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IMPACT OF STOCHASTIC LOADS AND GENERATIONS ON POWER SYSTEM
TRANSIENT STABILITY

by

THERESA AVOSUAHI ODUN-AYO

A DISSERTATION

Presented to the Faculty of the Graduate School of the
MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

In Partial Fulfillment of the Requirements for the Degree

DOCTOR OF PHILOSOPHY

In

ELECTRICAL ENGINEERING

2011

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PUBLICATION DISSERTATION OPTION

This dissertation has been prepared in the form of three papers for publication. The first paper consisting of pages 3 to 14 is a build up on work published in the proceedings of the Future Renewable Electric Energy Delivery and Management conference, 18-21 May 2009. This extended work has been published in the *Proceedings of the North American Power Symposium* 4-6 October 2009. The second paper consisting of pages from 15 to 36 has been accepted for publication in the *European Transactions on Electrical Power* in June 2011. The third paper consisting of pages from 37 to 56 has been submitted for review to the *IEEE Transactions on Power Systems* in July 2011.

ABSTRACT

Randomness in physical systems is usually ultimately attributed to external noise. Dynamic systems are driven not only by our own control inputs, but also disturbances which cannot be modeled deterministically. A linear system model is justifiable for a number of reasons, often such a model is adequate for the purpose at hand, and when non-linearities do exist, the typical engineering approach is to linearize about some nominal point or trajectory to achieve a perturbation or error model. However, in order for the resulting model to fit data generated by the real world, these disturbances need to be modeled stochastically.

The traditional approach to power system stability studies is based on a deterministic transient energy function. However, such a deterministic analysis does not provide a realistic evaluation of system transient performance where the intermittency and variability of energy production associated with any renewable technology needs to be reflected and accurately modeled in system stability and performance assessments.

In the papers that make up this dissertation, the random variations of system components is modeled by a Gaussian stationary process (white noise) with constant spectral density and the effect on the stability of the power system is examined. The stochastic perturbation of power loads has a significant effect on the transient stability of the power system. The load behavior is found in the random effect of system parameter arising from cumulative impacts of a number of independent events. The random load characteristic is considered to develop a structure-preserved power system transient stability using stochastic energy functions. The stochastic power system stability was analyzed both through the stochastic Lyapunov function and numerically using the Euler-Maruyama method.

ACKNOWLEDGEMENTS

I would like to thank God for His grace, blessing and provision. My deepest appreciation goes to my advisor, Dr. Mariesa L. Crow for her guidance, support, patience and kind words. Thank you for the hope that you gave to me.

Special thanks to my committee members, Dr. Badrul H. Chowdhury, Dr. Jaganathan Sarangapani, Dr. Mehdi Ferdowsi and Dr. Cihan Dagli, for their time and effort in reviewing this dissertation.

My heartfelt gratitude goes to my parents, brothers and sisters for their support and encouragement towards achieving this goal.

To all my friends and well wishers too numerous to mention, I want to say thank you.

Finally, this work is dedicated to my children David Oluwaseun, Daniel Oluwaseyi and Faith Titilayo, you are my inspiration.

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NOMENCLATURE

Symbol	Description
V_{TOT}	Total energy of the system
V_{KE}	Kinetic energy of the system
V_{PE}	Potential energy of the system
E_i	Magnitude of the i^{th} generator voltage behind transient reactance
P_{mi}	Generator mechanical output
P_i	Mechanical corrected power
$\sigma(x)$	Stochastic factor
M_i	Inertia constant of the i^{th} machine
M_T	COI inertia constant
B_{ij}	Transfer susceptance
G_{ij}	Transfer conductance
C_{ij}	Post fault parameter = $E_i E_j B_{ij}$
D_{ij}	Post fault parameter = $E_i E_j G_{ij}$
δ_i	Rotor angle
Y_{ij}	Admittance matrix element magnitude
ω_i	Angular frequency
D	Damping coefficient
ω_s	Synchronous speed
V_i	Bus voltage magnitude at bus i
V_j	Bus voltage magnitude at bus j
θ_i	COI bus angle
θ_i^S	Post-fault stable equilibrium bus angle in COI

t_{cl}	fault clearing time
t_{crit}	Critical clearing time
$V(x)$	Lyapunov function
σ	Gaussian standard deviation
σ^2	Gaussian variance
$W_i(t)$	Wiener process
α	Magnitude of applied noise
P_L	Deterministic load
\tilde{P}_L	Stochastic Load
h	Width of the integration time step
$E[x]$	Expected value of x
k	Shape factor
λ	Scale factor
μ	Actuation mean time
P_s	Probability of stability
P_{di}	Active load demand at bus i
Q_{di}	Reactive load demand at bus i
α_{pi}	Magnitude of active noise
α_{qi}	Magnitude of reactive noise
P_{di}^0	Mean values of active load at bus i
Q_{di}^0	Mean values of reactive load at bus i
f_R	Probability distribution of the recloser
f_{cct}	Probability distribution of the critical clearing times

1. INTRODUCTION

The power industry is going through a radical change with growing interests in obtaining energy from sustainable renewable energy resources such as wind, solar and plugged-in-hybrid electric vehicles. The way in which power is delivered to customers from central power plants through transmission and distribution networks, has been changing because of the deregulation of the power system and the integration of distributed generation.

With advancements in these technologies over the past decade, it is expected that a large amount of electric energy supply requirements be met by these non-conventional energy sources. Rising gas prices, carbon constraints, fuel economy standards, and the desire for energy independence are also driving the development of Plug-in Hybrid Electric Vehicles (PHEV) which are expected to achieve the equivalent of 100 miles per gallon of gasoline. If they achieve significant market potential, they will have a huge impact on the electric industry, increasing load by an amount which could put the grid at risk. The challenges faced by today's power system are severe. It is designed for moderate load increase due to long time investments in electricity generation, lines and cables but faces in the future a large new load with different patterns. Wind resource integration may have a significant impact on power system stability. Although deterministic stability studies are often used in generation interconnection studies, these deterministic studies lack the capability of considering the stochastic characteristics of wind and photovoltaic resources.

This dissertation consists of three papers; Paper 1 (Proceedings of the North America Power Symposium 2009) deals with an introduction and overview of the stability of system by using the potential energy generated at and around the equilibrium points. Preliminary results in the formulation of the Lyapunov function that allows the transient stability to be assessed by using the total energy at fault clearing were developed. Paper 2 (accepted for publication in the *European Transactions on Electrical Power*), develops an approach to analyze the impact of stochasticity on the transient stability of a power system. The stochastic power system stability was analyzed using both the stochastic Lyapunov function and numerically using the Euler-Maruyama

method. Paper 3 (submitted to the *IEEE Transactions on Power Systems*) builds upon the previous results of network-reduced power system models. This paper develops a structure-preserved power system transient stability using stochastic energy functions. In the context of system modeling, the network reduction power system models preclude consideration of load behaviors (i.e. voltage and frequency variations) at load buses. In addition, in the context of the physical explanation of results, reduction of the transmission network leads to loss of network topology and hence limits the study of transient energy shifts among different components of the entire power network.

The primary contributions of this dissertation are

- The formulation of a stochastic Lyapunov function that tests the stability of the power when uncertainties are present (Paper 1).
- A framework for constructing a stochastic transient energy function was developed for the classical power system (Paper 2)
- The formulation of a stochastic energy function that is used to determine the stochastic transient stability of the power system with random load. It also shows statistical analysis of results that can form the basis of risk assessment analysis of the power system in the presence of perturbation and uncertainties inherent in the integration of renewable and distributed energy sources (Paper 2).
- A structure preserving model was used to analyze the impact of random load and generation variations on the transient stability of a power system for a more realistic representation of power system components and load behavior (Paper 3).

I. AN ANALYSIS OF THE IMPACT OF PLUG-IN HYBRID ELECTRIC VEHICLES ON POWER SYSTEM STABILITY

Theresa Odun-Ayo, *Student Member, IEEE*, M. L. Crow, *Fellow, IEEE*

ABSTRACT

Rising gas prices, carbon constraints, fuel economy standards, and the desire for energy independence are driving the development of PHEVs which are expected to achieve the equivalent of 100 miles per gallon of gasoline. If they achieve significant market potential, they will have a huge impact on the electric industry, increasing load by an amount which could put the grid at risk. The challenges faced by today's power system are severe. It is designed for moderate load increase due to long time investments in electricity generation, lines and cables but faces in the future a large new load with different patterns. This paper investigates the stability of system by using the potential energy generated at and around the equilibrium points and an analysis of the stability of the power system using stochastic Lyapunov-like energy functions.

I. INTRODUCTION

Greater use of electricity as an energy source for transportation could substantially reduce oil consumption. Electric motors are inherently more efficient than internal combustion engines; they do not consume energy while vehicles are stationary and they provide the opportunity to recover energy from braking [1] [2] [3]. Current hybrid electric vehicle technology demonstrates some of the potential of this approach. The

introduction and wide spread use of plug-in hybrid technologies (PHEVs) with an all-electric range sufficient to meet average daily travel needs could reduce per-vehicle petroleum consumption by 50 percent, meaning half of the energy would come from electricity. Out of this ambience the promise for more efficient individual transportation is partly represented by PHEVs, mitigating vehicle technology to an increased electrification. However, the mitigation process intuitively entails several impacts for the transportation as well as for the power sector which need to be investigated and resolved. PHEVs are being developed around the world, with much work aiming to optimize engine and battery for efficient operation, both during discharge and when grid electricity is available for recharging. However, the general expectation has been that the grid will not be greatly affected by the use of PHEVs because the recharging will occur during off-peak hours, or the number of vehicles will grow slowly enough so that capacity planning will respond adequately. This expectation does not consider that drivers will control the timing of recharging, and their inclination will be to plug in when convenient, rather than when utilities would prefer.

II. CHALLENGES WITH PHEV INTEGRATION

PHEVs are a major potential load and energy storage on the grid. They are like regular hybrid vehicles but with larger batteries and the ability to recharge from an electric connection to the grid. It is important to understand the ramifications of adding load from PHEVs onto the grid. Depending on when and where the vehicles are plugged in, they could cause local or regional constraints on the grid [4]. They could require the addition of new electric capacity and increase the utilization of existing capacity. Usage patterns of local distribution grids will change, and some lines or substations may become overloaded sooner than expected. Furthermore, the type of generation used to meet the demand for recharging PHEVs will depend on the region of the country and the timing of recharging. References [3][5][6] look at the concept of vehicle-to-grid power when an electric-drive motor powered by batteries, a fuel cell, or a hybrid drivetrain generates or store electricity when parked and with appropriate connections can feed power to the grid. As PHEVs move toward commercialization, utilities, research institutions, and other organizations are attempting to analyze the possible impact that these new, high-power

loads could have on the electric grid in the future. PHEV technology also has the potential to provide peak load power during high demand periods, if a utility's electric distribution system provides vehicle-to-grid (V2G) capability through smart grid technologies. The concept has the potential of improving the sustainability and resilience of the transportation and electric power infrastructures. It will enable the grid to utilize PHEV batteries for storing excess renewable energy and then releasing this energy to grid customers when needed. Crucial changes for the transportation sector include behavioral pattern changes of the population as well as changes to the existing parking infrastructure, which are captured by transportation frameworks. Potential dangerous impacts which are intuitive to utilities are line congestion, transformer overloads and other not foreseen problems at the different grid levels, but mainly in distribution grids. Distribution grids will encounter the new load as a heavy impact even if it is small in the beginning, whereas the transmission and medium voltage grid will just see a slight load increase easily manageable when not occurring at peak times and in large quantities.

III. IMPACT OF PHEV INTEGRATION ON SYSTEM ENERGY BALANCE

As cars and light trucks begin a transition to electric propulsion, there is potential for a synergistic connection between such vehicles and the electric power grid [1] [3]. By itself, each vehicle will be small in its contribution to the power system, but in aggregate a large number of vehicles will represent significant storage or generating capacity. There is however the potential for these vehicles to have an effect on the voltage stability and control of the distribution system.

Small-signal stability analysis helps in predicting the system's response to persistent random fluctuations in load demands [7] [8]. The question of stability is whether for a given disturbance, the trajectories of pre-disturbance operating quantities of the system during the disturbance remain in the domain of attraction of the post-disturbance equilibrium when the disturbance is removed. This concept is one of transient stability. Transient instability in a power system is caused by severe disturbance which creates substantial imbalance between the input power supplied to the synchronous generators and their electrical outputs. Some of the severely disturbed generators will 'swing' far enough from their equilibrium positions losing synchronism in the process. Such a severe

disturbance may be due to a sudden and large change in load, generation, or network configuration. The transient energy function contains both kinetic and potential terms. The system kinetic energy, associated with the relative motion of machine rotors, is formally independent of the network. The system potential energy, associated with the potential energy of the post-fault system, whose stability is to be analyzed [9] [10]. Energy function based methods of determining transient stability are a special case of the more general Lyapunov methods of stability analysis. While a formal analysis using the Lyapunov's second method is possible, a more "physical" energy based analogy is quite helpful in understanding the mechanism of instability/stability. The fundamental goal of the energy approach is to calculate the transient energy that the post fault system is capable of absorbing and then finding the clearing time at which the faulted trajectory will introduce equal to or slightly less than the critical transient energy into the post-fault system [11]. The energy function of the post-fault system is given in (1).

$$V_{TOT} = \frac{1}{2} \sum_{i=1}^n M_i \omega_i^2 - \sum_{i=1}^n P_i (\theta_i - \theta_i^s) - \sum_{i=1}^{n-1} \sum_{j=i+1}^n [C_{ij} (\cos \theta_{ij} - \cos \theta_{ij}^s) - \int_{\theta_i^s + \theta_j^s}^{\theta_i + \theta_j} D_{ij} \cos \theta_{ij} d(\theta_i + \theta_j).] \quad (1)$$

where

$$\begin{aligned} P_i &= P_{mi} - E_i^2 G_{ii} \\ E_i E_j B_{ij} &= C_{ij} \\ E_i E_j G_{ij} &= D_{ij} \\ \theta_{ij} &= \theta_i - \theta_j \end{aligned}$$

and

θ_i^s is the post-fault stable equilibrium point

M_i = Inertia constant of the i^{th} machine

P_i = corrected mechanical power

P_{mi} = mechanical input power to the i^{th} machine

E_i = magnitude of the voltage behind the transient reactance of the i^{th} machine

$$V_{KE} = \frac{1}{2} \sum_{i=1}^n M_i \omega_i^2$$

is the kinetic energy tending to move the system away from synchronism, and

$$V_{PE} = - \sum_{i=1}^n P_i (\theta_i - \theta_i^s) - \sum_{i=1}^{n-1} \sum_{j=i+1}^n [C_{ij} (\cos \theta_{ij} - \cos \theta_{ij}^s) - \int_{\theta_i^s + \theta_j^s}^{\theta_i + \theta_j} D_{ij} \cos \theta_{ij} d(\theta_i + \theta_j).]$$

is the potential energy of the system, where

V_{TOT} is the sum of the kinetic and potential energy of the system.

C_{ij} and D_{ij} are the post-fault parameters and θ_i and ω_i are dynamic states.

B_{ij} and G_{ij} are the transfer susceptance and conductance in the reduced bus admittance matrix, respectively.

When a disturbance occurs in a power system the transient energy injected into the system during the disturbance increases and causes the machine to diverge from the rest of the system [12] [13][14]. When the disturbance is removed, and as machine continues to diverge from the rest of the system, its kinetic energy is being converted into potential energy. This motion will continue until the initial kinetic energy is totally converted into potential energy. When this takes place, the machine will converge towards the rest of the system. Figure 1 shows that even small changes in load can have a dramatic impact on stability.

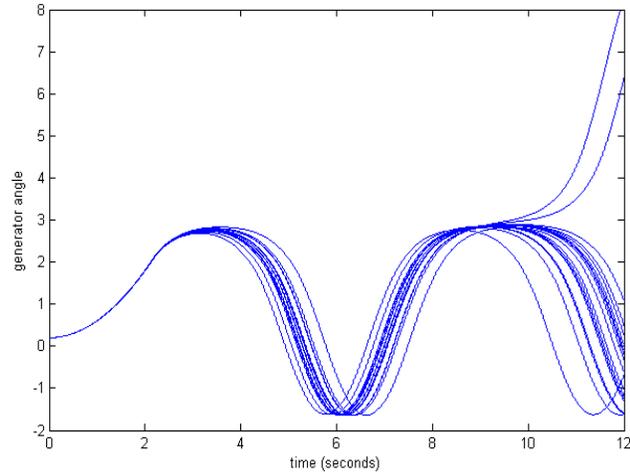


Fig. 1. Impact of load changes on stability.

The potential energy function can be viewed as an energy bowl in the state space of angles. The projection of the stable equilibrium point on the space of angles is located in the bottom of this bowl and corresponds to a minimum of potential energy on the surface. As shown in Figure 2, at the edge of the bowl, there are points of local maximum and saddle points. At these points, the gradient of the potential energy function is zero and, as a consequence, they correspond to unstable equilibrium points of the system.

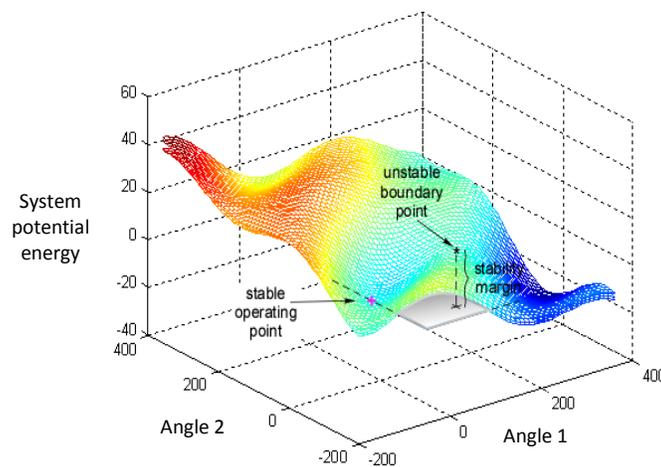


Fig. 2. Potential energy function as an energy bowl in the state space of angles.

IV. STOCHASTIC LYAPUNOV STABILITY FOR PHEVS

The subject of stochastic dynamics and control deals with response analysis and control design for dynamical systems with random uncertainties. Small magnitude disturbances in load are the result of aggregate behavior of many thousands of individual customer devices switching independently and can be expected to lead to a wide band disturbance term. Given that these stochastic load variations are the phenomena of interest, the question of how to model their effect becomes closely linked with the underlying load representation. The modeling of the stochastic component of the electrical network load has in some papers used different representations of the load distribution and correlation [15]. Reference [16] showed that most uncertainties of active and reactive daily peak loads in the system can be modeled by normal distributions. It also mentions the use of three probability density functions: normal, log-normal and beta distribution to model the load variations. Reference [17] introduces a systematic approach to the construction of stochastic models of electric power systems for small disturbance stability analysis. Even though a deterministic structure might be asymptotically stable, a small random force could cause its trajectories to reach an energy, beyond which it would collapse or enter a critical zone. The stability of the dynamic structure and the expected lifetime before it enters the critical zone is of interest. Stability and reliability of PHEV can be modeled stochastically. This method computes circuit loading and bus voltage probability distribution from a given load probability distribution. The output of the stochastic load flow is utilized to compute the conditional probability of system stability according to predefined criteria. It has been shown [18] that if the input to the system is represented by white noise, then in the absence of damping the dynamic system become unstable. In the presence of damping there is a critical noise-to-damping ratio below which the system is stable and above which the system becomes unstable. The random variations of system components can be modeled by a Gaussian stationary process (white noise) with constant spectral density. In this paper, the effect on stability of introducing some random perturbation into the system was examined. This was incorporated in equation (1) by replacing the P_i by $P_i + \alpha P_0$, where α is the white noise applied to the system. Figure 3 shows the changes in stability boundary of the potential energy of a three machine system.

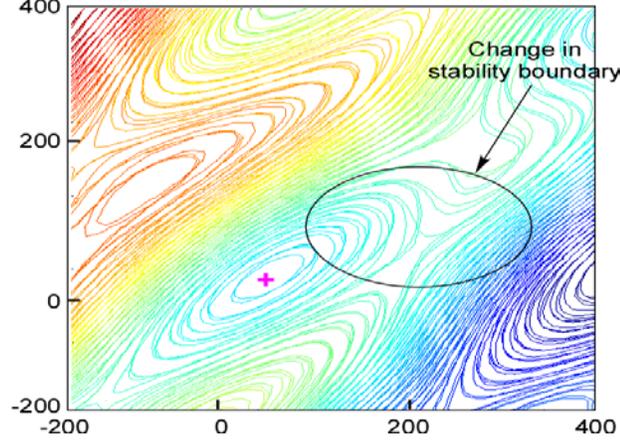


Fig. 3. Power system stability regions using stochastic Lyapunov functions.

V. FORMULATION OF THE LYAPUNOV FUNCTION

If there exists a continuously differentiable positive definite function v with a negative semi definite (or identically zero) derivative \dot{v} , then the equilibrium $x = 0$ of \dot{v} is stable. Point \hat{x} is the equilibrium point of the dynamic system described by a set of non-linear equations $\dot{x} = F(x)$ if $F(\hat{x}) = 0$. Lyapunov's stability theorem states that this equilibrium point is stable if there is a Lyapunov function such that: (i) $V(x)$ is positive definite with a minimum value at \hat{x} , and (ii) the time derivative $\dot{V} = dV/dt$ along the system trajectory $x(t)$ is semi-definite, i.e. $\dot{V} \leq 0$. If $\dot{V} < 0$ then the equilibrium point is asymptotically stable. The time derivative \dot{V} along the system trajectory $x(t)$ can be calculated as:

$$\dot{V} = \frac{dV}{dt} = \frac{\partial V}{\partial x_1} \frac{dx_1}{dt} + \frac{\partial V}{\partial x_2} \frac{dx_2}{dt} + \dots + \frac{\partial V}{\partial x_n} \frac{dx_n}{dt} \quad (2)$$

If \dot{V} is negative then the function $V(x)$ decreases with time and tends towards its minimum value, the system equilibrium point \hat{x} . The more negative the value of \dot{V} the faster the system returns to the equilibrium point \hat{x} . Consider a system of mass spring system with smooth functions $f(\cdot), g(\cdot)$ continuously differentiable and satisfying the following conditions

$$\sigma f(\sigma) \geq 0 \quad \forall \sigma \in [-\sigma_0, \sigma_0] \quad (3)$$

and

$$\sigma g(\sigma) \geq 0 \quad \forall \sigma \in [-\sigma_0, \sigma_0] \quad (4)$$

and equality is achieved when $\sigma = 0$. The candidate for the Lyapunov function is

$$\dot{V}(x) = \frac{x^2}{2} + \int_0^{x_1} g(\sigma) d\sigma \quad (5)$$

For a single machine infinite bus, where

$$\bar{\omega} = \omega - \omega_s$$

$$\bar{\delta} = \delta - \delta_0$$

the system model is given by

$$\dot{\bar{\delta}} = \bar{\omega} \quad (6)$$

$$\dot{\bar{\omega}} = p - k \sin(\bar{\delta} - \delta_0) - D\bar{\omega} \quad (7)$$

where ω = rotor speed, δ is the angle of the voltage behind transient reactance, indicative of generator rotor position, p is the mechanical power and D is the damping coefficient.

The transient energy function is given by

$$V(\delta, \omega) = \frac{1}{2} \bar{\omega}^2 + \int_0^{\bar{\delta}} g(\sigma) d\sigma \quad (8)$$

$$V(\delta, \omega) = \frac{1}{2} m(\omega - \omega_s)^2 - p(\delta - \delta_0) - k(\cos \bar{\delta} - \cos \delta_0) \quad (9)$$

where $k = C_{ij} = \frac{EV}{X}$

The transient stability can be directly assessed by comparing the critical energy $V(\delta_c, \omega_c)$ to the total energy at fault clearing $V(\delta_u, 0)$ i.e.

$$V(\delta_c, \omega_c) < V(\delta_u, 0) \quad \text{Stable}$$

$$V(\delta_c, \omega_c) = V(\delta_u, 0) \quad \text{Critically Stable}$$

$$V(\delta_c, \omega_c) > V(\delta_u, 0) \quad \text{Unstable}$$

and the stability margin can be calculated by

$$\Delta V = V(\delta_u, 0) - V(\delta_c, \omega_c)$$

Let us consider the nonlinear Ito stochastic system:

$$dx(t) = f(x(t))dt + \sigma(x(t))dw(t) \quad (10)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a continuous mapping; $\sigma(x(t)) \in \mathbb{R}^n \times \mathbb{R}^d$, the diffusion coefficients of $x(t)$; and $w(t) \in \mathbb{R}^d$, the standard Wiener process.

Assuming that the origin is an isolated equilibrium point and let

$$f(x) = (f_1(x), f_2(x), \dots, f_n(x))^T,$$

$$a(x) = \sigma(x) \cdot \sigma(x)^T = \begin{bmatrix} a_{11}(x) & \cdots & a_{1n}(x) \\ \vdots & \ddots & \vdots \\ a_{n1}(x) & \cdots & a_{nn}(x) \end{bmatrix}$$

Furthermore, the infinitesimal operator \mathcal{L} is expressed as

$$\mathcal{L}V(x) = V_x(x)f(x) + \frac{1}{2}\text{tr}V_{xx}(x) \cdot a(x) \quad (11)$$

If $\mathcal{L}V(x)$ is negative definite in the neighborhood of $x = 0$, then the equilibrium $x \equiv 0$ of the stochastic equation (10) is asymptotically stable in probability [19]. For the classical model, represented by (6) and (7), we get

$$\mathcal{L}V(x) = \frac{1}{2}\sigma^2 \left(m + \frac{EV}{X} \cos \delta \right) - D\bar{\omega}^2 \quad (12)$$

For $\sigma = 0$ (deterministic), equation (12) defaults back to the classical system stability. Obviously for large D or small σ this is satisfied and asymptotically stable. For small damping, or large noise, then the stability is indeterminate.

VI. FUTURE WORK

When there is a sufficient amount of data to form a sample space, uncertainties can be modeled as random variables or stochastic processes by means of statistical inference. Lyapunov's method is very useful for designing non-linear stochastic dynamical systems. The impact of PHEVs on power system stability can be further examined by taking an in depth look at the existence of a stochastic Lyapunov function which guarantees that the origins of a system are stable in probability. We also need to correlate the stability of the system with the amount of noise (magnitude of σ) and the noise probability function.

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II. An Analysis of Power System Transient Stability Using Stochastic Energy Functions

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ABSTRACT: This paper develops an approach to analyze the impact of stochasticity on the transient stability of a power system. The stochastic power system stability was analyzed both through the stochastic Lyapunov function and numerically using the Euler-Maruyama method. It was shown that increasing either (or both) the variance and the magnitude of the applied noise can have a destabilizing effect on the power system. This could potentially cause difficulties as more randomness is introduced into the power system through renewable energy sources and plug-in-hybrid vehicles.

I. INTRODUCTION

Small magnitude disturbances in load are the result of the aggregate behavior of many thousands of individual customer devices switching independently and can be expected to lead to a wide band disturbance. Plug-in-hybrid vehicles (PHEV) are a potential significant source of disturbance on the grid. PHEVs are like regular hybrid vehicles but with larger batteries and the ability to recharge from an electric connection to the grid. Furthermore renewable energy resources such as wind turbines or solar power can introduce additional uncertainty into the power system. The tandem effect of renewable resources and PHEVs may create uncertainties of such significant magnitude they may impact the operation of the power system. For example, the stochastic combination of wind generation and PHEVs in power system power flow analysis was recently considered in [1].

At the heart of the stochastic power system is the random perturbations of the load. The modeling of the stochastic component of the electrical network load has been studied

in several papers using different representations of the load distribution and correlation. References [2][3] showed that most uncertainties of active and reactive daily peak loads in the system can be modeled by normal distributions. In [4] a systematic approach to the construction of stochastic models of electric power systems is introduced for small disturbance stability analysis. In [4], it was shown that even though a deterministic power system might be stable, small random perturbations may cause the state trajectories to reach a critical point such that exceeding this point may cause the system to collapse or enter an undesirable operating state.

In addition to power flow studies, there has been renewed interest in stochastic power system stability analysis due to the projected increase in wind generation and PHEV penetration. The study and analysis of stochastic power system dynamic security is not a new topic; it has been studied for several decades [5]-[7], but has received renewed interest in recent years [8]-[10].

Transient stability assessment has at its core the necessity of a time-domain analysis: either through direct methods (such as Lyapunov-based energy functions) or through time-domain simulation [12]-[14]. Previous transient stability stochastic studies addressed uncertainty in the system model through a combination of deterministic simulation techniques with stochastic analyses [5]-[10]. Only [9] specifically addresses the impact of uncertainty in the time domain and proposes the probabilistic collocation method to develop a polynomial model to predict the outcome of interest. Both analytic and Monte Carlo simulation approaches have been discussed for the probabilistic assessment of transient stability. In fact, the basic idea for a Monte Carlo approach to transient stability assessment using transient energy functions was first proposed in [7], but the appropriate stochastic tools did not exist at that time to frame the stochastic energy function nor to numerically solve the stochastic differential equations.

In light of the renewed interest in stochastic power system stability analysis, we propose to extend the approach first presented in [7] specifically utilizing recent theoretical developments in

- stochastic transient energy functions, and the
- numerical simulation of the stochastic transient stability equations

II. LYAPUNOV TECHNIQUES FOR STOCHASTIC DIFFERENTIAL EQUATIONS

Consider the nonlinear stochastic system

$$dx = f(x, t)dt + g(x, t)\Sigma(t)dW(t) \quad x(0) = x_0 \in \mathbb{R}^n \quad (1)$$

whose solution can be written in the sense of Ito:

$$x(t) = x_0 + \int_0^t f(x, s)ds + \int_0^t g(x, s)\Sigma(s)dW(s) \quad (2)$$

where $x(t) \in \mathbb{R}^n$ is the state; $W(t)$ is an m -dimensional standard Wiener process defined on the complete probability space (Ω, \mathcal{F}, P) ; the functions (f, g) are locally bounded and locally Lipschitz continuous in $x \in \mathbb{R}^n$ with $f(0, t) = 0, g(0, t) = 0$ for all $t \geq 0$; and the matrix $\Sigma(t)$ is nonnegative-definite for each $t \geq 0$. The above conditions ensure uniqueness and local existence of strong solutions to equation (1) [15] [16].

The determination of stochastic system stability is not as straightforward as with deterministic systems. Consider for example the scalar stochastic process x_t given by the first order Ito stochastic differential equation

$$dx_t = rx_tdt + \alpha x_t dW_t \quad (3)$$

in which the randomness is multiplicative. The explicit solution to this equation is

$$x_t = x_0 \exp \left(\left(r - \frac{1}{2}\alpha^2 \right) t + \alpha W_t \right) \quad (4)$$

The qualitative behavior of the process as $t \rightarrow \infty$ is

- 1) If $r - \alpha^2/2 > 0$, then $x \rightarrow \infty$ with probability 1.0
- 2) If $r - \alpha^2/2 < 0$, then $x \rightarrow 0$ with probability 1.0
- 3) If $r - \alpha^2/2 = 0$, then x fluctuates between arbitrarily large and arbitrarily small values with probability 1.0

Note that the stability response is not governed by the deterministic boundary $r = 0$, but rather that sufficiently *large* magnitudes of randomness may actually improve the stability of the system. Fig. 1 shows the solutions to equation (3) for $r = 1$ and values of $\alpha = 1, \sqrt{2}, 2$ for the same dW_t in each run.

As with many nonlinear deterministic systems, Lyapunov functions may provide guidance regarding the stability of stochastic differential equation (SDE) systems. An SDE system is said to satisfy a Stochastic Lyapunov Condition at the origin if there

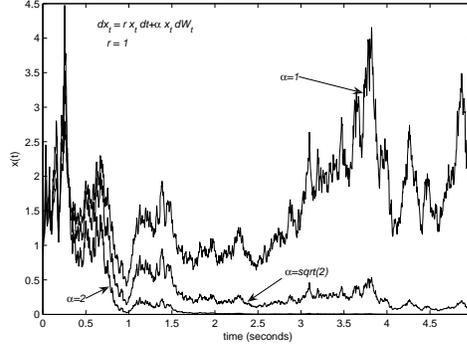


Fig. 1. Examples of different randomness levels in equation (3)

exists a Lyapunov function $V(x)$ defined in a neighborhood D of the origin in \mathbb{R}^n such that

$$\mathcal{L}V(x) \leq 0 \quad (5)$$

for any $x \in D \setminus \{0\}$. Then the equilibrium solution $x(t) \equiv 0$ of the stochastic differential equation (1) is stable in probability. Moreover, if $D = \mathbb{R}^n$ and the Lyapunov function $V(x)$ is proper, then the equilibrium solution $x(t) \equiv 0$ is *asymptotically stable in probability* provided

$$\mathcal{L}V(x) < 0 \quad (6)$$

for any $x \in D \setminus \{0\}$ [17]. The differential generator \mathcal{L} is given by

$$\mathcal{L}V(x, t) = \frac{\partial V}{\partial x} f(x, t) + \frac{1}{2} \text{Tr} \left\{ \Sigma(t)^T g(x, t)^T \frac{\partial^2 V}{\partial x^2} g(x, t) \Sigma(t) \right\} \quad (7)$$

To illustrate the application of the differential generator, consider the one-machine-infinite-bus system shown in Fig. 2 and described by the following equations:

$$\dot{\delta} = \omega \quad (8)$$

$$M\dot{\omega} = -D\omega - P_0 \sin \delta + P_m - P_L \quad (9)$$

If the deterministic load P_L is replaced with a stochastic load \tilde{P}_L that has an expected value of $E[\tilde{P}_L] = P_L^0$ with a stochastically varying component of magnitude α , then equation (9) can be written as a stochastic differential equation:

$$Md\omega = -D\omega dt - P_0 \sin \delta dt + P_m dt - P_L^0 dt - \alpha dW_t \quad (10)$$

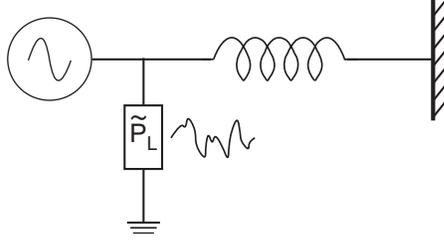


Fig. 2. Single-machine-infinite-bus system with stochastic load

Since the candidate Lyapunov function must satisfy the positive (semi-) definite criteria, a deterministic Lyapunov function is typically used. Therefore, a suitable Lyapunov function for this system is [18]

$$V(\delta, \omega) = \frac{1}{2}M\omega^2 - (P_m - P_L^0)\delta - P_0 \cos \delta \quad (11)$$

Note that the Lyapunov function is deterministic and not stochastic. From equation (6), this system will be stable in probability if

$$\mathcal{L}V(\delta, \omega) = \frac{1}{2}\alpha^2 M - D\omega \leq 0 \quad (12)$$

Obviously for large D or small α this condition is satisfied. For small damping or large load stochasticity, the stability of this system is indeterminate for this candidate Lyapunov function.

III. REVIEW OF TRANSIENT ENERGY FUNCTIONS FOR POWER SYSTEM TRANSIENT STABILITY

The concept of transient stability is based on whether for a given disturbance, the trajectories of the system states during the disturbance remain in the domain of attraction of the post-disturbance equilibrium when the disturbance is removed. Transient instability in a power system is caused by a severe disturbance which creates a substantial imbalance between the input power supplied to the synchronous generators and their electrical outputs. Some of the severely disturbed generators may “swing” far enough from their equilibrium positions to lose synchronism. Such a severe disturbance may be due to a sudden and large change in load, generation, or network configuration. Since large disturbances may lead to nonlinear behavior, Lyapunov functions are well-suited to determine power system transient stability. Since true Lyapunov functions do not exist

for lossy power systems, transient energy functions are frequently used to assess the dynamic behavior of the system [18].

The transient energy function contains both kinetic and potential terms. The system kinetic energy is associated with the relative motion of machine rotors. The potential energy is associated with the state of the post-fault system [19][20]. Energy function-based methods of determining transient stability are a special case of the more general Lyapunov methods of stability analysis. While a formal analysis using the Lyapunov's second method is possible, a more "physical" energy based analogy is quite helpful in understanding the mechanism of instability/stability. The fundamental goal of the energy approach is to calculate the transient energy that the post fault system is capable of absorbing and then finding the critical clearing time at which the energy of the faulted trajectory will be equal to or slightly less than the critical transient energy of the post-fault system. This approach is sometimes referred to as the "potential energy boundary surface" or PEBS method of transient stability.

For an electric power system modeled classically as

$$\begin{aligned}\dot{\delta}_i &= \omega_i - \omega_s \\ M_i \dot{\omega}_i &= P_{M_i} - E_i \sum_{j=1}^n E_j Y_{ij} \cos(\delta_i - \delta_j - \phi_{ij}) \quad i = 1, \dots, n\end{aligned}\tag{13}$$

where

δ_i	rotor angle
ω_i	angular frequency
M_i	inertia constant
P_{M_i}	mechanical output
E_i	constant voltage behind transient reactance
$Y_{ij} \angle \phi_{ij}$	(i, j) -th entry of the reduced admittance matrix
n	number of generators in the system
ω_s	synchronous speed in radians

The transient energy function of the post-fault system is given by:

$$V_{TOT} = \frac{1}{2} \sum_{i=1}^n M_i \tilde{\omega}_i^2 - \sum_{i=1}^n P_i (\theta_i - \theta_i^s)$$

$$-\sum_{i=1}^{n-1} \sum_{j=i+1}^n E_i E_j \left[B_{ij} (\cos \theta_{ij} - \cos \theta_{ij}^s) - \int_{\theta_i^s + \theta_j^s}^{\theta_i + \theta_j} G_{ij} \cos \theta_{ij} d(\theta_i + \theta_j) \right] \quad (14)$$

where

$$\begin{aligned} G_{ij} &= Y_{ij} \cos \phi_{ij} \\ B_{ij} &= Y_{ij} \sin \phi_{ij} \\ P_i &= P_{M_i} - E_i^2 G_{ii} \\ \theta_{ij} &= \theta_i - \theta_j \end{aligned}$$

and θ_i and $\tilde{\omega}_i$ are the transformed generator states in the center of inertia reference frame:

$$\begin{aligned} \theta_i &= \delta_i - \delta_0 \\ \tilde{\omega}_i &= \omega_i - \omega_0 \\ \delta_0 &= \frac{1}{M_T} \sum_{i=1}^n M_i \delta_i \\ \omega_0 &= \frac{1}{M_T} \sum_{i=1}^n M_i \omega_i \\ M_T &= \sum_{i=1}^n M_i \end{aligned}$$

and θ_i^s is the post-fault stable equilibrium point.

The closest unstable equilibrium point (UEP) and controlling UEP method are two common methods used to assess the system's stability [18]. The controlling UEP method consists of numerically integrating the system state and calculating the kinetic, potential, and total energy of the fault-on system until the point at which the potential energy reaches its maximum value. This maximum potential energy is at (or near) the UEP. The critical clearing time (CCT) of the system is then calculated by finding the time at which the total energy is equal to the maximum potential energy as shown in Fig. 3.

IV. A STOCHASTIC TRANSIENT ENERGY FUNCTION

Similar to the approach proposed in [4], the load and generation disturbances are modeled stochastically with varying magnitudes depending on bus location in the system. In this paper, we consider only the impact of Gaussian variation (normal distribution), but other

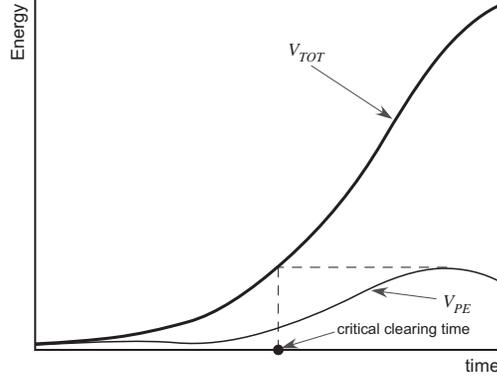


Fig. 3. V_{TOT} and V_{PE}

distributions can be incorporated. For example, wind generation is often modeled as a Weibull distribution [21], whereas PHEV distributions have been suggested to be Poisson distribution [1]. The load power is assumed to vary stochastically with an expected value of the base case loading. In the power system model of equation (14), the system loads are modeled as constant impedances. If both active and reactive powers at a bus are assumed to vary with the same level of randomness, then the load variation manifests itself in the diagonal elements of the reduced admittance matrix as

$$Y(i, i) = Y_{ii} (1 + \alpha_i dW_{t,i}) \angle \phi_{ii}$$

Note that only the magnitude varies; the power factor (and subsequently ϕ_{ii}) is considered to remain constant. The stochastic power system (SPS) equations become:

$$d\theta_i = \tilde{\omega}_i dt \quad (15)$$

$$M_i d\tilde{\omega}_i = \left(P_{M_i} - \frac{M_i}{M_T} P_{COI} - E_i \sum_{j=1}^n E_j [B_{ij} \sin \theta_{ij} + G_{ij} \cos \theta_{ij}] \right) dt - E_i^2 G_{ii} \alpha_i dW_{t,i} \quad (16)$$

$$i = 1, \dots, n$$

and

$$P_{COI} dt = \left(\sum_{i=1}^n (P_{M_i} - E_i^2 G_{ii}) - 2 \sum_{i=1}^{n-1} \sum_{j=1}^n E_i E_j G_{ij} \cos \theta_{ij} \right) dt - \sum_{i=1}^n E_i^2 G_{ii} \alpha_i dW_{t,i} \quad (17)$$

If the power system of equations (13)-(14) is lossless, then the energy function (14) is

a true Lyapunov function and [14]:

$$\dot{V}_{TOT} = \frac{\partial V_{TOT}}{\partial x} f(x) = 0 \quad (18)$$

and the Lyapunov stochastic stability is therefore determined by

$$\mathcal{L}V(x, t) = \frac{1}{2} \text{Tr} \left\{ \Sigma(t)^T g(x, t)^T \frac{\partial^2 V_{TOT}}{\partial x^2} g(x, t) \Sigma(t) \right\} \quad (19)$$

Applying the differential generator \mathcal{L} to equations (14)-(16) yields:

$$\mathcal{L}V(\theta, \tilde{\omega}) = \frac{1}{2} \sum_{i=1}^n (E_i^2 G_{ii} \alpha_i)^2 \left(\frac{1}{M_i} - \frac{1}{M_T} \right) \quad (20)$$

which in the absense of a damping term is always greater than zero for noise magnitude $\alpha_i \neq 0$, therefore the stochastic stability of this system is analytically indeterminant and must be determined numerically.

V. NUMERICAL SOLUTION OF STOCHASTIC DIFFERENTIAL EQUATIONS

The determination of the power system energy requires the numerical solution of the SDE system in equations (15)-(16). The numerical solution of SDEs is conceptually different from the numerical solution of deterministic ordinary differential equations. At the core of the numerical solution of SDEs is the representation of the standard Wiener process over the simulation interval $[0, T_{\max}]$. The random variable $W(t)$ satisfies the three following conditions [22]:

- 1) $W(0) = 0$ (with probability 1)
- 2) For $0 \leq s < t \leq T_{\max}$, the random variable given by the increment $W(t) - W(s)$ is normally distributed with mean zero and variance $t - s$; equivalently, $W(t) - W(s) \sim \sqrt{t - s} N(0, 1)$, where $N(0, 1)$ denotes a normally distributed random variable with zero mean and unit variance.
- 3) For $0 \leq s < t < u < v \leq T_{\max}$, the increments $W(t) - W(s)$ and $W(v) - W(u)$ are independent.

A standard Wiener process $W(t)$ can be numerically approximated in distribution on any finite time interval by a scaled random walk. A stepwise continuous random walk $H_N(t)$ can be constructed by taking independent, equally probable steps of length $\pm \sqrt{\Delta t}$ at the end of each subinterval.

For the ordinary differential equation

$$\dot{x} = f(x, t), \quad x(0) = x_0 \in \mathbb{R}^n$$

the well-known Euler's method can be applied to numerically approximate the solution over $[0, T]$ [23]:

$$x_j = x_{j-1} + \Delta t f(x_{j-1}, t_{j-1}), \quad j = 1, \dots, L \quad (21)$$

where $L\Delta t = T$ and L is a positive integer.

For the stochastic differential equation

$$dx = f(x, t)dt + g(x, t)\Sigma(t)dW(t) \quad x(0) = x_0 \in \mathbb{R}^n$$

a corresponding numerical integration method is the Euler-Maruyama (EM) method [22]:

$$\begin{aligned} x_j &= x_{j-1} + \Delta t f(x_{j-1}, t_{j-1}) + \\ &g(x_{j-1}, t_{j-1}) \Sigma(t_{j-1}) (W(\tau_j) - W(\tau_{j-1})) \\ &j = 1, \dots, L \end{aligned} \quad (22)$$

where $W(\tau_j), W(\tau_{j-1})$ are points on the Brownian path. The set of points $\{t_j\}$ on which the discretized Brownian path is based must contain the points $\{\tau_j\}$ at which the EM solution is computed. If the EM is applied using a stepsize $\Delta t = R\delta t$, then

$$(W(\tau_j) - W(\tau_{j-1})) = W(jR\delta t) - W((j-1)R\delta t) \quad (23)$$

$$= \sum_{k=jR-R+1}^{jR} dW_k \quad (24)$$

VI. ILLUSTRATIVE EXAMPLE

The Euler-Maruyama numerical integration method is applied to the stochastic power system equations of (15) and (16). The test system is the IEEE 3-machine, 9-bus (also known as the WSCC) system shown in Fig. 4. A three-phase fault is applied to bus 8 and then cleared at 0.15 seconds. The deterministic response of the system generator frequencies is shown in Fig. 5. One possible stochastic response for a given α is shown in Fig. 6. The loads are varied stochastically with a variance $\sigma^2 = h$ where h is the interval between samples (i.e. the time step). The magnitude of the variation is $\alpha = 0.0025$.

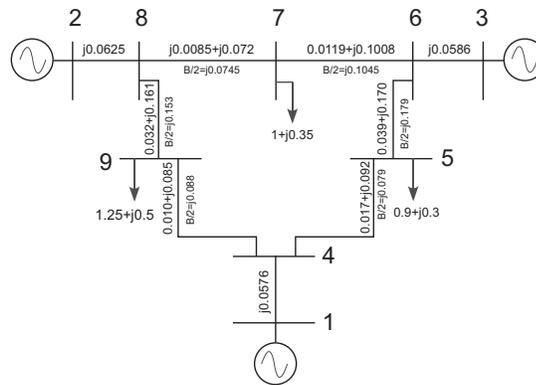


Fig. 4. 3-machine, 9-bus test system

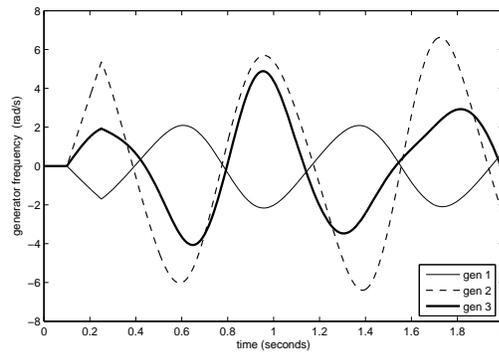


Fig. 5. Deterministic deviation of generator speed to a short circuit

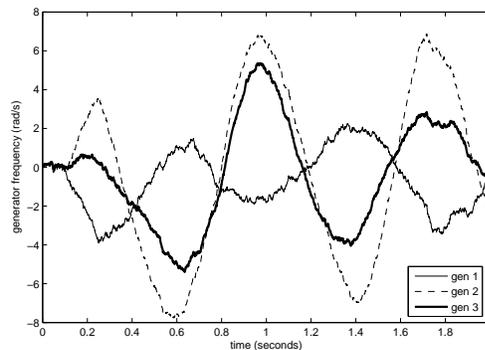


Fig. 6. Stochastic deviation of generator speed to a short circuit

To put this level of variation in context, the diagonal admittances (each $Y(i, i)$) are shown in Fig. 7. This indicates that for this choice of α , the variance in the magnitude of $Y(i, i) \leq 0.5\%$.

At this level of variance, the differences in the generator frequency can vary over a wide range. Ten consecutive simulations with the same α but different random walk sets

yields the set of responses for generator 1 shown in Fig. 8. From these responses, it is obvious that the stability of the power system may be affected by injecting stochasticity into the loads.

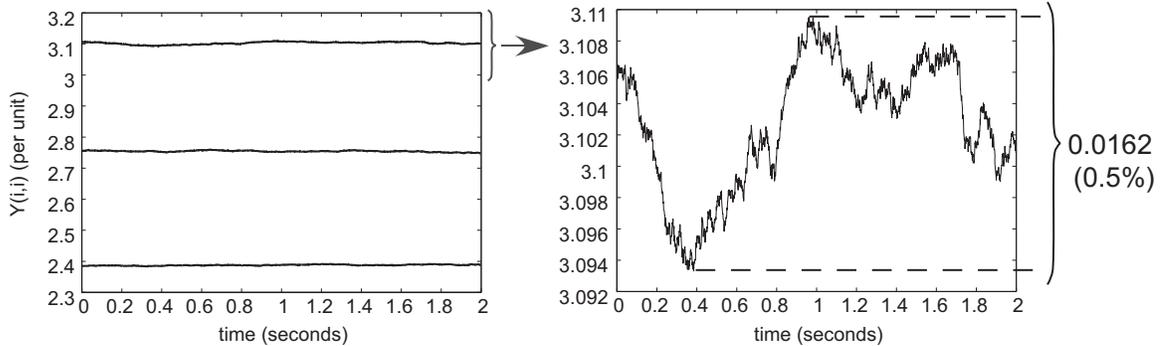


Fig. 7. Stochastic diagonal admittance variance

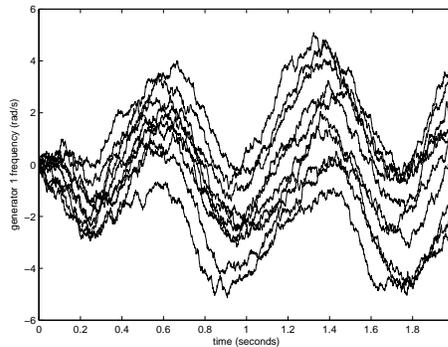


Fig. 8. Generator 1 deviation of generator speed (10 runs)

VII. STOCHASTIC POWER SYSTEM TRANSIENT STABILITY USING ENERGY FUNCTIONS

In the presence of stochasticity, both the kinetic and potential energy will exhibit random behavior. For example, consider Fig. 9 which shows a two-dimensional (top) view of the potential energy contours of the three-dimensional energy “bowl” of a three-machine system for five different runs. Looking closely at the elliptical energy contours, it can be seen that for the set of highest energy (outermost) contours enclosing the stable equilibrium point (SEP), one of the contours is open and approaches the next higher

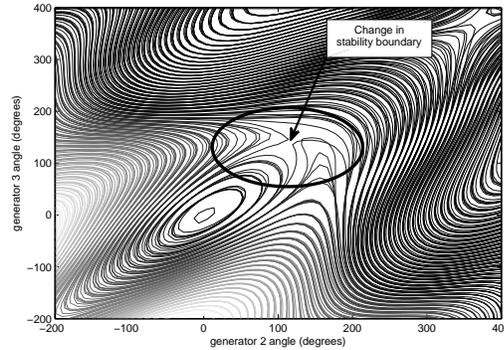


Fig. 9. Illustration of change in PEBS boundary (5 runs)

energy levels which do not enclose the SEP. These open contours indicate a saddle node point, such that if system state approaches this energy level the state may leave the energy well and the SEP cannot be attained post-fault. Thus, it can be seen that the inclusion of random load perturbations can affect the height of the energy well and possibly lead to instability.

The energy function approach for determination of transient stability will be applied to the system of stochastic differential equations and a Monte Carlo approach will be used to assess the critical clearing time of the stochastic system. The critical clearing time of a single run of a SPS will be governed by the magnitude and variance of the applied load perturbation. A single run will produce a critical clearing time that is distributed within a range of critical clearing times as shown in Fig. 10. This range of times will form a probability density function (PDF). Due to the nonlinearities inherent in power system dynamics, it will be shown that the PDF of the critical clearing time will not have the same characteristics as the load (i.e. the CCTs will not have a Gaussian distribution).

The deterministic critical clearing time for a short-circuit fault on bus 8 is 0.233 seconds. Fig. 11 shows a histogram of the critical clearing times obtained from 1000 runs of the SPS for this fault. For a large sample population, the histogram of critical clearing times predicts the shape of the probability density function. Of significant note is that the median value of the histogram is the same as the deterministic critical clearing time. This implies that half of the CCTs are greater than 0.233 seconds and half are smaller. Note however that even though the expected value is the same as the deterministic CCT,

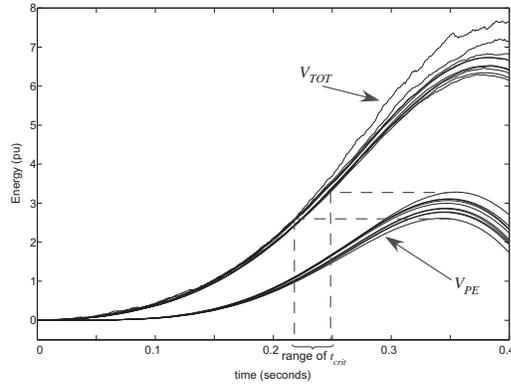


Fig. 10. Illustration of change in t_{crit} (10 runs)

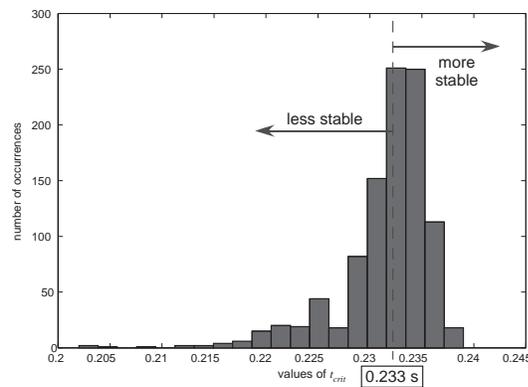


Fig. 11. Histogram of t_{crit} (1000 runs; $\alpha = 0.0025, \sigma^2 = h$)

the variance is not symmetric about the median even though the load perturbations were Gaussian distributed.

One way to interpret these results is that if the protection for this system was designed to act at 0.205 seconds, then according to the histogram, the system would be stable for 997 of the 1000 runs. This could be generalized in a statement that the system would be stable with a probability of 99.7%. These results could further be used in a risk assessment analysis.

Fig. 11 showed the results for a single level of perturbation magnitude α and variance. The next step in this analysis is to determine what impact different values of these parameters have on the stability. The Lyapunov analysis discussed earlier only accounted for the magnitude of the perturbation and not the variance. Fig. 12 shows the histogram of the same fault and the same magnitude of perturbation, but the perturbation is modeled

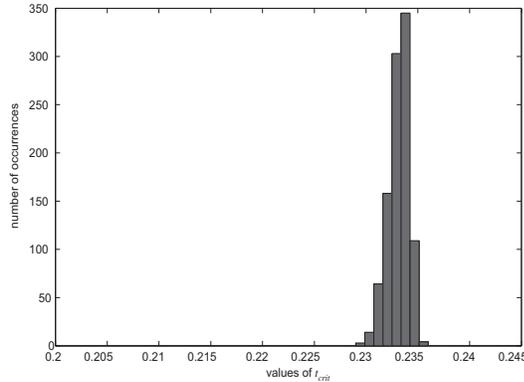


Fig. 12. Histogram of t_{crit} (1000 runs; $\alpha = 0.0025$, $\sigma^2 = h/2$)

with a much smaller variance (in this case, $\sigma^2 = h/2$). As might be expected, since the range of the perturbation is much smaller, so too is the range of the resulting CCTs. In this case, the median value is the same as the deterministic CCT and the values are tightly clustered around 0.233 seconds.

Alternately, Fig. 13 and Fig. 14 show the CCTs for the same fault except with an increase in variance and magnitude, respectively. In both these cases, the median value of the CCT histogram is smaller than the deterministic, and thus (in probability) the system is less stable. Fig. 15 illustrates the impact of magnitude and variance on the value of critical clearing time. Not surprisingly, an increase in variance and magnitude both decrease the expected value of the CCT. The shape of the energy well and the shape of the potential energy boundary surface used in transient energy functions both change as the loads in the system change. The UEPs, potential energy, kinetic energy, and trajectory of the system state are all randomly varying. That is why it is important to draw conclusions from the expected value of the Monte Carlo simulations as opposed to considering individual trajectories and CCTs. It is more informative to consider the expected value and the probability of stability (or instability) than a single random occurrence.

The inclusion of non-Gaussian variation also affects the CCT distribution. Most of the theoretical developments and the available numerical methods have been developed for zero-mean Gaussian perturbations. However, as mentioned previously, not all perturbations to the power system take the form of a Gaussian distribution. For example, wind variability is frequently modeled as having a Weibull probability distribution function as

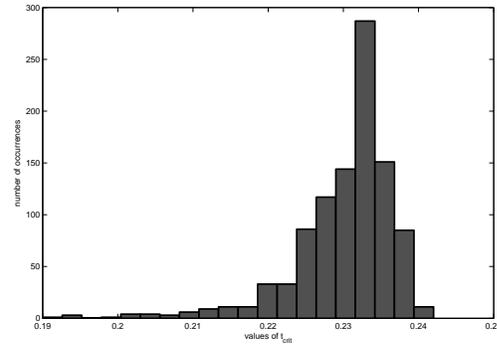


Fig. 13. Histogram of t_{crit} (1000 runs; $\alpha = 0.0025$, $\sigma^2 = 2h$, $E[t_{crit}] = 0.230$)

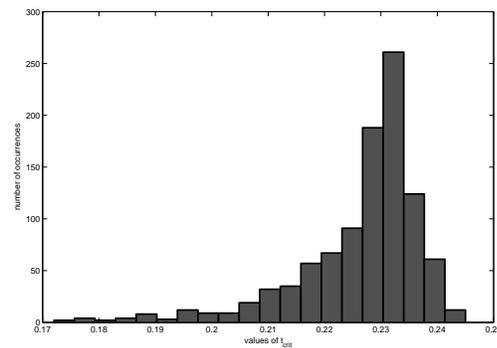


Fig. 14. Histogram of t_{crit} (1000 runs; $\alpha = 0.005$, $\sigma^2 = h$, $E[t_{crit}] = 0.227$)

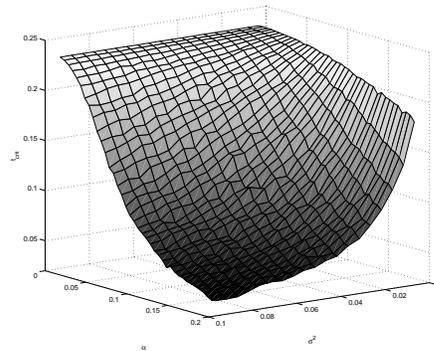


Fig. 15. Impact of magnitude (α) and variance (σ^2) on t_{crit} (1000 runs)

shown in Fig. 16. [21], and PEV distributions have been suggested to have a Poisson distribution [1]. On the other hand, no classical probability distribution function can be satisfactorily fitted to solar radiation [24]. As already seen for the classical model case, even if a Gaussian noise function is used, the resulting distribution function for the critical clearing times is significantly non-Gaussian. It is difficult to even predict how other noise

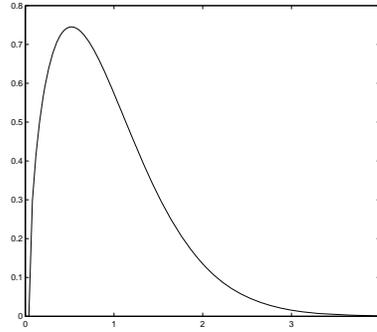


Fig. 16. A Weibull probability distribution function with $k = 1$ and $\lambda = 1.5$

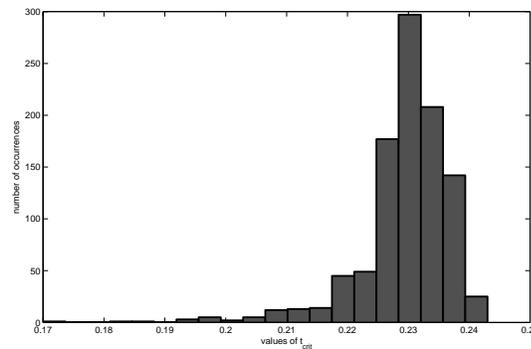


Fig. 17. Histogram of t_{crit} with Weibull distribution (1000 runs; $E[t_{crit}] = 0.230$)

functions may impact the stability of the system, but we have explored the possibility of the impact of Weibull distribution. Although it is difficult to perform a one-to-one comparison with Gaussian noise, we have endeavored to construct a Weibull distribution with a mean centered at the initial loading and with a maximum variation equivalent to twice the standard deviation of the Gaussian. The Weibull distribution is given by

$$f(x) = \begin{cases} \frac{k}{\lambda} \left(\left(\frac{x}{\lambda} \right)^{k-1} e^{-\left(\frac{x}{\lambda} \right)^k} \right) & x \geq 0 \\ 0 & x < 0 \end{cases} \quad (25)$$

where $\lambda > 0$ is the scale factor and $k > 0$ is the shape factor. In wind applications, k can range from 1 to 2.5, with regions with low wind having smaller k factors. In our example, we chose $k = 1.5$. Fig. 17 shows the CCT distribution when the load has a Weibull distribution as might occur if a small wind turbine were attached at a bus. While it is difficult to directly compare this distribution with one resulting from a Gaussian

distribution, it can be noted that the general trend is the same as with Fig. 14. The only difference is that the expected value is slightly larger for the Weibull than the Gaussian. This is to be expected since the Weibull distribution has only one “tail.” This asymmetry will cause the CCTs to trend slightly to one side (in this case towards stability since the Weibull distribution was used to model a generator contribution rather than a load).

VIII. DISCUSSION

Probably the first question that arises when considering the results of this example is why does the CCT probability density function take on a shape different than the shape of the noise perturbation? This can be easily understood by looking at the nature of the energy function. Consider again the SMIB stochastic power system of equations (8)-(9). During the fault, no power may flow to the system. The fault-on stochastic equations are (neglecting damping):

$$d\theta = \omega dt \quad (26)$$

$$Md\omega = (P_m - P_L^0) - \alpha dW_t \quad (27)$$

thus, during the fault:

$$\omega(t) = \frac{1}{M} (P_m - P_L^0) t - \frac{\alpha}{M} \int_0^t dW_s \quad (28)$$

and the kinetic energy of the fault-on system is

$$\begin{aligned} V_{KE} &= \frac{1}{2} M \omega(t)^2 \quad (29) \\ &= \frac{M}{2} \left[\left(\frac{(P_m - P_L^0)}{M} t \right)^2 \right. \\ &\quad \left. - 2 \frac{\alpha}{M} \int_0^t dW_s + \frac{\alpha^2}{M^2} \left(\int_0^t dW_s \right)^2 \right] \quad (30) \end{aligned}$$

The first term is the deterministic value of the kinetic energy, denoted \bar{V}_{KE} . The second term contains the integral:

$$\int_0^t dW_s$$

which is the Brownian motion (or random walk) term and has an expected value of zero. It is as likely to be positive as it is to be negative. The third term however contains the square of this random walk and the expected value is therefore always positive, regardless

of whether or not the point along the Brownian motion path is positive or negative. The result of this is that

$$E[V_{KE}] \geq \bar{V}_{KE} \quad (31)$$

This is illustrated in Fig. 18. Therefore the expected value of the kinetic energy will most probably be to the left of the deterministic value \bar{V}_{KE} .

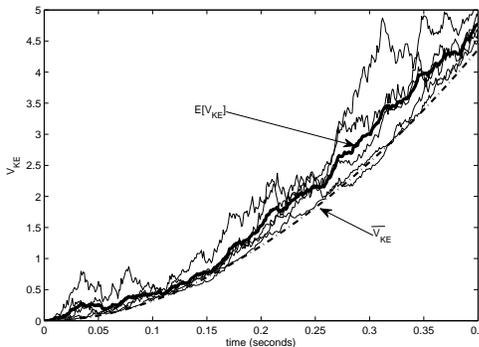


Fig. 18. Expected and deterministic values of V_{KE} (5 runs)

Furthermore, the angle δ is the integral of ω and therefore the large variations found in ω are smoothed. Since the potential energy is a function of angle only, there is not as wide a variance in the expected value of potential energy. This is why the values of t_{crit} stray much further to the left in the time domain, than to the right leading to the shifted probability distribution function.

IX. CONCLUSIONS AND FUTURE WORK

This paper develops an approach to analyze the impact of random load and generation variations on the transient stability of a power system. The well-known energy function method for power system transient stability is used as a basis to explore the stochastic power system stability through a stochastic Lyapunov stability analysis. Further, the method was extended numerically using the Euler-Maruyama method. It was shown that increasing either (or both) the variance and the magnitude of the applied variation can have a destabilizing effect on the power system. This could potentially cause difficulties as more randomness is introduced into the power system through renewable energy sources and plug-in-hybrid vehicles.

Further work will include an extension to the structure-preserving model so that the loads may be explicitly model rather than as constant impedances. Other considerations may include exploring the impact of non-Gaussian distributions on critical clearing times. An additional area of study would include modeling the stochastic behavior of generation scheduling.

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III. Structure-Preserved Power System Transient Stability Using Stochastic Energy Functions

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ABSTRACT: With the increasing penetration of renewable energy systems such as plug-in hybrid electric vehicles, wind and solar power into the power grid, the stochastic disturbances resulting from changes in operational scenarios, uncertainties in schedules, new demands and other mitigating factors become crucial in power system stability studies. This paper presents a new method for analyzing stochastic transient stability using the structure-preserving transient energy function. A method to integrate the transient energy function and recloser probability distribution functions is presented to provide a quantitative measure of probability of stability. The impact of geographical distribution and signal to noise ratio on stability is also presented.

I. INTRODUCTION

Electrical power system loads are functions of a myriad of active and reactive power demands that depend on a variety of factors including time, weather, geography, and economics. The result of the aggregate behavior of many thousands of individual customer devices switching independently are power system loads that are stochastic in nature. The variability of the electrical network loading has received increased attention in recent years due to the expansion of renewable resources and the likelihood of wide-spread adoption of plug-in electric vehicles (PEVs) [1]. Renewable energy resources such as wind turbines or solar power can introduce uncertainty into the power system as a result of atmospheric variations causing excursions in active power generation. Furthermore, plug-in electric vehicles are a potential significant source of disturbance on the grid due to their battery charge and discharge characteristics. The tandem effect of renewable resources and PEVs

may create uncertainties of such significant magnitude they impact the operation of the power system.

The study and analysis of stochastic power system dynamics is not a new topic; it has been studied for several decades [2]-[5], but has received renewed interest in recent years as the amount of uncertainty in the system has increased [6]-[9]. The inclusion of stochasticity in power systems may lead to very different stability results from a deterministic approach. For example, even though a deterministic power system might be stable, small random perturbations may cause the state trajectories to reach a critical point such that exceeding this point may cause the system to collapse or enter an undesirable operating state [10]. As power system loads and generation become increasingly non-deterministic, it is essential that analytical methods be developed to analyze the behavior of the stochastic system to better understand the inherent risks and provide sufficient protection against failures.

Power system transient stability is typically assessed either through direct methods (such as Lyapunov-based energy functions), or through time-domain simulation [11]-[15]. The inclusion of randomness into transient stability analysis most often requires the use of Monte Carlo methods to ascertain the behavior of the system over multiple trials. The basic idea for a Monte Carlo approach to transient stability assessment using transient energy functions was first proposed in [4], but the appropriate stochastic tools did not exist at that time to frame the stochastic energy function nor to numerically solve the stochastic differential equations.

Since the stochastic behavior of the power system is typically manifested through the variance of the loads, the choices of power system model and the particular transient stability assessment method are crucial. In many Lyapunov-based transient stability studies, the system energy function is developed for the “classical model” in which the load impedance is absorbed into an equivalent reduced network as viewed from the generator buses. In such a scheme the structure of the original network is lost. Although the classical model is frequently used in transient stability direct methods, this model is known to have several shortcomings: (i) it precludes the consideration of reactive power demand and voltage variation at the load buses; and (ii) the reduction of the impedance network leads to a loss of system topology and hence precludes the study

of how the transient energy varies among different components of the network [12]-[15]. An alternative approach is to adopt the structure preserving model in which the active and reactive demand at each load bus is explicitly represented. The use of a structure preserving model of the system, first proposed by Bergen and Hill [16] aims at overcoming some of the shortcomings of the classical model thereby allowing accurate modeling of loads. The structure preserved model maintains the original network and uses the unreduced admittance matrix resulting in a model that can be regarded as having structural integrity [17].

Since the time of [4], there has been considerable progress made in the development of the appropriate tools necessary to address stochastic transient stability. There have been numerous recent advances in the application of Lyapunov stability methods to stochastic differential equation systems [19]-[21]. Furthermore, the past decade has seen significant advances in the development of numerical integration methods to simulate stochastic (ordinary) differential equations [22]. *In this paper, these advances in stochastic Lyapunov stability methods and the numerical solution of systems of stochastic differential equations will be merged to present a novel approach to developing a quantitative measure of probability of stability that is suitable for power system risk assessment.*

II. STRUCTURE PRESERVED STOCHASTIC TRANSIENT ENERGY FUNCTIONS

The concept of transient stability is based on whether, for a given disturbance, the trajectories of the system states during the disturbance remain in the domain of attraction of the post-disturbance equilibrium when the disturbance is removed. Transient instability in a power system is caused by a severe disturbance which creates a substantial imbalance between the input power supplied to the synchronous generators and their electrical outputs. Some of the severely disturbed generators may “swing” far enough from their equilibrium positions to lose synchronism. Such a severe disturbance may be due to a sudden and large change in load, generation, or network configuration. Since large disturbances may lead to nonlinear behavior, Lyapunov functions are well-suited to determine power system transient stability. Since true Lyapunov functions do not exist for lossy power systems, so-called “transient energy functions” are frequently used to assess the dynamic behavior of the system [25]. From a modeling point of view the structure

preserved model allows a more realistic representation of power system components including load behaviors and generator dynamic models.

To better understand the how the structure preserved transient energy function will be developed and analyzed, a brief review of Lyapunov functions for stochastic differential equations is first presented.

Consider the nonlinear stochastic system

$$dx = f(x, t)dt + g(x, t)\Sigma(t)dW(t) \quad x(0) = x_0 \in \mathbb{R}^n \quad (1)$$

whose solution can be written in the sense of Ito:

$$x(t) = x_0 + \int_0^t f(x, s)ds + \int_0^t g(x, s)\Sigma(s)dW(s) \quad (2)$$

where $x(t) \in \mathbb{R}^n$ is the state; $W(t)$ is an m -dimensional standard Wiener process defined on the complete probability space (Ω, \mathcal{F}, P) ; the functions (f, g) are locally bounded and locally Lipschitz continuous in $x \in \mathbb{R}^n$ with $f(0, t) = 0, g(0, t) = 0$ for all $t \geq 0$; and the matrix $\Sigma(t)$ is nonnegative-definite for each $t \geq 0$. These conditions ensure uniqueness and local existence of strong solutions to (1) [19][26].

As with many nonlinear deterministic systems, Lyapunov functions can provide guidance regarding the stability of stochastic differential equation (SDE) systems. An SDE system is said to satisfy a Stochastic Lyapunov Condition at the origin if there exists a proper Lyapunov function $V(x)$ defined in a neighborhood D of the origin in \mathbb{R}^n such that

$$\mathcal{L}V(x) \leq 0 \quad (3)$$

for any $x \in D \setminus \{0\}$ where the differential generator \mathcal{L} is given by

$$\mathcal{L}V(x, t) = \frac{\partial V}{\partial x} f(x, t) + \frac{1}{2} \text{Tr} \left\{ \Sigma(t)^T g(x, t)^T \frac{\partial^2 V}{\partial x^2} g(x, t) \Sigma(t) \right\} \quad (4)$$

If equation (3) is satisfied, then the equilibrium solution $x(t) \equiv 0$ of the stochastic differential equation (1) is considered to be *stable in probability* [27].

To accurately include the effects of the loads in the system, the so called structure-preserved, center-of-inertia model of the power system is used, such that [18]:

$$\dot{\theta}_i = \tilde{\omega}_i \quad (5)$$

$$M_i \dot{\tilde{\omega}}_i = P_{M_i} - \sum_{j=1}^n B_{i,j} V_i V_j \sin(\theta_i - \theta_j) - \frac{M_i}{M_T} P_{COI} \quad (6)$$

$$i = 1, \dots, m$$

$$0 = P_{d_i} + \sum_{j=1}^n B_{i,j} V_i V_j \sin(\theta_i - \theta_j) \quad (7)$$

$$0 = Q_{d_i} + \sum_{j=1}^n B_{i,j} V_i V_j \cos(\theta_i - \theta_j) \quad (8)$$

$$i = m + 1, \dots, n$$

where

$$\theta_i = \delta_i - \delta_0$$

$$\tilde{\omega}_i = \omega_i - \omega_0$$

and

$$\delta_0 = \frac{1}{M_T} \sum_{i=1}^m M_i \delta_i; \quad \omega_0 = \frac{1}{M_T} \sum_{i=1}^m M_i \omega_i; \quad M_T = \sum_{i=1}^m M_i$$

$$P_{COI} = \sum_{i=1}^m \left(P_{M_i} - \sum_{j=1}^n B_{i,j} V_i V_j \sin(\theta_i - \theta_j) \right) \quad (9)$$

where

δ_i	generator rotor angle
θ_i	COI bus angle
ω_i	generator angular frequency
$\tilde{\omega}_i$	COI angular frequency
M_i	inertia constant
P_{M_i}	mechanical output
V_i	bus voltage
B_{ij}	(i, j) -th entry of the reduced lossless admittance matrix
m	number of generators in the system
n	number of total buses in the system
ω_s	synchronous speed in radians

and P_{d_i} and Q_{d_i} are the load demands at each bus i in the system.

The corresponding energy function is [18]:

$$\begin{aligned}
V(\tilde{\omega}_{gi}, \theta, V) = & \frac{1}{2} \sum_{i=1}^m M_i \tilde{\omega}_{gi}^2 - \sum_{i=1}^m P_{M_i} (\theta_i - \theta_i^s) + \sum_{i=1}^{n+m} P_{d_i} (\theta_i - \theta_i^s) \\
& - \frac{1}{2} \sum_{i=1}^{n+m} B_{ii} (V_i^2 - (V_i^s)^2) + \sum_{i=1}^{n+m} \frac{Q_{d_i}^s}{a (V_i^s)^a} (V_i^a - (V_i^s)^a) \\
& - \sum_{i=1}^{n+m-1} \sum_{j=i+1}^{n+m} B_{ij} (V_i V_j \cos(\theta_i - \theta_j) - V_i^s V_j^s \cos(\theta_i^s - \theta_j^s)) \quad (10)
\end{aligned}$$

where a is usually 2 and the superscript ‘ s ’ indicates the stable equilibrium point. In the structure preserved power system, the loads P_{d_i} and Q_{d_i} can be augmented to include the impact of uncertain and stochastic variations:

$$P_{d_i} = P_{d_i}^0 (1 + \alpha_{P_i} W_i(t)) \quad (11)$$

$$Q_{d_i} = Q_{d_i}^0 (1 + \alpha_{Q_i} W_i(t)) \quad (12)$$

where $P_{d_i}^0, Q_{d_i}^0$ are the mean values of the active and reactive load at bus i respectively and $\alpha_{P_i}, \alpha_{Q_i}$ are the magnitudes of the active and reactive noise. Note that the variance in the noise (i.e. standard deviation) is not explicitly represented but is inherent in the construction of the Weiner process $W_i(t)$.

Similar to the approach proposed in [10], the load and generation disturbances are modeled stochastically with varying magnitudes depending on bus location in the system. In this paper, we consider only the impact of Gaussian variation (normal distribution), but other distributions can be incorporated. For example, wind generation is often modeled as a Weibull distribution [23], whereas PHEV distributions have been suggested to be Poisson distribution [24]. The load power is assumed to vary stochastically with an expected value of the base case loading. The loads at each bus are subjected to random perturbations with Gaussian (white) variation ($dW(t)$ from equation (1)) as shown in Fig. 1(a). The resulting load variation takes the form of a Wiener process, also known as Brownian motion or a random walk, as shown in Fig. 1(b).

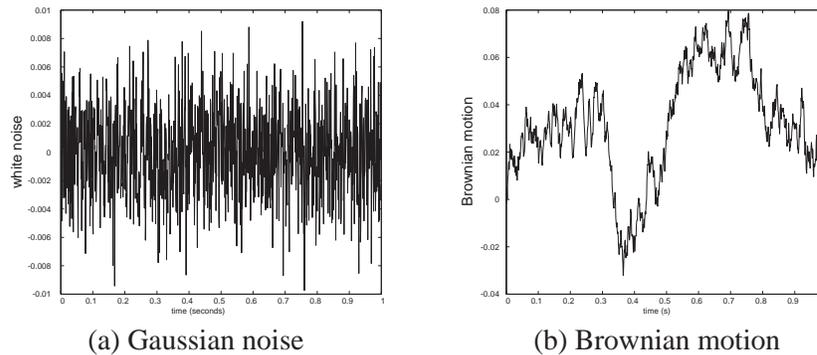


Fig. 1. Load Gaussian noise (a) and resulting Brownian motion (b)

III. METHODOLOGY

The closest unstable equilibrium point (UEP) and controlling UEP method are two common direct methods used to assess the system's stability [25]. The controlling UEP method consists of numerically integrating the system state and calculating the kinetic, potential, and total energy of the fault-on system until the point at which the potential energy reaches its maximum value. The critical clearing time (CCT) of the system is then calculated by finding the time at which the total energy is equal to the maximum potential energy as shown in Fig. 2.

The energy function approach for determination of transient stability is applied to the system of stochastic differential-algebraic equations and a Monte Carlo approach has

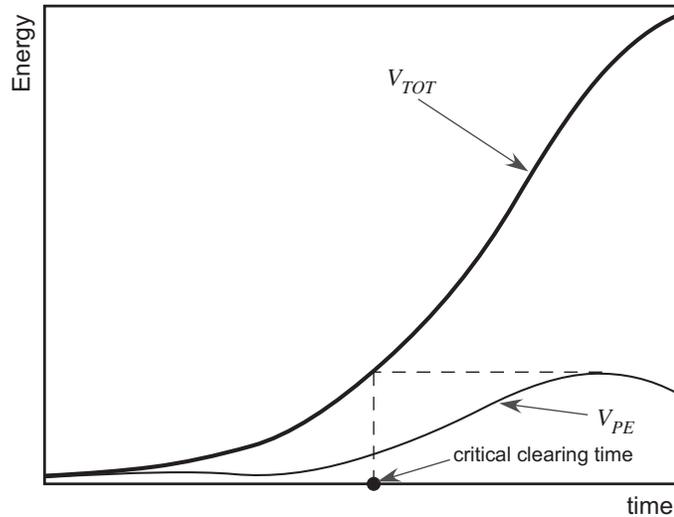


Fig. 2. The total energy V_{TOT} versus the potential energy V_{PE} .

been used to construct the probability distribution of the critical clearing time of the stochastic system. Ten consecutive simulations with the same Gaussian noise magnitude and variance but different noise sets yields the set of energies V_{TOT} and V_{PE} shown in Fig. 3.

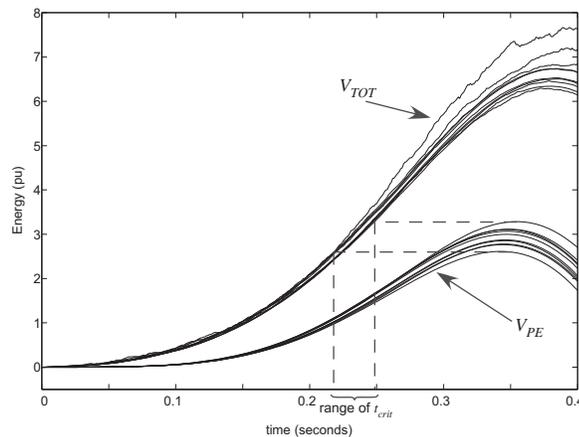


Fig. 3. Illustration of change in t_{crit} over the range of 10 runs – upper plots show the total energy V_{TOT} , lower plots show potential energy V_{PE} .

These responses demonstrate that the stability of the power system may be significantly affected by injecting stochasticity into the loads. Fig. 4 shows a histogram of the critical clearing times obtained from 1000 transient stability runs. This histogram was generated by calculating the critical clearing time of 1000 runs of the energy function method. This

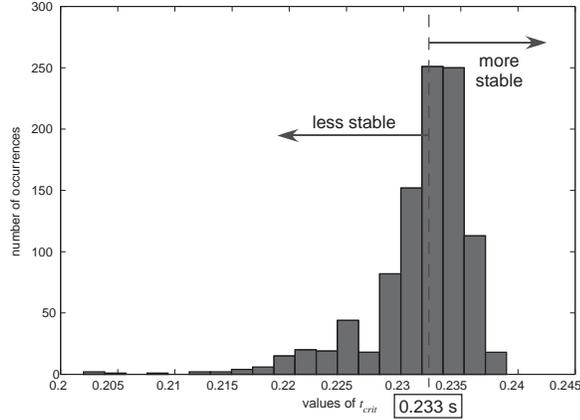


Fig. 4. Histogram of t_{crit} (1000 runs)

set of critical clearing times ranges from the least stable case of $t_{crit} = 0.2025$ seconds to the most stable case of $t_{crit} = 0.2385$ seconds with a mean value of $t_{crit} = 0.233$ seconds. Note that the mean CCT value 0.233 seconds is also the same CCT obtained from a single deterministic run of the energy method. Note that if another 1000 runs were performed with different noise sets, this histogram would most likely look slightly different, but would have the same general distribution and should yield the same mean value.

For a large sample population, the histogram of critical clearing times predicts the shape of the probability density function. Of significant note is that for a standard deviation and variance of 1.0, the median value of the histogram is the same as the deterministic critical clearing time. This implies that half of the CCTs are greater than 0.233 seconds and half are smaller. Note, however, that even though the expected value is the same as the deterministic CCT, the variance is not symmetric about the median even though the load perturbations are Gaussian distributed.

One way to interpret these results is to combine the critical clearing time distribution with a recloser distribution. The probability of maintaining stability P_S is then given by

$$P_S = \int_{\tau_r=0}^{\tau_r=\infty} \int_{\tau_d=0}^{\tau_d=\tau_r} f_{CCT}(\tau_r) f_R(\tau_d) d\tau_r d\tau_d \quad (13)$$

where f_R is the probability distribution of the recloser and f_{CCT} is the probability distribution function of the critical clearing times [28].

For example, consider a recloser probability distribution function shown in Fig. 5. The recloser action is a Gaussian distribution with an actuation mean time of 0.225 seconds and a one cycle standard deviation. The probability distribution of the critical clearing times cannot be represented by a closed form distribution, but the P_S can be estimated by:

$$P_S \approx \sum_{k_d=1}^N \sum_{k_r=1}^{k_d} \hat{f}_{CCT}(k_r) \hat{f}_R(k_d) \quad (14)$$

where \hat{f}_{CCT} and \hat{f}_R are the discretized distribution functions and N is the total number of samples. Applying this to the histogram of critical clearing times in Fig. 4, the probability of stability as a function of mean recloser time (with a one cycle standard deviation) is shown in Fig. 6.

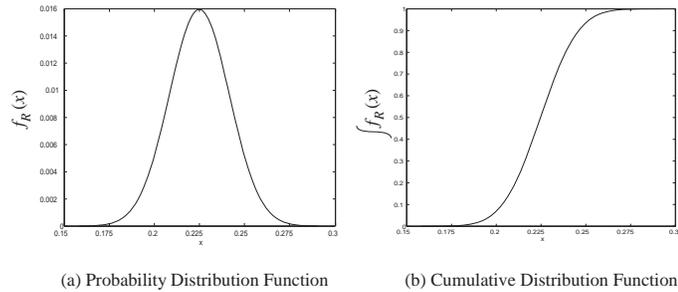


Fig. 5. Recloser distribution function with $\mu = 0.225$ s and $\sigma = 1/60$ s

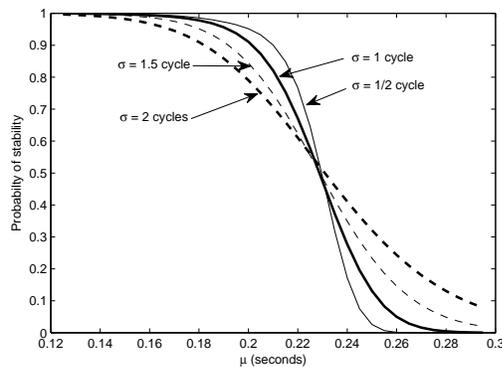


Fig. 6. Probability of stability as a function of recloser action expected value μ with varying σ

As the mean recloser time decreases, the probability that the system will be stable increases to 1.0 (100%) regardless of the standard deviation of the recloser action. This

implies that the more quickly the fault is cleared, the more likely the system is to be stable. However, as the standard deviation increases from $\frac{1}{2}$ cycle to 2 cycles, the slope of the probability curve decreases. This is intuitive since as the standard deviation increases, the spread of recloser action from the mean increases, allowing greater variation. As the standard deviation approaches zero, the slope approaches infinity at $\mu = 0.233$ seconds and 50% probability. Recall that the deterministic critical clearing time is 0.233s and is also the expected mean of the histogram of critical clearing times in Fig. 4. Therefore, as the standard deviation approaches 0, the probability distribution curve of the recloser action approaches a Dirac delta and will sample only a single point at the mean (which is 0.233 seconds). The process for determining the probability of stability is summarized in Fig. 7.

IV. NUMERICAL SOLUTION OF STOCHASTIC DIFFERENTIAL EQUATIONS

The determination of the power system energy requires the numerical solution of the SDE system. The numerical solution of SDEs is conceptually different from the numerical solution of deterministic ordinary differential equations. At the core of the numerical solution of SDEs is the representation of the standard Wiener process over the simulation interval $[0, T_{\max}]$. The random variable $W(t)$ satisfies the three following conditions [22]:

- 1) $W(0) = 0$ (with probability 1)
- 2) For $0 \leq s < t \leq T_{\max}$, the random variable given by the increment $W(t) - W(s)$ is normally distributed with mean zero and variance $t - s$; equivalently, $W(t) - W(s) \sim \sqrt{t - s}N(0, 1)$, where $N(0, 1)$ denotes a normally distributed random variable with zero mean and unit variance.
- 3) For $0 \leq s < t < u < v \leq T_{\max}$, the increments $W(t) - W(s)$ and $W(v) - W(u)$ are independent.

A standard Wiener process $W(t)$ can be numerically approximated in distribution on any finite time interval by a scaled random walk. A stepwise continuous random walk $H_N(t)$ can be constructed by taking independent, equally probable steps of length $\pm\sqrt{\Delta t}$ at the end of each subinterval.

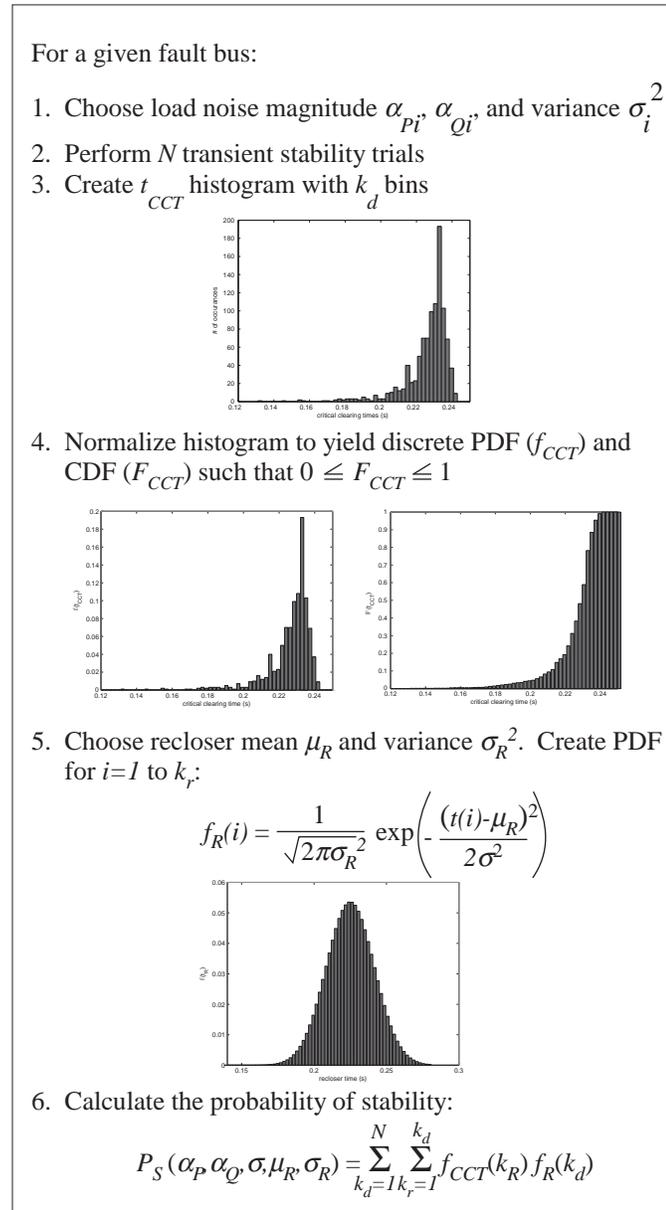


Fig. 7. Process for determining the stability of the system

For the ordinary differential equation

$$\dot{x} = f(x, t), \quad x(0) = x_0 \in \mathbb{R}^n$$

the well-known Euler's method can be applied to numerically approximate the solution over $[0, T]$ [30]:

$$x_j = x_{j-1} + \Delta t f(x_{j-1}, t_{j-1}), \quad j = 1, \dots, L \quad (15)$$

where $L\Delta t = T$ and L is a positive integer.

For the stochastic differential equation

$$dx = f(x, t)dt + g(x, t)\Sigma(t)dW(t) \quad x(0) = x_0 \in \mathbb{R}^n$$

a corresponding numerical integration method is the Euler-Maruyama (EM) method [22]:

$$\begin{aligned} x_j &= x_{j-1} + \Delta t f(x_{j-1}, t_{j-1}) + \\ &g(x_{j-1}, t_{j-1}) \Sigma(t_{j-1}) (W(\tau_j) - W(\tau_{j-1})) \\ &j = 1, \dots, L \end{aligned} \quad (16)$$

where $W(\tau_j), W(\tau_{j-1})$ are points on the Brownian path. The set of points $\{t_j\}$ on which the discretized Brownian path is based must contain the points $\{\tau_j\}$ at which the EM solution is computed. If the EM is applied using a stepsize $\Delta t = R\delta t$, then

$$(W(\tau_j) - W(\tau_{j-1})) = W(jR\delta t) - W((j-1)R\delta t) \quad (17)$$

$$= \sum_{k=jR-R+1}^{jR} dW_k \quad (18)$$

V. APPLICATION

To illustrate the application of the structure preserved stochastic energy function, the method is applied to the small power system shown in Fig. 8. This system was introduced in [29] for the study of structure preserving power systems.

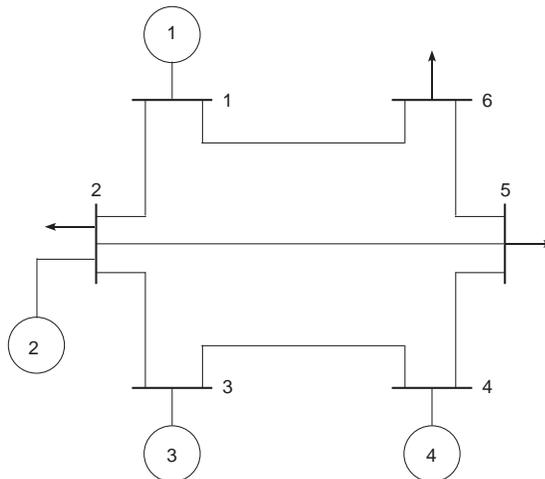


Fig. 8. 4-machine, 6-bus test system

As a benchmark, the deterministic system is subjected to a fault on bus 3 which is cleared at 0.46 seconds. The resulting generator angular frequencies and bus voltages are shown in Fig. 9 and Fig. 10, respectively.

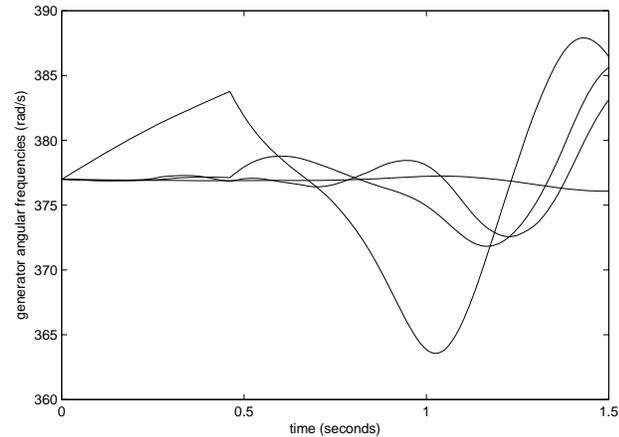


Fig. 9. Deterministic test system generator frequencies

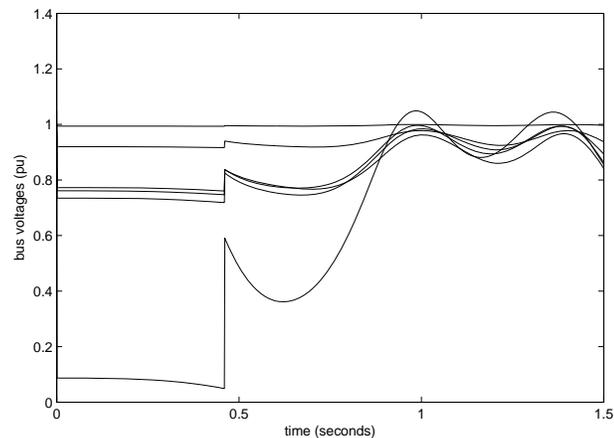


Fig. 10. Deterministic test system voltages

To illustrate the effect of the varying loads, ten different sets of noise with the same magnitude of variation and standard deviation are applied to the loads. The resulting noisy generator 4 frequency and bus 6 voltage are shown in Fig. 11 and Fig. 12, respectively. The mean, or expected, value of each set of responses is shown in bold. Note that the expected responses for both frequency and voltage are nearly identical to the deterministic

responses. The generator frequency is much smoother than the voltage because of the impact of the integration of the noise. Generator frequency (ω) is a state variable whereas voltage is an algebraic variable and changes in load are observed instantaneously.

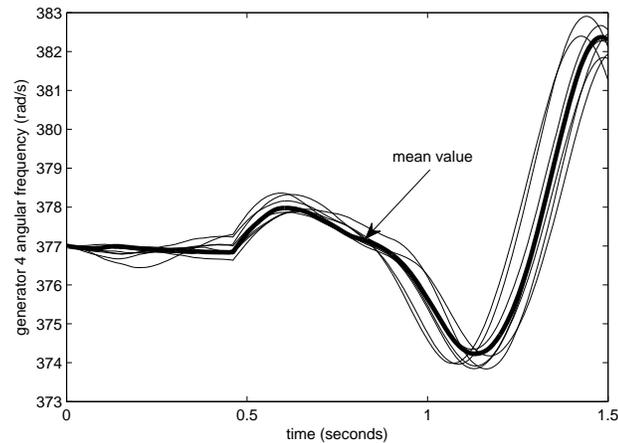


Fig. 11. Test system generator 4 frequency (10 runs)

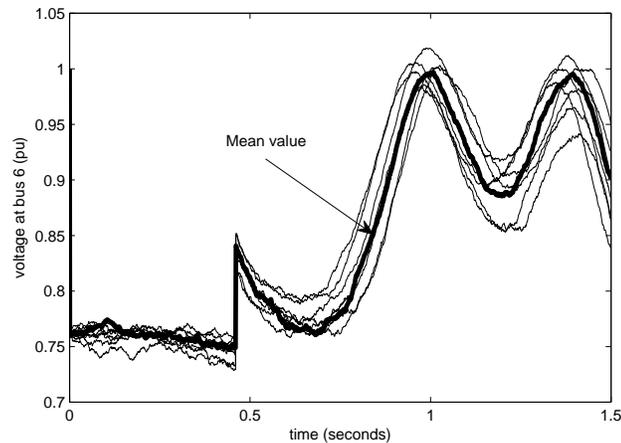


Fig. 12. Test system bus 6 voltages (10 runs)

For the test system, the deterministic critical clearing time is determined to be 0.74 seconds. To further elucidate the impact of noise on the critical clearing times, the critical clearing times resulting from 100 runs are plotted as a function of the inverse signal to noise ratio (i.e. SNR^{-1}) at a single bus (bus 5) in Fig. 13. As the level of noise in the signal decreases, the critical clearing times approach the deterministic CCT of 0.74 seconds. As

the noise level increases, the spectrum of CCTs increase in both the larger and smaller directions, but with a greater spread towards smaller CCTs. This is an indication that as the noise level increases, the system is more likely to become unstable.

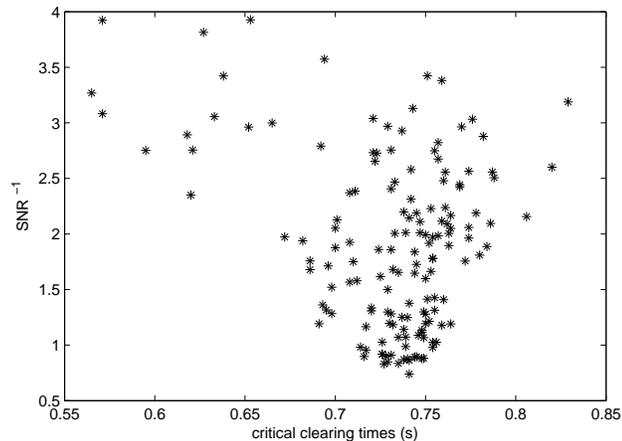


Fig. 13. Critical clearing times as a function of SNR^{-1} (100 runs)

To illustrate the impact of noise at different geographic locations, equal amounts of (expected) noise are added to the different load buses and the critical clearing times are plotted. Fig. 14 shows the impact of noise added at different locations on the critical clearing time. From the figure, it can be observed that the stability of the system is most sensitive to random load variations at bus 2 (for a fault on bus 3) and least sensitive to noise levels at bus 6. It is theorized that this sensitivity is due to the proximity of the buses to the fault bus. The closer the fault is to a bus, the more sensitive the critical clearing time is to random changes in load. If information regarding penetration of wind turbines, solar panels, or other randomly varying component is available, this information can be used to scale the noise magnitudes to provide a histogram of CCTs as a function of geographical differences.

VI. CONCLUSIONS AND FUTURE WORK

This paper develops an approach to analyze the impact of random load and generation variations on the transient stability of a structure preserved power system. The well-known energy function method for power system transient stability is used as a basis to explore the stochastic power system stability through a stochastic Lyapunov stability analysis.

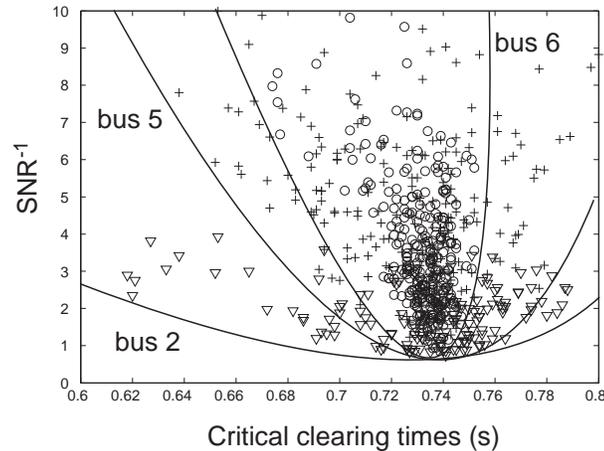


Fig. 14. Critical clearing times as a function of SNR^{-1} (100 runs); load changes at bus 2 (∇), bus 5 (+), and bus 6 (\circ)

Further, the method was extended numerically using the Euler-Maruyama method. It was shown that increasing the magnitude of the applied variation or changing the geographic location can have a destabilizing effect on the power system. This could potentially cause difficulties as more randomness is introduced into the power system through renewable energy sources and plug-in-hybrid vehicles.

Further work may include exploring the impact of non-Gaussian distributions on critical clearing times. An additional area of study would include modeling the stochastic behavior of generation scheduling.

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SECTION

2. CONCLUSIONS

Faced with the growing complexity of the future power grid and the stochastic disturbances caused by renewable energy sources such as PHEVS, wind and solar power, this dissertation deals with the issue of the stability of the power system and has presented contributions in the tools developed and analysis carried out to examine the stability of the power system when stochastic loads and generations are present. This will play an important role in the planning and operation of electric power systems. A new model for the study of stochastic power system stability using stochastic Lyapunov function was also developed.

The primary contributions of this research are the development of a stochastic energy function for power system transient stability analysis. The stochastic energy function was first developed for a classical model, reduced admittance matrix system and then extended to a classical model, structured preserved system. The proposed methodology produced a probability distribution function of critical clearing times for a given fault within a power system. The probability distribution function was determined through a Monte Carlo simulation approach. The critical clearing time probability distribution function was then shown to be used to determine the interdependent probability of stability of a system by combining the critical clearing time with the probability distribution function of a recloser. This approach can be further generalized to other power system components as well. The effect of using a non-Gaussian distribution was explored. Lastly, the effect of noise and geographic distribution of the randomly varying loads was illustrated.

Future work may include the stochastic effects of generator modeling and the particular random distribution of other types of loads such as photovoltaics.

VITA

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