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DYNAMIC INSTABILITY AND ULTIMATE CAPACITY OF INELASTIC SYSTEMS PARAMETRICALLY EXCITED

BY EARTHQUAKES

by

WU-HSIUNG TSENG, 1941-

A DISSERTATION

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ABSTRACT

A procedure of analysis is presented for determining the dynamic instability and response of framed structures subjected to pulsating axial loads, time-dependent lateral forces, or foundation movements. Included in the analytical work are the instability criterion of a structural system, the finite element technique of structural matrix formulation, and the computer solution methods.

The dynamic instability is defined by a region in relation to transverse natural frequency, longitudinal forcing frequency and the magnitude of axial dynamic force. The axial pulsating load is expressed in terms of static buckling load for ensuring that the applied load is not greater than the buckling capacity of a structural system. Consequently, the natural frequency and static instability analyses are also included. For static instability analysis, both the concentrated and uniformly distributed axial loads are investigated.

The displacement method is used in this research for structural matrix formulation for which the elementary matrices of mass, stiffness, and stability are developed using the Lagrangian equation and the system matrices are formulated using the equilibrium and compatibility conditions of the constituent members of a system.

Two numerical integration techniques of the fourth order

ii

Runge-Kutta method and the linear acceleration method are employed for the elastic and elasto-plastic response of continuous beams, shear buildings, and frameworks. The general considerations are the bending deformation, $p-\Delta$ effect, and the effect of girder shears on columns. For the elasto-plastic analysis, the effect of axial load on plastic moment is also included.

A number of selected examples are presented and the results are illustrated on a series of charts, tables, and figures from which the significant effect of pulsating load on the amplitude of transverse vibration is observed.

The work may be considered significant in the sense that the response behavior of parametric vibrations has been throughly studied and the computer programs developed can be used for various types of frameworks.

iii

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iv

TABLE OF CONTENTS

		P	age
ABSTRAC	r		ii
ACKNOWLI	EDGEM	ENTS	iv
LIST OF	FIGU	RES	i ii
LIST OF	TABL	ES	x
LIST OF	SYMB	OLS	xi
I.	INT	RODUCTION	1
	Α.	Purpose of Investigation	1
	в.	Scope of Investigation	2
II.	REV	IEW OF LITERATURE	4
	Α.	Structural Dynamic with Longitudinal Excitations	4
	в.	Structural Dynamics without Longitudinal Excitations	4
III.	MATI SYS'	RIX FORMULATION OF ELASTIC STRUCTURAL TEMS	7
	Α.	Governing Differential Equation	7
	в.	Derivation of Member Mass, Stiffness, Stability Matrices	13
	с.	System Matrix of Mass, Stiffness and Stability	17
	D.	Shear Building Subjected to Lateral Forces	20
IV.	STA	TIC AND DYNAMIC STABILITY	21
	Α.	Boundary of Dynamic Instability	21
	в.	Static Buckling Load and Natural Frequency	26
	с.	Static Buckling due to a Combined Action of Distributed and Concentrated Axial Forces	. 31

TABLE OF CONTENTS (continued)

		P	age
		1. Formulation of Stability Matrix	33
		2. Numerical Examples	39
ν.	NUM APP	ERICAL INTEGRATION METHODS AND THEIR LICATION TO DYNAMIC RESPONSE	48
	Α.	Fourth Order Runge-Kutta Method	48
	в.	Linear Acceleration Method	52
	с.	Modal Analysis	56
	D.	Application of Numerical Integration Methods to a Structure Subjected to a Ground Acceleration	56
VI.	DYN SYS	AMIC RESPONSE OF ELASTIC STRUCTURAL TEMS	71
	Α.	Numerical Examples	71
	в.	Discussion of Results	.73
VII.	MATI	RIX FORMULATION FOR ELASTO-PLASTIC UCTURAL SYSTEMS	83
	Α.	Idealized Elasto-Plastic Moment-Rotation Characteristics	83
	в.	Reduced Plastic Moment	84
	с.	Modified Elementary Mass, Stiffness, and Stability Matrices	86
	D.	System Matrix of Mass, Stiffness, and Stability	91
VIII.	DYN STR	AMIC RESPONSE OF ELASTO-PLASTIC UCTURES	92
	Α.	Transfer Matrix for Plastic Moments and Their Associated Shears	92
	в.	Calculation of Plastic Hinge Rotation	93

TALBE OF CONTENT (continued)

			Page
	с.	Numerical Examples	95
	D.	Discussion of Results	97
IX.	CON WOR	CLUSIONS AND RECOMMENDATIONS FOR FUTURE	100
	Α.	Summary and Conclusions	100
	в.	Recommendations for Future Work	101
BIBLIOGRAPHY	• • • •	• • • • • • • • • • • • • • • • • • • •	102
APPENDIX - Co	ompu	ter Programs	106
VITA		• • • • • • • • • • • • • • • • • • • •	145

LIST OF FIGURES

Figur	re	Page
3.1	General Problem	. 8
3.2	Loading on a Typical Member	8
3.3	Generalized Local Coordinates and Generalized Forces for a Typical Beam	. 14
4.1	Instability Region	. 24
4.2	Example 4.1	. 30
4.3	Dynamic Instability Region	. 32
4.4	Typical Bar Subjected to Concentrated Axial Load N and Uniformly Distributed Load q	. 35
4.5	Example 4.2	40
4.6	Example 4.3	42
5.1	Solutions of x and y of Example 5.1	. 59
5.2	Solutions of x, y and z of Example 5.2	. 62
5.3	Example 5.3	. 64
5.4	Diagrams for Example 5.3	. 65
5.5	Dynamic Response of y_1 and y_2 of Example 5.3	. 67
6.1	Example 6.1	. 74
6.2	Dynamic Instability Region of Example 6.1	. 75
6.3	Dynamic Response of Example 6.1	. 76
6.4	Example 6.2	. 77
6.5	Dynamic Instability Region of Example 6.2	. 78
6.6	y of Example 6.2 by Runge-Kutta Method	. 79
6.7	y ₂ of Example 6.2 by Runge-Kutta Method	. 80
6.8	y of Example 6.2 by Linear Acceleration Method	. 81

LIST OF FIGURES (continued)

Figu	re Page
6.9	y_2 of Example 6.2 by Linear Acceleration Method 82
7.1	Idealized Moment-Rotation Relationships
7.2	Generalized Local Coordinates and Generalized Forces of a Beam with j End Hinged
7.3	Generalized Local Coordinates and Generalized Forces of a Beam with i End Hinged
8.1	Dynamic Response of Example 8.196
8.2	Dynamic Response of y ₁ of Example 8.298
8.3	Dynamic Response of y ₂ of Example 8.299

LIST OF TABLES

Table	Page
I.	Buckling Load q with N Given of Example 4.2 44
II.	Buckling Load N _{cr} with q Given of Example 4.2 45
III.	Buckling Load q with N Given of Example 4.3 46
IV.	Buckling Load N _{cr} with q Given of Example 4.3 47
V.	Values of x and y of Example 5.1 by Direct Integration Method 57
VI.	Values of x and y of Example 5.1 by Runge- Kutta Method 58
VII.	Values of x, y, z of Example 5.2 by Direct Integration Method 60
VIII.	Values of x, y, z of Example 5.2 by Runge- Kutta Method
IX.	Modal Matrix Solution of Example 5.368
х.	Runge-Kutta Solution of Example 5.369
XI.	Linear Acceleration Solution of Example 5.370

LIST OF SYMBOLS

А	=	member cross-sectional area
[A _m]	=	equilibrium matrix relating internal moments to external nodal moments
[A _v]	=	equilibrium matrix relating internal shears to external nodal forces
[A _{ms}]	=	diagonal matrix involves the inertia forces due to joint displacements
^a 1, ^a 2, ^a 3, ^a 4	H	constants
dt	=	time increment
Ε	=	modulus of elasticity
{H _r }	=	plastic hinge rotations
{F _r }	=	external nodal moments
{F _s }	=	external nodal forces
I	=	moment inertia
[k _{ij}]	=	member stiffness matrix
[K]	=	structural stiffness matrix
L	=	length of member
m	=	member mass per unit length
[m _{ij}]	=	member mass matrix
[M]	=	structural mass matrix
N ₀	=	static buckling load
Ν	=	concentrated axial load
N _{cr}	=	concentrated buckling load
N(t)	=	time dependent axial force
đ	=	distributed load
q _{cr}	=	uniformly distributed buckling load

LIST OF SYMBOLS (continued)

Q _i	=	generalized forces
q _i	=	generalized coordinates
q _i	=	generalized velocities
 g _i	=	generalized acceleration
[s _{ij}]	=	member stability matrix
[S]	=	structural stability matrix
т	=	kinetic energy
U	=	strain energy of bending
v	=	potential energy due to axial force
W	=	work done by generalized external forces
{x }		global coordinates
{x _r }	=	global rotations
{x _s }	=	global displacements
{ x _{r} }	=	acceleration due to global rotations
{ x _{s} }	=	acceleration due to global displacements
y(x,t)	=	beam deflection
t	=	time
φ (x)	=	shape function
[]	=	matrix of dimension r X s
{ }	=	column matrix (vector) of dimension r X l
[] ^T	=	transpose of matrix
[] ⁻¹	=	inverse of square matrix
9	=	partial derivative operators
α, β	=	fractional factor

LIST OF SYMBOLS (continued)

γ	=	unit weight		
θ	=	longitudinal	forcing	frequency
ω	=	natural freq	uency	

I. INTRODUCTION

In recent years the theory of dynamic instability has become one of the newest branches of the structural dynamics and mechanics of deformable solids. The problems which are examined based on classical theory of vibrations and structural dynamics are emphasizing on response history due to lateral time-dependent excitations. It is known that when a rod is subjected to the action of longitudinal compressive force varying periodically with time, for a definite ratio of the longitudinal frequency to the transverse frequency, the transverse vibrations of the rod will have rapidly increasing amplitude. Thus the study of the formation of this type of vibrations and the methods for the prevention of their occurence are necessary in the various areas of mechanics, transportation, industrial construction, structures excited by earthquakes.

A. Purpose of Investigation

The purpose of this study is to develop an analytical method for determining the behavior of dynamic instability and response of structural systems subjected to longitudinal pulsating loads and lateral dynamic forces or foundation movements. The mathematical formulation is general for computer analysis of large structural systems with consideration of geometric and material nonlinearity.

1

B. Scope of Investigation

The scope of the study may be briefly stated as the derivation of instability criteria, finite element formulation of structural matrices and the numerical methods of a computer solution.

Chapter III presents the basic formulation of mass matrix, stiffness matrix, and stability matrix by using the energy concept and finite element technique. The governing differential equation is expressed in terms of a system matrix which is formulated based on structural geometric and equilibrium conditions.

In order to evaluate the dynamic instability regions, it is convenient to express the axial load in terms of static buckling load and the longitudinal forcing frequency in terms of natural frequency. Thus Chapter IV presents the techniques of finding natural frequencies, buckling loads and instability regions. For the buckling load case the uniform axial load is also investigated.

Two numerical integration techniques for dynamic response using the fourth order Runge-Kutta method and the linear acceleration method are presented in Chapter V in which the comparison of numerical solutions shows the accuracy of the presented methods.

Chapter VI contains dynamic response of various types of frameworks subjected to axial pulsating load and lateral forces or foundation movements. The elasto-plastic case is given in Chapters VII and VIII for the formulation of member matrices and system matrix; plastic hinge rotations and numerical solutions.

Two typical computer programs of elastic and elastoplastic analyses of general types of rigid frames are given in the Appendix.

II. REVIEW OF LITERATURE

A. Structural Dynamics with Longitudinal Excitations

The behavior of structural systems subjected to both lateral and longitudinal excitations is little known. Most of the research work has been concentrated on the problem of an elastic column subjected to a periodically varying axial load for the purpose of searching for the stability criteria of double symmetric columns (1) as well as nonsymmetric columns (2).

Sevin E. (3), among other investigators, studied the effect of longitudinal impact on the lateral deformation of initially imperfect columns. Recently, Cheng and Tseng (5) investigated the effect of static axial load on the Timoshenko beam-column systems.

It seems that very little work has been done for the criteria of dynamic instability and response behavior of framed structures subjected to dynamic lateral and longitudinal excitations.

B. Structural Dynamics without Longitudinal Excitations

The conventional structural dynamics problems have been generally solved by using three methods of lumped mass, distributed mass, and consistent mass. Before the computer facilities were available, the lumped mass model with a finite degree of freedom had been extensively studied by a number of investigators. With the advent of computers, the research works on multistory structures were performed by early investigators, namely N.M. Newmark, R.W. Clough, J.A. Blume (6,7,9,10,11,12,17), and later by Cheng (13), E.L. Wilson, I.P. King, etc. (14,15,16).

For the distributed mass system, the early research work was limited to single members (18), or one-story-frames (19). Later Levin and Hartz (20) used the dynamic flexibility matrix method to solve one and two-story rigid frames, Cheng (4,13,29) solved free and forced vibrations of continuous beams and rigid frames by using displacement method. The displacement and flexibility methods cited above may be considered exact in the sense that the members must be prismatic and the structural joints are rigid.

In recent years, the finite element technique has been extensively used for solving structural dynamics problems. The method was initially propose by Archer (21) for plane frameworks; Cheng (22) recently extended the technique to solve space frame problems. The model of the method is similar to the distributed mass system. The equation of motion, however, is expressed in an explicit form for which the solution effort is much less than that of the distributed mass model.

The fundamental behavior of dynamic response of elastoplastic systems may be found in standard texts (23,24). The elasto-plastic analysis method of beams and one-story frames

5

with distributed mass has appeared in references (25,26) in which the method is limited to simple structures.

For large structures, typical work may be referred to references (27,28). Berg and Dadeppo (27) investigated the response of a multistory elasto-plastic structure due to lateral dynamic forces. Walpole and Sheperd (28) studied the behavior of reinforced concrete frames subjected to earthquake movements.

III. MATRIX FORMULATION OF ELASTIC STRUCTURAL SYSTEMS

The displacement matrix method has been used for the structural system formulation for static and dynamic instability analysis, and dynamic response. The formulation involves deriving differential equations, element matrices of stiffness, mass, stability, and the matrix of general structural systems. The structures are plane frameworks of which the joints are rigid and the constituent members are prismatic. As shown in Fig. 3.1, the structure is subjected to time-dependent axial forces N(t) and lateral dynamic load F(t) or foundation movement G(t), and may have superimposed uniform mass m and concentrated mass M_i in addition to its own weight.

For the purpose of investigating large systems, the shears transmitted from girders to columns are taken into consideration and the members are assumed to have bending deformation only.

A. Governing Differential Equation

Consider an arbitrary member of a structural system as shown in Fig. 3.2. The governing differential equations for such an element can be obtained by using the Lagrangian equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathbf{T}}{\partial \dot{\mathbf{q}}_{\mathbf{i}}}\right) - \frac{\partial \mathbf{T}}{\partial \mathbf{q}_{\mathbf{i}}} + \frac{\partial \mathbf{U}}{\partial \mathbf{q}_{\mathbf{i}}} - \frac{\partial \mathbf{V}}{\partial \mathbf{q}_{\mathbf{i}}} = \frac{\partial \mathbf{W}}{\partial \mathbf{q}_{\mathbf{i}}} = Q_{\mathbf{i}}$$
(3.1)



Fig. 3.1 General Problem



Fig. 3.2 Loading on a Typical Member

in which

T = kinetic energy; U = strain energy of bending; V = potential energy done by axial force; Q_i = generalized forces; q_i = generalized coordinates at node i associated with Q_i ; q_i = generalized velocities; W = work done by generalized external forces. Let $\phi(x)$ be the shape function and q_i (t) be the time

function of the beam motion, then the displacement of the beam can be expressed as

$$y(x,t) = \sum_{i=1}^{n} q_i(t) \phi_i(x).$$
 (3.2)

The kinetic energy for lateral displacement of the member is

$$T = \frac{1}{2} \int_{0}^{L} m \left[\frac{\partial y(x,t)}{\partial t} \right]^{2} dx$$
(3.3)

where m is the mass per unit length.

The strain energy for bending of the member may be represented by

$$U = \frac{1}{2} \int_{0}^{L} EI \left[\frac{\partial^2 y(x,t)}{\partial x^2} \right]^2 dx$$
(3.4)

where E, I are elastic Young's modulus and moment of inertia, respectively.

The potential energy for the longitudinal force is

$$V = \frac{1}{2} \int_{0}^{L} N(t) \left[\frac{\partial y(x,t)}{\partial x} \right]^{2} dx$$
(3.5)

By the substitution of Eq. (3.2), one may obtain

$$T = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{dq_{i}}{dt} \frac{dq_{j}}{dt} \int_{0}^{L} m\phi_{i}(x)\phi_{j}(x)dx \qquad (3.6)$$

$$U = \frac{n}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{d^{2}\phi_{i}(x)}{d^{2}x} \frac{d^{2}\phi_{j}(x)}{d^{2}x} dx \qquad (3.7)$$

$$V = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} q_{i}q_{j} \int_{0}^{L} N(t) \frac{d\phi_{i}(x)}{dx} \frac{d\phi_{j}(x)}{dx} dx \qquad (3.8)$$

or

$$T = \frac{n}{2} \sum_{j=1}^{n} \sum_{i=1}^{n} q_{i} q_{j} = \frac{1}{2} \{q\}^{T} [m_{ij}] \{q\}$$
(3.9)

$$U = \frac{n}{2}\sum_{j}\sum_{i}k_{ij}q_{i}q_{j} = \frac{1}{2}\{q\}^{T}[k_{ij}]\{q\}$$
(3.10)

$$V = \frac{n}{2\Sigma} \sum_{ij}^{n} s_{ij}' q_{i} q_{j} = \frac{1}{2} \{q\}^{T} [s_{ij}'] \{q\}$$
(3.11)

where

$$m_{ij} = \int_{0}^{L} m\phi_{i}(x)\phi_{j}(x)dx \qquad (3.12)$$

$$k_{ij} = \int_{0}^{L} EI\phi''_{i}(x)\phi''_{j}(x)dx$$
(3.13)

$$s'_{ij} = \int_{0}^{L} N(t) \phi'_{i}(x) \phi'_{j}(x) dx \qquad (3.14)$$

To include the concentrated masses in the formulation of m_{ij} , let us consider masses $M_k(x_k)$ acting at the positions x_k , $k=1,2,\cdots,r$, then Eq. (3.12) should be expressed as

$$m_{ij} = \int_{0}^{L} m\phi_{i}(x)\phi_{j}(x)dx + \sum_{k=1}^{r} M_{k}(x_{k})\phi_{i}(x_{k})\phi_{j}(x_{k}) \qquad (3.15)$$

The work done by external forces acting at the general-ized coordinate ${\bf q}_{\rm i}$ is

$$W = \sum_{i=1}^{n} \left[\sum_{j=1}^{p} \{F_{j}(x_{j})\phi_{i}(x_{j})\} + \int_{0}^{L} f(x,t)\phi_{i}(x)dx]q_{i} \right]$$
(3.16)

where $F_j(x_j)$ is the concentrated forces acting at positions x_j , $j=1,2,\cdots,p$.

Let $N(t) = (\alpha + \beta \cos \theta t) N_0$, then Eq. (3.14) becomes

$$s'_{ij} = (\alpha + \beta \cos \theta t) s_{ij}$$
 (3.17)

where

$$s_{ij} = \int_{0}^{L} N_{0} \phi_{i}(x) \phi_{j}(x) dx.$$

Substituting Eqs. (3.9), (3.10), (3.11) and (3.17) into Eq. (3.1) and performing the operation shown in Eq. (3.1) lead to the following governing differential equations of motion

$$[m_{ij}]{\dot{q}} + [k_{ij}]{q} - (\alpha + \beta \cos \theta t) [s_{ij}]{q} = {f} (3.18)$$

in which the matrices [m_{ij}], [k_{ij}], and [s_{ij}] are the
matrices of mass, stiffness, and stability defined in Eqs.
(3.12), (3.13), (3.17), respectively. {f} is the vector of
equivalent generalized external forces. All the elements in
[m_{ij}], [k_{ij}], and [s_{ij}] will be derived in the next section.

For a structural system, the member matrices are assembled together by using the equilibrium and continuity conditions at nodal points and will be discussed in Section C. Similar to Eq. (3.18), the system matrix may be written as

$$[M] \{X\} + [K] \{X\} - (\alpha + \beta \cos \theta t) [S] \{X\} = \{F\}$$
(3.19)

in which {X} are global coordinates; [M], [K], and [S] are the matrices of total structural mass, stiffness, and

stability, respectively, and may be formulated through the procedure of displacement method. Eq. (3.19) is the governing differential equation of motion to be used in this study of the dynamic instability problem and dynamic response.

B. Derivation of Members Mass, Stiffness, Stability Matrices

For the displacement method, it is generally preferable to formulate the mass matrix, stiffness matrix, and stability matrix of a typical member based on a set of defined local coordinates; then the system matrices will be formulated by transfering local coordinates to global coordinates using equilibrium and compatibility conditions.

Let us consider a typical bar shown in Fig. 3.3 in which q_i (i=1,2,3,4) are local coordinates in positive direction and Q_i (i=1,2,3,4) are positive local generalized forces corresponding to q_i . The compressive axial force N(t) is considered to be positive. The displacements q_i are due to the application of the generalized forces Q_i . The displacement y(x,t) of the beam section at point x and time t may be written as

$$y(x,t) = \sum_{i=1}^{4} q(t)\phi(x)$$
 (3.20)

If bending deformation is considered only, then the differential equation of beam deflection is ϕ ""(x)=0 of which

.

13



Fig. 3.3 Generalized Local Coordinates and Generalized Forces for a Typical Beam

the solution may be expressed in cubic polynomials

$$\phi(x) = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

which is the shape function in Eq. (3.20). Let the coordinates q_i in Fig. 3.3 be displaced, one at each time, for a unit displacement; then $\phi(x)$ becomes

$$\phi_1(\mathbf{x}) = (\mathbf{x} - 2\mathbf{x}^2 / \mathbf{L} + \mathbf{x}^3 / \mathbf{L}^2)$$
(3.21)

$$\phi_2(\mathbf{x}) = (\mathbf{x}^3/\mathbf{L}^2 - \mathbf{x}^2/\mathbf{L})$$
(3.22)

$$\phi_3(\mathbf{x}) = (-1 + 3\mathbf{x}^2 / \mathbf{L}^2 - 2\mathbf{x}^3 / \mathbf{L}^3)$$
(3.23)

$$\phi_{A}(\mathbf{x}) = (3\mathbf{x}^{2}/L^{2}-2\mathbf{x}^{3}/L^{3}). \qquad (3.24)$$

Substituting Eqs. (3.21 to 3.24) into Eqs. (3.12 to 3.14) and performing the integration over the bar length, we can obtain [m_{ij}], [k_{ij}], and [s_{ij}] as follows:

$$\begin{bmatrix} Q_{1} \\ Q_{2} \\ Q_{3} \\ Q_{4} \end{bmatrix}_{m} \begin{bmatrix} \frac{4mL^{3}}{420} & \frac{-3mL^{3}}{420} & \frac{-22mL^{2}}{420} & \frac{13mL^{2}}{420} \\ \frac{-3mL^{3}}{420} & \frac{4mL^{3}}{420} & \frac{13mL^{2}}{420} & \frac{-22mL^{2}}{420} \\ \frac{-3mL^{3}}{420} & \frac{4mL^{3}}{420} & \frac{13mL^{2}}{420} & \frac{-22mL^{2}}{420} \\ \frac{-22mL^{2}}{420} & \frac{13mL^{2}}{420} & \frac{156mL}{420} & \frac{-54mL}{420} \\ \frac{13mL^{2}}{420} & \frac{-22mL^{2}}{420} & \frac{-54mL}{420} & \frac{156mL}{420} \\ \frac{13mL^{2}}{420} & \frac{-22mL^{2}}{420} & \frac{15mL}{420} & \frac{156mL}{420} \\ \frac{13mL^{2}}{420} & \frac{-22mL^{2}}{420} & \frac{-54mL}{420} & \frac{156mL}{420} \\ \frac{13mL^{2}}{420} & \frac{13mL^{2}}{420} & \frac{13mL^{2}}{420} & \frac{13mL}{420} & \frac{13mL}{420} \\ \frac{13mL^{2}}{420} & \frac{13mL^{2}}{420} & \frac{13mL}{420} & \frac{13mL}{420} & \frac{13mL}{420} & \frac{13mL}{420} \\ \frac{13mL^{2}}{420} & \frac{13mL^{2}}{420} & \frac{13mL^{2}}{420} & \frac{13mL}{420} & \frac{13mL}{420} \\ \frac{13mL^{2}}{420} & \frac{13mL}{420} & \frac{13m$$

$$\left\{ \begin{array}{c} Q_{1} \\ Q_{2} \\ Q_{2} \\ Q_{3} \\ Q_{4} \end{array} \right\}_{k} \left\{ \begin{array}{c} \frac{4 \text{EI}}{\text{L}} & \frac{2 \text{EI}}{\text{L}} & \frac{-6 \text{EI}}{\text{L}} & \frac{-6 \text{EI}}{\text{L}^{2}} & \frac{-6 \text{EI}}{\text{L}^{2}} \\ \frac{2 \text{EI}}{\text{L}} & \frac{4 \text{EI}}{\text{L}} & \frac{-6 \text{EI}}{\text{L}^{2}} & \frac{-6 \text{EI}}{\text{L}^{2}} \\ \frac{-6 \text{EI}}{\text{L}^{2}} & \frac{-6 \text{EI}}{\text{L}^{2}} & \frac{12 \text{EI}}{\text{L}^{3}} & \frac{12 \text{EI}}{\text{L}^{3}} \\ \frac{-6 \text{EI}}{\text{L}^{2}} & \frac{-6 \text{EI}}{\text{L}^{2}} & \frac{12 \text{EI}}{\text{L}^{3}} & \frac{12 \text{EI}}{\text{L}^{3}} \\ \frac{-6 \text{EI}}{\text{L}^{2}} & \frac{-6 \text{EI}}{\text{L}^{2}} & \frac{12 \text{EI}}{\text{L}^{3}} & \frac{12 \text{EI}}{\text{L}^{3}} \\ \frac{q_{4}} \\ \end{array} \right\}$$
(3.26)

$$\left\{ \begin{array}{c} Q_{1} \\ Q_{2} \\ Q_{2} \\ Q_{3} \\ Q_{4} \end{array} \right\}_{p} \left\{ \begin{array}{c} \frac{2L}{15} & \frac{-L}{30} & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-L}{30} & \frac{2L}{15} & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-L}{30} & \frac{-1}{15} & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-1}{10} & \frac{-1}{10} & \frac{6}{5L} & \frac{6}{5L} \\ \frac{-1}{10} & \frac{1}{10} & \frac{6}{5L} & \frac{6}{5L} \\ \frac{-1}{10} & \frac{$$

Note that Q_1 , Q_2 and Q_3 , Q_4 are corresponding to moments and shears, respectively; q_1 , q_2 and q_3 , q_4 are corresponding to

rotations and displacements, respectively; \ddot{q}_1 , \ddot{q}_2 and \ddot{q}_3 , \ddot{q}_4 are accelerations due to rotations and displacements, respectively. For convenience, let us rewrite Eqs. (3.25, 3.26, 3.27) in the following condensed forms

$$\begin{cases} Q_{m} \\ -- \\ Q_{v} \\ m \end{cases} = \begin{pmatrix} [MMR] & [MMY] \\ ---- \\ [MVR] & [MVY] \end{pmatrix} \begin{cases} \ddot{q}_{r} \\ -- \\ \ddot{q}_{s} \end{cases}$$
 (3.28)

$$\left\{ \begin{array}{c} Q_{m} \\ -- \\ Q_{v} \end{array} \right\}_{k} = \left[\begin{array}{c} [KMR] & [KMY] \\ ---- \\ [KVR] & [KVY] \end{array} \right] \left\{ \begin{array}{c} q_{r} \\ -- \\ q_{s} \end{array} \right\}$$
(3.29)

$$\left\{ \begin{array}{c} Q_{m} \\ -- \\ Q_{v} \end{array} \right\}_{p} = \left\{ \begin{array}{c} [SMR] & [SMY] \\ ---- \\ [SVR] & [SVY] \end{array} \right\} \left\{ \begin{array}{c} q_{r} \\ -- \\ q_{s} \end{array} \right\}$$
(3.30)

in which the subscripts m, k, p signify that the moments $\{Q_m\}$, shears $\{Q_v\}$ are associated with $[m_{ij}]$, $[k_{ij}]$, and $[s_{ij}]$, respectively; the subscripts r and s signify the joint rotations and displacements, respectively.

C. System Matrix of Mass, Stiffness and Stability The displacement method of formulating structural system matrix has been well documented (31,32). Following Cheng's recent work (13), one may rewrite the relationship between the generalized external forces {F} and generalized external displacement {X} as

+
$$\begin{bmatrix} \begin{bmatrix} A_{m} \end{bmatrix} \begin{bmatrix} KMR \end{bmatrix} \begin{bmatrix} A_{m} \end{bmatrix}^{T} & \begin{bmatrix} A_{m} \end{bmatrix} \begin{bmatrix} KMY \end{bmatrix} \begin{bmatrix} A_{v} \end{bmatrix}^{T} \\ \hline \begin{bmatrix} A_{v} \end{bmatrix} \begin{bmatrix} KVR \end{bmatrix} \begin{bmatrix} A_{m} \end{bmatrix}^{T} & \begin{bmatrix} A_{v} \end{bmatrix} \begin{bmatrix} KVY \end{bmatrix} \begin{bmatrix} A_{v} \end{bmatrix}^{T} \end{bmatrix} \begin{bmatrix} x_{r} \\ -- \\ x_{s} \end{bmatrix}$$

Knowing $\{F_r\}$ and $\{F_s\}$, one may find $\{X_r\}$, $\{X_s\}$, $\{X_r\}$, $\{X_r\}$, $\{X_s\}$ from Eq. (3.31) by using numerical integration to be presented in Chapter V. Consequently, the member end moments and end shears can be obtained as follows:

$$\begin{cases} \begin{array}{c} Q_{m} \\ -- \\ Q_{v} \end{array} \\ \end{array} = \left(\begin{array}{c} \left[MMR \right] \left[A_{m} \right]^{T} \\ \left[MVR \right] \left[A_{m} \right]^{T} \end{array} \right] \left[MVR \right] \left[A_{v} \right]^{T} \\ \left[MVR \right] \left[A_{m} \right]^{T} \end{array} \right] \left[MVR \right] \left[A_{v} \right]^{T} \\ \end{array} \\ + \left(\begin{array}{c} \left[KMR \right] \left[A_{m} \right]^{T} \\ \left[KVR \right] \left[A_{m} \right]^{T} \end{array} \right] \left[KMY \right] \left[A_{v} \right]^{T} \\ \left[KVY \right] \left[A_{v} \right]^{T} \end{array} \right] \left\{ \begin{array}{c} X_{r} \\ --- \\ X_{s} \end{array} \right\} \\ - \left(\begin{array}{c} \left[SMR \right] \left[A_{m} \right]^{T} \\ \left[SVR \right] \left[A_{m} \right]^{T} \end{array} \right] \left[SMY \right] \left[A_{v} \right]^{T} \\ \left[SVY \right] \left[A_{m} \right]^{T} \end{array} \right] \left[SVY \right] \left[A_{v} \right]^{T} \\ \end{array} \right] \left\{ \begin{array}{c} X_{r} \\ ---- \\ X_{s} \end{array} \right\}$$
(3.32)

in which

- [A_m] = equilibrium matrix relating internal moments to external nodal moments;
- [A_v] = equilibrium matrix relating internal shears to external nodal forces;

[F_r] = external nodal moments;

- [F_s] = external nodal forces;
- [X_r] = global rotations;
- [X_s] = global displacements;
- $[\ddot{x}_r]$ = acceleration due to global rotations;
- $[\ddot{x}_s]$ = acceleration due to global displacement;
- [A_{ms}] = diagonal matrix involves the inertia forces due to joint displacements; and
- T = transpose of matrix.

Eqs. (3.31, 3.32) have been explained in detail in SUBROUTINE ASATA, ASATB, SATMV shown in the Appendix.

D. Shear Building Subjected to Lateral Forces

In many practical cases the girder stiffnesses compared with those of columns are sufficiently large. Consequently, the structural joint rotations are very small and only the sway displacements are significant. Neglecting the global coordinates corresponding to structural joint rotations, one may rewrite Eq. (3.31) as

$$[M] \{X_{s}\} + [K] \{X_{s}\} - [S] \{X_{s}\} = \{F_{s}\}$$
(3.33a)

where

$$[M] = [A_{v}] [MVY] [A_{v}]^{T} + [A_{ms}]$$
$$[K] = [A_{v}] [KVY] [A_{v}]^{T}$$
$$[S] = [A_{v}] [SVY] [A_{v}]^{T}$$

When the axial load is $N(t) = (\alpha + \beta \cos \theta t) N_0$, then Eq. (3.33a) becomes

$$[M] \{ \ddot{X}_{s} \} + [K] \{ X_{s} \} - (\alpha + \beta \cos \theta t) [S] \{ X_{s} \} = \{ F_{s} \}$$
(3.33b)

IV. STATIC AND DYNAMIC STABILITY

A. Boundary of Dynamic Instability

When a structural framework is subjected to a transverse pulsating load, the framework will generally experience forced vibration with a certain frequency of the excitation. The amplitude of the vibration becomes larger and larger when the forcing frequency approaches closer to the natural frequency of the vibrating system. The behavior is called resonance. However, when the frame is subjected to pulsating axial load as shown in Eq. (3.19) an entirely different type of resonance will be observed, the resonance will occur when a certain relationship exists between the natural frequency, the frequency of longitudinal forces and their magnitude. This resonance is called parametric resonance. The behavior of parametric resonance may be studied by using the governing differential equations of motion Eq. (3.19).

Let us consider the time dependent axial forces only, then Eq. (3.19) becomes

$$[M] \{X\} + [[K] - (\alpha + \beta \cos \theta t) [S]] \{X\} = 0$$
(4.1)

which represents a system of second order differential equations with periodic coefficient of the known Mathieu-Hill type. It has been observed that the Mathieu-Hill equation similar to Eq. (4.1) has periodic solutions with period T and 2T $(T=2\pi/\theta)$ at the boundaries of the instability region (2). The regions of instability may be determined by finding the periodic solutions of Eq. (4.1) in the form of a trigonometric series. The instability regions are bounded by two solutions with same period and stability regions are bounded by two solutions with different periods. The critical values of parameters of α , β , θ contained in Eq. (4.1) are obtained from the condition that Eq. (4.1) has periodic solutions. The stability or instability solutions of Eq. (4.1) correspond to the stability or instability of the structural system. The above-mentioned statement may be illustrated by the following derivation.

For the solution with period 2T, let the trial solution be in the form of series

$$\{x\} = \sum_{k=1,3,5,\cdots}^{\infty} (A_k \sin \frac{k\theta t}{2} + B_k \cos \frac{k\theta t}{2})$$
(4.2)

in which A_k , B_k are vectors independent of time. Substituting Eq. (4.2) into Eq. (4.1), the following system of matrix equations will be obtain by a comparison of the coefficients of $\sin \frac{k\theta t}{2}$ and $\cos \frac{k\theta t}{2}$.

$$([K] - (\alpha - \frac{1}{2}\beta)[S] - \frac{1}{4}\theta^{2}[M])A_{1} - \frac{1}{2}\beta[S]A_{3} = 0$$

$$([K] - \alpha[S] - \frac{1}{4}k^{2}\theta^{2}[M])A_{k} - \frac{1}{2}\beta[S](A_{k-2} + A_{k+2}) = 0$$

$$(k = 3, 5, 7, \dots),$$
$$([K] - (\alpha + \frac{1}{2}\beta)[S] - \frac{1}{4}\theta^{2}[M])B_{1} - \frac{1}{2}\beta[S]B_{3} = 0$$

([K] - \alpha[S] - \frac{1}{4}k^{2}\theta^{2}[M])B_{k} - \frac{1}{2}\beta[S](B_{k-2}+B_{k+2}) = 0
(k = 3,5,7,...).

Solution having the period $2T=4\pi/\theta$ can occur if the following conditions are satified

(4.3)

Similarly, for the solution with period T, let the trial solution be represented by

$$\{x\} = \frac{1}{2}B_0 + \sum_{k=2,4,6,\cdots}^{\infty} (A_k \sin \frac{k\theta t}{2} + B_k \cos \frac{k\theta t}{2}). \qquad (4.4)$$

Substituting Eq. (4.4) into Eq. (4.1) yields Eqs. (4.5) and (4.6) for the solution having the period $T=2\pi/\theta$.

For finding the regions of instability as sketched in Fig. 4.1, one may solve Eqs. (4.3), (4.5) and (4.6) for the

critical values of the parameters $(\alpha, \beta, N_0, \theta)$. The first region of instability (Region A) is determined from Eq. (4.3). Similarly, the second region of instability (Region C) is determined from Eqs. (4.5) and (4.6). The stability region (Region B) is confined by Region A and Region C.



Fig. 4.1 Instability Region

$$\begin{bmatrix} K \end{bmatrix} - \alpha \begin{bmatrix} S \end{bmatrix} - \theta^{2} \begin{bmatrix} M \end{bmatrix} & -\frac{1}{2}\beta \begin{bmatrix} S \end{bmatrix} & 0 & \cdots \\ -\frac{1}{2}\beta \begin{bmatrix} S \end{bmatrix} & \begin{bmatrix} K \end{bmatrix} - \alpha \begin{bmatrix} S \end{bmatrix} - 4\theta^{2} \begin{bmatrix} M \end{bmatrix} & -\frac{1}{2}\beta \begin{bmatrix} S \end{bmatrix} & \cdots \\ 0 & -\frac{1}{2}\beta \begin{bmatrix} S \end{bmatrix} & \begin{bmatrix} K \end{bmatrix} - \alpha \begin{bmatrix} S \end{bmatrix} - 16\theta^{2} \begin{bmatrix} M \end{bmatrix} & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \end{bmatrix} \stackrel{i}{=} 0$$

(4.5)

 $\begin{bmatrix} K \end{bmatrix} - \alpha [S] & - \beta [S] & 0 & \cdots \\ -\frac{1}{2}\beta [S] & [K] - \alpha [S] - \theta^{2} [M] & -\frac{1}{2}\beta [S] & \cdots \\ 0 & -\frac{1}{2}\beta [S] & [K] - \alpha [S] - 4\theta^{2} [M] & \cdots \\ 0 & 0 & -\frac{1}{2}\beta [S] & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \end{bmatrix} = 0 \quad (4.6)$

In practice, only the finite number of terms in the determinant is used for studying the principal instability regions. Thus when the first term of the series of Eq. (4.2) is considered (i.e., k=1, $\{X\}=A_1\sin(\theta t/2)+B_1\cos(\theta t/2)$), one may have

$$|[K] - (\alpha \pm \frac{1}{2}\beta)[S] - \frac{\theta^2}{4}[M]| = 0$$
 (4.7)

which is corresponding to the first matrix element along the diagonal of Eq. (4.3). The solutions of Eq. (4.7) gives the principal regions of dynamic instability. Similarly, from Eq. (4.5) and Eq. (4.6) we may have

$$|[K] - \alpha[S] - \theta^{2}[M]| = 0$$

and

$$\begin{bmatrix} K \end{bmatrix} - \alpha \begin{bmatrix} S \end{bmatrix} - \beta \begin{bmatrix} S \end{bmatrix} = 0$$
$$-\frac{1}{2}\beta \begin{bmatrix} S \end{bmatrix} \begin{bmatrix} K \end{bmatrix} - \begin{bmatrix} S \end{bmatrix} - \theta^{2} \begin{bmatrix} M \end{bmatrix}$$

which give the secondary region (Region C of Fig. 4.1) of dynamic instability. Note that Eq. (4.7) is an eigenvalue equation which can be solved by a conventional method of expanding the determinant equation (Eq. (4.7)) into a polynomial equation for the eigenvalue and its associated eigenvector. For this research of studying large structural systems, a different technique of matrix iteration has been used by utilizing computer facilities (32).

B. Static Buckling Load and Natural Frequency

It may be observed from Eq. (4.7) that an instability region is confined by axial load and the ratio of the axial forcing frequency to the natural frequency. In order to ensure the amount of axial load to be applied is not greater than the elastic buckling capacity of the system, it is essential to express the applied load in terms of buckling load N_0 , as αN_0 and βN_0 , α and β are fractional numbers less than one. This section is to discuss the techniques of finding static buckling load and natural frequency.

Observing Eq. (4.1) one may obtain three groups of eigenvalue problems classified as (a), (b) and (c) shown below: (a). For static buckling case when $\{\ddot{x}\}=0$, $\beta \cos\theta t=0$, then Eq. (4.1) becomes

$$([K] - \alpha[S]) \{X\} = 0 \text{ or } |[K] - \alpha[S]| = 0$$
 (4.8)

(b). For free vibration of harmonic motions without external axial loads Eq. (4.1) may be written as

$$[M]{\ddot{x}} + [K]{x} = 0$$
(4.9)

Let ${X} = {Ae^{i\omega t}}$ then Eq. (4.9) becomes

$$|[K] - \omega^2[M]| = 0$$
 (4.10)

which gives the natural frequency ω .

(c). For the influence of static axial loads on the natural frequency one may rewrite Eq. (4.1) as

$$|[K] - \alpha[S] - \omega^{2}[M]| = 0$$
 (4.11)

from which one may observe that the compressive load will decrease the natural frequency and tensile force will increase the natural frequency.

Let Eqs. (4.7), (4.8), (4.10) and (4.11) be expressed in a standard eigenvlaue form as

$$\frac{1}{\lambda} \{X\} = [DM] \{X\}$$
(4.12)

where [DM] and λ in Eq. (4.7) signify

$$[DM] = [[K] - (\alpha + \frac{1}{2}\beta) [S]]^{-1} [M], \text{ and } \lambda = \theta^2/4 \qquad (4.13)$$

or

$$[DM] = [[K] - (\alpha - \frac{1}{2}\beta) [S]]^{-1} [M], \text{ and } \lambda = \theta^2/4 \qquad (4.14)$$

[DM] and λ in Eq. (4.8) represent

 $[DM] = [K]^{-1}[S]$, and $\lambda = \alpha$

For Eq. (4.10)

 $[DM] = [K]^{-1}[S]$, and $\lambda = \omega^2$

and for Eq. (4.11)

$$[DM] = [[K] - \alpha [S]]^{-1}[M]$$
, and $\lambda = \omega^2$

The matrix iteration method by Cheng (30) has been employed to obtain the eigenvalue λ and its associated eigenvector $\{X\}$.

Example 4.1. Consider a step beam given in Fig. 4.2a subjected to axial force $N(t) = \alpha N_0 + \beta N_0 \cos \theta t$. The cross

section of segments AB, BC are 8.375"x3.465" and 6.925"x 3.465", respectively. Let E=30x 10^{6} psi, γ =490 lbs/ft³, L_{AB}=144", L_{BC}=96". Find the dynamic instability region.

Solution: Using the local coordinates $\{q\}$ and global coordinates $\{X\}$ shown in Fig. 4.2b and 4.2c, respectively, one may find the equilibrium matrices $[A_m]$, $[A_v]$ tabulated in Fig. 4.2d and then manipulate Eq. (3.19) for

$$[M] = \begin{pmatrix} \frac{4m_{AB}L_{AB}^{3} + 4m_{BC}L_{BC}^{3}}{420} & \frac{-22m_{AB}L_{AB}^{2} + 22m_{BC}L_{BC}^{2}}{420} \\ \frac{-22m_{AB}L_{AB}^{2} + 22m_{BC}L_{BC}^{2}}{420} & \frac{156m_{AB}L_{AB} + 156m_{BC}L_{BC}}{420} \end{pmatrix}$$
(4.18)

$$[K] = \begin{pmatrix} \frac{4 E I_{AB}}{L_{AB}} + \frac{4 E I_{BC}}{L_{BC}} & \frac{-6 E I_{AB}}{L_{AB}^2} - \frac{6 E I_{BC}}{L_{BC}^2} \\ \frac{-6 E I_{AB}}{L_{AB}^2} + \frac{6 E I_{BC}}{L_{BC}^2} & \frac{12 E I_{AB}}{L_{AB}^3} - \frac{12 E I_{BC}}{L_{BC}^3} \\ \frac{12 E I_{AB}}{L_{AB}^3} - \frac{12 E I_{BC}}{L_{BC}^3} \end{pmatrix}$$
(4.19)



 and the second se		

(d) Equilibrium Matrices

Fig. 4.2 Example 4.1

$$[S] = \begin{pmatrix} \frac{2L_{AB}}{15} + \frac{2L_{BC}}{15} & 0 \\ 0 & \frac{6}{5L_{AB}} + \frac{6}{5L_{BC}} \end{pmatrix}$$
(4.20)

Thus substituting Eqs. (4.19) and (4.20) into Eq. (4.8) gives the static buckling load $N_0=2975$. kips. Using Eqs. (4.10) and (4.12) yields the natural frequency $\omega=28.95$ cps. Let $\alpha=0.$, 0.1, 0.2, 0.3, 0.4, 0.5, and $\beta=0.$, 0.1, 0.2, 0.3, 0.4, 0.5, then one may find various values of θ from Eqs. (4.12), (4.13), (4.14). Expressing θ in terms of θ/ω and then using parameters α and β one may draw the instability regions shown in Fig. 4.3.(12).

C. Static Buckling due to a Combined Action of Distributed and Concentrated Axial Forces

In the previous section, the static buckling load is assumed to be acting at the structural joints as a concentrated force. However, there are many cases where the longitudinal forces are distributed along the members. Typical examples may be the self-weight of chimneys, the self-weight of slender tall buildings and the weight of wall attached to columns. The stability matrix for above mentioned type of structures is different from that in Eq. (3.27).

31



Fig. 4.3 Dynamic Instability Region

It is well known that if the longitudinal compressive force is continuously distributed along a bar, the classical mathematical formulation becomes very sophisticated because the differential equation of the deflection curve of the buckled bar will no longer be an equation with constant coefficients. Consequently, the direct integration of the equation can only be applied to simple bars such as cantilever columns. It is the purpose of this section to present the stability matrix due to a combined action of distributed and concentrated axial forces.

1. Formulation of Stability Matrix

Consider the beam of Fig. 4.4a subjected to a concentrated axial force N, and a uniformly distributed axial load q. The generalized coordinates q_i and generalized forces Q_i are shown in Fig. 4.4b and c, respectively. Let N, q, Q_i , q_i are positive as shown, the displacement y(x) of the beam at point x due to q_i and Q_i may be expressed as

$$y(x) = \sum_{i=1}^{L} q_i \phi_i(x).$$
 (4.21)

For bending deformation only, the shape functions $\phi(x)$ of Eq. (4.21) are the same as Eqs. (3.21, 3.22, 3.23, 3.24) shown below:

33

$$\phi_{1}(x) = (x - 2x^{2}/L + x^{3}/L^{2})$$

$$\phi_{2}(x) = (x^{3}/L^{2} - x^{2}/L)$$

$$\phi_{3}(x) = (-1 + 3x^{2}/L^{2} - 2x^{3}/L^{3})$$

$$\phi_{4}(x) = (3x^{2}/L^{2} - 2x^{3}/L^{3})$$
(4.22)

The total potential energy V due to N and q is given by

$$V = V_{N} + V_{q}$$

where V_N is the virtual work done by the axial force N on displacement Δ , and V_q is the virtual work done by uniformly distributed axial load q on displacement Δ ; where Δ is the displacement resulting from the displacements q_i . For an element dx shown in Fig. 4.4d one may have

$$d\Delta = ds - dx \tag{4.23}$$

$$ds = dx \{1 + (dy/dx)^{2}\}^{\frac{1}{2}}$$
(4.24)

for small deflection, Eq. (4.24) becomes

$$ds = dx \{1 + \frac{1}{2} (dy/dx)^{2}\}$$
(4.25)

Substituting Eq. (4.25) into Eq. (4.23) and then integrating over the length yield

$$\Delta = \frac{1}{2} \int_{0}^{L} (dy/dx)^{2} dx$$

We can now write the work $V_{_{\rm N}}$ as









(d) Force-Deformation Relationship

Fig. 4.4 Typical Bar Subjected to Concentrated Axial Load N and Uniformly Distributed Load q

From Fig. 4.4d, $d\Delta = ds - dx = \frac{1}{2} (dy/dx)^2 dx$ the work done by the load acting on the right side of x on $d\Delta$ is

$$dV_{q} = (L-x)d\Delta = q(L-x)\{\frac{1}{2}(dy/dx)^{2}\}dx$$

Therefore, the total work produced by the distributed load over the length is

$$V_{q} = \int_{0}^{L} dV_{q} = \frac{1}{2} \int_{0}^{L} q (L-x) (dy/dx)^{2} dx \qquad (4.27)$$

The strain energy is

$$U = \frac{1}{2} \int EI\{y''(x)\}^2 dx$$
 (4.28)

The virtual work done by forces Q_i on q_i may be written as

$$W = \sum_{i} Q_{i} q_{i}$$
(4.29)

By Lagrange's equation

$$\frac{\partial U}{\partial q_i} - \frac{\partial V}{\partial q_i} = \frac{\partial W}{\partial q_i}$$
(4.30)

upon which the substitution of Eqs. (4.26), (4.27), (4.28), (4.29) leads

$$\{\nabla U\} - \{\nabla V_N\} - \{\nabla V_\alpha\} = \{\nabla W\}$$

$$(4.31)$$

From Eq. (4.21)

$$y'(x) = \sum_{i=1}^{\Sigma} \phi'(x)$$
(4.32)

$$y''(x) = \sum_{i=1}^{n} \phi_{i}''(x)$$
 (4.33)

thus substituting Eqs (4.32), (4.33) into Eqs (4.26), (4.27) and (4.28), respectively, gives

$$U = \frac{1}{2} \sum_{i j} \sum_{i j} k_{ij} q_{i} q_{j} = \frac{1}{2} \{q\}^{T} [k_{ij}] \{q\}$$
(4.34)

$$V_{N} = \frac{1}{2} \sum_{ij} \sum_{ij} s_{ij} q_{ij} = \frac{1}{2} \{q\}^{T} [s_{ij}] \{q\}$$
(4.35)

in which

$$k_{ij} = \int_{0}^{L} EI \phi_{i}^{"}(x) \phi_{j}^{"}(x) dx$$

$$s_{ij} = \int_{0}^{L} N \phi_{i}^{'}(x) \phi_{j}^{'}(x) dx$$

$$g_{ij} = \int_{0}^{L} q(L-x) \phi_{i}^{'}(x) \phi_{j}^{'}(x) dx.$$

Substituting Eqs. (4.34), (4.35), and (4.36) into Eq. (4.30) yields the results of Eq. (4.31) as

$$\{\nabla U\} = [k_{ij}] \{q\}$$

$$\{\nabla V_N\} = [s_{ij}] \{q\}$$

$$\{\nabla V_g\} = [g_{ij}] \{q\}$$

$$\{\nabla W\} = \{Q\}$$
(4.37)

Therefore Eq. (4.31) may be rewritten as

$$[k_{ij}] \{q\} - [s_{ij}] \{q\} - [g_{ij}] \{q\} = \{Q\}$$
 (4.38)

in which [k_{ij}], [s_{ij}] are exactly the same as Eqs. (3.26 and 3.27), [g] is the stability matrix due to uniformly ij distributed axial load and can be expressed as follows

$$\left\{ \begin{array}{c} Q_{1} \\ Q_{2} \\ Q_{2} \\ Q_{3} \\ Q_{4} \end{array} \right\} = \left\{ \begin{array}{c} \left[\frac{6qL^{2}}{60} & \frac{-qL^{2}}{60} & 0 & 0 \\ \frac{-qL^{2}}{60} & \frac{2qL^{2}}{60} & \frac{-qL}{10} & \frac{-qL}{10} \\ \frac{-qL}{60} & \frac{3q}{60} & \frac{3q}{10} \\ 0 & \frac{-qL}{10} & \frac{3q}{5} & \frac{3q}{5} \\ \end{array} \right\}$$
(4.39)

Through the displacement method discussed in Section C

of Chapter III, one can calculate the buckling load of a structure subjected to a simultaneous action of concentrated axial force N and distributed axial load q. The following examples are selected for the comparison of numerical solution obtained by the present method with that by Timoshenko's rigorous mathematical approach (34).

2. Numerical Examples

Example 4.2. Consider the uniform cantilever column shown in Fig. 4.5a with a concentrated axial force N acting at end B and a uniform load q acting along the axis. Find the critical load q_{cr} or the critical load N_{cr} . Let the member length L=240 in., the uniform cross section A=24 in², I=96 in⁴, and E=30x10⁶psi.

Solution: Let the column be divided into five segments as shown in Fig. 4.5a. The global coordinates and local coordinates are shown in Fig. 4.5b and 4.5c, respectively, from which the equilibrium matrices $[A_m]$ and $[A_v]$ are established as follows

	Pr	1	2	3	4	5	6	7	8	9	10
	1	0	1	1	0	0	0	0	0	0	0
	2	0	0	0	1	1	0	0	0	0	0
[A _m] =	3	0	0	0	0	0	1	1	0	0	0
	4	0	0	0	0	0	0	0	1	1	1
	5	0	0	0	0	0	0	0	0	0	0











(c) Local Coordinates

Fig. 4.5 Example 4.2

		P. S	1	2	3	4	5	6	7	8	9	10
		1	0	l	-1	0	0	0	0	0	0	0
		2	0	0	0	0	1	-1	0	0	0	0
[A _v]	=	3	0	0	0	0	0	1	-1	0	0	0
		4	0	0	0	0	0	0	0	l	-1	0
		5	0	0	0	0	0	0	0	0	0	l

The eigenvalue equation of this problem is similar to Eq. (4.8) with the inclusion of $[g_{ij}]$. Using the digital computer program based on the matrix iteration method (32) yields the solutions shown in Tables I and II in which the comparison of the present solution with Timoshenko's solution is very satisfactory.

Example 4.3. Consider the simply supported uniform beam shown in Fig. 4.6a with a concentrated axial force N acting at both ends A and B and a uniform load q acting along the axis. Find the critical load q_{cr} for given N and critical load N_{cr} for given q. Let L=240 in., A=30.2376 in², I=192 in⁴, and E=30x10⁶psi.

Solution: Let the beam be divided into five segments shown in Fig. 4.6a. The generalized global coordinates and generalized local coordinates are shown in Figs. 4.6b and 4.6c, respectively, from which the equilibrium matrices $[A_m]$ and $[A_v]$ are established as follows



(a) Loading



(b) Global Coordinates



(c) Local Coordinates

Fig. 4.6 Example 4.3

$$[A_{m}] = \begin{bmatrix} P_{m} \\ P_{m} \end{bmatrix} 1 2 3 4 5 6 7 8 9 10 \\ 1 1 0 0 0 0 0 0 0 0 0 0 0 0 \\ 2 0 1 1 0 0 0 0 0 0 0 0 0 \\ 3 0 0 0 1 1 0 0 0 0 0 0 \\ 4 0 0 0 0 0 1 1 0 0 0 0 \\ 5 0 0 0 0 0 0 0 1 1 0 0 0 \\ 5 0 0 0 0 0 0 0 0 0 1 1 0 \\ 6 0 0 0 0 0 0 0 0 0 0 0 0 1 \end{bmatrix}$$
$$\begin{bmatrix} P_{m} \\ P_{m} \end{bmatrix} = \begin{bmatrix} P_{m} \\ P_{m} \end{bmatrix} = \begin{bmatrix} P_{m} \\ P_{m} \end{bmatrix} = \begin{bmatrix} P_{m} \\ P_{m} \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

Similar to Example 4.2, the solutions obtained by using the computer program are shown in Tables III and IV in which a very good comparison between the present solution with Timoshenko's solution is shown. Timoshenko (34)

Present Method

b	$N=bEI/L^2$ (lbs)	a	q _{cr} =aπ ² EI/L ³ (lbs/in.)	q _{cr} (lbs/in.)
-2 / 4	100 070 00		0.00	0.00
π-/4.	123,370.00	0.00	0.00	0.00
2.28	114,000.00	0.25	128.51	130.99
2.08	104,000.00	0.50	257.02	269.06
1.91	95,500.00	0.75	385.53	385.42
1.72	86,000.00	1.00	514.04	514.70
0.96	48,000.00	2.00	1,028.08	1,019.80
0.15	7,500.00	3.00	1,542.13	1,538.97
0.00	0.00	3.18	1,634.66	1,632.92

Table II Buckling Load N_{cr} with q Given of Example 4.2

Timoshenko (34)

Present Method

a	$q=a\pi^2 EI/L^3$ (lbs/in.)	b	N _{CT} =bEI/L ² (lbs)	N _{cr} (lbs)
0.00 0.25 0.50	0.00 128.51 257.02	π ² /4. 2.28 2.08	123,370.00 114,000.00 104,000.00	123,372.92 114,184.90 104,898.80
0.75	385.53 514.04	1.91	95,500.00 86,000,00	95,499.12 86,064,87
2.00	1,028.08	0.96	48,000.00	47,358.83
3.18	1,634.66	0.00	0.00	0.00

Table III Buckling Load q_{cr} with N Given of Example 4.3

	Ti	moshenko	(34)	Present Method
b	N=bEI/L ² (lbs)	a	q _{cr} =a ² EI/L ³ (lbs/in.)	q _{cr} (lbs/in.)
π ²	986,965.00	0.00	0.00	0.00
8.63	836,000.00	0.25	1,028.09	1,025.64
7.36	736,000.00	0.50	2,056.18	2,057.84
6.08	608,000.00	0.75	3,084.26	3,084.26
4.77	477,000.00	1.00	4,112.35	4,112.62

Table IV Buckling Load N	er with q Given of Example 4.3
--------------------------	--------------------------------

a	$q=a_{\pi}^{2}EI/L^{3}$ (lbs/in.)	b	N _{cr} =bEI/L ² (lbs)	N _{cr} (lbs)
0.00	0.00	π ²	986,965.00	987,110.30
0.25	1,028.09	8.63	863,000.00	862,559.50
0.50	2,056.18	7.36	736,000.00	725,177.20
0.75	3,084.27	6.08	608,000.00	607,814.00

Timoshenko (34)

Present Method

V. NUMERICAL INTEGRATION METHODS AND THEIR APPLICATION TO DYNAMIC RESPONSE

In the analysis of dynamic response, an exact or rigorous mathematical approach may be possible for a very simple structure subjected to a force expressable in a mathematical function. For practical problems of complicated structures and loadings, the direct mathematic integration becomes tedious, or, perhaps impossible. Therefore, it is often desirable and sometimes imperative to solve the equations of motion by step-by-step numerical integration procedures which are designed to utilize the modern computational techniques.

Two well-known methods, the Runge-Kutta fourth order method and the linear acceleration method, have been employed in this research for general dynamic excitation of elastic as well as inelastic structures.

A. Fourth Order Runge-Kutta Method

Consider the following second order simultaneous differential equations

$$\left\{\frac{d^2x}{dt^2}\right\} = F(t,x,dx/dt)$$
(5.1)

of which the numerical integration by the fourth-order Runge-Kutta method may be expressed as (33)

$$\{x\}_{i+1} = \{x\}_{i} - (dt)\{x\}_{i} + (\frac{dt}{6})(\{\kappa_1\} + \{\kappa_2\} + \{\kappa_3\})$$
(5.2)

$$\{\dot{x}\}_{i+1} = \{\dot{x}\}_{i} - \frac{1}{6}(\{\kappa_{1}\}+2\{\kappa_{2}\}+2\{\kappa_{3}\}+\{\kappa_{4}\})$$
(5.3)

where

$$\{\kappa_{1}\} = (dt)F(t_{i}, \{x\}_{i}, \{\dot{x}\}_{i})$$

$$\{\kappa_{2}\} = (dt)F(t_{i} + \frac{dt}{2}, \{x\}_{i} + \frac{dt}{2}\{\dot{x}\}_{i}, \{\dot{x}\}_{i} + \frac{t_{2}}{2}\{\kappa_{1}\})$$

$$\{\kappa_{3}\} = (dt)F(t_{i} + \frac{dt}{2}, \{x\}_{i} + \frac{dt}{2}\{\dot{x}\}_{i} + \frac{dt}{4}\{\kappa_{1}\}, \{\dot{x}\}_{i} + \frac{t_{2}}{2}\{\kappa_{2}\})$$

$$\{\kappa_{4}\} = (dt)F(t_{i} + dt, \{x\}_{i} + dt\{\dot{x}\}_{i} + \frac{dt}{2}\{\kappa_{2}\}, \{\dot{x}\}_{i} + \{\kappa_{3}\})$$

From Eq. (3.19) or Eq. (3.31), one may write the acceleration equations as

$$\{\ddot{X}\} = [M]^{-1}(\{F\} - ([K] - (\alpha + \beta \cos \theta t)[S])\{X\})$$
(5.4)

Because of the similarity between Eq. (5.1) and Eq. (5.4), the solution of Eq. (5.4) can be obtained by applying the fourth order Runge-Kutta method.

The SUBROUTINE GFMKP in the appended computer programs is based on Eqs. (5.2 and 5.3) for which two examples are selected for the comparison of the numerical solution with the exact solution by direct integration.

Example 5.1. Find x and y of the following simultaneous second order differential equations by using (a) direct

integration and (b) the fourth order Runge-Kutta method.

$$\frac{d^{2}x}{dt^{2}} + \frac{dy}{dt} + x - y = sint$$

$$\frac{d^{2}y}{dt^{2}} + \frac{dx}{dt} + x - y = 2t^{2}$$
(5.5)

of which the initial conditions are

x=2., y=-4.5, dx/dt=-1., and dy/dt=-3.5 at t=0.

Solution: (a) Using the given initial conditions one may find the following solution to Eq. (5.5) by the direct integration technique.

$$x = 1+t-2t^{2}+\frac{2}{3}t^{3}-\frac{1}{6}t^{4}+e^{-t}-sint$$
$$y = -6-3t-4t^{2}-\frac{1}{6}t^{4}+e^{t}-e^{-t}-\frac{1}{2}sint-\frac{1}{2}cost$$

in which x and y are function of t. Let t be varied in an interval of 0.1 sec., then the values of x and y are tabulated in Table V.

(b) Let Eq. (5.5) be rewritten in the following matrix form

$$\begin{pmatrix} 1 & 0 \\ & \\ 0 & 1 \end{pmatrix} \begin{cases} \ddot{x} \\ \ddot{y} \end{cases} + \begin{pmatrix} 0 & 1 \\ & \\ 1 & 0 \end{pmatrix} \begin{cases} \dot{x} \\ \dot{y} \end{cases} + \begin{pmatrix} 1 & -1 \\ & \\ 1 & -1 \end{pmatrix} \begin{cases} x \\ y \end{cases} = \begin{cases} \text{sint} \\ 2t^2 \end{cases} (5.6)$$

Using the computer program GFMKP the solution of x and y in Eq. (5.6) has been found for the interval of time dt=0.004 sec. The result is shown in Table VI. Comparing Table V with Table VI reveals that the difference is negligible. x and y obtained in (a) and (b) are plotted in Fig. 5.1.

Example 5.2. Find x, y, z of the following simultaneous second order differential equations by using (a) direct integration method and (b) fourth order Runge-Kutta method.

$$d^{2}x/dt^{2} + d^{2}z/dt^{2} - x = 0$$

$$d^{2}y/dt^{2} + d^{2}z/dt^{2} - y = 0$$

$$d^{2}x/dt^{2} + y = 2cost$$
(5.7)

of which the initial conditions are x=0, y=0, z=3, dx/dy=0, dy/dt=0 and dz/dt=1.5 at t=0.

Solution: (a) The solutions to Eq. (5.7) are obtained by the direct integration method as

x = t sint

y = t sint

$$z = 1.5t - 2tsint - 2(1-cost) - 3$$

The numerical values of x, y, z are tabulated in Table VII. (b) Let Eq. (5.7) be rewritten in matrix form as

$$\begin{pmatrix} 1 & 0 & 1 \\ & & \\ 1 & 1 & 0 \\ & & \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \ddot{z} \\ \ddot{y} \\ \ddot{x} \\ \ddot{x} \end{pmatrix} + \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} z \\ y \\ x \\ x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 2cost \end{pmatrix}$$
(5.8)

The computer solution of Eq. (5.8) for dt=0.002 sec. is shown in Table VIII. The comparison between the results obtained by these two methods is very satisfactory. Fig. 5.2 shows the function of x, y, z vs time.

B. Linear Acceleration Method

The general expression of numerical integration of a second order differential equation may be rewritten as (17)

$$\{x\}_{t} = \{x\}_{t-dt} + \{\dot{x}\}_{t-dt} (dt) + (\dot{z}-B')\{\ddot{x}\}_{t-dt} (dt)^{2} + B'\{\ddot{x}\}_{t} (dt)^{2}$$

$$(5.9)$$

$$\{\dot{x}\}_{t} = \{\dot{x}\}_{t-dt} + \frac{1}{2}(\{\ddot{x}\}_{t-dt} + \{\ddot{x}\}_{t}) (dt)$$
(5.10)

in which the parameter B' to be chosen is to change the form

of the variation of acceleration in the time interval dt. When B'=1/6, the motion solution corresponds to a linear variation of acceleration in the time interval dt, and Eqs. (5.9), (5.10) become

$$\{x\}_{t} = \{x\}_{t-dt} + (dt)\{\dot{x}\}_{t-dt} + \frac{1}{3}\{\ddot{x}\}_{t-dt}(dt)^{2} + \frac{1}{6}\{\ddot{x}\}_{t}(dt)^{2}$$
(5.11)

$$\{\dot{x}\}_{t} = \{\dot{x}\}_{t-dt} + \frac{1}{2}(dt)\{\ddot{x}\}_{t-dt} + \frac{1}{2}(dt)\{\ddot{x}\}_{t}$$
(5.12)

in which the subscript t, and t-dt denote the response at time t and the previous t-dt, respectively. Thus the solution method is called linear acceleration method.

Let the governing differential equation of motion of Eq. (3.19) be rewritten as

$$[M] \{ \ddot{X} \} + ([K] - (\alpha + \beta \cos \theta t) [S]) \{ X \} = \{ F \}$$
(5.13)

which is actually a nonlinear differential equation, because the stability matrix $(\alpha+\beta\cos\theta t)$ [S] is time-dependent. The motion equation may be considered to be linear during a very short time duration, dt, for which Eq. (5.13) can be expressed in an incremental form as

$$[M] \{\Delta \mathbf{X}\} + ([K] - (\alpha + \beta \cos \theta t) [S]) \{\Delta \mathbf{X}\} = \{\Delta \mathbf{F}\}$$
(5.14)

in which

 $\{\Delta \ddot{\mathbf{x}}\}$ = incremental acceleration;

- $\{\Delta X\}$ = incremental displacement; and
- $\{\Delta F\}$ = incremental force.

From Eqs. (5.11) and (5.12) we have

$$\{\Delta \dot{x}\} = \{\dot{x}\}_{t} - \{\dot{x}\}_{t-dt} = 3/dt\{\Delta x\} + \{B\}$$
(5.15)

and

$$\{\Delta \ddot{x}\} = \{\ddot{x}\}_{t} - \{\ddot{x}\}_{t-dt} = 6/dt^{2}\{\Delta x\} + \{A\}$$
(5.16)

in which

$$\{\Delta x\} = \{x\}_{t} - \{x\}_{t-dt}$$
 (5.17)

$$\{A\} = -6/dt \{\dot{x}\}_{t-dt} - 3\{\ddot{x}\}_{t-dt}$$
(5.18)

$$\{B\} = -3\{\dot{x}\}_{t-dt} - dt/2\{\ddot{x}\}_{t-dt}$$
(5.19)

Substituting Eqs. (5.15 to 5.19) into Eq. (5.14) yields the following symbolic form

 $[K'] \{\Delta X\} = \{\Delta R\}$ (5.20)

in which

$$[K'] = 6/dt^{2}[M] + [K] - [S']$$
(5.21)

$$\{\Delta R\} = \{\Delta F\} - [M]\{A\}$$
 (5.22)

$$[S'] = (\alpha + \beta \cos \theta t) [S]$$
(5.23)

Thus Eq. (5.14) is reduced to the pseudo static form of Eq. (5.20) from which $\{\Delta X\}$ can be solved as

$$\{\Delta X\} = [K']^{-1}\{\Delta R\}$$
 (5.24)

Using the pseudo static form to find the dynamic response of a structure, one must repeatedly perform the following procedures.

$$\{A\} = -6/dt \{\dot{x}\}_{t-dt} - 3\{\ddot{x}\}_{t-dt}$$

$$\{B\} = -3\{\dot{x}\}_{t-dt} - dt/2\{\ddot{x}\}_{t-dt}$$

$$\{\Delta R\} = \{\Delta F\} - [M]\{A\}$$

$$[K'] = ([K] - [S'] + 6/dt^{2} [M])$$

$$\{\Delta x\} = [K']^{-1} \{\Delta R\}$$

$$\{x\}_{t} = \{x\}_{t-dt} + \{\Delta x\}$$

$$\{\dot{x}\}_{t} = \{\dot{x}\}_{t-dt} + \{\Delta \dot{x}\} = \{\dot{x}\}_{t-dt} - 3/dt \{\Delta x\} + \{B\}$$

$$\{\ddot{x}\}_{t} = \{\ddot{x}\}_{t-dt} + \{\Delta \ddot{x}\} = 6/dt^{2} \{x\}_{t-dt} + \{A\}$$

in which [S'] is different from time to time. Consequently, the structure is assumed to behave in a linear manner during each time increment, and the nonlinear response is obtained as a sequence of successive increments.

C. Modal Analysis

In analyzing the response of a structural system subjected to dynamic excitation, the governing differential equations of motion are usually composed of a set of coupled differential equations of second order. One of the approaches of solving these coupled equations is to uncouple the equations by using a technique of linear coordinate transformation. The linear transformation is obtained by assuming that the response is a superposition of the normal modes of a system multiplied by corresponding time-dependent generalized coordinates. The solutions to the uncoupled equations can be obtained by using Duhamel's integral. This analysis is called modal analysis (23,24).

D. Application of Numerical Integration Methods to a Structure Subjected to a Ground Acceleration

When a structure is excited by a ground acceleration, the motion equations of Eq. (3.19) may be expressed in terms of the following relative coordinates:

$$\{x_{s}\}_{relative} = \{x_{s}\} - \{x_{g}\}$$

$$\{x_{r}\}_{relative} = \{x_{r}\}$$

$$\{\ddot{x}_{s}\}_{relative} = \{\ddot{x}_{s}\} - \{\ddot{x}_{g}\}$$

$$\{\ddot{x}_{r}\}_{r}_{relative} = \{\ddot{x}_{r}\}$$

$$(5.25)$$

Table V Values of x and y of Example 5.1 by Direct Integration Method

Direct In	tegration Method
x (inch)	y (inch)
0.2000000E 01 0.1885653E 01	-0.4500000E 01 -0.4877422E 01
0.1745129E 01	-0.5309491E 01
0.1399307E 01	-0.6337336E 01
0.1200031E 01	-0.6933632E 01
0.9865822E 00	-0.7585610E 01
0.7610353E 00	-0.8294145E 01
0.5250612E 00	-0.9060350E 01
0.2799199E 00	-0.9885552E 01
0.2643967 = 01	-0.1077128E 02
-0.2349665E 00	-0.1171918E 02
-0.5043706E 00	-0.1273120E 02
-0.7822802E 00	-0.1380931E 02
-0.1069665E 01	-0.1495567E 02
-0.1367968E 01	-0.1617241E 02
-0.1679098E 01	-0.1746175E 02
-0.2005452E 0l	-0.1882587E 02
-0.2349898E 01	-0.2026689E 02
-0.2715786E 01	-0.2178680E 02
-0.3106950E 0 1	-0.2338741E 02
	Direct In x (inch) 0.2000000E 01 0.1885653E 01 0.1745129E 01 0.1581950E 01 0.1399307E 01 0.1200031E 01 0.9865822E 00 0.7610353E 00 0.5250612E 00 0.2799199E 00 0.2643967E-01 -0.2349665E 00 -0.5043706E 00 -0.7822802E 00 -0.1069665E 01 -0.1367968E 01 -0.1367968E 01 -0.2349898E 01 -0.2349898E 01 -0.2715786E 01 -0.3106950E 01

57

Table VI Values of x and y of Example 5.1 by Runge-Kutta Method

Time

sec.	x (inch)	y (inch)
0.0	0.200000E 01	-0.4500000E 01
0.1	0.1885633E 01	-0.4877402E 01
0.2	0.1745090E 01	-0.5309444E 01
0.3	0.1581895E 01	-0.5796010E 01
0.4	0.1399232E 01	-0.6337241E 01
0.5	0.1199939E 01	-0.9933517E 01
0.6	0.9864780E 00	-0.7585473E 01
0.7	0.7609386E 00	-0.8293986E 01
0.8	0.5249753E 00	-0.9060167E 01
0.9	0.2798458E 00	-0.9885345E 01
1.0	0.2638184E-01	-0.1077105E 02
1.1	-0.2350211E 00	-0.1171899E 02
1.2	-0.5044181E 00	-0.1273105E 02
1.3	-0.7823184E 00	-0.1380923E 02
1.4	-0.1069688E 01	-0.1495564E 02
1.5	-0.1367956E 01	-0.1617238E 02
1.6	-0.1679055E 01	-0.1746130E 02
1.7	-0.2005371E 01	-0.1882509E 02
1.8	-0.2349773E 01	-0.2026601E 02
1.9	-0.2715609E 01	-0.2178578E 02
2.0	-0.3106709E 01	-0.2338623E 02

Runge-Kutta Method


Table VII Value of x, y, z of Example 5.2 by Direct Integration Method

Time	Direct Integration	Meth	od	
sec.	x or y (inch)	z	(inch)	
0.0 0.1 0.2 0.3	0.0000000E 00 0.9983279E-02 0.3973359E-01 0.8865470E-01 0.1557643E 00).300).314).326).336	0000E 0006E 0354E 1946E 6251E	01 01 01 01
0.5	0.2397080E 00 0.3387790E 00 0.4509431E 00 0.5738719E 00).351).357).361	5290E 1613E 8241E 8622E	01 01 01
0.9 1.0 1.1	0.7049769E 00 0.8414487E 00 0.9802999E 00).369	6569E 6200E 1870E	01 01 01
1.2 1.3 1.4 1.5	0.1118392E 01 0.1252545E 01 0.1379519E 01 0.1496104E 01).390).400).411	9409E 0428E 5623E	01 01 01
1.6 1.7 1.8 1.9	0.1599156E 01 0.1685648E 01 0.1752727E 01 0.1797756E 01).425).443).464).490	9301E 5519E 7988E 0013E	01 01 01 01
2.0	0°TOTOTOTOT	1.019	1 I I I I	0 T

Table VIII Values of x, y, z of Example 5.2 by Runge-Kutta Method

Time	Runge-Kutta Met	chod
sec.	x or y (inch)	z (inch)
0.0	0.0000000E 00	0.3000000E 01
0.1	0.9983249E-02	0.3140024E 01
0.2	0.3973317E-01	0.3260397E 01
0.3	0.8865428E-01	0.3362012E 01
0.4	0.1557640E 00	0.3446340E 01
0.5	0.2397076E 00	0.3515406E 01
0.6	0.3387781E 00	0.3571754E 01
0.7	0.4509427E 00	0.3618406E 01
0.8	0.5738727E 00	0.3658814E 01
0.9	0.7049797E 00	0.3696787E 01
1.0	0.8414543E 00	0.3736449E 01
1.1	0.9803007E 00	0.3782142E 01
1.2	0.1118409E 01	0.3838374E 01
1.3	0.1252581E 01	0.3909723E 01
1.4	0.1379579E 01	0.4000764E 01
1.5	0.1496188E 01	0.4115977E 01
1.6	0.1599264E 01	0.4259674E 01
1.7	0.1685781E 01	0.4435905E 01
1.8	0.1752886E 01	0.4648388E 01
1.9	0.1797944E 01	0.4900426E 01
2.0	0.1818587E 01	0.5194835E 01



in which

 ${x_g}$ = ground displacement; and ${\ddot{x}_g}$ = ground acceleration. Substituting Eq. (5.25) into Eq. (3.19), the motion equations become

$$[M] \left\{ \begin{array}{c} \frac{\ddot{\mathbf{x}}_{\mathbf{r}}}{\ddot{\mathbf{x}}_{\mathbf{s}}} \right\}_{\mathbf{rel.}}^{+} \left([K] - (\alpha + \beta \cos \theta t) [S] \right) \left\{ \begin{array}{c} \frac{\mathbf{x}_{\mathbf{r}}}{\mathbf{x}_{\mathbf{s}}} \right\}_{\mathbf{rel.}}^{-} = -\ddot{\mathbf{x}}_{\mathbf{g}} [M] \left\{ \frac{\theta}{\mathbf{l}} \right\}$$

$$(5.26)$$

If the joint rotations are neglected, then Eq. (5.13) becomes

$$[M] {\ddot{x}_{s}}_{rel.} + [[K] - (\alpha + \beta \cos \theta t) [S]] {x_{s}}_{rel.} = -\ddot{x}_{g} [M] {1}$$
(5.27)

Example 5.3. Consider the shear building shown in Fig. 5.3 subjected to a ground acceleration $\ddot{x}_g = (-8.\pi^2 \sin 4\pi t)$ in./sec². The structure is assumed to be stationary at t=0. Find the relative displacements y_1 and y_2 .

Solution: Without considering the joint rotations, the diagrams of relative displacements and internal shears are shown in Fig. 5.4a and 5.4b, respectively. The governing differential equations of motion can be established as



 $M_2 = 177.1b - sec.^2 / in.$

Fig. 5.3 Example 5.3



(a) Relative displacements



(b) Internal Shears

Fig. 5.4 Diagrams for Example 5.3

$$\begin{pmatrix} 0.294 & 0 \\ 0 & 0.177 \end{pmatrix} \begin{cases} \ddot{y}_1 \\ \ddot{y}_2 \end{cases} + \begin{pmatrix} 113.4258 & -57.8703 \\ -57.8703 & 57.8703 \end{pmatrix} \begin{cases} y_1 \\ y_2 \end{cases}$$

$$= (-8\pi^{2}\sin 4\pi t) \begin{cases} -0.294 \\ -0.177 \end{cases}$$
 (5.28)

The solutions to Eq. (5.28) by modal matrix method are

$$y_{1} = (1.435726 \sin\omega t - 0.01522 \sin\omega t - 1.118190 \sin4\pi t) \text{ in.}$$

$$y_{2} = (2.077400 \sin\omega_{1} t + 0.01747 \sin\omega_{2} t - 1.695700 \sin4\pi t) \text{ in.}$$

in which ω_1 =10.0493 rad./sec. and ω_2 =24.7338 rad./sec.. Eq. (5.28) is also solved by Runge-Kutta method and linear acceleration method. The results obtained by using these three methods are shown in Tables IX, X, and XI. The values of y_1 and y_2 obtained by Runge-Kutta method are plotted in Fig. 5.5.



Modal Matrix Method

Time

sec.	y _l (inch)	y ₂ (inch)
0.00 0.25 0.50	0.0000000E 00 0.8464944E 00 -0.1363268E 01	0.0000000E 00 0.1220889E 01
0.75	0.1368720E 01 -0.8336927E 00 0.5519118E-03	0.1968805E 01 -0.1221648E 01 -0.1810213E-01
1.50	0.8590817E 00	0.1220779E 01
1.75	-0.1358700E 01	-0.1991364E 01
2.00	0.1373142E 01	0.1958570E 01
2.25	-0.8222809E 00	-0.1220102E 01
2.50	-0.5587449E-02	-0.3398962E-01
2.75	0.8692333E 00	0.1222534E 01
3.00	-0.1355897E 01	-0.1998659E 01
3.25	0.1374121E 01	0.1950198E 01
3.50	-0.8128962E 00	-0.1215090E 01
3.75	-0.5842257E-02	-0.4788642E-01
4.00	0.8772812E 00	0.1229915E 01
4.25	-0.1357949E 01	-0.2004099E 01
4.50	0.1373104E 01	0.1948336E 01
4.75	-0.8089312E 00	-0.1207948E 01
5.00	-0.1285170E-01	-0.5452869E-01

Time

Runge-Kutta Method

sec.	y (inch) l	y_2 (inch)
0.00	0.000000E 00	0,000000E 00
0.25	0.8465530E 00	0.1220697E 01
0.50	-0.1362926E 01	-0.1980345E 01
0.75	0.1368614E 01	0.1968013E 01
1.00	-0.8329155E 00	-0.1221237E 01
1.25	0.4706755E-03	-0.1900962E-01
1.50	0.8594314E 00	0.1220535E 01
1.75	-0.1358029E 01	-0.1991096E 01
2.00	0.1372542E 01	0.1957166E 01
2.25	-0.8211390E 00	-0.1219411E 01
2.50	-0.1396582E-02	-0.3526889E-01
2.75	0.8695287E 00	0.1223184E 01
3.00	-0.1355843E 01	-0.1997948E 01
3.25	0.1372791E 01	0.1949301E 01
3.50	-0.8122261E 00	-0.1212588E 01
3.75	-0.7707227E-02	-0.4857550E-01
4.00	0.8767487E 00	0.1231712E 01
4.25	-0.1358685E 01	-0.2002067E 01
4.50	0.1370356E 01	0.1948045E 01
4.75	-0.8087917E 00	-0.1203523E 01
5.00	-0.1628288E-01	-0.5468697E-01

Time	Linear Acceler	ation Method
sec.	y _l (inch)	y ₂ (inch)
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00 2.25 2.50 2.75 3.00 3.25 3.50 3.75 4.00 4.25 4.50 4.75	0.0000000E 00 0.8443995E 00 -0.1357155E 01 0.1362345E 01 -0.8228942E 00 -0.6071389E-02 0.8656081E 00 -0.1353907E 01 0.1364450E 01 -0.8034375E 00 -0.1523147E-01 0.8820280E 00 -0.1353487E 01 0.1362159E 01 -0.7889202E 00 -0.3058952E-01 0.8945796E 00 -0.1354414E 01 -0.7809602E 00	0.0000000E 00 0.1216910E 01 -0.1973529E 01 0.1956882E 01 -0.1209591E 01 -0.3182024E-01 0.1225681E 01 -0.1989300E 01 0.1941218E 01 -0.198106E 01 -0.1198106E 01 -0.2947738E-01 0.1237475E 01 -0.1997902E 01 -0.1997902E 01 -0.1931440E 01 -0.1180500E 01 -0.8214664E-01 0.1258290E 01 -0.2003111E 01 0.1928658E 01 -0.1157982E 01
5.00	-0.4974973E-01	-0.9619385E-01

VI. DYNAMIC RESPONSE OF ELASTIC STRUCTURAL SYSTEMS

The numerical integration techniques described in the preceding chapter will be used herein to study the instability behavior and displacement response of a structure subjected to time dependent axial forces as well as lateral forces or foundation movements. A number of selected examples given below have been studied by using digital computer programs based on the numerical integration techniques described in Chapter V.

A. Numerical Examples

Example 6.1. Consider a beam-column shown in Fig. 6.1a subjected to N_t at both ends and periodic lateral force F_t at point B. The periodic force F_t is shown in Fig. 6.1b and the axial force is $N_t = (\alpha + \beta \cos \theta t) N_0$. The member properties are

Cross sectional area: $A_{AB}=30.24 \text{ in}^2$, $A_{BC}=24. \text{ in}^2$. Member length : $L_{AB}=144. \text{ in., } L_{BC}=96. \text{ in.}$ Moment inertia : $I_{AB}=192. \text{ in}^4$, $I_{BC}=96. \text{ in}^4$. The static buckling load and natural frequency are found to be 2974.80 kips and 181.9423 rad./sec., respectively. The principal dynamic instability region for N₀=2974.80 kips,

 ω =181.9423 rad./sec. and α =0., β =0.2 is shown in Fig. 6.2. Two cases of dynamic response are investigated by using the Runge-Kutta method with time interval dt=0.004 sec.. As

indicated in Fig. 6.2, case A is for $\theta = 251.7872 \text{ rad./sec.}$ in the stability region and case B is for $\theta = 364.00 \text{ rad./sec.}$ in the principal instability region. The lateral deflections at point B corresponding to case A and case B are shown in Fig. 6.3.

Example 6.2. Consider a two-story steel framework shown in Fig. 6.4a in which the masses lumped at the floors, the length and moment inertia of the constituent members are given. The columns of the frame are subjected to time dependent axial force $N_t = (\alpha + \beta \cos \theta t) N_0$ and the base of the frame is excited by a ground acceleration $\ddot{X}_{a} = (-8\pi^{2}\sin 4\pi t)$ in./sec². After the static buckling load, $N_{0}\,,$ and natural frequency, $\omega\,,$ of the structural system have been found, the principal instability regions for $N_0=1001.626$ kips, $\omega=10.0494$ rad./sec. are investigated and the results are shown in Fig. 6.5 for various axial loads corresponding to $\alpha=0.$, 0.2, 0.4, and $\beta=0.1$, 0.2, 0.3, 0.4, 0.5. Two cases of dynamic response sketched in Fig. 6.5 have been studied in which case A is for $\alpha=0.$, $\beta=0.3$ and θ =15.0 rad./sec. in the stability region and case B is for $\alpha=0.$, $\beta=0.3$, and $\theta=20.1$ rad./sec. in the instability region. The Runge-Kutta method with time interval dt=0.025 sec. has been employed for studying the relative displacements y_1 and y_2 . The results associated with case A and case B are shown in Fig. 6.6 and Fig. 6.7. These two cases are also investigated by the linear acceleration method with time

interval dt=0.0125 sec.. The relative displacements y_1 and y_2 are shown in Fig. 6.8 and Fig. 6.9.

B. Discussion of Results

For the cases in the instability region, the deflection response grows exponentially with time. The deflection response associated with the cases in the stability region, however, is quite stable. The results obtained by the Runge-Kutta method agree satisfactorily with those obtained by the linear acceleration method.



Fig. 6.1 Example 6.1



Fig. 6.2 Dynamic Instability Region of Example 6.1





mm

(b) Relative Displacements

Fig. 6.4 Example 6.2



Fig. 6.5 Dynamic Instability Region of Example 6.2









VII. MATRIX FORMULATION FOR ELASTO-PLASTIC STRUCTURAL SYSTEMS

When the deflection of a structural framework becomes sufficiently large, the internal moments of the constituent members may exceed the elastic limit. Consequently, the elastic analysis will no longer be correct and the structure must be analyzed to include the inelastic deformation. Therefore the elementary mass, stiffness and stability matrices of a typical member must be derived to account for the deformation beyond elastic limit.

Α. Idealized Elasto-Plastic Moment-Rotation Characteristics Let us assume that the constituent members of a frame have an ideal elasto-plastic moment rotation characteristics as shown in Fig. 7.1. The typical moment-rotation diagram has a linear relationship called elastic branch which varies from zero moment to the reduced plastic moment M_{pc} . The reduced plastic moment will be evaluated to account for the effect of axial load on the plastic moment. For any further deformation, the member will have a plastic hinge at which the applied moment is M_{pC} . When the member has reverse deformation, the moment-rotation relationship becomes linear and parallel to the original elastic branch. The elastic behavior remains to be unchanged until the internal moment reaches M_{pc} . Consequently, a plastic hinge will be assumed and a constant moment will be applied at the hinge.

The cyclic process is sketched in Fig. 7.1.

B. Reduced Plastic Moment

The influence of axial force on plastic moment will be calculated according to ASCE manuals (35) as

(a) for wide-flange sections

when $0 \leq P \leq 0.15P_v$

$$M_{pc} = M_{p} = F_{y}Z$$
(7.1)

when $0.15p_{y} \leq P \leq P_{y}$

$$M_{pc} = 1.18 [1 - (P/P_y)]_{p}^{M}$$
(7.2)

(b) for rectangular section

$$M_{pc} = [1 - (P/P_y)^2]M_p$$
(7.3)

where

Z = plastic section modulus; $P_{y} = F_{y}A;$ $F_{y} = yielding stress of steel;$ A = cross sectional area; P = axial force; $M_{p} = F_{y}Z = plastic moment; and$ $M_{pc} = reduced plastic moment.$



Fig. 7.1 Idealized Moment-Rotation Relationships

C. Modified Elementary Mass, Stiffness, and Stability Matrices

An elastic analysis for dynamic response can only be carried out to the loading stage at which none of the internal moment reaches plastic moment. When an internal moment reaches plastic moment, the frame is then modified by inserting a real hinge at the location with a plastic moment applied at the hinge. Thus the mass, stiffness and stability matrices of that member must be modified according to the hinge location.

Let the typical member shown in Fig. 7.2 have a hinge at j, then the shape functions of the member are



Following the same procedure used in Chapter III, one can derive the mass, stiffness and stability matrices as

$$\left\{ \begin{array}{c} Q_{1} \\ Q_{2} \\ Q_{2} \\ Q_{3} \\ Q_{4} \end{array} \right\}_{m} = \left[\begin{array}{cccc} \frac{8mL^{3}}{420} & 0 \cdot \frac{-36mL^{2}}{420} & \frac{11mL^{2}}{280} \\ 0 \cdot & 0 \cdot & 0 \cdot & 0 \cdot \\ 0 \cdot & 0 \cdot & 0 \cdot & 0 \cdot \\ 0 \cdot & 0 \cdot & 0 \cdot & 0 \cdot \\ \frac{-36mL^{2}}{420} & 0 \cdot \frac{204mL}{420} & \frac{-39mL}{280} \\ \frac{-36mL^{2}}{420} & 0 \cdot \frac{204mL}{420} & \frac{-39mL}{280} \\ \frac{11mL^{2}}{280} & 0 \cdot \frac{-39mL}{280} & \frac{99mL}{420} \\ \frac{11mL^{2}}{280} & 0 \cdot \frac{-39mL}{280} & \frac{99mL}{420} \\ \frac{1}{4} \end{array} \right] \left\{ \begin{array}{c} \dot{\mathbf{q}}_{4} \\ \dot{\mathbf{q}}_{4} \end{array} \right\}$$



Similarly, let the typical member shown in Fig. 7.3 have a hinge at end i, then the shape functions for the boundary conditions of the member may be derived as

$$\phi_{1}(x) = 0$$

$$\phi_{2}(x) = (x^{3}/2L^{2} - x/2)$$

$$\phi_{3}(x) = (-x^{3}/2L^{3} + 3x/2L - 1)$$

$$\phi_{4}(x) = (-x^{3}/2L^{3} + 3x/2L)$$
(7.8)



Fig. 7.3 Generalized Local Coordinates and Generalized Forces of a Beam with i End Hinged

Consequently, the mass, stiffness and stability matrices become

$$\begin{cases} Q_{1} \\ Q_{2} \\ Q_{3} \\ Q_{4} \\ Q_{5} \\ Q_{5} \\ Q_{4} \\ Q_{4} \\ Q_{4} \\ Q_{5} \\ Q_{5} \\ Q_{4} \\ Q_{4} \\ Q_{5} \\ Q_{5} \\ Q_{4} \\ Q_{4} \\ Q_{5} \\ Q_{5} \\ Q_{6} \\ Q_{4} \\ Q_{4} \\ Q_{5} \\ Q_{5} \\ Q_{6} \\ Q_{4} \\ Q_{4} \\ Q_{5} \\ Q_{5} \\ Q_{5} \\ Q_{6} \\ Q_{4} \\ Q_{4} \\ Q_{5} \\ Q_{5} \\ Q_{6} \\ Q_{7} \\ Q_{1} \\ Q_{1} \\ Q_{2} \\ Q_{3} \\ Q_{4} \\ Q_{1} \\ Q_{2} \\ Q_{1} \\ Q_{2} \\ Q_{3} \\ Q_{4} \\ Q_{1} \\ Q_{2} \\ Q_{1} \\ Q_{2} \\ Q_{3} \\ Q_{4} \\ Q_{4} \\ Q_{4} \\ Q_{1} \\ Q_{2} \\ Q_{3} \\ Q_{4} \\ Q$$

If a member has both ends hinged, then the stiffness and stability matrices become null and the mass matrix is

D. System Matrix of Mass, Stiffness and Stability

Since the mass, stiffness and stability matrices of a member with one end or both ends hinged have been modified to account for the boundary conditions. Therefore the formulation of system mass, stiffness and stability matrices can be done by following the same procedure described in Section C of Chapter III.

VIII. DYNAMIC RESPONSE OF ELASTO-PLASTIC STRUCTURES

A. Transfer Matrix for Plastic Moments and Their Associated Shears

When an internal moment at the nodal point of a member is equal to or greater than the reduced plastic moment, then a plastic hinge will be assumed at the node with a constant moment M_{pc} applied at the hinge. Thus the plastic hinge will be treated as a real hinge and the member mass, stiffness, and stability matrices must be modified to satisfy the boundary conditions. If a plastic hinge forms at end i of member ij, the moment M_{pc} at that end must be carried over to end j with the magnitude of $M_{pc}f_{co}$ (f_{co} is the carry-over factor including the effect of axial force). Consequently, $M_{pc}f_{co}$ will be treated as the external moment at joint j. The shears due to M_{pc} and $M_{pc}f_{co}$ on the member ij are then transfered to the structural nodes and become the external forces.

Let {FEM}, {FEV} represent the plastic moments M_{pc} , $M_{pc}f_{co}$ and shears due to M_{pc} , $M_{pc}f_{co}$, respectively, then the transfer matrix may be expressed as

$$\{\mathrm{TF}\} = \begin{cases} \mathrm{TF}_{r} \\ \mathrm{TF}_{s} \end{cases} = \begin{cases} [\mathrm{A}_{m}] \{\mathrm{FEM}\} \\ [\mathrm{A}_{V}] \{\mathrm{FEV}\} \end{cases}$$
(8.1)

where {TF} = external load matrix transferred from plastic moments {TF_r} = $[A_m]$ {FEM}, and shears {TF_s} = $[A_V]$ {FEV}.

{TF} should be combined with the load matrix $\begin{cases} F_r \\ F_s \end{cases}$ in Eq. (3.31) for dynamic response of the elasto-plastic case.

The internal moments and shears can be evaluated from Eq. (3.32), and should be combined with moments {FEM}, and shears {FEV} for the final solution.

B. Calculation of Plastic Hinge Rotation

As discussed previously, when an internal nodal moment reaches the plastic moment capacity, a real hinge will be inserted at that node with a constant moment applied at the hinge which is allowed to rotate according to the material behavior shown in Fig. 7.1. When the hinge rotates in the direction of the plastic moment, the moment is assumed to be constant and the rotation can increase indefinitely. When the hinge rotation is in the opposite direction of the moment, however, the plastic moment will be removed and the member becomes elastic. Thus the plastic hinge rotation must be calculated at each step of numerical integrations and compared with the previous one, if any, in order to check the change of the sign of rotation. For a whole structural system, the hinge rotations may be otained as follows:

in which the first term of the right side of Eq. (8.2) is

composed of the force-deformation relationship of constituent members given in Eqs (3.29, 3.30) and the second term is due to external nodal rotations. The typical element in the first term may be derived from Eqs. (3.29, 3.30) as

$$\{q_r\} = [KMR+SMR]^{-1}\{Q_m\} + [KMY+SMY][A_v]^T\{x_s\}$$
 (8.3)

Thus the elements in Eq. (8.2) are

$$[FS] = \begin{pmatrix} [FM]_{1} \\ & [FM]_{2} \\ & & \tilde{[FM]}_{i} \\ & & & \tilde{[FM]}_{n} \end{pmatrix}$$
(8.4)

$$[FM]_{i} = \frac{1}{DET_{i}} \begin{pmatrix} \frac{4EI_{i}}{L_{i}} - \frac{2N_{i}L_{i}}{15} \end{pmatrix} - (\frac{2EI_{i}}{L_{i}} + \frac{N_{i}L_{i}}{30}) \\ - (\frac{2EI_{i}}{L_{i}} + \frac{N_{i}L_{i}}{30}) \end{pmatrix}$$
(8.5)

$$DET_{i} = \left(\frac{6EI_{i}}{L_{i}} - \frac{N_{i}L_{i}}{10}\right) \left(\frac{2EI_{i}}{L_{i}} - \frac{N_{i}L_{i}}{6}\right)$$
(8.6)


(8.7)

$$\begin{bmatrix} \mathbf{L} \end{bmatrix}_{\mathbf{i}} = \begin{pmatrix} \frac{-1}{\mathbf{L}_{\mathbf{i}}} & \frac{-1}{\mathbf{L}_{\mathbf{i}}} \\ \\ \frac{-1}{\mathbf{L}_{\mathbf{i}}} & \frac{-1}{\mathbf{L}_{\mathbf{i}}} \end{pmatrix}$$
(8.8)

Note that $\{Q_m\}$ is the vector of internal nodal moments due to nodal displacements. The subscript i denotes the number of members. The element i of the vector $\{H_r\}$ will have value only if a plastic hinge exists at node i.

C. Numerical Examples

Example 8.1 Example 6.1 is used to investigate the elasto-plastic dynamic response for $\alpha = 0.$, $\beta = 0.2$ and $\theta = 364$. rad./sec.. The deflections of point B for elastic and elasto-plastic cases are shown in Fig. 8.1.

Example 8.2 Example 6.2 is used to investigate the elasto-plastic dynamic response for $\alpha=0.$, $\beta=0.3$, and



 θ =20.1 rad./sec.. The lateral deflections of y_1 and y_2 for both elastic and elasto-plastic cases are shown in Fig. 8.2 and Fig. 8.3.

D. Discussion of Results

From these two examples, it may be observed that the behavior of elastic case is different from that of the elastoplastic case. The parametric resonance shows up clearly for the elastic case and can not be observed for the elastoplastic case. The reason is that the dynamic instability region is based on the assumption that the structure is elastic.





IX. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

A. Summary and Conclusions

An analytical method is presented for determining the behavior of dynamic instability and response of frameworks subjected to longitudinal pulsating loads and lateral dynamic forces or foundation movements. Some of the features of this work may be summarized as follows:

1. Dynamic instability criteria are discussed and formulated in relation to the magnitude of axial force, the longitudinal forcing frequency, and the transverse frequency.

2. The displacement method is employed for structural matrix formulation for which the typical member matrices of mass, stiffness, and stability are derived.

3. Eigenvalues of free vibrations and static instability are investigated in this work. The static instability analysis includes both concentrated and uniformly distributed loads.

4. The elastic and elasto-plastic frameworks are analyzed for the response of displacements, internal moments and shears due to dynamic lateral forces or ground accelerations. General considerations include bending deformation, geometric nonlinearity, the effect of girder shears on columns and the effect of axial loads on plastic moments. 5. Two numerical methods of fourth order Runge-Kutta method and the linear acceleration method are used for the solutions to nonlinear differential equations of motion. The comparison between the solutions obtained by these two methods is very satisfactory.

6. A number of selected examples are presented from which it may be observed that the deflection response corresponding to the instability region grows exponentially with time.

7. The dynamic instability analysis yields the stability and instability regions from which one may design a structure to avoid the occurrence of parametric resonance.

B. Recommendation for Future Work

 One may include the structural damping in the differential equations of motion to investigate the effect of damping on dynamic instability and response.

2. The structural material may be considered highly nonlinear in the form of Ramberg-Osgood or bilinear.

3. The static instability analysis method for distributed axial load may be applied to investigate the effect of structural self-weight on the buckling capacity of a structure.

4. The optimum design technique may be applied to parametrically excited structures with consideration of the constraints of longitudinal and transverse frequencies.

101

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APPENDIX

Computer Programs

LIST OF SYMBOLS USED IN COMPUTER PROGRAMS

AFV	= matrix relating girder shears to columns
AFP	= matrix relating vertical forces to axial force in columns
AF	= axial force in column
AM	= matrix [A _m]
AV	= matrix [A _v]
AMS	= matrix [A _{ms}]
AREA	= cross section area
A, B, C, D	<pre>= constants of K₁, K₂, K₃, K₄ for Runge-Kutta formula</pre>
ALPHA	= coefficient of axial load
BETA	= coefficient of axial load
DT	= small increment of time
FY	= yielding stress F _y
PSB	= static buckling load
PT	= time-dependent axial force
РМ	= plastic moment ZF_y
РҮ	= cross section area times F_y
NM	= number of member
NP	= number of degrees of freedom
NPR	= number of degrees of freedom in joint rota- tion
NPS	= number of degrees of freedom in side sway
NVP	= Number of vertical forces acting on columns
VA	$= \alpha$ value

VB	= β value
NPTS	= number of time steps
XL	= member length
ХМ	= mass per unit length
XI	= moment of inertia of cross section
XE	= elastic Young's modulus
х	= displacement
ХТ	= velocity
ХТТ	= acceleration
XEM	= internal end moments
XEV	= internal end shears
XXM	= system mass matrix [M]
ХХК	= system stiffness matrix [K]
XXP	= system stability matrix [S]
т	= time
ZP	= plastic modulus
ZETA	= θ value
NPH	= new plastic hinge
LPH	= old plastic hinge
NRH	= relieved plastic hinge
HR	= plastic hinge rotation $\{H_r\}$
FEM	= internal moment due to plastic moment
FEV	= internal shear due to plastic moment
COFR	= carry-over factor f _{co}

FRY1,	FRY2,	FRY3,	FRY4		element	of	matrix [FΥ]
FMVl,	FMV2,	FMV3,	FMV4	=	element	of	matrix [L]	
FRM1,	FRM2,	FRM3,	FRM4	=	element	of	matrix [FS]
PMR1,	PMR2,	PMR3,	PMR4	=	element	of	submatri	x	[SMR]
PMYl,	PMY2,	PMY3,	PMY4	=	element	of	submatri	x	[SMY]
pvrl,	PVR2,	PVR3,	PVR4	=	element	of	submatri	x	[SVR]
PVYl,	PVY2,	PVY3,	PVY4	=	element	of	submatri	x	[SVY]
SMR1,	SMR2,	SMR3,	SMR4	=	element	of	submatri	x	[KMR]
SMYl,	SMY2,	SMY3,	SMY4	=	element	of	submatri	x	[KMY]
svrl,	SVR2,	SVR3,	SVR4	=	element	of	submatri	x	[KVR]
svyl,	SVY2,	SVY3,	SVY4	=	element	of	submatri	x	[KVY]
xmrl,	XMR2,	XMR3,	XMR4	=	element	of	submatri	x	[MMR]
XMYl,	XMY2,	хмүз,	XMY4	=	element	of	submatri	x	[MMY]
xvrl,	XVR2,	XVR3,	XVR4	=	element	of	submatri	x	[MVR]
XVYl,	XVY2,	XVY3,	XVY4	=	element	of	submatri	x	[MVY]









Computer Programs

1 DIMENSION ALPHA(10), PAR(10), BETA(10), RATIO(13) 2 OIMENSION AM(12,18), AM(3,3), AV(12,18) 3 DIMENSION XL(10), XI(10), XM(10) 4 DIMENSION XK(10,10), XI(10), XM(10) 5 DIMENSION XK(10,10), XXM(10,10), XXP(10,10) 6 DIMENSION XK(10,10), XXM(10,10), XXP(10,10), SMV4(10) 7 DIMENSION XKR(10), SMR4(10), SMV4(10), SMV4(10) 8 OIMENSION XKR3(10), XMR4(10), SMV3(10), SMV4(10) 9 DIMENSION XKR3(10), XMR4(10), PMM2(10), PMV4(10) 10 DIMENSION XKR3(10), SMR4(10), PMM3(10), PMV4(10) 11 DIMENSION XKR1(10), SVR4(10), SVY1(10), SVY2(10) 12 DIMENSION XKR1(10), SVR4(10), SVY1(10), SVY2(10) 13 DIMENSION XKR1(10), SVR4(10), SVY1(10), SVY2(10) 14 DIMENSION XKR1(10), SVR4(10), SVY1(10), SVY2(10) 15 DIMENSION XKR1(10), XKR4(10), XKV1(10), SVY2(10) 16 OIMENSION XKR3(10), XKR4(10), XKV1(10), SVY2(10) 17 DIMENSION XKR3(10), XKR4(10), XKV1(10), XKV2(10) 16 DIMENSION XKR3(10), XKR4(10), XKV1(10), SVY2(10) 17 DIMENSION XKR3(10), XKR4(10), XKV1(10), XKV2(10) 18 DIMENSION XKR3(10), XKR4(10), XKV1(10), XKV1(10)		Ŧ	* ELASTIC DYNAMIC RESPONSE	¥
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2 01MENSILWA AM(12,181,AMS(3,3),AV(12,1M) 3 DI YENSILWA X(10),XXX(10),XXX(10) 4 01MENSION NPH(18) 5 DI YENSION XXX(10,10),XXM(10,10),XXY(10),SMY(10) 6 DI MENSION XXX(10,10),XXM(10,10),XXY(10),SMY(10) 7 DIMENSION XXX(10,10),XMX(10),XMY(10),SMY(10) 8 DI MENSION XXX(10),XMX(10),XMX(10),XMY(10) 9 DI MENSION XMR1(10),XMA(10),XMY(10),XMY(10) 10 DI MENSION XMR3(10),XMA(10),XMY(10),XMY(10) 11 DI MENSION YMR3(10),YMA(10),XMY(10),XMY(10) 12 DI MENSION YMR3(10),SVR4(10),XVY(10),YVY(10) 13 DI MENSION YMR3(10),YVR2(10),PVY(10),YVY(10) 14 DI MENSION YMR3(10),YVR4(10),PVY(10),YVY(10) 15 DI MENSION XVR1(10),XVR4(10),YVY(10),YVY(10) 16 DI MENSION XVR3(10),XVR4(10),YVY(10),XVY(10) 17 DI MENSION XMR3(10),XVR4(10),YVY(10),XVY(10) 18 DI MENSION XMR1(10),XVR1(10),XVY(10),YVY(10) 19 DI MENSION XX(10),XTI(10),XVT(10) 20 DI MENSION XX(10),XTI(10),XVT(10) 21 DI MENSION XX(10),XTI(10),XTI(10) 22 DI MENSION XA(10),XTI(10),XTI(10)	1		DIMENSION ALPHA(IU), PARTID), BETA(IU), RATID(IR)	
3 D1 MENSION APH(18) 4 DIMENSION ASAT(10,10),TMDEX(50) 6 DIMENSION XX(10,10),XXM(10),SMY(10),SMY(10),SMY(10) 7 DIMENSION XXX(10,10),XMZ(10),SMY(10),SMY(10) 8 D1 MENSION XXX(10,10),XMZ(10),XMY(10),XMY(10) 9 D1 MENSION XMR1(10),XMR2(10),XMY(10),XMY(10) 9 D1 MENSION XMR1(10),XMR2(10),XMY(10),XMY(10) 10 DIMENSION XMR1(10),YMR2(10),PMY3(10),PMY4(10) 11 DIMENSION YMR1(10),PMR2(10),PMY3(10),PMY4(10) 12 DIMENSION YMR1(10),YMR2(10),SVY3(10),PMY4(10) 13 DIMENSION YMR3(10),PWR2(10),SVY3(10),SVY4(10) 14 DIMENSION YWR3(10),YVR4(10),SVY3(10),PVY2(10) 15 DIMENSION YWR3(10),XVR4(10),SVY3(10),PVY2(10) 16 DIMENSION YWR3(10),XVR4(10),SVY3(10),PVY2(10) 17 DIMENSION XVR3(10),XVR4(10),XVY4(10) 18 DIMENSION XVR3(10),XVR4(10),XVY4(10) 19 DIMENSION XXR1(10),XVR4(10),XVY4(10),XVY4(10) 20 DIMENSION XXR1(10),XVR4(10),XVY4(10) 21 DIMENSION XXR1(10),XVR1(10),XVX(10),XVY4(10) 22 DIMENSION XXR1(10),XVR1(10),XVX(10),XVY4(10) 23 DIMENSION XXR1(10),XVR1(10),XVX(10),X	2		$\frac{1}{2} \frac{1}{2} \frac{1}$	
4 DIMENSION NPH(18) 5 DIMENSION XXK(10,10), XXM(10,10), XXP(10,10) 7 DIMENSION XXK(10,10), XXM(10), SVY(10), SVY2(10) 8 DIMENSION SMR1(10), SMR4(10), SMY3(10), SVY4(10) 9 DIMENSION XMR3(10), XMR4(10), XMY1(10), XVY2(10) 10 DIMENSION XMR3(10), XMR4(10), XMY1(10), XVY2(10) 11 DIMENSION XMR3(10), PMR4(10), PMY1(10), PVY2(10) 12 DIMENSION VMR3(10), PMR4(10), PMY1(10), PVY2(10) 13 DIMENSION SVR1(10), PVX2(10), PVY1(10), PVY2(10) 14 DIMENSION SVR3(10), PVX4(10), PVY2(10) 15 DIMENSION VR3(10), PVX4(10), PVY3(10), PVY2(10) 16 OIMENSION VR1(10), PVX3(10), PVY4(10) 17 DIMENSION XVR3(10), XVR4(10), XVY3(10), XVY4(10) 18 DIMENSION XXR(10), XVR4(10), XVY3(10), XVY4(10) 19 DIMENSION XXR(10), XVR4(10), XVY3(10), XVY4(10) 10 DIMENSION XXR(10), XVR(18,10), XKK(18,10) 21 DIMENSION XA(10), RXT(10), XTC(10) 22 DIMENSION XA(10), RXT(10), XTC(10) 23 DIMENSION XAC(10), XTA(10), XTC(10) 24 DIMENSION XEV(12), VYM(18,10), XTC(10) 25 DIMENSION XAC(10), FWY2(1	5		OIMENSION XL(10), XI(10), XM(10)	
DIMENSION ASAT(10,10), XNDEX(50) OIMENSION XXK(10,10), XNP(10,10), XXP(10,10) OIMENSION SNR1(10), SMR2(10), SMY1(10), SMY2(10) OIMENSION XMR1(10), XMR2(10), SMY1(10), SMY2(10) OIMENSION XMR1(10), XMR2(10), XMY1(10), XMY2(10) OIMENSION XMR1(10), PMR2(10), PMY1(10), PMY2(10) IDMENSION XMR3(10), PMR2(10), PMY3(10), PMY4(10) IDMENSION XMR3(10), PMR2(10), PMY3(10), PMY4(10) IDMENSION XMR3(10), PMR2(10), PMY3(10), PMY4(10) IDMENSION SVR3(10), SVR4(10), SVY3(10), SVY4(10) IDMENSION VR1(10), PVR2(10), PVY3(10), PVY4(10) IDMENSION VR3(10), PVR2(10), PVY3(10), PVY4(10) IDMENSION XVR3(10), PVR2(10), PVY3(10), PVY4(10) IDMENSION XVR3(10), PVR2(10), PVY3(10), PVY2(10) IDMENSION XVR3(10), PVR3(10), PVY3(10), PVY2(10) IDMENSION XVR3(10), PVR3(10), PVY3(10), PVY4(10) IDMENSION XX(10), RXT(10), RXT(10) IDMENSION XX(10), RXT(10), RXT(10) IDMENSION X1(10), FXT(10), RXT(10) IDMENSION X1(10), FXT(10), RXT(10), XTO(10) IDMENSION X1(10), FXT(10), RXT(10), FXT(10)	4		DIMENSION NPH(18)	
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7 DIMENSION SMR1(10), SMR2(10), SMY1(10), SMY2(10) 8 DIMENSION SMR1(10), SMR2(10), SMY1(10), SMY2(10) 9 DIMENSION XMR3(10), SMR4(10), SMY1(10), SMY2(10) 10 DIMENSION XMR3(10), PMR2(10), PMY3(10), SMY4(10) 11 DIMENSION SMR1(10), PMR2(10), PMY3(10), PMY4(10) 12 DIMENSION SMR1(10), SMR4(10), PMY3(10), PMY4(10) 13 DIMENSION SWR3(10), SWR4(10), SVY3(10), SVY4(10) 14 DIMENSION SWR3(10), PWR2(10), PVY3(10), SVY4(10) 15 DIMENSION SVR3(10), PVR2(10), PVY3(10), SVY4(10) 16 DIMENSION XVR3(10), PVR2(10), PVY3(10), SVY4(10) 17 DIMENSION XVR3(10), PVR3(10), PVY3(10), SVY4(10) 16 DIMENSION XVR3(10), PVR3(10), PVY3(10), SVY4(10) 17 DIMENSION XVR3(10), XVR2(10), XVY4(10) 18 DIMENSION XVR3(10), PVR3(10), PVY3(10), XVY4(10) 19 DIMENSION XMR(18,10), XVR4(10), XVY4(10) 20 DIMENSION XMR(18,10), XVR4(10), XVY4(10) 21 DIMENSION XMR(10,1, XTG(10), XTM1(10,1, XVY4(10) 22 DIMENSION XX(10), XTX(10), XVX(110), XVY4(10) 23 DIMENSION XX(10), XTX(10), XVX(110) 24 DIMENSION XX(10), XTX(10), XTT(10)	5		DIMENSION XXK(10,10),XXM(10,10),XXP(10,10)	
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10 DIMENSION XMR3(10), XMR4(10), XMY3(10), XMY4(10) 11 DIMENSION PMR1(10), PMR4(10), PMY3(10), PMY4(10) 12 DIMENSION SVR1(10), SVR2(10), SVY1(10), SVY2(10) 13 DIMENSION SVR1(10), SVR2(10), SVY1(10), SVY2(10) 14 DIMENSION SVR3(10), SVR2(10), SVY1(10), SVY2(10) 15 DIMENSION PVR3(10), PVR4(10), PVY3(10), PVY4(10) 16 OIMENSION XVR1(10), XVR2(10), PVY4(10), PVY2(10) 17 DIMENSION XVR1(10), XVR2(10), XVY1(10), PVY2(10) 18 DIMENSION XVR1(10), XVR4(10), PVY3(10), PVY4(10) 19 DIMENSION XVR1(10), XVR4(10), XVY3(10), XVY4(10) 19 DIMENSION XVR1(10), XVR4(10), XVY3(10), XVY4(10) 20 DIMENSION XVR1(10), XVR4(10), XVY3(10), XVY4(10) 21 DIMENSION XMK(16,10), XVR4(10), XVY3(10), XVY4(10) 22 DIMENSION XMK(16,10), XTC(10), XTI(10) 23 DIMENSION XA(10), RXT(10), XTC(10), XTD(10) 24 DIMENSION XAC(10), XTAC(10), XTC(10), XTD(10) 25 DIMENSION XAC(10), AFV1(0,18), AFP(10,18) 26 DIMENSION XAC(10), FMV2(10), FMV4(12) 27 DIMENSION XAC(10), XTAC(10) 28 DIMENSION XAC(10), AFP(10,18), AFP(10,18) 29 DIMENSION AFV(10), AFPP(10) </th <th>9</th> <th></th> <th>DIMENSION $XMR1(10), XMR2(10), XMY1(10), XMY2(10)$</th> <th></th>	9		DIMENSION $XMR1(10), XMR2(10), XMY1(10), XMY2(10)$	
11 DIMENSION PMR3(10), PMR2(10), PMR2(10), PMR2(10) 12 DIMENSION PMR3(10), PMR2(10), PMR2(10) 13 DIMENSION SVR1(10), SVR2(10), SVY1(10), SVY2(10) 14 DIMENSION SVR3(10), SVR2(10), SVY1(10), SVY2(10) 15 DIMENSION PVR3(10), PVR4(10), PVY3(10), PVY4(10) 16 DIMENSION PVR3(10), PVR4(10), XVY1(10), SVY2(10) 17 DIMENSION VR3(10), XVR2(10), XVY1(10), XVY2(10) 18 DIMENSION XVR3(10), XVR2(10), XVY1(10), XVY2(10) 19 DIMENSION XVR3(10), XVR2(10), XVY1(10), XVY2(10) 10 DIMENSION XVR3(10), XVR2(10), XVY1(10), XVY2(10) 20 DIMENSION XVR3(10), XVR(10), XVY1(10), XVY4(10) 21 DIMENSION XK(16,10), XVR(10), XVX1(10,10) 22 DIMENSION XX(10), RXT(10), XTT(10) 24 DIMENSION XAC(10), XTA(10), XTC(10), XTD(10) 25 DIMENSION XC(10), XTA(10), XEVM(18) 26 DIMENSION XC(10), ATAC(10) 27 DIMENSION XC(10), ATAC(10), XTC(10), XTD(10) 28 DIMENSION XEV(18), XEVM(18,10), XEVM(18) 29 DIMENSION XEV(10), AFP(10) 30 DIMENSION XEV(10), FMV2(10), FMV4(10) 31 DIMENSION XEV(10), AFP(10) 32 DIMENSION AFV(1	10		DIMENSION XMR3(10), XMR4(10), XMY3(10), XMY4(10)	
12 DIMENSION PMR3(10), PMR4(10), PMY4(10), PMY4(10) 13 DIMENSION SVR1(10), SVR2(10), SVY1(10), SVY2(10) 14 DIMENSION SVR3(10), PVR2(10), PVY3(10), SVY4(10) 15 DIMENSION PVR3(10), PVR2(10), PVY3(10), PVY2(10) 16 OIMENSION PVR3(10), PVR2(10), PVY3(10), PVY2(10) 17 DIMENSION XVR1(10), XVR2(10), XVY1(10), XVY2(10) 18 DIMENSION XVR3(10), XVR4(10), XVY3(10), XVY4(10) 19 DIMENSION XVR3(10), XVR4(10), XVY3(10), XVY4(10) 20 DIMENSION XMP(18,10), XVR4(10), XVY4(10) 21 DIMENSION XM(10,10), RXT(10), RXT1(10) 22 DIMENSION X(10), RXT(10), RXT1(10) 23 DIMENSION XA(10), XTS(10), XTC(10), XTD(10) 24 DIMENSION XAC(10), XTS(10), XTC(10), XTD(10) 25 DIMENSION XAC(10), XTS(10), XTC(10), XTD(10) 26 DIMENSION XEV(10), AFX(10), XEVM(18) 27 OIMENSION XEV(10), AFV(10), RVY(10), FMV4(10) 28 DIMENSION XEV(10), AFV(10), FMV3(10), FMV4(10) 29 DIMENSION AFUV(10), FMV2(10), FMV4(10) 30 DIMENSION AFUV(10), FMV2(10), FMV4(10) 31 DIMENSION AFUV(10), FMV2(10), FMV4(10) 32 DIMENSION AFUV(10), FMV2(10), FMV4(10)	11		DIMENSION PMR1(10), PMR2(10), PMY1(10), PMY2(10)	
13 DIMENSION SVRI(10), SVR2(10), SVY1(10), SVY2(10) 14 DIMENSION SVR3(10), SVR4(10), SVY3(10), SVY4(10) 15 DIMENSION PVR3(10), PVR2(10), PVY3(10), PVY2(10) 16 OIMENSION VR3(10), VR2(10), VVY1(10), PVY2(10) 17 DIMENSION XVR1(10), VR2(10), XVY1(10), XVY2(10) 18 DIMENSION XVR1(10), VR2(10), XVY1(10), XVY4(10) 19 DIMENSION XVR3(10), VR4(10), XVY3(10), XVY4(10) 19 DIMENSION XMP(18,10), VR4(10), XVY3(10), XVY4(10) 20 DIMENSION XMP(18,10), VR4(10), XVY3(10), XVY4(10) 21 DIMENSION XMP(18,10), XTC(10), XTM(10,10) 22 DIMENSION XX1(10), RX1(10), RXTT(10) 24 DIMENSION XA(10), XT1(10), XTC(10), XTD(10) 25 DIMENSION XA(10), XT4(10), XTC(10), XTD(10) 26 OIMENSION XEMPK(18), XEVPK(18), XEVM(18) 27 DIMENSION XEMPK(18), XEVPK(13), XEVM(18) 28 DIMENSION XEMPK(18), XEVPK(13), XEVM(18) 29 DIMENSION AFV(10), AFP(10,18), AFP(10,18), PT(10) 30 DIMENSION AFV(10), FMV2(10), FMV3(10), FMV4(10) 31 DIMENSION AFV(10), FMV2(10), FMV3(10), FMV4(12) 32 DIMENSION AFV(10), FMV2(10), FMV3(10), FMV4(12) 33 DIMENSION AFV(10), FMV2(10), FMV3(1	12		DIMENSION PMR3(10), PMR4(10), PMY3(10), PMY4(10)	
14 DIMENSION SVR3(10), SVR4(10), SVY4(10), SVY4(10) 15 DIMENSION PVR1(10), PVR2(10), PVY2(10) 16 DIMENSION PVR3(10), PVR4(10), PVY3(10), PVY2(10) 17 DIMENSION XVR3(10), XVR2(10), XVY1(10), XVY2(10) 18 DIMENSION XVR3(10), XVR4(10), XVY3(10), XVY2(10) 19 DIMENSION XVR3(10), XVR4(10), XVY3(10), XVY2(10) 19 DIMENSION XVR3(10), XVR4(10), XVY3(10), XVY2(10) 20 DIMENSION XMR(18,10), XVR4(18,10), XVK(18,10) 21 DIMENSION XMA(10), B(10), XVR1(10), TMI(10,10) 22 DIMENSION XA(10), XT3(10), XT1(10) 23 DIMENSION XA(10), XT3(10), XT1(10) 24 DIMENSION XA(10), XT4(10), XT1(10), XT1(10) 25 DIMENSION XA(10), XT4(10), XT1(10), XT0(10) 26 DIMENSION XA(10), XT4(10), XT4(10), XT0(10) 27 DIMENSION XEC(10), XT4(10), XEVM(18) 28 DIMENSION XEC(10), AFV2(10), XEVM(18) 29 DIMENSION AF(10), AFV2(10), FMV3(10), FMV4(10) 30 DIMENSION AF(10), FY(10), PY(10, ADPM(12) 31 DIMENSION AF(10), FY(10), PY(10), ADPM(12) 32 DIMENSION AF(10), FY(10), PY(10), ADPM(12) 33 DIMENSION AF(10), FMV2(10), FMV4(10) 34	13		DIMENSION SVR1(10), $SVR2(10)$, $SVY1(10)$, $SVY2(10)$	
15 DIMENSION PVR1(10),PVR2(10),PVY2(10) 16 DIMENSION PVR3(10),PVR2(10),PVY2(10) 17 DIMENSION XVR1(10),XVR2(10),XVY2(10) 18 DIMENSION XVR3(10),XVR2(10),XVY2(10) 19 DIMENSION XVR3(10),XVR2(10),XVY2(10) 19 DIMENSION XMR(18,10),XVR2(10),XVY4(10) 20 DIMENSION XMR(18,10),XVP(18,10),XVM(18,10) 21 DIMENSION XM(10),B(10),XTM(10),XXI(10,10) 22 DIMENSION XA(10),RXI(10),RXII(10) 23 DIMENSION XA(10),XTG(10),XTT(10) 24 DIMENSION XA(10),XTS(10),XTC(10),XTD(10) 25 DIMENSION XA(10),XTS(10),XTC(10),XTD(10) 26 OIMENSION XEV(18),XEVPK(18),10),XEVM(18) 27 DIMENSION XEV(10),AFP(10),18,AFP(10,18),PT(10) 28 DIMENSION AF(10),AFV(10),AFP(10,18),PT(10) 29 DIMENSION AFV(10),AFP(10),FMV3(10),FMV4(10) 30 DIMENSION AFV(10),AFPP(10) 31 DIMENSION AFV(10),AFPP(10) 32 DIMENSION AFV(10),FMV2(10),FMV4(10) 33 DIMENSION AFV(10),FMV2(10),FMV4(10) 34 DIMENSION AFV(10),FMV2(10),FMV4(10) 35 1 READ(1,4(10),FMV2(10),FMV4(10) 34 DIMENSION	14		DIMENSION SVR3(10), SVR4(10), SVY3(10), SVY4(10)	
16 OIMENSIGN PVR3(10), PVR4(10), PVV4(10), PVV4(10) 17 DIMENSIGN XVR1(10), XVR2(10), XVV1(10), XVV2(10) 18 DIMENSIGN XVR3(10), XVR4(10), XVV3(10), XVV4(10) 19 DIMENSIGN XMR(18,10), XVR4(10), XVV4(18,10) 20 DIMENSIGN XMK(18,10), XVR4(10), PVV4(10) 21 DIMENSIGN XMK(18,10), XVR4(10), PVV4(10) 22 DIMENSIGN XA(10), RXT(10), RXTT(10) 23 DIMENSIGN XA(10), XTG(10), XXTT(10) 24 DIMENSIGN XA(10), XTB(10), XCT(10), XTD(10) 25 DIMENSIGN XA(10), XTB(10), XCT(10), XTD(10) 26 OIMENSIGN XA(10), XTB(10), XCT(10), XTD(10) 27 OIMENSIGN XEV(18), XEVPK(18,10), XEVM(18) 28 DIMENSIGN XEV(10), AFPP(10) 29 DIMENSIGN AFV(10), AFPP(10) 30 DIMENSIGN AFV(10), FMV2(10), FMV3(10), FMV4(10) 31 DIMENSIGN MARIALF(18), PUM110), FMV3(10), FMV4(10) 32 DIMENSIGN AFA(10), FMV2(10), FMV3(10), FMV4(10) 33 DIMENSIGN AFA(10), FMV2(10), FMV4(10) 34 DIMENSIGN AXIALF(18), PUM110), FMV3(10), FMV4(10) 35 I READ(1,2) NO 36 IF(N0) 52,52,3 37 3 WRITE(3,1001) 38 <	15		DIMENSION $PVR1(10)$, $PVR2(10)$, $PVY1(10)$, $PVY2(10)$	
17 OIMENSION XVR1(10),XVR2(10),XVY2(10) 18 DIMENSION XVR3(10),XVR2(10),XVY2(10),XVY4(10) 19 DIMENSION XVR3(10),XVP(18,10),XVV4(18,10) 20 DIMENSION XMK(18,10),XVP(18,10),XVK(18,10) 21 DIMENSION XMK(10),RXT(10),RXTT(10),XMI(10,12) 22 DIMENSION X(10),RXT(10),RXTT(10) 23 DIMENSION X(10),RXT(10),RXTT(10) 24 DIMENSION XA(10),XB(10),XC(10),XD(10) 25 DIMENSION XA(10),XTB(10),XC(10),XD(10) 26 OIMENSION XA(10),XTB(10),XC(10),XD(10) 27 DIMENSION XEV(18),XVM(18,10),XEVM(18) 28 DIMENSION XEV(18),XVM(10,18,14,200(10),EMV4(10) 29 DIMENSION AFUV(10),AFPP(10) 30 DIMENSION AFVV(10),AFPP(10) 31 DIMENSION AFUV(10),FMV2(10),FMV4(10) 32 DIMENSION AFUV(10),FMV2(10),FMV4(10) 33 DIMENSION AFICHO,FWV(10),FMV3(10),FMV4(10) 34 DIMENSION AFICHO,FWV(10),FMV3(10),FMV4(10) 35 I READ(1,20) 36 DIMENSION AFICHO,FWV(10),FMV3(10),FMV4(10) 37 J MITE(3,1001) 38 MITE(3,1001) 39 READ(1,401) NM,NP,NPR,NPS,NVP 40	16		DIMENSION PVR3(10), PVR4(10), PVY3(10), PVY4(10)	
18 DIMENSION XVR3(10),XVV4(10),XVV4(10) 19 DIMENSION XMP(18,10),XVP(18,10),XEM(18) 20 DIMENSION XM(18,10),XMM(18,10),XVV(18,10) 21 DIMENSION A(10),B(10),C(10),D(10),XMI(10,10) 22 DIMENSION X(10),XT(10),XTT(10) 23 DIMENSION XA(10),XT(10),XTT(10) 24 DIMENSION XA(10),XTS(10),XTC(10),XTD(10) 25 DIMENSION XA(10),XTS(10),XTC(10),XTD(10) 26 DIMENSION XAC(10),XTS(10),XTC(10),XTD(10) 27 OIMENSION XAC(10),XTG(10),XTC(10),XTD(10) 28 DIMENSION XAC(10),AFV(10),XTC(10),XTD(10) 29 DIMENSION XEMPK(18),XEVPK(13),XEMM(18) 29 DIMENSION AF(10),AFV(10,18),AFP(10,18),PT(10) 30 DIMENSION AFV(10),FMV2(10),FMV3(10),FMV4(10) 31 DIMENSION AFV(10),FMV2(10),FMV3(10),FMV4(10) 32 DIMENSION ARFA(10),FY(10),PY(10),RDPM(18) 33 DIMENSION ARFA(10),FY(10),PY(10),RDPM(18) 34 DIMENSION AXIALF(18),PLIMIT(18),PEDUC(18) 35 READ(1,2) NO 36 IF(N0) 52,52,3 37 J WRITE(3,1001) 38 WRITE(3,1001) 49 READ(1,400) (XL(11),I=1,NM) <	17		DIMENSION XVR1(10), XVR2(10), XVY1(10), XVY2(10)	
19 DIMENSION XMP(18,10), XVP(18,10), XKM(18) 20 DIMENSION XMK(18,10), XMM(18,10), XVK(18,10) 21 DIMENSION X(10), R(10), C(10), C(10), XVK(18,10) 22 DIMENSION RX(10), RXT(10), RXT(10) 23 DIMENSION X(10), XT(10), XTT(10) 24 DIMENSION XA(10), XT(10), XTT(10) 25 DIMENSION XA(10), XTAC(10), XTC(10), XTD(10) 26 DIMENSION XA(10), XTAC(10), XTC(10), XTD(10) 27 DIMENSION XAC(10), XTAC(10), XTC(10), XTD(10) 28 DIMENSION XEV(18), XEVPK(13), XEMM(18) 29 DIMENSION AEMPK(18), XEVPK(13), XEMM(18) 29 DIMENSION AFPV(10), FMV2(10), FMV4(10) 30 DIMENSION AFVV(10), AFPP(10) 31 DIMENSION AFF(10), FMV2(10), FMV4(10) 32 DIMENSION ARFA(10), FMV2(10), FMV3(10), FMV4(10) 33 DIMENSION AXIALF(18), PLIMIT(18), PEDUC(18) 34 DIMENSION AXIALF(18), PLIMIT(18), PEDUC(18) 35 1 READ(1,2) NO 36 IF(NO) 52,52,3 37 3 WRITE(3,500) NO 39 READ(1,401) NM, NP, NPR, NPS, NVP 40 NEM=2*NM 41 READ(1,400)(XL(1), I=1, NM)	18		DIMENSION XVR3(10), XVR4(10), XVY3(10), XVY4(10)	
20 DIMENSION XMK(18,10),XMM(18,10),XVK(18,10) 21 DIMENSION A(10),B(10),C(10),NXMI(10,10) 22 DIMENSION X(10),RXT(10),XXT(10) 23 DIMENSION XA(10),XB(10),XC(10),XD(10) 24 DIMENSION XA(10),XB(10),XC(10),XD(10) 25 DIMENSION XA(10),XTB(10),XTC(10),XTD(10) 26 DIMENSION XTA(10),XTAC(10) 27 DIMENSION XTA(10),XTAC(10) 28 DIMENSION XEV(18),XEVM(18,10),XEVM(18) 29 DIMENSION AFU(10),AFP(10,18),AFP(10,18),PT(10) 30 DIMENSION AFU(10),AFP(10) 31 DIMENSION AFU(10),FMV2(10),FMV3(10),FMV4(10) 32 DIMENSION AFU(10),FMV2(10),FMV3(10),FMV4(10) 33 DIMENSION AREA(10),FY(10),PY(10),RDPM(12) 34 DIMENSION AREA(10),FY(10),PY(10),RDPM(12) 35 I READ(1,2) NO 36 IF(NO) 52,52,3 37 3 WRITE(3,1001) 38 WRITE(3,1001) 39 READ(1,400) (NPH(I),I=1,NEM) 42 READ(1,400) (XL (I),I=1,NM) 43 READ(1,400) (XL (I),I=1,NM) 44 READ(1,400) (XL (I),I=1,NM) 45 READ(1,400) (XP (I),I=1,NM)	19		DIMENSION XMP(18,10),XVP(18,10),XEM(18)	
21 DIMENSION A(10),B(10),C(10),O(10),XMI(10,10) 22 DIMENSION RX(10),RXT(10),RXTT(10) 23 DIMENSION X(10),RXT(10),RXTT(10) 24 DIMENSION XA(10),XB(10),XC(10),XD(10) 25 DIMENSION XA(10),XTA(10),XTC(10),XTD(10) 26 DIMENSION XAC(10),XTA(10),XTC(10),XTD(10) 27 DIMENSION XAC(10),XTAC(10) 28 DIMENSION XEV(18),XVM(18,10),XEVM(18) 29 DIMENSION XEWPK(18),XEVPK(13),XEMM(12) 29 DIMENSION AF(10),AFV(10,18),AFP(10,18),PT(10) 30 DIMENSION AF(10),FMV2(10),FMV3(10),FMV4(10) 31 DIMENSION AFFA(10),FMV2(10),FMV3(10),FMV4(10) 32 DIMENSION AFFA(10),FMV2(10),FMV3(10),FMV4(10) 33 DIMENSION AFFA(10),FMV2(10),FMV3(10),FMV4(10) 34 DIMENSION AFFA(10),FMV2(10),FMV4(10) 35 1 36 IF(NO) 52,52,3 37 3 38 WRITE(3,500) NO 39 READ(1,401) NM,NP,NPR,NPS,NVP 40 NEM=2*NM 41 READ(1,400)(XL(1),I=1,NM) 42 READ(1,400)(XL(1),I=1,NM) 43 READ(1,400)(XM(I),I=1,NM) 44	20		DIMENSION XMK(18,10),XMM(18,10),XVK(18,10)	
22 DIMENSION RX(10), RXT(10), RXTT(10) 23 DIMENSION X(10), RXT(10), XTT(10) 24 DIMENSION XA(10), XT(10), XTT(10) 25 DIMENSION XA(10), XTG(10), XTC(10), XTD(10) 26 DIMENSION XAC(10), XTA(10), XTC(10), XTD(10) 26 DIMENSION XEV(12), XTM(18,10), XEVM(18) 27 DIMENSION XEV(12), XTM(18,10), XEVM(18) 28 DIMENSION XEV(12), XTM(10), XEVM(18) 29 DIMENSION XEV(10), AFV(10,18), AFP(10,18), PT(10) 30 DIMENSION AFVV(10), AFV2(10), FMV3(10), FMV4(10) 31 DIMENSION AFVV(10), FMV2(10), FMV4(10) 32 DIMENSION AREA(10), FY(10), PY(10), FMV4(10) 33 DIMENSION AREA(10), FY(10), PY(10), FMV4(10) 34 DIMENSION AXIALF(18), PUIMIT(18), PEDUC(18) 35 1 36 IF(NO) 52,52,3 37 3 38 WRITE(3,1001) 38 WRITE(3,500) NO 39 READ(1,401) NM, NP, NPR, NPS, NVP 40 NEM=2*NM 41 READ(1,400) (XL(1), I=1, NM) 42 READ(1,400) (AREA(I), I=1, NM) 43 READ(1,400) (XL(1), I=1, NM) 44 </th <th>21</th> <th></th> <th>DIMENSION A(10), B(10), C(10), D(10), XMI(10, 10)</th> <th></th>	21		DIMENSION A(10), B(10), C(10), D(10), XMI(10, 10)	
23 DIMENSIGN X(10),XT(10),XTT(10) 24 DIMENSION XA(10),XB(10),XC(10),XD(10) 25 DIMENSION XA(10),XTB(10),XTC(10),XTD(10) 26 DIMENSION XAC(10),XTAC(10) 27 DIMENSION XEV(12),XVM(18,10),XEVM(18) 28 DIMENSION XEV(12),XVM(18,10),XEVM(18) 29 DIMENSION XEMPK(18),XEVPK(13),XEVM(18) 29 DIMENSION AF(10),AFP(10,18),AFP(10,18),PT(10) 30 DIMENSION AFVV(10),AFPV(10),AFPV(10),EMV4(10) 31 DIMENSION AFA(10),FY(10),PY(10),FMV4(10) 32 DIMENSION AREA(10),FY(10),PY(10),RDPM(18) 33 DIMENSION AXIALF(18),PLIMIT(18),PEDUC(18) 34 DIMENSION AXIALF(18),PLIMIT(18),PEDUC(18) 35 1 READ(1,2) NO 36 IF(NO) 52,52,3 37 3 WRITE(3,1001) 38 WRITE(3,1001) 39 READ(1,401) NM,NP,NPR,NPS,NVP 40 NEM=2*NM 41 READ(1,400) (XL(I),I=1,NM) 42 READ(1,400) (XL(I),I=1,NM) 43 READ(1,400) (XI(I),I=1,NM) 44 READ(1,400) (XM(I),I=1,NM) 45 READ(1,400) (XI(I),I=1,NM)	22		DIMENSION RX(10),RXT(10),RXTT(10)	
24 DIMENSION XA(10),XB(10),XC(10),XD(10) 25 DIMENSION XTA(10),XTB(10),XTC(10),XTD(10) 26 DIMENSION XAC(10),XTAC(10) 27 DIMENSION XEV(12),XVM(18,10),XEVM(18) 28 DIMENSION XEV(12),XVM(18,10),XEVM(18) 29 DIMENSION AF(10),AFV(10,18),AFP(10,18),PT(10) 30 DIMENSION AF(10),FMV2(10),FMV3(10),FMV4(10) 31 DIMENSION AFV(10),FMV2(10),FMV3(10),FMV4(10) 32 DIMENSION AFV(10),FMV2(10),FMV3(10),FMV4(10) 33 DIMENSION AFV(10),FMV2(10),FMV3(10),FMV4(10) 34 DIMENSION AFFA(10),FY(10),FMV3(10),FMV4(10) 35 INFORMATION AFFA(10),FY(10),FMV3(10),FMV4(10) 36 IF(N0) S2,52,3 37 3 36 IF(N0) 52,52,3 37 3 38 WRITE(3,1001) 39 READ(1,401) NM,NP,NPR,NPS,NVP 40 NEM=2*NM 41 READ(1,400) (XL(1),I=1,NM) 42 READ(1,400) (XL(1),I=1,NM) 43 READ(1,400) (XM(I),I=1,NM) 44 READ(1,400) (XM(I),I=1,NM) 45 READ(1,400) (ZP(I),I=1,NM) 46 READ(1,400) (ZP(I),I=1,NM) <th>23</th> <th></th> <th>DIMENSION X(10),XT(10),XTT(10)</th> <th></th>	23		DIMENSION X(10),XT(10),XTT(10)	
25 DIMENSION XTA(10), XTB(10), XTC(10), XTD(10) 26 DIMENSION XAC(10), XTAC(10) 27 DIMENSION XEV(12), XVM(18,10), XEVM(18) 28 DIMENSION XEMPK(18), XEVPK(13), XEMM(12) 29 DIMENSION AF(10), AFV(10,18), AFP(10,18), PT(10) 30 DIMENSION AFVV(10), AFPP(10) 31 DIMENSION FMV1(10), FMV2(10), FMV3(10), FMV4(10) 32 DIMENSION AFV(10), FMV2(10), FMV3(10), FMV4(10) 33 DIMENSION AFA(10), FMV2(10), FMV4(10) 34 DIMENSION AREA(10), FY(10), PY(10), RDPM(12) 35 I 36 IF(N0) 52,52,3 37 3 38 WRITE(3,1001) 39 READ(1,401) NM, NP, NPR, NPS, NVP 40 NEM=2*NM 41 READ(1,400) (NPH(I),I=1,NM) 42 READ(1,400) (XL(1),I=1,NM) 43 READ(1,400) (XK(I),I=1,NM) 44 READ(1,400) (XM(I),I=1,NM) 45 READ(1,400) (XM(I),I=1,NM) 46 READ(1,400) (ZP(I),I=1,NM) 47 READ(1,400) (ZP(I),I=1,NM) 48 READ(1,400) (APHA(I),I=1,NM) 49 READ(1,400) (APHA(I),I=1,NM) <th>24</th> <th></th> <th>DIMENSION XA(10), XB(10), XC(10), XD(10)</th> <th></th>	24		DIMENSION XA(10), XB(10), XC(10), XD(10)	
26 DIMENSION XAC(10),XTAC(10) 27 DIMENSION XEV(18),XVM(18,10),XEVM(18) 28 DIMENSION XEMPK(18),XEVPK(13),XEMM(18) 29 DIMENSION AF(10),AFV(10,18),AFP(10,18),PT(10) 30 DIMENSION AF(10),AFV(10),AFP(10) 31 DIMENSION AFV(10),FMV2(10),FMV3(10),FMV4(10) 32 DIMENSION AFV(10),FMV2(10),FMV3(10),FMV4(10) 33 DIMENSION AFV(10),FY(10),PY(10),RDPM(18) 34 DIMENSION AREA(10),FY(10),PY(10),RDPM(18) 35 I READ(1,2) NO 36 IF(NO) 52,52,3 37 J WRITE(3,1001) 38 WRITE(3,500) NO 39 READ(1,401) NM,NP,NPR,NPS,NVP 40 NEM=2*NM 41 READ(1,400) (XL(1),I=1,NM) 42 READ(1,400) (XL(1),I=1,NM) 43 READ(1,400) (XX(1),I=1,NM) 44 READ(1,400) (XM(I),I=1,NM) 45 READ(1,400) (XM(I),I=1,NM) 46 READ(1,400) (XM(I),I=1,NM) 47 READ(1,400) (APHA(I),I=1,NM) 48 READ(1,400) (BETA(I),I=1,NM) 49 READ(1,400) (BETA(I),I=1,NM) 49 READ(1,400) (BETA(I),I=1,NM)	25		DIMENSION XTA(10),XTB(10),XTC(10),XTD(10)	
27 DIMENSION XEV(18), XVM(18,10), XEVM(18) 28 DIMENSION XEMPK(18), XEVPK(13), XEMM(18) 29 DIMENSION AF(10), AFV(10,18), AFP(10,18), PT(10) 30 DIMENSION AF(10), AFP(10,18), AFP(10,18), PT(10) 31 DIMENSION AF(10), FMV2(10), FMV3(10), FMV4(10) 32 DIMENSION AFA(10), FMV2(10), FMV3(10), FMV4(10) 33 DIMENSION FMV1(10), FMV2(10), FMV4(10) 34 DIMENSION AFA(10), FY(10), PY(10), RDPM(18) 35 I READ(1,2) NO 36 IF(NO) 52,52,3 37 3 WRITE(3,1001) 38 WRITE(3,500) NO 39 READ(1,401) NM,NP,NPR,NPS,NVP 40 NEM=2*NM 41 READ(1,400) (XL(I),I=1,NM) 42 READ(1,400) (XL(I),I=1,NM) 43 READ(1,400) (XL(I),I=1,NM) 44 READ(1,400) (XM(I),I=1,NM) 45 READ(1,400) (XM(I),I=1,NM) 46 READ(1,400) (XL(I),I=1,NM) 47 READ(1,400) (ALPHA(I),I=1,NM) 48 READ(1,400) (BETA(I),I=1,NM) 49 READ(1,400) (BETA(I),I=1,NM) 49 READ(1,400) (BETA(I),I=1,NM) 49 READ(1,601)	26		DIMENSION XAC(10),XTAC(10)	
28 DIMENSION XEMPK(18), XEVPK(13), XEMM(18) 29 DIMENSION AF(10), AFV(10), AFP(10,18), AFP(10,18), PT(10) 30 DIMENSION AF(10), AFV(10), FMV3(10), FMV4(10) 31 DIMENSION AFEA(10), FY(10), FMV3(10), FMV4(10) 32 DIMENSION AREA(10), FY(10), PY(10), RDPM(18) 33 DIMENSION AREA(10), FY(10), PY(10), RDPM(18) 34 DIMENSION AREA(10), FY(10), PY(10), RDPM(18) 35 1 36 IF(NO) 52, 52, 3 37 3 38 WRITE(3, 1001) 38 WRITE(3, 500) NO 39 READ(1, 401) NM, NP, NPR, NPS, NVP 40 NEM=2*NM 41 READ(1, 400) (NPH(I), I=1, NM) 42 READ(1, 400) (XL(I), I=1, NM) 43 READ(1, 400) (XK(I), I=1, NM) 44 READ(1, 400) (XM(I), I=1, NM) 45 READ(1, 400) (XM(I), I=1, NM) 46 READ(1, 400) (YI) (I), I=1, NM) 47 READ(1, 400) (YI) (I), I=1, NM) 48 READ(1, 400) (ALPHA(I), I=1, NM) 49 READ(1, 400) (BETA(I), I=1, NM) 49 READ(1, 400) (BETA(I), I=1, NM) 50 READ(1, 601)	27		DIMENSION XEV(18), XVM(18,10), XEVM(18)	
27 DIMENSION AF(10), AFV(10,18), AFP(10,18), PT(10) 30 DIMENSION AFVV(10), AFPP(10) 31 DIMENSION AFVV(10), FMV2(10), FMV3(10), FMV4(10) 32 DIMENSION ARFA(10), FY(10), PY(10), RDPM(18) 33 DIMENSION ARFA(10), FY(10), PY(10), RDPM(18) 34 DIMENSION ARFA(10), FY(10), PY(10), RDPM(18) 35 IRENSION ARFA(10), FY(10), PY(10), RDPM(18) 34 DIMENSION ARFA(10), FY(10), PY(10), RDPM(18) 35 IRENSION ARFA(10), FY(10), PY(10), RDPM(18) 36 DIMENSION ARFA(10), FY(10), PY(10), RDPM(18) 35 IRENSION ARFA(10), FY(10), PY(10), RDPM(18) 36 DIMENSION ARFA(10), FY(10), PY(10), RDPM(18) 37 JIMENSION AXIALF(18), PH(18), ZP(10) 36 IF(NO) 52,52,3 37 JRITE(3,1001) 38 WRITE(3,500) NO 39 READ(1,401) NM, NP, NPR, NPS, NVP 40 NEM=2*NM 41 READ(1,400) (XL(1),I=1,NM) 42 READ(1,400) (XL(1),I=1,NM) 43 READ(1,400) (XI(1),I=1,NM) 44 READ(1,400) (ZP(1),I=1,NM) 45 READ(1,400) (ALPHA(I),I=1,NM) 46 READ(1,40	28		DIMENSION XEMPK(18), XEVPK(18), XEMM(18)	
30 DIMENSION AFVV(10), AFPP(10) 31 DIMENSION FMV1(10), FMV2(10), FMV3(10), FMV4(10) 32 DIMENSION ARFA(10), FY(10), PY(10), RDPM(12) 33 DIMENSION ARFA(10), FY(10), PY(10), RDPM(12) 34 DIMENSION ARFA(10), FY(10), PY(10), RDPM(12) 35 DIMENSION ARFA(10), FY(10), PY(10), RDPM(12) 34 DIMENSION ARFA(10), FY(10), PY(10), RDPM(12) 35 I. READ(1, 2) NO 36 IF(NO) 52, 52, 3 37 BRITE(3, 1001) 38 WRITE(3, 500) NO 39 READ(1, 401) NM, NP, NPR, NPS, NVP 40 NEM=2*NM 41 READ(1, 400) (NPH(I), I=1, NM) 42 READ(1, 400) (AREA(I), I=1, NM) 43 READ(1, 400) (AREA(I), I=1, NM) 44 READ(1, 400) (XI(I), I=1, NM) 45 READ(1, 400) (XM(I), I=1, NM) 46 READ(1, 400) (XP(I), I=1, NM) 47 READ(1, 400) (ALPHA(I), I=1, NM) 48 READ(1, 400) (ALPHA(I), I=1, NM) 49 READ(1, 400) (BETA(I), I=1, NM) 49 READ(1, 400) (BETA(I), I=1, NM) 50 READ(1, 601) PSB, XE 51	27		DIMENSION AF(10), AFV(10,18), AFP(10,18), PT(10)	
31 DIMENSION FMV1(10), FMV2(10), FMV3(10), FMV4(10) 32 DIMENSION AREA(10), FY(10), PY(10), RDPM(18) 33 DIMENSION AEEA(10), FY(10), PY(10), RDPM(18) 34 DIMENSION AEEA(10), FY(10), PY(10), RDPM(18) 35 DIMENSION AXIALF(18), PEIMIT(18), PEDUC(18) 35 1 36 IF(N0) 52, 52, 3 37 3 38 WRITE(3, 1001) 38 WRITE(3, 500) NO 39 READ(1, 401) NM, NP, NPR, NPS, NVP 40 NEM=2*NM 41 READ(1, 400) (NPH(I), I=1, NM) 42 READ(1, 400) (XL(I), I=1, NM) 43 READ(1, 400) (AREA(I), I=1, NM) 44 READ(1, 400) (XM(I), I=1, NM) 45 READ(1, 400) (ZP(I), I=1, NM) 46 READ(1, 400) (ZP(I), I=1, NM) 47 READ(1, 400) (ALPHA(I), I=1, NM) 48 READ(1, 400) (ALPHA(I), I=1, NM) 49 READ(1, 400) (BETA(I), I=1, NM) 49 READ(1, 400) (BETA(I), I=1, NM) 50 READ(1, 601) PSB, XE 51 READ(1, 400) VA, VB, ZETA	30		DIMENSION AFVV(10), AFPP(10)	
32 DIMENSION AREA(10), FY(10), PY(10), RDPM(18) 33 DIMENSION EDPM(18), PM(18), ZP(10) 34 DIMENSION AXIALF(18), PLIMIT(18), PEDUC(18) 35 1 36 IF(N0) 52,52,3 37 3 38 WRITE(3,1001) 38 WRITE(3,500) NO 39 READ(1,401) NM,NP,NPR,NPS,NVP 40 NEM=2*NM 41 READ(1,1009) (NPH(I),I=1,NM) 42 READ(1,400)(XL(I),I=1,NM) 43 READ(1,400)(XREA(I),I=1,NM) 44 READ(1,400)(XM(I),I=1,NM) 45 READ(1,400)(ZP(I),I=1,NM) 46 READ(1,400)(ZP(I),I=1,NM) 47 RFAD(1,400)(ALPHA(I),I=1,NM) 48 READ(1,400)(BETA(I),I=1,NM) 49 READ(1,601) PSB,XE 50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	31		DIMENSION FMV1(10), FMV2(10), FMV3(10), FMV4(10)	
33 DIMENSION EDPM(18), PM(18), ZP(10) 34 DIMENSION AXIALF(18), PLIMIT(18), PEDUC(18) 35 1 READ(1,2) NO 36 IF(NO) 52,52,3 37 3 WRITE(3,1001) 38 WRITE(3,500) NO 39 READ(1,401) NM, NP, NPR, NPS, NVP 40 NEM=2*NM 41 READ(1,1009) (NPH(I),I=1,NEM) 42 READ(1,400)(XL(I),I=1,NM) 43 READ(1,400)(AREA(I),I=1,NM) 44 READ(1,400)(XM(I),I=1,NM) 45 READ(1,400)(ZP(I),I=1,NM) 46 READ(1,400)(ZP(I),I=1,NM) 47 READ(1,400)(ALPHA(I),I=1,NM) 48 READ(1,400)(ALPHA(I),I=1,NM) 49 READ(1,400)(BETA(I),I=1,NM) 49 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	32		DIMENSION AREA(10), FY(10), PY(10), RDPM(18)	
34 DIMENSION AXIALF(18),PLIMIT(18),PEDUC(18) 35 1 READ(1,2) NO 36 IF(NO) 52,52,3 37 3 WRITE(3,1001) 38 WRITE(3,500) NO 39 READ(1,401) NM,NP,NPR,NPS,NVP 40 NEM=2*NM 41 READ(1,1009) (NPH(I),I=1,NEM) 42 READ(1,400)(XL(I),I=1,NM) 43 READ(1,400)(AREA(I),I=1,NM) 44 READ(1,400)(XI(I),I=1,NM) 45 READ(1,400)(XM(I),I=1,NM) 46 READ(1,400)(ZP(I),I=1,NM) 47 RFAD(1,400)(FY(I),I=1,NM) 48 READ(1,400)(BETA(I),I=1,NM) 49 READ(1,400)(BETA(I),I=1,NM) 50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	33		DIMENSION EDPM(18), PM(18), ZP(10)	
35 1 READ(1,2) NO 36 IF(NO) 52,52,3 37 3 WRITE(3,1001) 38 WRITE(3,500) NO 39 READ(1,401) NM,NP,NPR,NPS,NVP 40 NEM=2*NM 41 READ(1,1009) (NPH(I),I=1,NEM) 42 READ(1,400)(XL(I),I=1,NM) 43 READ(1,400)(AREA(I),I=1,NM) 44 READ(1,400)(XI(I),I=1,NM) 45 READ(1,400)(XM(I),I=1,NM) 46 READ(1,400)(ZP(I),I=1,NM) 47 RFAD(1,400)(FY(I),I=1,NM) 48 READ(1,400)(ALPHA(I),I=1,NM) 49 READ(1,400)(BETA(I),I=1,NM) 50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	34		DIMENSION AXIALF(18), PLIMIT(18), REDUC(18)	
36 IF(NO) 52,52,3 37 3 WRITE(3,1001) 38 WRITE(3,500) NO 39 READ(1,401) NM,NP,NPR,NPS,NVP 40 NEM=2*NM 41 READ(1,1009) (NPH(I),I=1,NEM) 42 READ(1,400)(XL(I),I=1,NM) 43 READ(1,400)(AREA(I),I=1,NM) 44 READ(1,400)(XI(I),I=1,NM) 45 READ(1,400)(XM(I),I=1,NM) 46 READ(1,400)(ZP(I),I=1,NM) 47 RFAD(1,400)(FY(I),I=1,NM) 48 READ(1,400)(ALPHA(I),I=1,NM) 49 READ(1,400)(BETA(I),I=1,NM) 50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	35	1	READ(1,2) NO	
37 3 WRITE(3,1001) 38 WRITE(3,500) NO 39 READ(1,401) NM,NP,NPR,NPS,NVP 40 NEM=2*NM 41 READ(1,1009) (NPH(I),I=1,NEM) 42 READ(1,400)(XL(I),I=1,NM) 43 READ(1,400)(AREA(I),I=1,NM) 44 READ(1,400)(XI(I),I=1,NM) 45 READ(1,400)(XM(I),I=1,NM) 46 READ(1,400)(ZP(I),I=1,NM) 47 RFAD(1,400)(FY(I),I=1,NM) 48 READ(1,400)(BETA(I),I=1,NM) 49 READ(1,400)(BETA(I),I=1,NM) 50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	36		IF(NO) 52,52,3	
38 WRITE(3,500) NO 39 READ(1,401) NM,NP,NPR,NPS,NVP 40 NEM=2*NM 41 READ(1,1009) (NPH(I),I=1,NEM) 42 READ(1,400)(XL(I),I=1,NM) 43 READ(1,400)(AREA(I),I=1,NM) 44 READ(1,400)(XI(I),I=1,NM) 45 READ(1,400)(XM(I),I=1,NM) 46 READ(1,400)(ZP(I),I=1,NM) 47 READ(1,400)(FY(I),I=1,NM) 48 READ(1,400)(BETA(I),I=1,NM) 49 READ(1,400)(BETA(I),I=1,NM) 50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	37	3	WRITE(3,1001)	
39 READ(1,401) NM,NP,NPR,NPS,NVP 40 NEM=2*NM 41 READ(1,1009) (NPH(I),I=1,NEM) 42 READ(1,400)(XL(I),I=1,NM) 43 READ(1,400)(XI(I),I=1,NM) 44 READ(1,400)(XM(I),I=1,NM) 45 READ(1,400)(ZP(I),I=1,NM) 46 READ(1,400)(FY(I),I=1,NM) 47 RFAD(1,400)(FY(I),I=1,NM) 48 READ(1,400)(BETA(I),I=1,NM) 49 READ(1,400)(BETA(I),I=1,NM) 50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	38		WRITE(3,500) NO	
40 NEM=2*NM 41 READ(1,1009) (NPH(I),I=1,NEM) 42 READ(1,400)(XL(I),I=1,NM) 43 READ(1,400)(AREA(I),I=1,NM) 44 READ(1,400)(XM(I),I=1,NM) 45 READ(1,400)(ZP(I),I=1,NM) 46 READ(1,400)(FY(I),I=1,NM) 47 RFAD(1,400)(FY(I),I=1,NM) 48 READ(1,400)(ALPHA(I),I=1,NM) 49 READ(1,400)(BETA(I),I=1,NM) 50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	37		READ(1,401) NM,NP,NPR,NPS,NVP	
41 READ(1,1009) (NPH(I),I=1,NEM) 42 READ(1,400)(XL(I),I=1,NM) 43 READ(1,400)(AREA(I),I=1,NM) 44 READ(1,400)(XI(I),I=1,NM) 45 READ(1,400)(ZP(I),I=1,NM) 46 READ(1,400)(FY(I),I=1,NM) 47 READ(1,400)(FY(I),I=1,NM) 48 READ(1,400)(ALPHA(I),I=1,NM) 49 READ(1,400)(BETA(I),I=1,NM) 50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	40		NEM=2*NM	
42 READ(1,400)(XL(I),I=1,NM) 43 READ(1,400)(AREA(I),I=1,NM) 44 READ(1,400)(XI(I),I=1,NM) 45 READ(1,400)(ZP(I),I=1,NM) 46 READ(1,400)(FY(I),I=1,NM) 47 RFAD(1,400)(FY(I),I=1,NM) 48 READ(1,400)(ALPHA(I),I=1,NM) 49 READ(1,400)(BETA(I),I=1,NM) 50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	41		READ(1, 1009) (NPH(I), I = 1, NEM)	
43 READ(1,400) (AREA(I),I=1,NM) 44 READ(1,400) (XI(I),I=1,NM) 45 READ(1,400) (ZM(I),I=1,NM) 46 READ(1,400) (ZP(I),I=1,NM) 47 READ(1,400) (FY(I),I=1,NM) 48 READ(1,400) (ALPHA(I),I=1,NM) 49 READ(1,400) (BETA(I),I=1,NM) 50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	42		READ(1, 400)(XL(I), I=1, NM)	
44 READ(1,400) (XI(I),I=1,NM) 45 READ(1,400) (XM(I),I=1,NM) 46 READ(1,400) (ZP(I),I=1,NM) 47 READ(1,400) (FY(I),I=1,NM) 48 READ(1,400) (ALPHA(I),I=1,NM) 49 READ(1,400) (BETA(I),I=1,NM) 50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	43		READ(1, 400)(AREA(I), I=1, NM)	
45 READ(1,400) (XM(I),I=1,NM) 46 READ(1,400) (ZP(I),I=1,NM) 47 READ(1,400) (FY(I),I=1,NM) 48 READ(1,400) (ALPHA(I),I=1,NM) 49 READ(1,400) (BETA(I),I=1,NM) 50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	44		READ(1,400)(XI(I),I=1,NM)	
46 READ(1,400)(ZP(I),I=1,NM) 47 RFAD(1,400)(FY(I),I=1,NM) 48 READ(1,400)(ALPHA(I),I=1,NM) 49 READ(1,400)(BETA(I),I=1,NM) 50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	45		READ(1,400)(XM(I),I=1,NM)	
47 RFAD(1,400)(FY(I),I=1,NM) 48 READ(1,400)(ALPHA(I),I=1,NM) 49 READ(1,400)(BETA(I),I=1,NM) 50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	45		READ(1, 400)(ZP(I), I=1, NM)	
48 READ(1,400)(ALPHA(I),I=1,NM) 49 READ(1,400)(BETA(I),I=1,NM) 50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	47		RFAD(1,400)(FY(I),I=1,NM)	
49 READ(1,400)(BETA(I),I=1,NM) 50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	48		READ(1, 400)(ALPHA(I), I=1, NM)	
50 READ(1,601) PSB,XE 51 READ(1,400) VA,VB,ZETA	49		READ(1,400)(BETA(I), $I=1, NM$)	
51 READ(1,400) VA,VB,ZETA	50		READ(1,601) PSB,XE	
	51		READ(1,400) VA, VB, ZETA	

52		WRITE(3,700)
53	×	WRITE(3,701)(XL(I),I=1,NM)
54		WRITE(3,702)
55		WRITE(3,701)(XI(I),I=1,NM)
50		WRITE(3,703)
57		WRITE(3,701)(XM(I),I=1,NM)
58		WRITE(3,704)
59		WRITE(3,701)($ALPHA(I)$, I=1, NM)
60		WRITE(3,705)
61		WRITE(3,701)(BETA(I),I=1,NM)
62		WRITE(3,706)
63		WRITE(3,701) PSB,XE
64		WRITE(3,4321) VA,VB,ZETA
65		WRITE(3,3348)
66		DC 3346 $I=1,NM$
67		PY(I)=AREA(I)*FY(I)
68		$PM(I) = ZP(I) \neq FY(I)$
69		WRITE(3,3347) I,AREA(I),FY(I),PY(I),ZP(I),
		6PM(I)
70	3346	CONTINUE
. 71		DD 402 I=1,NPR
72		DO 402 J=1,NEM
73	402	AM(I,J)=0.
74		DO 407 I=1,NPS
75		DO 407 J=1,NEM
75	407	AV(I,J)=0.
77		DO 414 $I=1,NM$
78		DO 414 J=1,NEM
79	414	AFV(I,J)=0.
80		DO 415 I=1,NM
81		DO 415 J=1,NVP
82	415	AFP(I, J)=0.
83	406	READ(1,403) I,J,AMIJ
84		IF(I) 404,404,405
85	405	AM(I, J) = AMIJ
86		GO TO 406
87	404	READ(1,403) I, J, AVIJ
88		IF(I) 408,408,409
89	409	AV(I, J) = AVIJ
90	· ·	GO TO 404
91	408	DO 410 I=1, NPS
92		DO 410 J=1, NPS
93	410	AMS(I, J)=0.
94	413	READ(1,403) I, J, AMSIJ
95		IF(I) 411,411,412
96	412	AMS(I, J) = AMSIJ
97		GO TO 413
98	411	READ(1,403) I, J, AFVIJ
99		IF(I) 417,417,416
100	416	AFV(I, J) = AFVIJ
101		GO TO 411
102	417	READ(1,403) I,J,AFPIJ

103	IF(I) 418,419,419
104	419 AFP(I,J)=AFPIJ
105	GO TO 417
105	418 WRITE(3,650)
107	WRITE(3,603) ((AM(I,J),J=1,NEM),I=1,NPR)
108	WRITE(3,651)
1 29	WRITE(3.603) ((AV(I.J).J=1.NEM).I=1.NPS)
110	WRITE(3.652)
111	WRITE(3.633)((AMS(I.J).J=1.NPS).I=1.NPS)
112	WQITE(3,653)
113	WRITE(3,603)((AFV(I,J),J=1,NEM),I=1,NM)
114	WRITE(3,654)
115	WRITE(3,603)((AFP(I,J),J=L,NVP),I=L,NM)
	C FORMULATE MASS & STIFF. MATRIX
116	DO 1000 $I = 1 \cdot NM$
117	MN=I
118	CALL STIFFA(SMR1.SMR2.SMR3.SMR4.
	65MY1 . 5MY2 . 5MY 3 . 5MY4 . 5VR1 . 5VR2 . 5VR3 . 5VR4 .
	£\$VY1.\$VY2.\$VY3.\$VY4.XMR1.XMR2.XMR3.XMR4.
	£XMY1, XMY2, XMY3, XMY4, XVR1, XVR2, XVR3, XVR4,
	£XVY1,XVY2,XVY3,XVY4,MN,XF,X1,X1,XM)
119	1000 CONTINUE
120	CALL ASATA(NPR.NPS.NM.AM.AV.
	ESMEL SMEZ SMEZ SMEZ SMEL SMEL SMEL SMEL SMEL SMEL
	5 VR1 - SVR2 - SVR3 - SVR4 - SVY1 - SVY2 - SVY3 - SVY4 - XXK
121	
1 4 1	EXMPL XMP2 XMP3 XMP4 XMY1 XMY2 XMY3 XMY4
	f X V R I = X V R 2 = X V R 3 = X V R 4 = X V Y 1 = X V Y 2 = X V Y 3 = X V Y 4 =
122	CALL ASATM(NP-XXM-XMT)
166	
123	CALL SATEVINER NPS.NM.SMR1.SMR2.SMR3.
123	SCMP4. SMY1. SMY2. SMY3. SMY4. AM. AV. XMK)
124	CALL SATEVINER NPS.NM.SVR1.SVR2.SVR3.
124	ESVR4-SVY1-SVY2-SVY3-SVY4-AM-AV-XVK)
125	CALL SAT MV (NPR .NPS.NM. XMR1. XMR2. XMR3.
123	SYMPA- XMY1 - XMY2 - XMY3 - XMY4 - AM - AV - XMM)
126	CALL SAT MV (NPR .NPS .NM .XVR1 .XVR2 .XVR3 .
120	EXVR4-XVY1-XVY2-XVY3-XVY4-AM-AV-XVM)
	C DEFINE THE INITIAL CONDITION
1 2 7	PEAD(1, 900)(X(T), I=1,NP)
129	READ(1,900)(XT(1),I=1,NP)
120	PEAD(1, 900)(XTT(1), 1=1, NP)
129	$N_{1} = 1 \cdot N_{1} = N_{1} = 1 \cdot N_{1} = $
1 2 1	
121	
122	
133	1 = 0
134	NP15=201
132	WKIIE(3)70171
130	UU YUUU I=IYNM UU YUUU I=IYNM
137	MKT16(2+402) VIII+VIIII

138	9000	CONTINUE
139		DT=0.002
140		DO 9999 KK=2,NPTS
141		DO 930 I=1.NP
142		RX(I) = X(I)
143		RXT(I) = XT(I)
144		RXTT(I) = XTT(I)
145	930	CONTINUE
146		BT=T
	c	
147	C	7T=7FT A*T
149		CT = COS(TT)
140		$\frac{1}{1} = \frac{1}{1} = \frac{1}$
150		
151	455	
152	055	
152		
155		
174		DU ODO J = L NCM
100		AFVV(1)=AFVV(1)+AFV(1,3)+XEV(3)
155	000	
157		DU 667 1=1,NM
158		
109		UU CO (J=1)NVP
150		
171	001	
102		$DU 008 I=L_{1}NM$
100		AF(1) = AF(1) + AF(1)
104	008	
100		U = 1 = 1 + NM
155		
157		
	e e	APMY1, PMY2, PMY3, PMY4, PVR1, PVR2, PVR3, PVR4,
		LMN , XL , AF)
158	1100	CUNTINUE ACATACINOD NOS NO AN AV
159		CALL ASALA(NPK, NPS, NM, AM, AV, NPS, NM, AM, AV, NPS, NM, AM, AV, NPS, NPS, NPS, NPS, NPS, NPS, NPS, NPS
	٤	APMRI, PMRZ, PMR3, PMR4, PMT1, PMT2, PMT3, PMT4,
	2	APVR1, PVR2, PVR3, PVR4, PVY1, PVY2, PVY3, PVY4, AP7
	C	CALCULATE STAT FUR P
175		$CALL \qquad SAIMV(NPR, NPS, NM, PMR1, PMR2, PMR3, DAVA DAVA DAVA DAVA DAVA DAVA DAVA DAV$
	٤	SPMR4, PMY1, PMY2, PMY3, PMY4, AM, AV, AMP7
171		(ALL SAIMV(NPP, NPS, NM, PVRI, PVR2, PVR3, NM, PVR1, PVR2, PVR3, NM, PVR2, PVR3, PVR2, PVR3, NM, PVR2, PVR3, NM, PVR2, PVR2, PVR3, NM, PVR2, PVR2, PVR3, NM, PVR2, PVR2, PVR3, NM, PVR2, PVR3, PVR3, NM, PVR2, PVR3, PVR3, NM, PVR2, PVR3, NM, PVR2, PVR3, NM, PVR2, PVR3, PVR3, NM, PVR2, PVR3, PVR
	3	APVR4, PVY1, PVY2, PVY3, PVY4, A*, AV, XVP1
	C	CALCULATE A,B,C,D VECTOR
172		D() 3001 1=1,NP
173		XA(I) = PX(I)
174		X A(I) = RX (I)
175	3001	CUNTINUE
176		IA=KI
177		UALL GEMKPLIA, UI, NP, NPK, VA, VD, ZETA, PSD, XA, XXP,
	8	SXXK, XMI, A)
173		TB = RT + DT/2.
179		DO 931 I=1,NP

182	931 CONTINUE
193	CALL GEMKP(TB,DT,NP,NPR,VA,VB,ZFTA,PSB,X3,XXP,
	£XXK,XMI,B)
184	TC = BT + DT/2
135	DO 934 T=1.NP
194	
107	$\mathbf{X} = \mathbf{X} + $
107	XIU(1)=RXI(1)+U.5*B(1)
138	934 LUNTINUE
189	CALL GFMKP(IC, DI, NP, NPR, VA, VB, ZEIA, PSB, XL, XXP,
	EXXK,XMI,C)
190	TD=RT+DT
191	DD 936 I=1,NP
192	XD(I)=RX(I)+DT*RXT(I)+(DT/2.)*B(I)
193	XTO(I) = RXT(I) + C(I)
194	936 CONTINUE
195	CALL GEMKP(TD, DT, NP, NPR, VA, VB, ZETA, PSB, XD, XXP,
	EXXK.XMI.D)
196	DO 038 t=1.NP
107	$\mathbf{v}(\mathbf{I}) = \mathbf{p}\mathbf{v}(\mathbf{I}) + \mathbf{D}\mathbf{T} \neq \mathbf{p}\mathbf{v}(\mathbf{I}) + (\mathbf{D}\mathbf{T}/\mathbf{A}_{1}) \neq (\mathbf{A}(\mathbf{I}) + \mathbf{B}(\mathbf{I}) + C(\mathbf{I}))$
197	$x_{1} = x_{1} + y_{1} + y_{1} + y_{1} + y_{1} + y_{2} + y_{1} + y_{1} + y_{2} + y_{1} + y_{1} + y_{2} + y_{1} + y_{1} + y_{1} + y_{1} + y_{2} + y_{1} + y_{1$
198	X((1)=RX)((1)+(1)+(1)+(A(1)+2)+O(1)+(2)+(2)+(2)+(2)+(2)+(2)+(2)+(2)+(2)+(2
199	938 CUNTINUE
200	TAC=RT+DT
201	DO 939 I=1,NP
202	XAC(I) = X(I)
203	XTAC(I) = XT(I)
204	939 CONTINUE
205	CALL GEMKP(TAC, DT, NP, NPR, VA, VB, ZETA, PSB, XAC,
	EXXP.XXK.XMI.XTT)
206	DO 941 I = 1.NP
207	xTT(I) = xTT(I) / 0T
209	
200	
209	
210	
211	$\frac{1}{100} = \frac{1}{100} = \frac{1}$
212	$WRITE(3,903) \times (1) \times (1) \times (1)$
213	9100 CUNTINUE
	C CALCULATE END FURCES
214	DO 890 I=1,NEM
215	XEM(I)=0.
216	XEV(I)=0.
217	DO 890 J=1,NP
218	XEM(I)=XEM(I)+(XMK(I,J)-XMP(I,J))*X(J)+
	(L)TX*(L.I)MMX3
219	$X \in V(I) = X \in V(I) + (XVK(I,J) - XVP(I,J)) + X(J) +$
217	$EXVM(T, I) \neq XTT(J)$
220	POD CONTINUE
220	$\frac{1}{10} = \frac{1}{10} $
221	WK1161390717 '
222	UU 9001 1-19NCH UDITE(2 002) I.YEM(1),YEV(1)
223	WKIIELDIDZEJ IJACHLIJJACTI

XB(I)=RX(I)+(DT/2.)*RXT(I) XTB(I)=RXT(I)+0.5*A(I)

180

181

```
224
        9001 CONTINUE
225
             T = RT + DT
225
        9999 CONTINUE
227
           2 FORMAT(15)
         400 FORMAT(6F10.4)
228
229
         401 FORMAT(515)
         403 FORMAT(215,F10.4)
230
231
         500 FORMAT(//10X, 'NO. OF PROGRAMS =', I5)
232
         601 FORMAT(2F10.2)
233
         603 FORMAT(12F10.4)
234
         633 FORMAT(2F10.4)
         650 FORMAT(//10X . AM
                                  MATRIX')
235
         651 FORMAT(//10X, 'AV MATRIX')
236
        652 FORMAT(//10X, 'AMS MATRIX')
237
                                   MATRIX )
238
         653 FORMAT(//10X, *AFV
        654 FORMAT(//10X, AFP
                                   MATRIX )
239
         700 FORMAT(//10X, 'MEMBER LENGTH')
240
        701 FORMAT(3E16.7)
241
         702 FORMAT(//10X, 'MEMBER MOMENT INERTIA')
242
         703 FORMAT(//10X, 'MEMBER MASS')
243
        704 FORMAT(//10X, 'ALPHA VALUE')
244
        705 FORMAT(//10X, 'BETA VALUE')
245
         706 FORMAT(//10X, 'LOAD P AND ELASTIC MODULUS')
246
         892 FORMAT(//2X, 'PT', 12, 2X, E16.7, 4X, E16.7, 4X, E16.7
247
            £,4X,E16.7,4X,E16.7)
         891 FORMAT(//10X, 'END MOMENT, END SHEAR AT TIME=',
248
            &F10.7)
         900 FORMAT(6F10.4)
249
         903 FORMAT (//10X, E16.7, 10X, E16.7, 10X, E16.7)
250
        1001 FORMAT(1H1)
251
         901 FORMAT(//10X, *X, XT, XTT AT TIME T=*, F10.7)
252
        1009 FORMAT(615)
253
        3347 FORMAT(//10X,15,5E16.7)
254
        3348 FORMAT(//10X, *MEMBER NO.*, 10X, *AREA*, 10X, *FY*,
255
            £10X, 'PY', 10X, 'ZP', 10X, 'PM')
        4321 FORMAT (//10X, 'VA=', F10.4, 5X, 'VB=', F10.4, 5X,
256
            &'ZETA=', F10.4)
          52 STOP
257
             END
258
```

		* ELAST	O-PLASTIC DYNAMIC RESPONSE	*
1			AM (1 2 . 1 2) AMS (4 . 4) AV (1 2 . 1 2)	
2		DIMENSION	A(12) (2) (2) (4) (4) (4) (12) (2) (2) (2) (2) (2) (2) (2) (2) (2) (
2		DIMENSION	ASAT([0,10],INDEX([0],X]([0],X]([0],X]([0]))	
		DIMENSION	$\frac{1}{2} = \frac{1}{2} = \frac{1}$	
-		DIMENSION	SMP1(10) = SMP2(10) = SMV1(10) = SMV2(10)	
5		DIMENSION	SMP3(10) + SMP4(10) + SMP3(10) + SMP4(10)	
7		DIMENSION	XMP = 1(10) + XMP = 2(10) + XMV = 1(10) + XMV = 2(10)	
		DIMENSION	$XMR_{1}(10), XMR_{1}(10), XMR_{1}(10), XMR_{1}(10)$	
5		DIMENSION	PMP1(10), PMP2(10), PMV1(10), PMV2(10)	
10		DIMENSION	PMR1(10), PMR2(10), PMY1(10), PMY2(10)	
10		DIMENSION	SVR1(10), $SVR2(10)$, $SVY1(10)$, $SVY2(10)$	
1 2		DIMENSION	SVR3(10), SVR4(10), SVY3(10), SVY4(10)	
12		DIMENSION	PVPI(10) = PVP2(10) = PVVI(10) = PVV2(10)	
1/		DIMENSION	PVR3(10) = PVR4(10) = PVY3(10) = PVY4(10)	
15		DIMENSION	XVE1(10) + XVR2(10) + XVY1(10) + XVY2(10)	
14		DIMENSION	XVR3(10) - XVR4(10) - XVY3(10) - XVY4(10)	
10		DIMENSION	XVRJ(10),XVR(10),XVIJ(10),XVIJ(10)	
17		DIMENSION	XMF(18,10), XMM(18,10), XVK(18,10)	
18		DIMENSION	$\lambda(10) = P(10) = C(10) = D(10) = XMI(10, 10)$	
19		DIMENSION	P(10) = P(10) = P(10) = P(10)	
20		DIMENSION	$\mathbf{X}(10), \mathbf{X}(10), \mathbf{X}(10)$	
21		DIMENSION	X(10), X(10), X(10), X(10)	
22		DIMENSION	$X_{1}(10), X_{1}(10), X_{1}(10), X_{1}(10), X_{1}(10)$	
23		DIMENSION	XFX(10), XVM(18,10), XEVM(18)	
24		DIMENSIUM		
25		DIMENSION	PR(18), PV(18), EAVT(18, 18), ENDR(18)	
20		DIMENSION	EPM1(10) - EPM2(10) - EPM3(10) - ERM4(10)	
21		DIMENSION	$FRY1(10) \cdot FRY2(10) \cdot FRY3(10) \cdot FRY4(10)$	
23		DIMENSION	$AE(10) \cdot AEV(10, 18) \cdot AEP(10, 18) \cdot PT(10)$	
29		DIMENSION	ENV1(10) - ENV2(10) - ENV3(10) - ENV4(10)	
50		DIMENSION	EEM(12), $EEV(12)$, $PE(10)$, $RSET(10)$	
51		DIMENSION	APEA(10) - EY(10) - PY(10) - RDPM(18)	
32		DIMENSION	NPH(18), IPH(18), PRHR(18), NPH(18)	
33		DIMENSION	1 PHP (18) -1 PHRD(18) - FDPM(18) - PM(18)	
.34		DIMENSION	COER(10) - BXEV(20) - MNPH(20) - MNPH(20)	
.35		DIMENSION	SPVY1(10), SPVY2(10), SPVY3(10)	
30		DIMENSION	$SAVT(20, 20) \cdot SECUV(20) \cdot SPVY4(10)$	
31		DIMENSION	$DET(10) \cdot AEVV(10) \cdot AEPP(10)$	
38		DIMENSION	$PSE(10) \cdot XS(10) \cdot ZP(10)$	
37		DIMENSION	$AYTALF(18) \cdot PLIMIT(18)$	
40		DIMENSION	$REDUC(20) \cdot HRATIO(18)$	
41		OTMENSTON	$XAC(10) \cdot XTAC(10)$	
42		DIMENSION	AMTX(18).HR(18)	
43		DEAD(1.2)		
44	L	TEINOL ET	-52-3	
45	2	JEINUI DA	001)	
40	د	WALLELDIL	00) NC	
47		DEAD[1.40	1) NM .NP .NPR , NPS , NVP	
48		NEM-DANM	• Construction of the second second	
49			5 .	
50.		ALOWM-1.0	0	
51		ALUWR-0.J	\sim	

52	READ(1, 400)(XL(I), I=1, NM)
53	PEAD(1, 400)(AREA(1), I=1, NM)
54	READ(1.400)(XI(I).I=1.NM)
55	E = AD(1 + 400)(XM(T) + T = 1 + NM)
55	READ(1.400)(7P(1).I=1.NM)
57	BEAD(1, 400)(EY(1), I = 1, NM)
54	$READ(1, 400)(A) PH(1), \mathbf{I} = 1, NM)$
50	READ(1) + 00)(RETA(1), 1 - 1) + 00)
60	$\frac{2}{2} = \frac{2}{2} = \frac{2}$
61	READ(1,400) VA.VB.ZETA
47	MOITE/3.700)
62	$\frac{1}{1} \frac{1}{1} \frac{1}$
55	WF1/C(3,/01/(AC(1/)1-1)///)
45	WRI!E(3)/UZ/ UDITE(3 701)/VI/I\ I~1 NMA
60	WRITE(3,701)(X1(1),1-1,100)
00	WR11E(3,703)
67	WR11E(3,703)
63	WRITE(3,701)(XM(1),1=1,NM)
59	WRITE(3,704)
70	WRITE(3,701)(ALPHA(I),I=1,NM)
71	WRITE(3,705)
72	WRITE(3,701)(BETA(I),I=1,NM)
73	WRITE(3,706)
74	WRITE(3,701) PSB,XE
75	WRITE(3,4321) VA,VB,ZETA
75	WRITE(3,3348)
77	DO 3346 I=1, MM
73	PY(I) = AREA(I) * FY(I)
79	$PM(I) = ZP(I) \neq FY(I)$
30	WRITE(3,3347) I,AREA(I),FY(I),PY(I),ZP(I),
	SPM(I)
81	3346 CONTINUE
	C CALCULATE REDUCED PLASTIC MOMENT
82	DO 1007 I = 1.0M
83	$\Delta XIALF(I) = (ALPHA(I) * VA + BETA(I) * VB) * PSB$
84	PIIMIT(I)=0.15*PY(I)
35	REDUC(I) = ((ALPHA(I) * VA + BETA(I) * VB) * PSB) / PY(I)
86	RDPM(I) = PM(I) * (1 - REDUC(I) * REDUC(I))
37	$TI = 2 \times T - 1$
00	$10 = 2 \times 1$
20	EDPM(TL) = ROPM(T)
22	
40	
91	
92	
93	$\frac{DD}{DT} = \frac{1}{2} \frac{DT}{T} =$
94	WRITE(3,5347) ITANIALITITY CITITITY OF CIT
95	3447 CONTINUE
95	
97	$DU 402 J=L_{P}NEM$
93	402 AM(I, J) = 0.
99	Dn 407 I=1,NPS
100	DC 407 J=1, NEM
101	407 AV(I,J)=0.
19 1	

102		DC 414 I=1,NM
103		DO 414 J=1,NEM
104	414	AFV(I,J)=0.
105		DO 415 I=1,NM
106		DO 415 J=1,NVP
107	415	AFP(I,J)=0.
103	405	READ(1.403) I.J.AMIJ
109		IF(I) 404,404,405
110	405	AM(I,J) = AMIJ
111		GO TO 406
112	404	READ(1.403) I.J.AVIJ
113		IF(I) 403.408.409
114	409	$\Delta V(I,J) = A V I J$
115		GO TO 404
116	408	00 410 I=1.NPS
117		00 410 J=1 NPS
118	410	AMS(I, I) = 0
110	413	READ(1,403) T.J.AMSTJ
120	, 41 5	IE(I) 411.411.412
120		MS(T, I) = MSTI
121	412	CO TO 413
122	6.1.1	PEAD(1, 403) I = 1 = AEVI I
123	411	READ(1)+057 100000000000000000000000000000000000
124	() (
125	410	AFV(1,J) - AFVIJ
120		
127	417	$\begin{array}{c} READ(1,403) 1, J, AFFIJ \\ J \in (I,I,I,I,I,I,I,I,$
128		1F(1) 418,418,419
129	419	
130		GO TO 417
131	418	WRITE($3,650$)
132		WRITE(3,603) ((AM(1, J), J-L, VEM)) - 1, VEM)
133		WRITE (3,651)
134		WRITE(3,603) ((AV(1,J),J=1,NEM),1-1,NPS)
135		WRITE(3,652)
136		WRITE(3,633)((AMS(1,J),J=1,NPS))
137		WRITE(3,653)
133		WRITE(3,603)((AFV(1, J), J=1, NEM), 1=1, NM)
139		WRITE(3,654)
140		WRITE(3,603)((AFP(I,J),J=1,NVP),I=1,NM)
141		DO 561 $I=1,NM$
142		FMV1(I) = -1./XL(I)
143		FMV2(I) = -1./XL(I)
144		FMV3(I) = -1./XL(I)
145		FMV4(I) = -1./XL(I)
146	561	CONTINUE
	С	DEFINE THE INITIAL CONDITION
147		DO 567 I=1,NEM
148		NPH(I)=0
149		MNPH(I)=0
150		NRH(I)=0
151		MNRH(I)=0
152		IPH(I)=0
6.72		

153		PRHR(I)=0.
154		HRATIO(I)=0.
155		LPHRD(I)=0
156	567	CONTINUE
157		DO 4446 I=1.NPS
153		XS(I)=0.
159	4445	CONTINUE
150		00 6001 I=1,NEM
161		SECDV(I)=0.
132	6001	CONTINUE
1.53		DO 9449 I = 1, NP
164		PSE(I)=0.
165	9449	CONTINUE
166		DO 560 I=1.NEM
167		FEV(I)=0.
168		FEM(I)=0.
169	560	CONTINUE
170		READ(1.900)(X(I),I=1.NP)
171		READ(1.900)(XT(I),I=1,NP)
172		READ(1,900)(XTT(1),I=1,NP)
173		DO 671 I=1.NEM
174		$X \in V(I) = 0$
175	671	CONTINUE
176		WRITE(3.674)
177		WRITE(3,900)(XEV(1),I=1,NE4)
178		T=0.
170		DT = 0.004
120		K7EPO=0
101		
1.27		DO = 9000 I = 1.NP
1 8 3		WRITE(3.903) X(I).XT(I).XTT(I)
124	9000	CONTINUE
185	,000	WRITE(3,1920)(LPH(I),I=1,NEM)
136		NPTS=51
187		DD 9999 KK=2.NPTS
189	1599	WRITE(3,1001)
139		WRITE(3.1919) (LPH([).I=1.NEM)
127		DO = 930 I=1.NP
191		BX(I) = X(I)
192		RXT(I) = XT(I)
133		RXTT(I) = XTT(I)
134	930	CONTINUE
105	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	DO 5682 J=1.NEM
104		$R \times F \vee (A) = X F \vee (A)$
193	5682	CONTINUE
102	1002	RT=T
120		PRT=RT
2.10		WRITE(3.901) PRT
200		DO 1972 I = 1. NP
2.72		WRITE(3,903) PX(I), RXT(I), RXT(I)
202	1972	CONTINUE
	c	FIND AXIAL FORCE

.

.

204		ZT=ZETA≭T
205		CZT=COS(ZT)
206		00.655 I = 1.NVP
2:17		PT(I)=VA+PSB+VP+PSB+C7T
2.12	655	CONTINUE
203		$DD = 665 T = 1 \cdot NM$
210		
210		DD + 44 + 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1
211		$\Delta \mathbf{E} \mathbf{V} \mathbf{V} \mathbf{I} \mathbf{V} = \mathbf{V} \mathbf{E} \mathbf{V} \mathbf{V} \mathbf{I} \mathbf{V} \mathbf{I} \mathbf{V} \mathbf{E} \mathbf{V} \mathbf{I} \mathbf{I} \mathbf{V} = \mathbf{V} \mathbf{V} \mathbf{V} \mathbf{I} \mathbf{V} \mathbf{I} \mathbf{V} \mathbf{E} \mathbf{V} \mathbf{I} \mathbf{I} \mathbf{V}$
212		
213	000	
214		$\frac{1}{1}$
215		
215		DU 667 J=LINVP
217		AFPP(1)=AFPP(1)+AFP(1,J)*P((J)
218	667	CONTINUE
219		DO 668 I = 1, NM
220		AF(I) = AFVV(I) + AFPP(I)
221	668	CONTINUE
	С	CALCULATE CARRY OVER FACTOR
222		DO 1601 I=1,NM
223		COFP(I) = (2 * XE * XI(I) / XL(I) + AF(I) * XL(I) / 30 ·) /
	1	&(4.*XE*XI(I)/XL(I)-2.*AF(I)*XL(I)/15.)
224	1601	CONTINUE
225		DO 1020 I=1,NM
220		IL=2*I-1
227		JR = 2 * I
229		MIL=LPH(IL)
229		MJR=LPH(JR)
230		IF(MIL) 2011,2011,2012
231	2011	IF (MJP) 2013,2013,2014
232	2013	FEM(IL)=0.
233	2015	FFM(JR)=0
234		GD TO 1020
215	2014	EEM(II) = EEM(JR) *COER(I)
232	2014	EEM(IR) = EEM(JR)
200		
221	2012	TE(MIR) 2015.2015.2016
230	2012	EEM(TL) = EEM(TL)
231	2019	$FEM(1E) = FEM(1E) \neq COFR(1)$
240		
241	2014	
242	2016	FEM(1L)-FEM(1L)
243		CONTINUE
244	1020	
	C	FINDED END STEAR
245		DO = 562 K = 1 NM
245		
247		M=2*K
248		FEV(L)=FMV1(K)+FEM(L)+FMV4(K)+FFM(M)
249	1966	FEV(M)=FMV3(K)#FEM(L)FFMV4(K)FEEM(M)
250	562	CONTINUE
251		DO 1000 $I = 1, NM$
252		MN = I

•

253	$IL = 2 \neq I - 1$
254	$JR = 2 \neq I$
255	NIL=LPH(IL)
235	NJR=LPH(JP)
257	IF(NTL)1011,1011,1002
258	1011 IF(NJR) 1003,1003,1004
257	LOO3 CALL STIEPA(PMR1.PMR2.PMR3.PMR4.
	EPMY1. PMY2. PMY3. PMY4. PVR1. PVR2. PVP3. PVR4.
	EPVY1. PVY2. PVY3. PVY4. 11. XI. AF)
250	$GO_{TO} = 1000$
251	1004 CALL STIEPB(PMR1.PMR2.PMR3.PMR4.
	EPMY1. PMY2. PMY3. PMY4. PVE1. PVR2. PVP3. PVR4.
	EPVY1.PVY2.PVY3.PVY4.MN.XL.AF)
262	
263	1002 LE(NIR) 1005-1005-1006
265	1005 CALL STIEPC(PMR1.PMR2.PMR3.PMR4.
204	CDNVF_DNV2_DNV3_DNV4_DVR1_DVR2_DVR3_DVR4_
	CDVV1_DVV2_DVV3_DVV4_MN_X1_AF)
745	CO TO 1000
200	
200	
	$\frac{2PMTL}{PMTZ} = \frac{PMTJ}{PMT} + \frac{PMTJ}{PMT} + \frac{PMTJ}{PMTZ} = \frac{2PMTJ}{PMTZ} + \frac{PMTJ}{PMTZ} + \frac{PMTJ}{PMTZ$
3/7	APVYL PVYZ PVYD PVYH PAI ALPANI
257	LUUU GUNTINUE CALL ACATAINDE NDS NM AM AV
268	
	EPMRI, PMRZ, PMR3, PMR4, PMr1, PMr2, PMr3, PMr4,
	EPVR1, PVR2, PVR3, PVR4, PVY1, PVY2, PVY3, PVY4, XXP1
259	CALL SATMVINPRINPS, NM, PMRL, PMRZ, PMP3,
	EPMR4, PMY1, PMY2, PMY3, PMY4, AM, AV, XMP)
270	CALL SATAVIAPR, NPS, MM, PVR1, PVR2, PVP3,
	&PVR4, PVY1, PVY2, PVY3, PVY4, AM, AV, XVP1
271	DO 8000 I=1,NM
272	MN=I
273	IL = 2 * I - 1
274	JR = 2 * I
275	IF(KZERO.EQ.O) GO TO 8101
275	IF(LPHRD(IL)-LPH(IL)) 8101,8102,8101
277	3102 IF(LPHRD(JR)-LPH(JR)) 8101,8000,8101
278	3101 NIL=LPH(IL)
279	NJR=LPH(JR)
280	IF(NIL)8011,8011,8002
231	3011 IF(NJR) 8003,8003,8004
232	9003 CALL STIFFA(SMR1, SMR2, SMR3, SMR4,
	ESMY1, SMY2, SMY3, SMY4, SVR1, SVR2, SVR3, SVR4,
	&SVY1,SVY2,SVY3,SVY4,XMR1,XMP2,XMR3,XMR4,
	EXMY1, XMY2, XMY3, XMY4, XVP1, XVR2, XVR3, XVR4,
	EXVY1, XVY2, XVY3, XVY4, MN, XE, XI, XL, XM)
233	GD TD 8000
234	8004 CALL STIFFB(SMR1, SMR2, SMR3, SMR4,
1	ESMY1, SMY2, SMY3, SMY4, SVP1, SVP2, SVP3, SVR4,
	55VY1,5VY2,5VY3,5VY4,XMP1,XMP2,XMP3,XMP4,
	5XMY1.XMY2,XMY3,XMY4,XVR1,XVR2,XVP3,XVR4,
	EXVY1.XVY2.XVY3.XVY4.MN,XE,XI,XL,XM)
	CUALTA ALL CLUTTER FOR THE FOR

285	60 TO 8000
295	3002 IF(NJR) 8005,8005,8006
237	3005 CALL STIFFC(SMR1,SMR2,SMP3,SMR4,
	&SMY1, SMY2, SMY3, SMY4, SVR1, SVR2, SVR3, SVR4,
	ESVY1, SVY2, SVY3, SVY4, XMR1, XMR2, XMP3, XMP4,
	EXMY1, XMY2, XMY3, XMY4, XVR1, XVR2, XVR3, XVR4,
	& XVY1, XVY2, XVY3, XVY4, M N, XE, XI, XL, XM)
298	GO TO 8000
237	3006 CALL STIFFD(SMR1,SMR2,SMR3,SMR4,
	&SMY1,SMY2,SMY3,SMY4,SVP1,SVP2,SVR3,SVR4,
	\mathcal{E} SVY1,SVY2,SVY3,SVY4,XMR1,XMR2,XMR3,XMR4,
	&XMY1,XMY2,XMY3,XMY4,XVR1,XVR2,XVR3,XVR4,
	& xvy1, xvy2, xvy3, xvy4, mn, xe, xI, xL, xm)
290	8000 CONTINUE
291	IF(KZERO.EQ.0) GO TO 8204
292	DO 8201 I=1,NEM
293	IF(LPHRD(I)-LPH(I)) 3204,8201,3204
294	8201 CONTINUE
295	GO TO 8203
296	8204 CALL ASATA(NPR, NPS, NM, AM, AV,
	ESMR1, SMR2, SMR3, SMR4, SMY1, SMY2, SMY3, SMY4,
	ESVR1, SVR2, SVR3, SVR4, SVY1, SVY2, SVY3, SVY4, XXK)
297	CALL ASAIB(NPR, NPS, NM, AM, AV,
	EXMR1, XMR2, XMR3, XMR4, XMY1, XMY2, XMY3, XMY4,
	$\mathbf{E} \mathbf{X} \mathbf{V} \mathbf{E} \mathbf{I} \mathbf{Y} \mathbf{V} \mathbf{Z} \mathbf{I} \mathbf{X} \mathbf{V} \mathbf{Z} \mathbf{I} \mathbf{X} \mathbf{V} \mathbf{X} \mathbf{Y} \mathbf{I} \mathbf{I} \mathbf{Y} \mathbf{V} \mathbf{Y} \mathbf{Z} \mathbf{I} \mathbf{X} \mathbf{V} \mathbf{Y} \mathbf{I} \mathbf{I} \mathbf{Y} \mathbf{V} \mathbf{Y} \mathbf{Z} \mathbf{I} \mathbf{X} \mathbf{V} \mathbf{Y} \mathbf{I} \mathbf{I} \mathbf{Y} \mathbf{V} \mathbf{Y} \mathbf{Z} \mathbf{I} \mathbf{X} \mathbf{V} \mathbf{Y} \mathbf{I} \mathbf{I} \mathbf{Y} \mathbf{V} \mathbf{Y} \mathbf{Z} \mathbf{I} \mathbf{X} \mathbf{V} \mathbf{Y} \mathbf{I} \mathbf{I} \mathbf{Y} \mathbf{V} \mathbf{Y} \mathbf{Z} \mathbf{I} \mathbf{X} \mathbf{V} \mathbf{Y} \mathbf{I} \mathbf{I} \mathbf{Y} \mathbf{V} \mathbf{Y} \mathbf{Z} \mathbf{I} \mathbf{X} \mathbf{V} \mathbf{Y} \mathbf{Z} \mathbf{I} \mathbf{X} \mathbf{V} \mathbf{Y} \mathbf{X} \mathbf{X} \mathbf{X} \mathbf{X} \mathbf{X} \mathbf{X} \mathbf{X} X$
298	
299	CALL = SATEV(NPR(NPS(NPS(NPS(NPS(NPS(NPS(NPS(NPS(NPS(NPS
2.10	CALL CAT NVINDE NDE SVR1 SVR2 SVR3
500	CONDA SUVI SVV2 SVV3 SVV4 AM AV XVK)
2.11	CALL SAT MVINPR .NPS .NM . XMR1 . XMR2 . XMR3 .
201	SYMPL, XMY1, XMY2, XMY3, XMY4, AM, AV, XMM)
302	CALL SATMVINPR • NPS • NM • XVR1 • XVR2 • XVR3 •
302	SXVR4, XVV1, XVV2, XVY3, XVY4, AM, AV, XVM)
	C = RECORD (PH(T))
303	8203 1081 I=1•NEM
3.74	I PHRD(I) = LPH(I)
305	1081 CONTINUE
	C CHECK IF THERE IS ANY MEMBER BOTH ENDS HINGED
305	4444 DG 5011 I=1,NM
307	IL = 2 * I - 1
303	JR=2≭I
309	NMIL=LPH(IL)
310	NMJR=LPH(JR)
311	IF(NMIL) 5011,5011,5012
312	5012 IF(NMJR) 5011,5011,5013
313	5011 CONTINUE
314	GO TC 4445
	C CALCULATE SECONDARY SHEAR
315	5013 DO 4020 I=1,NM
315	$IL = 2 \times I - 1$

317		JR = 2 * I
313		MMIL=LPH(IL)
319		MMJR=LPH(JR)
320		IF(MMIL) 4012,4012,4011
321	4011	IF(MMJR) 4012,4012,4013
322	4013	SPVY1(I)=AF(I)*(-1./XL(I))
323		$SPVY2(I) = AF(I) \neq (-1./XL(I))$
324		SPVY3(I) = AF(I) + (-1./XL(I))
325		SPVY4(I) = AF(I) * (-1./XL(I))
326		GO TO 4020
327	4012	$SPVY1(I) = AF(I) \neq 0.$
323		SPVY2(I) = AF(I) * 0.
329		SPVY3(I) = AF(I) * 0.
330		$SPVY4(I) = AF(I) \neq 0.$
331	4020	CONTINUE
332		DO 4470 J=1, NPS
333		DO 4470 $K=1, NM$
334		$L=2 \neq K-1$
335		M=2×K
336		SAVT(M,J) = SPVY3(K) * AV(J,L) + SPVY4(K) * AV(J,M)
337	<i>e</i>	$SAVT(I \cdot J) = SPVY1(K) \neq AV(J \cdot L) + SPVY2(K) \neq AV(J \cdot M)$
338	4470	CONTINUE
339		DD 4480 I=1.NEM
340		SECDV(I)=0.
341		DO 4480 J=1.NPS
342		SECDV(I) = SECDV(I) + SAVT(I,J) + XS(J)
343	4480	CONTINUE
515	C	TRANSFER SECONDARY SHEAR TO EXTERNAL JOINT
344	Ũ	00 4564 T=1.NPS
345		II=I+NPR
345		PSE(II)=0
347		DC 4564 J=1.NEM
348		PSE(II) = PSE(II) + AV(I,J) + SECOV(J)
349	4564	CONTINUE
350	1001	WRITE(3,903)($PSE(LL)$, $LL=1$, NP)
351	4445	CALL GEXTPLAM.AV.NP.NPR.NM.FEM.FEV.PSE.BSET)
	C	CALCULATE A.B.C.D VECTOR
352	0	DD 3001 I = 1.NP
353		$X \Delta (T) = P X (T)$
354		$XT\Delta(I) = RXT(I)$
155	3001	CONTINUE
356	J 00 L	TA=BT
357		CALL GEMKPITA.DT.NP.NPR.VA.VB.7FTA.PSB.XA.XXP.
571	\$	CYXK, YMI, RSET, A)
252		TB=PT+DT/2
150		DO = 0.1 T = 1.NP
360		$XB(T) = RX(T) + (DT/2) \Rightarrow RXT(T)$
161		$XTB(I) = RXT(I) + O_5 \neq A(I)$
763	021	CONTINUE
302	721	CALL GEMKP(TB.DT.NP.NPR.VA.VB.ZETA.PSB.XB.XYP.
177	ſ	XXK.XMI.RSFT.B)
364	ę	TC=PT+DT/2.
104		

365	DC 934 I=1,NP
365	XC(I)=RX(I)+(DT/2•)*(RXT(I))+(DT/4•)*(A(I))
367	$XTC(I) = PXT(I) + O \cdot 5 + B(I)$
353	934 CONTINUE
359	CALL GEMKP(TC,DT,NP,NPR,VA,VB,ZETA,PSB,XC,XXP,
	£XXK,XMI,RSFT,C)
370	TD =RT+DT
371	DO 936 I=1,MP
372	$XO(I) = RX(I) + OT \Rightarrow RXT(I) + (DT/2) \Rightarrow B(I)$
373	XTD(I) = PXT(I) + C(I)
374	936 CONTINUE
375	CALL GEMKP(TD, DT, NP, NPR, VA, VB, ZETA, PSB, XD, XXP,
	£XXK,XMI,RSFT,D)
375	DC 938 I=1,NP
377	X(I) = RX(I) + DT * RXT(I) + (DT/6.) * (A(I) + B(I) + C(I))
378	XT(I)=RXT(I)+(1./6.)*(A(I)+2.*B(I)+2.*C(I)+
	((1))
377	938 CONTINUE
390	TAC=RT+DT
331	DO 939 I=1,NP
382	$X \land C(I) = X(I)$
333	XTAC(I) = XT(I)
384	939 CONTINUE
385	CALL GEMKPITAC, DI, NP, NPR, VA, VB, ZETA, PSB, XAC,
	EXXP,XXK,XMI,RSFI,XII)
336	DC 941 I=1, NP
387	XTT(I) = XTT(I) / DI
333	941 CUNTINUE
339	107446 I=1,NP5
390	
391	XS(1) = X(11)
392	7448 CONTINUE
343	4448 =R +D UDITE(2,00))T
394	
392	$\frac{1}{1} = \frac{1}{1} = \frac{1}$
222	
391	
222	
170	
400	$X \in V \cap K(I) = 0$
4 31	XEMM(I)=0.
402	$X \in VM(I) = 0$
403	
405	$X \in MPK(I) = X \in MPK(I) + (XMK(I,J) - XMP(I,J)) * X(J)$
405	$X \in V \in (I) = X \in V \in (I) + (X \vee K (I, J) - X \vee P (I, J)) * X (J)$
426	XEMM(I) = XEMM(I) + XMM(I, J) * XTT(J)
407	$X \in VM(I) = X \in VM(I) + XVM(I, J) * XTT(J)$
403	890 CONTINUE
439	DC 1890 I=1, NEM
410	$X \in MPK(I) = X \in MPK(I) + F \in M(I)$
411	XEVPK(I) = XEVPK(I) + FEV(I) + SECDV(I)

412	1890	CONTINUE
	С	OFFORMATION CHECK
413		DO 450 I=1,NM
414		<pre>DFT(I)=(6.*XE*XI(I)/XL(I)-AF(I)*XL(I)/10.)*</pre>
		1(2.*XE*XI(I)/XL(I)-AF(I)*XL(I)/6.)
415		FRM1(I)=((4.*XE*XI(I)/XL(I))-(2.*AF(I)*XL(I)/
		615.))/DET(I)
415		FRM4(I)=((4.*XE*XI(I)/XL(I))-(2.*AF(I)*XL(I)/
	1	£15.))/DET(I)
417		FRM2(I)=-((2.*XE*XI(I)/XL(I))+(1.*AF(I)*XL(I)/
	1	530.))/DET(I)
419		FRM3(I) =- ((2.*XE*XI(I)/XL(I))+(1.*AF(I)*XL(I)/
	,	530.))/DET(I)
419		FRY1(I) = -1./XL(I)
420		FRY2(I) = -1./XL(I)
421		$FRY3(I) = -1 \cdot / XL(I)$
422		FRY4(I)=-1./XL(I)
423	450	CONTINUE
424		DG 460 K=1,NM
425		L=2*K-1
426		M=2.≭K
427		DP(L)=FRM1(K)*XEMPK(L)+FRM2(K)*XEMPK(M)
428		DR(M)=FRM3(K)*XFMPK(L)+FRM4(K)*XEMPK(M)
429	460	CONTINUE
430		DO 470 J=1,NPS
431		DC 470 K=1,NM
432		L=2*K-1
433		M=2*K
434		FAVT(L,J)=FRY1(K)*AV(J,L)+FRY2(K)*AV(J,M)
435		FAVT(M,J)=FRY3(K)*AV(J,L)+FRY4(K)*AV(J,M)
436	470	CONTINUE
437		DD 480 I=1, NEM
433		DY(I)=0.
439		DO 480 J=1,NPS
440		JJ=J+NPR
441		DY(I)=DY(I)+FAVT(I,J)+X(JJ)
442	480	CONTINUE
443		DC 490 I=1,NEM
444		ENDR(I)=DR(I)-DY(I)
445	490	CONTINUE
445		DO 491 I=1,NEM
447		AMTX(I)=0.
443		DO 499 J=1,NPR
449		AMTX(I) = AMTX(I) + AM(J,I) + X(J)
450	499	CONTINUE
451		HR(I) = ENDR(I) - AMTX(I)
452	491	CONTINUE
453		DO 492 I=1,NEM
454		[F[ABS(HR(I)).LE.0.000001] HR(17-0.
455	492	CONTINUE
455		WRITE(3,550)
457		DC 551 I=1,NEM

458		WRITE(3,552) I, HR(I), ENDR(I), AMTX(I), PRHP(I)
457	551	CONTINUE
45)		DD 893 I=1,NEM
461		XEM(I)=XEMPK(I)+XEMM(I)
462		XEV(I)=XEVPK(I)+XEVM(I)
453	893	CONTINUE
464		WRITE(3,891) T
455		DO 9001 I=1.NEM
466		WRITE(3.392) I.XEM(I).XEV(I)
467	9001	CONTINUE
	C	CHECK : IS THERE ANY NEW PLASTIC HINGE FORMED
458	C C	DO 569 I=1.NEM
469		IE(MNRH(I)) 7856.7857.7857
470	7856	IF (ABS(XFM(I)/EDPM(I)).GT.ALDWM) NPH(I)=1
471		GO TO 569
472	7857	IE(IPH(I)_EQ.1) GO TO 569
473	1051	TE(ARS(XEM(T)/EDPM(T)),GT.1) NPH(I)=1
474	569	CONTINUE
475	207	DO 590 I=1-NEM
476		IE(NPH(I), E0, 0) G0 T0 590
477		I PH(T) = I PH(T) + NPH(T)
472		MNPH(T) = NPH(T)
470		$IE(XEM(I), IT_0) = GO TO 596$
413		
400		CO TO 581
401	504	
402	591	C1 CEM(I)=KC*EDPM(I)
400	261	
405		$IE(I = C = 2 \times NOM)$ CD TO 582
102		I = (I = E(I = E + 1) = E(I = E)
400		
457	507	$I_{F}(I_{F}) = I_{F}(I_{F}) = F_{F}(I_{F}) = F_{F}(I_{F})$
+15	502	
409	590	DD 599 J=1 NEM
490		$\frac{1}{10} \frac{1}{10} \frac$
491		1 (NPA(17) LQ+07 00 10 277
492		
4.93	3500	
494	2599	
495		1=KI DO 5678 1=1-NP
490		
497		
498		$X = \{J\} = \{X\} = \{J\}$
499		
500	5678	
501		
502		
503	5680	
504		DU 5693 J=LINYS
505		11=1+NHK 11=1+NHK
506		XS(J)=KX(JJ)
507	5683	CONTINUE
508		GU TU 1599
500		CONTINUE
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203	223	CUNITNUE
	L	CHECK IS THERE ANY OLD PLASTIC HINGE PELIEVED
510		DG 2005 I=1,NEM
511		IF(MNPH(I).EQ.1) GO TO 2005
512		IF(LPH(I).EQ.O) GO TO 2005
513		IF(HR(I)) 2000,2001,2001
514	2000	IF(PRHR(I)) 2004,2003,2003
515	2001	IF(PRHR(I)) 2003,2004,2004
516	2004	HRATID(I) = (PRHR(I) - HP(I)) / PRHR(I)
517	1	IF(HRATIO(I).GT.ALOWR) GC TO 2003
518		GO TO 2005
519	2003	NRH(T) = -1
520		MNRH(I) = NRH(I)
521	2005	CONTINUE
522	1.000	DO 571 I=1-NEM
523		IE (NRH(I), EO, O) GO TO 571
524		$WDTTF(2,FQF) = T_{A}NPH(T)$
525		
525		
526		NUM=(1+1)/2
521		IF(I.EQ.2*NUM) GU TU 572
528		IF(LPH(I+I).EQ.I) 60 16 575
529		FEM(1+1)=0.
530		FEM(1)=0.
531		GO TO 571
532	573	FEM(I)=FEM(I+1)*COFR(NOM)
533		GO TO 571
534	572	IF(LPH(I-1).EQ.1) GO TO 574
535		FEM(I-1)=0.
536		FEM(I)=0.
537		GO TO 571
533	574	FEM(I)=FEM(I-1)*COFR(NCM)
539	571	CONTINUE
540		DD 594 I=1,NEM
541		IF (NRH(I) . EQ.0) GO TO 594
542		00 3599 J=1.NEM
543		NBH(J)=0
544	3599	CONTINUE
545		T=RT
546		DD 5679 K=1.NP
547		X(K) = b X(K)
547		XT(K) = BXT(K)
540		XTT(K) = PXTT(K)
549	E 4 7 0	
550	2019	DO 5491 1-1-NEM
551		
552	F / 0 1	
253	2081	
554		
55 5		
555		XS(J) = KX(JJ)
557	5684	CONTINUE
558		DO 5689 L=1,NP
557		PSE(L)=0.

```
560
       5689 CONTINUE
561
             GD TO 1599
562
         594 CONTINUE
      С
             CHOOSE THE MAX HINGE ROTATION
563
             DO 583 I=1, NEM
564
             IF(LPH(I).EQ.0) GO TO 583
565
             IF(HR(I)) 585,586,586
555
         535 IF(PRH3(I)) 587,588,588
567
         586 IF(PRHR(I)) 588,587,587
563
         587 IF(ABS(HR(I)).GT.ABS(PRHR(I))) PRHR(I)=HR(I)
557
             GO TO 583
         538 PRHR(I)=HR(I)
570
571
         583 CONTINUE
572
             DO 580 I=1,NEM
573
             LPHR(I) = LPH(I)
574
             NPH(I)=0
575
             MNPH(I)=0
             NRH(I)=0
576
577
             MNRH(I)=0
        580 CONTINUE
578
579
             DO 5685 J=1,NPS
580
             JJ=J+NPR
581
             (L) X = (L) ZX
582
       5685 CONTINUE
             KZERO=KK
583
             T=RT+DT
584
       9999 CONTINUE
585
           2 FORMAT(15)
585
        400 FORMAT(6F10.4)
537
        401 FORMAT(515)
588
537
        403 FORMAT(215,F10.4)
        500 FORMAT(//IOX, 'NO. OF PROGRAMS =', I5)
590
        550 FORMAT(//5X, 'HINGE ROTATION', 18X, 'HR', 18X, 'DF'
j91
            E, 10X, "AMTX", 13X, "MAX.
                                     PRHR!)
         552 FORMAT(//10X, 'POINT(', 15, ')', 10X, 4E16.7)
592
         595 FORMAT(//10X, PLASTIC HINGE RELIEVED AT POINT',
593
            &15,2X,15)
        601 FORMAT(2F10.2)
594
        603 FORMAT(12F10.4)
595
        633 FORMAT(2F10.4)
596
        650 FORMAT (//10X, 'AM
                                MATRIX!)
597
        651 FORMAT(//10X, 'AV MATRIX')
598
        652 FORMAT(//10X, "AMS MATRIX")
599
        653 FORMAT(//10X, AFV
                                  MATRIX )
500
        654 FORMAT(//10X, AFP
                                  MATRIX )
501
        674 FORMAT(//10X, 'INITIAL XEV')
602
        700 FORMAT(//10X, 'MEMBER LENGTH')
503
        701 FORMAT(3E16.7)
504
        702 FORMAT(//10X, 'MEMBER MOMENT INERTIA')
605
        703 FORMAT(//10X, 'MEMBER MASS')
606
        704 FORMAT(//10X, 'ALPHA VALUE')
607
        705 FORMAT(//10X, BETA VALUE')
508
```

509	892 FORMAT(//2X, PT*, I5, E16.7, 4X, E16.7, 4X, E16.7,
	&4X,E16.7,4X,E16.7)
510	706 FORMAT(//10X, LOAD P AND ELASTIC MODULUS!)
511	901 FORMAT(//10X,'X,XT,XTT AT TIME T=',F10.7)
512	891 FORMAT(//lox,'END MOMENT,END SHEAR AT TIME=',
	£F10.7)
613	900 FORMAT(6F10.4)
614	903 FORMAT(//10X,E16.7,10X,E16.7,10X,E16.7)
515	1001 FORMAT(1H1)
615	1920 FORMAT(//10X, 'PLASTIC HINGE LOCATION(INITIAL)'
	£,615)
517	1919 FORMAT(//10X, PLASTIC HINGE LOCATION, 615)
513	3348 FORMAT(//10X, 'MEMBER NO.', 10X, 'AREA', 10X, 'FY',
	&10X, 'PY', 10X, 'ZP', 10X, 'PM')
519	4321 FURMAT(//10X, 'VA=', F10.4, 5X, 'VB=', F10.4, 5X,
	E *ZETA=*, F10.4)
520	3347 FURMAT(//10X, 15, 5E16.7)
621	3349 FORMAT(//10X, 'NO. OF MEMBER', 2X, 'AXIALF', 2X,
	E'PLIMIT', 2X, "RDPM")
622	52 STOP
623	END
624	SUBROUTINE ASATA(NPR,NPS,NM,AM,AV,
	EAMR1, AMR2, AMR3, AMR4, AMY1, AMY2, AMY3, AMY4,
	EAVR1, AVR2, AVR3, AVR4, AVY1, AVY2, AVY3, AVY4, XXA)
625	DIMENSION AMR1(10), AMR2(10), AMR3(10), AMP4(10)
626	DIMENSION AMY1(10), AMY2(10), AMY3(10), AMY4(10)
627	DIMENSION AVR1(10), AVR2(10), AVR3(10), AVR4(10)
629	DIMENSION AVY1(10), AVY2(10), AVY3(10), 4VY4(10)
620	DIMENSION AM(12.12) AV(12.12), XXA(10,10)
0.07	C FORMULATE FRAME STIFFNESS & STABILITY MATRIX
630	DO 419 I=1.NPR
631	DO 419 J=1.NPR
632	XXA(I,I)=0
633	DO 419 K=1.NM
636	$I = 2 \pm K - I$
635	M=2×K
536	$XXA(T, J) = XXA(I, J) + AM(I, L) \times (AMR1(K) + AM(J, L) +$
000	$E \land M \Rightarrow 2 (K) * \land M (J, M) + \land M (I, M) * (\land M \Rightarrow 3 (K) * \land M (J, L) +$
	$\mathcal{E}_{AMR4}(K) \neq AM(J.M))$
: 77	ALO CONTINUE
221	$D_{1} = 1.NPS$
6.20	DD 420 1=1.0P3
6 1 9	
040	YXA(TI = I) = 0
541	AA(11)07-00
642	
5+3	
544	$\frac{1}{2} = 2\pi N$
5+5	CAUP2(K) # AM(1. M)) + AV(T. M) * (AVR3(K) * AM(J.L) +
645	

547	DU 421 I = 1, NPR	
543	DO 421 J=1.NPS	
649	JJ = J + MPR	
650	XXA(I,JJ)=0.	
651	DO 421 K=1.NM	
652	$L = 2 \neq K - 1$	
653	M=2+4	
654	XXA(I,JJ) = XXA(I,JJ) + AM(I,L) * (AMY1(K) * AV(J,	L)+
021	EAMY2(K) *AV(J.M))+AM(I.M)*(AMY3(K)*AV(J.L)+	+
	(X, W) = (X, W)	
655	421 CONTINUE	
656	DD 422 I=1.NPS	
657	DO 422 $J=1.NPS$	
653		
659	1.1=.1+NPR	
660	XXA(TI, LL)=0	
561	DD 422 K=1.NM	
552	$1 = 2 \times K - 1$	
663	$M = 2 \times K$	
654	XXA(TT, III) = XXA(TT, III) + AV(I, I) * (AVY1(K) * AV((J.L)
034	$(+ \Delta V Y 2 (K) + \Delta V (J, M)) + \Delta V (I, M) + (\Delta V Y 3 (K) + \Delta V (J, L))$	+
	$(\Delta V Y 4 (K) \neq \Delta V (J \cdot M))$	
665	422 CONTINUE	
666	DETURN	
203	END	
0.31		
658	SUBROUTINE ASATB (NPR, NPS, NM, AM, AV, AMP1, AMP	2,
000	EAMR3 . AMR4 , AMY1 , AMY2 , AMY3 , AMY4 , AVR1 , AVR2 , AV	/R3,
	EAVR4, AVY1, AVY2, AVY3, AVY4, AMS, XXA)	
	C FORMULATE FRAME MASS MATRIX	
669	DIMENSION AMR1(10), AMR2(10), AMR3(10), AMR4(10)
570	DIMENSION AMY1(10), AMY2(10), AMY3(10), AMY4(10)
671	DIMENSION AVR1(10), AVR2(10), AVR3(10), AVR4(10)
672	DIMENSION AVY1(10), AVY2(10), AVY3(10), AVY4(10)
673	DIMENSION AM(12,12), AV(12,12), XXA(10,10)	
674	DIMENSION AMS(4,4)	
675	DO 419 I=1,NPR	
676	DO 419 J=1,NPR	
677	XXA(I,J)=0.	
678	DO 419 K=1,NM	
679	L=2*K-1	
580	M=2*K	
681	XXA(I,J)=XXA(I,J)+AM(I,L)*(AMR1(K)*AM(J,L)	+
	EAMR2(K) *AM(J, M))+AM(I, N)*(AMR3(K)*AM(J,L)+	F
	$EAMR4(K) \neq AM(J, M))$	
682	419 CONTINUE	
683	DO 420 I=1,NPS	
684	DO 420 J=1,NPR	
685	II=I+NPR	
636	XXA(II,J)=0.	
587	DO 420 K=1,NM	
688	L = 2 * K - 1	
000		

683	M=2×K
690	XXA(II,J)=XXA(II,J)+AV(I,L)*(AVR1(K)*AM(J,L)+
	$EAVR2(K) \neq AM(J,M) + AV(I,M) \neq (AVR3(K) \neq AM(J,L) +$
	$EAVR4(K) * AM(J \cdot M))$
671	420 CONTINUE
692	DO 421 I=1. MP3
403	20.421.1=1.NPS
694	
505	$XX \Lambda (I - 11) = 0$
696	0.0 421 K = 1.5 M
630	1 = 2kk - 1
609	$M = 2 \times K$
600	$XXA(T_{1}) = XXA(T_{1}) + AM(T_{1}) * (AMY1(K) * AV(J_{1}) + AM(T_{1}) * (AMY1(K) * AV(J_{1})) + AM(T_{1}) * (AMY1(K)) + AM(T_{1})) + AM(T_{1}) + AM(T_{1}) + AM(T_{1})) + AM(T_{1}) + AM(T_{1}) + AM(T_{1}) + AM(T_{1})) + AM(T_{1}) + AM(T_{1}) + AM(T_{1}) + AM(T_{1}) + AM(T_{1}) + AM(T_{1})) + AM(T_{1}) + AM(T_{1$
033	$(\Delta M \vee 2 (K) \times A \vee (I, M)) + \Delta M (I, M) \times (\Delta M \vee 3 (K) \times A \vee (I, I)) + (\Delta M \vee 3 (K) \vee A \vee (I, I)) + (\Delta M \vee 3 (K) \vee A \vee (I, I)) + (\Delta M \vee 3 (K) \vee A \vee (I, I)) + (\Delta M \vee A \vee$
	(AMYZ(K) = AV(1, M))
700	
700	
701	
702	
103	
704	
105	XXA(11)JJ = A(3(1)J)
705	DU = 422 K = 10 NM
707	
708	21年2年K 2224777 - 111-2224777 - 11146777 - 1 1★(*//Y1(K)#*//(1-1)
709	
	$\xi + \Delta V Y Z (K) + \Delta V (J, M) + \Delta V (J, M) + (\Delta V + J (K) + \Delta V (J, M))$
	£AVY4(K) ₩AV(J,M))
710	422 CONTINUE
711	RETURN
712	END
	CHOROLITANE ACATMINE ASAT ASATI
713	SUBRICULINE ASATMINE, $4SATMINE, 4SATMINE, 4SATMINE, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10$
714	DIMENSION ASATTOTOTARATICIO COTTOTA
715	
715	16 INDEX(I)=0
717	17 AMAX = -1.
713	101 18 1=1000
713	$I \vdash (I \land U \vdash X \land I) \land I \land $
720	19 TEMP=ABS(ASA)(1,1/7)
721	IF (TEMP-AMAX) 10,10,20
722	20 ICCL=1
723	AMAX=IEMP
724	18 CONTINUE
725	IF (AMAX) 21, 29, 22
726	22 INDEX(ICUL) = 1
727	PIV0T=ASAT(TCOL)TCOL
723	ASAT(ICUL,ICUL)=1.0
729	
730	DD = 23 J = 1 PPP
731	23 ASAT(ICUL, J)=ASAT(ICUL, J)=1 VOT
732	DO 24 I = 1, NP
733	IF(I-ICOL) 23,24,63

734	25	TEMP=ASAT(I,ICOL)
735		ASAT(I,ICOL)=0.0
736		DC = 26 J = 1, NP
737	25	ASAT(I,J)=ASAT(I,J)-ASAT(ICCL,J)*TEMP
739	24	CONTINUE
739		60 TO 17
740	21	DO 27 I = 1, NP
741		DO 27 J=1,NP
742	27	ASATI(I,J)=ASAT(I,J)
743		GO TO 28
744	29	WRITE(3,100)
745	100	FORMAT(//IOX, 'SINGULAR MATRIX OCCURS')
745	28	RETURN
747		END
743		SUBROUTINE SATMV(NPR,NPS,NM,AMR1,AMR2,AMR3,
	3	AMR4, AMY1, AMY2, AMY3, AMY4, AM, AV, AMK)
	C	FORMULATE S*AT
749	-	DIMENSION AM(12,18), AV(12,18), AMK(18,10)
750		DIMENSION AMR1(10), AMR2(10), AMR3(10), AMR4(10)
751		DIMENSION AMY1(10), AMY2(10), AMY3(10), AMY4(10)
752		DO 1500 J=1,NPR
753		DO 1500 K=1, NM
754		L=2*K-1
755		M=2*K
756		AMK(L, J) = AMR1(K) * AM(J, L) + AMR2(K) * AM(J, M)
757		AMK(M, J) = AMR3(K) * AM(J, L) + AMR4(K) * AM(J, M)
758	1500	CONTINUE
759		DO 600 J=1,NPS
760		DO 600 K=1, NM
751		L=2*K-1
752		M=2*K
763		JJ=J+NPR
764		AMK(L,JJ) = AMY1(K) * AV(J,L) + AMY2(K) * AV(J,M)
755		AMK(M,JJ) = AMY3(K) * AV(J,L) + AMY4(K) * AV(J,M)
766	600	CONTINUE
767		RETURN
768		END
759		SUBROUTINE STIFFA(SMR1,SMR2,SMR3,SMR4,
/	ខ	SMY1, SMY2, SMY3, SMY4, SVR1, SVR2, SVF3, SVR4,
	3	SVY1, SVY2, SVY3, SVY4, XMP1, XMP2, XMR3, XMR4,
	ĩ	XMY1, XMY2, XMY3, XMY4, XVP1, XVR2, XVR3, XVP4,
	5	XVY1, XVY2, XVY3, XVY4, I, XE, XI, XL, XM)
770		DIMENSION XI(10), XL(10), XM(10)
771		DIMENSION SMR1(10), SMR2(10), SMY1(10), SMY2(10)
772		DIMENSION SMR3(10), SMR4(10), SMY3(10), SMY4(10)
773		DIMENSION XMR1(10), XMR2(10), XMY1(10), XMY2(10)
174		DIMENSION XMR3(10), XMR4(10), XMY3(10), XMY4(10)
775		DIMENSION SVR1(10), SVR2(10), SVY1(10), SVY2(10)
775		DIMENSION SVR3(10), SVR4(10), SVY3(10), SVY4(10)
777		DIMENSION XVR1(10), XVR2(10), XVY1(10), XVY2(10)

773	DIMENSION XVR3(10), XVR4(10), XVV3(10), XVV4(10)
773	SM21(T) = (4 + x)Exy(T))/y(T)
78.)	
700	
78L 730	S(TX)/1/-(2**AC*A(1))/XL(()
702	5MR4(1)=(4.************************************
733	$S^{MYI}(1) = (-6 \cdot *XE * XI(1)) / (XL(1) * XL(1))$
784	SMY2(I)=(-6.*XE*XI(I))/(XL(I)*XL(I))
735	SMY3(I)=(-6.*XE*XI(I))/(XL(I)*XL(I))
750	SMY4(I)=(-6.*XE*XI(I))/(XL(I)*XL(I))
737	SVR1(I)=(-6.*XE*XI(I))/(XL(I)*XL(I))
788	SVR2(I)=(-6.*XE*XI(I))/(XL(I)*XL(I))
789	SVR3(I)=(-6.*XE*XI(I))/(XL(I)*XL(I))
790	SVR4(I)=(-6.*XE*XI(I))/(XL(I)*XL(I))
791	SVY1(I)=(12.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
792	SVY2(I) = (12.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
793	SVY3(I) = (12, *XF*XI(I))/(XI(I)*XI(I)*XI(I))
794	VV4(T) = (12, *XF*XT(T)) / (XI(T) *XI(T) *XI(T))
795	XMR1(T) = (4.23) M(T) XI(T)
796	$YMP2(T) = (-3. \times YM(T) \times Y(T) \times Y(T) \times Y(T) \times Y(T)) / 4200$
707	$YMP2(I) + (-3 \pm YM(I) \pm YI(I) \pm YI) + YI(I) \pm YI) + YI(I) \pm YI(I) \pm YI(I) \pm YI(I) \pm YI) + YI(I) \pm YI(I) \pm YI(I) \pm YI) + YI(I) \pm YI) + YI(I) + YI(I) + YI) + YI) + YI + YI) + \mathsf$
700	X = X = (x + y) + (x + y
795	$X^{m} + (1) - (+ \bullet + A^{m} + 1) + A \cup (1) + $
199	$X M Y L (L) = (-22 \bullet A M (L) \bullet A L (L) \bullet A L (L) A A L (L) A A A A A A A A A A A A A A A A A A A$
800	X = X = (1) = (1) = (1) = X = (1) = X = (1) = X = (1) = (1
301	XMY3(1) = (+13. *XM(1) + XL(1) * XL(1)) + (1)
302	XMY4(1)=(-22.*XM(1)*XL(1)*XL(1))/420.
803	$XVR1(I) = (-22 \cdot XM(I) \neq XL(I) \neq XL(I))/420$.
304	XVR2(I)=(+13.*XM(I)*XL(I)*XL(I))/420.
805	XVR3(I)=(+13.*XM(I)*XL(I)*XL(1))/420.
305	XVR4(I)=(-22.*XM(I)*XL(I)*XL(I))/420.
9.37	XVYl(I)=(156.*XM(I)*XL(I))/420.
308	XVY2(I)=(-54.*XM(I)*XL(I))/420.
809	XVY3(I)=(-54.*XM(I)*XL(I))/420.
310	XVY4(I)=(156.*XM(I)*XL(I))/420.
911	RETURN
912	END
566	
213	SUBROUTINE STIEFB(SMR1.SMR2.SMR3.SMR4.
010	SCMV1_SMV2_SMV3_SMV4_SVR1_SVR2_SVR3+SVR4+
	CCVV1, CVV2, CVV3, CVV4, YMR1, XMR2, XMR3, XMR4,
	CYNYL YMYZ YMYZ YMYZ YWYZ YWEL YWEZ XWEZ XWEZ
	CVVVI VVVO VVVO VVVA T YE YI YI YM)
	$a_X v_Y L_y X v_Y Z_y X v_Y J_y X v_Y v_Y L_y X L_y A L_y $
314	$\frac{DIMENSION}{CONTACTOTACTOTACTOTACTOT}$
315	$UIMENSION SMRIIII_{SMRIII} SMRIII_{SMRIII} SMRIII $
316	$\frac{1}{100} \frac{1}{100} \frac{1}$
917	DIMENSION XMR1(10), XMR2(10), XMY1(10), XMY2(10)
818	DIMENSION XMR3(10), XMR4(10), XMY3(10), XMY4(10)
319	DIMENSION SVR1(10), SVR2(10), SVY1(10), SVY2(10)
320	DIMENSION SVR3(10), SVR4(10), SVY3(10), SVY4(10)
321	DIMENSION XVP1(10), XVP2(10), XVY1(10), XVY2(10)
322	DIMENSION XVR3(10),XVR4(10),XVY3(10),XVY4(10)
323	SMRl(I) = (3. *XE * XI(I)) / XL(I)
324	SMR2(I) = 0.

975	0-11/564
323	SMR/(I)=0.
020 2 7	3 "K4(1)-U.
221	5471(1)=(->.*XE*X!(1))/(XE(1))*XE(1))
320	$S(1) = (-5 \cdot \pi X = (-1) / (X = (1) / (X = (1)))$
329	SMY3(1)=0.
330	SMY4(1)=0.
531	SVR1(1)=(-3.*XE*X1(1))/(XL(1)*XL(1))
832	SVR2(1)=0.
833	SVR3(1)=(-3.*XF*X1(1))/(XL(1)*XL(1))
834	SVR4(I)=0.
835	SVY1(I)=(3.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
836	SVY2(I)=(3.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
837	SVY3(I)=(3.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
838	SVY4(I)=(3.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
839	XMR1(I)=(8.*XM(I)*XL(I)*XL(I)*XL(I))/420.
840	XMR4(I)=0.
941	XMR2(I)=0.
842	XMR3(I)=0.
843	XMY1(I)=(-36.*XM(I)*XL(I)*XL(I))/420.
344	XMY2(I)=(+11.*XM(I)*XL(I)*XL(I))/280.
845	XMY3(I)=0.
845	XMY4(I)=0.
347	XVR1(I)=(-36.*XM(I)*XL(I)*XL(I))/420.
848	XVR2(I)=0.
849	XVR3[I]=(+11.*XM(I)*XL(I)*XL(I))/280.
850	XVR4(I)=0.
851	$XVY1(I) = (+204 \cdot *XM(I) *XL(I))/420$
852	XVY2(I) = (-39.*XM(I)*XL(I))/230.
853	XVY3(I) = (-39. *XM(I) * XL(I))/280.
854	XVY4(I) = (+99, *XM(I) * XL(I))/420.
355	RETURN
855	END
0,00	
857	SUBROUTINE STIFFCISMR1.SMR2.SMR3.SMR4.
0.21	ESMY1. SMY2. SMY3. SMY4. SVE1. SVR2. SVR3. SVR4.
	ESVY1. SVY2. SVY3. SVY4. XMR1. XMR2. XMR3. XMR4.
	EXMV1, XMV2, XMV3, XMV4, XVR1, XVR2, XVR3, XVR4,
	5 X V Y 1 - X V Y 2 - X V Y 3 - X V Y 4 - I - X F - X I - X L - X M)
259	DIMENSION $XI(10)$, $XI(10)$, $XI(10)$
950	DIMENSION SMR1(10).SMR2(10).SMY1(10).SMY2(10)
36.3	DIMENSION SMR3(10), SMR4(10), SMY3(10), SMY4(10)
30U 041	DIMENSION SHRPT(10), $XMR2(10)$, $XMY1(10)$, $XMY2(10)$
301	DIMENSION XMR3(10) XMR4(10) XMY3(10) XMY4(10)
302	DIMENSION SVP1(10) SVP2(10) SVY1(10) SVY2(10)
000	DIMENSION SVR2(10), SVR2(10), SVY3(10), SVY4(10)
334	$\frac{1}{10} \frac{1}{10} \frac$
305	DIMENSION XVR2(10), XVR2(10), XVY3(10), XVY4(10)
355	
567	SMR711/-12+TALTAINI/ALTI/
853	5MK2(117=0)
869	5MK5(1)=U.
870	CMM/(T)=(=0. **C**T(T))/(Y)(T)*Y((T)) SMKI(T)=(-
371	5M14(1)=(-3.#XE#X1(1)///////////////////////////////////

872	\$443(1)=(-3, *XF*XI(1))/(X)(1)*()
372	
374	S + 12 (1) = 0
014	3/3TIV1/-U. CV3//TV-/-3
070	5VR4(1)=(-3.*XE#XI(1))/(XL(1)*XL(1))
375	SVR3([]=].
377	SVR2(I)=(-3.*XE*XI(I))/(XL(I)*XL(I))
378	SVPl(I)=0.
879	SVY1(I)=(3.*XF*XI(I))/(XL(I)*XL(I)*XL(I))
380	SVY2(I)=(3.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
331	SVY3(I)=(3.*XE*XI(I))/(XL(T)*XL(I)*XL(I))
882	SVY4(I)=(3.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
333	XMR4(I)=(8.*XM(I)*XL(I)*XL(I)*XL(I))/420.
834	XMR1(I)=0.
385	XMR2(I)=0.
886	XMR3(I)=0.
88 7	XMY4(I)=(-36.*XM(I)*XL(I)*XL(I))/420.
388	XMY3(I)=(+11.*XM(I)*XL(I)*XL(I))/280.
839	XMY2(I)=0.
390	XMY1(I)=0
391	XVR4(I) = (-36 * XM(I) * XL(I) * XL(I))/420
392	XVB3(I)=0
893	XVR2(I) = (+11, *XM(I) * XL(I) * XL(I))/280.
904	XVP1(T)=0
305	XVY4(T) = (+204 + XM(T) + XI(T))/420
806	XVY2(1) = (-39. * XM(1) * XI(1)) / 280.
990 997	$Y(Y_3(I) = (-39 + Y_3(I) + Y_1(I))/280$
371	X Y = (-37) + X = (-37) + (-37) + X = (-37) + (-37)
070	AVILL1/-(+97• *AN(1)*AL\1/// +20• RETURN
579	RE I URIN
400	t N D
	CHAROLTINE CTICEN/CND1 CND2 CND2 CMD4
301	SUBRUUTINE STIFFULSMRI, SMRZ, SMRZ, SMRZ,
	45MT1,5MT2,5MT3,5MT4,5VR1,5VR2,5VR3,5VR4,
	XVY1, XVY2, XVY3, XVY4, 1, XE, X1, AL, A. T
902	$\frac{1}{1}$
903	DIMENSION SMR1(10), SMR2(10), SMR1(10), SMR2(10)
904	DIMENSION SMR3(10), SMR4(10), SMY3(10), SMY4(10)
205	DIMENSION XMR1(10), XMR2(10), XMY1(10), XMY2(10)
905	DIMENSION $XMR3(10)$, $XMR4(10)$, $XMY3(10)$, $XMY4(10)$
907	DIMENSION SVR1(10), SVR2(10), SVY1(10), SVY2(10)
903	DIMENSION SVR3(10), SVR4(10), SVY3(10), SVY4(10)
909	DIMENSION XVR1(10), XVR2(10), XVY1(10), XVY2(10)
910 .	DIMENSION XVR3(10),XVR4(10),XVY3(10),XVY4(10)
911	SMR2(I)=0.
912	SMR1(I) = 0.
913	SMR3(I)=0.
914	$5^{4}R4(I)=0.$
915	SMY1(T)=0.
916	SMY2(I)=0.
917	SMY3(I)=0.
918	SMY4(I)=0.

,	
919	SVR1(I)=0.
920	SVR2(I)=0.
921	SVR4(I)=0.
922	SVR3(I)=0.
923	SVY1(I)=0.
924	SVY2(I)=0.
725	SVY3(I)=0.
926	SVY4(I)=0.
927	XMR1(I)=0.
929	XMR2(I)=0.
929	XMR3(I)=0.
730	XMR4(I)=0.
931	XMY1(I) = 0.
932	XMY2(I)=0.
933	XMY3(I)=0.
934	XMY4(I)=0.
935	XVR1(I)=0.
935	XVR2(I)=0.
937	XVR3(I)=0.
938	XVR4(T)=0
939	XVY1(I) = (2, *XM(I) * XI(I))/3,
940	XVY2(I) = (-1.*XM(I)*XL(I))/3.
941	XVY3(I) = (-1, *XM(I) * XI(I))/3
942	XVY4(I) = (2 * X'(I) * XL(I))/3
943	RETURN
944	FND
945	SUBBOUTINE STIEPA(PMR1.PMR2.PMR3.PMR4.
	EPMY1. PMY2. PMY3. PMY4. PVR1. PVR2. PVR3. PVR4.
*	EPVY1.PVY2.PVY3.PVY4.I.XL.AF)
946	DIMENSION $XI(10) \cdot XI(10) \cdot AF(10)$
247	DIMENSION PMR1(10) • PMR2(10) • PMY1(10) • PMY2(10)
943	DIMENSION PMR3(10), PMR4(10), PMY3(10), PMY4(10)
340	DIMENSION PVR1(10) \cdot PVR2(10) \cdot PVY1(10) \cdot PVY2(10)
350	DIMENSION PVR3(10) PVR4(10) PVY3(10) PVY4(10)
951	$PMR2(I) = AF(I) \times (-XI(I)/30_{-})$
952	$PMR3(I) = \Delta F(I) * (-XI(I)/30_{-})$
053	$PMR4(T) = \Delta F(T) \neq (2 \neq X) (T) / (15)$
254	$PMR1(T) = \Delta F(T) + (2 + X) (T) / (15)$
255	$PMY1(I) = AF(I) \neq (-1 \cdot / 10 \cdot)$
956	$PMY2(1) = \Delta F(1) + (-1)/10$
057	$PMY3(I) = AF(I) * (-1 \cdot / 10 \cdot)$
059	$PMY4(1) = AF(1) \neq (-1 \cdot / 10 \cdot)$
325	$P_{1}(T) = A_{F}(T) + (-1) / (0)$
323	$PVP2(1) = AF(1) \times (-1) / (0)$
960	$PVR3(1) = \Delta F(1) * (-1 / 10)$
701	$PVPA(T) = AF(T) \times (-1 \cdot / 10 \cdot)$
702	$PVY1(T) = \Delta F(T) * (A_1/(S_*X) (T))$
703	DVV2(T)=AF(T)*(6./(5.**((T)))
754	PVV3(I)=AF(I)*(6./15.*X1(I))
707	PVT3(1)-4F(1)±(6./15.±4)/11)
956	FV14(1)-4F11/=100/120=AL11///
957	KEIUKN

.

7	53	FND

939)	SUBROUTINE STIFPR(PMR1, PMR2, PMP3, PMR4,
		&PMY1, PMY2, PMY3, PMY4, PVR1, PVR2, PVR3, PVR4,
		&PVY1, PVY2, PVY3, PVY4, I, XL, AF)
970)	DIMENSION XI(10), XL(10), AF(10)
771		DIMENSION PMR1(10), PMR2(10), PMY1(10), PMY2(10)
972		DIMENSION PMR3(10), PMR4(10), PMY3(10), PMY4(10)
973		DIMENSION PVR1(10), PVR2(10), PVY1(10), PVY2(10)
74	•	DIMENSION PVR3(10), PVR4(10), PVY3(10), PVY4(10)
975	5	PMR1(I)=AF(I)*(1.*XL(I)/5.)
776		PMP2(I)=AF(I)*O.
977		$PMR3(I) = AF(I) \neq 0$.
978		PMR4(I) = AF(I) * 0.
979)	$PMY1(I) = AF(I) \neq (-1./5.)$
930) .	PMY2(I) = AF(I) * (-1./5.)
9 81		PMY3(I) = AF(I) * 0.
932		$PMY4(I)=AF(I) \neq 0.$
933	5	$PVR1(I) = AF(I) \neq (-1./5.)$
934	•	PVR2(I) = AF(I) * 0.
935	5	PVR3(I) = AF(I) * (-1./5.)
935		PVR4(I)=AF(I)≠0.
7 87		PVY1(I)=AF(I)*(6./(5.*XL(I)))
938	8	PVY2(I)=4F(I)*(6./(5.*XL(I)))
933		PVY3(I)=AF(I)*(6./(5.*XL(I)))
390)	PVY4(I)=AF(I)*(6•/(5•*XL(I)))
<u> </u>	•	RETURN
992		END
202	Ł	SUBDOUTINE STIEDC(PMR1.PMR2.PMR3.PMR4.
	,	EDMY1. DMY2. DMY3. DMY4. DVE1. DVE2. DVE3. DVE4.
		EPVYL PVY2 PVY3 PVY4 T.XI AF)
394		DIMENSION $XI(10) \cdot XI(10) \cdot \Delta F(10)$
395		DIMENSION PMR1(10) \cdot PMP2(10) \cdot PMY1(10) \cdot PMY2(10)
996	·. ·	DIMENSION PMR3(10), PMR4(10), PMY3(10), PMY4(10)
397		DIMENSION PVR1(10) . PVR2(10) . PVY1(10) . PVY2(10)
335	ł	DIMENSION PVR3(10) • PVR4(10) • PVY3(10) • PVY4(10)
000		$PMR1(I) = AF(I) * O_{\bullet}$
1200		PMP2(I) = AF(I) * 0.
1001		PMR3(I) = AF(I) * O.
1002		PMR4(I) = AF(I) * (1.*XL(I)/5.)
1007		$PMY1(I) = AF(I) \neq 0$.
1004		$PMY2(I) = AF(I) \neq 0$.
1005		PMY3(I) = AF(I) * (-1./5.)
1004		PMY4(I) = AF(I) * (-1./5.)
1207		PVR1(I) = AF(I) * 0.
10:08	L.	PVR2(I) = AF(I) * (-1./5.)
1000)	$PVR3(I) = AF(I) \neq 0$.
1010		PVR4(I) = AF(I) * (-1./5.)
1011		
		PVY1(I)=AF(I)*(6./(5.*XL(I)))
1012		PVY1(I)=AF(I)*(6./(5.*XL(I))) PVY2(I)=AF(I)*(6./(5.*XL(I)))

1014	PVY4(I)=AF(I)*(6./(5.*XL(I)))
1.115	DETUDN

1015	RETURN
1010	E 11/2

1016 END

1217	SUBPOUTINE STIEPD(PMB1, PMP2, PMP3, PMB4,
	EPMY1 - PMY2 - PMY3 - PMY4 - DVD1 - DVD2 - DVD3 - DVD4
1.119	$d^2 + (1 + (1 + (1 + (1 + (1 + (1 + (1 + ($
1010	DIMENSION AIVIG/ALVID/APVID/
1019	DIMENSION PMRI(10) PMR2(10) PMY1(10) PMY2(10)
1020	DIMENSION PMR3(10), PMR4(10), PMY3(10), PMY4(10)
1251	DIMENSION PVR1(10), PVR2(10), PVY1(10), PVY2(10)
1022	DIMENSION PVR3(10), PVR4(10), PVY3(10), PVY4(10)
1023	$PMR1(I) = AF(I) \neq 0$.
1024	$PMR2(I)=AF(I)\neq0$.
1025	$PMR3(I) = AF(I) \neq 0$.
1025	PMR4(I)=AF(I)*0.
1027	PMY1(I)=AF(I)=0
1029	$PMY2(I) = \Delta F(I) \neq 0$
1029	$PMY3(I) = \Delta F(I) \neq 0$
1030	$DNVA(I) = AF(I) \neq 0$
1030	
1031	PVPD(I) = AF(I) + 0
1032	PVN2(1)=AF(1)+0
1033	$PVR3(1) = \Delta F(1) \neq 0.$
1034	PVR4(I)=AF(I)*0.
1035	PVY1(I) = AF(I) * 0.
1036	$PVY2(I) = AF(I) \neq 0$.
1037	PVY3(I)=4F(I)*0.
1038	PVY4(I)=AF(I)*0.
1739	RETURN
1140	FND
1.741	SUBROUTIME GEXTPLAM.AV.NP.NPR.NM.EEM.EEV.PSE.
1,41	SOURCE TI COLUMN STATE OF THE S
1042	$DIMENSION = EV(12) \cdot EEM(12) \cdot PE(10) \cdot PSET(10)$
1042	DIMENSION AN($12,12$) AV($12,12$) DSE(10)
1043	DIMENSION AM(12)12/94V(12)12/9F3C(10)
1344	NEM=NM#Z
1045	NPS=NP-NPR
1746	DO 563 $I=1, NPR$
1047	PE(I)=0.
1048	DD 563 $J=1$, NEM
1349	$PE(I) = PE(I) + AM(I,J) \neq FEM(J)$
1250	563 CONTINUE
1051	DO 564 I=1.NPS
1152	
1053	PE(II)=0
1054	DO 564 J=1-NEM
1024	
1 1 5 5	$DC(TT) \rightarrow DC(TT) \rightarrow AV(T, 1) = EV(1)$
1055	PE(II) = PE(II) + AV(I, J) + EV(J)
1055	PE(II)=PE(II)+AV(I,J)*FEV(J) 564 CONTINUE
1055 1055 1057	PE(II)=PE(II)+AV(I,J)*FEV(J) 564 CONTINUE DO 565 I=1,NP
1055 1055 1057 1059	PE(II)=PE(II)+AV(I,J)*FEV(J) 564 CONTINUE DO 565 I=1,NP RSFT(I)=-PE(I)-PSE(I)
1055 1055 1057 1059 1059	PE(II)=PE(II)+AV(I,J)*FEV(J) 564 CONTINUE DD 565 I=1,NP RSFT(I)=-PE(I)-PSE(I) 565 CONTINUE

1061		END
1052		SUBROUTINE GEMKP(T,DT,NP,NPP,VA,VB,ZETA,P38,X,
	8	IXXP,XXK,XMI,RSFT,G)
1053		DIMENSION XP(10,10), XMF(10), XXMPK(10),
	8	XMPK(10,10),XXK(10,10)
1064		DIMENSION FT(10),X(10),XXP(10,10),XMJ(10,10),
	8	EG(10), DG(10)
1365		DIMENSION RSFT(10), SFT(10)
1065		NPS=NP-NPR
1057		TS1=0.02
1058		152=0.02
1069		1D1=0.04
1370		TD2=0.08
1971		103=0.12
1972		104=0.16
1373		TD5=0.20
1074		TD6=0.24
1075		F01=10000.
1076		FO2=10000.
1077		SLOPI=FUI/ISI
1078		SLOP2=F02/1S2
1079		IF(T-TD1) 9006,5006,9007
1030	9006	SFI(1)=F01-SLUP1*1
1031		GD 10 9008
1082	9007	IF(I-TD2) 9009,5009,9010
1083	9009	$SFT(1) = -FU2 + SLUP2 \neq (1 - TD1)$
1084		GU IU 9008
1085	9010	1F(1-103) 9011,9011,9012
1385	9011	SFI(1)=FU1-SLUP1*(1-102)
1.0.37	0010	GU 10 9008 JELT TOLD 2012 0013 0016
1000	9012	1F(1-104) 9013,9013,9014
1039	9013	SFI(1) ====02+SLUP2+(1=105)
1090	0014	50 TO 9008
1091	9014	IF(1-103) = 9013,9013,9010
1992	9015	SFI(1)=F01-5L0F1+(1-104)
1093	001/	GU TU 9000 TELT-TDAN 0017 0017-0019
1394	9010	2 = 1 = 1007 = 3017 =
10.75	9017	3FIL1+-FULT3LUFLTLIDJ1
1095	0010	60 10 9005 CET(1)-0
1097	3012	DD 0004 I-1 NDP
1348	9003	CT(1)-005T(1)
1044	000/	
1101	9004	
1102		TN =NDD+T
1102		ET(TN) = SET(T) + RSET(TN)
11.34	2005	
1105	1005	
		YNE(1)=0-
11.07		XXMPK(I) = 0.
1105		
1100		

1109		XMPK(I,J)=0.
1110		DO 908 K=1,NP
1111	908	$XMPK(I,J)=XMPK(I,J)+XMI(I,K) \neq (XXP(K,J)-XXK(K,J))$
1112		XXMPK(I) = XXMPK(I) + XMPK(I, J) * X(J)
1113	907	XMF(I) = XMF(I) + XMI(I,J) + FT(J)
1114		DG(I) = XMF(I) + XXMPK(I)
1115		G(I)=OG(I)*OT
1116	906	CONTINUE
1117		RETURN
1113		END

VITA

Wu Hsiung Tseng was born on February 19, 1941, in Kangshan, Kaohsiung, Taiwan. He attended Kaohsiung High School in Kaohsiung, Taiwan, and graduated in June, 1960. He receive a bachelor of Science degree in Civil Engineering from Cheng Kung University, Tainan, Taiwan, in June, 1964. In September, 1967, he enrolled in the Graduate School at the University of Missouri at Rolla, and completed the requirements for a Master of Science degree in Civil Engineering in January, 1969. He was admitted as a candidate for the degree of Doctor of Philosophy in Civil Engineering in February, 1970, and has since so enrolled in the Graduate School at the University of Missouri at Rolla.

145