Decision Making Based on the Aggregation Operator and the Intuitionistic Fuzzy Reduction Method of Intuitionistic Fuzzy Parameterized Intuitionistic Fuzzy Soft Sets

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***Abstract*—In this paper, we introduce the aggregation operator of IFPIFS set and apply this operator in a hypothetical decision making problem involving attributes and parameters that are subjective in nature. Specifically, this operator is applied in a decision making problem involving the selection of the best candidate for a vacant position in an organization. Next, we introduce an algorithm called the intuitionistic fuzzy (IF) reduction method which involves the reduction of the original IFPIFS set into an IF set and subsequently a fuzzy set which would then be used to determine the optimal solution for the problem. We demonstrate the application of this algorithm in an object recognition problem which involves subjective and uncertain data.**

***Index Terms*—Fuzzy Parameterized Intuitionistic Fuzzy Soft Expert Set; Fuzzy Parameterized Soft Expert Set; Fuzzy Soft Expert Set; Soft Set; Decision Making.**

I. Introduction

Molodtsov [1] introduced the theory of soft sets as a general mathematical tool to deal with uncertainties and vagueness in a more effective and accurate manner compared to the existing tools that have been traditionally used to deal with uncertainties and vagueness such as fuzzy set theory by Zadeh [2], rough set theory by Pawlak [3] and probability theory. However, all of these theories have their inherent weaknesses as pointed out by [1]. Maji et al. then generalized the notion of soft sets to establish the notion of fuzzy soft sets [4] and intuitionistic fuzzy soft sets [5] and also presented an application of these concepts in decision making problems. Cagman et al. then introduced the notion of fuzzy parameterized fuzzy soft sets [6] and fuzzy parameterized soft sets [7] and studied some of their basic properties. Alkhazaleh et al. [8] then generalized the concept introduced in [6] to introduce the notion of fuzzy parameterized interval-valued fuzzy soft sets. They also gave an application of this concept in a decision making problem. Selvachandran & Salleh [9] then introduced the notion of fuzzy parameterized intuitionistic fuzzy soft expert set theory as a generalization of [8] and fuzzy soft expert set theory (see [10]). Thus far, most of the research in this area has revolved around the generalization of fuzzy parameterized fuzzy soft sets until the introduction of the concept of intuitionistic fuzzy parameterized intuitionistic fuzzy soft set (IFPIFSS) by Karaaslan, Cagman and Yilmaz in [12]. Following in this direction, Deli & Cagman [13] introduced the concept of intuitionistic fuzzy parameterized soft sets (IFPSS) and applied this concept in various decision making problems.

In this paper, we study the decision making methods pertaining to the concept of IFPIFSS. The IFPIFSS is an improvement to [6] and [8] and utilizes the concept of intuitionistic fuzzy sets which has been proven to be superior to fuzzy sets and thus can better reflect the imprecision, uncertainties and fuzziness which are characteristics that are pervasive in describing probability parameters and the data involved in most real-life problems (we refer the readers to [11]).

We introduce the aggregation operator of IFPIFSS, which is then applied in a decision making problem related to the selection of the best candidate in a job interview which is set in an imprecise environment. Lastly, we introduce an algorithm called the intuitionistic fuzzy (IF) reduction method which is subsequently used to form a systematic and practical decision making algorithm based on the concept of IFPIFSS. The application of this concept and this method in decision making is then illustrated using a hypothetical example related to an object recognition problem.

II. Preliminaries

In this section, we recall some definitions and properties pertaining to intuitionistic fuzzy sets, soft sets and its hybrid structures.

Definition 2.1 ([1]). A pair (F, A) is called a *soft set* over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-elements of the soft set (F, A) or as the ε-approximate elements of the soft set.

Definition 2.2 ([11]). An *intuitionistic fuzzy set A* defined over a universe of discourse U is an object in the following form:

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in U \} \quad (1)$$

where the function $\mu_A : U \to [0,1]$ and $\nu_A : U \to [0,1]$ are the membership function and non membership function respectively of every element $x \in U$ to set A and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for every $x \in U$. In the event that $0 \leq$

$\mu_A(x) + \nu_A(x) < 1$, there is a degree of uncertainty that exists for element x with respect to set A. This degree of uncertainty, denoted as $\pi_A(x)$ is defined as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$. In general, a high degree of uncertainty implies that there are a lot things that are unknown about element x with respect to set A.

From now on, let A and B be intuitionistic fuzzy sets defined over a universal set U and are as defined below:

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in U \} \quad (2)$$
$$B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in U \} \quad (3)$$

Definition 2.3 ([11]). The *subset* and *equality* of two intuitionistic fuzzy sets A and B are as defined below:

(a) $A \subset B \leftrightarrow \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in U$

(b) $A = B \leftrightarrow A \subset B$ and $B \subset A$.

Definition 2.4 ([11]). The *complement*, *union* and *intersection* of two intuitionistic fuzzy sets A and B are as defined below:

(a) $\bar{A} = \{\langle x, \nu_A(x), \mu_A(x) \rangle : x \in U\}$

(b) $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) \rangle : x \in U\}$

(c) $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle : x \in U\}$

Definition 2.5 ([5]). Consider U and E as a universe set and a set of parameters respectively. Let $P(U)$ denote the set of all intuitionistic fuzzy sets of U. Let $A \subseteq E$. A pair (F, E) is an *intuitionistic fuzzy soft set* over U, where F is a mapping given by $F : A \to P(U)$.

Definition 2.6 ([7]). Let U be an initial universe, $P(U)$ be the power set U, E be the set of all parameters and X be a fuzzy set over E with membership funtion $\mu_X : E \to [0, 1]$. Then a *fuzzy parameterized soft set* (fps-set) F_X over U is a set defined by a function f_X representing a mapping

$$f_X : E \to P(U) \quad (4)$$

such that $f_X(x) = \emptyset$ if $\mu_X(x) = 0$. Here f_X is called an approximate function of the fps-set F_X and the value $f_X(x)$ is a set called the x-element of the fps-set for all $x \in E$. Thus, an fps-set F_X over U can be represented by the set of ordered pairs.

$$F_X = \left\{ \left(\frac{\mu_X(x)}{x}, f_X(x) \right) : x \in E, f_X(x) \in P(U), \mu_X(x) \in [0, 1] \right\} \quad (5)$$

Definition 2.7 ([8]). Let U be an initial universe, E the set of all parameters and X a fuzzy set over E with membership function

$$\mu_X : E \to [0,1], \quad (6)$$

and let τ_X be a fuzzy set over U for all $x \in E$. Then a *fuzzy parameterized fuzzy soft set* (fpfs-set) ρ_X over U is a set defined by a function $\tau_X(x)$ representing a mapping $\tau_X : E \to F(U)$ such that $\tau_X(x) = \emptyset$ if $\mu_X(x) = 0$. Here, τ_X is called a fuzzy approximate function of the fpfs-set ρ_X and the value $\tau_X(x)$ is a set called x −element of the fpfs-set for all $x \in E$. Thus, an fpfs-set ρ_X over U can be represented by the set of ordered pairs:

$$\rho_X = \left\{ \left(\frac{\mu_X(x)}{x}, \tau_X(x) \right) : x \in E, \tau_X(x) \in F(U), \mu_X(x) \in [0,1] \right\} \quad (7)$$

III. INTUITIONISTIC FUZZY PARAMETERIZED INTUITIONISTIC FUZZY SOFT SETS

The approximate functions of a soft set is defined from crisp subsets of the universal set. Karaaslan et al. [12] defined the approximate functions of an intuitionistic fuzzy parameterized intuitionistic fuzzy soft set (IPPIFSS) from an IFS of parameters to the IF subsets of a universal set.

Definition 3.1 ([12]). From now on, let U be an initial universe, $P(U)$ be the power set U, E be the set of all parameters and X be an IFS over E with membership funtion $\mu_X : E \to [0, 1]$ and non-membership function $\gamma_X : E \to [0, 1]$ and let δ_X be an IFS over U for all $x \in E$. Then an *intuitionistic fuzzy parameterized intuitionistic fuzzy soft set* (IFPIFSS) F_X over U is a set defined by function δ_X representing a mapping:

$$\delta_X : E \to IFS(u) \quad (8)$$

such that $\delta_X(x) = \emptyset$ if $\mu_X(x) = 0$. Here $\delta_X = [\mu_{\delta_X}(x), \gamma_{\delta_X}(x)]$ is called an IF approximate function of the IFPIFSS F_X and the value of $\delta_X(x)$ is a set called x- element of the IFPIFSS for all $x \in E$. Thus an IFPIFSS F_X over U can be represented by the set of ordered pairs :

$$F_X = \left\{ \left(\frac{x}{[\mu_X(x), \gamma_X(x)]}, \delta_X(x) \right) : \in E, \right. \quad (9)$$

$$\left. \delta_X(x) \in IFS\,(u), \mu_X(x) \in [0, 1], \gamma_X(x) \in [0,1] \right\}. \quad (10)$$

The set of all IFPIFSS over U will be denoted by $IFPIFSS\,(U)$.

Example 3.2 Let $U = \{u_1, u_2, u_3, u_4\}$ be an initial universal set, $E = \{x_1, x_2, x_3\}$ be a set of parameters, $\mu_X : E \to [0, 1]$ and $\gamma_X : E \to [0, 1]$. Suppose $X = \left\{ \frac{x_1}{[0.1, 0.6]}, \frac{x_2}{[0.4, 0.5]}, \frac{x_3}{[0.8, 0.2]} \right\}$ and $\delta_X(x)$ is an IFS defined as given below:

$$\begin{aligned} \delta_X(x_1) &= \left\{ \frac{u_1}{[0.3, 0.6]}, \frac{u_2}{[0, 1]}, \frac{u_3}{[0.5, 0.5]}, \frac{u_4}{[1, 0]} \right\}, \\ \delta_X(x_2) &= \left\{ \frac{u_1}{[0.3, 0.7]}, \frac{u_2}{[0.2, 0.6]}, \frac{u_3}{[1, 0]}, \frac{u_4}{[0.1, 0.75]} \right\}, \\ \delta_X(x_3) &= \left\{ \frac{u_1}{[0.2, 0.8]}, \frac{u_2}{[0.4, 0.6]}, \frac{u_3}{[0.9, 0.05]}, \frac{u_4}{[0.6, 0.3]} \right\}. \end{aligned} \quad (11)$$

Then the IFPIFSS F_X is given by:

$$F_X = \left\{ \left(\frac{x_1}{[0.1, 0.6]}, \{\delta_X(x_1)\} \right), \left(\frac{x_2}{[0.4, 0.5]}, \{\delta_X(x_2)\} \right), \left(\frac{x_3}{[0.8, 0.2]}, \{\delta_X(x_3)\} \right) \right\} \quad (12)$$

Definition 3.3 ([12]). Let F_X and F_Y be two $IFPIFSS\,(U)$. Then F_X is said to be an *IFPIFS subset* of F_Y if

(a) $\mu_X(x) \leq \mu_Y(x)$ and $\gamma_X(x) \geq \gamma_Y(x)$ for all $x \in E$,

(b) $\delta_X(x) \leq \delta_Y(x)$ for all $x \in E$.

This relationship is denoted as $F_X \precsim F_Y$.

Definition 3.4 ([12]). Let $F_X \in IFPIFSS\,(U)$. Then the *complement* of F_X, denoted by ${F_X}^C$, is defined by $\tilde{c}[\mu_X(x), \gamma_X(x)]$ and $\tilde{c}(\delta_X(x))$, $\forall_x \in E$, where $\tilde{c}$ is the intuitionistic fuzzy complement.

Definition 3.5 ([12]). Let F_X and F_Y be two $IFPIFSS\ (U)$. Then the union of F_X and F_Y, denoted by $F_X\ \tilde{\cup}\ F_Y$, is defined by:

$$\mu_{X\tilde{\cup}Y}(x) = \max\{\mu_X(x), \mu_Y(x)\},\quad \gamma_{X\tilde{\cup}Y}(x) = \min\{\gamma_X(x), \gamma_Y(x)\}, \tag{13}$$

and

$$\delta_{X\tilde{\cup}Y}(x) = \delta_X(x)\ \tilde{\cup}\ \delta_Y(x),$$

where $\tilde{\cup}$ is the intuitionistic fuzzy union.

Definition 3.6 ([12]). Let F_X and F_Y be two $IFPIFSS\ (U)$. Then the intersection of F_X and F_Y, denoted by $F_X\ \tilde{\cap}\ F_Y$, is defined by:

$$\mu_{X\tilde{\cap}Y}(x) = \min\{\mu_X(x), \mu_Y(x)\},\quad \gamma_{X\tilde{\cap}Y}(x) = \max\{\gamma_X(x), \gamma_Y(x)\}, \tag{14}$$

and

$$\delta_{X\tilde{\cap}Y}(x) = \delta_X(x)\ \tilde{\cap}\ \delta_Y(x), \tag{15}$$

where $\tilde{\cap}$ is the intuitionistic fuzzy intersection.

IV. AGGREGATION OPERATOR OF IFPIFSS

In this section, we define an aggregate intuitionistic fuzzy set of an IFPIFSS. We then introduce an IFPIFSS-aggregation operator that produces an aggregate intuitionistic fuzzy set from an IFPIFSS and its intuitionistic fuzzy parameter set. Lastly, we present an application of this operator in a decision making problem.

Definition 4.1 Let $F_X \in IFPIFSS\ (U)$. Then an *IFPIFSS-aggregation operator*, denoted by $IFPIFSS_{agg}$, is defined by:

$$IFPIFSS_{agg} : F(E) \times IFPIFSS(U) \to IFS(U),\quad IFPIFSS_{agg}(X, F_X) = {F_X}^*, \tag{16}$$

where:

$${F_X}^* = \left\{\frac{u}{[\mu_{F_X^*}(u), \gamma_{F_X^*}(u)]} : u \in U\right\} \tag{17}$$

which is an intuitionistic fuzzy set over U. The value ${F_X}^*$ is called an aggregate intuitionistic fuzzy set of F_X. Here the membership function $\mu_{F_X^*}(u)$ and non-membership function $\gamma_{F_X^*}(u)$ of $u \in U$ are defined as follows:

$$\mu_{F_X^*}(u) = \frac{1}{|E|}\sum_{x \in E} \mu_X(x)\ \mu_{\delta_X(x)}(u) \tag{18}$$

and

$$\gamma_{F_X^*}(u) = \frac{1}{|E|}\sum_{x \in E} \gamma_X(x)\ \gamma_{\delta_X(x)}(u), \tag{19}$$

where $|E|$ is the cardinality of E.

Example 4.2 Suppose that company Y is looking to hire a person to fill in the vacancy for a position in their company. Out of all the people who applied for the position, four candidates were shortlisted and these three candidates form the universe of elements, $U = \{u_1, u_2, u_3, u_4\}$. The hiring committee consists of the hiring manager, head of department and the HR director of the company considers a set of parameters, $E = \{x_1, x_2, x_3, x_4\}$, where the parameters $x_i\ (i = 1, 2, 3, 4)$ represent the characteristics or qualities that the candidates are assessed on, namely “relevant job experience”, “excellent academic qualifications in the relevant field”, “attitude and level of professionalism” and “technical knowledge” respectively. After interviewing all the three candidates and going through their certificates and other supporting documents, the hiring committee constructs the intuitionistic fuzzy set

$$X = \left\{\frac{x_1}{[0.9, 0]}, \frac{x_2}{[0.7, 0.3]}, \frac{x_3}{[0.8, 0.2]}, \frac{x_4}{[0.5, 0.5]}\right\} \tag{20}$$

and subsequently uses it to construct the following IFPIFSS over U.

Step 1
Let the IFPIFSS constructed by the selection committee F_X, be defined as follows:

$$F_X = \left\{\left(\frac{x_1}{[0.9, 0]}, \left\{\frac{u_1}{[0.8, 0.1]}, \frac{u_2}{[0.4, 0.5]}, \frac{u_3}{[0.6, 0.4]}, \frac{u_4}{[0.3, 0.7]}\right\}\right), \left(\frac{x_2}{[0.7, 0.3]}, \left\{\frac{u_1}{[0, 1]}, \frac{u_2}{[0.3, 0.5]}, \frac{u_3}{[0.2, 0.75]}, \frac{u_4}{[0.9, 0.1]}\right\}\right), \left(\frac{x_3}{[0.8, 0.2]}, \left\{\frac{u_1}{[0.2, 0.7]}, \frac{u_2}{[1, 0]}, \frac{u_3}{[0.5, 0.5]}, \frac{u_4}{[0.8, 0.2]}\right\}\right), \left(\frac{x_4}{[0.5, 0.5]}, \left\{\frac{u_1}{[0.25, 0.75]}, \frac{u_2}{[0.05, 0.9]}, \frac{u_3}{[0.4, 0.4]}, \frac{u_4}{[1, 0]}\right\}\right)\right\}.$$

Step 2
The aggregate intuitionistic fuzzy set can be computed and is as given below:

$${F_X}^* = \left\{\frac{u_1}{[0.25125, 0.20375]}, \frac{u_2}{[0.34875, 0.15]}, \frac{u_3}{[0.32, 0.13125]}, \frac{u_4}{[0.51, 0.0175]}\right\}$$

Step 3
$\forall_{u_i} \in U$, compute the score r_i of u_i where

$$r_i = \sum_{u_j \in U}\left((u_i - u_j) + (\gamma_i - \gamma_j)\right).$$

Then we obtain the following scores:
$r_1 = (-0.425) + (0.3125) = -0.1125$
$r_2 = (-0.035) + (0.0975) = 0.0625$
$r_3 = (-0.15) + 0.0225 = -0.1275$
$r_4 = 0.61 + (-0.4325) = 0.1775$

Step 4
The decision is any one of the elements in S, where $S = \max_{u_i \in U}\{r_i\}$. In this example, candidate u_4 is the best choice as $\max_{u_i \in U}\{r_i\} = r_4$. Hence candidate u_4 should be selected for the job.

V. THE IF REDUCTION METHOD FOR IFPIFSS

In this section, we introduce an algorithm called the intuitionistic fuzzy (IF) reduction method for IFPIFSS. We also define the concepts of the reduced IFS of an IFPIFSS and the reduced fuzzy set of the reduced IFS. All these concepts are then used together with the algorithm to solve a hypothetical object recognition problem involving uncertain and subjective data.

Definition 5.1 Let $F_P \in IFPIFSS\ (U)$. Then a reduced IFS of F_P, denoted by P_{RIF}, is defined as follows:

$$P_{RIF} = \left\{\frac{u}{[\mu_{P_{RIF}}(u), \gamma_{P_{RIF}}(u)]} : u \in U\right\}, \quad (21)$$

where

$$\mu_{P_{RIF}} : U \to [0,1], \mu_{P_{RIF}} = \frac{1}{|U|} \sum_{x \in E, u \in U} \mu_X(x)\, \mu_{\delta_X(x)}(u) \quad (22)$$

and

$$\nu_{P_{RIF}} : U \to [0,1], \nu_{P_{RIF}} = \frac{1}{|U|} \sum_{x \in E, u \in U} \nu_X(x)\, \nu_{\delta_X(x)}(u), \quad (23)$$

where $|U|$ is the cardinality of U.

Definition 5.2 Let $F_P \in IFPIFSS\ (U)$ and P_{RIF} be a reduced IFS of F_P. Then a reduced fuzzy set of P_{RIF}, denoted by P_{RF} is a fuzzy set over U, which is defined as follows:

$$P_{RF} = \left\{\frac{u}{\mu_{P_{RF}}(u)} : u \in U\right\}, \quad (24)$$

where

$$\mu_{P_{RF}} : U \to [0,1], \mu_{P_{RF}} = \mu_{P_{RIF}}(u)\left(1 - \nu_{P_{RIF}}(u)\right) \quad (25)$$

Next we introduce an algorithm for the IF reduction method. This algorithm involves the reduction of the intuitionistic fuzzy sets in the original IFPIFSS to a single intuitionistic fuzzy set and subsequently reduces this intuitionistic fuzzy set to a fuzzy set, which can then be used to obtain the optimal solution.

This algorithm is as given below:

Step 1
An IFPIFS set is constructed by the set of experts dealing with the problem.

Step 2
Compute the reduced IF set P_{RIF} of the IFPIFS set.

Step 3
Compute the reduced fuzzy set P_{RF} of the IFPIFS set.

Step 4
Determine the element of the reduced fuzzy set P_{RF} that has maximum membership degree. Then the decision is to choose this element as the optimal solution to the problem. If there are more than one element with the highest membership degree, then any one of the elements can be chosen as the optimal solution.

Next we apply this algorithm in an object recognition problem.

Step 1
Consider an object recognition problem where a student is considering a set of objects in an experiment. Let $U = \{k_1, k_2, k_3, k_4, k_5, k_6\}$ be the set of objects having different colours, sizes and price features. The parameter set E which describes the attributes of these objects is given by:

$$E = \{x_1 = \text{dark coloured}, x_2 = \text{small}, x_3 = \text{expensive}, x_4 = \text{round shaped}, x_5 = \text{shiny}\}.$$

Now suppose that the IFPIFS set F_P describes the set of objects with the attributes described above. The student constructs the following IFPIFS sets with information obtained from her experiments and the IFPIFSS are as defined below:

$$F_P = \left\{ \left(\frac{x_1}{[0.7, 0.25]}, \left\{\frac{k_1}{[0.8, 0.2]}, \frac{k_2}{[0.4, 0.5]}, \frac{k_3}{[0.6, 0.4]}, \frac{k_4}{[0.7, 0.2]}, \frac{k_5}{[0.1, 0.8]}, \frac{k_6}{[0.5, 0.5]}\right\}\right), \left(\frac{x_2}{[0.2, 0.8]}, \left\{\frac{k_1}{[0, 0.7]}, \frac{k_2}{[0.3, 0.65]}, \frac{k_3}{[0.2, 0.5]}, \frac{k_4}{[0.5, 0.3]}, \frac{k_5}{[0.9, 0]}, \frac{k_6}{[0.6, 0.4]}\right\}\right), \left(\frac{x_3}{[0.4, 0.4]}, \left\{\frac{k_1}{[0.3, 0.5]}, \frac{k_2}{[0.8, 0]}, \frac{k_3}{[0.5, 0.5]}, \frac{k_4}{[0.7, 0.1]}, \frac{k_5}{[0.9, 0]}, \frac{k_6}{[0.2, 0.5]}\right\}\right), \left(\frac{x_4}{[1, 0]}, \left\{\frac{k_1}{[0.5, 0.5]}, \frac{k_2}{[0.1, 0.2]}, \frac{k_3}{[0.3, 0.5]}, \frac{k_4}{[0, 0.45]}, \frac{k_5}{[0, 0.9]}, \frac{k_6}{[0.8, 0.1]}\right\}\right), \left(\frac{x_5}{[0.5, 0.3]}, \left\{\frac{k_1}{[0.4, 0.6]}, \frac{k_2}{[0.95, 0]}, \frac{k_3}{[0.75, 0.25]}, \frac{k_4}{[0.6, 0.3]}, \frac{k_5}{[0.25, 0.7]}, \frac{k_6}{[0.35, 0.5]}\right\}\right) \right\}. \quad (26)$$

Step 2
The reduced IF set P_{RIF} of F_P is computed and is as given below:

$$P_{RIF} = \left\{\frac{k_1}{[0.23, 0.165]}, \frac{k_2}{[0.2058, 0.1075]}, \frac{k_3}{[0.2225, 0.1292]}, \frac{k_4}{[0.195, 0.07]}, \frac{k_5}{[0.1225, 0.0683]}, \frac{k_6}{[0.2542, 0.1325]}\right\}. \quad (27)$$

Step 3
The reduced fuzzy set P_{RF} of P_{RIF} is computed and is as given below:

$$P_{RIF} = \left\{\frac{k_1}{0.1921}, \frac{k_2}{0.1837}, \frac{k_3}{0.1938}, \frac{k_4}{0.1814}, \frac{k_5}{0.1141}, \frac{k_6}{0.2205}\right\}. \quad (28)$$

Step 4
The element with the highest degree of membership is k_6. As such, object k_6 is the object that the student was looking for as it the element with the highest membership value.

VI. CONCLUSION

In this paper, we studied the decision making methods involving the concept of IFPIFSS. We introduced the concept of the aggregation operator of intuitionistic fuzzy parameterized intuitionistic fuzzy soft sets (IFPIFSS) as well as the intuitionistic fuzzy reduction method for IFPIFSS. These two decision making methods were then applied in two hypothetical problems related to decision making and object recognition respectively. Both of these problems are problems set in imprecise environments and involve data which are subjective and uncertain in nature.

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