DEVELOPMENT OF A STRICT FEEDBACK TWO-MASS HORIZONTAL AXIS WIND TURBINE MODEL WITH EMPIRICAL POWER COEFFICIENT AND EXTERNAL STIFFNESS

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ABSTRACT: For a variable speed wind turbine, the control approach requires the exact dynamical model of such wind turbine to be under controlled. The established wind turbine dynamics neglect the external stiffness as an integrator to the system dynamics is introduced. In addition, many approaches assume a constant value for the coefficient, which is practically not a valid choice. Hence, this paper proposed a strict feedback model of a two-mass wind turbine system that focused on external stiffness, and also embedded empirical power coefficient in the system dynamics. The model represented in this study is a "ready-to-be-used" form that benefits the control system engineers when the feedback control approaches are taken into account.

KEYWORDS: *Wind turbine, modeling, strict feedback model, power coefficient, tip-speed-ratio.*

1.0 INTRODUCTION

The issue arises from the fact that fossil energy and awareness of the environmental protection are influencing people to use wind as an alternative energy source. For example, coal-fired power plants emit greenhouse gases such as carbon dioxide and sulphur dioxide that contribute to global climate change. As such, control techniques to achieve wind turbine efficacy have been designed, for instance a fixed-pitch variable speed wind turbine [1-4] and variable-pitch variable speed wind turbine [5]. In modern wind energy conversion systems, research in variable speed wind turbine is flourishing. In a constant speed wind turbine, the rotor speed remains constant for all wind speeds. As the size of turbine increases and due to wind intermittent, the inherent problems of the constant speed wind turbine becomes more pronounced. On the other hand, variable speed wind turbine allows the rotor and wind speed to be matched in order to maintain its optimum tip-speed-ratio for maximum efficiency. For control engineers and designers, the knowledge of a numerical representation of a dynamical wind turbine model is important. In addition, prominent aero-turbine controls inside the nacelle require the exact model of the turbine that includes the mechanical dynamics of the turbine and generator. However, most research overlook some fundamental aspects in the system dynamic. For instance, in various literatures [1-2, 6-7], the lumped rotor stiffness and the generator stiffness have been neglected. The presence of the stiffness introduces an integrator in the system model, which is difficult to handle and trivial to controller design. Hence, this paper focused on the stiffness in the wind turbine dynamics for practicality.

The aerodynamic torque produced by the turbine is proportional to the square of the rotational speed of the blades [8] and the power coefficient. The power coefficient is a unique nonlinear function that depends on turbine types [9-10]. As such, the value of power coefficient is provided through look-up-table by each turbine manufacturer. Beltran et al. [1] states that there is no accurate way to determine the power coefficient. Ozbay et al. [11] sets a constant value for the coefficient, which is practically not a valid choice. Alternatively, this paper embeds the empirical value of power coefficient in the system model. The empirical expression of the power coefficient is the established expression that has been published [9-10].

This paper produces a strict feedback model of a two-mass horizontal axis wind turbine system. Strict feedback model is known as a standard model that eases control engineers when the controller has to be designed by using full state feedback approaches such as back-stepping [12], pole placement [13-15] or optimal type such as Linear-Quadratic-Regulator / Gaussian [16-19]. Figure 1 shows the structure of a two-mass wind turbine system. Table 1 tabulates wind turbine parameters. The model consists of a turbine side, a low-speed shaft and a high-speed shaft. The term of two-mass model emerges from the turbine structure which imposes a gear box that separates a low-speed mechanical dynamic and a high-speed mechanical dynamic.

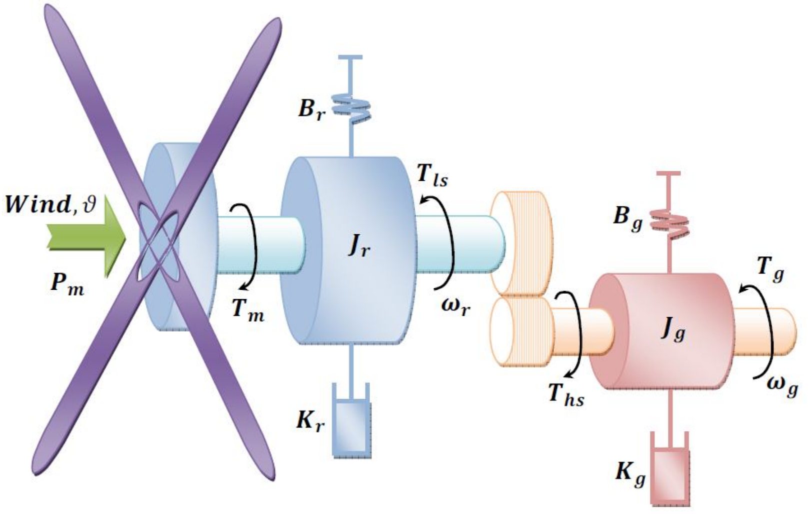

Figure 1: Two-mass wind turbine structure

Table 1: Nomenclature

Symbols	Definition	Symbols	Definition
R	Rotor blade radius (m)	K_g	Generator external damping $(N.m.rad^{-1}.s^{-1})$
υ	Wind speed $(m.s^{-1})$	K_r	Generator external damping $(N.m.rad^{-1}.s^{-1})$
ρ	Air density $(Kg.m^{-3})$	B_r	Rotor stiffness $(N.m.rad^{-1})$
$C_p(\lambda,\beta)$	Power coefficient	B_g	Generator stiffness $(N.m.rad^{-1})$
λ	Tip speed ratio	T_m	Aerodynamic torque $(N.m)$
β	Pitch angle (deg)	T_g	Generator torque/Electromagnetic torque $(N.m)$
γ	Gearing ratio	T_{hs}	High-speed shaft torque $(N.m)$
ω_r	Rotor speed $(rad.s^{-1})$	T_{ls}	Low-speed shaft torque $(N.m)$
ω_g	Generator speed $(rad.s^{-1})$	θ_g	Generator-side angular deviation (rad)
J_r	Rotor inertia $(Kg.m^2)$	θ_r	Rotor-side angular deviation (rad)
J_g	Generator inertia $(Kg.m^2)$		

2.0 WIND TURBINE MODEL

A mechanical structure of wind turbine comprises rotor model and aero-turbine model. Turbine rotor consists of blades, hub and pitch. The aero-turbine consists of a brake, gear box, rotor dynamics comprising a low speed shaft, high speed shaft, and mechanical dynamics of the generator. The yaw drive and yaw motor are located inside the tower-end. The anemometer with the wind vane is attached at the nacelle-end. The electrical dynamics of the generator can be modeled separately and only beneficial for power electronics and electrical engineers, and therefore, is not the focus of this paper.

2.1 Rotor Model

Wind turbines operates by converting the kinetic energy from the wind into rotational energy in the turbine. The rotational energy is then converted into electrical energy. The energy conversion depends on the wind speed and the swept area of the turbine. Figure 2 shows the swept area; that is the region where the turbine can capture the kinetic energy.

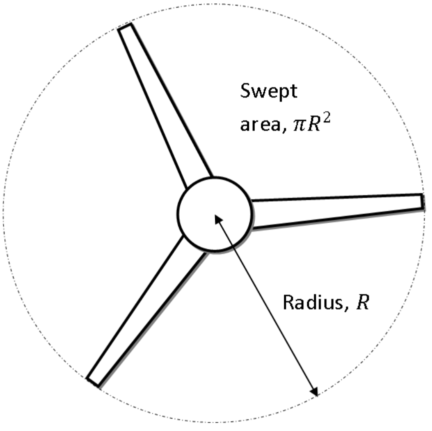

Figure 2: Turbine swept area

The aero-dynamic power produced by the turbine can be expressed as:

$$P_m = P_{wind} C_p(\lambda, \beta) \tag{1}$$

where

$$P_{wind} = \frac{1}{2}\rho\pi R^2 v^3 \tag{2}$$

is the instantaneous power produced by the wind, and

$$\lambda = \frac{R\omega_r}{v} \tag{3}$$

is the tip-speed-ratio.

The power coefficient $C_p(\lambda, \beta)$ can be obtained through the look-up-table by each turbine manufacturer. For a variable speed wind turbine, the control scheme aims at the optimum tip-speed-ratio $\lambda = \lambda_{opt}$, as such that the maximum power coefficient $C_p = C_{p_{max}}$ can be obtained. Beltran et al. [1] states that there is no accurate way to determine the power coefficient. Ozbay et al. [11] sets a constant value for the coefficient, which is practically not a valid choice. As an alternative approach, this paper embeds the empirical value of power coefficient in the strict feedback wind turbine model. In [9-10], the empirical power coefficient model is represented by

$$C_p(\lambda,\beta) = 0.5\left(116\frac{1}{\phi} - 0.4\phi\beta - 5\right)e^{-21\frac{1}{\phi}} \tag{4}$$

where the function ϕ is given as:

$$\frac{1}{\phi} = \frac{1}{\lambda + 0.08\beta} - \frac{0.035}{1 + \beta^3} \tag{5}$$

Figure 3 shows power coefficient characteristic for various β. For a regulated pitch angle at $\beta = 0^\circ$, one may obtain the expression for power coefficient as:

$$C_p = 0.5\left(\frac{116}{\lambda + 0.0001} - 9.06\right)e^{-\frac{21}{\lambda+0.0001}+0.735} \tag{6}$$

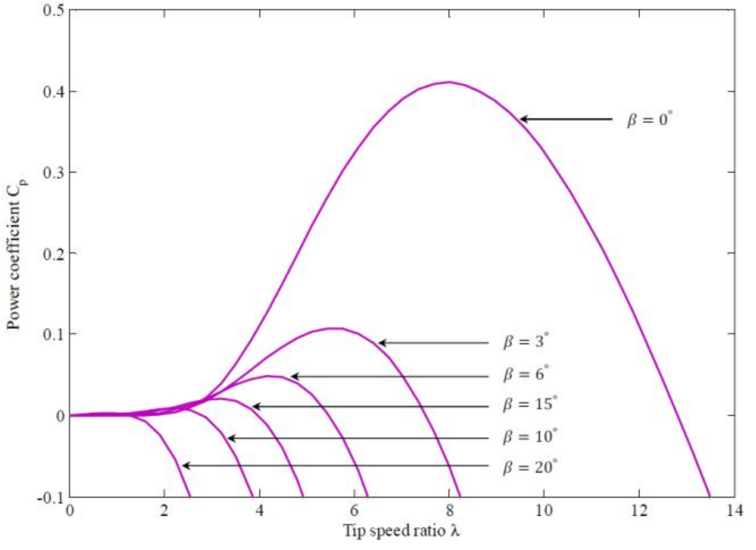

Figure 3: Power coefficient characteristic

2.2 Aero-turbine Model

From Figure 1, the torque produced by the turbine (low speed shaft) can be deduced as

$$T_m = T_{ls} + J_r\dot{\omega}_r + K_r\omega_r + B_r\theta_r$$
$$T_m - T_{ls} = J_r\dot{\omega}_r + K_r\omega_r + B_r\theta_r \tag{7}$$

The transmission output torque (high speed shaft) is devised as

$$T_{hs} = T_g + J_g\dot{\omega}_g + K_g\omega_g + B_g\theta_g$$

$$T_{hs} - T_g = J_g\dot{\omega}_g + K_g\omega_g + B_g\theta_g \tag{8}$$

The gearing ratio

$$\gamma = \frac{\omega_g}{\omega_r} = \frac{T_{ls}}{T_{hs}} \tag{9}$$

From equation (9), $T_{hs} = T_{ls}\left(\omega_r / \omega_g\right)$, and substitute into equation (8) renders

$$T_{ls}\frac{\omega_r}{\omega_g} - T_g = J_g\dot{\omega}_g + K_g\omega_g + B_g\theta_g \tag{10}$$

Knowing from equation (9) that $\omega_g = \gamma\omega_r$, plugging into equation (10) yields

$$\frac{T_{ls}}{\gamma} - T_g = J_g\gamma\dot{\omega}_r + K_g\gamma\omega_r + B_g\theta_g$$

$$T_{ls} = \gamma T_g + J_g\gamma^2\dot{\omega}_r + K_g\gamma^2\omega_r + B_g\gamma\theta_g$$

$$= \gamma T_g + J_g\gamma^2\dot{\omega}_r + K_g\gamma^2\omega_r + B_g\gamma\int \omega_g\, d(t)$$

$$= \gamma T_g + J_g\gamma^2\dot{\omega}_r + K_g\gamma^2\omega_r + B_g\gamma^2\int \omega_r\, d(t) \tag{11}$$

Substitute (11) into (7) yields

$$T_m - \gamma T_g + J_g\gamma^2\dot{\omega}_r + K_g\gamma^2\omega_r + B_g\gamma^2\int \omega_r\, d(t)$$

$$= J_r\dot{\omega}_r + K_r\omega_r + B_r\int \omega_r\, d(t)$$

$$T_m - \gamma T_g = \left(J_r + \gamma^2 J_g\right)\dot{\omega}_r + \left(K_r + \gamma^2 K_g\right)\omega_r + \left(B_r + \gamma^2 B_g\right)\int \omega_r\, d(t)$$

$$J\dot{\omega}_r + K\omega_r + B\int \omega_r\, d(t) = T_m - \gamma T_g \tag{12}$$

where

The lumped inertia, $$J = J_r + \gamma^2 J_g \tag{13}$$

The lumped external stiffness, $B = B_r + \gamma^2 B_g$ (14)

The lumped external damping, $K = K_r + \gamma^2 K_g$ (15)

To reach the strict-feedback model of the turbine, let consider Proposition 1.

Proposition 1: *For a maximum power capture of the wind turbine in equation (12), the rotor speed ω_r should be regulated at $\omega_r^* = (\lambda_{opt}/R)\upsilon$. Therefore, let the rotor tracking error be*

$$y_1 = \omega_r - \frac{\lambda_{opt}}{R}\upsilon \tag{16}$$

Also, let the acceleration of the rotor is defined as

$$y_2 = \dot{\omega}_r \tag{17}$$

Then, a strict feedback wind turbine model can be obtained as

$$\dot{y}_1 = y_2 + \xi_1(\upsilon) \tag{18}$$

$$\dot{y}_2 = a_1 y_2 + a_2 y_1 + a_3 y_1 y_2 + u + \xi_2(y_2, \upsilon) \tag{19}$$

where the control input

$$u = a_4 \dot{T}_g \tag{20}$$

and the lumped parameters

$$a_1 = -\frac{K}{J} \tag{21}$$

$$a_2 = -\frac{B}{J} \tag{22}$$

$$a_3 = \frac{\rho\pi R^5 C_{p_{max}}}{J\lambda_{opt}} \tag{23}$$

$$a_4 = -\frac{\gamma}{J} \tag{24}$$

The sum of parameter uncertainties and exogenous disturbances due to wind intermittent are defined as

$$\xi_1(\upsilon) = -\frac{\lambda_{opt}}{R}\dot{\upsilon} \tag{25}$$

$$\xi_2(y_2, \upsilon) = \frac{(a_2 + a_3)\lambda_{opt}\upsilon}{R} y_2 \tag{26}$$

Proof of Proposition 1:

From equation (12),

$$\dot{\omega}_r = -\frac{K}{J}\omega_r - \frac{B}{J}\int \omega_r \, d(t) + \frac{T_m}{J} - \frac{\gamma}{J}T_g \tag{27}$$

The aero-dynamic torque applied to the hub of the wind turbine is denoted as

$$T_m = \frac{P_m}{\omega_r} \tag{28}$$

By substituting equations (1)-(2) into equation (28) renders

$$T_m = \frac{\rho \pi R^2 C_p \upsilon^3}{2\omega_r} \tag{29}$$

Recall the tip-speed-ratio in equation (3), then equation (29) becomes

$$T_m = \frac{\rho \pi R^5 C_p \omega_r^2}{2\lambda^3} \tag{30}$$

By substituting equation (30) into equation (27), the wind turbine dynamic can be extended as

$$\dot{\omega}_r = -\frac{K}{J}\omega_r - \frac{B}{J}\int \omega_r \, d(t) + \frac{\rho \pi R^5 C_p}{2\lambda^3 J}\omega_r^2 - \frac{\gamma}{J}T_g \tag{31}$$

From the equation (16) in Proposition 1, let the tracking error dynamics be

$$\begin{aligned} \dot{y}_1 &= \dot{\omega}_r - \frac{\lambda_{opt}}{R}\dot{\upsilon} \\ &= y_2 - \frac{\lambda_{opt}}{R}\dot{\upsilon} \end{aligned} \tag{32}$$

and

$$\begin{aligned} \dot{y}_2 &= \ddot{\omega}_r \\ &= -\frac{K}{J}\dot{\omega}_r - \frac{B}{J}\omega_r + \frac{\rho \pi R^5 C_p}{\lambda_{opt}^3 J}\omega_r \dot{\omega}_r - \frac{\gamma}{J}\dot{T}_g \end{aligned}$$

$$= -\frac{K}{J}y_2 - \frac{B}{J}\left(y_1 + \frac{\lambda_{opt}}{R}\upsilon\right) + \frac{\rho\pi R^5 C_p}{\lambda_{opt}^3 J}\left(y_1 + \frac{\lambda_{opt}}{R}\upsilon\right)y_2 - \frac{\gamma}{J}\dot{T}_g$$

$$= -\frac{K}{J}y_2 - \frac{B}{J}y_1 + \frac{\rho\pi R^5 C_p}{\lambda_{opt}^3 J}y_1 y_2 - \frac{B}{J}\left(\frac{\lambda_{opt}}{R}\upsilon\right)$$

$$+ \left(\frac{\rho\pi R^5 C_p}{\lambda_{opt}^3 J}\right)\left(\frac{\lambda_{opt}}{R}\upsilon\right)y_2 - \frac{\gamma}{J}\dot{T}_g$$

$$= -\frac{K}{J}y_2 - \frac{B}{J}y_1 + \frac{\rho\pi R^5 C_p}{\lambda_{opt}^3 J}y_1 y_2 + \left(-\frac{B}{J} + \frac{\rho\pi R^5 C_p}{J\lambda_{opt}^3}\right)\frac{\lambda_{opt}}{R}\upsilon y_2 - \frac{\gamma}{J}\dot{T}_g \qquad (33)$$

Recall equations (20)-(26) in Proposition 1, dynamical system in equations (32)-(33) becomes:

$$\dot{y}_1 = y_2 + \xi_1(\upsilon) \qquad (34)$$

$$\dot{y}_2 = a_1 y_2 + a_2 y_1 + a_3 y_1 y_2 + u + \xi_2(y_2, \upsilon) \qquad (35)$$

which is identical to the strict feedback system in equations (18)-(19). Hence , completing the proof.

3.0 DISCUSSION

Dynamical model in equation (12) is a standard two-mass wind turbine model. However, in extant literatures [1-2, 6-7], the external stiffness has been neglected in easing control design. Therefore, the model appears [1-2, 6-7] as

$$J\dot{\omega}_r + K\omega_r = T_m - \gamma T_g \qquad (36)$$

The power coefficient is assumed as a constant value [11]. As an alternative approach, this paper embeds the empirical value of power coefficient, and also includes the stiffness in the system model. The results are shown in equations (34)-(35), which confirm the strict feedback model (18)-(19) of Proposition 1. With a strict feedback model in equations (34)-(35), a variable speed controller using a state feedback approach can be readily applied.

Dynamical system (34)-(35) is a nonlinear system. The representation of the system in matrix form is illustrated in equation (37).

$$\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\end{bmatrix} = \begin{bmatrix}0 & y_2\\ a_2 & a_1 + a_3 y_1\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u + \begin{bmatrix}\xi_1(\upsilon)\\ \xi_2(y_2, \upsilon)\end{bmatrix} \qquad (36)$$

Wind intermittent $\xi_1(\upsilon)$ is mismatched to y_2, and $\xi_2(y_2, \upsilon)$ is appeared to be matched with the control signal u.

With a rated generator torque of 10KNm, the model is tested with wind speed around $\pm 18\, m.s^{-1}$. The wind speed profile is shown in Figure 4.

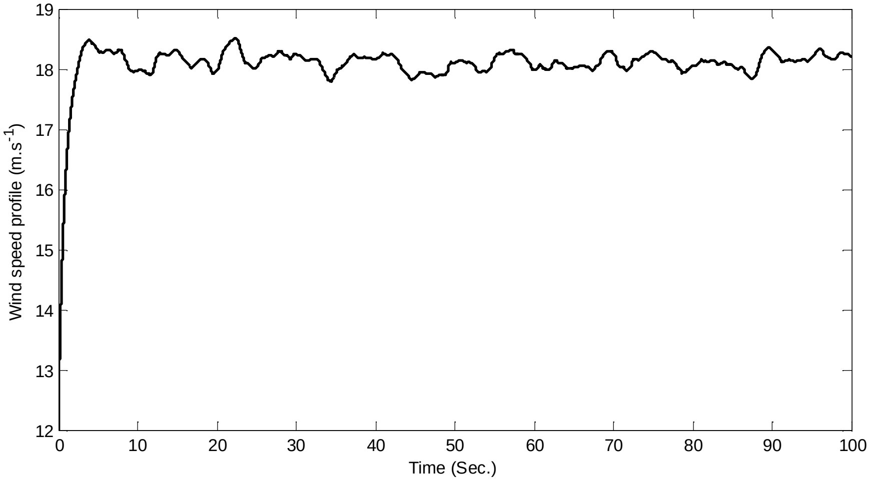

Figure 4: Wind speed profile

Figure 5 shows the rotor speed for both wind turbine with stiffness and without stiffness. It can be observed that the stiffness opposes the exerted torque by the generator. The effect of the stiffness is rather small and some researchers neglect its appearance. However, for a large aero-turbine system, the appearance of the stiffness would affect the generated output power.

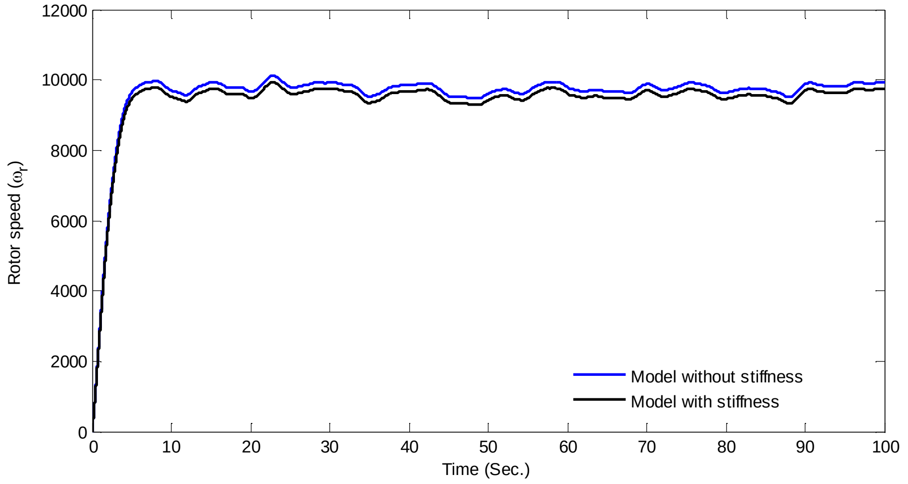

Figure 5: Rotor speed

4.0 CONCLUSION

In this paper, a strict feedback form of a two-mass wind turbine system has been developed. It is apparent that inclusion of external stiffness and power coefficient expression in the wind turbine model is possible for an advanced control approach. The model appears as a nonlinear wind turbine with a mixed match-mismatch wind intermittent. The model presented in this paper can be further applied by control engineers for controller design. The mismatch $\xi_1(\upsilon)$ gives challenge to control engineers, hence, the matching condition should be taken into account prior to design phase.

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