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Differences between regular and random order of updates in damage-spreading simulations

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We investigate the spreading of damage in the three-dimensional Ising model by means of large-scale Monte Carlo simulations. Within the Glauber dynamics we use different rules for the order in which the sites are updated. We find that the stationary damage values and the spreading temperature are *different* for *different update order*. In particular, random update order leads to larger damage and a lower spreading temperature than regular order. Consequently, damage spreading in the Ising model is nonuniversal not only with respect to different update algorithms (e.g., Glauber vs heat-bath dynamics) as already known, but even with respect to the order of sites. [S1063-651X(98)12312-7]

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Damage spreading (DS) investigates how a small perturbation in a cooperative system changes during the time evolution. It was first studied in theoretical biology [1] in the context of genetic evolution. Later the DS concept found its way into the physics of cooperative systems [2–4]. In order to study DS two replicas of the system are considered which evolve stochastically under the same noise realization (i.e., the same random numbers are used in a Monte Carlo procedure). The difference in the microscopic configurations of the two replicas constitutes the ''damage.'' Depending on the Hamiltonian, the dynamic rules, and the external parameters a small initial amound of damage will either spread or heal with time (or remain finite in a finite spatial region). Initially, it was believed that the DS behavior can be used to distinguish chaotic and regular phases of the model. However, it was realized early that the properties of DS depend sensitively on the update rule employed in the Monte Carlo procedure. For instance, in the Ising model with Glauber dynamics [4] the damage heals at low temperatures and spreads at temperatures above a certain spreading temperature T_s. In contrast, the Ising model with heat-bath dynamics [3] shows qualitatively different behavior: the damage heals at high temperatures but it may freeze at low temperatures. Thus DS appears to be uniquely defined only if one specifies the Hamiltonian *and* the dynamic rule. (Note that it was suggested [5] to obtain an unambiguous definition of DS for a particular model by considering all possible dynamic rules which are consistent with the physics of a single replica.) The differences between Glauber and heat-bath dynamics which can be traced back to different use of the random numbers in the update rules [6] can be understood already on the basis of a mean-field theory for DS [7].

In addition to this dependence of DS on the update rule (i.e., the way the random numbers are used in the simulation) it was also found [8] that in some systems DS can be completely different for parallel instead of sequential updates of the lattice sites. This is not too surprising since even the equilibrium probability distributions are different for parallel and sequential updates.

In this Brief Report we investigate the dependence of DS on another detail of the Monte Carlo procedure employed in the simulation, viz., the order of sites within a sequential update scheme. In general, different update schemes define different dynamical systems which will show different dynamical behavior. While all update schemes which differ only in the order of the sites will lead to the same stationary (equilibrium) state for a *single* replica (thanks to detailed balance) the same is not *a priori* true for DS, which is a nonequilibrium phenomenon. To the best of our knowledge the question of whether the stationary state of DS (i.e., the stationary state of the *pair* of replicas) does depend on the site order in the update scheme has not been investigated before [9]. Most of the published work on DS in the Ising model seems to (implicitly) assume that at least the stationary damage (and thus the spreading temperature) do not depend on the site order. In this Brief Report we provide numerical evidence that this assumption is mistaken.

We have studied DS in the Glauber-Ising model on a cubic lattice with $N=L^3$ sites. The Hamiltonian is given by

$$H=-\frac{1}{2}\sum_{ij} J_{ij} S_i S_j , \tag{1}$$

where $S_i=\pm 1$ is the Ising variable at site *i*, and J_{ij} is the exchange energy, which we take to be one for nearest-neighbor sites and zero otherwise. The Glauber dynamics is given by the stochastic map

$$S_i(t+1)=\text{sgn}\left\{ v[h_i(t)]-\frac{1}{2}+S_i(t)\left[\xi_i(t)-\frac{1}{2}\right]\right\}, \tag{2}$$

with the transition probability

$$v(h)=e^{h/T}/(e^{h/T}+e^{-h/T}). \tag{3}$$

Here $h_i(t)=\Sigma_j J_{ij} S_j(t)$ is the local magnetic field at site *i* and (discretized) time *t*, *T* denotes the temperature, and $\xi_i(t)\in[0,1)$ is a random number which is identical for the two copies of the system considered in a DS simulation. As in any DS simulation the central quantity studied is the Hamming distance (damage) *D* as a function of time *t*,

$$D(t)=\frac{1}{2N}\sum_i |S_i^{(1)}(t)-S_i^{(2)}(t)|, \tag{4}$$

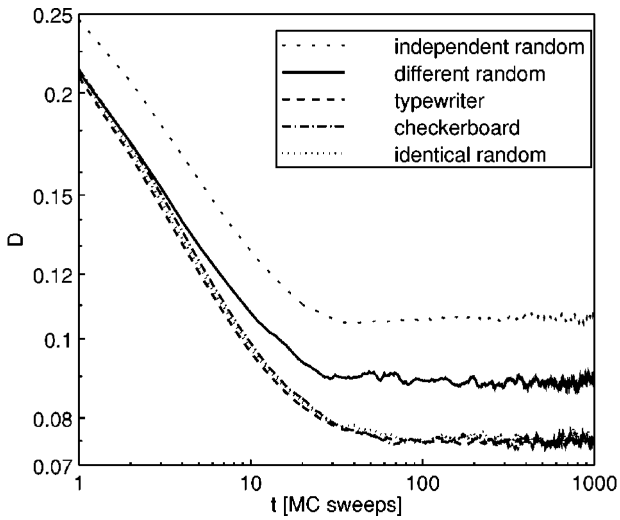

FIG. 1. Time evolution of the damage for a 27^3 system at temperature $T=4.25$. The two copies were prepared independently with an initial magnetization of $m_0=0.6$ which corresponds to initial damage $D_0=(1-m_0^2)/2=0.32$. The curves represent averages over 400 noise realizations.

where the upper index of the spin variable distinguishes the two replicas.

DS in the Glauber Ising model has been intensively investigated both numerically [4,10–13] and using an effective field theory [7,14]. The most precise estimate of the spreading temperature T_s (above which the Hamming distance remains finite in the long-time limit) in three dimensions was obtained in Ref. [12] for systems with up to $309\times 309\times 310$ sites using helical boundary conditions and a checkerboard update scheme. The result was a spreading temperature of $T_s/T_c=0.9225\pm 0.0005$ ($T_s=4.162$) where $T_c=4.5115$ is the equilibrium critical temperature of the ferromagnetic phase transition (all temperatures are measured in units of the nearest-neighbor interaction).

We have carried out extensive DS simulations for systems with up to $N=101^3$ sites with periodic and helical boundary conditions giving both the time evolution of the damage and its asymptotic stationary value. Different update sequences have been used: typewriter (regularly going from one site to the next), checkerboard (regularly going from one site to its next nearest neighbor, effectively updating first one sublattice, then the other), and three different types of random sequences. For the first random sequence the site to be updated is chosen independently for each time step. In the second random scheme each site is updated exactly once during each sweep (a sweep consists of N Monte-Carlo updates), but the (random) order is different from sweep to sweep. In the third random procedure, we use identical (random) order in all sweeps.

In Fig. 1 we show an example for the time evolution of the damage averaged over 400 runs with different noise realizations. The temperature $T=4.25$ is slightly below $T_c=4.5115$. The figure shows that not only the approach to the stationary state but also the stationary damage itself depend on the order of sites in the update process.

The short-time behavior is comparatively easy to understand: If the sites to be updated are chosen independently some sites will be updated twice or even several times while some will not be updated at all during the first sweep through the lattice. In contrast, for all other update sequences each site is updated exactly once during each Monte Carlo sweep. Now, in the example in Fig. 1 the initial damage is higher than its stationary value. Thus, the damage has to be reduced during the first few sweeps. However, if some sites are not updated at all, their damage cannot heal and consequently the case of independently chosen sites leads to slower decrease of the damage within the first few sweeps. In accordance with this explanation Fig. 1 shows that after the first sweep the damage is identical for all sequences that update each site exactly once in each sweep.

Let us now turn to the stationary states. Figure 1 indicates that the stationary state of the *pair* of replicas is indeed different for different site order in contrast to the stationary state of a *single* replica which is independent of the site order as discussed above. A closer inspection of Fig. 1 shows that the stationary damage for all those schemes for which the order of sites does not change from sweep to sweep (typewriter, checkerboard, and identical random) is the same within the statistical accuracy. A significantly higher stationary damage value is obtained if we use different random sequences but still update each site exactly once in each sweep. Finally, for a completely uncorrelated sequence of sites the stationary damage value is largest. We also note that the mean-field theory [7] cannot explain this new dependence of DS on the update sequence since within the mean-field theory the problem is reduced to a single-site problem.

We have carried out high precision calculations at different temperatures using the various update schemes discussed above in order to obtain the temperature dependence of the average stationary damage values. In these calculations the two replicas are prepared with a small initial amount of damage. The time evolution is monitored and after a stationary regime has been reached the damage is averaged over a large number (10^4) of Monte Carlo sweeps. The results for the typewriter and independent random update schemes are shown in Fig. 2. Analogous calculations have been carried out for the other update schemes. In the paramagnetic phase ($T>T_c$) the average stationary damage value is 0.5 for all update sequences investigated. In the ferromagnetic phase, however, the results are different. The three schemes that use the same sequence of sites in all sweeps (typewriter, checkerboard, and identical random) give identical stationary damage averages within the statistical accuracy. For these schemes we obtain a spreading temperature of $T_s=4.1625\pm 0.0050$, i.e., $T_s/T_c=0.9225\pm 0.0010$. This is exactly the value obtained by Grassberger [12] (using the checkerboard update scheme). In contrast, for the independent random sequence the spreading temperature is significantly lower. We obtain $T_s=4.0950\pm 0.0050$, i.e., $T_s/T_c=0.9075\pm 0.0010$. The results shown in Fig. 2 also indicate that the critical behavior at the spreading transition is the same for the update schemes investigated. Since DS in the Glauber-Ising model has two equivalent absorbing states (corresponding to $D=0$ and $D=1$), the critical behavior should be in the parity conserving (PC) universality class [15]. It has been suggested [16] that the model with two absorbing states in three dimensions is already above its upper critical dimension. It should then have a critical exponent $\beta=\beta_{mf}=1$, see, e.g., Ref. [7] [β is defined by $D(T)\sim(T-T_c)^{\beta}$]. The data in Fig.

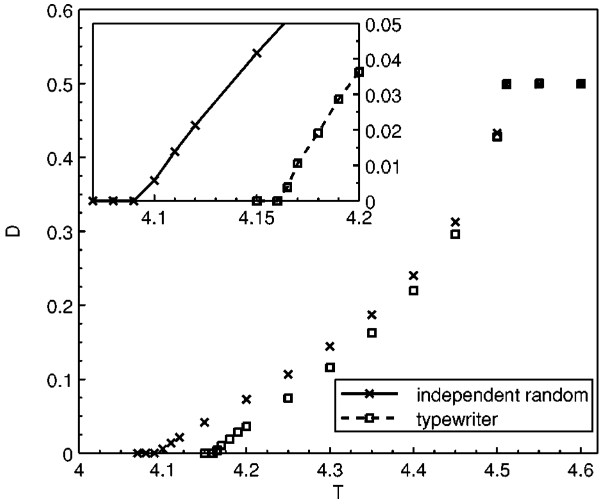

FIG. 2. Temperature dependence of the average stationary damage for typewriter and independent random site sequences. The curves represent averages over ten runs of a system with 101^3 sites. In each run the damage is averaged over 10 000 Monte Carlo sweeps after a stationary regime has been reached. The inset shows the spreading transition region. The statistical error is smaller than the symbol size in the main figure and approximately given by the symbol size in the inset.

2 are roughly consistent with this prediction for both update schemes although the inset seems to suggest a slightly smaller exponent. We plan to publish a systematic investigation of the critical behavior elsewhere [17].

All the results reported so far have been obtained using periodic boundary conditions. For comparison we have also investigated the influence of helical boundary conditions. Within the statistical accuracy the results of both boundary conditions are the same.

Furthermore, we have also checked whether the choice of the random number generator does play any role. Three very different random number generators have been used in the simulations: a combined linear congruential generator (RAN2 from Ref. [18]), a very simple linear feedback shift register generator (R250, see Ref. [19]), and a state-of-the-art combined linear feedback shift register generator (LFSR113 from Ref. [20]). All random number generators lead to the same results in our DS simulations. From this we exclude any errors due to poor random numbers.

To summarize, we have studied the dependence of damage spreading in the three-dimensional Glauber-Ising model on the order of the sites in the Monte Carlo update scheme. By using five different update schemes we have provided numerical evidence that the stationary damage and thus the spreading temperature are different for different site order. For all schemes which use the same site sequences in each sweep (typewriter, checkerboard, identical random) we have obtained a spreading temperature of $T_s/T_c = 0.9225 \pm 0.0010$ in good agreement with results from the literature [12]. For completely uncorrelated random site sequences we have obtained a significantly lower spreading temperature of $T_s/T_c = 0.9075 \pm 0.0010$. To our knowledge there are no published data for DS in the case of a random site sequence. (In Refs. [10,11] regular site order was used. Moreover, the accuracy would not have been high enough to distinguish the different cases.)

From our results we conclude that the stationary state of DS is very sensitive to changes in the details of the Monte Carlo procedure even if they do not influence the stationary state of a single replica. For the ferromagnetic Glauber-Ising model in three dimensions a change of the site order only leads to a shift of the spreading temperature T_s. For more complicated systems it appears to be possible, however, that changing the site order leads to qualitative changes of DS as was found for the change from sequential to parallel updates [8]. Investigations in this direction are in progress.

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