finite, perpendicular rectangles having a common edge. (B) The configuration factors for finite parallel, opposed rectangles.

The emissivities corresponding to the temperatures are approximated by using the Lagrange interpolation formula. All of the calculations were performed with the aid of an IBM 1620 Model II Digital Computer.

DISCUSSION

PART I: GENERAL CONCEPTION

1. ASSUMPTIONS:

In this analysis the following assumptions are made for convenience in solving the problem:

(A) The condition of steady state has been assumed, i.e., all conditions are independent of time.

(B) Conduction along the wall of the duct and convection in the duct are neglected, only radiation is considered.

(C) The temperature profile down the wall of the duct is assumed to be a straight line. The temperatures of both ends of the duct are equal to the values of the ends of the temperature profile, respectively.

(D) In the calculations the mean temperature is used and is based on the assumption that the temperature is uniform over the entire section concerned.

(E) The emissivity and reflectivity depend upon the mean temperature.

Other assumptions will be made in the following discussion.

2. FUNDAMENTALS OF THERMAL RADIATION

Stefan-Boltzmann established a law that the energy density of the radiation is proportional to absolute temperature to the fourth power:

$$E_b = \alpha T^4, \tag{1-1}$$

where E_b is the energy radiated per unit time and per unit area by an ideal radiator, i.e., a black body. α is the Stefan-Boltzmann constant. The value of α is 0.1714×10^{-8} Btu/hr-ft^2-$^\circ$R^4, when E_b is in Btu per hour per square foot, and T is in degrees Rankine. For a gray body the emissive power is:

$$E = \epsilon \alpha T^4, \tag{1-2}$$

where ϵ is the emissivity.

When radiant energy strikes the surface of a material, part of the radiation is reflected, part is absorbed, and part is transmitted, then

$$q_i = q_r + q_a + q_\tau$$

or $$1 = q_r/q_i + q_a/q_i + q_\tau/q_i = r + a + \tau$$

where the fraction r is reflectivity, a is absorptivity, and τ is transmissivity. Many solid materials do not transmit thermal radiation, for the case

$$r + a = 1, \tag{1-3}$$

Another useful tool was developed by Kirchhoff. His identity shows that

$$\epsilon = a, \tag{1-4}$$

when the system is in thermodynamic equilibrium.

3. THE RADIATION SHAPE FACTOR

Consider two finite black surfaces A_1 and A_2 which are in view of each other. The energy exchange between these surfaces, when they are maintained at different temperatures, depends on the spatial arrangement of the surfaces. Hence the shape or configuration factor is instrumental in the analysis.

The configuration factor from A_1 to A_2, written F_{12}, may be defined as the fraction of the total radiant energy leaving surface A_1 which is incident upon surface A_2. The general expression, F_{mn} is defined as the fraction of energy leaving surface A_m that is incident upon surface A_n. The limiting values are then zero and unity.

The configuration factor is a function of the geometry of the two surfaces A_1 and A_2 and depends on the directional distribution of the radiant emission. The emission has been assumed to follow Lambert's cosine law. This law states that the intensity, the radiant energy emitted per unit time per unit solid angle subtended at emitting element, is a constant throughout the half-space above the emitting element. This law implies that the radiant heat flux in the space varies inversely as the square of the distance from the emitting surface and directly with the cosine of the angle made with the normal to the surface. Experiments indicate that most engineering materials do not exactly follow Lambert's cosine principle. The error introduced by using Lambert's law in the calculation of radiant heat transfer has been assumed to be too small, in comparison with other calculation errors tolerated in practice, to warrant the complication introduced by the use of a more accurate form of the directional distribution function. The configuration factor is denoted as

$$F_{12} = \frac{1}{\pi A_1} \int_{A_2} \int_{A_1} \cos\phi_1 \cos\phi_2 \frac{dA_1 dA_2}{r^2} , \qquad (1-5)$$

where ϕ_1 and ϕ_2 are the acute angles measured between a normal to the surface and the connecting line r between the area elements.

The total heat transfer per unit time leaving A_1 which reaches A_2 is

$$q_{12} = \frac{E_{b1}}{\pi} \int_{A_2} \int_{A_1} \cos\phi_1 \cos\phi_2 \frac{dA_1 dA_2}{r^2} . \qquad (1-6)$$

It now becomes desirable to develop two special configuration factors in a general form.

(A) The configuration factor for finite, perpendicular rectangles with a common edge:

Fig. 1-1 indicates a rectangle, which will be called A_1, of the dimensions X by Y located normal to rectangle A_2 with the dimensions X by Z. The line X is then the common edge.

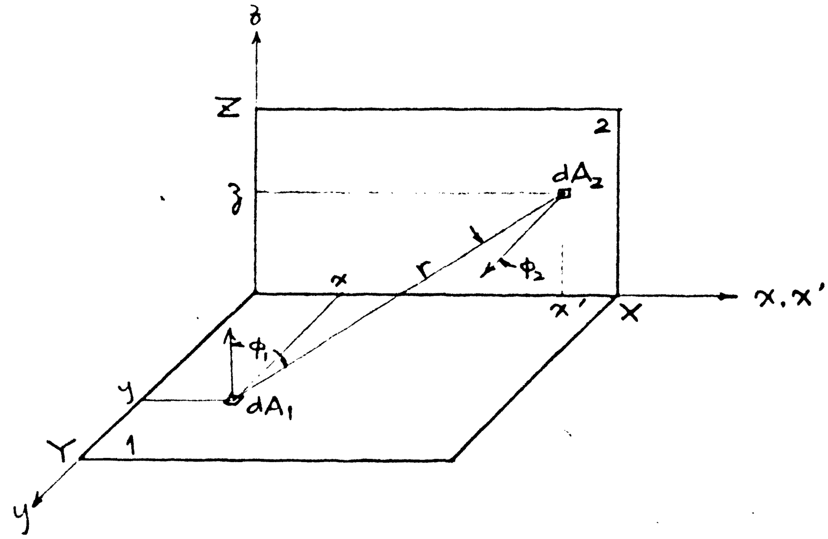


Fig. 1-1 Configuration factor notation in perpendicular rectangles

The quantities needed to evaluate F_{12} are given below:

$dA_1 = dxdy$

$dA_2 = dx'dz$

$r^2 = (x'-x)^2 + y^2 + z^2$

$\cos\phi_1 = z/r$

$\cos\phi_2 = y/r.$

The configuration factor is expressed as

$$F_{12} = \frac{1}{\pi X Y}\int_0^Y\int_0^X\int_0^X\int_0^Z \frac{yz\,dz\,dx'\,dx\,dy}{[(x'-x)^2+y^2+z^2]^2}\,.$$

Integration of the above equation yields

$$F_{12} = \frac{1}{\pi}\Big[\tan^{-1}(X/Y) + (Z/Y)\tan^{-1}(X/Z)$$

$$-(\sqrt{Y^2+Z^2}/Y)\tan^{-1}(X/\sqrt{Y^2+Z^2}) - (X/4Y)\ln\frac{(X^2+Y^2+Z^2)X^2}{(X^2+Y^2)(X^2+Z^2)} \qquad (1-7)$$

$$+(Y/4X)\ln\frac{(X^2+Y^2+Z^2)Y^2}{(X^2+Y^2)(Y^2+Z^2)} + (Z^2/4XY)\ln\frac{(X^2+Y^2+Z^2)Z^2}{(X^2+Z^2)(Y^2+Z^2)}\Big].$$

(B) The configuration factor for finite, parallel, opposed rectangles:

Fig. 1-2 shows two rectangles X by Y in size and separated by a distance D.

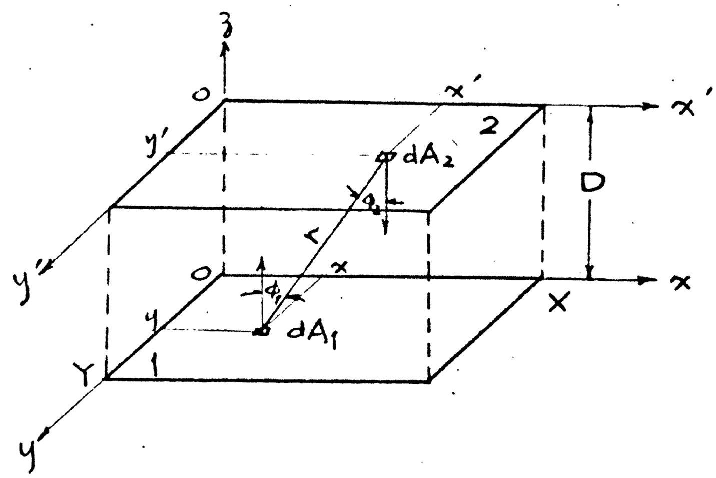


Fig. 1-2 Configuration factor notation in parallel rectangles

The quantities needed to evaluate F_{12} are given below:

$dA_1 = dxdy$

$dA_2 = dx'dy'$

$r^2 = D^2 + (x'-x)^2 + (y'-y)^2$

$\cos\phi_1 = \cos\phi_2 = D/r$.

Therefore,

$$F_{12} = \frac{D^2}{\pi XY}\int_0^X\int_0^Y\int_0^X\int_0^Y \frac{dx\,dy\,dx'\,dy'}{[D^2+(x'-x)^2+(y'-y)^2]}$$

The result of the above equation, in terms of the dimensionless ratios X/D and Y/D is

$$F_{12}=\frac{1}{\pi}\Big[\frac{1}{R_1R_2}\ln\frac{(1+R_1^2)(1+R_2^2)}{(1+R_1^2+R_2^2)}-\frac{2}{R_1}\tan^{-1}R_2$$

$$-\frac{2}{R_2}\tan^{-1}R_1+2\sqrt{1+(1/R_1^2)}\,\tan^{-1}(R_2/\sqrt{1+R^2}) \qquad (1\text{-}8)$$

$$+\,2\sqrt{1+(1/R_2^2)}\,+\tan^{-1}(R_1/\sqrt{1+R_2^2})\Big],$$

where $R_1 = X/D$, $R_2 = Y/D$.

In some cases, the evaluation of the configuration factor of a particular configuration by means of the Eq. (1-5) is difficult or even impossible. Sometimes it may be possible, however, to evaluate the required configuration factor by means of "configuration factor algebra". This method makes use of four principles which are summarized here for convenience.

(A) Basic reciprocity law:

The product of an area A_1 and the configuration factor of A_1 relative to another area A_2; i.e., F_{12}, is related to the product of A_2 and F_{21} by the relation

$A_1F_{12} = A_2F_{21}$. (1-9)

To simplify this relation the geometric factor G_{12}, numerically equal to the product of A_1F_{12}, is introduced, hence

$G_{12} = G_{21}$. (1-10)

(B) Summation law:

If the interior surface of a completely enclosed space is subdivided into parts having area A_1, A_2, ..., A_n and each area is irradiated, then the following relationship holds:

$$\sum_{i=1}^{n} F_{ij} = 1 \qquad \text{where} \quad i = 1,\ 2,\ \ldots,\ n \qquad \text{and} \quad j = 1,\ 2,\ \ldots,\ n. \tag{1-11}$$

(C) Decomposition law:

Given two surfaces A_1 and A_2, if surface A_1 is subdivided into A_3 and A_4, then the total configuration factor F_{12} is related to the two subsidiary configuration factors F_{32} and F_{42} by the relation

$$A_1F_{12} = A_3F_{32} + A_4F_{42}, \tag{1-12}$$

or

$$G_{12} = G_{32} + G_{42}. \tag{1-13}$$

(D) Modified reciprocity law:

For rectangular geometric systems, if two planes intersect, the product of a corner area in plane A and its configuration factor with respect to the opposite corner area in plane B is equal to the product of the other corner area in plane A and its configuration factor with respect to the other corner area in plane B, irrespective of the angle between planes. This law plays an important role in this study, the illustrations are as follows:

From Fig. 1-3 the quantities of Eq. (1-5) in terms of x, x', y, z are developed as

$$dA_1 = dx\,dy$$

$$dA_2 = dx'\,dz$$

$$r^2 = (x'-x)^2 + y^2 + z^2$$

$$\cos\phi_1 = z/r$$

$$\cos\phi_2 = y/r.$$

Hence, Eq. (1-5) yields

$$G_{12} = F_{12} A_1 = \frac{1}{\pi} \int_0^Z \int_0^X \int_0^Y \int_0^a \frac{yz \, dx \, dy \, dx' dz}{[(x'-x)^2 + y^2 + z^2]^2} ,$$

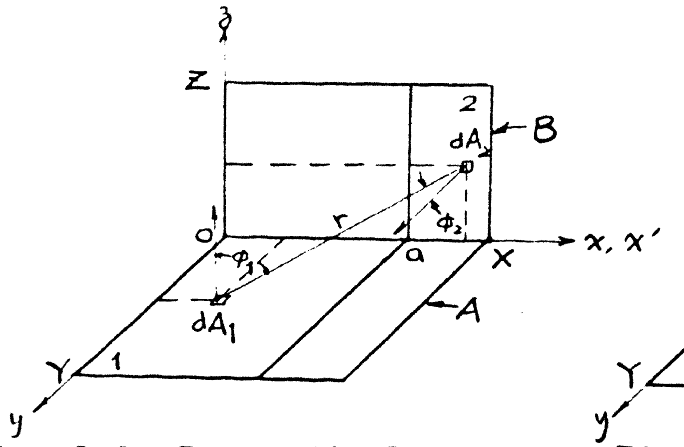


Fig. 1-3 Perpendicular shape factor geometry

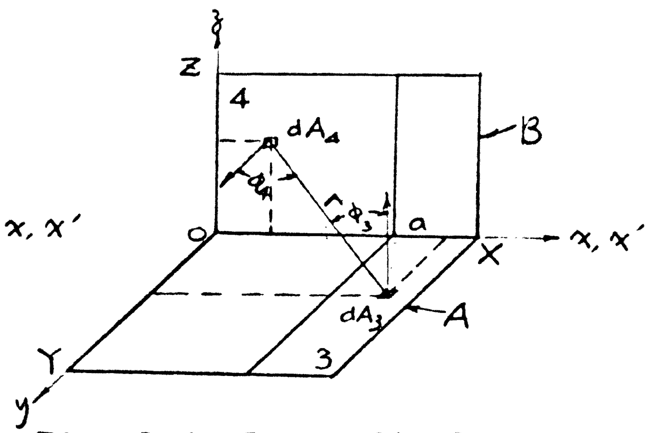


Fig. 1-4 Perpendicular shape factor geometry

and from Fig. 1-4

$dA_3 = dxdy$

$dA_4 = dx'dz$

$r^2 = (x'-x)^2 + y^2 + z^2$

$\cos\phi_1 = z/r$

$\cos\phi_2 = y/r.$

Therefore, $G_{34} = F_{34} A_3 = \frac{1}{\pi} \int_0^Z \int_0^a \int_0^Y \int_0^X \frac{yz \, dx \, dy \, dx' dz}{[(x'-x)^2 + y^2 + z^2]^2}$.

The two integrals are of identical form except for the order of integration. Since the nature of the integrand permits the interchange of the order of the integration, the reciprocity formula is obtained

$$G_{12} = G_{34}. \tag{1-14}$$

For parallel rectangles, the reciprocity formula also holds.

Applying the previous laws the following useful relations for determining the configuration factors are developed.

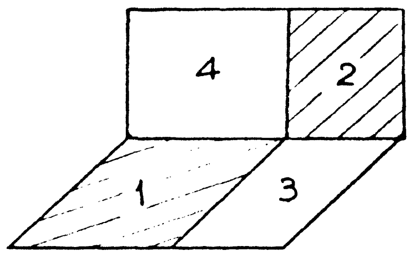


Fig. 1-5 Perpendicular shape factor geometry

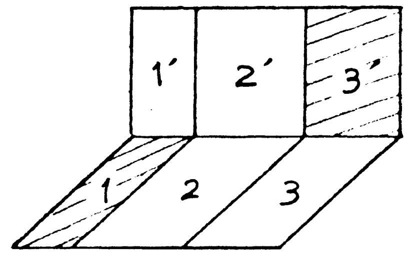


Fig. 1-6 Perpendicular shape factor geometry

Denoting $G_{mn} = A_m F_{mn}$ and $G_{m^2} = A_m F_{mm}$, one can develop

$$G_{12} = 1/2 \left[G_{(1+3)(2+4)} - G_{14} - G_{32} \right], \qquad (1\text{-}15)$$

corresponding to Fig. 1-5, and

$$G_{13'} = 1/2 \left\{ \left[G_{(1+2+3)^2} - G_{1^2} - G_{(2+3)^2} \right] - \left[G_{(1+2)^2} - G_{1^2} - G_{2^2} \right] \right\}$$

$$= 1/2 \left[G_{(1+2+3)^2} + G_{2^2} - G_{(2+3)^2} - G_{(1+2)^2} \right]. \qquad (1\text{-}16)$$

corresponding to Fig. 1-6. The above formulas are also applicable for parallel rectangles.

4. RADIATION HEAT TRANSFER CALAULATIONS

There are many methods to determine the radiation heat exchange among the surfaces. In this analysis, the unified method for radiation exchange calculations developed by Gebhart is used. This method treats all diffuse-radiation configurations, including those which involve special features such as windows, openings, and surfaces in radiant balance. The symbol q_j denotes the rate of energy transfer from an arbitrary surface A_j participating in the radiative exchange process, while the rate of emission from area A_j is equal to E_jA_j.

Consider an enclosure which contains no emitting or absorbing medium, and is formed of areas A_1, A_2, ..., A_n having emissivities ϵ_1, ϵ_2, ..., ϵ_n and emissive powers E_1, E_2, ..., E_n. The radiant energy absorbed by A_j per unit time from each area is then given by $B_{1j}E_1A_1$, $B_{2j}E_2A_2$, ..., $B_{jj}E_jA_j$, ..., $B_{nj}E_nA_n$. Where B_{ij} is the absorption factor, defined as that fraction of the radiant energy emitted by surface A_i which is absorbed by surface A_j. This fraction is to include radiation along all paths by which portions of E_iA_i reaches A_j and is absorbed by A_j. In general, B_{jj} is not zero because some of the energy emitted by A_j may be reabsorbed by A_j. The rate of energy loss from A_j is equal to the rate of emission minus the total amount of radiant energy absorbed by A_j per unit time. Hence,

$$q_j = E_jA_j - B_{1j}E_1A_1 - B_{2j}E_2A_2 - \cdots - B_{jj}E_jA_j - \cdots - B_{nj}E_nA_n = E_jA_j - \sum_{i=1}^{n} B_{ij}E_iA_i \, , \tag{1-17}$$

or $$q_j = \epsilon_j \sigma T_j^4 A_j - \sum_{i=1}^{n} \sigma B_{ij} \epsilon_j T_i^4 A_i . \qquad (1\text{-}18)$$

The n values of B_{ij} necessary to compute q_j may be determined by summing absorption rates at j due to the emission rates of $A_1, A_2, \ldots, A_n$. For example, E_1A_1 is emitted at A_1 and reaches the n surfaces in fractions given by the configuration factors $F_{11}, F_{12}, \ldots, F_{1n}$. The fraction of energy emitted by area A_1 and absorbed by A_j is $F_{1j}\epsilon_j$, while $F_{1j}r_j$ is reflected. In general, $F_{1i}r_i$ is reflected by the ith surface. The fraction of $F_{1i}r_i$ which is absorbed at A_j is the same as the fraction of E_iA_i which is absorbed at A_j if the incident energy $F_{1i}E_1A_1$ is uniformly distributed over A_i and is diffusely reflected. Assuming uniform distribution, the fraction of E_1A_1 absorbed at A_j because of reflection off A_i is then $B_{ij}F_{1i}r_i$. So the total fraction of E_1A_1 absorbed at A_j, that is, that is, B_{1j}, is

$$B_{1j} = F_{1j}\epsilon_j + F_{11}r_1B_{1j} + F_{12}r_2B_{2j} + F_{13}r_3B_{3j} + \cdots + F_{1n}r_nB_{nj} .$$

Similarly, the absorption factors for each of the other surfaces $A_2, A_3, \ldots, A_n$ are

$$B_{2j} = F_{2j}\epsilon_j + F_{21}r_1B_{1j} + F_{22}r_2B_{2j} + F_{23}r_3B_{3j} + \cdots + F_{2n}r_nB_{nj}$$

$$B_{3j} = F_{3j}\epsilon_j + F_{31}r_1B_{1j} + F_{32}r_2B_{2j} + F_{33}r_3B_{3j} + \cdots + F_{3n}r_nB_{nj}$$

$$\vdots$$

$$B_{nj} = F_{nj}\epsilon_j + F_{n1}r_1B_{1j} + F_{n2}r_2B_{2j} + F_{n3}r_3B_{3j} + \cdots + F_{nn}r_nB_{nj} .$$

This set of n equations with the n unkown values B_{1j}, $B_{2j}, \ldots, B_{nj}$ are rearranged as the set of following equations

$$
\begin{aligned}
(F_{11}r_1-1)B_{1j}+F_{12}r_2B_{2j}+F_{13}r_3B_{3j}+\cdots+F_{1n}r_nB_{nj}&=-F_{1j}\epsilon_j\\
F_{21}r_1B_{1j}+(F_{22}r_2-1)B_{2j}+F_{23}r_3B_{3j}+\cdots+F_{2n}r_nB_{nj}&=-F_{2j}\epsilon_j\\
F_{31}r_1B_{1j}+F_{32}r_2B_{2j}+(F_{33}r_3-1)B_{3j}+\cdots+F_{3n}r_nB_{nj}&=-F_{3j}\epsilon_j\\
&\vdots\\
F_{n1}r_1B_{1j}+F_{n2}r_2B_{2j}+F_{n3}r_3B_{3j}+\cdots+(F_{nn}r_n-1)B_{nj}&=-F_{nj}\epsilon_j
\end{aligned}
\qquad (1\text{-}19)
$$

The absorption factor B_{ij} for the jth surface can be solved by determinants, i.e., Cramer's rule. When the number of equations is large, Cramer's rule is inefficient, since it requires evaluating determinants of high order. For this reason and because of the convenience in using the subroutine on IBM 1620 Digital Computer, the method of Gauss-Jordan reduction is used.

PART II: RADIATION HEAT TRANSFER ANALYSIS IN THE RECTANGULAR CONFIGURATIONS

In this section two cases are discussed and the programs used on IBM 1620 Model II Digital Computer are included. The programs are constructed to be as general as possible. Here the method of solution to the problem is by means of finite differences. For convenience, equal intervals will be used in the solution, i.e., the wall will be divided into equal finite sections. The more subdivisions used the greater the accuracy obtained. The thermal properties of each subdivided section of the wall are assumed to be uniform and to satisfy all the conditions and results of PART I. The mean temperature of each subdivided section is determined and made uniform over the section.

CASE A: RADIATION HEAT TRANSFER ANALYSIS IN RECTANGULAR DUCT

Assume a rectangular duct with the dimensions of (XX+W) by YY by ZZ and open at each end of the dimension (XX+W). The portion of the duct XX by YY by ZZ is denoted as the source and has the temperature range T_1 to T_2 at either end. The temperature profile of the wall of this duct from T_1 to T_2 is assumed to be a straight line along the dimension XX (for convenience only). The temperature of the portion other than the source, here denoted as the sink, is T_3 and uniform over that section. The construction is shown in the following. The rate of energy loss from both the source and the sink is determined and that of each subdivided section is also investigated.

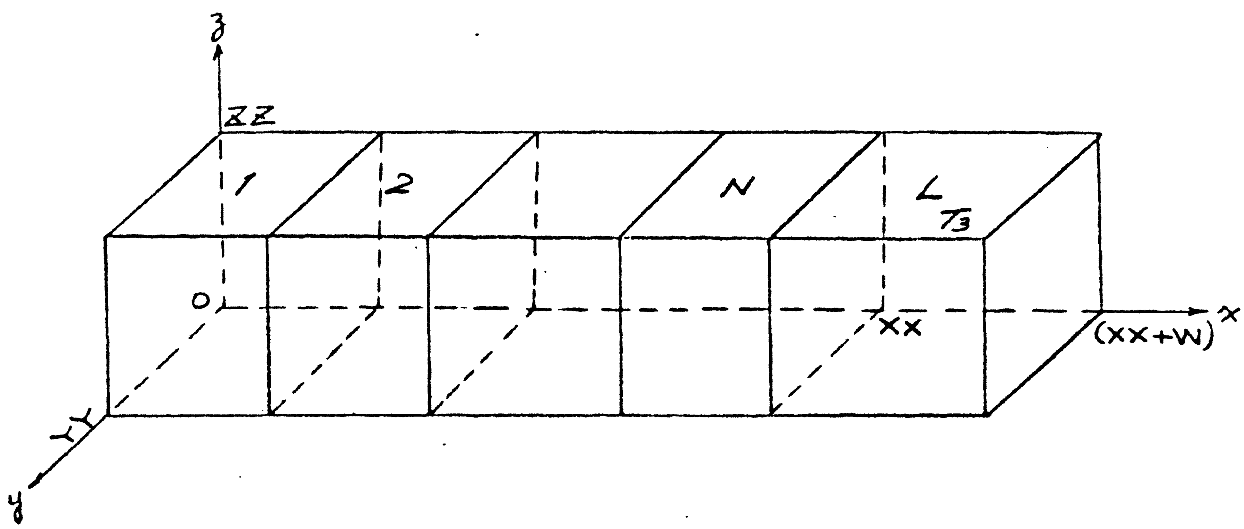


Fig. 2-1 Dimensions of the rectangular duct.

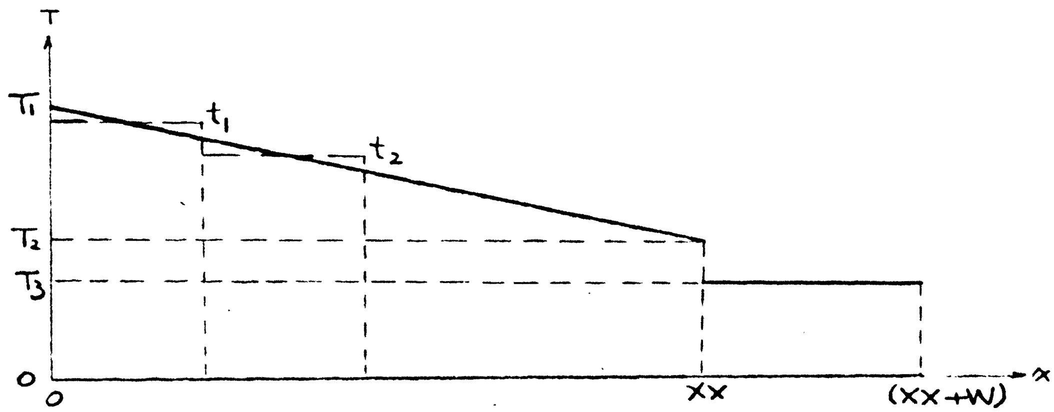


Fig. 2-2 Temperature profile down the wall of the duct.

The source section XX is divided equally into N parts, the mean temperatures of the subdivisions, say t_1, $t_2, \ldots, t_n$ are determined as follows,

$$t_1 = T_1 - (T_1 - T_2)/2N \tag{2-1}$$

and
$$t_i = t_{(i-1)} - (T_1 - T_2)/N \, , \tag{2-2}$$

where $i = 2, 3, \ldots, N$.

The emissivity and reflectivity are dependent on the temperature distribution. From Fig. 13-10, P.375, "Heat and Mass Transfer" by E. R. Eckert, the total reflectivity and absorptivity of different materials for incident black radiation at the indicated temperature are obtained. (The emissivities are obtained by means of Kirchhoff's identity $\epsilon = a$.)

Since the number of subdivisions are uncertain in the solution, an approximate numerical method for evaluating the required emissivities and reflectivities is introduced. The method used is the Lagrange interpolation formula, given by:

$$y = f(x) = \sum_{i=1}^{n} l_i(x) f(x_i), \tag{2-3}$$

where

$$l_i(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_i-x_1)(x_i-x_2)\cdots(x_i-x_n)}$$

and the terms $(x-x_i)$ and (x_i-x_i) are omitted. To use this formula, one first reads several sets of data, say 5, for the emissivities corresponding to the different temperatures as

ϵ_i where $i = 1, 2, \ldots, 5$

and T_i where $i = 1, 2, \ldots, 5,$

then

$$\epsilon = \sum_{i=1}^{5} l_i(T) \epsilon_i, \tag{2-4}$$

where

$$l_i(T) = \frac{(T-T_1)(T-T_2)\cdots(T-T_5)}{(T_i-T_1)(T_i-T_2)\cdots(T_i-T_5)}$$

Again, the terms of $(T-T_i)$ and (T_i-T_i) are omitted, and the identity $r = 1-\epsilon$ is used for evaluating the reflectivities.

Applying the formula derived in PART I the desired quantities are calculated. The two openings absorb energy and reradiate none, so the reflectivity of both openings is zero. The procedure used to develop these quantities is as follows: First evaluate the geometric factors, letting G_{ij} denote the geometric factors, where

$i = 1, 2, \ldots, N+1,$

and $j = 1, 2, \ldots, N+1.$

The number $N+1$ denotes the sink section. Next the configuration factors are evaluated, letting F_{ij} denote the configuration factors, where

$i=1, 2, \ldots, N+1,$

and $j=1, 2, \ldots, N+1.$

Following the configuration factors the absorption factors are determined by means of the Gauss-Jordan reduction method for solving a set of simultanious equations, letting B_{ij} denote the absorption factors, where

$i=1, 2, \ldots, N+1,$

and $j=1, 2, \ldots, N+1.$

With the aid of Eq.(1-2), the energy loss of each section is obtained. Denote the radiant flux of each section by Q_j, where

$j=1, 2, \ldots, N+1,$

and the total energy loss of the sink section is Q_{N+1}. The total energy loss of the source section is then

$$Q=\sum_{j=1}^{N} Q_j . \qquad (2\text{-}5)$$

The procedures are clearly seen from the computer program, and the descriptions are made in detail following the program.

COMPUTER PROGRAM I

```
C       PROGRAM I RADIATION HEAT TRANSFER ANALYSIS IN THE RECTANGULAR DUCT
        DIMENSION FA(10),FAB(10),FAAL(12),GA(12,12),FBD(2),FB(10),FDDL(2),
       1FBBL(12),GB(12,12),G(12,12),F(12,12),D(12,14),GC(12,12),T(12),EMI(
       212),REF(12),TT(10),EE(10),QASUB(12),Q(12)
        PI=3.1415926
        STBOC=.1714E-8
        READ 101, NN
        READ 100, (TT(I),I=1,NN),(EE(I),I=1,NN)
        PRINT 101, NN
        PRINT 100,(TT(I),I=1,NN),(EE(I),I=1,NN)
        READ 100, XX,YY,ZZ,W,T1,T2,T3
        PRINT 100, XX,YY,ZZ,W,T1,T2,T3
      1 READ 101, N
        PRINT 101, N
        L=N+1
        L1=N+2
        L2=N-1
C       EVALUATION OF THE GEOMETRIC FACTORS
C       FOR THE PERPENDICULAR FORM
        R=N
        H=XX/R
        DO 30 M=1,10
        GO TO (11,12,13,14,15,16,17,18,19,20),M
     11 X=YY
        Y=H
        Z=ZZ
        GO TO 25
     12 X=YY
        Y=XX+W
        Z=ZZ
        GO TO 25
     13 X=YY
        Y=XX-H+W
        Z=ZZ
        GO TO 25
     14 X=ZZ
        Y=H
        Z=YY
        GO TO 25
     15 X=ZZ
        Y=XX+W
        Z=YY
        GO TO 25
     16 X=ZZ
        Y=XX-H+W
```

```
      Z=YY
      GO TO 25
   17 X=YY
      Y=W
      Z=ZZ
      GO TO 25
   18 X=YY
      Y=XX
      Z=ZZ
      GO TO 25
   19 X=ZZ
      Y=W
      Z=YY
      GO TO 25
   20 X=ZZ
      Y=XX
      Z=YY
C     EQUATION OF THE GEOMETRIC FACTOR FOR THE PERPENDICULAR RECTANGLES
C     WITH A COMMON EDGE
   25 C1=X**2
      C2=Y**2
      C3=Z**2
      CC1=C1+C2+C3
      CC2=C1+C2
      CC3=C2+C3
      CC4=C1+C3
      CSR=SQRTF(CC3)
      FC1=X*Y*ATANF(X/Y)
      FC2=X*Z*ATANF(X/Z)
      FC3=-(X*CSR*ATANF(X/CSR))
      FC4=-(C1*LOGF(CC1*C1/(CC2*CC4)))/4.0
      FC5=(C2*LOGF(CC1*C2/(CC2*CC3)))/4.0
      FC6=(C3*LOGF(CC1*C3/(CC3*CC4)))/4.0
      FA(M)=(FC1+FC2+FC3+FC4+FC5+FC6)/PI
   30 CONTINUE
      GA1=2.0*(FA(1)+FA(2)-FA(3)+FA(4)+FA(5)-FA(6))
      GAN1=2.0*(FA(7)+FA(2)-FA(8)+FA(9)+FA(5)-FA(10))
      X=0.0
      M=N+L
      DO 40 K=1,M
      IF (K-N) 31,31,32
   31 X=X+H
      Y=YY
      Z=ZZ
      GO TO 35
   32 IF (K-L) 33,33,34
```

```
   33 X=W
   34 Y=YY
      Z=ZZ
      I=K-N
   35 C1=X**2
      C2=Y**2
      C3=Z**2
      CC1=C1+C2+C3
      CC2=C1+C2
      CC3=C2+C3
      CC4=C1+C3
      CSR=SQRTF(CC3)
      FC1=X*Y*ATANF(X/Y)
      FC2=X*Z*ATANF(X/Z)
      FC3=-(X*CSR*ATANF(X/CSR))
      FC4=-(C1*LOGF(CC1*C1/(CC2*CC4)))/4.0
      FC5=(C2*LOGF(CC1*C2/(CC2*CC3)))/4.0
      FC6=(C3*LOGF(CC1*C3/(CC3*CC4)))/4.0
      IF (K-N) 36,36,37
   36 FAB(K)=(FC1+FC2+FC3+FC4+FC5+FC6)/PI
      GO TO 40
   37 FAAL(I)=(FC1+FC2+FC3+FC4+FC5+FC6)/PI
      X=X+H
   40 CONTINUE
      GA2=4.0*(FAAL(L)-FAAL(L-1)-FAB(1))
      GAN2=4.0*(FAAL(L)-FAB(L-1)-FAAL(1))
      GA(1,2)=(FAB(2)-2.0*FAB(1))*4.0
      IF (N-2) 44,44,42
   42 DO 43 I=3,N
   43 GA(1,I)=(FAB(I)-2.0*FAB(I-1)+FAB(I-2))*4.0
   44 GA(L,1)=(FAAL(2)-FAAL(1)-FAB(1))*4.0
      DO 45 I=2,N
   45 GA(L,I)=(FAAL(I+1)+FAB(I-1)-FAB(I)-FAAL(I))*4.0
C     FOR THE PARALLEL FORM
      X=H
      M=N+L
      DO 60 K=1,M
      Y=YY
      Z=ZZ
      IF (K-N) 53,53,50
   50 IF (K-L) 51,51,52
   51 X=W
   52 J=K-N
   53 DO 57 I=1,2
C     EQUATION OF THE GEOMETRIC FACTOR FOR THE PARALLEL AND OPPOSED
C     RECTANGLES
```

```
      R1=X/Z
      R2=Y/Z
      C1=R1**2
      C2=R2**2
      C3=1.0+C1
      C4=1.0+C2
      CC1=LOGF((C3*C4)/(C2+C3))
      CC2=SQRTF(C3)
      CC3=SQRTF(C4)
      FD1=CC1/(R1*R2)
      FD2=-(2.0*ATANF(R2))/R1
      FD3=-(2.0*ATANF(R1))/R2
      FD4=(2.0*CC2*ATANF(R2/CC2))/R1
      FD5=(2.0*CC3*ATANF(R1/CC3))/R2
      IF (K-N) 54,54,55
   54 FBD(I)=(FD1+FD2+FD3+FD4+FD5)*X*Y/PI
      GO TO 56
   55 FDDL(I)=(FD1+FD2+FD3+FD4+FD5)*X*Y/PI
   56 Y=ZZ
   57 Z=YY
      IF (K-N) 58,58,59
   58 FB(K)=FBD(1)+FBD(2)
      GO TO 60
   59 FBBL(J)=FDDL(1)+FDDL(2)
   60 X=X+H
C     EVALUATION OF THE GEOMETRIC FACTORS AND THE CONFIGURATION FACTORS
C     BETWEEN ANY TWO SUBDIVISIONS
      GB1=FBBL(L)-FB(1)-FBBL(N)
      GBN1=FBBL(L)-FB(N)-FBBL(1)
      GB(1,2)=(FB(2)-2.0*FB(1))
      IF (N-2) 64,64,62
   62 DO 63 I=3,N
   63 GB(1,I)=(FB(I)-2.0*FB(I-1)+FB(I-2))
   64 GB(L,1)=(FBBL(2)-FB(1)-FBBL(1))
      DO 65 I=2,N
   65 GB(L,I)=(FBBL(I+1)+FB(I-1)-FB(I)-FBBL(I))
      AN=2.0*H*(YY+ZZ)
      ANL=2.0*W*(YY+ZZ)
      F(1,1)=1.0-(GA1+GA2+GB1)/AN
      G(1,1)=F(1,1)*AN
      DO 71 I=2,N
   71 G(1,I)=(GA(1,I)+GB(1,I))
      DO 72 I=1,N
   72 G(I,1)=G(1,I)
      DO 74 M=2,N
      K=N
```

```
73 G(M,K)=G(M-1,K-1)
   K=K-1
   IF (K-1) 74,74,73
74 CONTINUE
   F(L,L)=1.0-(GAN1+GAN2+GBN1)/ANL
   DO 75 I=1,N
75 GC(L,I)=GA(L,I)+GB(L,I)
   K=N
   DO 76 I=1,N
   G(I,L)=GC(L,K)
76 K=K-1
   DO 77 I=1,N
77 G(L,I)=G(I,L)
   DO 78 I=1,N
   DO 78 J=1,L
78 F(I,J)=G(I,J)/AN
   DO 80 J=1,N
80 F(L,J)=G(L,J)/ANL
   PRINT 102
   PRINT 100,((F(I,J),J=1,L),I=1,L)
C       EVALUATION OF THE MEAN TEMPERATURES OF THE SUBDIVISIONS
   T(1)=T1-(T1-T2)/(2.0*R)
   DO 81 I=1,L2
81 T(I+1)=T(I)-(T1-T2)/R
   T(L)=T3
   PRINT 103
   PRINT 100, (T(I),I=1,L)
C       EVALUATION OF THE MEAN EMISSIVITIES OF THE SUBDISIONS BY MEANS OF
C       LAGRANGIAN INTERPOLATION FORMULA
   DO 86 J=1,L
   FEM=0.0
   DO 85 K=1,NN
   FNU=1.0
   FNO=1.0
   DO 84 I=1,NN
   IF (I-K) 83,84,83
83 FNU=FNU*(T(J)-TT(I))
   FNO=FNO*(TT(K)-TT(I))
84 CONTINUE
85 FEM=FEM+FNU/FNO*EE(K)
86 EMI(J)=FEM
   PRINT 104
   PRINT 100, (EMI(J),J=1,L)
C       EVALUATION OF THE MEAN REFLECTIVITIES OF THE SUBDIVISIONS BY MEANS
C       KIRCHHOF'S IDENTITY
   DO 87 I=1,L
```

```
   87 REF(I)=1.0-EMI(I)
      PRINT 105
      PRINT 100, (REF(I),I=1,L)
      DO 98 J=1,L
      PRINT 106, J
C     SOLUTION OF THE ABSORPTION FACTORS BY MEANS OF GAUSS-JORDAN
C     REDUCTION METHOD
      DO 89 I=1,L
   89 D(I,I)=F(I,I)*REF(I)-1.0
      DO 92 I=1,L
      DO 91 M=1,L
      IF (I-M) 90,91,90
   90 D(I,M)=F(I,M)*REF(M)
   91 CONTINUE
   92 CONTINUE
      DO 93 I=1,L
   93 D(I,L+1)=-(F(I,J)*EMI(J))
      CALL GAUJOR (D,L,L1,12,14)
      PRINT 107
      PRINT 100, (D(I,L1),I=1,L)
C     EVALUATION OF THE ABSORPTION ENERGY FROM SURFACE I TO SURFACE J
      DO 94 I=1,N
   94 QASUB(I)=D(I,L1)*STBOC*T(I)**4*EMI(I)*AN
      QASUB(L)=D(L,L1)*STBOC*T(L)**4*EMI(L)*ANL
      PRINT 108
      PRINT 100, (QASUB(I),I=1,L)
C     EVALUATION OF THE TOTAL ABSORPTION ENERGY OF SURFACE J
      QABSO=0.0
      DO 95 I=1,L
   95 QABSO=QABSO+QASUB(I)
      PRINT 109
      PRINT 100, QABSO
C     EVALUATION OF THE RADIANT HEAT TRANSFER FROM SURFACE J
      IF (J-N) 96,96,97
   96 Q(J)=STBOC*T(J)**4*EMI(J)*AN-QABSO
      GO TO 98
   97 Q(J)=STBOC*T(J)**4*EMI(J)*ANL-QABSO
   98 CONTINUE
      PRINT 110
      PRINT 100,(Q(J),J=1,L)
C     EVALUATION OF THE TOTAL HEAT TRANSFER OF THE SOURCE
      QLOSS=0.0
      DO 99 I=1,N
   99 QLOSS=QLOSS+Q(I)
      PRINT 111
      PRINT 100, QLOSS
```

```
      PRINT 112
      PRINT 100, Q(L)
      GO TO 1
100   FORMAT (4F18.8)
101   FORMAT (4I18)
102   FORMAT (10X44HCONFIGURATION FACTORS ((F(I,J),J=1,L),I=1,L))
103   FORMAT (10X41HMEAN TEMPERATURES (T(I),I=1,L) IN RANKINE)
104   FORMAT (10X28HMEAN EMISSIVITY OF EACH PART)
105   FORMAT (10X25HREFLECTIVITY OF EACH PART)
106   FORMAT (10X2HJ=,I2)
107   FORMAT (10X33HABSORPTION FACTORS (B(I,J),I=1,L))
108   FORMAT (10X39HABSORPTION ENERGY FROM EACH PART BTU/HR)
109   FORMAT (10X41HTOTAL ENERGY ABSORRED BY EACH PART BTU/HR)
110   FORMAT (10X45HENERGY LOSS OF EACH PART (Q(J),J=1,N2 BTU/HR))
111   FORMAT (10X38HTOTAL ENERGY LOSS OF THE SOURCE BTU/HR)
112   FORMAT (10X37H TOTAL ENERGY LOSS OF THE SINK BTU/HR)
      END
```

DETAILED DESCRIPTION OF THE PROGRAM

Geometric factors in perpendicular form:

Statement 11 to line 11 + 3, is the evaluation of the geometric factor from 1 to 2 as shown. This is obtained by using Eq.(1-7), where FA(1) is the program variable.

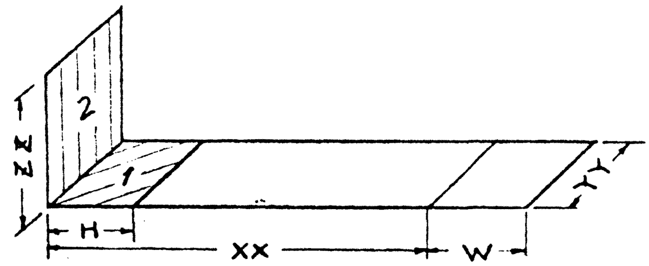


Fig. 2-3 Geometric factor notation.

Statement 12 to line 12 + 3, is the evaluation of the geometric factor from 1 to 2 as shown. This value is given by FA(2) in the program.

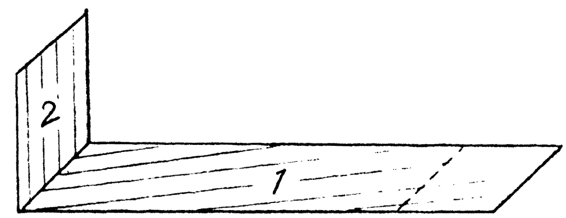


Fig. 2-4 Geometric factor notation.

Statement 13 to line 13 + 3, is the evaluation of the geometric factor from 1 to 2 as shown. This value is given by FA(3) in the program.

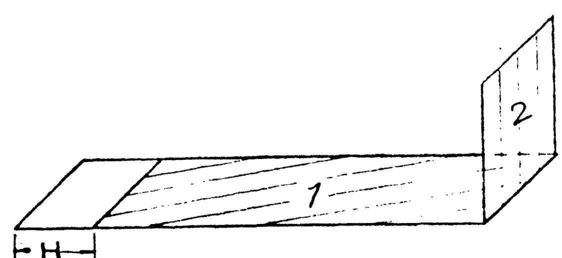


Fig. 2-5 Geometric factor notation.

Statement 14 to line 14 + 3, is the evaluation of the geometric factor from 1 to 2 as shown. This value is given by FA(4) in

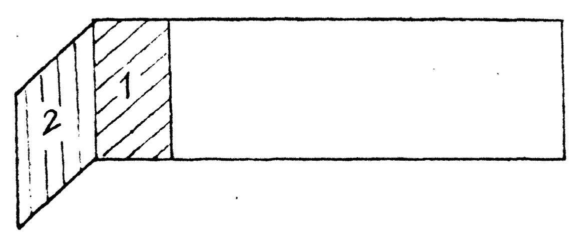


Fig. 2-6 Geometric factor notation.

the program.

Statement 15 to line 15 + 3, is the evaluation of the geometric factor from 1 to 2 as shown. This value is given by FA(5) in the program.

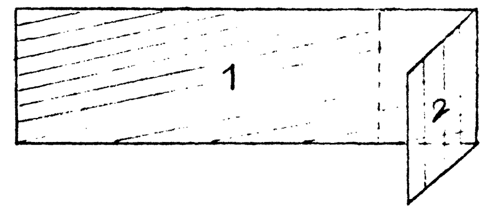


Fig. 2-7 Geometric factor notation.

Statement 16 to line 16 + 3, is the evaluation of the geometric factor from 1 to 2 as shown. This value is given by FA(6) in the program.

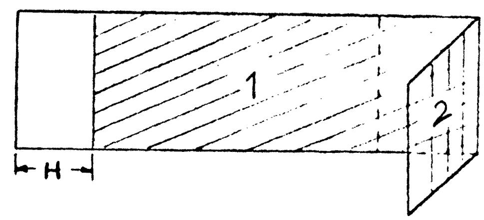


Fig. 2-8 Geometric factor notation.

Statement 17 to line 17 + 3, is the evaluation of the geometric factor from 1 to 2 as shown. This value is given by FA(7) in the program.

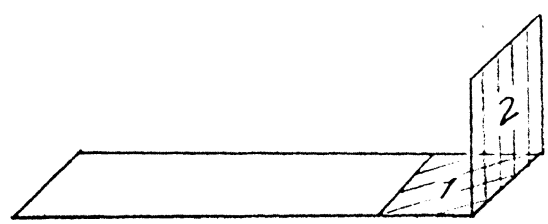


Fig. 2-9 Geometric factor notation.

Statement 18 to line 18 + 3, is the evaluation of the geometric factor from 1 to 2 as shown. This value is given by FA(8) in the program.

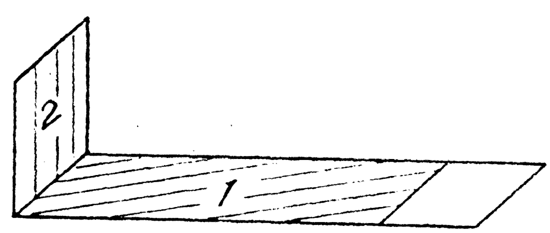


Fig. 2-10 Geometric factor notation.

Statement 19 to line 19 + 3, is the evaluation of the geometric factor from 1 to 2 as shown. This value is given by FA(9) in the program.

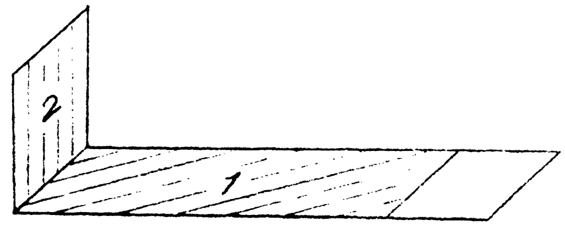


Fig. 2-11 Geometric factor notation.

Statement 20 to statement 30 is the evaluation of the geometric factor from 1 to 2 as shown. This value is given by FA(10) in the program.

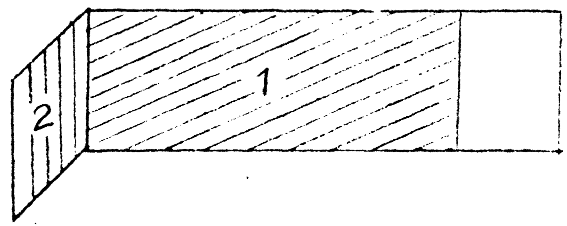


Fig. 2-12 Geometric factor notation.

Statement 30 + 1 line is the evaluation of the geometric factor from 1 to 2 as shown. Where surface 1 is the ring of width H and surface 2 is the two end plates. This value is given by GA1 in the program.

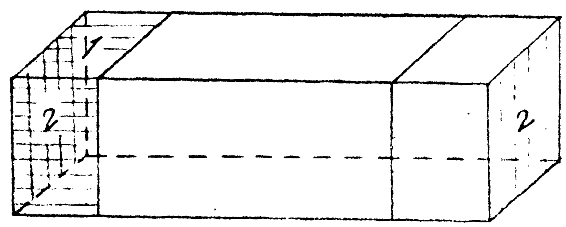


Fig. 2-13 Geometric factor notation.

Statement 30 + 2 lines is the evaluation of the geometric factor from 1 to 2 as shown. Where surface 1 is a ring of width W and surface 2 is the two end plates. This value is given by GAN1 in the program.

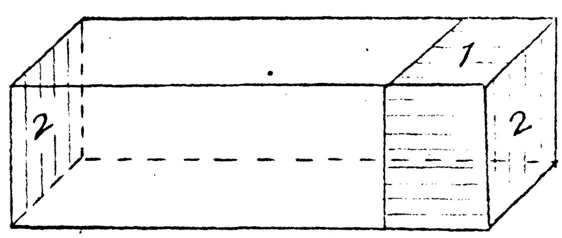


Fig. 2-14 Geometric factor notation.

Statement 30+3 lines to statement 40 is the evaluation of the geometric factors from 1 to 2 and 1' to 2' as shown. Where the quantity X in Eq.(1-7) is changing from H to XX with the increment H. The same procedure is then followed from W to (XX+W) with the increment H. These values are given by (FAB(K), K=1,N) and (FAAL(I),I=1,N+1) respectively in the program.

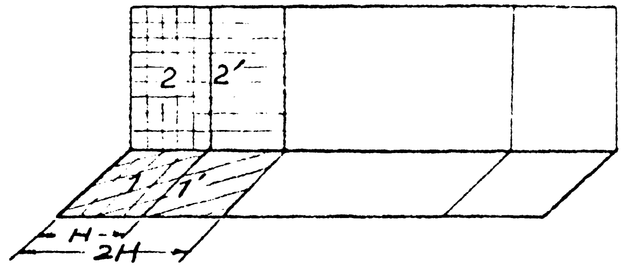


Fig. 2-15 Geometric factor notation.

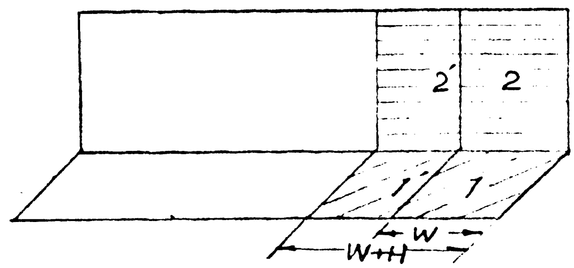


Fig. 2-16 Geometric factor notation.

Statement 40+1 line is the evaluation of the geometric factor from 1 to 2 as shown. Where surface 1 is a ring of width H and surface 2 is a ring of width (XX+W-H). This value is given by GA2 in the program. It should be noted that this is only in the perpendicular form, the complete geometric factor for the ring 1 to the ring 2 would include the factor for parallel geometry.

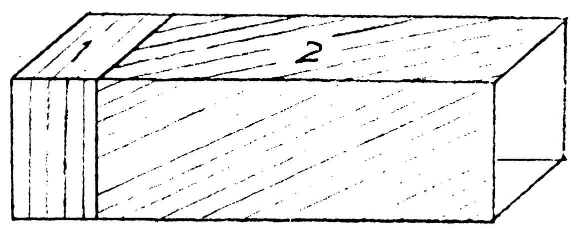


Fig. 2-17 Geometric factor notation.

Statement 40+2 lines is the evaluation of the geometric factor from 1 to 2 as shown. Where surface 1 is a ring of width W and surface 2 is a ring of width XX. This value is given by GAN2 in the program. Again, it should be noted that this is only in the perpendicular form.

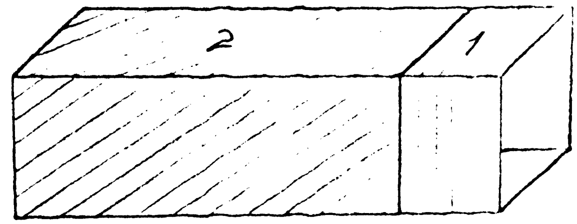


Fig. 2-18 Geometric factor notation.

Statement 40+3 lines is the evaluation of the geometric factor from ring 1 to ring 2 as shown. Where 1 and 2 are both of width H. This value is given by GA(1,2) in the program. Again, it should be noted that this is only in the perpendicular form.

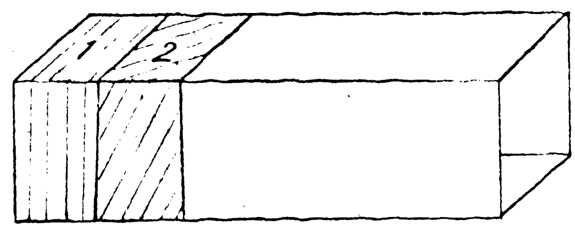


Fig. 2-19 Geometric factor notation.

Statement 42 to statement 43 is the evaluation of the geometric factor from ring 1 to ring I as shown. Where 1 and I are both of width H. I denotes the ring number 3, 4, ..., N. These values are given by (GA(1,I),I=3,N) in the program. Again, it should be noted that this is only in the perpendicular

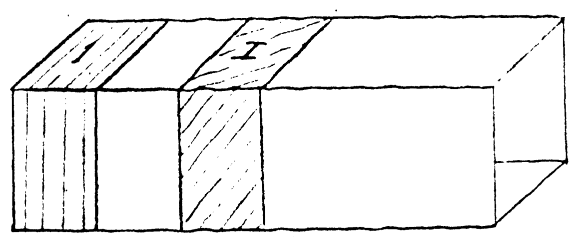


Fig. 2-20 Geometric factor notation.

form.

Statement 44 is the evaluation of the geometric factor from ring L to ring 1 as shown. Where ring L is of width W and ring 1 is of width H. This value is given by GA(L,1) in the program. It should be noted that this is only in the perpendicular form.

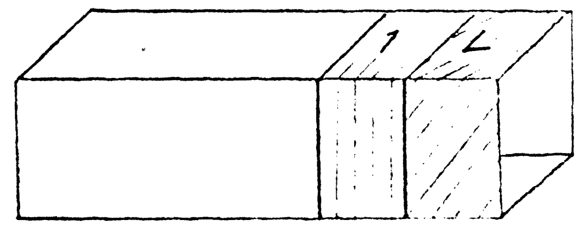


Fig. 2-21 Geometric factor notation.

Statement 44+1 line to statement 45 is the evaluation of the geometric factors from L to I as shown. Where surface L is a ring of width W and surface I is a ring of width H. These values are given by (GA(L,I),I=2,N) in the program. Again, it should be noted that this is only in the perpendicular form.

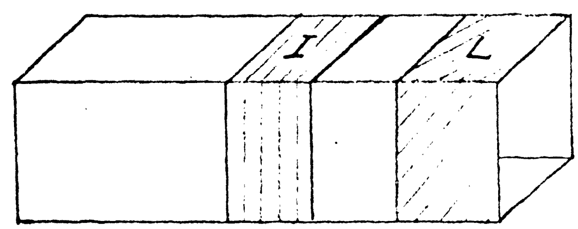


Fig. 2-22 Geometric factor notation.

Geometric factors in parallel form:

Statement 55+1 line to statement 60 is the evaluation of the geometric factors from 1 to 2 as shown by means of Eq.(1-8). Where surface 1 is the half of a ring having a width that varies from H to XX with the increment H, and surface 2 is the other half of the ring. The procedure is then reversed, where surface 1 and surface 2 are of width varing from W to (XX+W), with the increment H. These values are given by (FB(K),K=1,N) and (FBBL(J), J=1,L) in the program. It should be noted that this is only in the parallel form.

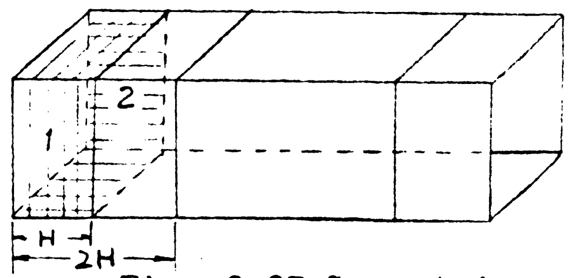


Fig. 2-23 Geometric factor notation.

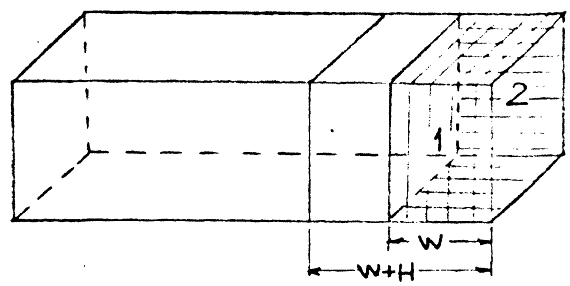


Fig. 2-24 Geometric factor notation.

Statement 60+1 line is the evaluation of the geometric factor from 1 to 2 as shown. Where surface 1 is a ring of width H and surface 2 is a ring of width (XX+W-H). This value is given by GB1 in the program. It should be noted that

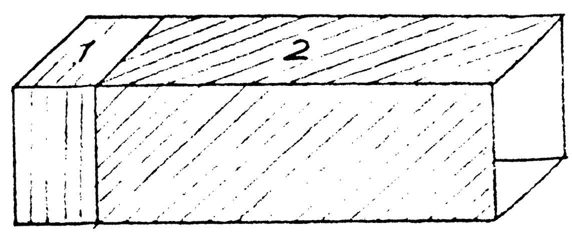


Fig. 2-25 Geometric factor notation.

this is only in the parallel form, the complete geometric factor for ring 1 to the ring 2 would include the factor for perpendicular form.

Statement 60+2 lines is the evaluation of the geometric factor from 1 to 2 as shown. Where surface 1 is a ring of width W and surface 2 is a ring of width XX. This value is given by GBN1 in the program. Again, it should be noted that this only is the parallel form.

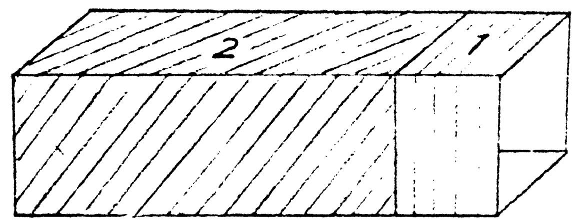


Fig. 2-26 Geometric factor notation.

Statement 60+3 lines is the evaluation of the geometric factor from 1 to 2 as shown. Where surface 1 and surface 2 are the ring of width H. This value is given by GA(1,2) in the program, and is only for the parallel form.

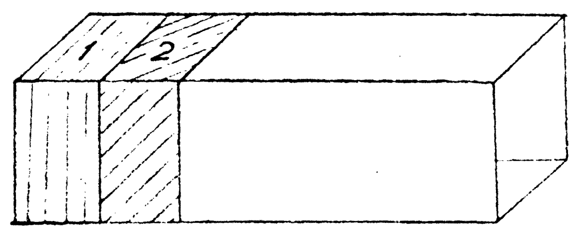


Fig. 2-27 Geometric factor notation.

Statement 62 to statement 63 is the evaluation of the geometric factor from 1 to I as shown.. Where surface 1 and surface I are rings of width H. These values are given by (GB(1,I),

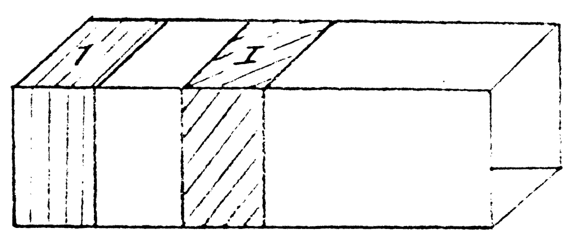


Fig. 2-28 Geometric factor notation.

I=3,N) in the program. It should be noted that this is only in the parallel form.

Statement 64 is the evaluation of the geometric factor from L to 1 as shown. Where surface L is a ring of width W and surface 1 is a ring of width H. This value is only of the parallel form, given by GB(L,1) in the program.

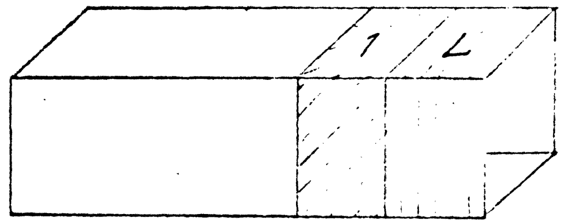


Fig. 2-29 Geometric factor notation.

Statement 64+1 line to statement 65 is the evaluation of the geometric factors from L to I as shown. Where surface L is a ring of width W and surface I is a ring of width H. These values are given by (GB(L,I),I=2,N) in the program. Again, it should be noted that this is only in the parallel form, the complete geometric factor would include the factor for perpendicular form.

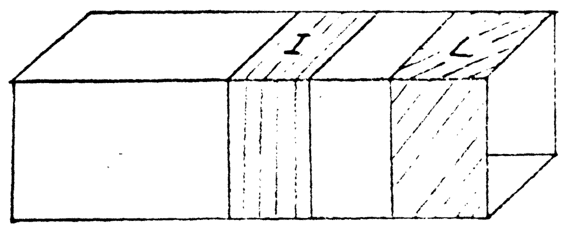


Fig. 2-30 Geometric factor notation.

Statement 65+1 line and statement 65+2 lines are the evaluation of the surface area of both the ring of width H and W, respectively. These values are given by AN and ANL, respectively, in the program.

Complete geometric factors and configuration factors:

In the following, 1, 2, ..., N, L will denote the subdivided sections as shown in Fig.(2-1) unless otherwise specified.

Statement 65+3 lines and statement 65+4 lines are the evaluation of the complete configuration factor F_{11} and the complete geometric factor G_{11}, respectively, where surface 1 is a ring of width H. These values are given by F(1,1) and G(1,1), respectively, in the program.

Statement 65+5 lines to statement 71 is the evaluation of the complete geometric factors G_{1i}. Where surface 1 and surface i are rings of width H. These values are given by (G(1,I),I 2,N) in the program.

Statement 71+1 line to statement 74 is the evaluation of the complete geometric factors G_{ij}, where surface i and surface j are rings of width H, i denotes 1, 2, ..., N and j denotes 1, 2, ..., N. These values are evaluated by means of the following relations, since the source section is divided equally. The values of each column are the same and the values of each row are symetrical to G_{ii}, that is, $G_{i(i+N)} = G_{i(i-N)}$ for $N \leq i$. These values are given by (G(L,1),I=1,N) and G(M,K), where $M = 2, 3, \ldots, N$ and $K = N, N-1, \ldots, 2$ in the program.

$$\begin{matrix} G_{11} & G_{12} & G_{13} & \cdots\cdots & G_{1(N-1)} & G_{1N} \\ G_{21} & G_{22} & G_{23} & G_{24} & \cdots\cdots & G_{2N} \\ G_{31} & G_{32} & G_{33} & G_{34} & G_{35} \cdots & G_{3N} \\ \vdots \\ G_{N1} & G_{N2} & \cdots\cdots & G_{NN} \end{matrix} \qquad (2\text{-}6)$$

Statement 74+1 line is the evaluation of the complete configuration factor F_{LL}, where surface L is a ring of width W, i.e., the ring of the sink. This value is given by F(L,L) in the program.

Statement 74+2 lines to statement 75 is the evaluation of the complete geometric factors from L to I as shown. Where surface L is a ring of width W and surface I is a ring of width H. These values are given by (GC(L,I),I=1,N) in the program. It should be noted that I denotes the ring number 1, 2, ..., N which is specified from the right hand side to the left hand side.

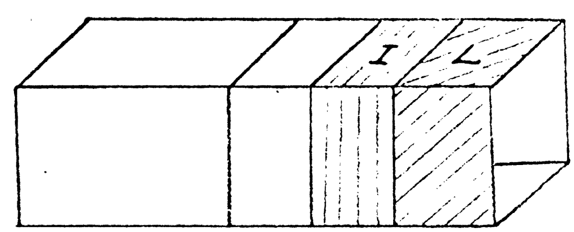


Fig. 2-31 Geometric factor notation.

Statement 75+1 line to statement 76 is the evaluation of the complete geometric factors G_{iL}. Where surface i is a ring of width H and surface L is a ring of width W. These values are given by (G(I,L),I=1,N) in the program.

Statement 76+1 line to statement 77 is the evaluation of the complete geometric factors G_{Li}. Where surface L is a ring of width W and i is a ring of width H. These values are given by (G(L,I),I=1,N) in the program.

Statement 77+1 line to statement 80 is the evaluation of the complete configuration factors F_{ij}, where i=1, 2, ..., N+1 and j=1, 2, ..., N+1. These values are given by ((F(I,J),J=1,L),I=1,L) in the program.

The completion of the program requires the use of Eq.(2-1), Eq.(2-2), Eq.(2-4) and Eq.(2-5).

The other symbols used in the program are defined as follows:

PI	π.
STBOC	Stefan-Boltzmann constant σ, Btu/hr-ft^2-$^\circ$R.
N	Number of subdivided sections along XX.
NN	Number of sets of emissivities and temperatures.
TT(I)	Temperatures corresponding to NN.
EE(I)	Emissivities corresponding to TT(I).
T1, T2, T3.	Temperatures corresponding to the two ends of the source and the sink, respectively, as defined previously.
T(I)	Mean temperatures of subdivisions corresponding to N.
L	Denotes the sink section.
L1, L2.	Variables as defined.
R	The value of N in the floating point.
X, Y, Z.	Variables defined in Eq.(1-7) and Eq.(1-8).
H	Subdivided interval width in the XX direction.
C1, C2, C3, C4: CSR; FC1, FC2, ..., FC6; R1, R2, CC1, CC2, CC3; FD1, FD2, ..., FD5; FNO, FNU and FEM.	Variables as defined in the program.
FBD(I), FDDL(I).	The geometric factors for the two sets of opposed rectangles defined in the program.
AN	Surface area of each subdivision.

ANL Surface area of the sink section.

EMI(I), REF(I).

Emissivities and reflectivities corresponding to T(I).

D(I,J) Matrix of the coefficients of Eq.(1-9).

QASUB(I) Energy absorbed by subdivided surface J from surface I.

QABSO Total energy absorbed by subdivided surface J.

Q(J) The energy loss of subdivided surface J.

QLOSS The total energy loss of the source section.

All other symbols have the same meanings as defined previously.

An example is given in the following.

Material: Al, XX = 10 ft., YY = ZZ = W = 1 ft.,

(A) $T_1 = 560\ ^oR$, $T_2 = 540\ ^oR$, $T_3 = 530\ ^oR$.

(B) $T_1 = 800\ ^oR$, $T_2 = 600\ ^oR$, $T_3 = 570\ ^oR$.

(C) $T_1 = 1000\ ^oR$, $T_2 = 650\ ^oR$, $T_3 = 600\ ^oR$.

(D) $T_1 = 2000\ ^oR$, $T_2 = 1000\ ^oR$, $T_3 = 800\ ^oR$.

The emissivities corresponding to the temperatures are given by,

TT(I)	600	800	1000	1500	2000 ^oR
EE(I)	.08	.095	0.10	0.12	0.16

The results are tabulated and the curves are plotted in the following pages.

TABLE I

The energy loss of the source and sink of Case A

(A) $T_1 = 560\ ^oR$, $T_2 = 540\ ^oR$, $T_3 = 530\ ^oR$.

N	Q_{source} Btu/hr	Q_{sink} Btu/hr
2	158.65	20.28
3	152.23	20.49
4	146.13	20.80
5	141.15	21.09
6	137.23	21.34
7	134.19	21.54
8	131.81	21.69
9	129.95	21.82
10	128.47	21.92

(B) $T_1 = 800\ ^oR$, $T_2 = 600\ ^oR$, $T_3 = 570\ ^oR$.

N	Q_{source} Btu/hr	Q_{sink} Btu/hr
2	490.28	13.30
3	480.16	16.18
4	465.72	18.25
5	452.62	19.75
6	441.82	20.85
7	433.18	21.67
8	426.33	22.29
9	420.88	22.76
10	416.53	23.12

TABLE I (Continued)

(C) $T_1 = 1000\ ^oR$, $T_2 = 650\ ^oR$, $T_3 = 600\ ^oR$.

N	Q_{source} Btu/hr	Q_{sink} Btu/hr
2	1027.71	-5.88
3	1022.57	2.02
4	999.96	7.24
5	976.70	10.83
6	956.56	13.37
7	940.02	15.21
8	926.69	16.57
9	915.96	17.60
10	907.32	18.37

(D) $T_1 = 2000\ ^oR$, $T_2 = 1000\ ^oR$, $T_3 = 800\ ^oR$.

N	Q_{source} Btu/hr	Q_{sink} Btu/hr
2	14365.57	-628.64
3	15105.72	-469.36
4	15213.27	-374.27
5	15134.14	-313.86
6	15004.90	-273.54
7	14872.59	-245.54
8	14752.72	-225.46
9	14649.05	-210.63
10	14561.37	-199.43

PLATE 1-A

Energy loss of the source section of the duct

$T_1 = 560\ ^{o}R$

$T_2 = 540\ ^{o}R$

$T_3 = 530\ ^{o}R$

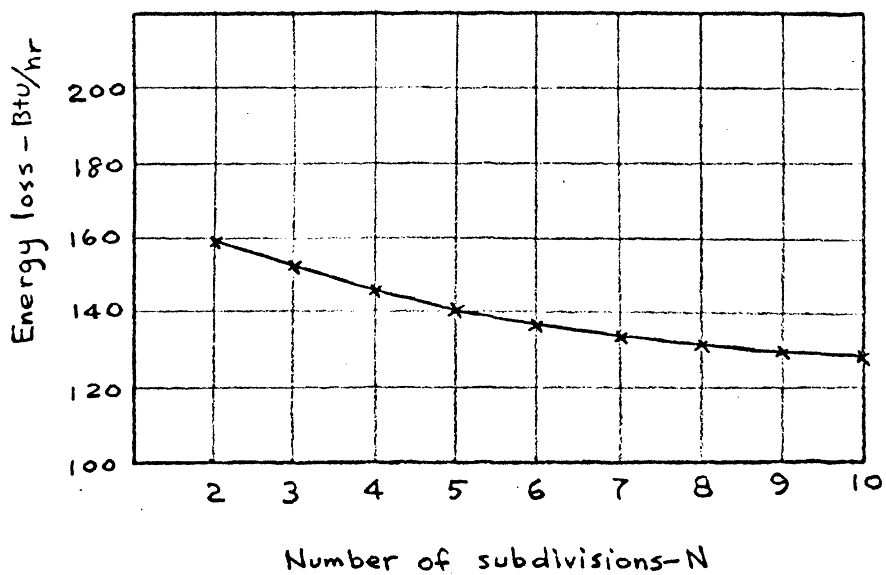


PLATE 1-B

Energy loss of the source section of the duct

$T_1 = 800\ ^oR$

$T_2 = 600\ ^oR$

$T_3 = 570\ ^oR$

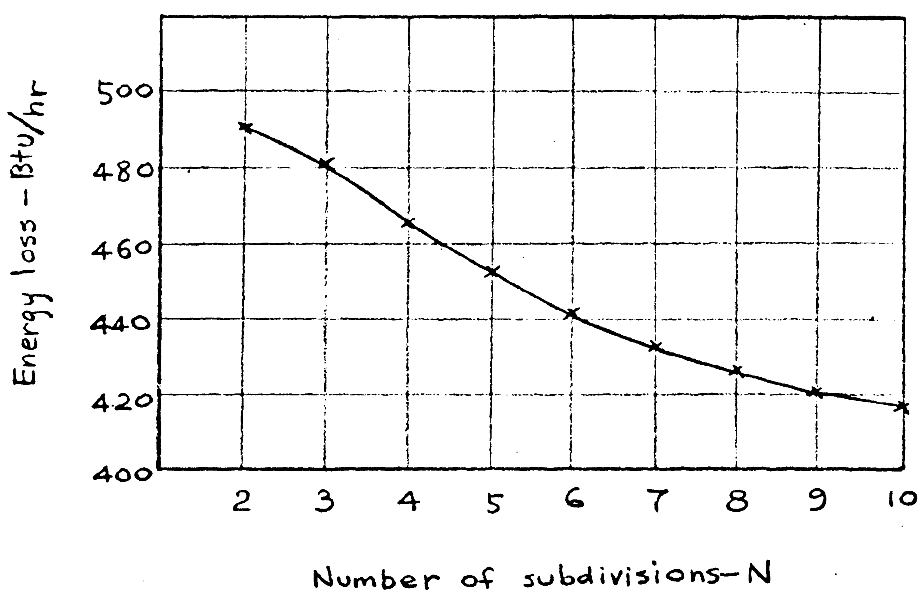


PLATE 1-C

Energy loss of the source section of the duct

$T_1 = 1000\ ^oR$

$T_2 = 650\ ^oR$

$T_3 = 600\ ^oR$

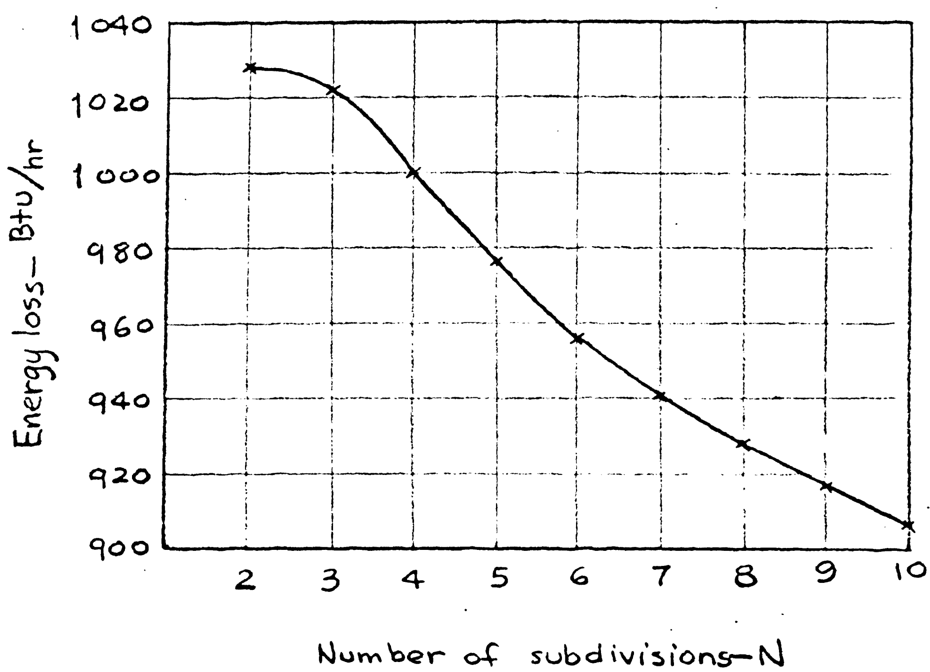


PLATE 1-D

Energy loss of the source section of the duct

$T_1 = 2000\ ^oR$

$T_2 = 1000\ ^oR$

$T_3 = 800\ ^oR$

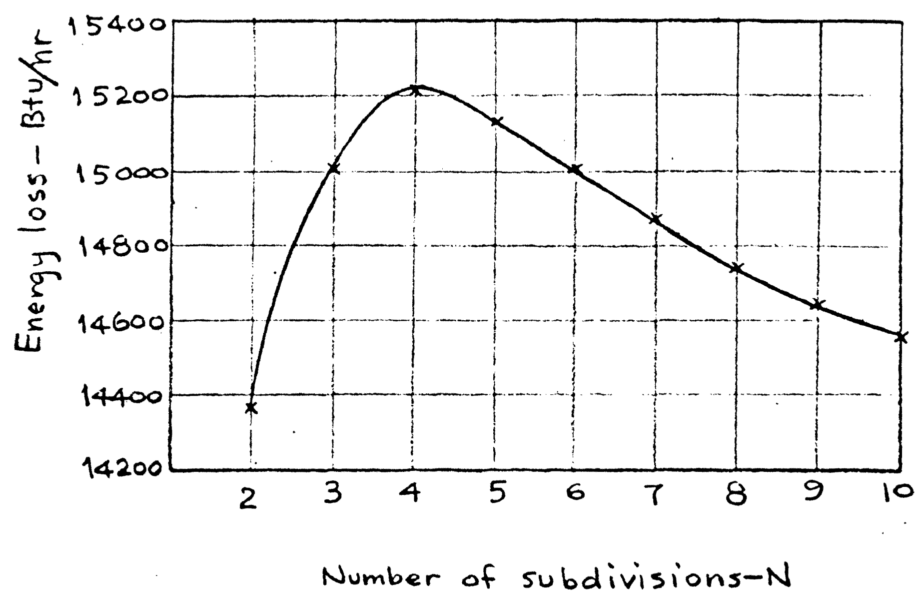


PLATE 2-A

Energy loss of the sink section of the duct

$T_1 = 560\ ^oR$

$T_2 = 540\ ^oR$

$T_2 = 530\ ^oR$

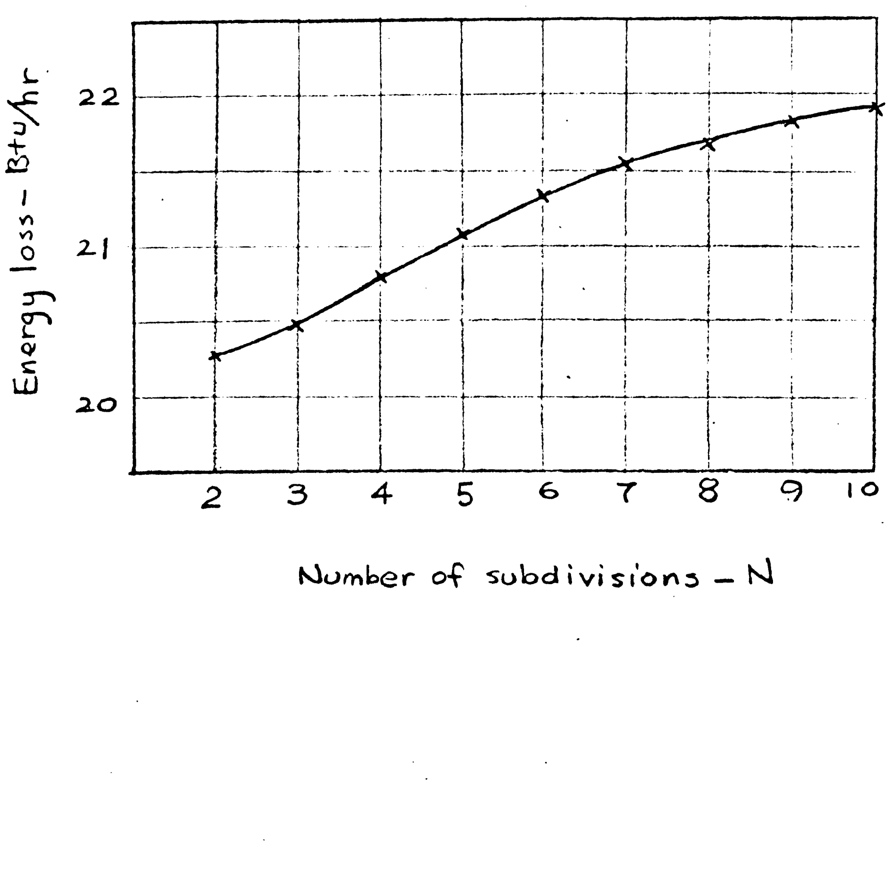


PLATE 2-B

Energy loss of the sink section of the duct

$T_1 = 800\ ^oR$

$T_2 = 600\ ^oR$

$T_3 = 570\ ^oR$

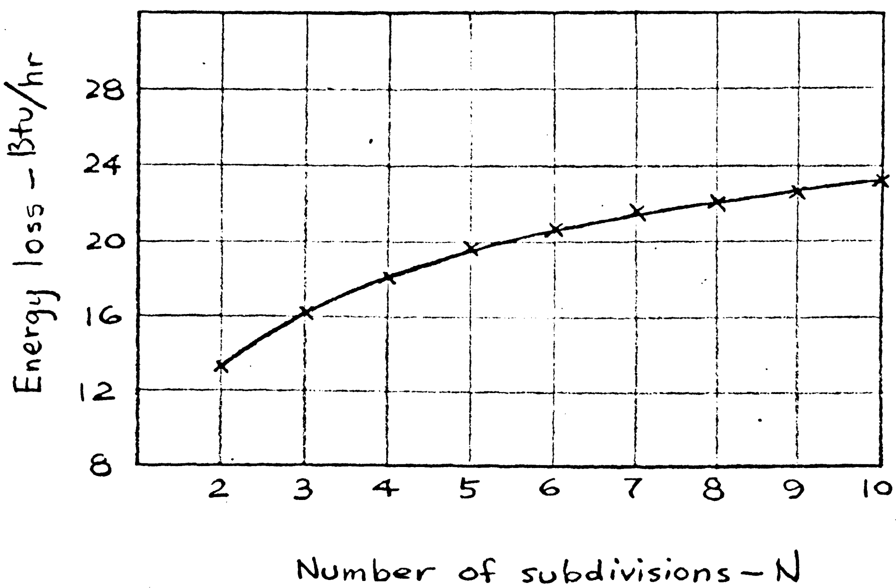


PLATE 2-C

Energy loss of the sink section of the duct

$T_1 = 1000\ ^{o}R$

$T_2 = 650\ ^{o}R$

$T_3 = 600\ ^{o}R$

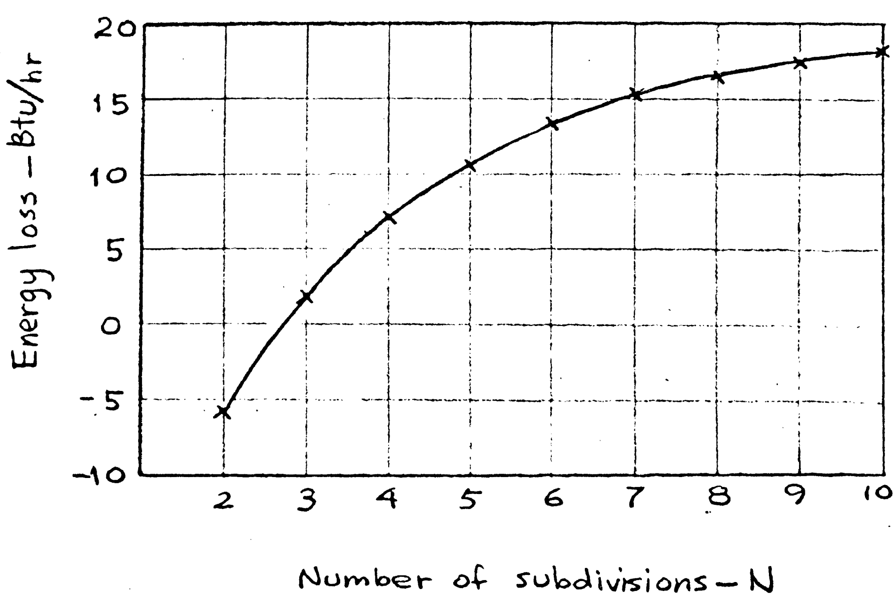


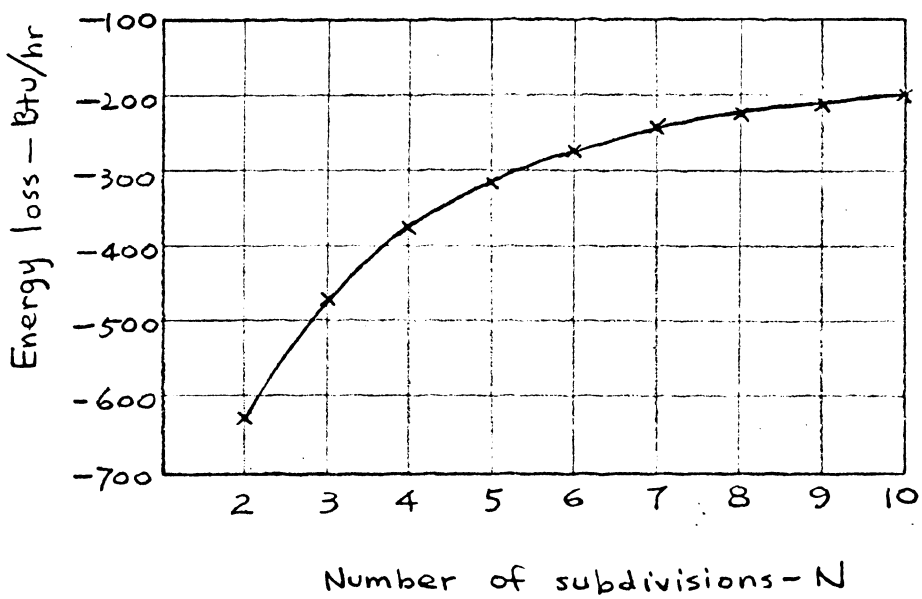
PLATE2-D

Energy loss of the sink section of the duct

$T_1 = 2000\ ^oR$

$T_2 = 1000\ ^oR$

$T_3 = 800\ ^oR$



CASE B: THE RADIATION HEAT TRANSFER ANALYSIS IN RECTANGULAR ENCLOSURE

Assume a box with the dimensions XX by YY by ZZ and with temperatures T_1 and T_2 at the surfaces at the ends of XX. Denote these surfaces as the source and the sink, respectively. The temperature profile along the wall in the dimension XX is assumed to be a straight line (for convenience only). The radiant heat transfer of both end plates is investigated. The diagram of the dimensions of the box and of temperature profile are constructed as shown below.

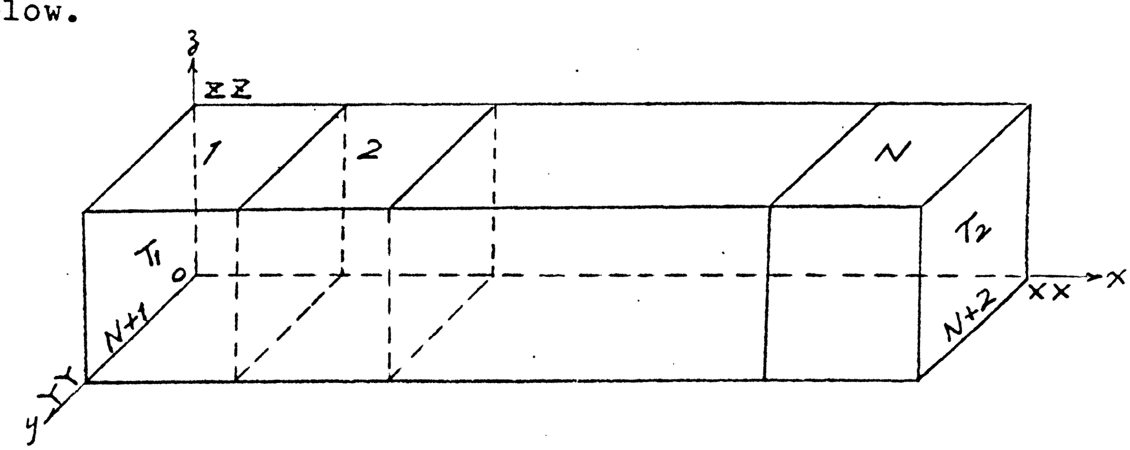


Fig. 2-32 The dimensions of the rectangular box.

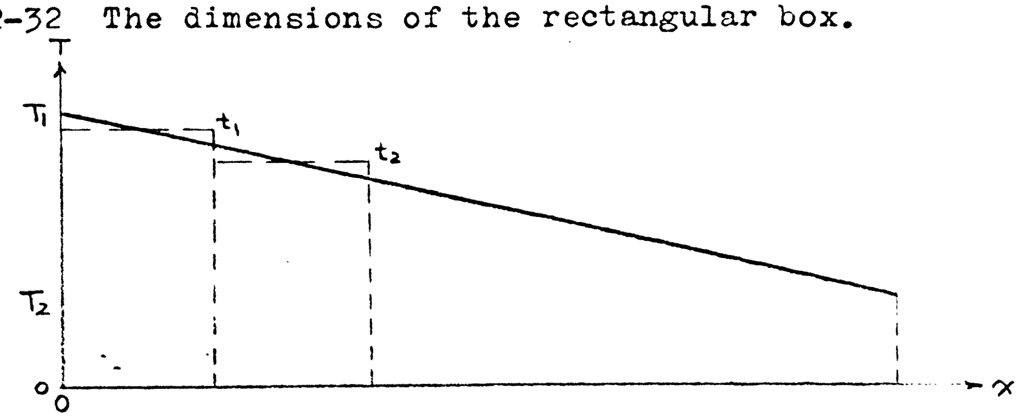


Fig. 2-33 The temperature profile of the wall of the box along XX.

The procedures of solution are similar to that of CASE A. Divide the box along the dimension XX into N equal sections, say A_1, A_2, ..., A_N. The surfaces at both ends of XX are denoted as A_{N+1}, the source, and A_{N+2}, the sink. The mean temperatures of the subdivided sections, denoted t_1, t_2, ..., t_N, are determined by Eq.(2-1) and Eq.(2-2). The temperatures of both end plates are assumed to be uniform and equal to T_1 and T_2, respectively, i.e., $T_{N+1} = T_1$ and $T_{N+2} = T_2$. Then by the Lagrange interpolation formula, Eq.(2-4), the emissivity and the reflectivity of each section corresponding to its temperature may be approximated. These are given by:

ϵ_i where $i = 1, 2, ..., N+2$

and r_i where $i = 1, 2, ..., N+2$.

By applying the formulas derived in PART I, the required quantities can be obtained. First evaluate the geometric factors. The symbol G_{ij} denotes the geometric factors,

where $i = 1, 2, ..., N+2$

and $j = 1, 2, ..., N+2$.

After determining geometric factors, the configuration factors are determined. The symbol F_{ij} denotes the configuration factors, where

$i = 1, 2, ..., N+2$

and $j = 1, 2, ..., N+2$.

The absorption factors are evaluated by the Gauss-Jordan reduction method. This method allows the set of equations developed by Gebhart to be solved and the values of B_{ij} to be obtained, where

$$i = 1,\ 2,\ \ldots,\ N+2$$

and $$j = 1,\ 2,\ \ldots,\ N+2.$$

The heat loss of section j is obtained from Eq.(1-18),

$$Q_j = \epsilon_j \alpha T_j^4 A_j - \sum_{i=1}^{N+2} \epsilon_i \alpha T_i^4 B_{ij} A_i$$

where $j = 1,\ 2,\ \ldots,\ N\ 2.$

The total heat loss of the box is then given by,

$$Q = \sum_{j=1}^{N+2} Q_j = 0 \qquad (2\text{-}7)$$

since the complete enclosure is involved.

The procedures may be more clearly seen from the computer program following. The descriptions are given in detail following the computer program.

COMPUTER PROGRAM II

```
C       PROGRAM II  RADIATION HEAT TRANSFER ANALYSIS IN THE RECTANGULAR
C       ENCLOSURE
        DIMENSION FAAL(12),GA(12,12),FABL(2),FBBL(12),GAAL(12,12),FBCL(2),
       1FCCL(12),GB(12,12),F(14,14),G(14,14),T(14),D(14,16),Q(14),EMI(12),
       2REF(12),TT(10),EE(10),QASUB(12)
        STBOC=.1714E-8
        PI=3.1415926
        READ 101, NN
        READ 100, (TT(I),I=1,NN),(EE(I),I=1,NN)
        PRINT 101, NN
        PRINT 100, (TT(I),I=1,NN),(EE(I),I=1,NN)
        READ 100, XX,YY,ZZ,T1,T2
        PRINT 100, XX,YY,ZZ,T1,T2
      1 READ 101, N
        PRINT 101, N
        N1=N+1
        N2=N+2
        N3=N+3
        NM1=N-1
C       EVALUATION OF THE GEOMETRIC FACTORS
C       FOR THE PERPENDICULAR FORM
        R=N
        H=XX/R
        X=0.0
        Y=YY
        Z=ZZ
        DO 20 I=1,N
        X=X+H
C       EQUATION OF THE GEOMETRIC FACTOR FOR THE PERPENDICULAR RECTANGLES
C       WITH A COMMON EDGE
        C1=X**2
        C2=Y**2
        C3=Z**2
        CC1=C1+C2+C3
        CC2=C1+C2
        CC3=C2+C3
        CC4=C1+C3
        CSR=SQRTF(CC3)
        FC1=X*Y*ATANF(X/Y)
        FC2=X*Z*ATANF(X/Z)
        FC3=-(X*CSR*ATANF(X/CSR))
        FC4=-(C1*LOGF(CC1*C1/(CC2*CC4)))/4.0
        FC5=(C2*LOGF(CC1*C2/(CC2*CC3)))/4.0
        FC6=(C3*LOGF(CC1*C3/(CC3*CC4)))/4.0
     20 FAAL(I)=(FC1+FC2+FC3+FC4+FC5+FC6)/PI
```

```
      GA(1,2)=(FAAL(2)-2.0*FAAL(1))*4.0
      IF (N-2) 26,26,24
   24 DO 25 I=3,N
   25 GA(1,I)=(FAAL(I)-2.0*FAAL(I-1)+FAAL(I-2))*4.0
   26 Y=H
      DO 40 I=1,N
      X=YY
      Z=ZZ
      DO 30 K=1,2
      C1=X**2
      C2=Y**2
      C3=Z**2
      CC1=C1+C2+C3
      CC2=C1+C2
      CC3=C2+C3
      CC4=C1+C3
      CSR=SQRTF(CC3)
      FC1=X*Y*ATANF(X/Y)
      FC2=X*Z*ATANF(X/Z)
      FC3=-(X*CSR*ATANF(X/CSR))
      FC4=-(C1*LOGF(CC1*C1/(CC2*CC4)))/4.0
      FC5=(C2*LOGF(CC1*C2/(CC2*CC3)))/4.0
      FC6=(C3*LOGF(CC1*C3/(CC3*CC4)))/4.0
      FABL(K)=(FC1+FC2+FC3+FC4+FC5+FC6)/PI
      X=ZZ
   30 Z=YY
      FBBL(I)=FABL(1)+FABL(2)
   40 Y=Y+H
      GAAL(1,N1)=FBBL(1)*2.0
      DO 45 I=2,N
   45 GAAL(I,N1)=(FBBL(I)-FBBL(I-1))*2.0
C     FOR THE PARALLEL FORM
      X=H
      DO 60 I=1,N
      Y=YY
      Z=ZZ
      DO 50 K=1,2
C     EQUATION OF THE GEOMETRIC FACTOR FOR THE PARALLEL AND OPPOSED
C     RECTANGLES
      R1=X/Z
      R2=Y/Z
      C1=R1**2
      C2=R2**2
      C3=1.0+C1
      C4=1.0+C2
      CC1=LOGF((C3*C4)/(C2+C3))
```

```
      CC2=SQRTF(C3)
      CC3=SQRTF(C4)
      FD1=CC1/(R1*R2)
      FD2=-(2.0*ATANF(R2))/R1
      FD3=-(2.0*ATANF(R1))/R2
      FD4=(2.0*CC2*ATANF(R2/CC2))/R1
      FD5=(2.0*CC3*ATANF(R1/CC3))/R2
      FBCL(K)=(FD1+FD2+FD3+FD4+FD5)*X*Y/PI
      Y=ZZ
   50 Z=YY
      FCCL(I)=FBCL(1)+FBCL(2)
   60 X=X+H
      GB(1,2)=FCCL(2)-2.0*FCCL(1)
      IF (N-2) 66,66,64
   64 DO 65 I=3,N
   65 GB(1,I)=FCCL(I)-2.0*FCCL(I-1)+FCCL(I-2)
C     EVALUATION OF THE GEOMETRIC FACTORS AND THE CONFIGURATION FACTORS
C     BETWEEN ANY TWO SUBDIVISIONS
   66 X=ZZ
      Y=YY
      Z=XX
      R1=X/Z
      R2=Y/Z
      C1=R1**2
      C2=R2**2
      C3=1.0+C1
      C4=1.0+C2
      CC1=LOGF((C3*C4)/(C2+C3))
      CC2=SQRTF(C3)
      CC3=SQRTF(C4)
      FD1=CC1/(R1*R2)
      FD2=-(2.0*ATANF(R2))/R1
      FD3=-(2.0*ATANF(R1))/R2
      FD4=(2.0*CC2*ATANF(R2/CC2))/R1
      FD5=(2.0*CC3*ATANF(R1/CC3))/R2
      G(N1,N2)=(FD1+FD2+FD3+FD4+FD5)*X*Y/PI
      AREA=2.0*H*(YY+ZZ)
      GPERP=4.0*(FAAL(N)-FAAL(N-1)-FAAL(1))
      GPARA=FCCL(N)-FCCL(N-1)-FCCL(1)
      GPEND=GAAL(1,N1)+GAAL(N,N1)
      F(1,1)=1.0-(GPERP+GPARA+GPEND)/AREA
      G(1,1)=F(1,1)*AREA
      ENDAR=YY*ZZ
      DO 70 I=2,N
   70 G(1,I)=GA(1,I)+GB(1,I)
      DO 75 I=1,N
```

```
   75 G(I,1)=G(1,I)
      DO 90 M=2,N
      K=N
   80 G(M,K)=G(M-1,K-1)
      K=K-1
      IF (K-1) 90,90,80
   90 CONTINUE
      DO 95 I=1,N
      DO 95 J=1,N
   95 F(I,J)=G(I,J)/AREA
      F(N1,N1)=0.0
      F(N2,N2)=0.0
      F(N1,N2)=G(N1,N2)/ENDAR
      F(N2,N1)=F(N1,N2)
      DO 96 I=1,N
   96 F(N1,I)=GAAL(I,N1)/ENDAR
      K=N
      DO 97 I=1,N
      F(N2,K)=F(N1,I)
   97 K=K-1
      DO 98 I=1,N
   98 F(I,N1)=GAAL(I,N1)/AREA
      K=N
      DO 99 I=1,N
      F(I,N2)=F(K,N1)
   99 K=K-1
      PRINT 102
      PRINT 100, ((F(I,J),J=1,N2),I=1,N2)
C     EVALUATION OF THE MEAN TEMPERATURES OF THE SUBDIVISIONS
      T(1)=T1-(T1-T2)/(2.0*R)
      DO 150 I=1,NM1
  150 T(I+1)=T(I)-(T1-T2)/R
      T(N1)=T1
      T(N2)=T2
      PRINT 103
      PRINT 100, (T(I),I=1,N2)
C     EVALUATION OF THE MEAN EMISSIVITIES OF THE SUBDISIONS BY MEANS OF
C     LAGRANGIAN INTERPOLATION FORMULA
      DO 170 J=1,N2
      FEM=0.0
      DO 165 L=1,NN
      FNU=1.0
      FNO=1.0
      DO 164 I=1,NN
      IF (I-L) 161,164,161
  161 FNU=FNU*(T(J)-TT(I))
```

```
      FNO=FNO*(TT(L)-TT(I))
  164 CONTINUE
  165 FEM=FEM+FNU/FNO*EE(L)
  170 EMI(J)=FEM
      PRINT 104
      PRINT 100,(EMI(J),J=1,N2)
C     EVALUATION OF THE MEAN REFLECTIVITIES OF THE SUBDIVISIONS BY MEANS
C     KIRCHHOF S IDENTITY
      DO 175 I=1,N2
  175 REF(I)=1.0-EMI(I)
      PRINT 105
      PRINT 100, (REF(I),I=1,N2)
      DO 199 J=1,N2
      PRINT 106, J
C     SOLUTION OF THE ABSORPTION FACTORS BY MEANS OF GAUSS-JORDAN
C     REDUCTION METHOD
      DO 180 I=1,N2
  180 D(I,I)=F(I,I)*REF(I)-1.0
      DO 182 I=1,N2
      DO 182 M=1,N2
      IF (I-M) 181,182,181
  181 D(I,M)=F(I,M)*REF(M)
  182 CONTINUE
      DO 183 I=1,N2
  183 D(I,N3)=-(F(I,J))*EMI(J)
      CALL GAUJOR (D,N2,N3,14,16)
      PRINT 107
      PRINT 100, (D(I,N3),I=1,N2)
C     EVALUATION OF THE ABSORPTION ENERGY FROM SURFACE I TO SURFACE J
      DO 185 K=1,N
  185 QASUB(K)=D(K,N3)*STBOC*T(K)**4*EMI(K)*AREA
      DO 186 K=N1,N2
  186 QASUB(K)=D(K,N3)*STBOC*T(K)**4*EMI(K)*ENDAR
      PRINT 108
      PRINT 100, (QASUB(I),I=1,N2)
C     EVALUATION OF THE TOTAL ABSORPTION ENERGY OF SURFACE J
      QABSO=0.0
      DO 187 I=1,N2
  187 QABSO=QABSO+QASUB(I)
      PRINT 109
      PRINT 100, QABSO
C     EVALUATION OF THE RADIANT HEAT TRANSFER FROM SURFACE J
      IF (J-N1) 191,192,192
  191 Q(J)=STBOC*T(J)**4*AREA*EMI(J)-QABSO
      GO TO 199
  192 Q(J)=STBOC*T(J)**4*ENDAR*EMI(J)-QABSO
```

```
  199 CONTINUE
      PRINT 110
      PRINT 100, (Q(J),J=1,N2)
C     EVALUATION OF THE TOTAL HEAT TRANSFER OF THE SOURCE
      QLOSS=0.0
      DO 200 IJK=1,N2
  200 QLOSS=QLOSS+Q(IJK)
      PRINT 111
      PRINT 100, QLOSS
      PRINT 112
      PRINT 100, Q(N1),Q(N2)
      GO TO 1
  100 FORMAT (4F18.8)
  101 FORMAT (4I18)
  102 FORMAT (10X46HCONFIGURATION FACTORS ((F(I,J),J=1,N2),I=1,N2))
  103 FORMAT (10X42HMEAN TEMPERATURES (T(I),I=1,N2) IN RANKINE)
  104 FORMAT (10X28HMEAN EMISSIVITY OF EACH PART)
  105 FORMAT (10X25HREFLECTIVITY OF EACH PART)
  106 FORMAT (10X2HJ=,I2)
  107 FORMAT (10X34HABSORPTION FACTORS (B(I,J),I=1,N2))
  108 FORMAT (10X39HABSORPTION ENERGY FROM EACH PART BTU/HR)
  109 FORMAT (10X41HTOTAL ENERGY ABSORBED BY EACH PART BTU/HR)
  110 FORMAT (10X45HENERGY LOSS OF EACH PART (Q(J),J=1,N2 BTU/HR))
  111 FORMAT (10X24HTOTAL ENERGY LOSS BTU/HR)
  112 FORMAT (10X45HENERGY LOSS OF THE SOURCE AND THE SINK BTU/HR)
      END
```

DETAILED DESCRIPTION OF PROGRAM II

Geometric factors in perpendicular form:

Statement 1+10 lines to statement 20 is the evaluation of the geometric factors from 1 to 2 as shown. These values are obtained by using Eq.(1-7) with the increment H, and (FAAL(I), I=1,N) is the program variable.

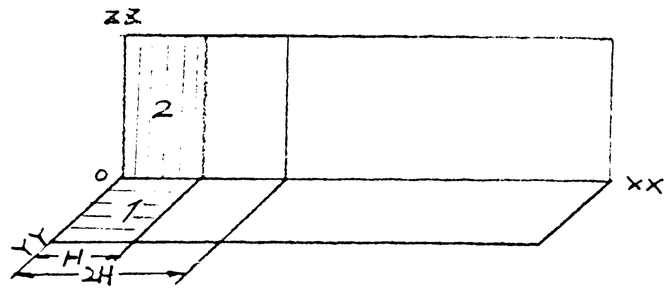


Fig. 2-34 Geometric factor notation.

Statement 20+1 line is the evaluation of the geometric factor from 1 to 2 as shown. Here surface 1 and surface 2 are the rings of width H. This value is given by GA(-1,2) in the program. It should be noted that this is only in the perpendicular form, the complete geometric factor for ring 1 to ring 2 would include the factor for parallel geometry.

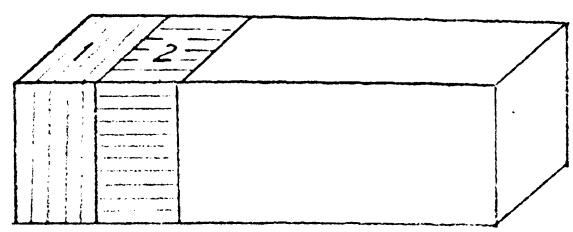


Fig. 2-35 Geometric factor notation.

Statement 24 to statement 25 is the evaluation of the geometric factors from 1 to I as shown. Here surface 1 and surface I are the rings of width H,

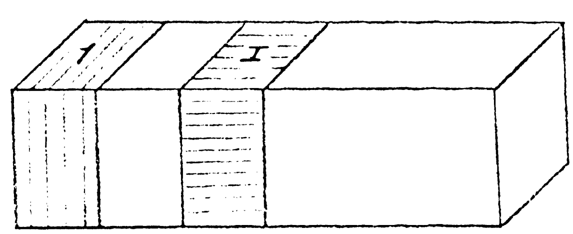


Fig. 2-36 Geometric factor notation.

I denotes the ring number as 3, 4, ..., N. These values are given by (G(1,I),I=3,N) in the program. Again, it should be noted that these values are only for the perpendicular form.

Statement 26 to statement 40 is the evaluation of the geometric factors from 1 to 2 as shown. Surface 2 is the end plate A_{N+1} and surface 1 is the half ring of changing width from H in increments of H to XX. These values are given by (FBBL(I),I=1,N) in the program.

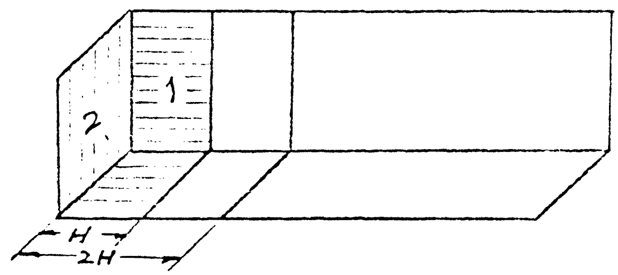


Fig. 2-37 Geometric factor notation.

Statement 40 + 1 line is the evaluation of the geometric factor from 1 to N1 as shown. Surface 1 is the ring of width H and surface N1 is the end plate A_{N+1}. This value is given by GAAL(1,N1) in the program.

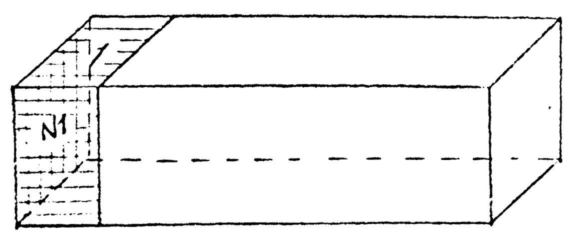


Fig. 2-38 Geometric factor notation.

Statement 40 + 2 lines to statement 45 is the evaluation of the geometric factors from I to N1 as shown. Here surface N1 is the end plate A_{N+1} and

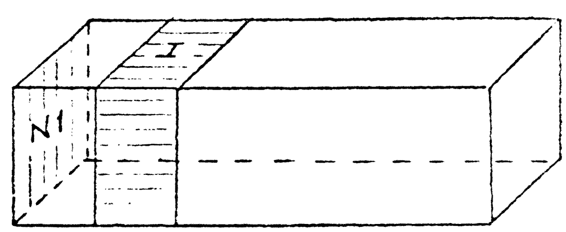


Fig. 2-39 Geometric factor notation.

surface I is the ring of width H, I denotes the ring number as 2, 3, ..., N. These values are given by (GAAL(I,N1),I=2,N) in the program.

Geometric factors in parallel form:

Statement 45+1 line to statement 60 is the evaluation of the geometric factors from 1 to 2 as shown. This is evaluated by using Eq.(1-8). Here surface 1 is the half ring of changing width from H to XX with the increment H and surface 2 is the other half ring corresponding to surface 1. These values are given by (FCCL(I),I=1,N) in the program. It should be noted that these values are only for the parallel form, the complete geometric factor for surface 1 to surface 2 would include the factor for the perpendicular geometry.

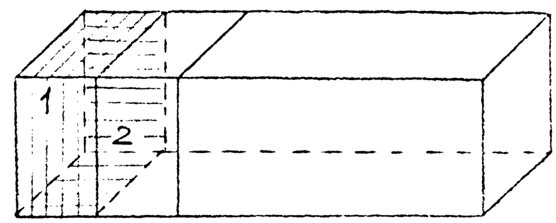


Fig. 2-40 Geometric factor notation.

Statement 60+1 line is the evaluation of the geometric factor from 1 to 2 as shown. Where surface 1 and surface 2 are rings of width H. It, also, should be noted that this is only in

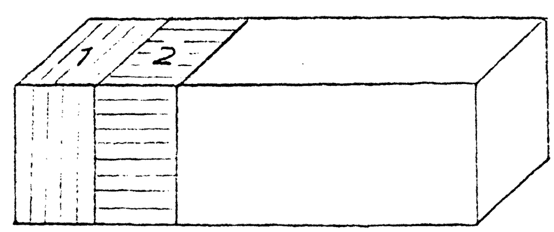


Fig. 2-41 Geometric factor notation.

the parallel form. This value is given by GB(1,2) in the program.

Statement 64 to statement 65 is the evaluation of the geometric factor from 1 to I as shown. Where surface 1 and surface I are rings of width H, and I denotes the ring number as 3, ..., N.

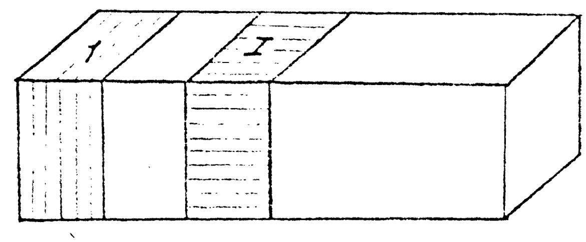


Fig. 2-42 Geometric factor notation.

These values are given by (GB(1,I),I=3,N) in the program. Again, it should be noted that these values are only for the parallel form.

Statement 66 to statement 66+17 lines is the evaluation of the geometric factor from N1 to N2 as shown. Where surface N1 and surface N2 are the end plates A_{N1} and A_{N2}, respectively.

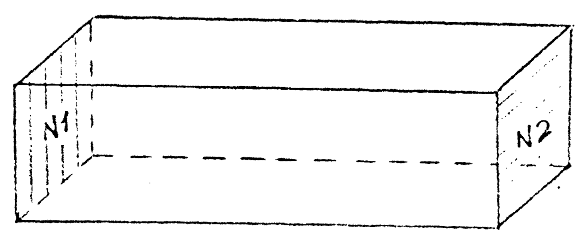


Fig. 2-43 Geometric factor notation.

This value is given by G(N1,N2) in the program.

Statement 66+18 lines is the evaluation of the surface area of each subdivision, i.e., the area of the ring of width H. This value is given by AREA in the program.

Statement 66+19 lines is the evaluation of the geometric factor from 1 to 2 as shown in Fig.(2-44). Where surface 1 is the ring of width H and surface 2 is the ring of width (XX−H). This value is given by

GPERP in the program. It should be noted that this is only in the perpendicular form.

Statement 66 + 20 lines is the evaluation of the geometric factor from 1 to 2 as shown. However, this is only in the parallel form. This value is given by GPARA in the program.

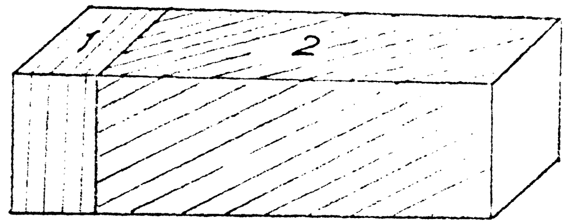


Fig. 2-44 Geometric factor notation.

Complete geometric factors and configuration factors:

In the following, 1, 2, ..., N, N1, N2 will denote the subdivisions as shown in Fig.2-32 unless otherwise specified.

Statement 66 + 21 lines is the evaluation of the geometric factor from ring 1 to both end plates. This value is given by GPEND in the program.

Statement 66+22 lines to statement 66+23 lines is the evaluation of the complete configuration factor F_{11} and the complete geometric factor G_{11} of the ring of width H. These values are given by F(1,1) and G(1,1), respectively.

Statement 66 + 25 lines to statement 70 is the evaluation of the geometric factors from 1 to I as shown. Here surface 1 and surface I are rings of width

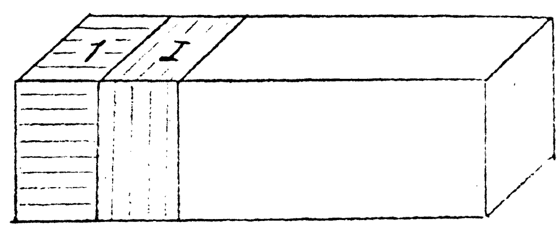


Fig. 2-45 Geometric factor notation.

H, and I denotes the ring number as 2, 3, ..., N. These values are for complete geometry and given by (G(1,I),I=2,N) in the program.

Statement 70+1 line to statement 90 is the evaluation of the complete geometric factors G_{ij} where $i=1, 2, \ldots, N$ and $j=1, 2, \ldots, N$ by using Eq.(2-6). These values are given by (G(I,1),I=1,N) and G(M,K) where M = 2, 3, ..., N and K = N, N-1, ..., 2 in the program.

Statement 90+1 line to statement 99 is the evaluation of the complete configuration factors F_{ij}, where $i=1, 2, \ldots, N+2$ and $j=1, 2, \ldots, N+2$. These values are given by ((F(I,J),J=1,N2),I=1,N2) in the program.

The program is completed by the use of Eq.(2-1), Eq.(2-2), Eq.(2-4) and Eq.(2-5).

The other symbols used in the program are defined as follows:

N1, N2, N3, NM1.

Fixed point variables defined in the program.

FABL(K) The geometric factor for the perpendicular form from any subdivided section of the wall of the box in XX direction to the end plate.

FBCL(K) The geometric factors for the two sets of the opposed, parallel rectangles.

All other symbols involved have the same meanings as defined previously.

An example is given in the following.

Material: Al, XX = 10 ft., YY = ZZ = 1 ft.,

(A) $T_1 = 560\ ^oR$, $T_2 = 530\ ^oR$.

(B) $T_1 = 800\ ^oR$, $T_2 = 570\ ^oR$.

(C) $T_1 = 1000\ ^oR$, $T_2 = 600\ ^oR$.

(D) $T_1 = 2000\ ^oR$, $T_2 = 1000\ ^oR$.

The emissivities corresponding to the temperatures are given in CASE A. The results are tabulated and the curves are plotted on the following pages.

TABLE II

The energy loss of the source and sink of Case B

(A) $T_1 = 560\ ^oR$, $T_2 = 530\ ^oR$.

N	Q_{source}	Q_{sink}	Q_{total} (Btu/hr)
2	0.99	-0.87	.00027
3	0.84	-0.74	-.00085
4	0.75	-0.66	.00001
5	0.69	-0.61	.00039
6	0.65	-0.57	.00141
7	0.62	-0.55	-.00084
8	0.60	-0.53	.00013
9	0.59	-0.52	.00079
10	0.58	-0.51	-.00054

(B) $T_1 = 800\ ^oR$, $T_2 = 570\ ^oR$.

N	Q_{source}	Q_{sink}	Q_{total} (Btu/hr)
2	22.56	-11.71	.00029
3	18.90	- 9.72	-.00183
4	16.79	- 8.52	.00072
5	15.45	- 7.77	.00037
6	14.55	- 7.26	.00391
7	13.92	- 6.91	-.00201
8	13.47	- 6.66	.00065
9	13.13	- 6.48	.00212
10	12.88	- 6.34	-.00131

TABLE II (Continued)

(C) $T_1 = 1000\ ^oR$, $T_2 = 600\ ^oR$.

N	Q_{source}	Q_{source}	Q_{total} (Btu/hr)
2	73.87	-30.11	.00084
3	61.93	-24.64	-.00470
4	55.07	-21.39	.00076
5	50.72	-19.35	.00325
6	47.79	-18.00	.00706
7	45.75	-17.08	-.00453
8	44.27	-16.42	.00060
9	43.17	-15.95	.00351
10	42.33	-15.59	-.00198

(D) $T_1 = 2000\ ^oR$, $T_2 = 1000\ ^oR$.

N	Q_{source}	Q_{sink}	Q_{total} (Btu/hr)
2	2089.06	-546.32	.01845
3	1695.51	-427.58	-.06282
4	1476.91	-360.92	.01346
5	1341.74	-320.84	.04292
6	1252.43	-295.25	.08579
7	1190.62	-278.09	-.04197
8	1146.33	-266.11	-.00102
9	1113.67	-257.49	.04862
10	1088.99	-251.07	-.02725

PLATE 3-A

Energy loss of the source section of the box

$T_1 = 560\ ^oR$

$T_2 = 530\ ^oR$

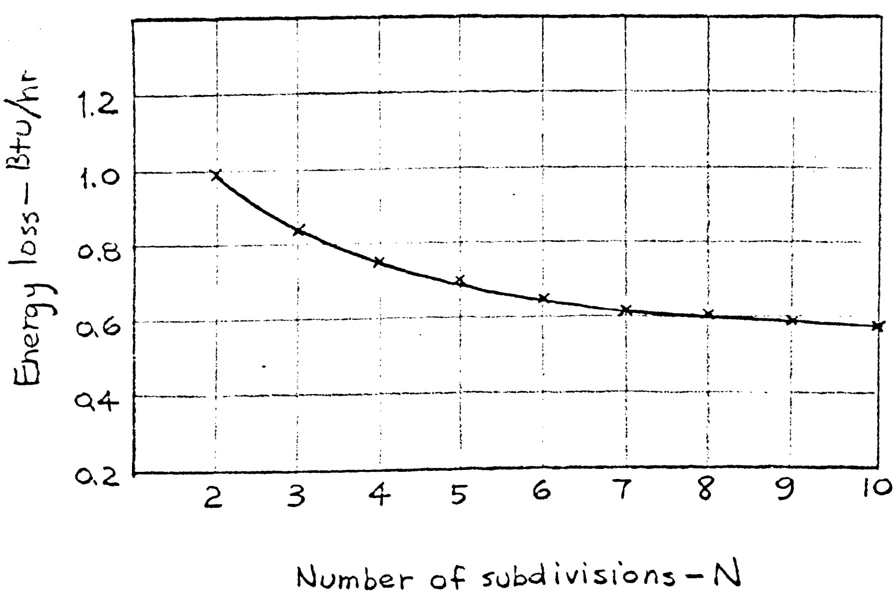


PLATE 3-B

Energy loss of the source section of the box

$T_1 = 800\ ^oR$

$T_2 = 570\ ^oR$

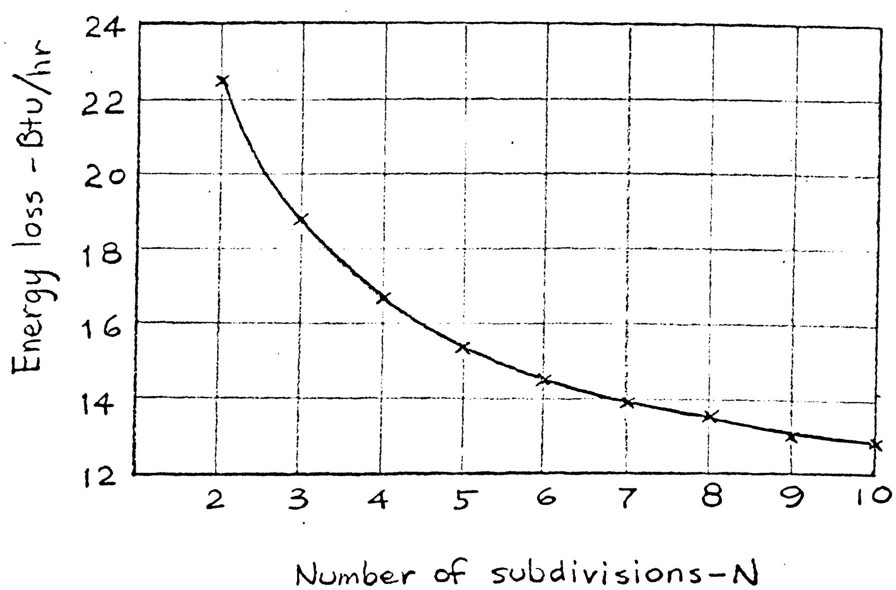


PLATE 3-C

Energy loss of the source section of the box

$T_1 = 1000\ ^oR$

$T_2 = 600\ ^oR$

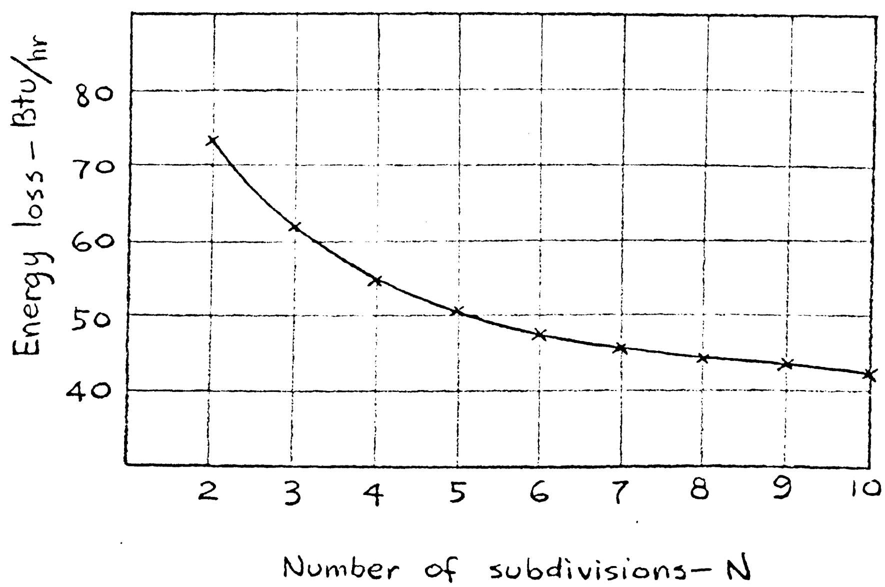


PLATE 3-D

Energy loss of the source section of the box

$T_1 = 2000\ ^{o}R$

$T_2 = 1000\ ^{o}R$

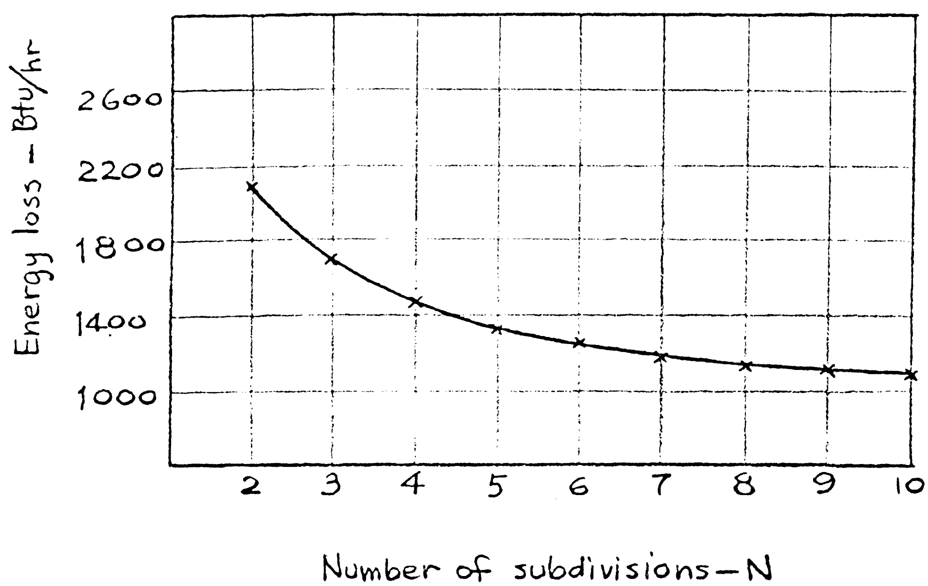


PLATE 4-A

Energy loss of the sink section of the box

$T_1 = 560\ ^oR$

$T_2 = 530\ ^oR$

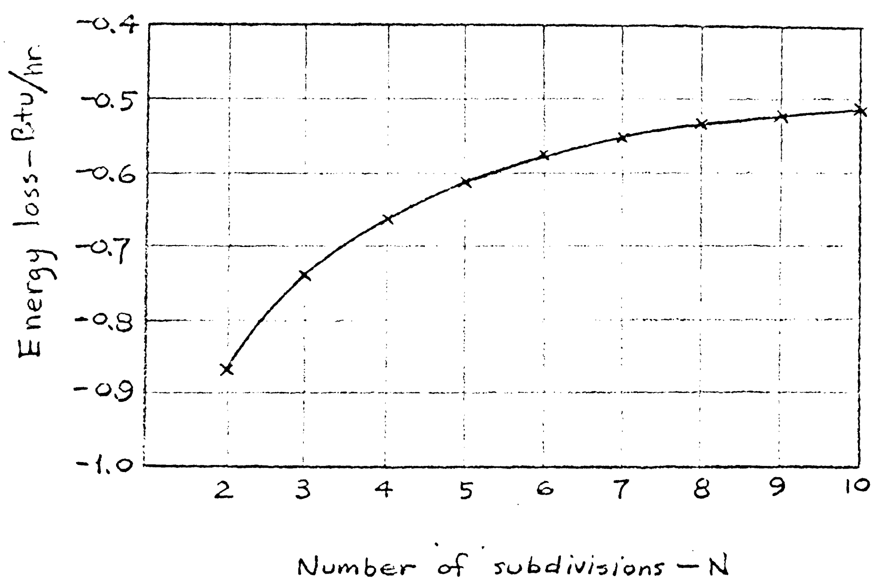


PLATE 4-B

Energy loss of the sink section of the box

$T_1 = 800\ ^oR$

$T_2 = 570\ ^oR$

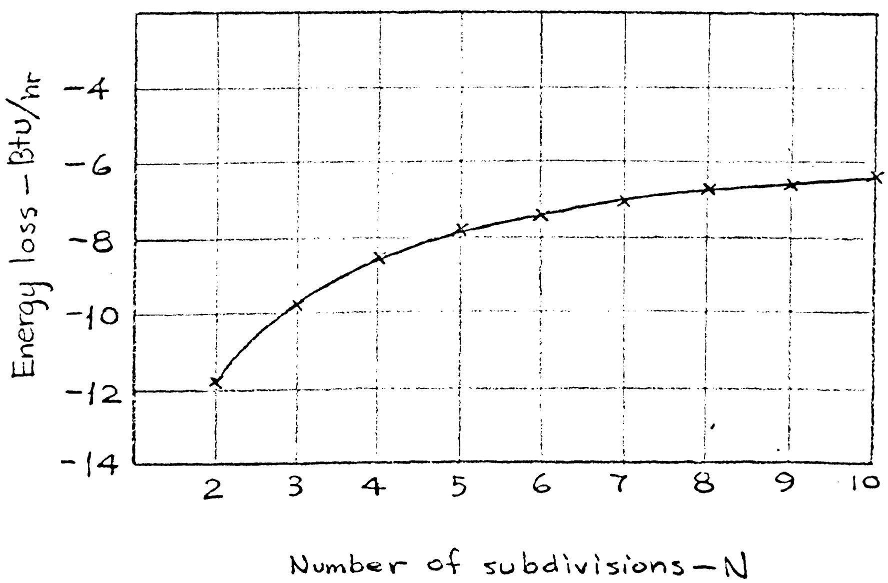


PLATE 4-C

Energy loss of the sink section of the box

$T_1 = 1000\ ^oR$

$T_2 = 600\ ^oR$

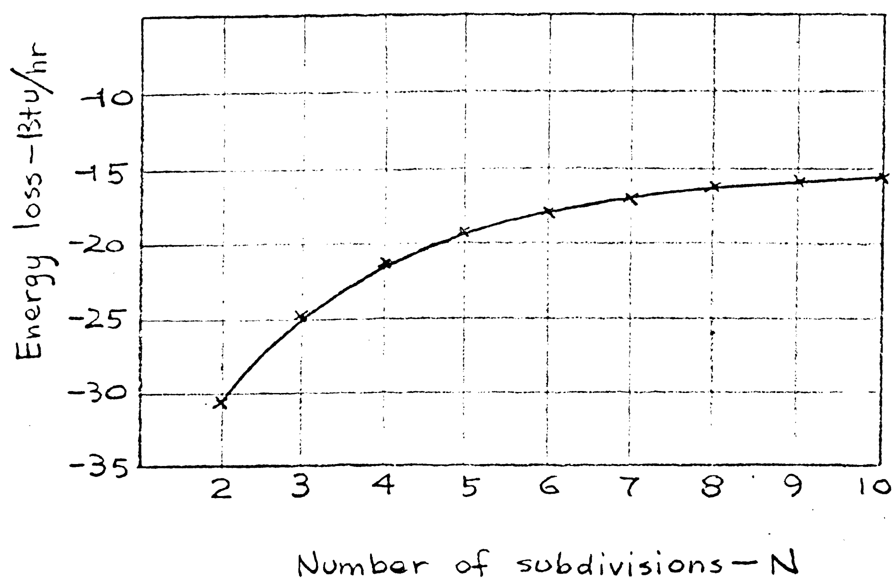
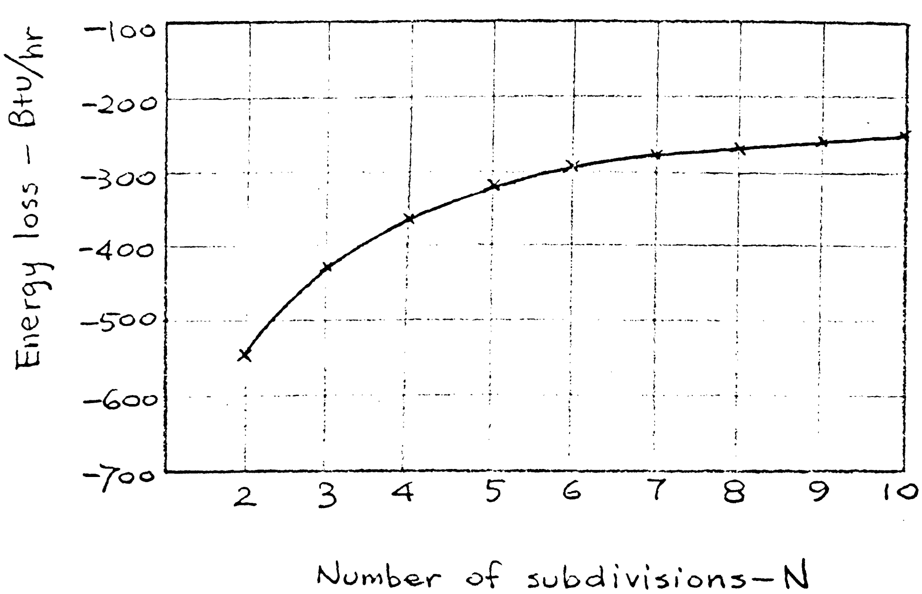


PLATE 4-D

Energy loss of the sink section of the box

$T_1 = 2000\ ^oR$

$T_2 = 1000\ ^oR$



CONCLUSION

The plot of the results of the source section in the first case for high temperatures shows that the slope increases and then decreases to become nearly horizontal as the number of subdivisions increases. This is a result of the summation of the product of the mean temperature to the fourth power and the subdivided section area, since the summation of mean temperature to the fourth power increases with the number of subdivisions, while the area of each subdivision decreases. If the number of the subdivisions is large enough, the results approach a constant value.

The results of the sink section in the first case and the results of the source and the sink in the second case indicate that these both approach a constant value as the number of subdivisions increases. The curves keep on increasing or decreasing because the areas and the temperatures are kept constant at these sections.

The computer programs were checked to be correct by putting $XX = YY = ZZ = 10$ ft., $W = 1$ ft. and $T_1 = T_2 = T_3 = 1000^oR$, for the first program. The results show that $Q_1 = Q_{sink}$, $Q_2 = Q_{10}$, ..., $Q_i = Q_{(N-i+2)}$, where $2 \leq i \leq 10$ and $2 \leq N \leq 10$. For the second program, $XX = 11$ ft., $YY = ZZ = 10$ ft., $T_1 = T_2$ 1000^oR and $r_{N+1} = r_{N+2} = 0$. The results show that the energy loss of the duct like section of the box is approximately equal to the energy loss of the duct, $Q_{duct\ like\ (II)} =$ 64448 Btu/hr, $Q_{duct} =$ 64446 Btu/hr, for $N = 10$.

The first program was run for a constant temperature, 600°R, in the source section and the sink section. The dimensions of the duct were XX = YY = ZZ = 10 ft., W = 1 ft.. The results for the energy loss of the source and the sink were 6248.4 Btu/hr and 643.7 Btu/hr for N = 2, 6237.9 Btu/hr and 643.7 Btu/hr for N = 10 respectively. The maximum errors were .17% and .003% respectively. The results for $T_1 = T_2 = 2000^oR$, $T_3 = 600^oR$ were 1398337.2 Btu/hr and -11682.5 Btu/hr for N = 2, and 1394496.3 Btu/hr and -11634.0 Btu/hr with the maximum errors .28% and .41% respectively.

The program is limited for N varying from N = 2 to N = 10 because of the programming and the capacity of the computer. If a larger number of the subdivisions is required, the program can be used by separating it into several parts and rearranging the DIMENSION statement and the INPUT data.

BIBLIOGRAPHY

1. Hamilton, D. C. & Morgan, W. R., "Radiation-Interchange Configuration Factors". NACA, TN2836, 1952.

2. Hottel, H., "Radiant Heat Transmission" in McAdams, W. B. "Heat Transmission". McGraw-Hill Co. Inc., New York, 1954.

3. Oppenheim, A. K., "Radiation Analysis by Network Method". Trans. ASME 78, 725, 1956.

4. Ishimoto, T. & Bevans., "Method of Evaluating Script F for Radiant Exchange within an enclosure". AIAA, Vol. I, No.6, P.1428, 1963.

5. Gebhart, B., "Unified Treatment for Thermal Radiation Transfer Process". Paper 57-A-34, ASME, 1957.

6. Gebhart, B., "Heat Transfer". McGraw-Hill Co. Inc., New York, 1961.

7. Chapman, A. J., "Heat Transfer", Macmillan Co., New York, 1960.

8. Eckert, E. R. G. & Drake, R. M., "Heat and Mass Transfer". McGraw-Hill Co. Inc., New York, 1959.

9. Holman, J. P., "Heat Transfer". McGraw-Hill Co. Inc., New York, 1963.

10. Jakob, M.,

"Heat Transfer", Vol. II. John Wiley & Sons, Inc., New York, 1957.

11. Kreith, F.,

"Radiation Heat Transfer for Spacecraft and Solar Power Plant Design". International Textbook Co., Scranton, Pennsylvania, 1962.

12. Moon, P.,

"The Scientific Basis of Illuminating Engineering". Dover Publications, Inc., New York, 1963.

VITA

The author was born on December 10, 1938 to Mr. & Mrs. K. T. Cu in Tainan, Taiwan, China.

He received his primary and secondary education in Tainan, Taiwan. He has received his college education in National Taiwan University, Taipei, Taiwan and received a Bachelor of Science Degree in Mechanical Engineering in September, 1961. He came to the United States in September, 1963 and enrolled in the Graduate School of the University of Missouri School of Mines and Metallurgy.