

# Review of Modern Vehicle Modelling

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**Abstract**—Earlier models were a simplified form of the complex vehicle system, which was governed by the important parameters and states. These parameters and states were either obtained indirectly through some simplified equations or directly measured using instrumentation. In either case, the models were designed on the basis of parameters and state availability or importance, and not to have a complete model representation and maximum parameter utilization through estimation and instrumentation. Since a complete vehicle model can lead to a better controller. Without considering all the important parameters, a complete model is not obtained to design better controllers. All types of controllers, classical, optimal, nonlinear and linear controllers use the basic equations that govern the vehicle dynamics. This paper reviews all the previous models with greater insight into each system. The aim is to provide a better understanding of each model, its shortcomings and how it can represent the complex vehicle model.

**Index Terms**—Parameter Estimation; Vehicle Models; Tire Modelling; Estimation.

## I. INTRODUCTION

The first cars used was a crude form of modern vehicles, having thin bicycle like tires, and a basic chassis with rigid suspension that resembled the horse driven carts [1]. The vehicle parts were a modified form of the bicycle technology [1]. Based on the model, the insight into the system helped the industry to modernize the design into modern cars. Since most of the design and control work was done by purely mechanical engineers, earlier cars were a marvel of mechanical design only [2-3]. As electronics made its way into highly complex system like airplanes and ships, their importance in automobiles was realized [3]. The first use of electronics in cars was in the firing of engine plugs to have smooth ride. At that time, passive electrical systems were introduced. The first active electronic system was in the design of antilock braking system [3]. Its utilization helped save the lives of many people. It also influenced the industry to have active electrical systems.

The following sections provide a brief overview of the historical achievements that has resulted into modern cars and has provided greater insight to build better ones in the future.

## II. VEHICLE MODELLING

There are several types of models available, such as, bicycle model, half car model and full car model [1,3]. Following is a short description of some of the commonly used models.

### A. Bicycle Model

In bicycle model the front wheel and the rear wheel of the automobile are considered as the two wheels of a bicycle. The distance between the wheels is considered to be “ $l$ ” distance. The center of gravity of the bicycle is considered to lie between the front and the rear wheels at distance “ $a$ ” from the front wheel and distance “ $b$ ” from the rear wheel.

The net forces acting on the bicycle are resolved along the direction of motion ( $F_x$ ) and perpendicular to the direction of motion ( $F_y$ ) as shown in Figure. 1.

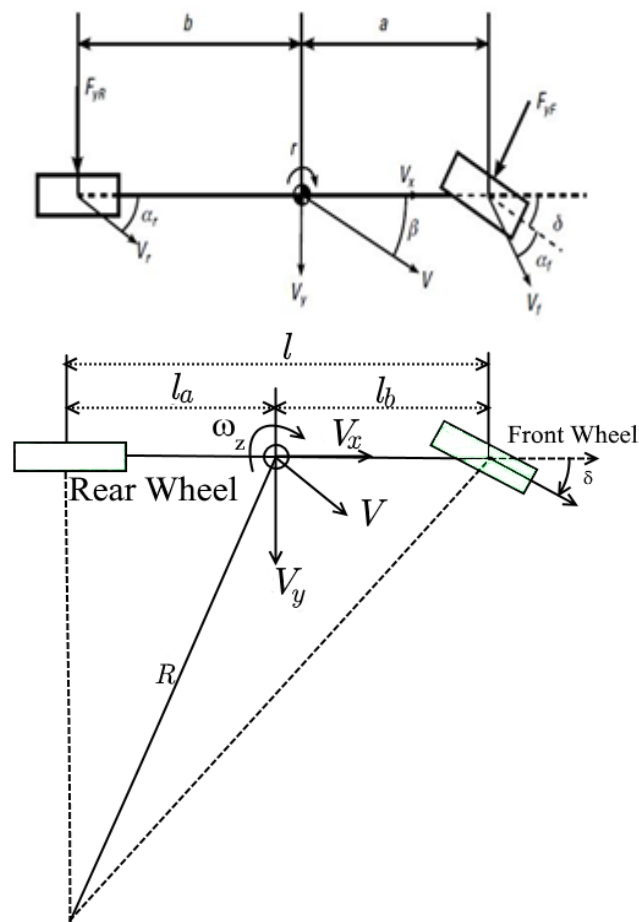


Figure 1: Force diagram of the bicycle model

In Figure. 1, the bicycle is moving with a velocity “ $V$ ” and is turning with a steering angle ( $\delta$ ). Due to the motion, the center of gravity (c.g) is traveling with the velocity “ $V$ ” at a

net direction ( $\beta$ ), caused by the interaction of the steering angle ( $\delta$ ) and the front and rear tire slip angles  $[\alpha_x \alpha_y]$ . The non-zero slip angle ( $\beta$ ) causes the bicycle to turn at a turn rate of ( $\omega_z$ ) rad/s. The state space of the bicycle model is given in (1)[1-5]. Consider:

$$C_0 = C_{af} + C_{ar}$$

$$C_1 = aC_{af} + bC_{ar}, C_2 = a^2C_{af} + b^2C_{ar}$$

$$\begin{bmatrix} \dot{\beta} \\ \dot{\omega}_z \end{bmatrix} = \begin{bmatrix} -C_0 & -C_1 \\ mV_x & mV_x^2 \\ -C_1 & -C_2 \\ I_z & I_z V_x \end{bmatrix} \begin{bmatrix} \beta \\ \omega_z \end{bmatrix} + \begin{bmatrix} C_{af} \\ mV_x \\ aC_{af} \\ I_z \end{bmatrix} \delta \quad (1)$$

The model given in (1) uses the tire coefficients of the front and rear wheels. It considers the automobile has full freedom in roll, to compensate the centripetal forces acting on the model during turns. This assumption is not true for automobiles, therefore, the model is not suitable for complete vehicle analysis.

### B. Full Car Model

The full car model is more accurate than the bicycle model, since it considers the 4-wheel traction and braking forces when modeling. The forces acting on the vehicle are resolved along the body longitudinal, lateral and vertical axis. The angular velocity along the longitudinal axis ( $\dot{\phi}$ ), lateral axis ( $\dot{\theta}$ ) and vertical axis ( $\dot{\psi}$ ) are also included in the analysis. A typical block diagram of a four-wheel car model is given in Figure. 2.

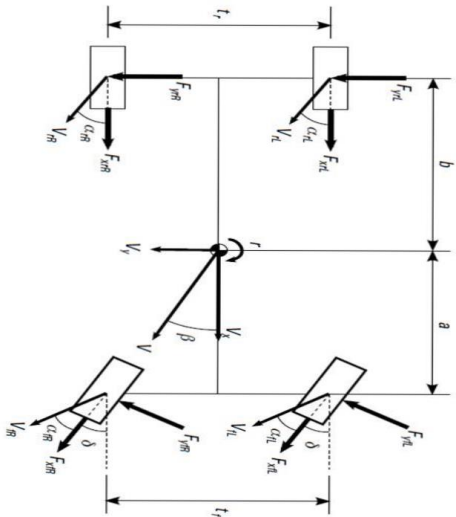


Figure 2: Force diagram of a full car model [1].

In Figure 2, the vehicle front wheels are turning with a steering angle ( $\delta$ ) causing lateral force components to act on each tire. This results into tire saturation causing slips in each wheel at a slip angle ( $\alpha_i$ ). The resulting motion of the vehicle causes it to turn with a yaw rate ( $\omega_z$ ). The net forces and

moments acting in the longitudinal and lateral direction are given by (2) [1-7,33-40].

$$\begin{aligned} \dot{a} F_x &= (F_{flx} + F_{frx}) \cos d + F_{rlx} + F_{rrx} - \\ & (F_{fly} + F_{fry}) \sin d - A_p V_x^2 \text{sign}(V_x) \\ \dot{a} F_y &= (F_{flx} + F_{frx}) \sin d + (F_{fly} + F_{fry}) \cos d + \\ & F_{rly} + F_{rly} - A_p V_y^2 \text{sign}(V_y) \\ I_{xx} \omega_z &= a[F_{flx} + F_{frx} \sin d + F_{fly} + F_{fry} \cos d] - \\ & b(F_{rly} + F_{rry}) + e[(F_{flx} \cos d - F_{fly} \sin d) + \\ & (F_{fry} \sin d - F_{frx} \cos d)] + e(F_{rrx} - F_{rlx}) \end{aligned} \quad (2)$$

Similarly, the simplified roll model is represented in state space form in Equation (3) (assuming  $\phi$  is small) [1], [4], [5], [7], [17].

In this model, the center of gravity is considered to be ( $h_{cg}$ ) high from the road floor with roll stiffness ( $K_\phi$ ) and roll damping coefficient ( $B_\phi$ ). The parameters and states used in the model are defined in Figure (3).

$$\begin{bmatrix} \ddot{\phi} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ K_\phi - mgh_{cg} & B_\phi \end{bmatrix} \begin{bmatrix} \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ mh_{cg} \end{bmatrix} a_y \quad (3)$$

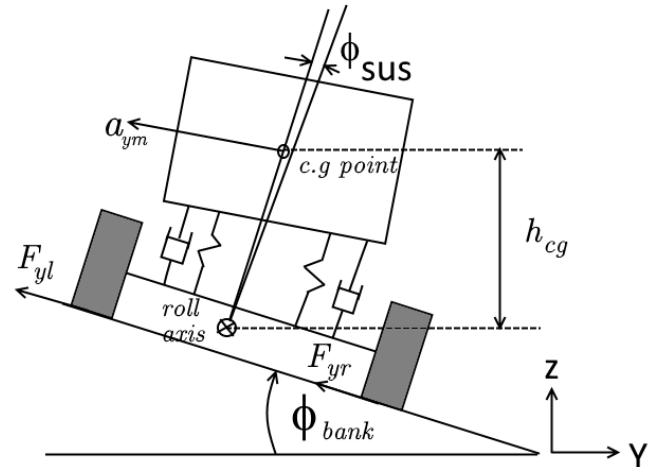


Figure 3: Vehicle Roll Model (ISO8855) [5].

Equation (3) is important for roll dynamics of the vehicle, which is related to suspension dynamics. In automobile, the purpose of the suspension system is to absorb vibrations caused by the road surface irregularities, engine vibrations, adjust terrain angles to ensure better ride comfort and improve the contact surface between tires to ensure stability and improved mileage [8-12].

Earlier, suspension models were usually obtained by considering the quarter car model (one wheel only). The equations for a quarter car model are given in Equation (4) [8-10].

$$\begin{aligned} m\ddot{x}_1 &= -K(x_1 - x_5) - B(\dot{x}_1 - \dot{x}_5) - K_t(x_1 - x_t) \\ M\ddot{x}_5 &= K(x_1 - x_5) + B(\dot{x}_1 - \dot{x}_5) \end{aligned} \quad (4)$$

In Equation. 4,  $x_1$  represents the vertical displacement of the suspension lower part and  $x_5$  represents the vertical displacement of the suspension upper part. While  $x_t$  represents the disturbance input through the tire with spring constant  $K_t$ . The spring and damper coefficients  $K$  are  $B$  and respectively. A full car suspension system considers the suspension having four wheels. A detailed suspension system includes the 4-wheel suspension system model as given in Figure. 3

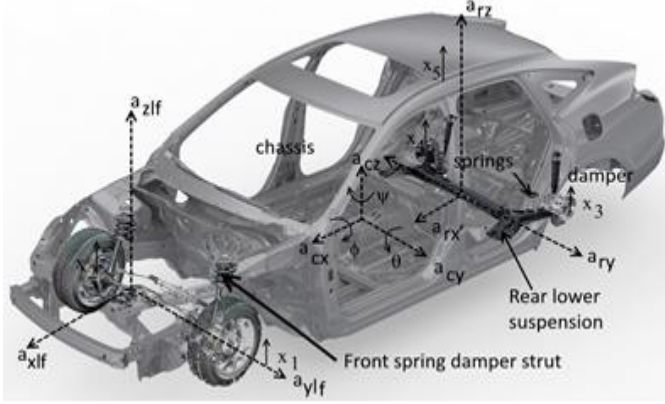


Figure 3: Full car model with states defined

In Figure. 3, the front lower suspension moves with the terrain with accelerations  $[a_{fx} \ a_{fy} \ a_{fz}]$  along the vertical axis of the lower suspension. The rear suspension is also moving with the terrain with accelerations  $[a_{rx} \ a_{ry} \ a_{rz}]$ . The chassis has engine vibrations, terrain gradients and induced road grade vibrations via the lower suspensions with accelerations  $[a_{cx} \ a_{cy} \ a_{cz}]$  [11-12].

In full car suspension model, the linear displacements  $x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]$  represent the front left, front right, rear left and rear right lower suspension and chassis displacements respectively, while the angular displacements  $[\phi \ \theta] = [x_6 \ x_7]$  represent the roll and pitch of the chassis [28-30]. The set of spring constants  $K = [k_1 \ k_2 \ k_3 \ k_4]$  and  $B = [b_1 \ b_2 \ b_3 \ b_4]$  represent the individual spring constants and damping coefficients of each damper. The full car suspension system is given in (5) [1], [3], [10,13-15].

Equation (5) shows the performance of the suspension depends upon the performance of the dampers. In dampers, the significant source of the disturbance is due to the road surface, which has a spectral content in the 1 Hz to 12 Hz range, making it difficult to reduce using fixed dampers [9], since they are tuned to reduce only a limited range of disturbance frequencies [9-15].

An effective method to reduce the road surface induced disturbance is to use semi-active dampers in comparison to other semi-active and fully active or passive suspensions [9-15], due to their failsafe and lower power requirements [10]. The suspension system is nonlinear [16-18], so suspension

dynamics must be considered in the suspension dynamic model. A completely nonlinear model is suitable for proper control of the suspension [18]. Since suspensions are in contact with the terrain through tires, the performance of the suspension and automobile dynamics depends upon the tire models.

$$\begin{aligned} m_1\ddot{x}_1 &= -k_1(x_1 - x_5 - r_{11}\phi - r_{12}\theta) - b_1(\dot{x}_1 - \dot{x}_5 - r_{11}\dot{\phi} - r_{12}\dot{\theta}) - k_{t1}(x_1 - x_{t1}) \\ m_2\ddot{x}_2 &= -k_2(x_2 - x_5 - r_{21}\phi + r_{22}\theta) - b_2(\dot{x}_2 - \dot{x}_5 - r_{21}\dot{\phi} + r_{22}\dot{\theta}) - k_{t2}(x_2 - x_{t2}) \\ m_3\ddot{x}_3 &= -k_3(x_3 - x_5 + r_{31}\phi - r_{32}\theta) - b_3(\dot{x}_3 - \dot{x}_5 + r_{31}\dot{\phi} - r_{32}\dot{\theta}) - k_{t3}(x_3 - x_{t3}) \\ m_4\ddot{x}_4 &= -k_4(x_4 - x_5 + r_{41}\phi + r_{42}\theta) - b_4(\dot{x}_4 - \dot{x}_5 + r_{41}\dot{\phi} + r_{42}\dot{\theta}) - k_{t4}(x_4 - x_{t4}) \\ M\ddot{x}_5 &= k_1(x_1 - x_5 - r_{11}\phi - r_{12}\theta) + b_1(\dot{x}_1 - \dot{x}_5 - r_{11}\dot{\phi} - r_{12}\dot{\theta}) + k_2(x_2 - x_5 - r_{21}\phi + r_{22}\theta) + b_2(\dot{x}_2 - \dot{x}_5 - r_{21}\dot{\phi} + r_{22}\dot{\theta}) + k_3(x_3 - x_5 + r_{31}\phi - r_{32}\theta) + b_3(\dot{x}_3 - \dot{x}_5 + r_{31}\dot{\phi} - r_{32}\dot{\theta}) + k_4(x_4 - x_5 + r_{41}\phi + r_{42}\theta) + b_4(\dot{x}_4 - \dot{x}_5 + r_{41}\dot{\phi} + r_{42}\dot{\theta}). \\ I_{xx}\ddot{\phi} &= k_1r_{11}(x_1 - x_5 - r_{11}\phi - r_{12}\theta) + b_1r_{11}(\dot{x}_1 - \dot{x}_5 - r_{11}\dot{\phi} - r_{12}\dot{\theta}) - k_2r_{21}(x_2 - x_5 - r_{21}\phi + r_{22}\theta) - b_2r_{21}(\dot{x}_2 - \dot{x}_5 - r_{21}\dot{\phi} + r_{22}\dot{\theta}) + k_3r_{31}(x_3 - x_5 + r_{31}\phi - r_{32}\theta) + b_3r_{31}(\dot{x}_3 - \dot{x}_5 + r_{31}\dot{\phi} - r_{32}\dot{\theta}) - k_4r_{41}(x_4 - x_5 + r_{41}\phi + r_{42}\theta) + b_4r_{41}(\dot{x}_4 - \dot{x}_5 + r_{41}\dot{\phi} + r_{42}\dot{\theta}). \\ I_{zz}\ddot{\theta} &= k_1r_{12}(x_1 - x_5 - r_{11}\phi - r_{12}\theta) + b_1r_{12}(\dot{x}_1 - \dot{x}_5 - r_{11}\dot{\phi} - r_{12}\dot{\theta}) + k_2r_{22}(x_2 - x_5 - r_{21}\phi + r_{22}\theta) + b_2r_{22}(\dot{x}_2 - \dot{x}_5 - r_{21}\dot{\phi} + r_{22}\dot{\theta}) - k_3r_{32}(x_3 - x_5 + r_{31}\phi - r_{32}\theta) - b_3r_{32}(\dot{x}_3 - \dot{x}_5 + r_{31}\dot{\phi} - r_{32}\dot{\theta}) - k_4r_{42}(x_4 - x_5 + r_{41}\phi + r_{42}\theta) - b_4r_{42}(\dot{x}_4 - \dot{x}_5 + r_{41}\dot{\phi} + r_{42}\dot{\theta}). \end{aligned} \quad (5)$$

### III. TIRE MODELS

Tires provide the necessary friction forces required by the vehicle against externally applied forces of traction, braking and lateral forces. When externally applied forces become excessive, the tires can no longer provide sufficient friction forces to overcome their effect, so they start slipping [19]. The tire slips are different when the vehicle is accelerating, when it is braking or moving along a curved path as given by Equation 6 [1,3,19]. Slipping in tires causes the vehicle to move at slower speeds and in arbitrary directions (tire slip angle). The tire force-slip angle relationship is anti-symmetric with hysteresis [1,3,19].

There are several models available for tires. The first tire model was the Fiala model presented in 1954 [20], in which the tire stiffness was assumed to be uniform along the tire slip angle  $C_\alpha$  and tire center line  $C_\sigma$ .

The Fiala tire model is important because it describes the relationship of the reaction force  $\mu F_z$  on tire adhesion, measured in terms of stiffness coefficients along the longitudinal and lateral direction. It does not provide the relationship between tire and slip [1-3,20].

$$\lambda_x = \begin{cases} \frac{R\Omega_w - V_x}{R\Omega_w} & \text{when accelerating} \\ \frac{V_x - R\Omega_w}{V_x} & \text{when braking} \\ \frac{V_x}{R\Omega_w} \tan\alpha & \text{when turning} \end{cases} \quad (6)$$

$$\text{while } \mu F_z = 3\theta\sigma - \frac{1}{3}\theta\lambda^2 + \frac{1}{27}(3\theta\lambda^3) + \dots$$

The Dug off tire was the first model that provided the analytical relationship of slip with force as given in Equation (7).

$$F_{xi} = \begin{cases} J_i I_i & \text{when } a \geq 0 \\ J_i I_i (I_i + 1) & \text{otherwise} \end{cases} \begin{cases} \ddot{u} \\ \dot{y} \\ \ddot{b} \end{cases}$$

$$F_{yi} = \begin{cases} u_i \tan(a_i) & \ddot{u} \\ u_i (I_i + 1) \tan(a_i) & \dot{y} \\ \ddot{b} \end{cases} \quad (7)$$

$$\text{where } J_i = \frac{C_{xi} I_i}{1 + I_i} f(g_i), \quad u_i = \frac{C_{yi} \tan(a_i)}{1 + I_i} f(g_i)$$

$$g_i = \frac{mF_z(1 + I_i)}{2\sqrt{(C_{xi} I_i)^2 + C_{yi} \tan a_i}}, \quad f(g_i) = \begin{cases} 1 & \text{if } g \geq 1 \\ (2 - g_i)g_i & \text{otherwise} \end{cases}$$

This model provides a comprehensive analytical model for tires. It does not however, consider the direction of motion effect on the performance of tires. The geometrical properties of the tires are also not sufficiently explained for obtaining the tires stiffness.

A more thorough model was the magic formula model presented in delft, October 1991 [21], which was later modified in 1997 [22]. According to the magic tire model, the forces exerted in the longitudinal ( $F_x$ ) and lateral ( $F_y$ ) direction are different and related with the tire shape, curvature, longitudinal slip angle and lateral slip angle as given by Equation (8)[1, 3, 21-22].

$$x(k + S_H) = n \sin[\nu \tan^{-1} V(k + S_H)] - E \{V(k + S_H) - \tan^{-1} V(k + S_H)\}; \quad F_x = x(k + S_H) + S_H \quad (8)$$

$$y(a + S_H) = n \sin[\nu \tan^{-1} V(a + S_H)] - E \{V(k + S_H) - \tan^{-1} V(a + S_H)\}; \quad F_y = y(a + S_H) + S_H$$

$$C_k = \frac{V_x}{J} \quad C_a = \frac{V_x}{J}, \quad \text{where } J = \max(V, e_y)$$

The output behavior of this equation is modified with the parameters [ $\nu, \varsigma, k, E, S_v, S_H$ ]. The various shape parameters used in the magic formula model are defined in Figure 4.

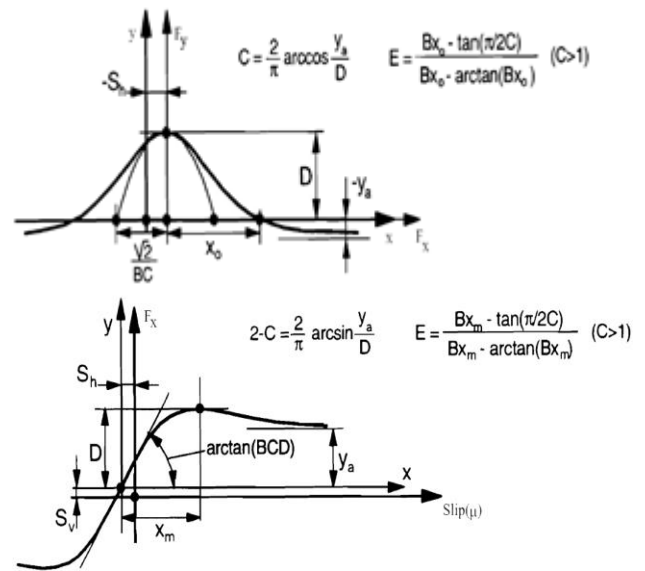


Figure 4: Definition of various parameters of Magic Formula Model [21]

Since, tires are responsible for providing the necessary traction and braking forces required by the automobile to be controllable [1,3]. They also help in overcoming the excessive lateral forces exerted on the automobile during heavy turns [23-27]. A better tire model can help in designing a controller that can help in better yaw control. The magic tire model is a promising model [45].

#### IV. STATE SPACE REPRESENTATION OF FULL CAR MODEL

The full car model can be linearized and represented in the state space representation form. The process of linearization starts with calculation of the wheel slip angle [28-30,43,44]. The 4-wheel slip angles are resolved using the steering angle inputs and velocity components. The electronic steering system used to model the steering input is also included. Similarly, the slips in each wheel are modeled using the wheel traction equation for angular velocity [28].

The space matrix representation of the full car model is given in Equation (9)[28-30].

$$a_{11} = \frac{A_p |V_x|}{M};$$

$$a_{12} = w_z + \frac{K_1 \sin d}{M(V_x + ew_z)} + \frac{K_2 \sin d}{M(V_x - ew_z)};$$

$$a_{13} = \frac{aK_1 \sin d}{M(V_x + ew_z)} + \frac{aK_2 \sin d}{M(V_x - ew_z)};$$

$$a_{15} = \frac{J_1 \cos d}{M};$$

$$a_{14} = \frac{(K_1 + K_2) \sin d}{M};$$

$$a_{23} = \frac{aK_1 \cos d}{M(V_x + ew_z)} + \frac{aK_2 \cos d}{M(V_x - ew_z)} - \frac{bK_3}{M(V_x - ew_z)} - \frac{bK_4}{M(V_x + ew_z)};$$

$$a_{16} = \frac{J_2 \cos d}{M}; \quad a_{17} = \frac{J_3}{M}; \quad a_{18} = \frac{J_4}{M}; \quad a_{21} = -w_z; \quad (9)$$

$$\begin{aligned}
 a_{37} &= -e \frac{J_3}{I_z}, & a_{38} &= e \frac{J_4}{I_z}, & a_{68} &= -\frac{(1+l_2)}{V_x} a_{18} \\
 a_{83} &= -\frac{(1+l_4)}{V_x} a_{14}, & a_{44} &= -\frac{1}{t}, & a_{63} &= -\frac{(1+l_2)}{V_x} a_{14} \\
 a_{22} &= \frac{A_p |V_y|}{M} + \frac{K_1 \cos d}{M(V_x + eW_z)} + \frac{K_2 \cos d}{M(V_x - eW_z)} + \\
 &\quad \frac{K_3}{M(V_x - eW_z)} + \frac{K_4}{M(V_x + eW_z)}, \\
 a_{86} &= -\frac{(1+l_4)}{V_x} a_{16} \\
 a_{87} &= -\frac{(1+l_4)}{V_x} a_{17}, & a_{24} &= \frac{(K_1 + K_2)}{M}, & a_{25} &= \frac{J_1 \sin d}{M}; \\
 a_{26} &= \frac{J_2 \cos d}{M}, & a_{25} &= \frac{J_1 \sin d}{M}, & a_{26} &= \frac{J_2 \cos d}{M}; \\
 a_{88} &= -\frac{(1+l_4)}{V_x} a_{18} - \frac{R_{w4}^2 J_4}{I_w V_x}, & a_{32} &= \frac{K_1(a \cos d - e \sin d)}{I_z(V_x + eW_z)} + \\
 &\quad \frac{K_2(a \cos d + e \sin d)}{I_z(V_x - eW_z)} - \frac{bK_3}{I_z(V_x - eW_z)} - \frac{bK_4}{I_z(V_x + eW_z)}; \\
 a_{81} &= -\frac{(1+l_4)}{V_x} a_{11}, \\
 a_{33} &= \frac{K_1(a \cos d - e \sin d)}{I_z(V_x + eW_z)} + \\
 &\quad \frac{K_2(a \cos d + e \sin d)}{I_z(V_x - eW_z)} - \frac{b^2 K_3}{I_z(V_x - eW_z)} - \frac{b^2 K_4}{I_z(V_x + eW_z)}; \\
 a_{78} &= -\frac{(1+l_3)}{V_x} a_{18}, & a_{15} &= \frac{R_{w1}^2 J_1}{I_w V_x}, & a_{55} &= -\frac{(1+l_1)}{V_x} \\
 a_{34} &= \frac{K(a \cos d - e \sin d) + K(a \cos d + e \sin d)}{I_z}, \\
 a_{35} &= \frac{J_1(a \sin d + e \cos d)}{I_z}, & a_{36} &= \frac{J_2(a \sin d - e \cos d)}{I_z}; \\
 a_{51} &= -\frac{(1+l_1)}{V_x} a_{11}, & a_{52} &= -\frac{(1+l_1)}{V_x} a_{12}, & a_{53} &= -\frac{(1+l_1)}{V_x} a_{13}, \\
 a_{54} &= -\frac{(1+l_1)}{V_x} a_{14}, & a_{84} &= -\frac{(1+l_4)}{V_x} a_{14}, & a_{85} &= -\frac{(1+l_4)}{V_x} a_{15}, \\
 a_{56} &= -\frac{(1+l_1)}{V_x} a_{16}, & a_{57} &= -\frac{(1+l_1)}{V_x} a_{17}, & a_{58} &= -\frac{(1+l_1)}{V_x} a_{18}, \\
 a_{61} &= -\frac{(1+l_2)}{V_x} a_{11}, & a_{62} &= -\frac{(1+l_2)}{V_x} a_{12}, & a_{64} &= -\frac{(1+l_2)}{V_x} a_{14}, \\
 a_{65} &= -\frac{(1+l_2)}{V_x} a_{15}, & a_{66} &= -\frac{(1+l_2)}{V_x} a_{16} - \frac{R_{w2}^2 J_2}{I_w V_x}, \\
 a_{67} &= -\frac{(1+l_2)}{V_x} a_{17}, & a_{76} &= -\frac{(1+l_3)}{V_x} a_{16}, & a_{71} &= -\frac{(1+l_3)}{V_x} a_{11}, \\
 a_{72} &= -\frac{(1+l_3)}{V_x} a_{12}, & a_{73} &= -\frac{(1+l_3)}{V_x} a_{14}, & a_{74} &= -\frac{(1+l_3)}{V_x} a_{14},
 \end{aligned}$$

$$\begin{aligned}
 a_{75} &= -\frac{(1+l_3)}{V_x} a_{15}, & a_{77} &= -\frac{(1+l_3)}{V_x} a_{17} - \frac{R_{w3}^2 J_3}{I_w V_x} \\
 a_{82} &= -\frac{(1+l_4)}{V_x} a_{12},
 \end{aligned}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} & a_{38} \\ 0 & 0 & 0 & a_{44} & 0 & 0 & 0 & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & 0 & 0 \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} & 0 \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77} & a_{78} \\ a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & a_{88} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

The model given by (9), considers the automobile traction system, the electronic steering system response, and the tire dynamics. It also considers the mass distribution of the automobile. The model therefore, is more accurate. Since the model includes several sub-models, traction and braking model, tire model, full car model, the model consists of a large number of states and parameters. The proper use of this model is possible if the required parameters and states are available. Since many of the parameters are not measurable [1,3], estimators of various types are suggested.

## V. STATE AND PARAMETER OBSERVER

State and parameter estimators are used to estimate unknown states using either automobile kinematics equations, called "Kinematics based Estimation", or automobile model equations, called "Model based Estimation". Tables 1 and 2, list both types of observers used for the estimation of various parameters and states.

Table 1  
List of Some Kinematics Based Observers

Variables Measured	Variables Estimated	Method Used
$\Omega_w, a_y, \delta$	$\dot{\phi}, \dot{\beta}$	Nonlinear Observer [4], [31], [37], [42]
$a_x, a_y, a_z$	$V_x, V_y$	Kalman Filter [2], [26]
$V_{GPS}, P_{GPS}$	$V_x, V_y, M, \theta, \phi$	Kalman Filter [35], [42]
$T_s, \omega_x, \omega_y, \omega_z$	$M, \mu$	Recursive least square [11], [33]
$A_x, A_y, V$	$M_z$	Fuzzy Logic [25]
$V_{GPS}, \phi_{GPS}, \theta_{GPS}$	$V, \psi$	SO3 Filter [32]
Received Mobile Signals	$V$	Triangulation [34]
$\psi_{GPS}, \theta_{GPS}, Image$	$p(x), V$	Particle Filter [36]

Table 2  
List of Parameters And States Estimated Using Model Based Estimation

Variables Measured	Variables Estimated	Model Used	Method Used
$\delta, \omega_z, \psi_{GPS}$	$V_x, V_y, \omega_{bx}, \alpha_f, \alpha_r$	Bicycle	Kalman Filter [5]
$a_x, a_y, \psi$	$V_x, V_y, \theta, \mu$	Full Car	Nonlinear Observer [6], [12], [41]
$\delta, \psi, a_y$	$\beta, h_{cg}, CG$	Bicycle	Multiple Models with switch [38]
$\delta, a_y, V_x, M$	$\beta$	Full Car	DHME [39]
$a_x, a_y, a_z, \omega_x, \omega_y, \omega_z$	$V_x, V_y, V_z, \phi, \theta$	Bicycle	Kalman Filter [40]
$\omega_z, \delta, \beta$	$M_z$	Full Car	Fuzzy Logic [25]
$\Omega_w, a_x, a_z, \omega_x, \omega_z$	$\mu, F_z$	Full Car	Extended Kalman Filter [42]

In either form of estimation, various sensors are used with complex equations. The performance of the estimators depends upon the performance, speed and cost of the algorithm and on the accuracy of the sensors [1, 3, 24, 46]. The above tables list the number of sensors used in respective observers. Among the estimation schemes, the most suitable observer is using SO (3) based observer, since its computational complexity is significantly less than the kalman based and non-linear observer based schemes [32, 46].

## VI. CONCLUSION

This paper presents a conclusive review of the different vehicle models used for the control and analysis of automobiles. The bicycle model presented in the simplest model but it does not include the interaction of the four wheels in obtaining the slips. The full car model is a complete representation of the vehicle chassis motion but it is incomplete until the suspension dynamics and tire dynamics are included, making the model nonlinear. Linear methods, therefore, become inadequate for controller design and estimation. Nonlinear schemes like sliding mode [43-44] optimal control [28-30] and intelligent control [24-25] are possible candidates.

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## REFERENCES

- [1] D. M. Bevy and S. Cobb, *GNSS for Vehicle Control*, Artech House, 3<sup>rd</sup> Edition, 2010, pp. 81-89.
- [2] Farrelly, Jim, and Peter Wellstead. "Estimation of vehicle lateral velocity." *Proceedings of the IEEE International Conference on Control Applications*, 1996, pp. 552-557.
- [3] Rajamani, R., *Vehicle dynamics and control*. Springer Science & Business Media. 2<sup>nd</sup> Edition, May2005

- [4] Tchamna, R., and I. Youn. "Yaw rate and side-slip control considering vehicle longitudinal dynamics." *International Journal of Automotive Technology*, vol 14, no. 1, 2013, pp. 53-60.
- [5] Bevy, David M., Jihan Ryu, and J. Christian Gerde. "Integrating INS sensors with GPS measurements for continuous estimation of vehicle sideslip, roll, and tire cornering stiffness." *IEEE Transactions on Intelligent Transportation Systems*, vol 7, no. 4, 2006, pp. 483-493.
- [6] Oh, Jiwon, and Seibum B. Choi. "Design of a new composite observer for vehicle velocity and attitude estimation." *IASTED Int. Conf on Control and Applications*, Vancouver, BC, Canada, vol. 102109, 2011.
- [7] Cho, Kwanghyun, Hyunwoo Son, Seibum B. Choi, and Sukchang Kang. "Lateral acceleration compensation of a vehicle based on roll angle estimation." *IEEE International Conference on Control Applications (CCA)*, 2010, pp. 1363-1368.
- [8] Yi, K., and B. S. Song. "A new adaptive sky-hook control of vehicle semi-active suspensions." *Proceedings of the Institution of Mechanical Engineers, part D: Journal of automobile engineering*, vol 213, no. 3, 1999, pp. 293-303.
- [9] Lai, Chun Yu, and Wei-Hsin Liao. "Vibration control of a suspension system via a magnetorheological fluid damper." *Journal of Vibration and Control*, vol 8, no. 4, 2002.
- [10] Yao, G. Z., F. F. Yap, G. Chen, Who Li, and S. H. Yeo. "MR damper and its application for semi-active control of vehicle suspension system." *Mechatronics*, vol 12, no. 7, 2002, pp. 963-973.
- [11] Ray, Laura R. "Nonlinear tire force estimation and road friction identification: simulation and experiments." *Automatica*, vol. 33, no. 10 1997, pp. 1819-1833.
- [12] Bae, Hong S., Jihan Ryu, and J. Christian Gerdes. "Road grade and vehicle parameter estimation for longitudinal control using GPS." *Proceedings of the IEEE Conference on Intelligent Transportation Systems*, 2001, pp. 25-29.
- [13] Zareh, Seiyed Hamid, Atabak Sarrafan, Ali Fella Jahromi, and Amir Ali Khayyat. "Linear quadratic Gaussian application and clipped optimal algorithm using for semi-active vibration of passenger car." *IEEE International Conference on Mechatronics (ICM)*, 2011, pp. 122-127.
- [14] Sandhu, Fargham, Hazlina Selamat, and Yahya Sam. "Linear quadratic regulator and skyhook application in semiactive MR damper full car model." *IEEE 10th Asian Control Conference (ASCC)*, 2015, pp. 1-7.
- [15] Jahromi, Ali Fella, and A. Zabihollah. "Linear quadratic regulator and fuzzy controller application in full-car model of suspension system with magnetorheological shock absorber." *IEEE/ASME International Conference on Mechatronics and Embedded Systems and Applications (MESA)*, 2010, pp. 522-528.
- [16] Fallah, M. S., R. Bhat, and W. F. Xie. "New model and simulation of Macpherson suspension system for ride control applications." *Vehicle System Dynamics*, vol 47, no. 2, 2009, pp. 195-220.
- [17] Stenonson, Annika, Christer Asplund, and Lennart Karlsson. "The nonlinear behavior of a Macpherson strut wheel suspension." *Vehicle system dynamics*, vol 23, no. 1, 1994, pp. 85-106.
- [18] Chen, Kaiyu, and David G. Beale. "Base dynamic parameter estimation of a MacPherson suspension mechanism." *Vehicle System Dynamics*, vol 39, no. 3, 2003, pp. 227-244.
- [19] Kachroo, Pushkin, and Masayoshi Tomizuka. "Vehicle traction control and its applications." *California Partners for Advanced Transit and Highways (PATH)*, 1994.
- [20] Fiala, E. "Lateral forces on rolling pneumatic tires." *Zeitschrift VDI 96*, no. 29, 1954.
- [21] Pacejka, Hans B., and Egbert Bakker. "The magic formula tire model", *Vehicle system dynamics*, vol 21, no. S1, 1992, pp. 1-18.
- [22] Pacejka, H. B., and I. J. M. Besselink. "Magic formula tire model with transient properties." *Vehicle system dynamics*, vol 27, no. S1, 1997, pp. 234-249.
- [23] Acarman, Tankut. "Nonlinear optimal integrated vehicle control using individual braking torque and steering angle with on-line control allocation by using state-dependent Riccati equation technique." *Vehicle System Dynamics*, vol 47, no. 2, 2009.
- [24] Mauer, Georg F. "A fuzzy logic controller for ABS braking system." *IEEE Transactions on Fuzzy Systems*, vol 3, no. 4, 1995, pp. 381-388.
- [25] Boada, B. L., M. J. L. Boada, and V. Diaz. "Fuzzy-logic applied to yaw moment control for vehicle stability." *Vehicle System Dynamics*, vol 43, no. 10, 2005, pp. 753-770.
- [26] Esmailzadeh, E., A. Goodarzi, and G. R. Vossoughi. "Optimal yaw moment control law for improved vehicle handling." *Mechatronics*, vol 13, no. 7, 2003, pp. 659-675.

- [27] N. Hamzah, Y.M. Sam, H. Selamat & M.K. Aripin, "GA-based Sliding Mode Controller for Yaw Stability Improvement." *Asian control conference*, 24-27 June, Turkey, 2013.
- [28] Sandhu, Fargham, Hazlina Selamat, and Vahid Behtaji Siahkal Mahalleh. "Direct Yaw Control of Vehicle using State Dependent Riccati Equation with Integral Terms." *Advances in Electrical and Computer Engineering* 16.2, 2016.
- [29] Nagai, Masao, Yutaka Hirano, and Sachiko Yamanaka. "Integrated control of active rear wheel steering and direct yaw moment control." *Vehicle System Dynamics* 27, no. 5-6, 1997, pp. 357-370.
- [30] Cho, Wanki, Jangyeol Yoon, Jeongtae Kim, Jaewoong Hur, and Kyongsu Yi. "An investigation into unified chassis control scheme for optimized vehicle stability and manoeuvrability." *Vehicle System Dynamics* vol 46, no. S1, 2008, pp. 87-105.
- [31] Yu, J. S. "A robust adaptive wheel-slip controller for antilock brake system." *Proceedings of the IEEE Conference on Decision and Control*, vol. 3, 1997, pp. 2545-2546.
- [32] Sandhu, Fargham, Hazlina Selamat, and Yahaya Md Sam. "Antilock Braking System Using Dynamic Speed Estimation." *Jurnal Teknologi* vol 72, no. 2, 2015.
- [33] Vahidi, Ardalan, Maria Druzhinina, Anna Stefanopoulou, and Huei Peng. "Simultaneous mass and time-varying grade estimation for heavy-duty vehicles." *Proceedings of the American Control Conference*, vol. 6, 2003, pp. 4951-4956.
- [34] Chandrasekaran, Gayathri, Tam Vu, Alexander Varshavsky, Marco Gruteser, Richard P. Martin, Jie Yang, and Yingying Chen. "Vehicular speed estimation using received signal strength from mobile phones." *Proceedings of the ACM international conference on Ubiquitous computing*, ACM, 2010, pp. 237-240.
- [35] Bevely, David M. "Global positioning system (GPS): A low-cost velocity sensor for correcting inertial sensor errors on ground vehicles." *Journal of dynamic systems, measurement, and control*, vol. 126, no. 2, 2004, pp. 255-264.
- [36] Won, Dae Hee, Sebum Chun, Sangkyung Sung, Young Jae Lee, Jeongho Cho, Jungmin Joo, and Jungkeun Park. "INS/vSLAM system using distributed particle filter." *International Journal of Control, Automation and Systems*, vol. 8, no. 6, 2010.
- [37] Imsland, Lars, Håvard Fjær Grip, Tor A. Johansen, Thor I. Fossen, Jens C. Kalkkuhl, and Avshalom Suissa. "Nonlinear observer for vehicle velocity with friction and road bank angle adaptation-validation and comparison with an extended Kalman filter." *SAE Technical Paper 2007-01-0808*, 2007.
- [38] Solmaz, Selim, Mehmet Akar, and Robert Shorten. "Online center of gravity estimation in automotive vehicles using multiple models and switching." *International Conference on Control, Automation, Robotics and Vision*, 2006, pp. 1-7.
- [39] Canale, Massimo, Lorenzo Fagiano, and Carlo Novara. "Vehicle sideslip angle estimation using a direct MH estimator," *IEEE International Conference on Control Applications*, 2010.
- [40] Oh, Jiwon, and Seibum B. Choi. "Vehicle roll and pitch angle estimation using a cost-effective six-dimensional inertial measurement unit." *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, vol 227, no. 4, 2013.
- [41] Grip, Håvard Fjær, Lars Imsland, Tor A. Johansen, Jens C. Kalkkuhl, and Avshalom Suissa. "Vehicle sideslip estimation." *Control Systems, IEEE*, vol 29, no. 5, 2009, pp. 36-52.
- [42] Bevely, David M., J. Christian Gerdes, Christopher Wilson, and Gengsheng Zhang. "The use of GPS based velocity measurements for improved vehicle state estimation." *Proceedings of the American Control Conference*, vol. 4, 2000, pp. 2538-2542.
- [43] Abdulrahim, Mujahid. "On the dynamics of automobile drifting." *SAE Technical Paper*, vol 1, 2006, pp. 10-19.
- [44] Johansen, Tor A., Idar Petersen, Jens Kalkkuhl, and Jens Lüdemann. "Gain-scheduled wheel slip control in automotive brake systems." *IEEE Transactions on Control Systems Technology*, vol 11, no. 6, 2003, pp. 799-811.
- [45] Ding, Nenggen, and Saied Taheri. "An adaptive integrated algorithm for active front steering and direct yaw moment control based on direct Lyapunov method." *Vehicle System Dynamics*, vol. 48, no. 10, 2010, pp. 1193-1213.
- [46] Chandrasekar, J., A. J. Ridley, and D. S. Bernstein. "A comparison of the extended and unscented Kalman filters for discrete-time systems with non-differentiable dynamics." *Proceedings of the 2007 American Control Conference*, 2007, vol. Rn 2, 2007, pp. 4-8.