Parametric Studies on Resonance Frequency Variation for Piezoelectric Energy Harvesting With Varying Proof Mass and Cantilever Length

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***Abstract*—This paper demonstrates the potential of the resonance frequency for a piezoelectric beam clamped on one of its end to be altered to a higher or lower range by introducing additional proof mass or by reducing the piezoelectric cantilever length. When the effective mass of the cantilever is increased, the resonance frequency of the cantilever is expected to shift to lower region, while when the stiffness of the cantilever is increased, the resonance frequency of the cantilever is expected to shift to higher region. These statements have been proven and validated in this paper. Overall, the experimental result showed good agreement with the theoretical result, however, there was still 20% and 35% error different for the proof mass experiment and length reduction experiment respectively.**

***Index Terms*—Micro-Generator; Energy Harvester; Frequency Variation; Natural Frequency.**

I. Introduction

Piezoelectric beam clamped on one of its end (piezoelectric cantilever) is known for harvesting energy from vibration. It uses mechanical energy from the source of ambient vibration that is translated into a cantilever and converts it into electrical energy. Electrical energy produced by piezoelectric can be used to power up low power electronic devices and appliances [1]. It can be used to replace small scale battery. By using piezoelectric energy harvester and eliminating the use of battery, a system can be self-powered and independent to wired power source. This made electronic or appliances that were not possible with wall plug or traditional battery based power source become practical. Piezoelectric has been used in many areas from unattended sensor network applications, automotive applications, building technologies, medical implants to consumer electronics [2-4]. By supplying the appliances with power from natural source, it eliminates the needs for maintenances throughout the life span of the appliance itself. This makes the overall system more reliable and sustainable.

However, due to the fact that the quality factor (Q-factor) of piezoelectric cantilever is high, the operating frequency of piezoelectric cantilever is relatively narrow [5]. This generally means that the piezoelectric cantilever is only capable to generate optimum energy at a small frequency range. At a frequency level, which is far from its natural frequency, the piezoelectric cantilever generates relatively small amount of energy. However, this small amount of energy may not be sufficient to power up electronic devices. Therefore, before the piezoelectric cantilever is used to power up devices, its resonance frequency need to be characterized and tuned to match the frequency of the vibration from the surrounding. After tuning, the piezoelectric cantilever will then able to generate optimum energy from that particular environment.

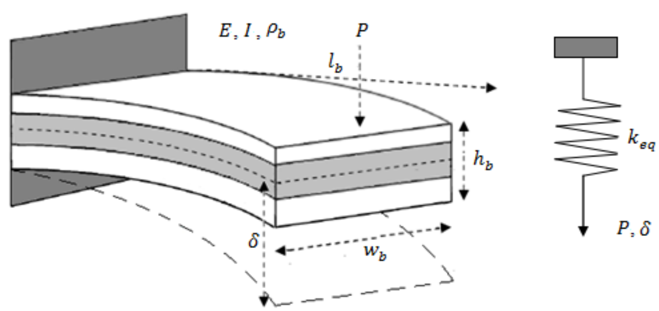


Figure 1: Illustration of Measurement Variables of Piezoelectric Cantilever

According to a number of research papers, the resonant frequency of a piezoelectric cantilever is closely related to its effective mass and effective stiffness of the cantilever. According to Inman [6], the relationship of the resonant frequency, effective stiffness and effective mass is as shown in Equation 1, while the measurement variables are illustrated in Figure 1.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m_{eq}}} \quad (1)$$

where f_n is the natural frequency of the cantilever beam. k_{eq} represents the effective stiffness and m_{eq} represents the effective mass of the cantilever beam. According to Yi et al. [7], m_{eq} is 23.6% of the total beam massm_b., whereas total beam mass is the product of the density p_b, width w_b, total thickness h_b and length of the beam l_b.

$$m_{eq} = 0.236 m_b = 0.236\, p_b w_b h_b l_b \tag{2}$$

By substituting Equation 2 into Equation 1, the detailed equation of f_n is as shown in Equation 3.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{0.236\, p_b w_b h_b l_b}} \tag{3}$$

A. *Effect of Proof Mass Variation*

The effective mass of a piezoelectric cantilever can be altered by introducing proof mass to the tip of the beam. The new resonant frequency, f_{new} of a piezoelectric cantilever after its proof mass is added can be calculated by using the Equation 4, which is a derivate from Equation 1.

$$f_{new} = f_n \sqrt{\frac{m_{eq}}{m'_{eq}}} \tag{4}$$

where, m'_{eq} is the total beam mass, including the proof mass, as shown in Equation 5.

$$m'_{eq} = m_{eq} + weight\ of\ proof\ mass \tag{5}$$

By adding proof mass to the tip of the cantilever, it increases the effective mass of the cantilever, thus the resonant frequency will drop to a lower region based on Equation 4 and 5. This alternation would allow the cantilever to become a suitable micro-power generator for low frequency applications.

B. *Effect of Cantilever Length Variation*

According to Equation 1, the resonant frequency of a piezoelectric cantilever can shift to a higher region by increasing the stiffness of the cantilever. Although it is impossible to alter the stiffness of readymade piezoelectric cantilevers, the stiffness can be increased by reducing the length of the cantilevers. This can be easily done by shifting the clamping area of the cantilevers more towards the beam. This will shorten the length of flexible beam. According to Hua Yu et al. [8], by referring to Equation 6, the stiffness of the cantilever can be increased by reducing the effective length of the cantilever.

$$k_{eq} = \frac{3EI}{{l_b}^3} \tag{6}$$

where l_b is the length of the beam, *E* is the modulus of elasticity and *I* is the area moment of inertia. Using Equation 6, an equation for the new stiffness of the cantilever is obtained.

$$k_{new} = \frac{3EI}{{l_{new}}^3} \tag{7}$$

$$k_{new} = \left(\frac{3EI}{{l_b}^3}\right)\left(\frac{{l_b}^3}{{l_{new}}^3}\right) \tag{8}$$

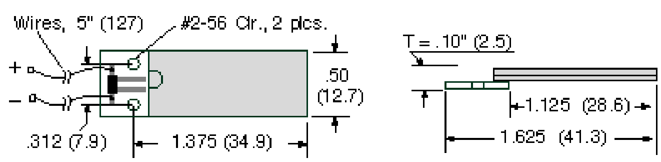
$$k_{new} = k_{eq}\left(\frac{l_b}{l_{new}}\right)^3 \tag{9}$$

where l_{new} is the new length of the beam after length alteration. Hence, the new resonant frequency, f_{new} of a piezoelectric cantilever after its length is altered can be calculated by using Equation 10.

$$f_{new} = \frac{1}{2\pi}\sqrt{\frac{k_{new}}{m_{eq}}} \tag{10}$$

II. EXPERIMENTAL SET-UP

The piezoelectric cantilever used in this research is a standard quick-mount bending generator with pre-mounted and wired at one end (Q220-A4-303YB) from Piezo Systems [9], Inc. The dimension of the piezoelectric cantilever is as shown in Figure 2. Off-the-shelf piezoelectric cantilever is used in this research in order to demonstrate the potential of altering the piezoelectric material available in the market to suite the requirement of the designed application.



(a) Top View *(b) Side View*

Figure 2: Dimension of Piezoelectric Cantilever (Q220-A4-303YB) adapted from Piezo Systems Inc [9].

The experiment set-up of this research is as shown in Figure. 3, which consists of an oscilloscope, a function generator, a G-link wireless sensor and receiver, a gain amplifier, an electrodynamics shaker, and piezoelectric cantilevers. In order to generate a controllable artificial vibration for test purpose, function generator, gain amplifier, electrodynamics shaker, and G-link wireless accelerometer were used. The function generator was used to supply AC input power to the electrodynamics shaker. Since the power supplied by the function generator alone is not sufficient to generate vibration with high acceleration level (g-force), the gain amplifier was used to amplify the power before supplying it to the electrodynamics shaker.

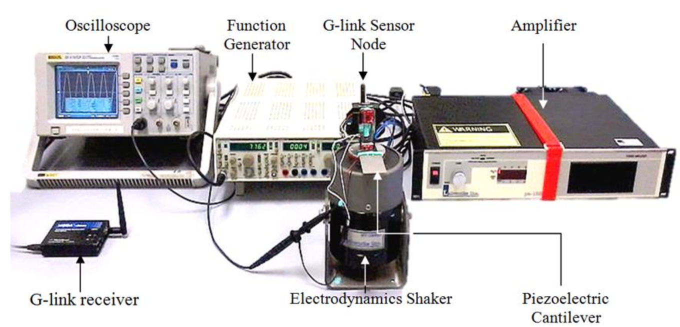


Figure 3: Experimental Set-up

In order to observe the influence of additional effective mass toward the resonant frequency of the cantilever, experiment with the set-up as shown in Figure 4 is constructed. The voltage output of the cantilever with matching resistor, 10 kΩ was recorded based on the vibration from the electrodynamics shaker at a range of frequency from 10 Hz to 500 Hz with the acceleration level (g-force) fixed at constant magnitude 1-g (9.81 m/s^2). Then, the experiment continued by adding proof masses weighing from 0.05 grams to 1.50 grams to the tips of the cantilever, as shown in Figure 4.

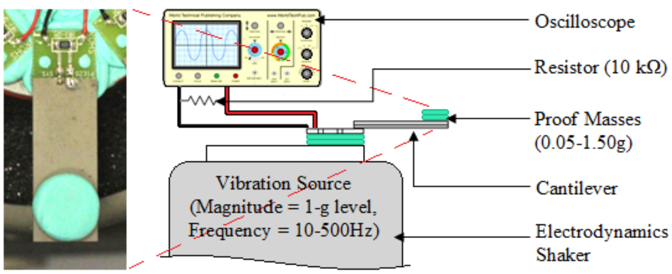


Figure 4: Illustration of the Experiment Set-up for Proof Mass Variation Method

In order to observe the effect of effective stiffness alteration toward the resonant frequency of the cantilever, the experiment was continued without proof mass attached to the cantilever, but the effective length of the cantilever was reduced. The length of the cantilever was reduced by clamping the cantilever over than its clamping base towards its flexible beam, as illustrated in Figure 5.

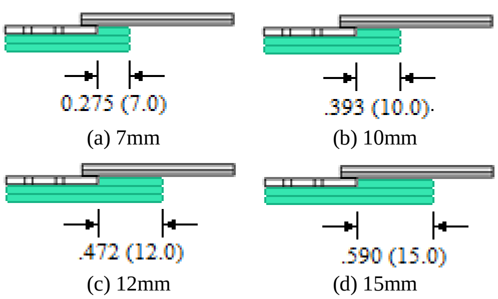


Figure 5: Illustration for Reducing Cantilever Length by Clamping Over

The result obtained from both experiments were plotted in graphs against the theoretical value obtained from the calculation using the mathematical equations stated in the previous section for comparison purpose. The values of the constant parameters used in the theoretical calculation are shown in Table 1.

Table 1
Parameters for Theoretical Value Calculation

Symbol	Parameter	Unit	Value
f_n	Natural Frequency	Hz	300
p_b	Density	kg/m^3	7800
w_b	Width	m	0.0127
h_b	Height	m	0.0051
l_b	Length	m	0.0286

III. Experimental Results

The result in Figure 6 shows that when proof masses were added, the resonant frequency of the cantilever changes. With the increasing proof masses, the resonant shifted more towards the left side of the graph, indicating that the resonant frequency decreased. The maximum voltage also increased when the weight of the proof mass is increased. The output voltage increased 52% when 1.50g proof mass is added as compared to the cantilever without the addition of proof mass. The reason for this is because the proof mass on the tip of the cantilever allows the cantilever beam to deflect even more, hence the stress that acts on the piezoelectric material increases. As a result, more charge is produced.

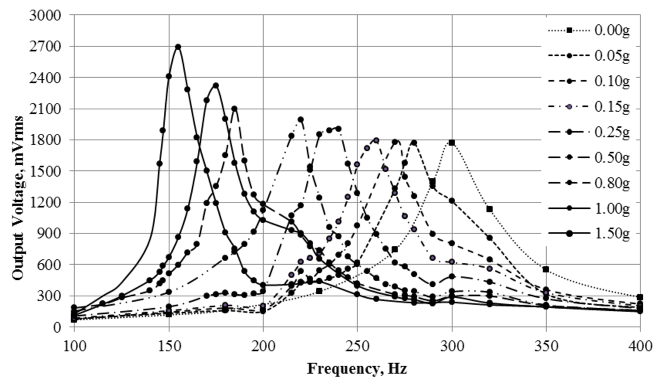


Figure 6: Frequency Responses (Voltage) of the Cantilever with Different Proof Mass is added

By substituting the value in Table 1 to Equation 3, 5 and 6, the theoretical values for shifting of resonant frequency for the cantilever are obtained and plotted in Figure 7. The experimental result shows good agreement with the theoretical result. It shows that the resonant frequency indeed shifted to lower frequencies when proof mass is added. Heavier proof mass would reduce the resonant frequency even more. However, the actual resonant frequency did not reduce to the exact theoretical value. There was still 20% error compared to the theoretical value, as shown in Figure 7. In a nutshell, this experiment verifies that adding proof mass to the tip of the cantilever does indeed reduce the resonant frequency.

Whilst, the length of the cantilever was reduced to 0.7cm, 1.0cm, 1.2cm and 1.5cm respectively from the actual length, the new frequency response of the length reduced the cantilever as shown in Figure 8. The result shows that decreasing the cantilever length, increased the resonant frequency of the cantilever. Resonant frequency of cantilever at the normal condition, namely 300Hz, increased to 320Hz after the length reduction of 0.7cm, increased to 400Hz after the length reduction of 1.0cm, 500Hz at the reduction of 1.2cm, and lastly 780Hz at the reduction of 1.5cm. Again, altering the stiffness of the cantilever does not only alter its resonance frequency, it affects the generated output voltage. The voltage reduces when the effective length of the cantilever is reduced.

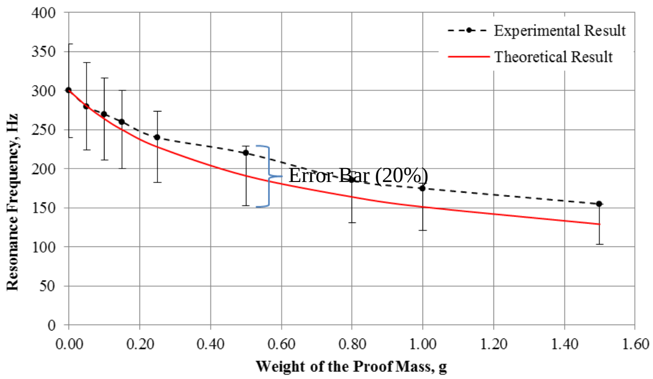


Figure 7: Comparison of Experimental and Theoretical Resonant Frequency Shift based on Proof Mass Weight

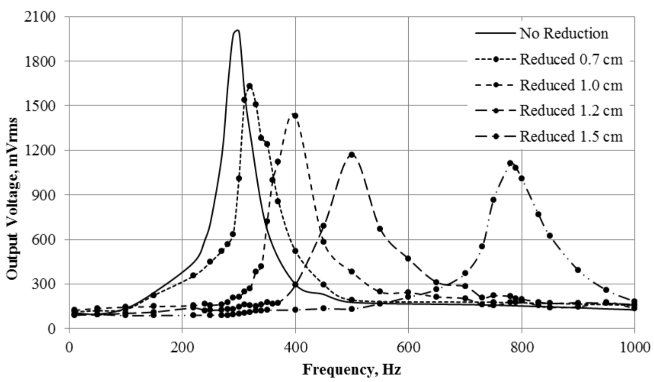


Figure 8: Frequency Response of the Cantilever after Length Reduction

The experimental result was plotted and compared against the theoretical value in Figure 9. The experimental result was within 35% error different as compared to the theoretical result. The error different between experimental result and theoretical result might be due to the fact that the actual stiffness of the laboratory clamped piezoelectric cantilever was lower than the value as shown in its datasheet, which was measured when the piezoelectric cantilever was rigidly clamped. However, both the experiment and theoretical result support the fact that the decreasing of the cantilever length increases the resonant frequency of the cantilever. This is due to the fact that the stiffness of the cantilever increases when its length is reduced, hence its resonant frequency shifts to a higher region.

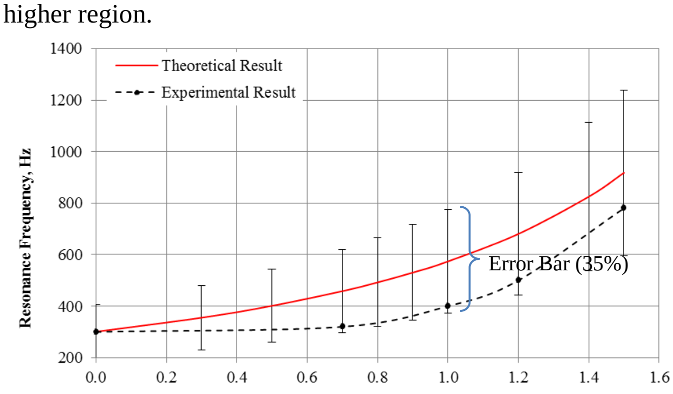


Figure 9: Theoretical and Experimental Results for Effect of Length Reduction of Cantilevers toward its Resonant Frequency

IV. CONCLUSION

This paper has proven that by altering the effective mass and stiffness of the piezoelectric cantilever, its resonant frequency can be altered to suit the requirement of the designed application. Adding proof mass or increasing the clamping length of the cantilever can change the resonance frequency of readily available cantilever in the market. The result shows that when additional proof mass was added to the tips of the cantilevers, its resonant frequency shifted to a lower frequency region. A piezoelectric cantilever with its natural frequency initially at 300Hz, shifted to 270Hz when 0.10g proof mass was attached to the tip. It further reduced to 220 Hz when 0.50g proof mass was attached, and reached 155Hz when 1.50g proof mass was attached. Attaching proof mass to the piezoelectric cantilever does not only show the effect on its resonance frequency range, but it also increases the output voltage generated by the piezoelectric cantilever. The output voltage increased 52% when 1.50g proof mass is added as compared to the cantilever without proof mass added. This experimental result is in good agreement with the theoretical value. However, the actual resonant frequency does not reduce to the exact theoretical value. There was still a 20% error compared to the theoretical value. On the other hand, mathematical equation shows that when the stiffness of the cantilever is increased, the resonance frequency of the cantilever is expected to shift to a higher region. This statement is has also been proven in this paper. In this research the stiffness of the cantilever is changed by altering the length of the cantilever. The result shows that when the length of the cantilever is reduced, the resonance frequency shifted to higher region, which is in good agreement with the theoretical result. A piezoelectric cantilever that has natural frequency initially at 300 Hz, shifted to 400 Hz when 1.0 cm length from its structure is reduced, but increase to 780 Hz when 1.5 cm length is reduced. Again, altering the stiffness of the cantilever does not only alter its resonance frequency, but it also affects the output voltage generated. The voltage reduces when the length of the cantilever is reduced. The experimental result was plotted and compared against the theoretical value. The experimental result was within 35% error different as compared to the theoretical result. However, both the experiment and theoretical result supported the fact that with the decrease in the cantilever length, the resonant frequency of the cantilever increases. In nutshell, this research has proven that the resonance frequency of the piezoelectric cantilever can be altered by altering the effective mass and length of the cantilever.

ACKNOWLEDGMENT

The authors would like to thank the Ministry of Science, Technology and Innovation of Malaysia to sponsor this research project under the Science Fund research grant no. 06-01-14-SF0087/L00017 as well as the facility support by the Advanced Sensors and Embedded Control System (ASECs) Research Group, Centre for Telecommunication Research and Innovation (CeTRI), UTeM.

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