Evaluation of New Gaussian Wavelet Functions in Signal Edge Detection

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***Abstract*— Edge detection is a fundamental tool in image processing, machine vision and computer vision, particularly in the areas of feature detection and feature extraction. The same problem of finding discontinuities in 1D signals is known as step detection and the problem of finding signal discontinuities over time is known as change detection. In this paper, a new set of wavelet basis functions for the edge detection issue is introduced in 1D space. First, we develop the Gaussian wavelet and present new bases by the derivation of Gaussian smoothing filter. It is proven that these filters have the necessities of the wavelet basis. After that, for proposed wavelet functions, three Canny criteria (signal-to-noise ratio, localization and low spurious response) and spatial and frequency width, which are surveys for edge detectors are discussed and formulated. For the better understanding the behavior of bases, the formulas are presented in the parametric form and compared with each other in relevant tables. The unit step and line edge are modeled as two particular types of edges and detected in the wavelet domain via introduced wavelet functions. Moreover, the effect of smooth filtering as a denoising preprocessing stage in the edge detection is discussed, and relevant formulas are derived.**

***Index Terms*— Canny Criteria; Edge Detection; Gaussian Filter; Multiscale Analysis; Step Response; Wavelet Transform.**

I. Introduction

In many cases, when a signal summarized by its edges, the complexity of the problem would be reduced and a general form obtained with fewer amounts of data. There are two main criteria for edge detectors. Edge detection should be implemented easily and have low cost computing. Edges can be detected by finding local maxima of the first derivation of the function or zero-crossing of second derivation, which named inflection points. Zero-crossing of the function has a drawback too. As definition, we want to find maximum points of the first derivation and note it as the edge. In this process, minimum points left as the ordinary point and should not be considered; because they indicate the slow variation of the function. However, zero-crossing responds to both maximum and minimum points, i.e. an inflection point could be a maximum or minimum. So, extra computation is imposed to the scheme to distinguish local maxima's.

A technique to derived edges is filtering method. Choosing the size of filter, sensitivity to noise and fine localization have been considered as critical problems in edge detectors. A wide filter is less sensitive to random noise, but its localization response is not good [1]. A low size filter could exploit edges with admissible displacement. However, its effect is poor in noisy conditions and yields broken and twisted edges. So, filter-based edge detection suffers from two major problems: localization and accuracy.

In the wavelet domain, selecting a large scale misses weak edges, but reduces noise influence and with selecting a low scale, details would be achieved, but with error edge displacement. For example, in high scale, line edges detected with greater localization error. It is very difficult to select a single scale to have the lowest localization error and the highest noise suppression [2]. Hence, multiresolution analysis has been introduced to present signals in the coarse-to-fine levels. In this procedure, the combination of various levels is used to present edge's information. The significant challenge is methods of the combination which can retain most real edges and stop spurious responses. Multiresolution analysis has a wide range of applications in image segmentation and edge tracking [3-8].

Another way to refine the random noise is the use of smooth filter as a pre-processing stage. This method has a drawback too. The localization of detected edges is degraded by increasing the degree of blurring signals. On the other hands, the sharpen filters improve the spatial resolution, but reinforce the noise ability too.

These trade-offs have been led to introduce optimal edge detectors, which compromise between displacement and true detection. There are many works that have been performed in the optimum edge detection [9-12]. One of the most bolded research is the work of Canny [13]. He introduced Gaussian filter as optimal edge detectors and presented three optimal criteria for designing edge filters based on local maxima, which have been used until now. Recently, various edge detections have been proposed, including statistical method [14], gradient-differentiation methods [15-17] and fuzzy-based method [18].

This paper is organized as follows: Section 2 describes new wavelet functions based on Gaussian wavelet derivations. It is shown that proposed bases have the necessities of wavelet functions. Section 3 is dedicated to the study of three Canny edge detector criteria and evaluation of proposed wavelet bases. The signal-to-noise ratio, localization and low spurious response, which defined by canny as useful tools to compare edge detectors' performance, are presented and relevant formulas of proposed bases are studied and compared with each other. In this part, the formulas are derived parametrically. In section 4, two basic edge types (step and line) are modeled. After that, the evolution across bases is studied. In this section, the effect of pre-smoothing stage is discussed and relevant relationship for so-call edge models, are derived. Finally, the conclusion and discussion of paper are devoted in section 5.

II. PROPOSED WAVELET BASES

Gaussian filter has some special characteristics, which has been widely considered in the edge detection issue. In this paper, we use it as smoothing function. Gaussian filter has a symmetric shape in the time and frequency domain. Furthermore, it can be separated in *x*, *y* direction in Cartesian coordination, which can reduce the amount of calculations. Assume $g(x)$ be a Gaussian filter with variance σ^2 and zero mean and $g_s(x)$ be a Gaussian filter at the scale *s*.

$$g(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \qquad (1-a)$$

$$g_s(x) = \frac{1}{s} g(\frac{x}{s}) \qquad (1-b)$$

The Gaussian filter is a smooth or primitive function. Because it's integral over R is 1 and reaches to zero in infinity. Canny used the first derivative of the Gaussian filter for edge detecting and introduced it as the optimal edge detector [13]. We develop this idea to the higher-order derivation of the Gaussian filter. Let wavelet $\psi^n(x)$ be the *nth* order derivative of $g(x)$ and $\psi_s(x)$ be the scaled function of $\psi(x)$. With these assumptions, we have

$$\psi^n(x) = \frac{d^n g(x)}{dx^n} \qquad n = 1,2, \ldots \qquad (2-a)$$

$$\psi_s^n(x) = s^n \frac{d^n g_s(x)}{dx^n} \qquad (2-b)$$

A function $f(x)$ would be a wavelet basis if it has two properties. First, its average over Hilbert space is equal to 0 i.e.

$$\int_{-\infty}^{+\infty} f(x) dx = 0 \;\; or \;\; F(\omega = 0) = 0 \qquad (3)$$

Where the $F(\omega)$ is the Fourier transform of $f(x)$. Proposed wavelet functions satisfy this condition. The Fourier Transform of $g(x)$ would be driven as

$$G(\omega) = e^{-\frac{\sigma^2\omega^2}{2}} \qquad (4)$$

According to the relationship between the smooth function and wavelet functions, the Fourier Transform of $\psi^n(x)$ would be obtained as

$$\Psi^n(\omega) = (j\omega)^n G(\omega), \Psi^n(\omega)|_{\omega=0} = 0 \qquad (5)$$

Second, the wavelet basis should satisfy following admissibility condition:

$$\int_{-\infty}^{+\infty} \frac{|\Psi(\omega)|^2}{|\omega|} d\omega < \infty \qquad (6-a)$$

$\psi^n(x)$ has this necessity too. It can be written

$$\int_{-\infty}^{+\infty} \frac{|\Psi^n(\omega)|^2}{|\omega|} d\omega = \int_{-\infty}^{+\infty} \frac{\omega^{2n} e^{-\sigma^2\omega^2}}{|\omega|} d\omega < \infty \qquad (6-b)$$

Canny indicated that an edge detector must be antisymmetric to find local maxima in edge detection applications [13]. i.e. $\psi(-x) = -\psi(x)$.$\psi^n(x)$ would be symmetric with even *n* and antisymmetric with odd *n*. So, all of the derivations of Gaussian smooth filter can be regarded as wavelet bases and odd derivations can be used as edge detectors. Figure 1 shows first six derivations of Gaussian filter.

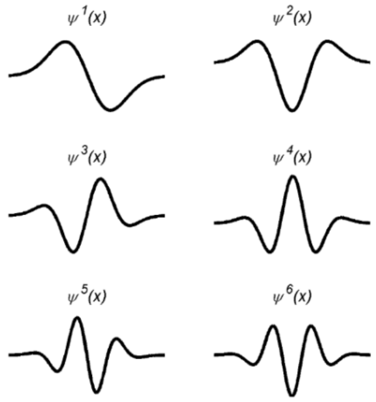

Figure 1: First six Gaussian derivations as wavelet bases

Another name of $\psi^2(x)$ is Mexican hat wavelet. As *n* increases, the vibration of the function increases. It means that there are more peaks in the higher order of derivation. These tips are classified as follows:

According to this Figure, $\psi^1(x)$ has 2 local extrema and 1 cross-zero point. $\psi^2(x)$ has 3 local extrema and 2 cross-zero points. This manner continues to higher order of *n*. In general form, $\psi^n(x)$ has $n+1$ local extrema ($\frac{n+1}{2}$ minimum and $\frac{n+1}{2}$ maximum for odd *n* and $\frac{n}{2}$ minimum and $\frac{n+2}{2}$ maximum for even *n*) and n cross-zero. $\psi^n(x)$ with even *n*, has a dominant peak in the center. $\psi^n(x)$ with odd *n*, has two dominant peaks with the same amplitude, which are antisymmetric into zero-crossing points. In continue, edge detector criteria studied and proposed wavelet bases are compared with each other across the *n*.

III. CANNY PERFORMANCE MEASURES

In the previous section, we defined new wavelet bases by using Gaussian derivations. In this section, Canny criteria are discussed for new wavelet bases. Canny focused on the step edge detection with and without the noise presence. He assumed that the noise has a model of AGWN. He assumed the edge detector filter to be antisymmetric and has been achieved by the derivation of a smooth function $g(x)$. An edge occurs in a signal where there would be a local maximum in first-order derivation or equivalently a zero-crossing in the second derivation [19].

Canny criteria are good candidates to compare the filters' performances. Hence, they have been considered in much research. McIlhagga [20] and Jeong [21] revised his idea, and Demigny [22] developed it to the discrete space domain. There are three criteria which Canny introduced for designing an optimal edge filter:

A. Good Detection. Canny attempted to maximize the signal to noise ratio in the edge detection process and introduced "good detection" criterion by selecting proper filter *f(x)*, which could maximize the SNR. According to the good detection criterion, increasing the SNR, would reduce the number of wrong detected edges. Signal to noise ratio is defined as

$$SNR = \frac{\left|\int_{-\infty}^{+\infty} g(x)f(x)dx\right|}{\sigma\sqrt{\int_{-\infty}^{+\infty} f^2(x)dx}} \tag{7}$$

Where $g(x)$ is the edge function, and σ is the noise standard deviation in the normal distribution. Assume the edge function be a unit step ($g(x) = u_{-1}(x)$). For *n=1,3,5,7* SNR can be calculated as the first column of Table 1. This Table shows that the SNR has a direct relationship with the square root of the scale $\sqrt{s}$ and an inverse relationship with the noise standard deviation σ. This means that we have a better signal-to-noise ratio in coarser scales.

Table 1
The SNR and the Localization of $\psi_s^n(x)$

n	SNR	Localization
1	$\frac{\sqrt{8\pi^{\frac{3}{2}}}}{2\pi}\frac{\sqrt{s}}{\sigma}$	$\frac{\sqrt{\frac{16}{3}\pi^{\frac{3}{2}}}}{2\pi}\frac{1}{\sigma\sqrt{s}}$
3	$\frac{\sqrt{\frac{32}{15}\pi^{\frac{3}{2}}}}{2\pi}\frac{\sqrt{s}}{\sigma}$	$\frac{3\sqrt{\frac{64}{105}\pi^{\frac{3}{2}}}}{2\pi}\frac{1}{\sigma\sqrt{s}}$
5	$\frac{3\sqrt{\frac{128}{945}\pi^{\frac{3}{2}}}}{2\pi}\frac{\sqrt{s}}{\sigma}$	$\frac{15\sqrt{\frac{256}{10395}\pi^{\frac{3}{2}}}}{2\pi}\frac{1}{\sigma\sqrt{s}}$
7	$\frac{15\sqrt{\frac{512}{135135}\pi^{\frac{3}{2}}}}{2\pi}\frac{\sqrt{s}}{\sigma}$	$\frac{105\sqrt{\frac{1024}{2027025}\pi^{\frac{3}{2}}}}{2\pi}\frac{1}{\sigma\sqrt{s}}$

B. Good Localization. Another Canny criterion in the edge detector is the good localization. It means that detected edges should be as near as possible to the true edges. One of the parameters which leads to make the error and edge displacement, is the interference of the noise in signals and images. He proved that localization *L* has an inverse relationship with the mean distance of the detected edge and the actual one. He has defined this criterion as

$$L = \frac{\left|\int_{-\infty}^{+\infty} g'(-x)f'(x)dx\right|}{\sigma\sqrt{\int_{-\infty}^{+\infty} f'^2(x)dx}} \tag{8}$$

And attempt to find the filter impulse response *f(x)* that maximizes *L* as the best edge detector. This formula has been derived by assuming the impulse response be odd and derivable. Localization can be calculated for different $\psi_s^n(x)$ as the second column of the Table 1. The results indicate that Localization of $\psi_s^n(x)$ has an inverse relationship with the *scale* and noise standard deviation σ. Refer to the Table 1, we find out Localization is approximately constant over n.

C. Low Spurious Response. Third Canny edge detector survey is "one responding to one edge". When the edge detector is applied to a single edge, it is clear that there should not be more than one response as the result. It can be defined the measure for the suppression of false edge detection to be proportional to the mean distance between the neighbored maxima of the filter responding to AWGN noise. This criterion expresses as

$$x_{max} = kw = 2\pi\sqrt{\frac{\int_{-\infty}^{+\infty} f'^2(x)dx}{\int_{-\infty}^{+\infty} f''^2(x)dx}} \tag{9}$$

Where, *w* is the operator width, and *k* is a fraction factor. Canny has made an estimate of *k* named *multiple response criterion* for probability of spurious edges. This parameter is calculated for wavelet basis $\psi_s^n(x)$ as

$$x_{max}^n = 2\pi s\sqrt{\frac{2}{2n+3}} \tag{10}$$

Where *n* refers to the notation of $\psi_s^n(x)$ and *nth* order derivative of the Gaussian scaling function. The first column of the Table 2 shows calculations of x_{max}^n . Sarkar and Boyer [9] modified the multiple response criterion as follows:

$$MRC = k = 2\pi\sqrt{\frac{\int_{-\infty}^{+\infty} f'^2(x)dx}{\int_{-\infty}^{+\infty} f''^2(x)dx}}\sqrt{\frac{\int_{-\infty}^{+\infty} f^2(x)dx}{\int_{-\infty}^{+\infty} x^2 f^2(x)dx}} \tag{11}$$

Calculations of the MRC for Gaussian bases are listed in the last column of the Table 2. Clearly, the MRC is independent of scale, i.e. MRC is identical for a basis function in a specified scale. As a desire, MRC should be as great as possible. It guarantees the distance between the fake maxima.

Table 2
Summary of ψ_s^n characteristics

n	$xmax_s^n$	Spatial width W_s^n	$k = MRC^n$
1	$2\pi s\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{2}}s$	$2\pi\sqrt{\frac{4}{15}}$
2	$2\pi s\sqrt{\frac{2}{7}}$	$\sqrt{\frac{7}{6}}s$	$2\pi\sqrt{\frac{12}{49}}$
3	$2\pi s\sqrt{\frac{2}{9}}$	$\sqrt{\frac{11}{10}}s$	$2\pi\sqrt{\frac{20}{99}}$

n	$2\pi s\sqrt{\frac{2}{2n+3}}$	$\sqrt{\frac{4n-1}{4n-2}}s$	$2\pi\sqrt{\frac{n-\frac{1}{2}}{(n+\frac{3}{2})(n-\frac{1}{4})}}$

Another parameter is a measure of the spread function in the frequency domain which noted by Ω and identified as [23]

$$\Omega = \sqrt{\frac{\int_{-\infty}^{+\infty}(\omega-\varpi)^2|\Psi_s^n(\omega)|^2 d\omega}{\int_{-\infty}^{+\infty}|\Psi_s^n(\omega)|^2 d\omega}} \tag{12}$$

Where ϖ is

$$\varpi = \int_{-\infty}^{+\infty}\omega|\Psi_s^n(\omega)|^2 d\omega \tag{13}$$

The frequency filter bandwidth is calculated for proposed wavelet basis functions as

$$\Omega_s^n = \sqrt{\frac{\int_{-\infty}^{+\infty}\omega^2(s^n\omega^n G_s(\omega))^2 d\omega}{\int_{-\infty}^{+\infty}(s^n\omega^n G_s(\omega))^2 d\omega}} = \sqrt{\frac{2n+1}{2s^2}} \tag{14}$$

Similarly, the spatial width of the edge detector filter $f(x)$ is defined as [23]

$$W = \sqrt{\frac{\int_{-\infty}^{+\infty}(x-\bar{x})^2 f^2(x)dx}{\int_{-\infty}^{+\infty}f^2(x)dx}} \tag{15}$$

Where $\bar{x}$ is

$$\bar{x} = \int_{-\infty}^{+\infty}xf^2(x)dx \tag{16}$$

For every ψ_s^n ,W will be obtained as follows

$$W_s^n = \sqrt{\frac{4n-1}{4n-2}}s \tag{17}$$

Second column of Table 2, shows numerical calculated W_s^n for first six bases. If the filter has a large width in the spatial domain, it is not clear whether the output would be due to a single edge or multiple edges. This means that we should extend the filter frequency width to overcome this problem; but in the frequency domain, it is better to reduce the filter bandwidth to restrict the noise ability. So it is a tradeoff between the width of the spatial and the width of the frequency domain. In the other words, we cannot reduce the spatial filter width (to have a unique response) and the frequency filter width (to inhibit the noise bandwidth) simultaneously. Hence, the width of the filter becomes a criterion to evaluate edge detectors' performance. The smaller the spatial width, the better the detector. We denote ΩW as production of frequency and spatial widths. ΩW is independent of the scale and is calculated for $\psi_s^n(x)$ as

$$\Omega W = \sqrt{\frac{(2n+1)(4n-1)}{4(2n-1)}} \tag{18}$$

Figure 2 illustrates the spatial and the frequency of ψ_s^n bandwidth. If these diagrams are multiplied together (blue diagram in Figure 2), the result will be more than 1 that shows uncertainly spatial-frequency relationship. Minimum of ΩW occurs in $n = 1$ as shown in Figure 2. This means that the first-order derivation of the Gaussian filter has the best spatial-frequency condition.

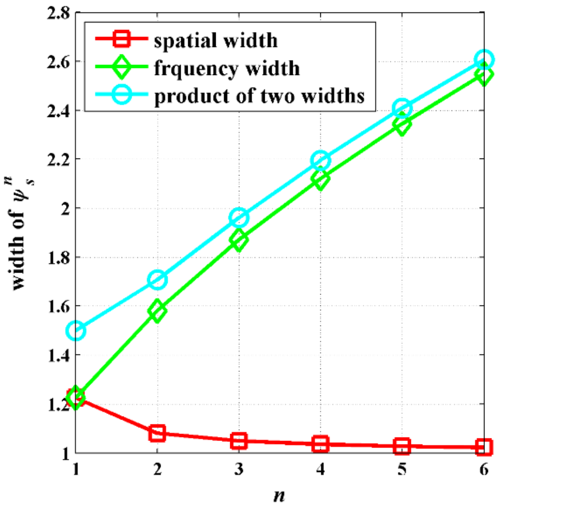

Figure 2: Width of ψ_s^n as a function of n in the spatial and frequency domain

Another trade-off in edge detector filters is between SNR and the width of the filter. The larger filter results the better SNR, because of noise diluting; and the smaller filter results the better edge localization performance.

IV. Edge Detection

In the previous section, the details of Canny criteria for new wavelet basis functions were illustrated. The next step is to find out wavelet functions' behavior to the basic edges. In this section, the step and line function as two significant edge shapes are modeled and relevant wavelet coefficients calculated. The contents of this section divided into two main parts. In the first part, the defined wavelet bases $\psi_s^n(x)$ will be used to derive corresponding wavelet transform of the unit step function. In the second part, the response of the line edge is described. The effect of the smoothing function as a preprocessing stage is obtained from each edge type. To reach multiresolution edge detection of these two types of edge, the behavior of wavelet maxima across n (degree of the wavelet basis) and s (scale) is studied.

A. Step Response

When the two regions with different amplitudes meet each other, a broken edge occurs in their boundary region. Step function is a basic shape which has been considered in much research [10, 24-28]. We consider the uniform Heaviside $u_{-1}(x)$ as a basic edge and formulate its wavelet coefficients with and without a smooth function. For the unit step the coefficients are calculated as

$$W_s^n f(x) = f(x) * \psi_s^n(x) = s^n \frac{d^{n-1} g_s(x)}{dx^{n-1}} \quad (19)$$

W_s^n is symmetric for odd n and antisymmetric for even n. Also it has a peak in odd n and a zero-crossing in even n in $x = 0$. So, for the edge detection theory, it can be used to find the peak in W_s^n with odd n or the zero-crossing with even n in zero point (break point). Figure 3 shows the step edge wavelet coefficient response for odd n in scale s=1. As focus on this Figure, we find out ψ_s^n has sharper peaks and higher attenuation with the increase of n.

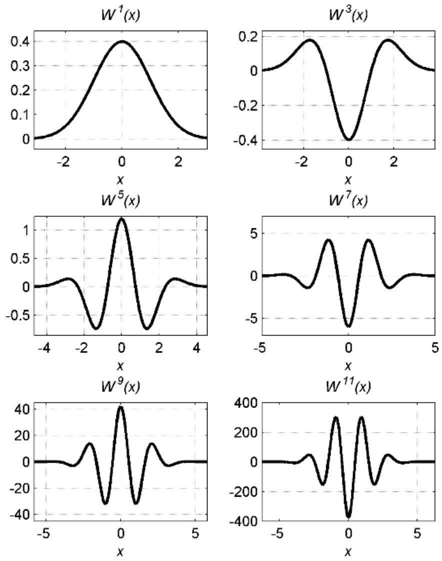

Figure 3: The step edge response in the scale s=1

Smooth filter is used as a preprocessing unit to reduce the noise influence in the filter-based edge detection. One of the motives that makes smoothing as an essential preprocessing stage is band-limiting of signals and images. Thus, the noise ability is inhibited in high frequency. It has a drawback too. Noise and details have both high-frequency characteristics. The blurring process dilutes the details of signals and images. This means that we lose some information. Indeed, it is not possible to eliminate noise without any damage to data. Gaussian function is known as a smoothing filter. Let $g_\sigma(x)$ be a Gaussian function with variance σ^2 as a smoothing filter.

$$g_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \quad (20)$$

Therefore, the coefficient of the blurred step function will be

$$\begin{aligned} W_s^n f(x) &= f(x) * g_\sigma(x) * \psi_s^n(x) \\ &= s^n \frac{\partial^{n-1} G_{\sqrt{s^2+\sigma^2}}(x)}{\partial x^{n-1}} \end{aligned} \quad (21)$$

The results with n=1 is similar to Ducottet work [26]. The response of the edge is related with the degree of smoothing σ and scaling parameter s. The effect of pre-smoothing is agreed with the effect on the scale in wavelet coefficients. i.e. Both s and σ increase the degree of blurring signals.

B. Line Edge Response

In many cases, pulse shapes exist in signals. In the previous section, step response with and without smoothing function as a preprocessing method was discussed. In this section, the line edge response to proposed wavelet bases is presented. For beginning, let $\Pi\left(\frac{x}{\Delta S}\right)$ be a pulse shape with zero center and width of ΔS. It can be introduced by shifting step function as follows

$$\Pi\left(\frac{x}{\Delta S}\right) = u_{-1}\left(x + \frac{\Delta S}{2}\right) - u_{-1}\left(x - \frac{\Delta S}{2}\right) \quad (22)$$

Figure 4 shows a principle line edge shape with width of ΔS [29, 30].

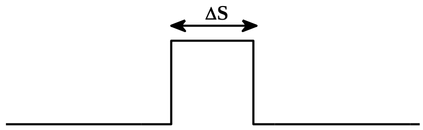

Figure 4: A line edge shape with width of ΔS

For defined new Gaussian wavelet functions, we have

$$\begin{aligned} W_s^n f(x) &= \Pi\left(\frac{x}{\Delta S}\right) * \psi_s^n(x) \\ &= s^n \left(\frac{d^{n-1} g_s\left(x + \frac{\Delta S}{2}\right)}{dx^{n-1}} - \frac{d^{n-1} g_s\left(x - \frac{\Delta S}{2}\right)}{dx^{n-1}} \right) \end{aligned} \quad (23)$$

W_s^n consists of two Gaussian derivations with the distance of ΔS from each other. Two Gaussian functions with the similar variance and the centric distance d have a $e^{-\frac{d^2}{4\sigma^2}}$ percent surface overlap.

$$\begin{aligned} overlap\ percentage &= \frac{\int_{-\infty}^{+\infty} g_1(x) g_2(x) dx}{\int_{-\infty}^{+\infty} g_1^2(x) dx} \\ &= e^{-\frac{d^2}{4\sigma^2}} \end{aligned} \quad (24)$$

With increasing n, the width of $\psi_s^n(x)$ will decrease and the overlap of two shifted Gaussian derivations are reduced.

Figure 5 illustrates the wavelet coefficients of the line edge for s=1, ΔS =5s.

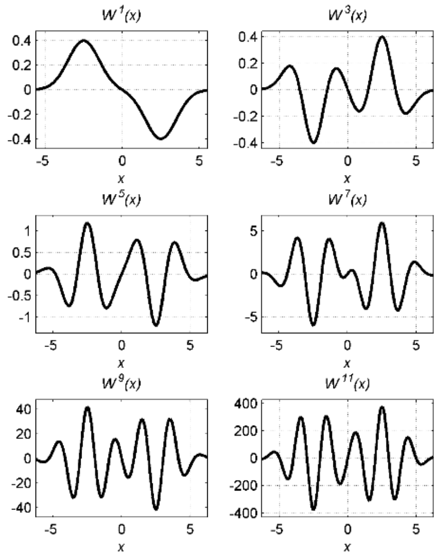

Figure 5: Wavelet coefficients of line edge for s=1, ΔS =5s

The vibration in responses, increases with higher n. But in all of them, a zero-crossing occurs in the center point $x = 0$.

V. CONCLUSION

In this paper, we developed the Canny edge detector and introduce new wavelet functions by derivations of the Gaussian smoothing filter. We proved that these new functions could be wavelet bases and have necessities of the wavelet functions. New wavelet bases are evaluated by Canny criteria (signal-to-noise ratio, localization and low spurious response) and results are given in relevant figures and tables. There are trade-off between the special and frequency filter width. To understand performance of introduced basis functions, the responses of two main edges, step and line are presented and effects of smoothing pre-filter on the edge detection are formulated and discussed.

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