MATHEMATICAL MODELLING OF THE PASSIVE AND SEMI-ACTIVE AUTOMOBILE SUSPENSION SYSTEMS IN FORD SCORPIO CAR MODEL

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ABSTRACT: The suspension is a system of spring or shock absorbers connecting the wheels and axles at the chassis of a vehicle. In this study, Ford Scorpio car with passive and semi-active suspension system are formulated using Second Order Differential Equations (ODE). These models were solved analytically using the undetermined coefficient and Cramer rule methods. The comparison between passive and semi-active suspension system also conducted to measure such as displacement, frequency and time by plotting graphs. The passive system resulted that a constant displacement roughly at 0.035 m while 0.025 m was obtained by a semi-active suspension system. Semi-active system took $t = 1$ s to yield a constant displacement while for the passive suspension system required $t = 1.5$ s. The comparison showed the semi-active results were better than a passive suspension system.

KEYWORDS: *Passive Suspension; Semi-Active Suspension; Mathematical Modelling; ODE; Displacement*

1.0   INTRODUCTION

The suspension is a term given to the system of springs, shock absorbers and linkages that connects a vehicle to its wheels. Suspension systems serve a dual purpose which is contributing to the car's road-holding/ handling and braking for good active safety and driving pleasure, and keeping vehicle occupants comfortable and reasonably well isolated

from road noise, bumps, vibrations, and others. The suspension needs to keep the road wheel in contact with the road surface as much as possible because all the forces acting on the vehicle do so through the contact patches the tires. In the engineering term, the suspension system can be categorized into passive, semi-active and active suspension system according to the external power input to the system and/or a control bandwidth [16].

A passive suspension system is an ordinary suspension system consists of a non-controlled spring and shocking-absorbing damper. The commercial vehicles today use a passive suspension system to control the dynamics of a vehicle's vertical motion as well as pitch and roll. Passive indicates that the suspension elements cannot supply energy to the suspension system. The good automotive suspension system can absorb road shocks rapidly and could return to its normal position slowly while maintaining optimal tire to road contact [2-4]. However, this condition is difficult if using the passive automobile suspension system. Passive automobile suspension systems will give too much movement if the soft spring is being used and a hard spring causes passenger discomfort due to road irregularities. The passive automobile suspension system incorporates the hydraulic damper and mechanical spring. The disadvantage of the passive suspension systems is the design that only achieves certain condition. The characteristics of these suspension systems also are fixed and cannot be adjusted by any mechanical part. Sam and Osman [8] stated that the problem of passive suspension systems is evident when the vehicle travels at the low speed on a rough road or the high speed in a straight line, where it will be perceived as a harsh road. Then, if the suspension is designed lightly damped, it will give the comfortable ride. Unfortunately, this design will reduce the stability of the vehicle in making turn and lane changing.

The semi-active suspension has the same elements as a passive suspension system but the damper has two or more selectable damping rate. Semi-active systems can only change the viscous damping coefficient of the shock absorber without any energy generated in the suspension system. The semi-active suspension system also can reduce the acceleration of sprung mass continuously. This ability will improve the tire grip with the road surface, thus, brake, traction control and vehicle manoeuvrability can be considerably improved [1]. The damper or spring in semi-active systems is interceding by the force actuator which has its task to add the energy from the suspension system. The force actuator is controlled by various types of controller determined by the designer. The dynamic behaviour of the system in nature can

be observed by neglecting of the effect of force and damping elements [13]. Few published studies have been conducted to determine the semi-active suspension systems provide better result compared to the passive suspension systems. Bhise et al. [6] identified that traditionally or passive automotive suspension systems are designed with three criteria which are load capacity, passengers' comfort and road handling. For a better comfort ride and vehicle stability, it is important to use the correct control strategy [15]. Sam [9] described the active suspensions differ from the conventional passive suspensions to give the energy into the suspension systems.

This study aims to formulate the passive and semi-active suspension system using Second Order Differential Equation and Cramer rule. The proposed models continue to be used in the Ford Scorpio car model and finally, the comparison will be made by plotting a graph using Matlab. Many researchers have tended to employ the numerical and simulation approaches in predicting the fast response of the models [18]. The main focus of the study is on how the semi-active automobile suspension system can perform and improve the stability of the vehicle and ride comfort significantly. A basic car such as the Ford Scorpio model is employed to know the applicability of the mathematical model of passive and semi-active suspension systems [5]. The contribution of this study obviously in the resulting outcomes capitalized as the guidelines of having better automobile suspension systems. Many studies also have been done in the field, for example, the optimization design of the suspension systems in improving the ride comfort. Griffin [10] has performed a detailed experimental work to determine the effects of road excitation on ride comfort. Several methods are also developed in the field such as rotating square evolutionary operation (ROVOP), gradient method, genetic algorithm and sequential search method to determine the ride comfort. Shirahatti et al. [12] have used a genetic algorithm to find out the minimum acceleration and road holding and where the results were compared with the Simulink model. Numerous advantages for the semi-active suspension such as high strength, good controllability, wide dynamic range, fast response rate, low energy consumption and contains a simple structure [7]. Bhise et al. [6] also stated that semi-active using Magnetorheological (MR) damper offered a better control performance than a passive system. Past studies showed the magneto-rheological fluids were applied in various engineering devices such as damper, clutches, brakes and valves [17]. Moreover, MR fluids also applied in the semi-active suspension system for giving the actuators controlled the produced vibration effectively. MR fluid damper is a semi-active control device applied the MR fluids to produce the controllable damping force [20]. Surprisingly, the performance

between passive and semi-active automobile suspension system is not comprehensively studied. Many of the studies on the semi-active suspension system are only focusing on the applicability of the system.

This paper is organized as follows: Introduction to passive and semi-active automobile suspension systems in Section 1. Section 2 focuses on the mathematical modelling of passive and semi-active automobile suspension systems. Section 3 derives the development of theories for passive and semi-active systems of Ford Scorpio car model. Result and discussion are in Section 4 while a concluding remark presented in Section 5.

2.0 MATHEMATICAL MODELLING OF AUTOMOBILE SUSPENSION SYSTEM

2.1 Passive System

A passive suspension system is an ordinary suspension system consists of a non-controlled spring and shocking-absorbing damper. The commercial vehicles today use a passive suspension system to control the dynamics of a vehicle's vertical motion as well as pitch and roll. The passive element does not supply energy to the suspension system. The passive suspension system controls the motion of the body and wheel by limiting their relative velocities to a rate that gives the desired ride characteristics. This is achieved by using some type of damping element placed between the body and the wheels of the vehicle, such as hydraulic shock absorber.

In this section, the passive automobile suspension system is formulated by using a simple car of Ford Scorpio model. The model uses spring (k), damper (c) that is connected to the wheel mass (m). The model is shown in Figures 1.
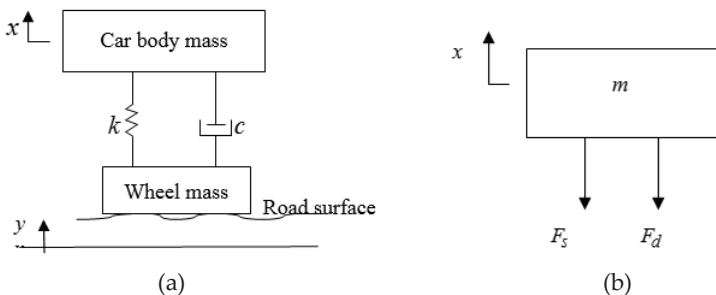

Figure 1: (a) Simple model car suspension and (b) its simplified model [11]

Newton's second and Hooke's laws of motion in a vertical direction were applied to derive the mathematical equations for describing the motion. By assuming the car body and the wheels are moving vertically upwards with the spring and damper expressed as

$$X(t) = x - y \qquad (1)$$

Where $X(t)$ is an extension of the spring; x is the vertical displacement of the car body above its equilibrium position; y is the vertical displacement of the wheel (due to the road surface measured concerning some fixed horizontal line). Hooke's law denoted as

$$F_S = kx(t) \qquad (2)$$

and the spring force represented as

$$F_S = k(x - y) \qquad (3)$$

where F_s is a force of the spring; k is a constant parameter and x-y is an extension of the spring. Shock absorbers provide damping of the motion experienced by the vehicle's wheels as they move up and down over uneven road surface. A hydraulic system used to provide resistance to the kinetic energy produced by the wheels. The damping force often being a model when the viscosity or drag is not negligible in a system. For this passive system, its only deal with the linear damping force. Given the damper force such as

$$F_d = cv \qquad (4)$$

where F_d is resisting force for damper, c is damping coefficient and v is relative velocity of the housing and the piston.

$$v = \frac{dX(t)}{dt} = \frac{d(x - y)}{dt} \qquad (5)$$

where $X(t)$ is an extension of the spring; x is the vertical displacement of the car body above its equilibrium position and y is the vertical displacement of the wheel (due to the road surface measured with respect for some fixed horizontal line). The first-order derivative of Equation (5) yields as

$$v = x'(t) - y'(t) \tag{6}$$

Newton's second law also can be written as a second-order of an ordinary differential equation (Figure 1 (b)) such as

$$m''(t) = -F_s(t) - F_d(t) \tag{7}$$

Yu et al. [19] noted that road surface profile can be denoted by the sinusoidal curve for example

$$y = h\sin(\alpha z) \tag{8}$$

where h is the amplitude of the sinusoidal; y is the vertical displacement of the wheel due to the road surface measured concerning some fixed horizontal datum line and z is horizontal displacement. Assume that the car is travelled with an average horizontal speed (V) in the road profile with

$$z = Vt \tag{9}$$

Combine Equations (8) and (9) to form

$$y(t) = h\sin\left(\frac{\pi V t}{d}\right) \tag{10}$$

Differentiate of y(t) to yield

$$y'(t) = h\left(\frac{\pi V}{d}\right)\cos\left(\frac{\pi V t}{d}\right) \tag{11}$$

Equation (11) is recognized as the passive suspension system model and will be applied for Ford Scorpio car model.

2.2 Semi-Active System

Figure 2 shows the models employed to develop the mathematical modelling for the semi-active suspension system. The model consists of semi-active suspension (k_w), damper (c_w) and wheel (m_w).
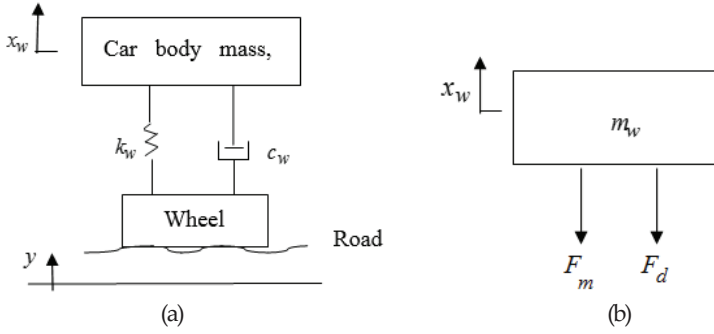
Figure 2: (a) Simple model of semi-active suspension and (b) its forces diagram model [6]

From Figure 2, the force of the semi-active suspension system for Newton's Second Law of motion is applied vertically upwards. The second order of ordinary differential equation for Newton's second law can be written such as

$$m''(t) = -F_m(t) - F_d(t) \tag{12}$$

Resistance for motion due to the damper effect denoted by

$$F_d = k(x(t) - x_w(t)) \tag{13}$$

The semi-active suspension system proposed with an MR damper. Bingham model used as a modelled for dynamics of the damper to describe the behaviour of MR damper. Bhise et al. [6] noted that the Bingham model is a simple model where only two parameters are needed to characterize the MR damper behaviour and making the analysis simpler. The Bingham model is expressed as

$$F_m = f_c \operatorname{sgn} \dot{x} + c_o \dot{x} + f_o \tag{14}$$

where f_c is the friction force; f_0 is the force due to the presence of the accumulator and c_0 is the viscous damping parameter. By substituting Equations (13) and (14) into Equation (12) therefore

$$mx''(t) = -k(x(t) - x_w(t)) - f_c \operatorname{sgn} \dot{x} + c_o \dot{x} + f_o \tag{15}$$

Re-arrange Equation (15) becomes

$$m_w x_w''(t) = k(x_w(t)) - k(x\ (t)) - f_c \operatorname{sgn} \dot{x} + c_o \dot{x} + f_o \tag{16}$$

Equation (16) is also known as the mathematical model for the semi-active suspension system. This equation will be utilized in the Ford Scorpio car for the validation purpose.

3.0 DEVELOPMENT OF THEORIES

The proposed model is verified using the parameter values of the Ford Scorpio Car. The following parameters of the Ford Scorpio car model as referred to [14] are such as

$m = 275kg;\ k = 25000 Nm^{-1};\ c = 1200 Nsm^{-1};$

$V = 14ms^{-1};\ d = 2m$ and $h = 0.1m.$

3.1 Passive System of Ford Scorpio Car

Equation (7) is also displayed as

$$m''(t) = -k(x - y) - c(x'(t) - y'(t)) \tag{17}$$
$$m''(t) + cx'(t) + kx(t) = ky + cy'(t) \tag{18}$$

Substitute Equations (10) and (11) into Equation (18) producing

$$m''(t) + cx'(t) + kx(t) = k.h \sin\left(\frac{\pi v t}{d}\right) + c.h\left(\frac{\pi v}{d}\right)\cos\left(\frac{\pi v t}{d}\right) \tag{19}$$

Input parameters from Ford Scorpio car model are substituted into Equation (19) obtain

$$275 x''(t) + 1200 x'(t) + 2500 x(t) = 2500 \sin(7\pi t) + 840\pi \cos(7\pi t) \tag{20}$$

Employ the method of the undetermined coefficient for solving Equation (20) and finally, the passive system model is displayed as

$$X(t) = e^{-\frac{24}{11}t}\left[0.0284\cos\left(\frac{2\sqrt{2606}}{11}\right)t + 0.045\sin\left(\frac{2\sqrt{2606}}{11}\right)t\right]$$
$$- 0.0284\cos(7\pi t) - 0.0162\sin(7\pi t) \tag{21}$$

3.2 Semi-Active System of Ford Scorpio Car

Substitute the input parameter of Ford Scorpio into Equation (16) to yield

$$X(t) = e^{-\frac{24}{11}t}\left[0.0284\cos\left(\frac{2\sqrt{2606}}{11}\right)t + 0.045\sin\left(\frac{2\sqrt{2606}}{11}\right)t\right]$$
$$- 0.0284\cos(7\pi t) - 0.0162\sin(7\pi t) - f_c\,\text{sgn}\,\dot{x} + c_o\dot{x} + f_o \tag{22}$$

$$275x''_w(t) = -1200(x'_w(t)) - 25000(x_w(t)) \tag{23}$$

$$m_w x''_w(t) = 25000(x_w(t)) + 1200(x'_w(t)) - 25000(x_w(t)) \\ - y(t)) - 140(x'_w(t) - y'(t) \tag{24}$$

Equations (23) and (24) can also be written as

$$Ms^2X(s) = -C(sX(s) - sX_w(s)) - K_1(X(s) - X_w(s)) \tag{25}$$

$$M_w s^2 X_w(s) = K_2 Y(s) - C(sX_w(s) - sX(s)) \\ - K_1(X_w(s) - X(s)) - K_2 X_w(s) \tag{26}$$

Re-arrange Equations (25) and (26) to develop

$$(M_1 s^2 + Cs + K_1)X(s) - (Cs + K_1)X_w(s) = 0 \tag{27}$$

$$-(Cs + K_1)X(s) + (M_2 s^2 + Cs + K_1 + K_2)X_w(s) = K_2 Y(s) \tag{28}$$

Convert Equations (27) and (28) into a matrix system such as

$$\begin{bmatrix} (M_1 s^2 + Cs + K_1) & -(Cs + K_1) \\ -(Cs + K_1) & (M_2 s^2 + Cs + K_1 + K_2) \end{bmatrix} \begin{bmatrix} X(s) \\ X_w(s) \end{bmatrix} = \begin{bmatrix} 0 \\ K_2(s) \end{bmatrix} Y(s) \tag{29}$$

Solve Equation (29) using Cramer rule and include the input parameter of Ford Scorpio car model to yield

$$x(t) = 0.00256\cos(7\pi t) + 5.65450 \times 10^{-33}\,e^{-0.5446t}\,(1.09197 \times 10^{30}$$
$$\cos(6.64811)t + 2.900278 \times 10^{-30} + 2.900278 \times 10^{-30}\,e^{-16.5261t}$$
$$(-1.247 \times 10^{27}\cos(29.35963t) + 1.01072 \times 10^{27}\sin(29.35963t)) \tag{30}$$

The mathematical model of the semi-active automobile suspension system for Ford Scorpio car is represented by Equation (30).

3.0 RESULTS AND DISCUSSION

Mathematically, *x(t)* developed from the previous section or also known as displacement is the main focus of the study. *x(t)* continues to be employed in graphing them using MATLAB software. Both graphs either passive or semi-active suspension system uses a range of time from 0 to 6s because of the stability and consistency reason. The assessment of validating the developed models is based on the suspension travel limits given by Ford Scorpio car model. Amin et al. [14] noted that the oscillation of the suspension system must lie within ±0.1 m to give a suitable and comfortable ride among the user.

Figure 3 shows the maximum displacement occurred at 0.07 m due to the spring is expanded while the minimum displacement at -0.07 m because the spring is in shrank mode. Both reactions happened due to the spring functioned to stabilize the body of a car. Figure 6 also depicts the passive suspension system contained a constant value starting from $t = 1.5$ s at $x(t) = 0.035$ m. The frequency produced by the passive system equivalents to 1.2732 Hz for $t = 0$ to $t = 6$ s.
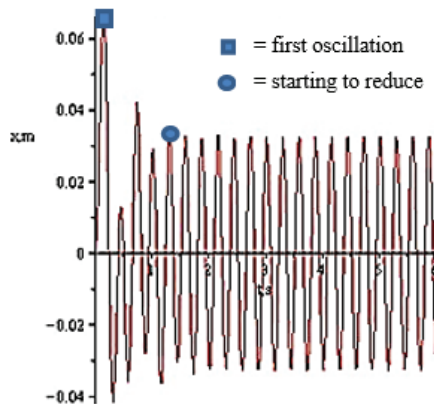

Figure 3: Graph of displacement (*x*) against time (*t*) for Ford Scorpio passive suspension system

Figure 4 depicts the oscillation of the car body's vertically in damped displacement. At 0.06 m, the displacement is peak while the minimum displacement occurred at -0.06 m. Figure 4 also shows the semi-active system has a constant value at 0.025 m starting from $t = 1$ s until the end.

The frequency of the semi-active suspension system produced roughly 1.0610 Hz occurred from the displacement ($t = 0$ to $t = 6$ s).
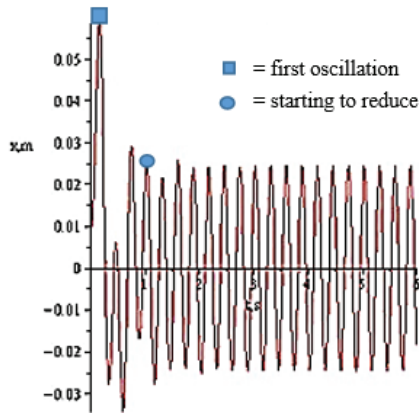

Figure 4: Graph of displacement (x) against time (t) for Ford Scorpio semi-active suspension system

Moreover, Figures 6 and 7 showed that the displacement of passive and semi-active suspension system lies within ±0.1 m with a passive system at ±0.035 m and semi-active system at ±0.025 m. Semi-active system consisted of lower displacement values compared to the passive system. Therefore, the semi-active system has lower frequency value than a passive system because the relationship between frequency and displacement was inversely proportional with the influencing of velocity. Furthermore, good stability also observed from the semi-active suspension system as the system depicted the constant displacement produced rapidly when compared with the passive system. The statement was also agreed by Bhise et al. [6] that the smooth response of controlling the quarter car vehicle travelling on a rough road was from the semi-active suspension system. Therefore, the semi-active suspension system in all vehicles has the potential to increase the safety and comfort among the users. The semi-active damper performance showed a better value of displacement, frequency and time than the passive damper system.

4.0   CONCLUSION

The mathematical model of passive and semi-active suspension systems has been successfully developed and applied for Ford Scorpio car. Second-order differential equation and Cramer rule are the employed methods during mathematical modelling development. Semi-active suspension system produced the lower displacement than

passive system and therefore, the semi-active suspension produced a low vibration. A low vibration gives a low frequency as depicted in the semi-active suspension system. The semi-active suspension system also obtained a faster time to yield a constant displacement when compared to the passive suspension system. From this finding, the semi-active showed a good in terms of stability and comfortability from the passive system. Continuously, the semi-active system gives the user or passenger with comfort travel and also offers to reduce the car suspension system from any damage. Further investigation and experimentation into semi-active automobile suspension systems are strongly recommended especially for Malaysian vehicles such as PROTON. It would be more interesting to assess the combination of semi-active suspension systems such as electromagnetic and Magneto-Rheological for having better results in the controlling performance of the automobile suspension system.

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