Mathematical Modelling of Surface Discharge on the Contaminated Surface of Insulator Using Nernst's Planck Equation

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Abstract— **The outdoor insulator is exposed to the surface degradation due to the continuous electrical and environmental stresses. The contaminant flow due to the pollution that mixed with water like dew or rainwater will provide a conductive path that allowed the leakage current flow. This leakage current flow will heat-up the insulator surface and causing the formation of carbonize path due to the surface discharge. Thus, leakage current measurement had been widely used in the monitoring of surface discharge phenomena and to understand the insulator surface condition. However, study on the physical process of discharge phenomenon that leads to the conductive path and breakdown on the insulator are not well concerned and understood. Therefore, this study is concerned with modelling of surface discharge on the contaminated surface of insulator including the charge transport and generation mechanism. This model is used electro-migration model and considered the electric field dependent molecular ionization as the generation mechanism of charge carriers. The charge carrier generation and transport mechanism are accounted with the Nernst Planck theory to model the behaviour of the charge carriers while Poisson's equation is used to determine the distribution of electric field on the insulator surface. A mathematical model of surface discharge on the insulator based on the Nernst Planck Theory is then discussed.**

Index Terms— **Electric Discharge; Leakage Current; Nernst's Planck equation.**

I. INTRODUCTION

The insulation is the most important part of the high voltage application and can be exposed to some stresses like mechanical, thermal, electrical and environmental stress that acts permanently or temporary. Each stress will give the difference effect of the deterioration and ageing effect to the insulators. A study is conducted by Rusu-Zagaret. al. [1], has shown that insulation suffers more pronounced ageing under the action of multifactor stresses. This result showed that precaution action must be taken in order to reduce the failure factor of insulation by the presence of the stresses. The good insulator must be able to avoid the flow of current to the undesired path.

Furthermore, the stress could exceed the electrical strength of air and spark occurred, causing discharges along small portions of the insulation. This discharge can lead to the formation of carbonized path and tracking between the high voltage electrode and the ground electrode due to the heat generated on the surface of the insulator due to the flow of leakage current. Although the leakage current flow is quite small, the long-term of leakage current flow due to the

contaminant flow may lead to the insulator degradation. The increasing of the leakage current would cause the dry band formation and the development of arcing [2]. The development of the arcing then leads to the probability of flashover on the insulator. Thus, the good insulator must be able to prevent the flow of current to the undesired path.

In order to understand the propagation of the surface discharge on the insulator, the mechanism for the charge carrier generation and transportation must be known. There are several types of mechanisms for the charge carrier generation and transportation that can lead to the breakdown of the insulators. Apart of the mechanism that involves in the charge carrier generation are the impact ionization [3], field emission [4], secondary electron emission [5], ionic dissociation [6] and molecular ionization [7]. Besides that, recombination of free charge carriers like electrons, positive ions, and negative ions with each other and the surrounding media also can give the contribution in the charge carrier generation on the insulator surface [4]. There are possibilities of positive ion/negative ion recombination, positive ion/electron recombination and electron attachment to the neutral molecule can occur. The electrons attachment with neutral molecules will generate negative ions and reduce the number of electrons[4].

Therefore, by modelling the leakage current on the contaminated surface of the insulator including the surface discharge activities can give the benefit in order to prevent the failure in the system. This model has included the charge transport continuity equation by using electro-migration model and generation mechanism of electric field dependent molecular ionization. For the charge transport continuity equation, Nernst Planck theory can be used to understand the charge carrier behavior on the insulator [8] and this equation will be coupled with Poisson's equation to determine the field distribution.

II. SURFACE DISCHARGE

For the initial phase of surface flashovers, streamer discharges can be stated as the main factors of the flashovers occurred [9]. This flashover can cause the failure of the high voltage equipment by the formation of conductive path from the streamer development. Electric discharge also can be contributed to the high amplitude of the leakage current and the formation of streamer on the dielectric surface. Therefore, there are many studies that were correlated the discharge phenomena with the insulator breakdown. Lee et. al. [10],

examined the initiation and propagation of surface discharge of solid insulator that immersed in a liquid insulator. In this study, when the surface discharge propagates to the grounded electrode, the total current was increased abruptly. The discharge can continue to occur until the conductive path are formed and lead to the flashover and breakdown of solid-liquid insulator [8]. In the study for the outdoor insulator, it was shown that the flow of leakage current might heat up the insulator surface and lead to the formation of dry-band. Thus, the surface discharge activities will cause the formation of carbonize path and tracking on the surface of the insulator and caused deterioration of the insulator [11].

Leakage current measurement is widely used as the parameters to indicate the deterioration and condition of the insulators [12],[13]. Meanwhile, from the study that conducted by Zhichenget. al. [14], it is shown that there is a strong relation between discharge phenomenon and leakage current. When there is heavy pollution, strong discharge occurs and leads to the high amplitude of the leakage current. Therefore, leakage current measurement has been widely used in order to study and analyse the surface discharge phenomenon for the outdoor insulator [15],[16]. The magnitude of leakage current was seen proportional to the carbon path development due to the intense surface discharge activities [13]. Piahet. al., [15] showed that the characteristics of surface discharge are determined by the amount of leakage current that flows on the insulator surface. Equivalent circuit model is one of the techniques to model the surface discharge in order to understand and predict the discharge activities when a leakage current occurs. A work was done by Waluyoet. Al. [17], investigated several equivalent circuits in which representing the insulators under three different conditions (clean, polluted and polluted with dry-band discharge).

A. Charge Transport Continuity Equation

In order to model the leakage current on the contaminated surface of the insulator, the charge transport continuity equation accounted the Nernst Planck theory was used to model the behaviour of the charge carrier transport like positive ions, negative ions and electrons in the insulator system. Generally, the charge transport continuity equation can be written as [4] shown in equation (1):

$$\frac{\partial N_i}{\partial t} + \nabla . J_i = G_i - R_i \tag{1}$$

where $i = n, p, e$ indicate the species of the charge; positive ion, negative ion and electron respectively in the dielectric system. $J_i [mol. m^{-2}. s^{-1}]$ is a total current density due to the movement of each species i across the electrode at distance $x [m]$, $N_i [mol. m^{-3}]$ is the concentration of each charge carrier, $G_i [mol. m^{-3}. s^{-1}]$ is the generation rate and $R_i [mol. m^{-3}. s^{-1}]$ is the recombination rate of the ions species.

The total current density, J_i in equation (1) can be written as follows:

$$J_i = -(D_i \frac{\partial N_i}{\partial x} \pm \frac{z_i F}{RT} D_i N_i \frac{\partial \emptyset}{\partial x} + N_i v) \tag{2}$$

where $D_i [m^2. s^{-1}]$, z_i and $N_i [mol. m^{-3}]$ are the diffusion

coefficient, the valence and the concentration for the species i respectively, v $[m. s^{-1}]$ is the rate with which a volume element moves in solution; $\frac{\partial N_i}{\partial x}$ is the concentration gradient; and $\frac{\partial \emptyset}{\partial x}$ is the potential gradient. The diffusion coefficient can be described as $D_i = \mu_i \frac{RT}{F}$. R is the gas constant $(8.31440 J. mol^{-1}. K^{-1})$, T is the temperature $(296.15 K)$ and F is the Faraday constant $(96485.3 C. mol^{-1})$. The $\pm$ sign in equation (2) accounts for the direction of charge migration, whereby the $+$ sign is used for positive ion and $-$ is used for negative ion and electron. The first term on the right hand side of equation (2) represent the Fick's law equation which related to the diffusion of ion due to the concentration gradient, the second term accounts for the migration of the ion species in the presence of electric field and the last term represents the convection movement of the ion species induced by the bulk fluid motion, concentration gradients or velocity of the fluid [18].

In order to determine the electric field distribution, the charge transport continuity equation for each charge obeys Poisson's equation [19] as shown in equation (3):

$$\nabla . (-\varepsilon_0 \varepsilon_r \vec{E}) = (N_p - N_n - N_e) q N_A \tag{3}$$

where $\vec{E} [V. m^{-1}]$ is the electric field vector, $\varepsilon_0 [F. m^{-1}]$ is permittivity of free space charge, ε_r is relative permittivity of material, $N_p, N_n, and N_e$ are the density of each charge carrier determined from the charge transport continuity in equations (1), $q [C]$ is the elementary charge and $N_A [mol^{-1}]$ is the Avogadro's number.

B. Charge Generation Mechanism

Charge generation mechanism of electric field dependent molecular ionization was used in the modelling of leakage current on the contaminated surface of the outdoor insulator in the presence of surface discharge. Electric field dependent molecular ionization is a direct ionization mechanism that extracts an electron from the neutral molecule and then generating free electron and positive ion [7]. Zener theory [7] of tunneling electron in solid that was correlated with molecular ionization mechanism in the presence of high electric field was used in the modelling of leakage current on the insulator surface with the presence of surface discharge. This theory had been used by the researchers [20],[21] because this mechanism is well suited to model the streamer propagation and electric field distribution along the surface of oil/pressboard interface. Hence, the ionization rate [4] can be expressed as shown in equation (4):

$$G|\vec{E}| = \alpha_I exp(-\beta_I) \tag{4}$$

where q is the electronic charge, a is the molecular separation, $|\vec{E}|$ is the magnitude of electric field, h is the Planck's constant, m^* is the effective electron mass, Δ is the molecular ionization potential and N_0 is the density of

ionizable charge. Where, $\alpha_I = \frac{q^2 N_0 a |\vec{E}|}{h}$ and $\beta_I = \frac{\pi^2 m^* a \Delta^2}{q h^2 |\vec{E}|}$.

From the electric field dependent molecular ionization, electron and positive ion are extracted from a neutral molecule when there are given high electric field. Thus, the charge generation term for positive ion and electron in equation (1) can be expressed same as equation (4). Meanwhile, for negative ion, the charge generation mechanism is the electron attachment to the neutral molecules. This is because by the electric field molecular ionization, only electron and positive ion will be extracted from the neutral molecule. This mechanism is suitable for the modeling of the leakage current on the contaminated surface of the insulator due by the flowing of the contaminant along the insulator surface; the electric field distribution will be propagating from the high voltage electrode to the ground electrode.

Besides charge generation, charge recombination between the free charge carriers with each other and surrounding media was also contributed to the charge generation. Based on equation (1), there are two recombination rate for positive ion, which is the recombination of electron/positive ion recombination (R_{pe}) and positive/negative ion recombination (R_{pn}). For the negative charge, there is a recombination of positive/negative ion recombination (R_{pn}). Lastly for the electron, there are two recombination rate occur that are electron/positive ion recombination (R_{pe}) and electron attachment(EA).

The charge recombination rate can be written as:

$$R_{pe} = N_p N_e K_{rpe} \quad R_{pn} = N_p N_n K_{rpn} \quad EA = \frac{N_e}{\tau_a} \tag{5}$$

where, K_{rpn} and K_{rpe} are recombination coefficient of positive ion/negative ion and positive ion/ electron respectively and τ_a is the electron attachment time constant. The recombination coefficient can be expressed by using Langevin's relation that based on the diffusion equation [4],[22] as follows:

$$K_{rpn} = \frac{q(\mu_p + \mu_n)}{\varepsilon \varepsilon_0} \quad K_{rpe} = \frac{q(\mu_p + \mu_e)}{\varepsilon \varepsilon_0} \tag{6}$$

where, $\mu_p, \mu_n, and \mu_e$ is the mobility of positive ion, negative ion and electron respectively, , ε_0 and ε is an elementary charge, relative permittivity and permittivity of free space charge respectively.

III. PHYSICAL MODEL OF SURFACE DISCHARGE ACTIVITIES

The physical model based on the practical problems of leakage current on the contaminated surface of artificially polluted insulation was established. It had been assumed that the surface discharge process was occurred along the insulator surface. Physical model in this modelling was involved the mechanism of charge transport based on electro-migration term and charge generation due to the electric field molecular ionization, recombination and electron attachment as had been discussed earlier. In order to model 1 dimensional model of the leakage current on the surface of the outdoor

insulator, there are two part that must be understood. There are governing equations and boundary condition for the model. The schematic diagram of the contaminated surface of high voltage insulator was shown in Figure 1. Based on this figure, the contaminant flow on the surface of the solid insulator will be causing the insulator to become conductive. The contaminant flow can cause the conducting path for the leakage current flow along the insulator surface. Meanwhile, from the studies about the surface discharge on the outdoor insulator, the formation of carbonization path are occurred due to the drying out process by the flow of leakage current [11] and lead to the flashover and breakdown of solid-liquid insulator [8].
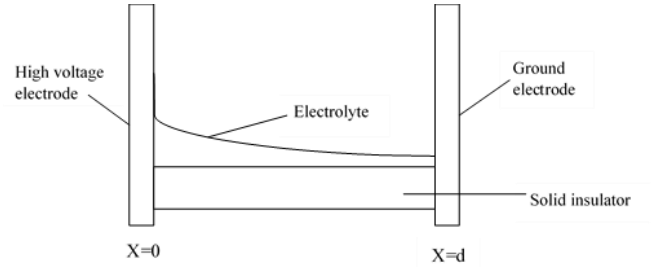

Figure 1: Schematic diagram of contaminated surface of high voltage insulator

A. Charge Transport Continuity Equation

In order to model the leakage current on the contaminated surface of the outdoor insulator that involves the surface discharge activities, the charge transport continuity equations accounted the Nernst Planck theory was used. In the charge continuity equation, the generation rate, recombination rate and electron attachment as mentioned before in equation (4)-(5) were substituted into equation (1) and can be written as equation (7)-(8):

$$\frac{\partial N_p^*}{\partial t^*} - \nabla^* \cdot \left(D_p \nabla N_p^* + N_p \mu_p \nabla V^* \right) \\ = \alpha_I^* exp(-\beta_I^*) - N_p N_n K_{rpn}^* \\ - N_p N_e K_{rpe}^* \tag{7}$$

$$\frac{\partial N_n^*}{\partial t^*} - \nabla^* \cdot (D_n \nabla N_n^* - N_n \mu_n \nabla V^*) = \frac{N_e^*}{\tau_a^*} - N_p N_n K_{rpn}^* \tag{8}$$

$$\frac{\partial N_e^*}{\partial t^*} - \nabla^* \cdot (D_e \nabla N_e^* - N_e \mu_e \nabla V^*) \\ = \alpha_I^* exp(-\beta_I^*) - N_p N_e K_{rpe}^* - \frac{N_e^*}{\tau_a^*} \tag{9}$$

Note that the symbol of '*' represents the dimensional variables.

In this model, the total current density only considered the diffusion-migration term of each charge carrier due to the influence of high electric field and the concentration gradient on the surface of the contaminated insulator. Meanwhile, the convection term in equation (2) is neglected because, in this modelling, the contamination flow rate will be assumed constant. However, in the case for the high electric field influence, researchers had been only considered the migration term in the charge continuity equations. Such as Hwang et. al., in the study of a mechanism for positive streamer, only migration term had been considered. This is due to the high electric field applied in the system [23]. Meanwhile, Sullivan

et. al., in their study on the effect of ionic dissociation on charge transport in transformer oil also only considered the migration term [24].

For the electric field equation, it can be expressed as, $\vec{E} = -\nabla V$, where V is the electric potential. Thus, to determine the electric field distribution, the Poisson's equation was used and can be expressed as [19]:

$$\nabla \cdot (\varepsilon_0 \varepsilon_r \nabla V^*) = (N_p{}^* - N_n{}^* - N_e{}^*) q N_A \tag{10}$$

where $V[V.m^{-1}]$ is the electric potential, $\varepsilon_0[F.m^{-1}]$ is permittivity of free space charge, ε_r is relative permittivity of material, $N_p, N_n, and\ N_e$ are the concentration of each charge carrier, $q\ [C]$ is the elementary charge and $N_A\ [mol^{-1}]$ is the Avogadro's number. In order to determine the conduction current, the sum of integral of total current density was used as:

$$I = \int (\vec{J_p} + \vec{J_n} + \vec{J_e})\, dx \tag{11}$$

where x is the distance between highvoltage electrode and ground electrode and J_i is the current density.

B. Boundary Conditions

The boundary condition that will be applied to the surface discharge model can be expressed as:

Poisson's equation: The high voltage electrode was set to a supply voltage V_0 at the boundary of $x = 0$. This boundary was supplied with DC voltage. Meanwhile, for the ground voltage at $x = d$, it was set to zero normal electric field component.

$$\hat{n}(V) = V_0 \quad , \quad at\ x = 0 \tag{12}$$

$$\hat{n}(V_i) = 0 \quad , \quad at\ x = d \tag{13}$$

Charge Transport Continuity Equations: The boundary equation at the high voltage electrode and the ground electrode are set to zero normal flux as follow:

$$\hat{n}(\nabla N_i) = 0 \quad at\ x = 0\ and\ x = d \tag{14}$$

For the initial guess for the electric potential, the value of the function of apply voltage was used. Then, that was assumed that the initial condition for the charge concentration as:

$$N_i(x^*, 0) = N_0 \tag{15}$$

where N_0 is the initial ions concentration in the electrolyte.

IV. MATHEMATICAL RESULTS

All the charge continuity equation and Poisson's equation was solved using a dimensionless framework to reduce the difficulty while doing the simulation. The appropriate scaling of dimensional equations to obtain the dimensionless

equations are as follows:

$$V^* = V_0 V \qquad \nabla^* = \frac{\nabla}{d} K_{rpe}{}^* = K_{rpn}{}^* = \frac{F\mu_p}{\varepsilon} K_{rpn}$$
$$N^* = \frac{\varepsilon V_0}{d^2} N \qquad \mu_i{}^* = \mu_p \mu_i \tau_a{}^* = \frac{d^2}{\mu_p V_0} \tau_a \tag{16}$$

The timescale for the leakage current flow in the contaminated surface with the surface discharge activities was defined as:

$$t^* = \frac{d^2}{V_0 \mu_p} t \tag{17}$$

Hence, we obtain the dimensionless parameters:

$$\Gamma = \frac{RT}{FV} \alpha_I{}^* = \frac{\mu_p \varepsilon V_0}{h} \alpha_I \beta_I{}^* = \frac{V_0}{d} \beta_I \tag{18}$$

where, α_I is the pre-exponential term charge generation $\alpha_I = \frac{qN_0 a}{h}$ and β_I is the exponential term of the charge generation $\beta_I = \frac{\pi^2 m a \Delta^2}{qh^2}$ and * sign is for the dimensional parameter in the equations. The dimensionless model can be expressed as follow:

$$\nabla \cdot (\nabla V) = (N_p - N_n - N_e) \tag{19}$$

$$\frac{\partial N_p}{\partial t} = \nabla(\Gamma \mu_p \nabla N_p + N_p \mu_p \nabla V) + \alpha_I |\nabla V| \exp\left(-\frac{\beta_I}{|\nabla V|}\right) \\ - N_p N_e K_{rpe} - N_p N_n K_{rpn} \tag{20}$$

$$\frac{\partial N_n}{\partial t} = \nabla(\Gamma \mu_n \nabla N_n - N_n \mu_n \nabla V) + \frac{N_e}{\tau_a} - N_p N_n K_{rpn} \tag{21}$$

$$\frac{\partial N_e}{\partial t} = \nabla(\Gamma \mu_e \nabla N_e - N_e \mu_e \nabla V) + \alpha_I |\nabla V| \exp\left(-\frac{\beta_I}{|\nabla V|}\right) \\ - N_p N_e K_{rpe} - \frac{N_e}{\tau_a} \tag{22}$$

From the dimensionless model, each size of the dimensionless parameter was calculated from the actual value of each parameter. Based on the dimensionless model, Γ is the dimensionless parameter for the diffusion terms. From the calculation, Γ was estimated to be extremely small (10^{-6}) by using the value of voltage potential $4kV$[25]. This condition allows the model from equation (22)-(24) to be written without the diffusion term. This result is well suited with the previous studies [23],[24], which neglected the diffusion term due to the high electric field influenced.

V. PATTERN OF THE SIMULATION GRAPH

The simulation of the mathematical modelling is solved by using the method of line technique (MOL). This technique then is solved by using finite difference method in MATLAB software. Then, the results will be compared to the results from the previous study that was presented by Sullivan [4] that was modelled the propagation of streamer in transformer oil for study the pattern of the graph. In addition, for the streamer modelling, Sullivan was using the electric field

dependent ionization as the generation mechanism of the charge carrier. Figure 2 shows the comparison graph pattern of the electric field for streamer modelling and surface discharge on the insulator modelling. Figure 2(a) shows the electric field simulation result for the streamer development in the transformer oil while Figure 2(b) shows the result of electric field simulation for the surface discharge development on the insulator surface by using the dimensionless parameter.
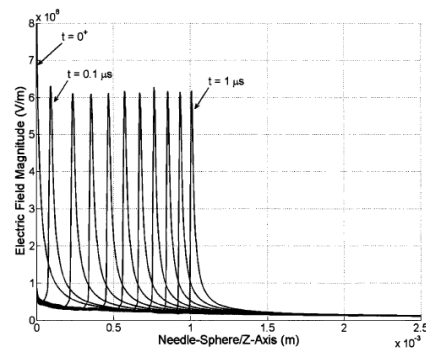
(a) Streamer development
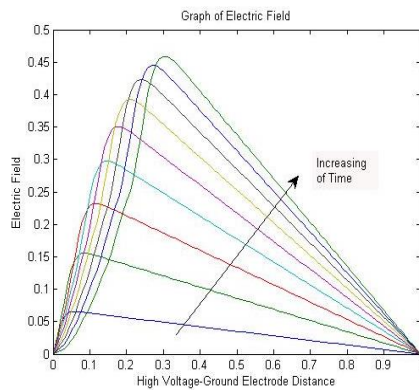
(b) Surface discharge

Figure 2: Comparison graph pattern of the electric field

From the Figure 2(a), it can be shown that there is the development of an electric field wave, which propagates from the needle electrode's tip towards the spherical electrode. Compared to the electric field graph's pattern for the surface discharge on the insulator surface, the propagation of the electric field wave is not insignificant. This is due to the surface discharge activities are occurred along the surface of the insulator from the high voltage to the ground electrode compared to the transformer oil that propagates toward the spherical electrode. Although the pattern of the graph is much different but both graph showed that the electric field dependent ionization can be used in the modelling of the streamer in the transformer oil and surface discharge on the insulator surface. The pattern of the electric field gives the same results with the physical model discussed previously in the case of surface discharge on the insulator surface. Thus, this modelling is suitable for modelling the surface discharge on the insulator surface.

VI. CONCLUSION

It can be concluded that study on the physical process of leakage current flow that included the charge transport and

generation is important to know the behavior of charge on the insulator surface that caused the deterioration of the insulator surface. By modelling the surface discharge on the contaminated surface of the insulator, the surface condition of the insulator will be known to prevent the system failure. The Nernst Planck equation had been used to describe the charge continuity equation and coupled with Poisson's equation to determine the electric field distribution. For the future work, the results for the leakage current will be simulated in order to study the condition of the insulator surface.

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