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A First-Principles-Based Model for Crack Formation in a Pressurized Tank Following an MMOD Impact

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Abstract

Most robotic spacecraft have at least one pressurized vessel on board, usually a liquid propellant tank. One of the design considerations of such spacecraft is the anticipation and mitigation of the possible damage that might occur from on-orbit impacts by micro-meteoroids or orbital debris (MMOD). While considerable effort has been expended in the study of the response of non-pressurized spacecraft components to MMOD impacts, relatively few studies have been conducted on the pressurized elements of such spacecraft. In particular, since it was first proposed nearly 45 years ago, NASA's current evaluation methodology for determining impact-induced failure of pressurized tanks has undergone little scrutiny. This paper presents a first-principles based model that has been developed to predict whether or not cracking might start or a through-crack might be created under an impact crater in a thin plate. This model was used to examine the effect of penetration depth on crack formation and whether or not the crack might grow through the tank wall thickness. The predictions of the model are compared to experimental data with encouraging results. The paper also develops some suggestions for future work in this area, including the extension of the first-principles model to include 3-D crack initiation modelling.

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Keywords: Pressure vessel, MMOD, fracture, hypervelocity impact, analytical model

1. Introduction

Most spacecraft have at least one pressurized vessel on board. For robotic spacecraft, it is usually a liquid propellant tank. For human spacecraft, there are also pressurized living quarters (or modules). Because of the potential of serious mission-threatening damage that might result following an on-orbit MMOD impact, one of the primary design considerations of such spacecraft is the anticipation and mitigation of the possible damage that might occur in the event of such an impact; to prevent mission failure and possibly loss of life, protection against perforation by high-speed impacts must be included.

The performance of a hypervelocity impact shield is typically characterized by its ballistic limit equation (BLE), which defines the threshold particle size that causes damage, such as perforation of, or detached spall from, the shield as a function of velocity, impact angle, particle density, shield and inner wall thicknesses and particle shape. These BLEs are typically drawn as lines of demarcation between regions of inner-wall perforation and no perforation (P / NP) in two-dimensional projectile diameter-impact velocity space and when graphically represented, are often referred to, in this form, as ballistic limit curves (BLCs).

The BLEs used to characterize the impact response of common spacecraft wall configurations are typically anchored by either tests or numerical simulations (or both) using unstressed plates to simulate the inner or main wall of the structural

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component. However, fuel tanks and habitable modules are pressurized internally, and so their main walls will develop bi-axial stress fields because of that internal pressurization. Considerable energy and effort has been expended in the study of the response of non-pressurized spacecraft components to MMOD impacts, but technical and safety challenges have limited the number of tests that have been conducted on the pressurized elements of such spacecraft. An Apollo-era design criterion [1,2] is still sometimes used by NASA for pressurized tanks operating in the MMOD environment, but there is no known documentation in the open literature of the tests on which the criterion was based [3].

A similar criterion appears in an older document [4], but it gives only a cursory description of the underlying tests, and an incomplete explanation of the proper implementation of the criterion. Subsequent tests of pressurized vessels have concentrated on differentiating between conditions that cause leaks rather than rupture (see, e.g. [5]), but not the conditions under which a leak is initiated. For robotic spacecraft, where pressurized vessels typically have no redundant unit as back-up, and leaks cannot be repaired, a leak usually constitutes loss of the spacecraft. It is important to note that the boundary between impact-induced leak and burst is pertinent when addressing the possibility of impact-induced debris generation (see, for example, requirement SD-DE-08 in Ref. [6] concerning end-of-mission removal of a space vehicle's stored energy).

This paper presents a first-principles based model that has been developed to predict whether or not cracking might start or a through-crack might result under a crater in a thin plate. This model was used to examine the effect of penetration depth on crack formation and whether or not the crack might grow through the tank wall thickness. The predictions of the model are compared to experimental data with encouraging results. The paper also develops some suggestions for future work in this area, including the extension of the first-principles model to include 3-D crack initiation modeling.

2. Analytical model of crack formation

Pressurized tank wall failure following a high-speed impact can occur in any number of ways. In the event of a tank wall perforation, these failure mechanisms include

a. a through-hole on the front-facing tank wall (with only pitting on the inside of the rear surface),

b. a through-hole on the front and on the rear walls, or

c. so-called 'wing cracks' that emanate from a through-hole on either the front or rear wall (these cracks could lead to catastrophic failure of the tank, but not necessarily).

In the event that tank perforation does not occur (that is, the front wall sustains only crater damage), it is possible that a crack emanates from the floor of one of the deeper craters, and, under the right conditions, grows through the tank wall until it reaches the rear free surface of the front wall. The failure modes associated with tank perforation (and longitudinal cracks that might develop from the edges of such perforations) have been studied extensively by other investigators (see, e.g., [7] – [10]), while tank failure resulting from a crack forming at the floor of a deep crater or pit has not. Previous investigators have also explored conditions under which pressurized vessels could burst following perforation and cracking (see, e.g., [11] for ISS-type pressurized modules and [12] for highly pressurized tanks).

It is apparent that tank failure due to cracking without perforation appears to be the failure mode addressed by the design criteria originally proposed by Cour-Palais [4] and currently used by NASA and its contractors to design pressurized tanks [3]. As such, the objective of our effort was to develop a first-principles based model that can be used to assess the effect of penetration depth on the possibility of a crack forming under a crater, and whether or not the crack might grow through the tank wall thickness. Fig. 1 below highlights the various components of this model and shows the flow of the calculations performed. The various components of the model are discussed in more detail in the following sub-sections.

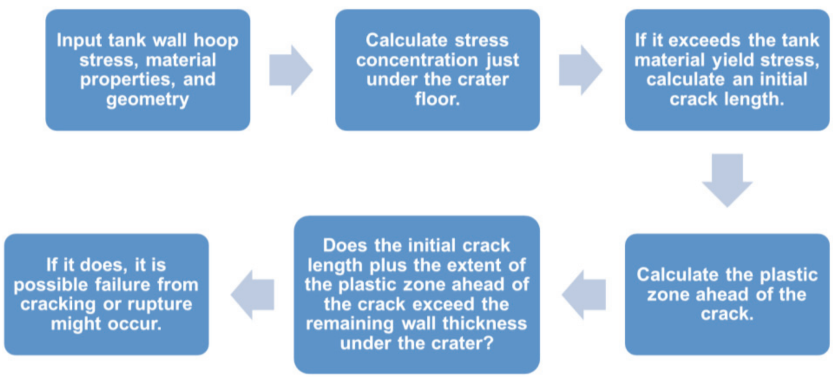


Fig. 1. Flow of calculations in crack formation model

2.1. Tank wall hoop stress

The hoop stress in either a thin-walled spherical or cylindrical tank is found through the application of the linear elastic theory of plates and shells. It is given by

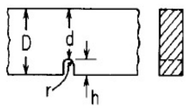
$$\sigma_h = f_s(pr / t_w) \tag{1}$$

where p is the internal pressure, r is the tank radius, t_w is the tank wall thickness, and the shape factor f_s is equal to 1 for a cylindrical tank and 0.5 for a spherical tank.

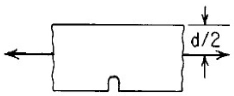
2.2. Crater floor stress concentration

Much as the presence of a circular hole in a thin plate under a tensile load gives a stress concentration near the hole, the presence of a dimple or pit in the tank wall gives rise to a stress concentration just under the dimple floor, with the maximum stress concentration being located just under the dimple's deepest point. Fig. 2 below provides the equation necessary to calculate the stress concentration for a two-dimensional representation of the geometry of interest (i.e. for a circular-profile notch, as a bounding case for a circular dimple) [13].

3. One U-notch in a member of rectangular section



3a. Elastic stress, axial tension



$$K_t = C_1 + C_2\left(\frac{h}{D}\right) + C_3\left(\frac{h}{D}\right)^2 + C_4\left(\frac{h}{D}\right)^3$$

where for $0.5 \leqslant h/r \leqslant 4.0$

$$C_1 = 0.721 + 2.394\sqrt{h/r} - 0.127h/r$$
$$C_2 = 1.978 - 11.489\sqrt{h/r} + 2.211h/r$$
$$C_3 = -4.413 + 18.751\sqrt{h/r} - 4.596h/r$$
$$C_4 = 2.714 - 9.655\sqrt{h/r} + 2.512h/r$$

For the semicircular notch ($h/r = 1$)

$$K_t = 2.988 - 7.300\left(\frac{h}{D}\right) + 9.742\left(\frac{h}{D}\right)^2 - 4.429\left(\frac{h}{D}\right)^3$$

Fig. 2. Stress concentration factor for a notch in a thin plate under a tensile load

We note that the value of the stress under the notch, σ_A, is given by

$$\sigma_A = K_t \sigma_{nom} \tag{2}$$

where σ_{nom} is the tensile stress resulting from the application of the full load over a reduced cross-sectional area, that is, Wd = W(D-r) per notch section of length W. That is, the relationship between σ_{nom} and an applied load σ_o is given as follows:

$$\sigma_o DW = \sigma_{nom}(D-r)W \tag{3}$$

From which we find that

$$\sigma_{nom} = \sigma_o \frac{D}{D-r} = \sigma_o \frac{1}{1-r/D} \tag{4}$$

Thus, in our problem, if in Equation (4) we set σ_o equal to the hoop stress ($\sigma_o = \sigma_h$), and let $D = t_w$, we are able to use Equation (2) to estimate the value of the (concentrated) stress just under the floor of a pit or dimple in the wall of a pressurized tank.

If the value of the concentrated stress given by Equation (2) exceeds an appropriate pre-determined value (for example, the yield stress of the tank wall material, or its ultimate stress), we can put forth the proposition that a crack may form under the pit or dimple in the tank wall. In this case, we move on to the next calculation – that of the initial crack length. However, if that pre-determined value is not exceeded, it is presumed that no crack will form and the calculations cease.

We also reiterate that the information in Fig. 2 above is for a 2-D notch geometry in a thin plate whereas the actual situation is fully three-dimensional. However, it is interesting and instructive to note that for $r/t_w = 0.25$ (i.e. a 25% indentation depth), Equations (2) and (4) predict the stress under the crater floor to be approx. 2.27 times the far-field applied load. This result compares very favorably with the values obtained in two studies of the 3-D geometry.

(1) In Ref. [14], investigators determined (using NASTRAN) that the stress concentration for a hemispherical cavity in a thin plate at a 25% indentation depth was approx. 2.3.

(2) In Ref. [15], investigators found a series form solution for the stress field around a hemi-spherical pit (i.e. they solved a fully 3-D problem) at the free surface of a semi-infinite elastic body. They found that the ratio of the stress just below the crater floor to the far-field applied load to be 2.23.

In the first case, there was a difference of approx. 1.5% from the plane stress value of 2.27 calculated using this model. And, in the second, the difference was less than 2% from the plane stress model value. These are highly encouraging results, and indicate that the use and combination of fundamental 2-D solutions (such as that in Fig. 2) can lead to an increased understanding of the possible formation of cracks emanating from the floor of a tank wall crater or pit.

2.3. Initial crack length

If the value of the tensile stress concentration under the crater in the tank wall does exceed some pre-determined stress value (such as the yield stress or the ultimate stress of the material), a Mode I crack is presumed to be formed under the crater in the direction of tank center (i.e. through the thickness of the tank wall). We can bound the length of such a crack by equating the stress intensity at the tip of the crack under the action of that pre-determined stress value to the fracture toughness of the material, and then solving for the crack length. While a closed-form, fully three-dimensional expression for the Mode I stress intensity factor for such a situation would be desirable, it has, unfortunately, proved elusive and remains unfound. Thus, the two-dimensional analog of this situation shown in Fig. 3 below is used instead [16].

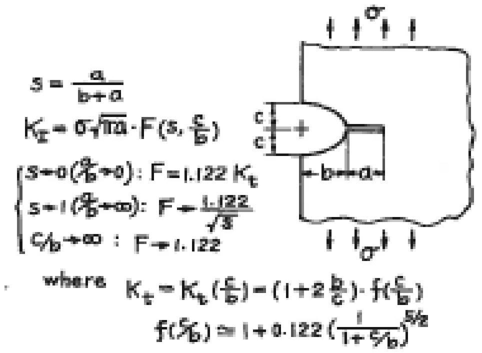


Fig. 3.2 Mode I stress intensity factor for a crack emanating from a semi-circular edge notch under a tensile load

In our particular situation, b = c, and since we would expect an initial crack to be very small, we have the situation where $a/b \rightarrow 0$. This allows us to use simplified versions of the equations given in Ref. [16] and highlighted in Figure 3. Under these conditions we have

$$f\left(c/b=1\right)=1.0215 \rightarrow K_t = 3.0647 \rightarrow F = 3.4386 \quad (5)$$

so that the stress intensity K_I at the tip of the crack is

$$K_I = 3.4386\sigma\sqrt{\pi a} \quad (6)$$

If we presume that an initial crack of length a_o will appear when the stress under the crater floor reaches, say, the yield stress of the material, then we can write Equation (6) as

$$K_{Ic} = 3.4386\sigma_y\sqrt{\pi a_o} \quad (7)$$

from which the value of a_o can be readily obtained.

2.4. Plastic zone ahead of the crack

The plastic zone ahead of a crack time is illustrated in Fig. 4 below.

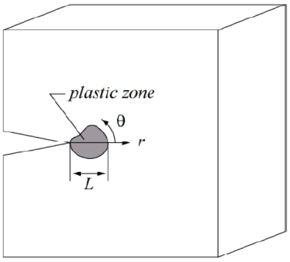


Fig. 4. Sketch of the plastic zone ahead of a crack tip

The extent 'L' of the plastic zone can be found using 2-D linear elastic fracture mechanics [17], and is given as follows:

$$L = (1/2\pi)(K_I/\sigma_y)^2 \tag{8}$$

In our particular problem the length of the plastic zone is calculated with $K_I = K_{Ic}$.

3. Model predictions

Equations (1) – (8) constitute the first-principles-based model developed to assess whether or not a crack might form and/or propagate through the thickness of a pressurized tank wall. After the length of the plastic zone is calculated using Equation (8) it is added to the length of the initial crack calculated using Equation (7). If that sum exceeds the tank wall thickness remaining under the dimple, it is said that a crack might form and propagate through the remaining thickness. If it does not, then a crack might still form, but would probably not propagate through the remaining wall thickness.

Figs. 5 and 6 show plots obtained by running the model for two different tank wall materials (Al 2219-T87 and Ti-6Al-4V) for a series of penetration depths relative to tank wall thickness, and a series of operating conditions (hoop stress relative to material yield stress). Fig. 5 also highlights the nominal operating condition for the International Space Station, while Fig. 6 highlights that of the SMAP satellite currently being built, and shows where the results of some experimental tests from Refs. [18] and [19] might lie when the model is run using the conditions of those tests.

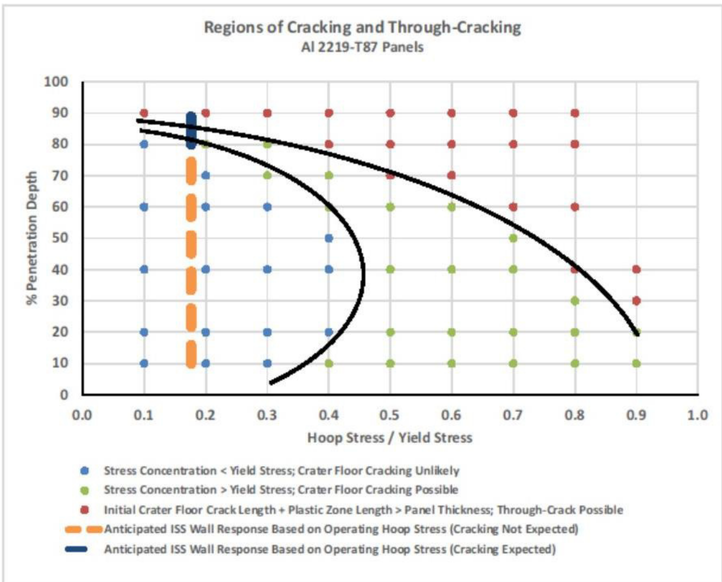


Fig. 5. Model predictions of regions of cracking and thru-cracking for Al 2219-T87 tank walls

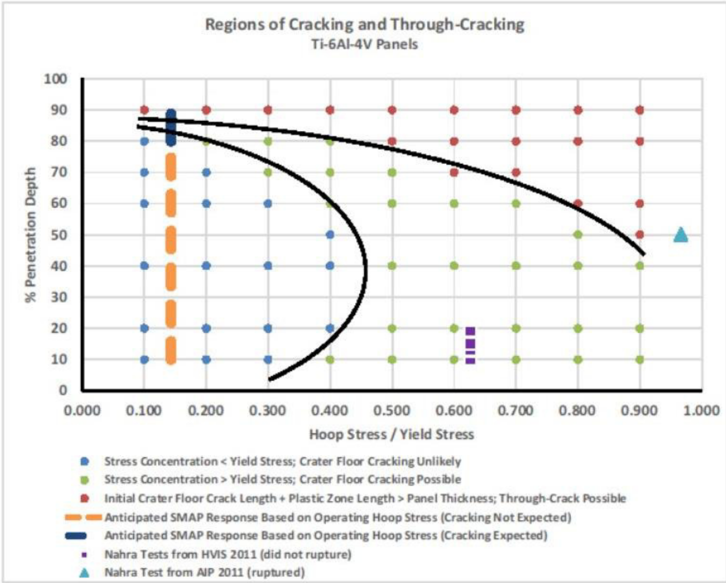


Fig. 6. Model predictions of regions of cracking and thru-cracking for Ti-6Al-4V tank walls

In Fig. 6 we see that the phenomenology evident in experimental data agrees with the predictions of the model developed herein:

a. the tests (purple square markers) that resulted in tank wall cratering without rupture (but possibly cracking under the crater) [18] fall in the regime where the model predicts that cracking might occur under a crater, but not loss of pressurization, and

b. the test (teal triangle) that resulted in tank rupture [19] falls in the regime where the model predicts that the length of the initial crack plus the plastic zone ahead of it will likely exceed the thickness of the tank wall material under the crater.

4. Concluding thoughts and suggestions for future work

This paper has presented the development of a first-principles based model that predicts whether or not cracking might start or a through-crack might result under a crater in a thin plate created by a high-speed impact. Based on the work performed and the encouraging preliminary results obtained thus far, it would appear that extending this model to 3-D crack initiation by using stress concentration factors for '3-D dimples' or stress intensity factors for surface-breaking semi-elliptical cracks (e.g., see Fig. 7 below).

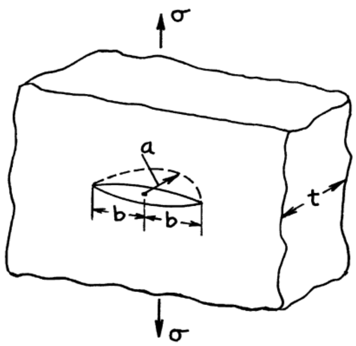


Fig. 7. Illustration of a surface-breaking semi-elliptical crack

Stress intensity factors for such configurations are readily available (see, e.g. [20] and [21]), and can be used to estimate initial crack length as a function of penetration depth in a manner similar that which was used in the present 2-D analysis.

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